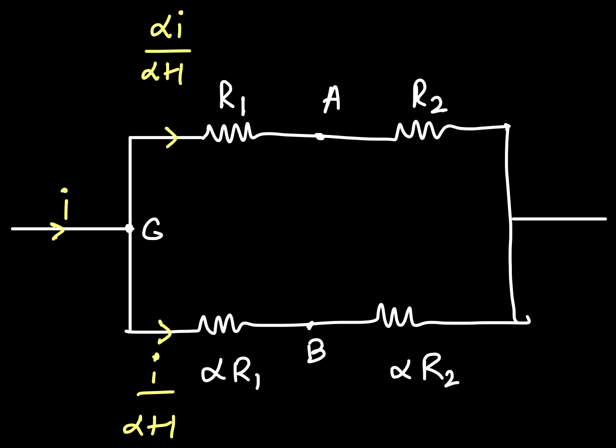
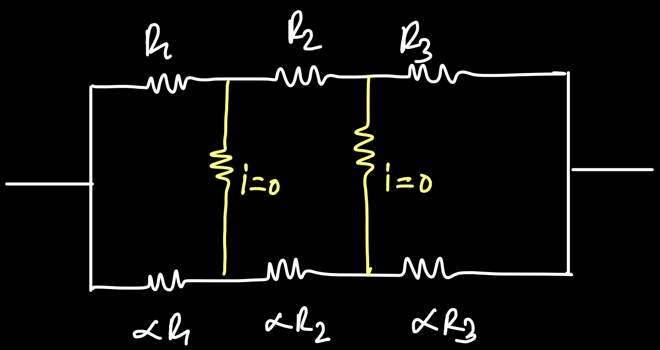
Wheatstone bridge

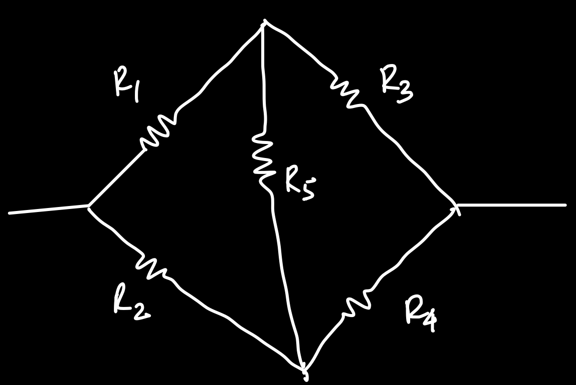


$$V_C - V_A = \frac{\alpha i}{\alpha+1}\cdot R_1$$

$$V_C - V_B = \frac{i}{\alpha+1}\cdot \alpha R_1$$

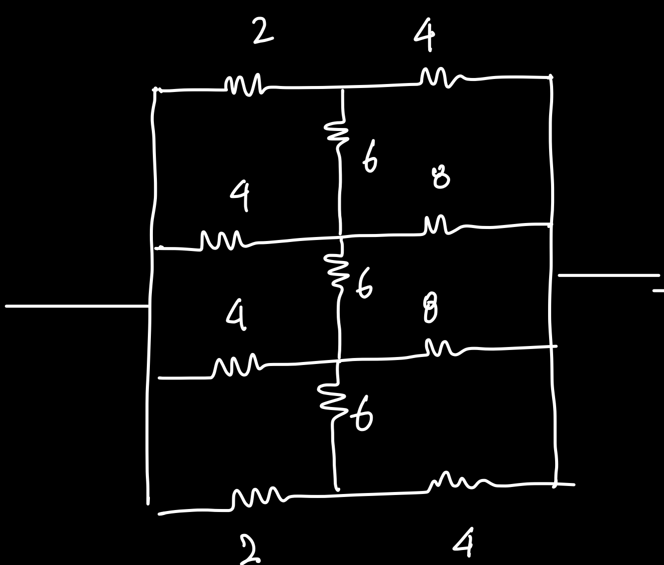
$$\boxed{V_A = V_B}$$

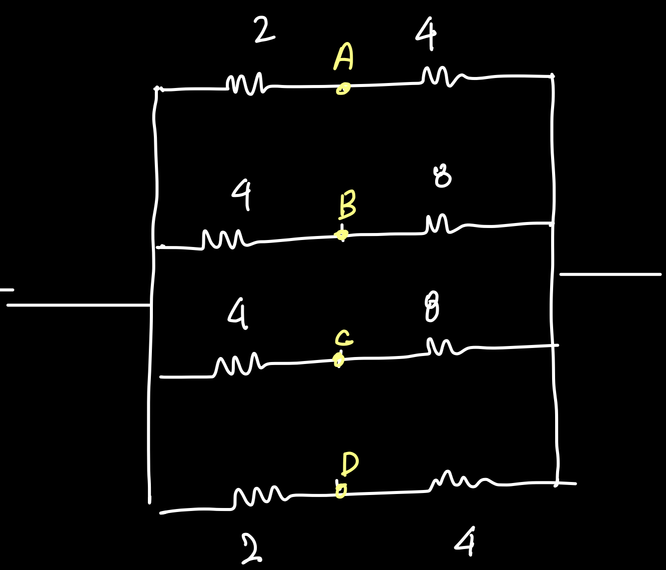




If $\frac{R_1}{R_2} = \frac{R_3}{R_4} \Rightarrow$ no current through R_5

It can be removed from circuit

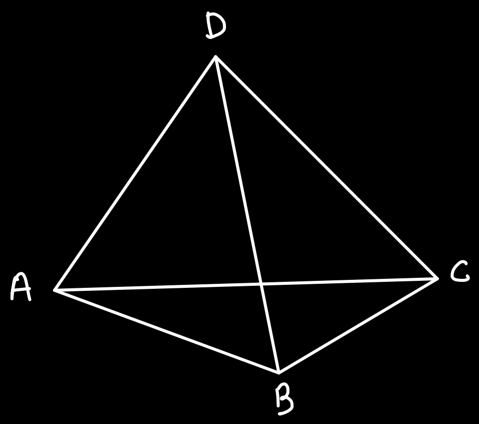




Without 6Ω resistances

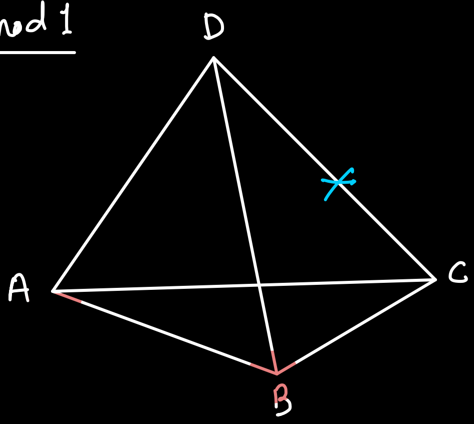
$$V_A = V_B = V_C = V_D$$

Problems related to symmetric circuits



Find equivalent resistance between A & B

method 1



method of equipotential points

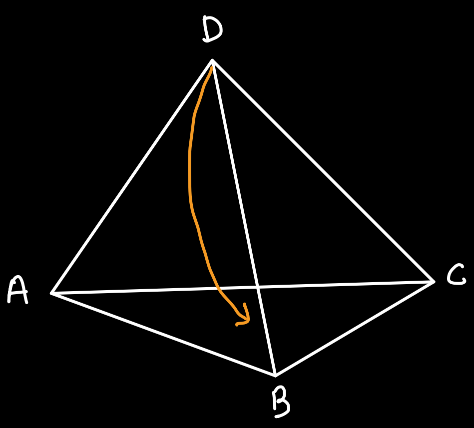
path of A is identical for C & D

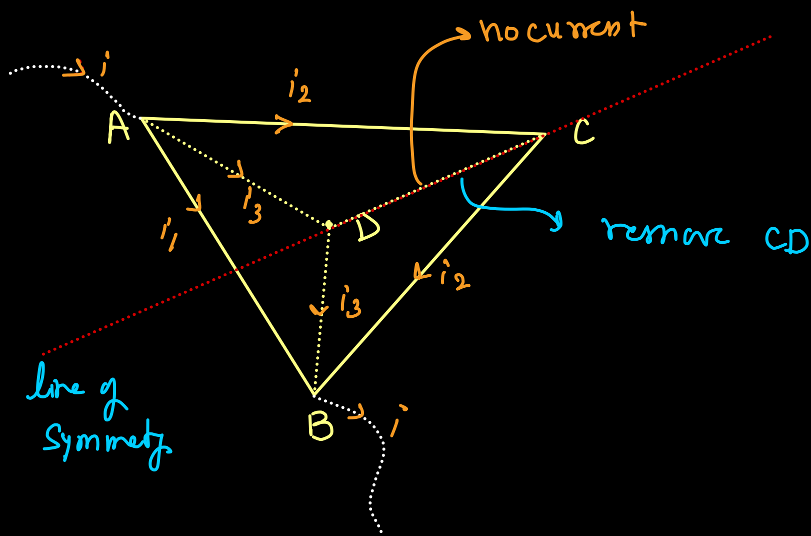
path of B is identical for C & D

$$V_C = V_D$$

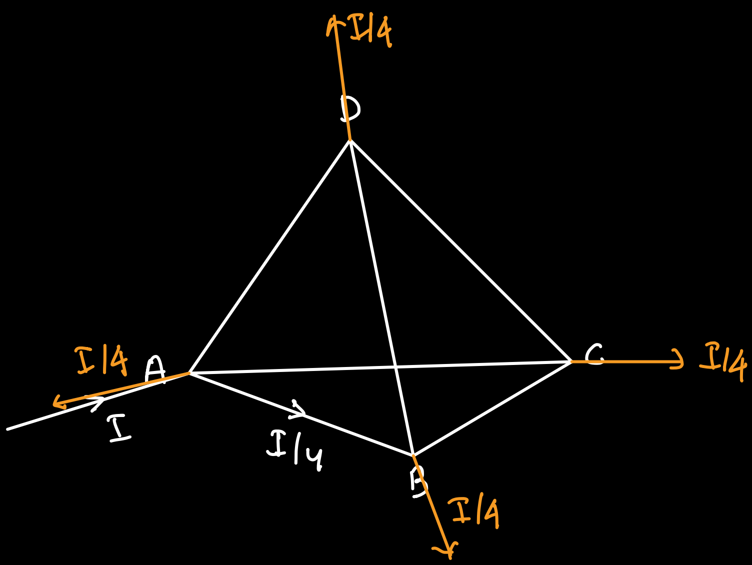
P & Q are equipotential if path to A is same for both P & Q. And path to B is same for both P & Q

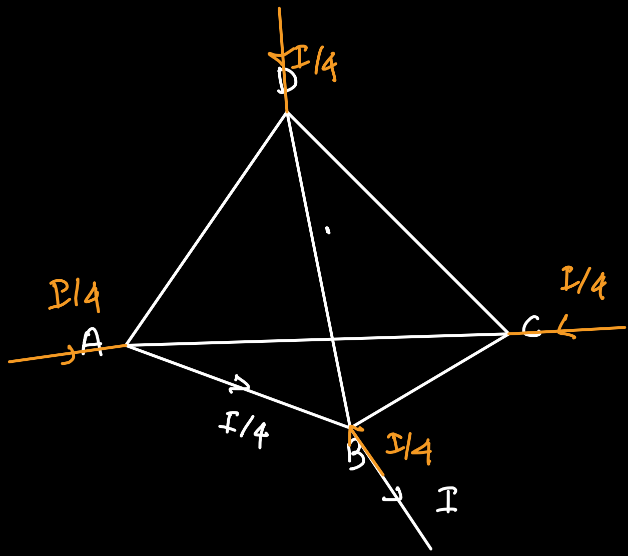
method 2: flatten 2D → 3D & observe symmetry



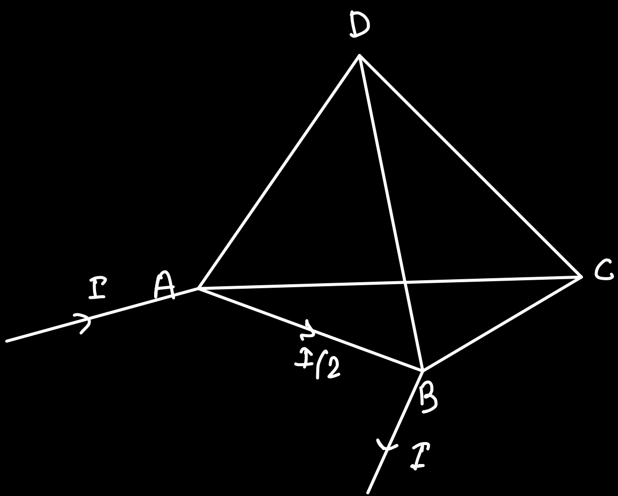


method 3   method of superposition



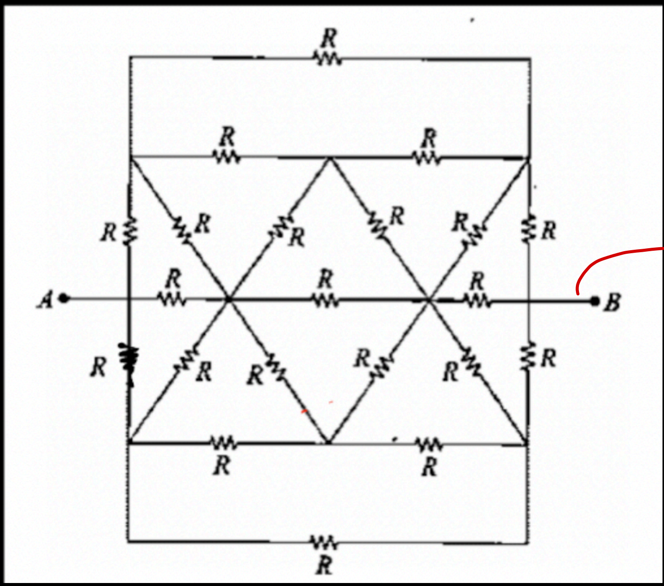


on superposition



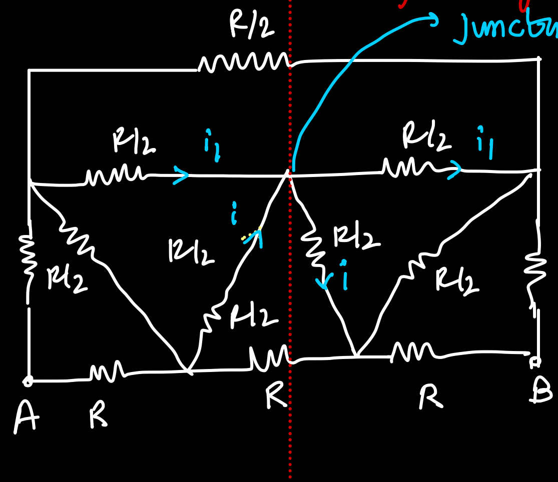
$\Delta V_{AB} = I R_{eq} = \frac{I}{2} R \Rightarrow R_{eq} = \frac{R}{2}$

Ex.

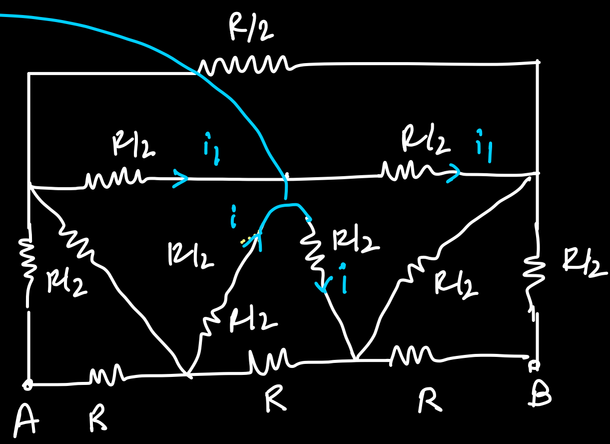


Line of symmetry → folding symmetry

Junction is useless



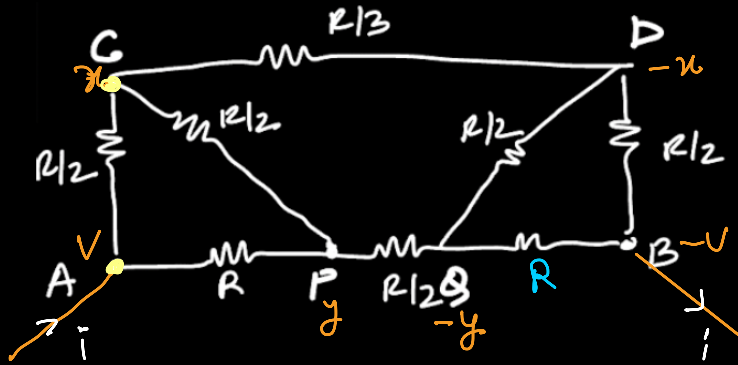
Junction can be detached



What do we want

$$2V = kiR = i(KR)$$

equivalent resistance



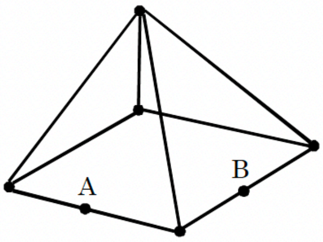
$$i = \frac{V-x}{R/2} + \frac{V-y}{R}$$

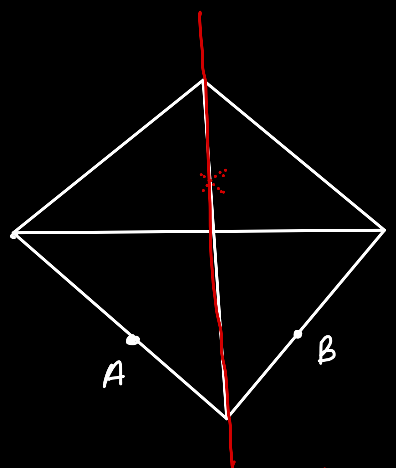
At C
$$\frac{x-V}{R/2} + \frac{x-y}{R/2} + \frac{2x}{R/3} = 0$$

At P
$$\frac{y-V}{R} + \frac{y-x}{R/2} + \frac{2y}{R/2} = 0$$

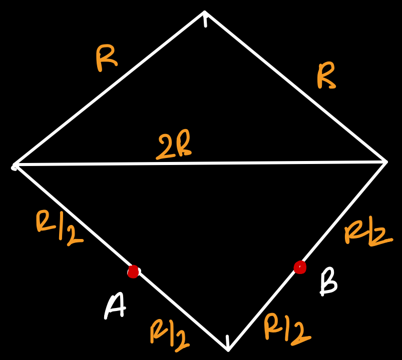
Solve for x & y

A regular pyramid of square is made of eight identical wires each resistance 12 Ω shown in the figure. The terminals A and B are the mid-points of the corresponding sides. Find equivalent resistance between the terminals A and B in ohms.





Symmetry

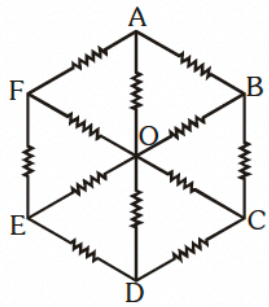


$R_{AB} = 8\,\Omega$

HW

Paragraph for Questions no. 12 to 14

Twelve wires of equal resistance R are placed in a given arrangement as shown in figure. Then find



2. Equivalent resistance between A & B is

(A) $\frac{11R}{20}$ (B) $\frac{10R}{20}$ (C) $\frac{R}{20}$ (D) $\frac{3R}{2}$

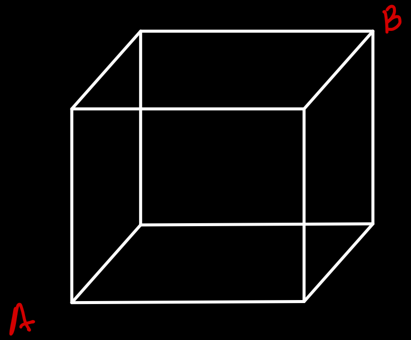
3. Equivalent resistance between A & C is

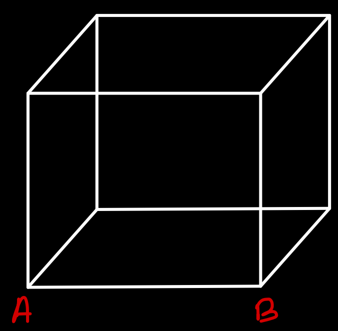
(A) $\frac{3R}{4}$ (B) $\frac{R}{4}$ (C) $\frac{5R}{6}$ (D) $\frac{3R}{5}$

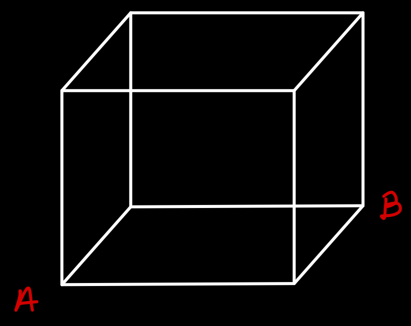
4. Equivalent resistance between A & O is

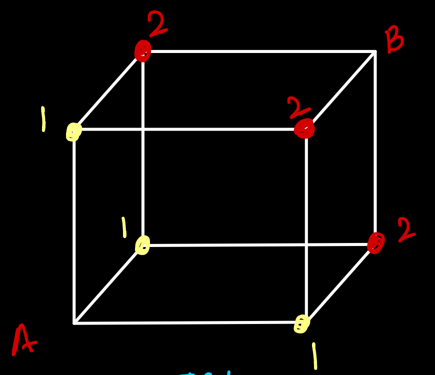
(A) $\frac{19R}{41}$ (B) $\frac{20R}{41}$ (C) $\frac{18R}{23}$ (D) $\frac{9R}{20}$

Resistance cubes

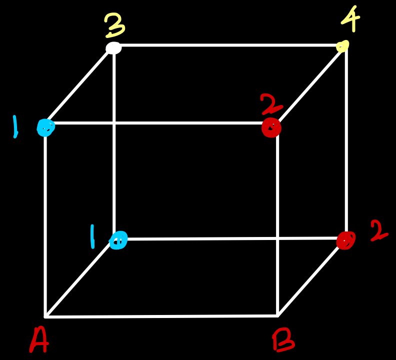




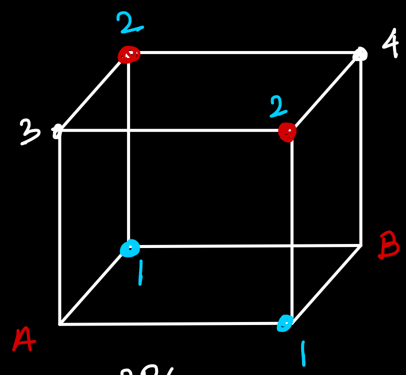




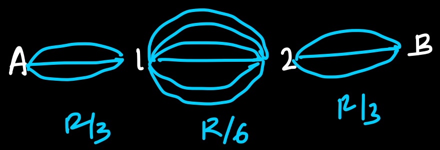
$5R/6$

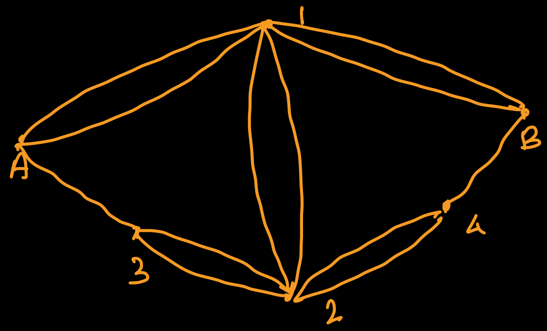


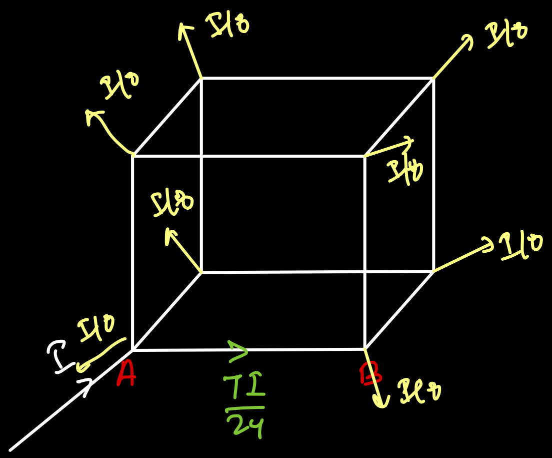
$\frac{7R}{12}$

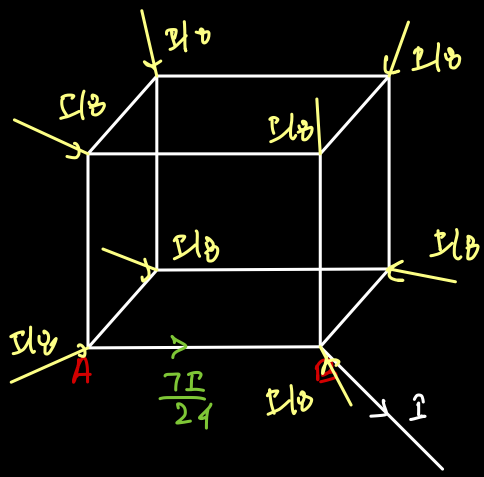


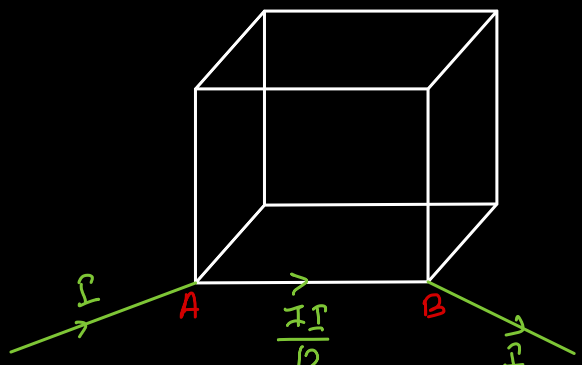
$3R/4$







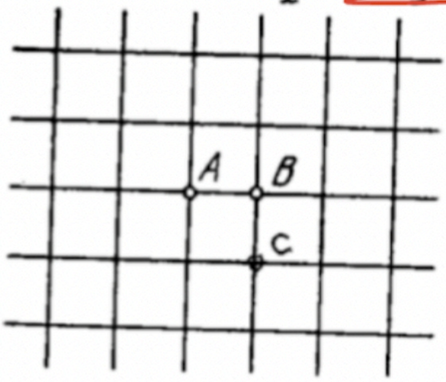


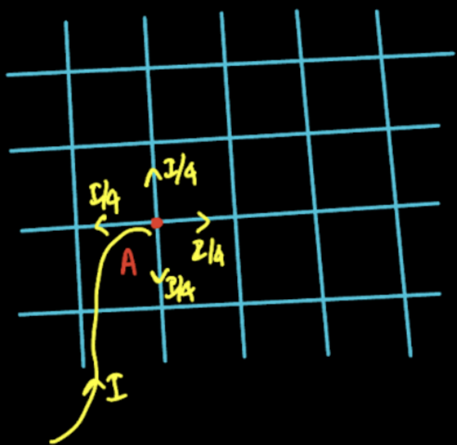


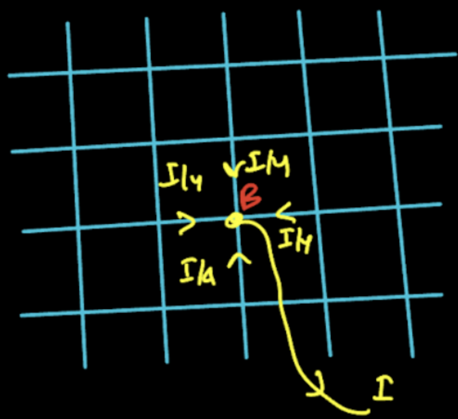
3.153. There is an infinite wire grid with square cells (Fig. 3.38). The resistance of each wire between neighbouring joint connections is equal to R_0. Find the resistance R of the whole grid between points A and B.

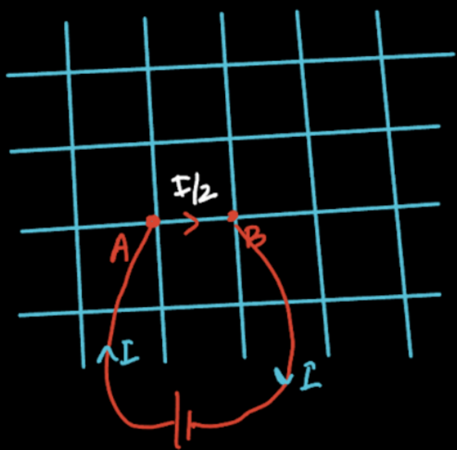
Instruction. Make use of principles of symmetry and superposition.

$R_{AB} = \frac{R_0}{2}$,





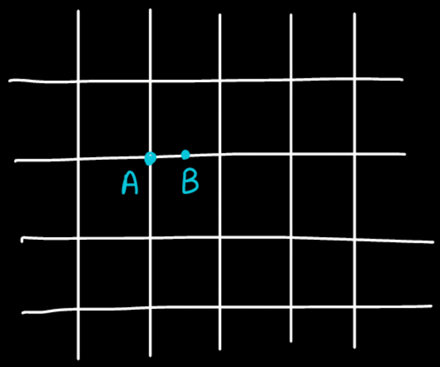


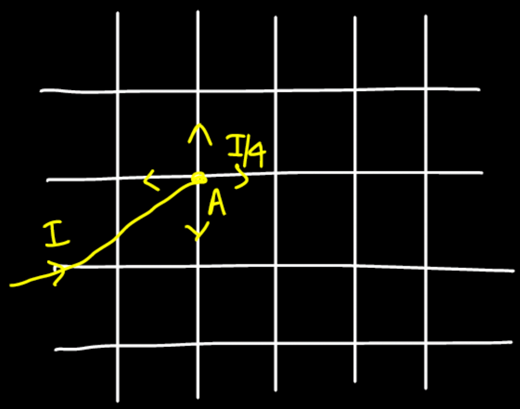


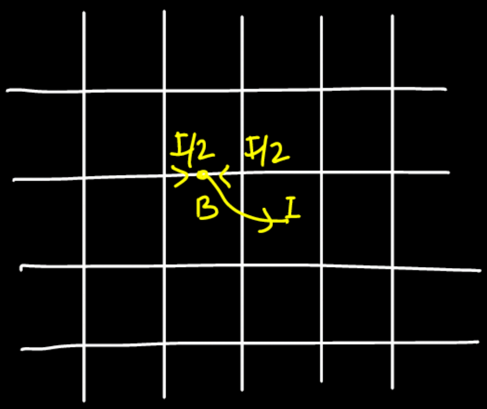
$\mathcal{E} = I R_{eq}$

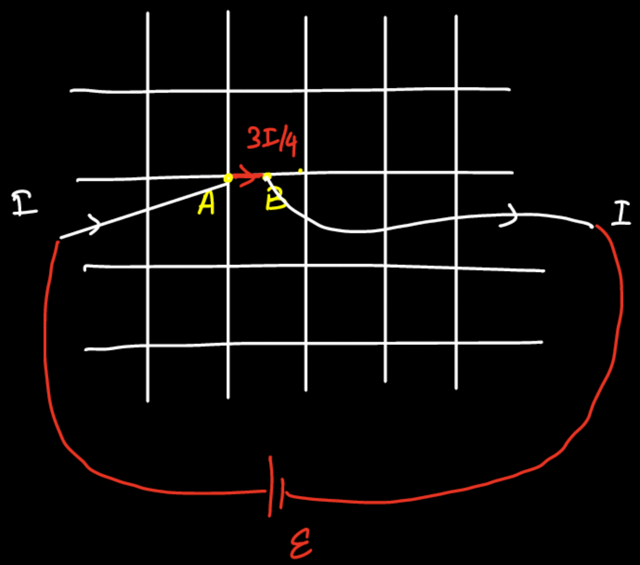
$\mathcal{E} = I R/2$

$R_{eq} = R/2$





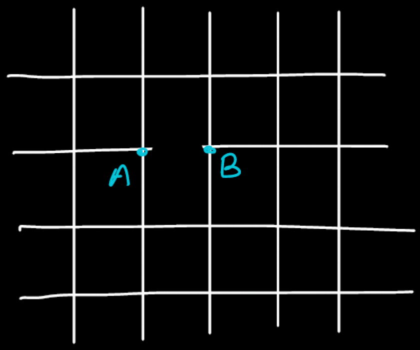




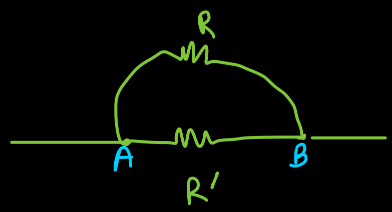
$$\varepsilon = \frac{3I}{4} \cdot \frac{R}{2} = I R_{eq}$$

$$R_{eq} = \frac{3R}{8}$$

Ex.



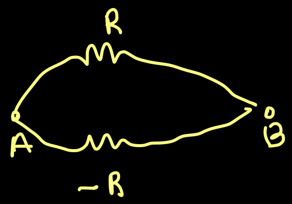
method 1 $R_{AB} = R'$



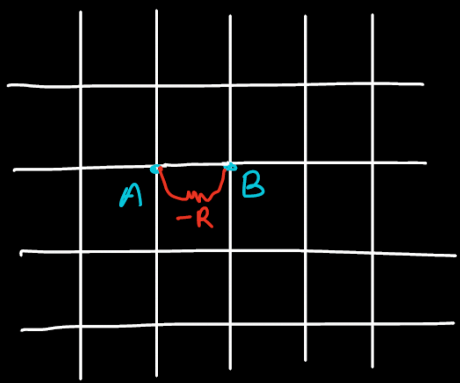
$$\frac{RR'}{R+R'} = \frac{R}{2}$$

$$R' = R$$

method 2

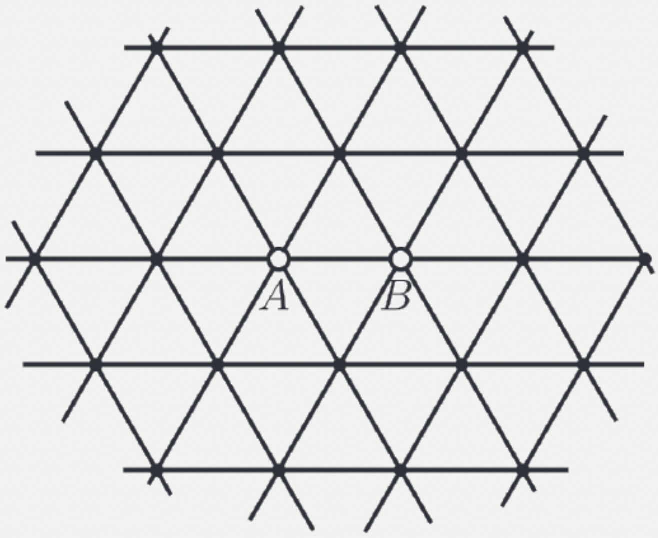


$$R_{eq} = \infty$$



$$\Rightarrow \quad R_{AB} = \frac{(R/2)(-R)}{(R/2) - R} = R$$

pr 31. Determine the resistance between two neigbouring nodes in an infinite triangular lattice, as shown in the figure below. The resistance of the wire connecting any two neighbouring nodes is 1 Ω. *Instructions:* Let C be a node that is infinitely far from nodes A and B. Now consider the currents flowing between these nodes.



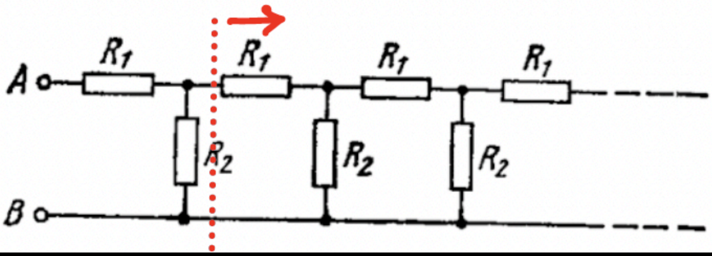
Ans : $R/3$

pr 33. Solve Problem 31 if the wire connecting nodes A and B is cut (i.e. current cannot flow through that wire).

Ans ! $R/2$

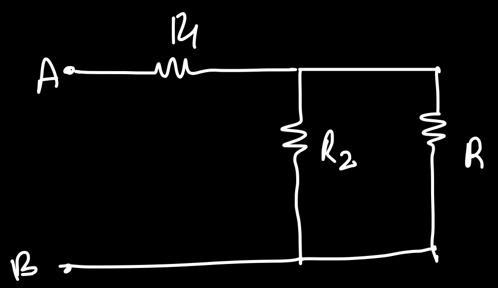
Infinite ladder circuits

3.152. Fig. 3.37 shows an infinite circuit formed by the repetition of the same link, consisting of resistance $R_1 = 4.0\,\Omega$ and $R_2 = 3.0\,\Omega$. Find the resistance of this circuit between points A and B.



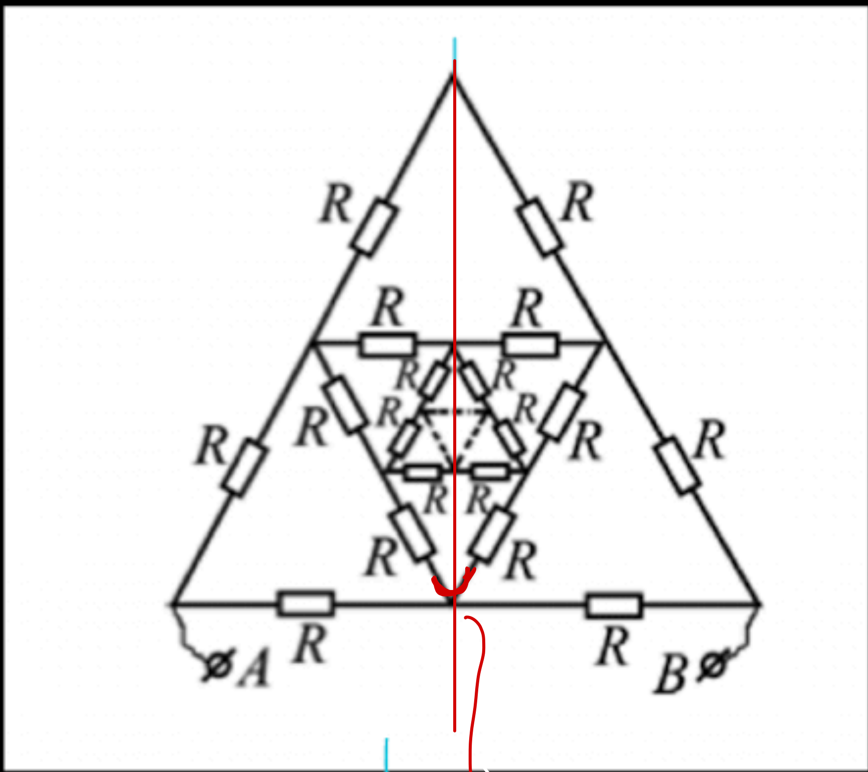
Ans: $6\,\Omega$

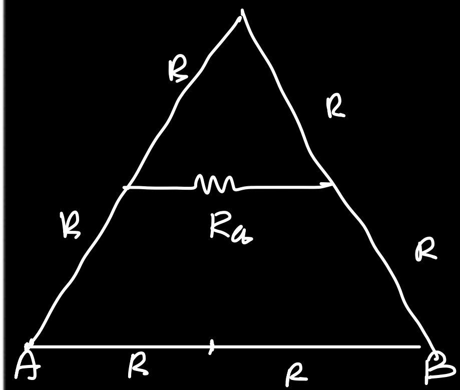
$R_{AB} = R$



$$\left\{\frac{R R_2}{R+R_2}\right\} + R_1 = R$$

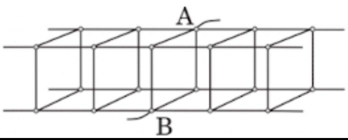
Ex Find R_{eq} across AB



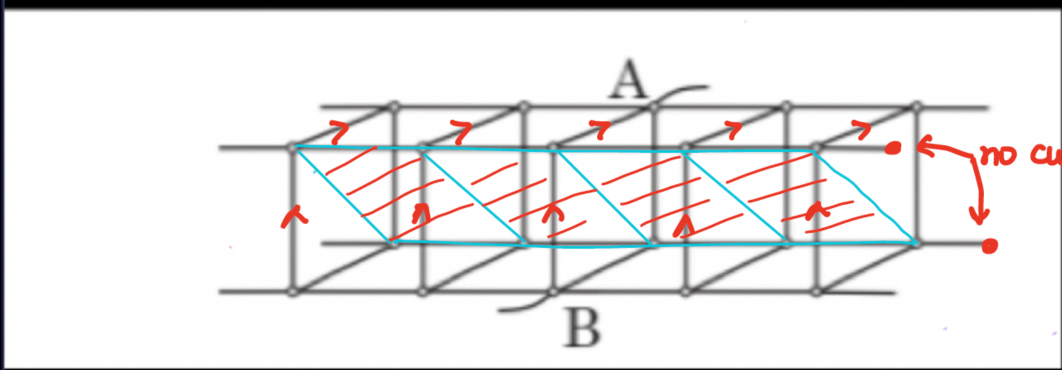


$R_{eq} = \frac{2R}{\sqrt{3}}$

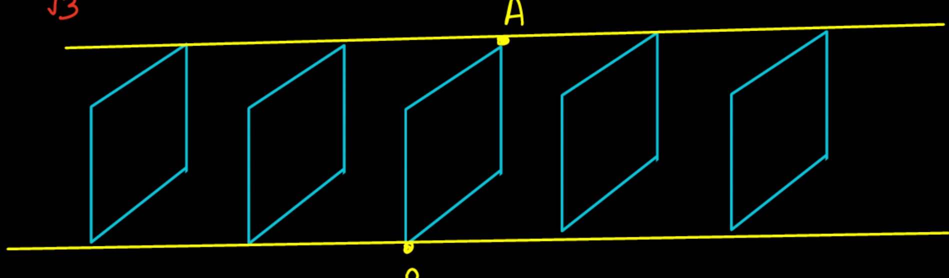
→ node can be detached

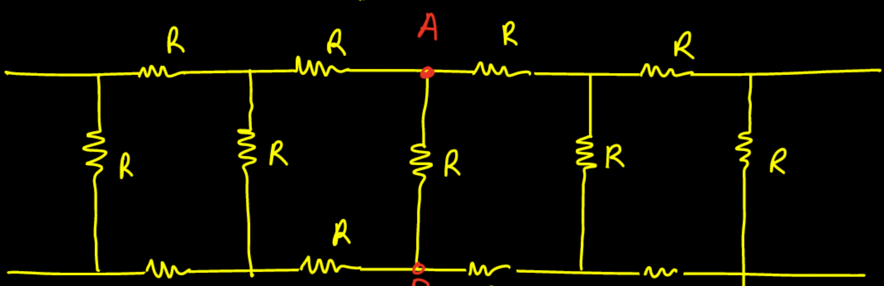


20. The network shown consists of an infinitely large number of identical sections. Resistance of each branch is R. Find resistance of the whole network measured between the terminals A and B.

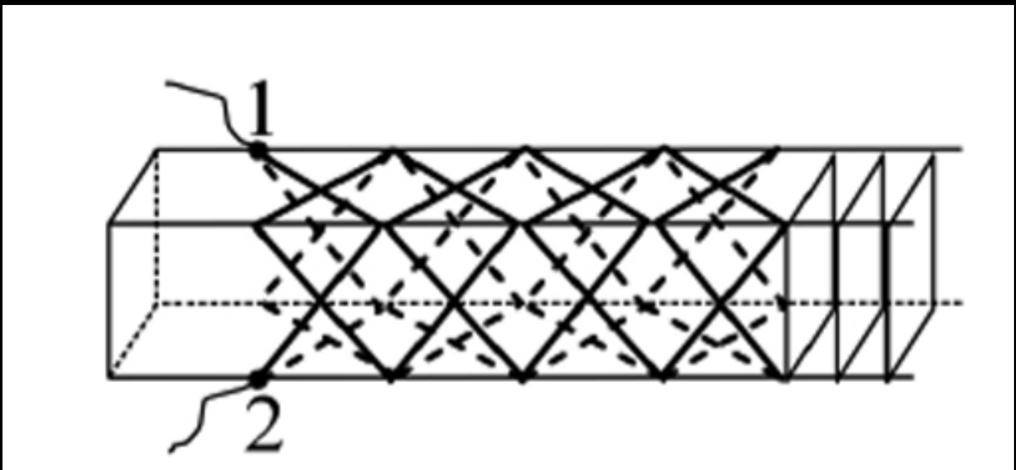


Ans $\frac{R}{\sqrt{3}}$



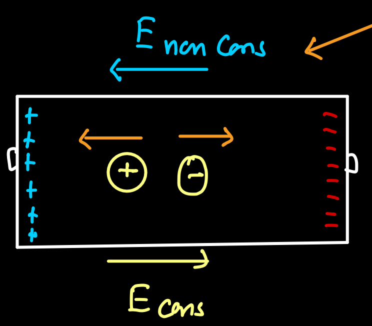


Problem 66. *(MOSH, 2010, 11)* Eight identical long wires are wound around a long (semi-infinite) wooden block. The beginnings of these wires are connected in pairs, as shown in the figure. Find the resistance R_{12} between points 1 and 2 of this infinite "stocking-like" chain pulled over the block, if the resistance of a wire segment located between two adjacent intersections of wires is equal to R. There is electrical contact at all intersection points between the wires.



Ans: $R\sqrt{2}$

Batteries / cells



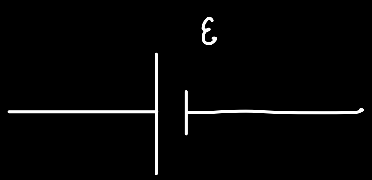
Electric field inside cell is non electrostatic

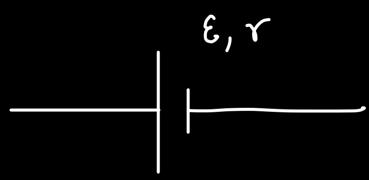
Emf of battery $\varepsilon = \int \vec{E}_{non\ con} \cdot \vec{dl}$

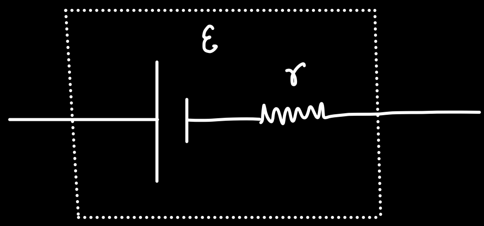
potential difference $\Delta V = -\int E_{Cons} \cdot \vec{dl}$

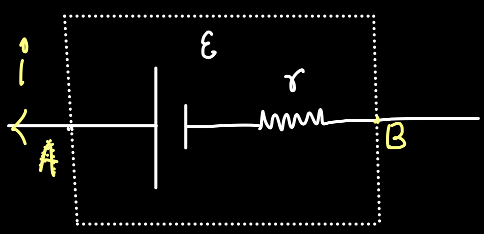
for an ideal battery $\boxed{\Delta V = \varepsilon}$

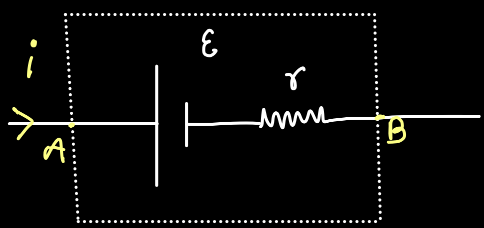
Non ideal battery, battery with internal resistance











$V_A - \varepsilon + ir = V_B$

$V_A - V_B = \varepsilon - ir$

Battery supplies energy

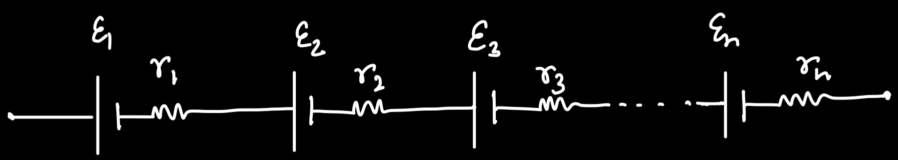
$V_A - \varepsilon - ir = V_B$

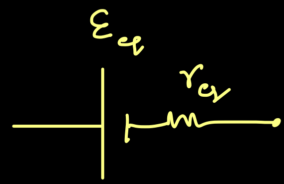
$V_A - V_B = \varepsilon + ir$

Battery absorbs energy

Combination of batteries

Batteries in series

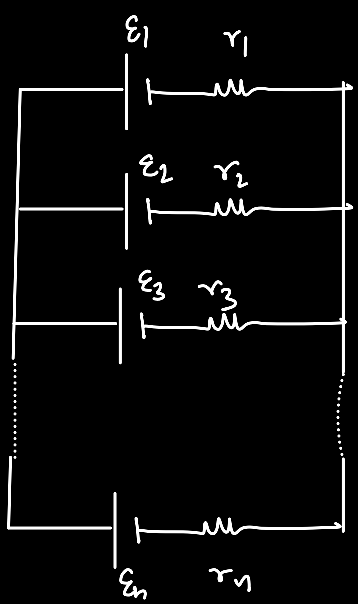




$$\varepsilon_{q} = \sum \varepsilon_i$$

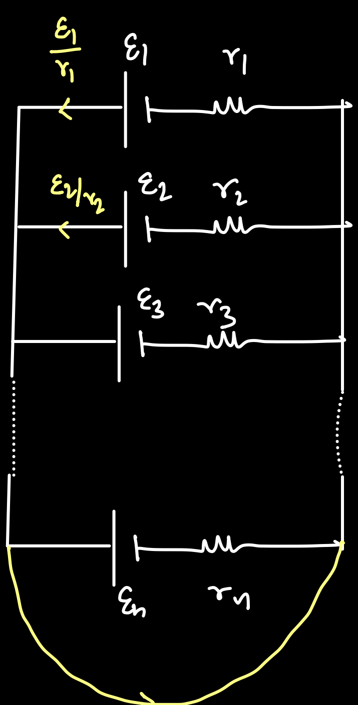
$$r_{q} = \sum r_i$$

Batteries in parallel



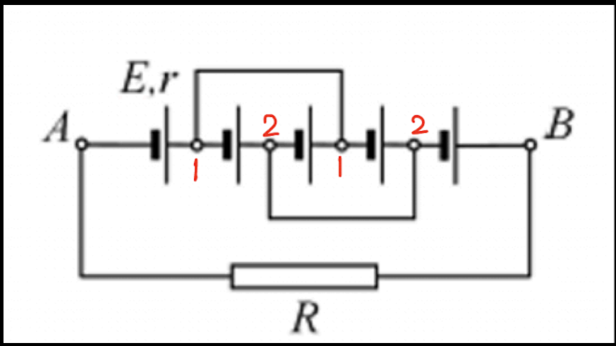
$$\varepsilon_{q} = \frac{\frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2} + \cdots + \frac{\varepsilon_n}{r_n}}{\frac{1}{r_1} + \frac{1}{r_2} + \cdots + \frac{1}{r_n}}$$

$$\frac{1}{r_q} = \frac{1}{r_1} + \frac{1}{r_2} + \cdots + \frac{1}{r_n}$$



$$i = \frac{\varepsilon_q}{r_q} = \frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2} + \cdots + \frac{\varepsilon_n}{r_n}$$

PROBLEM 12. (Moscow School of Electrical Engineering, 2017, 11) Find the current I through a resistor with a resistance $R = 5$ Ohm in the circuit shown in the figure. All sources are the same and have an EMF $E = 15$ V and an internal resistance $r = 2$ Ohm. Neglect the resistance of the connecting wires.



Ans: $\frac{7E}{3R+7r}$