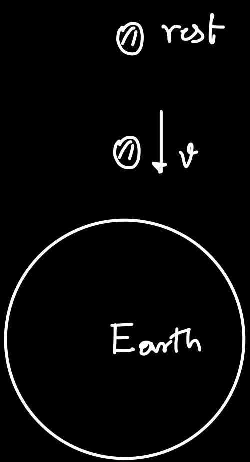
Potential Energy



As the body falls from rest near the earth surface. Explain why does speed increase.

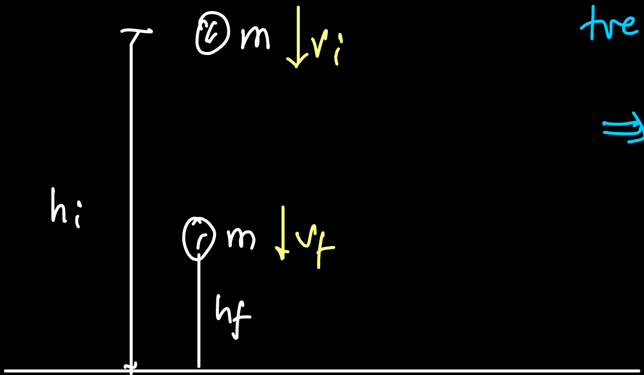
Explanation 1: Due to gravitational force of earth the mass moves down with increasing velocity

$$F = ma$$

$$v^2 = u^2 + 2as$$

Explanation 2: Work done by gravity $= \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

+ve



$$\Rightarrow v_f > v_i$$

$$+mg(h_i - h_f) = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

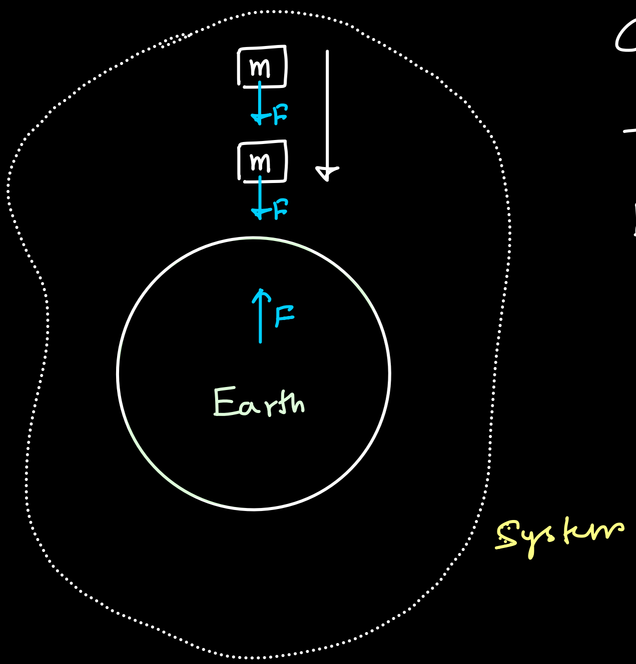
$$mgh_i + \frac{1}{2}mv_i^2 = mgh_f + \frac{1}{2}mv_f^2$$

Explanation 3 : As the body falls its KE increases. This increase may be because of decrease in some other type of energy. This other type of energy is called gravitational potential Energy

Potential Energy

Energy associated with internal forces between two bodies which can be converted to other forms of energy as the separation / shape of bodies change

Gravitational potential Energy

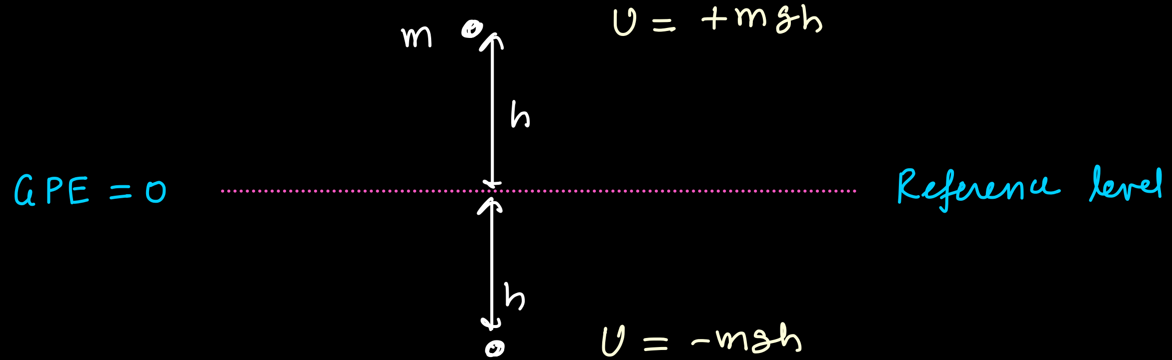


Consider mass & earth as a system

There is an attractive force acting between them → gravitational force

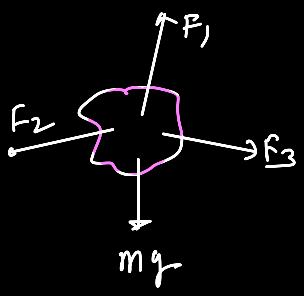
As the separation between mass & earth decreases kinetic energy of mass increases

How is gravitational P.E defined

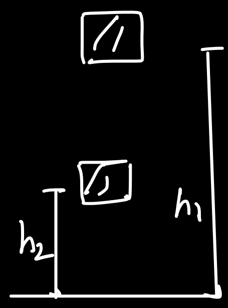


Take an horizontal level & assign the gravitational P.E at that level to be zero.

Conservation of mechanical Energy



Work Energy theorem



Total work done by all forces = change in K.E

$$W_{other} + W_{gravity} = KE_f - KE_i$$

$$W_{other} + mg(h_1 - h_2) = KE_f - KE_i$$

$$W_{other} + U_i - U_f = KE_f - KE_i$$

Mechanical Energy

$$W_{other} = (KE_f + U_f) - (KE_i + U_i)$$

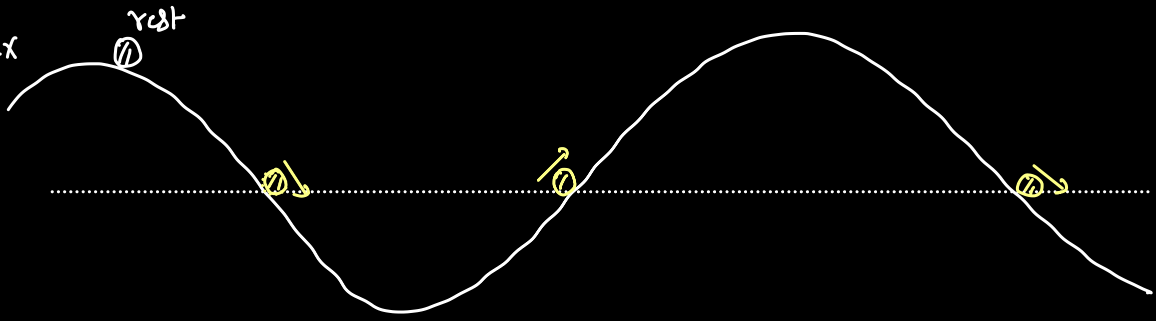
$$W_{other} = E_f - E_i$$

If work done by all forces except gravity is zero

$$0 = E_f - E_i$$

$$\boxed{E_f = E_i} \longrightarrow \text{mechanical energy does not change}$$

Ex



If there is no air resistance / friction show that whenever body is at the same horizontal level it has the same speed

Same KE $\Rightarrow v_f = v_i$

$$\frac{1}{2} m v_i^2 + mgh_i = \frac{1}{2} m v_f^2 + mgh_f$$

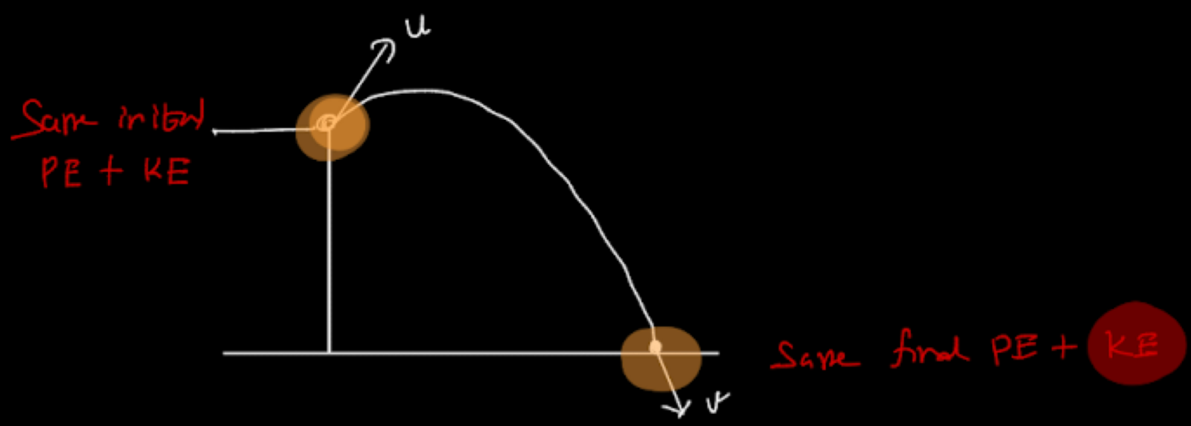
Same level $\Rightarrow$ Same gravitational PE

12) A girl throws a stone from a bridge. Consider the following ways she might throw the stone. The speed of the stone as it leaves her hand is the same in each case, and air resistance is negligible.

Case A: Thrown straight up.
Case B: Thrown straight down.
Case C: Thrown out at an angle of 45° above horizontal.
Case D: Thrown straight out horizontally.

In which case will the speed of the stone be greatest when it hits the water below?

A) Case A
B) Case B
C) Case C
D) Case D
E) The speed will be the same in all cases.



Initially $\longrightarrow$ Same KE & Same PE

Finally $\longrightarrow$ on the ground Same P.E & hence they must have Same KE

3) Two stones, one of mass *m* and the other of mass 2*m*, are thrown directly upward with the same velocity at the same time from ground level and feel no air resistance. Which statement about these stones is true?

A) The heavier stone will go twice as high as the lighter one because it initially had twice as much kinetic energy.

B) Both stones will reach the same height because they initially had the same amount of kinetic energy.

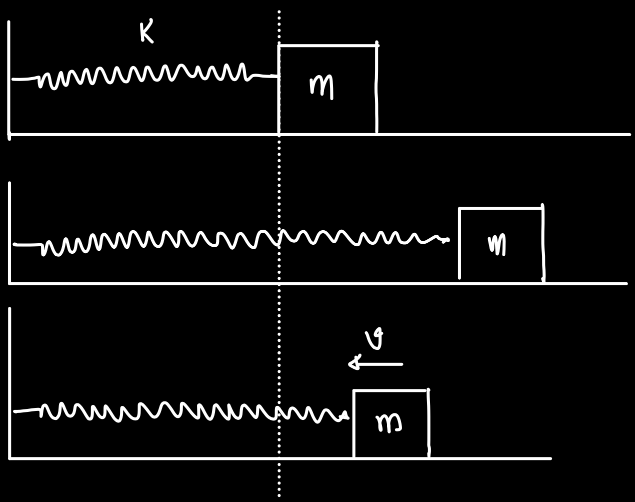
C) At their highest point, both stones will have the same gravitational potential energy because they reach the same height.

D) At its highest point, the heavier stone will have twice as much gravitational potential energy as the lighter one because it is twice as heavy.

E) The lighter stone will reach its maximum height sooner than the heavier one.

$\frac{1}{2} 2mu^2 = 2mgh_1$ $\qquad$ $\frac{1}{2} mu^2 = mgh_2$ $\qquad$ $h_1 = h_2$

Elastic potential Energy

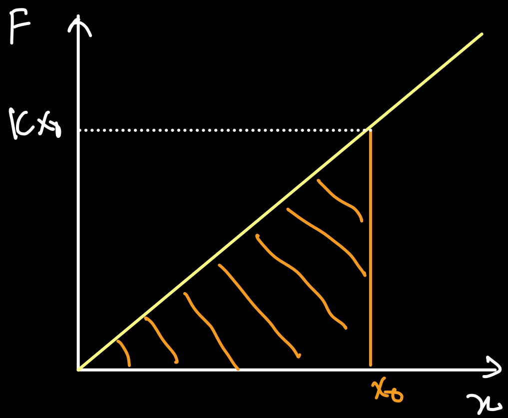


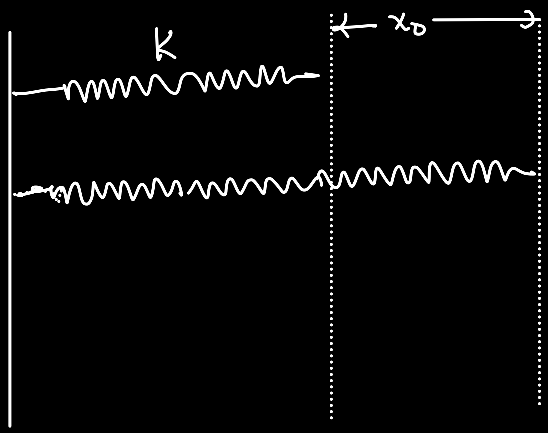
If we elongate the spring & release the block.

The KE of the block increases

In the elongated state spring has a potential energy that decreases as elongation decreases and converts to KE of block

Both Compressed & Elongated Springs will have potential Energy. This energy is called Elastic potential Energy.





$$F = kx$$

Elastic potential Energy in spring = Work done to stretch the Spring

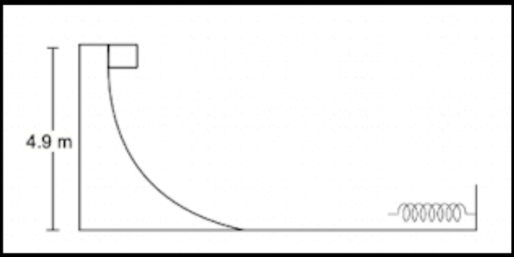
= Area under $F-x$ graph

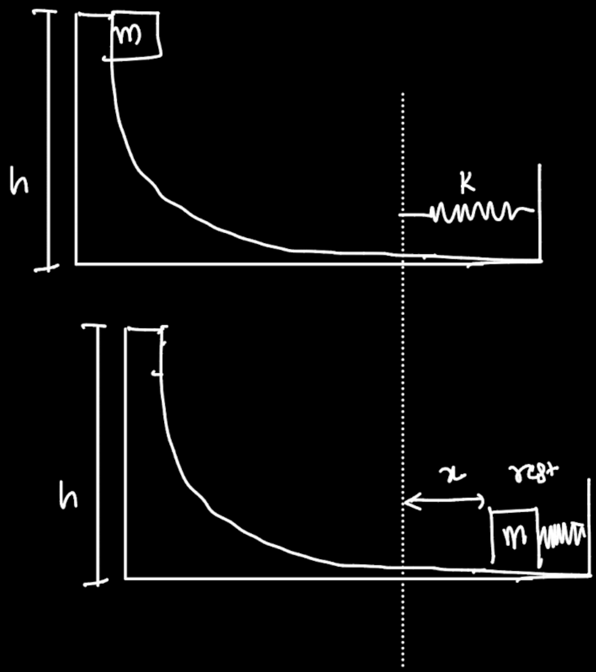
$$= \frac{1}{2} x_0 \cdot kx_0$$

$= \frac{1}{2} k x_f^2$

Imp

Figure (8-W8) shows a smooth curved track terminating in a smooth horizontal part. A spring of spring constant 400 N/m *is attached at one end to a wedge fixed rigidly with the horizontal part. A* 40 g *mass is released from rest at a height of* 4·9 m *on the curved track. Find the maximum compression of the spring.*





maximum Compression is at the instant when mass just comes to rest

$mgh = \frac{1}{2} k x^2$

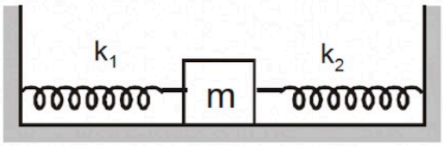
initial

find

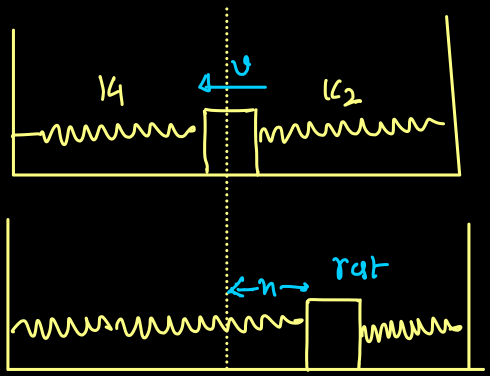
$x = \sqrt{\frac{2mgh}{k}} = 9.8 \text{ cm}$

$g = 9.8 \text{ m/s}^2$

A block of mass m is attached to two spring of spring constant k_1 and k_2 as shown in figure. The block is displaced by x towards right and released. The velocity of the block when it is at its mean position.



(A) $\sqrt{\frac{(k_1+k_2)x^2}{2m}}$ (B) $\sqrt{\frac{3}{4}\frac{(k_1+k_2)x^2}{m}}$ (C) $\sqrt{\frac{(k_1+k_2)x^2}{4m}}$ (D) $\sqrt{\frac{(k_1+k_2)x^2}{m}}$

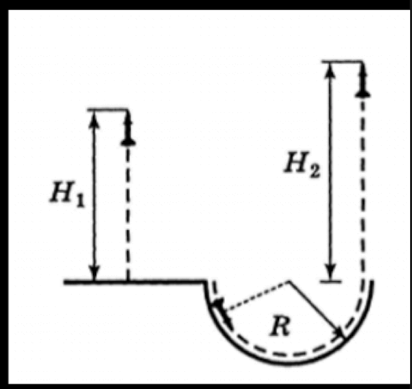


$\frac{1}{2}mv^2$

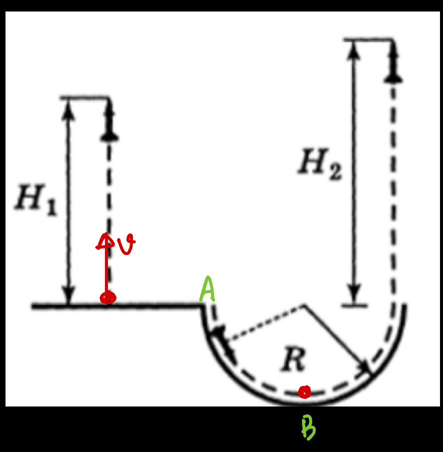
$\frac{1}{2}k_1 x^2 + \frac{1}{2}k_2 x^2$

$$\frac{1}{2}k_1 x^2 + \frac{1}{2}k_2 x^2 = \frac{1}{2}mv^2$$

PROBLEM 3. (All-Russian Olympiad, 1998, Experimental Stage, Grade 11)
Someone proposed a new method for launching rockets. Instead of launching them upward, they recommended releasing the rockets downward along guide rails forming an arc of a large radius R (see figure). At a certain point of motion along the guides, the engine was to be turned on. The inventor claimed that with such a launch, the rocket's maximum ascent height H_2 would exceed the height H_1 achievable during a conventional launch (vertically upward). Assuming H_1 and R are given, find the maximum possible value of the height H_2. Assume that the rocket engine operates for a short period of time, and air resistance and friction between the rocket body and the guides can be neglected.



$$H_2 = H_1 + 2\sqrt{H_1 R}$$



$\frac{v^2}{g} = H_1 \longrightarrow v = \sqrt{2gH_1}$ launch velocity in 1st case

$mgR = \frac{1}{2}mv_1^2$ $v_1 = \sqrt{2gR}$

After launch $v_2 = \sqrt{2gR} + \sqrt{2gH_1}$

$$\frac{1}{2} m v_2^2 = mg(R + H_2)$$