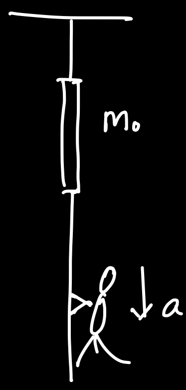
A monkey of mass m-kg slides down a light rope attached to a fixed spring balance with an acceleration a. the reading of the spring is M_0 kg (g = acceleration due to gravity). Then:

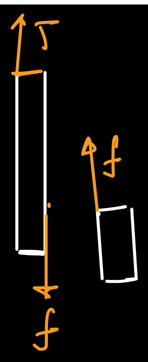
(A) the tension in the rope is M_0g

(B) the force of friction exerted by the rope on the monkey is $m(g-a)N$

(C) $m = \frac{M_0 g}{(g-a)}$

(D) $m = M_0\left(1+\frac{a}{(g)}\right)$

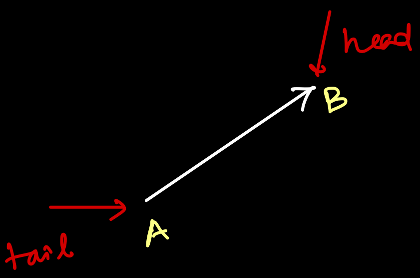


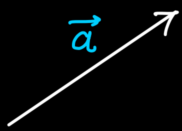


$f = T$

Vectors

Geometric representation of a vector

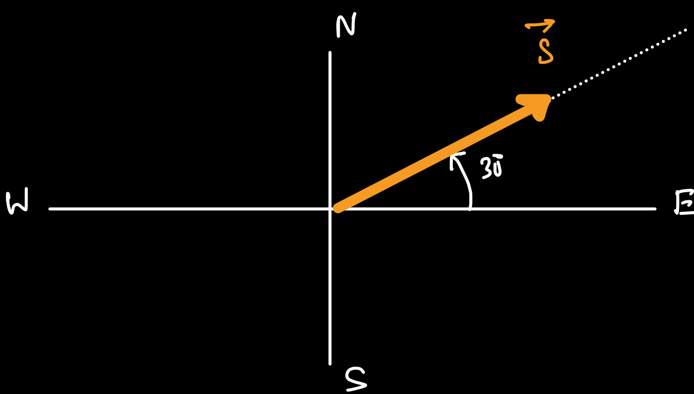




$\vec{AB}$

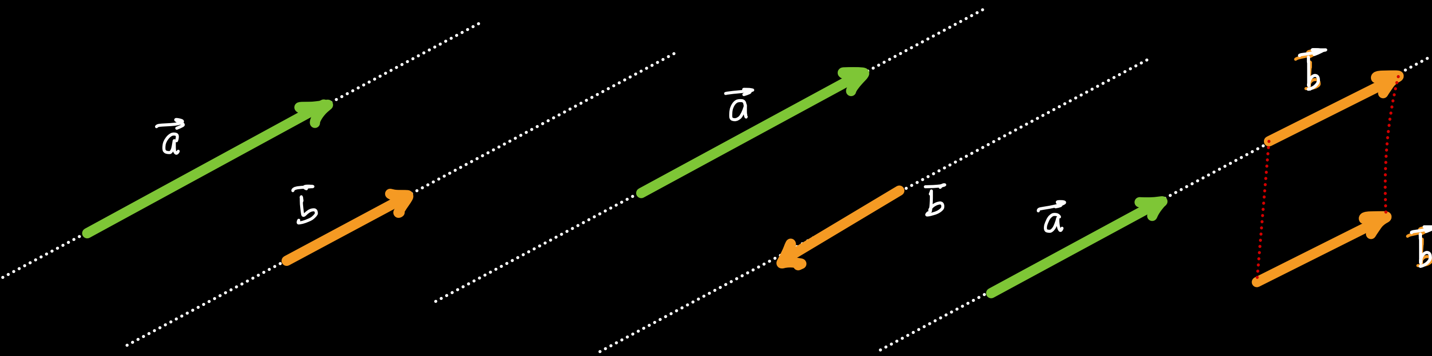
Magnitude of a vector : $|\vec{AB}|$ or $|\vec{a}|$

Ex A person moves 50m in a direction 30° North of east



Magnitude of $\vec{s}$ $\boxed{|\vec{s}| = 50\,m}$

Parallel or collinear vectors



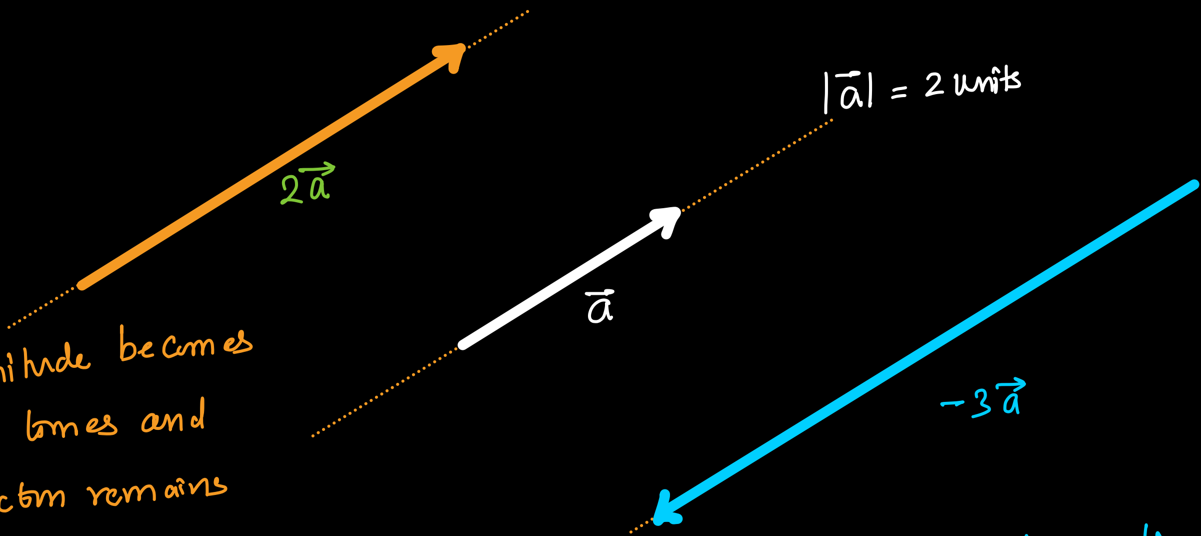
parallel

Antiparallel

Since we can translate a vector parallel to itself, collinear implies parallel

Multiplication of a vector by a scalar

(a) $\hat{a}$



Magnitude becomes two times and direction remains same

magnitude becomes three times but the direction reversed

Conditions for vectors to be parallel / Collinear

If $\vec{b} \parallel \vec{a}$ then $\vec{b} = m\,\vec{a}$

$m \rightarrow$ a number

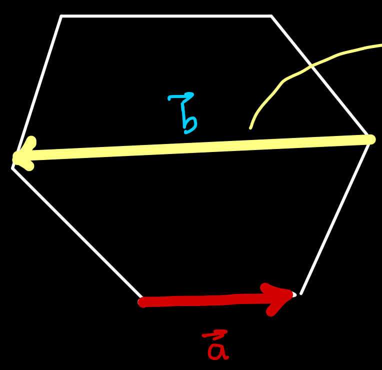
Also if m is a +ve number $\longrightarrow$ Same direction

Also if m is a −ve number $\longrightarrow$ opp direction

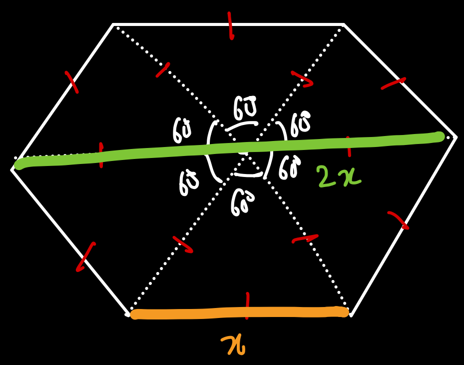
Given a regular hexagon

Ex

Write this vector in terms of $\vec{a}$

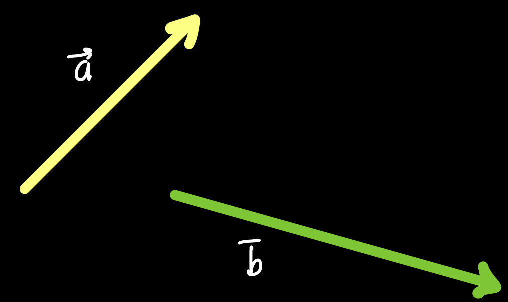


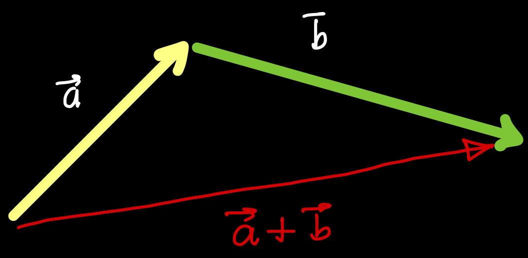
$\vec{b} = -2\vec{a}$ Ans



Addition of vectors

Triangle law



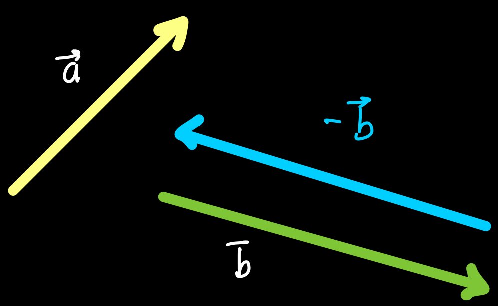


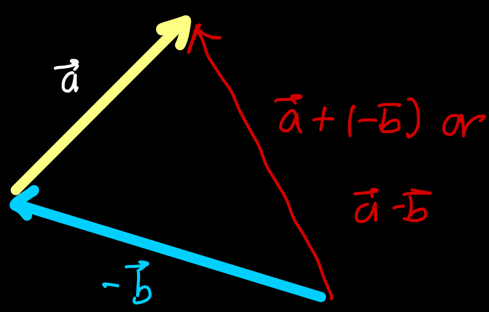
Place vectors head to tail

Vector drawn from free tail to the free head represents the vector sum

Subtraction of two vectors using triangle law

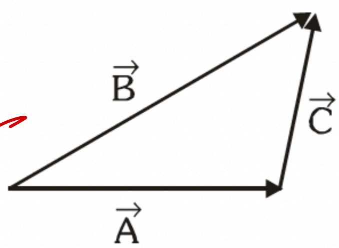
$$\vec{a}-\vec{b} = \vec{a} + (-\vec{b})$$





Which of the following sketches satisfies the vector equation $\vec{A}=\vec{B}-\vec{C}$?

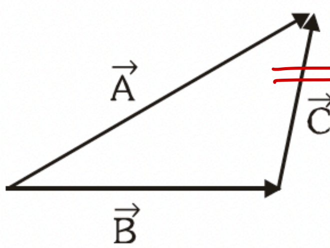
(A)



$\vec{A} = \vec{B} - \vec{C}$

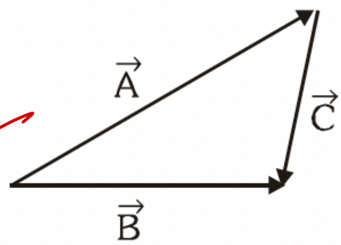
$\vec{A} + \vec{C} = \vec{B}$

(B)

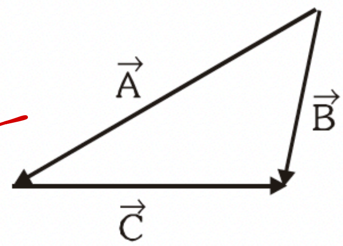


$\vec{B}+\vec{C}=\vec{A}$

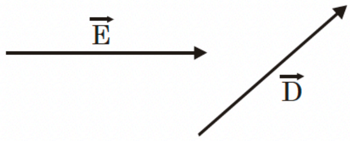
(C)



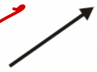
(D)

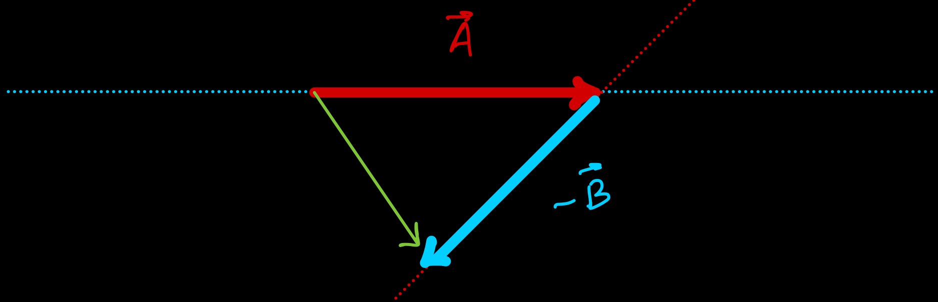


Two vectors $\vec{A}$ and $\vec{B}$ of unknown magnitudes are along $\vec{E}$ and $\vec{D}$ (as shown below) respectively :



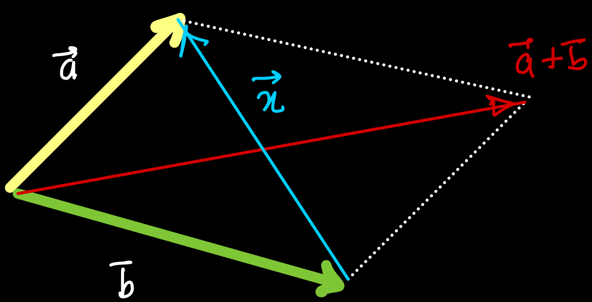
Then $(\vec{A}-\vec{B})$ cannot be :-

(A)  (B)  ✓(C)  (D) 



we can't get (C)

Parallelogram law

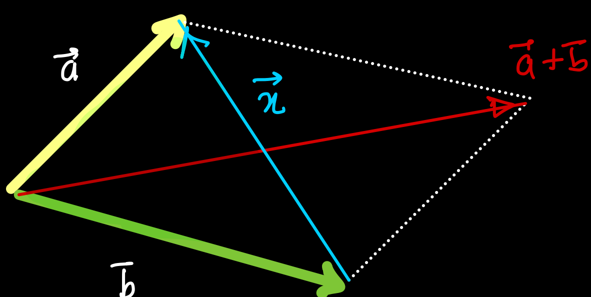


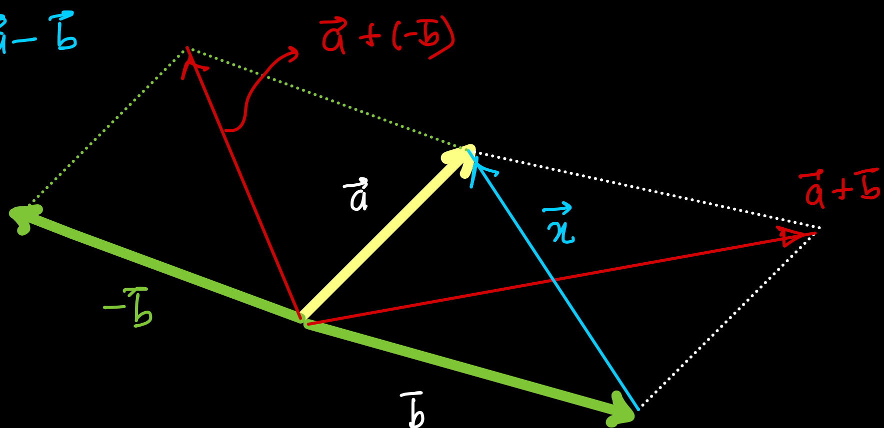
Place the vectors tail to tail

Complete the parallelogram

Vector along the middle diagonal represents $\vec{a}+\vec{b}$

The other diagonal drawn from head of $\vec{b}$ to head of $\vec{a}$ is $\vec{a}-\vec{b}$

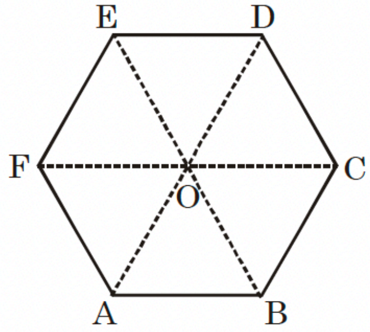




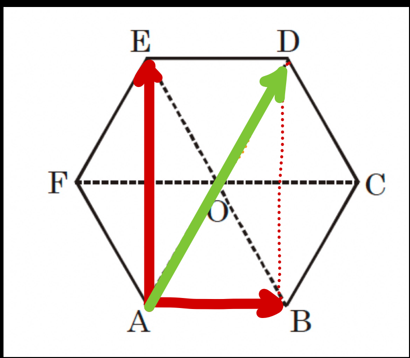
$\vec{b}+\vec{x}=\vec{a}$

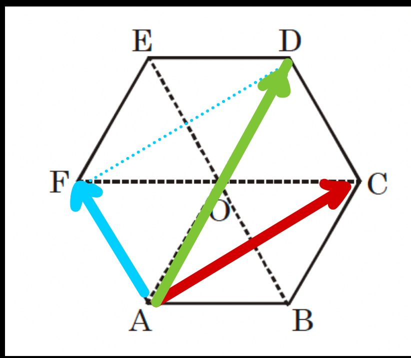
$\vec{x}=\vec{a}-\vec{b}$

Figure shows ABCDEF as a regular hexagon. What is the value of $\overrightarrow{AB}+\overrightarrow{AC}+\overrightarrow{AD}+\overrightarrow{AE}+\overrightarrow{AF}$:-



(A) $\overrightarrow{AO}$ (B) $2\overrightarrow{AO}$ (C) $4\overrightarrow{AO}$ (D) $6\overrightarrow{AO}$

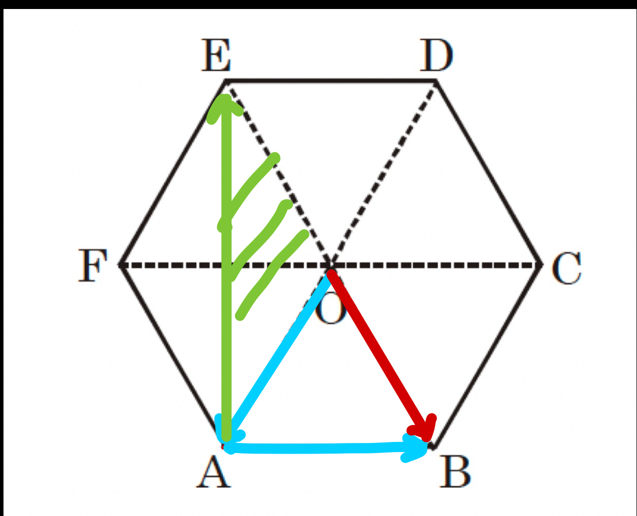




$\vec{AB} + \vec{AE} = \vec{AD}$

$\vec{AF} + \vec{AC} = \vec{AD}$

$\overrightarrow{AB}+\overrightarrow{AC}+\overrightarrow{AD}+\overrightarrow{AE}+\overrightarrow{AF}$:- $= 3\vec{AD} = 3(2\vec{AO}) = 6\vec{AO}$



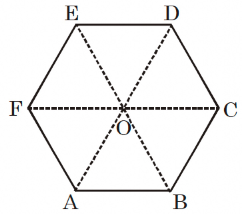
$\vec{OA} + \vec{AB} = \vec{OB}$

$\vec{OA} + \vec{AE} = \vec{OE}$

$2\vec{OA} + \vec{AB} + \vec{AE} = 0$

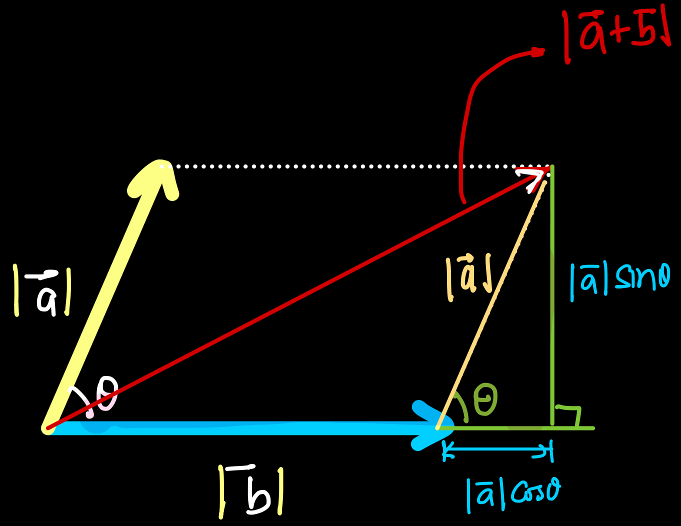
$\vec{AB} + \vec{AE} = -2\vec{OA} = 2\vec{AO}$

Choose the correct statement. Assume ABCDEF to be a regular hexagon.



(A) $\overrightarrow{ED} + \overrightarrow{DB} + \overrightarrow{BE} = \overrightarrow{0}$ (B) $\overrightarrow{FE} = \overrightarrow{BC}$ (C) $\overrightarrow{AD} = 2\overrightarrow{FE}$ (D) $\overrightarrow{DC} = -\overrightarrow{AF}$

Magnitude of the resultant using parallelogram law



By pythagoras theorem

$$|\vec{a}+\vec{b}|^2 = (|\vec{a}|\sin\theta)^2 + (|\vec{b}| + |\vec{a}|\cos\theta)^2$$

$$|\vec{a}|^2(\sin^2\theta + \cos^2\theta)$$

$$|\vec{a}+\vec{b}|^2 = |\vec{a}|^2\sin^2\theta + |\vec{b}|^2 + |\vec{a}|^2\cos^2\theta + 2|\vec{a}||\vec{b}|\cos\theta$$

$$\boxed{|\vec{a}+\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}|\cos\theta}$$

The resultant of two forces 3P and 2P is R. If the first force is doubled, the resultant is also doubled. Then find the angle (in degree) between the two forces [Use cos 120° = – 1/2]

$$R^2 = (3P)^2 + (2P)^2 + 2(3P)(2P)\cos\theta$$

$$(2R)^2 = (6P)^2 + (2P)^2 + 2(6P)(2P)\cos\theta$$

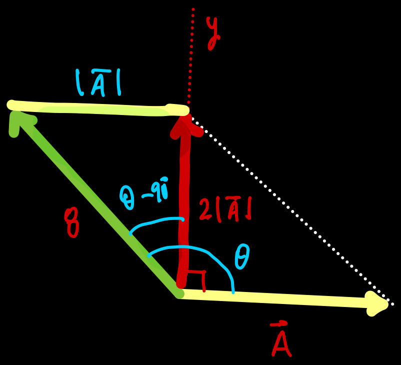
$$R^2 = 13P^2 + 12P^2\cos\theta \quad \times 4$$

$$-\ 4R^2 = 40P^2 + 24P^2\cos\theta$$

$$0 = 12P^2 + 24P^2\cos\theta$$

$$\cos\theta = -\frac{1}{2} \qquad \theta = 120^\circ$$

A vector $\vec{B}$ which has a magnitude 8.0 is added to a vector $\vec{A}$ which lie along the x-axis. The sum of these two vectors is a third vector which lie along the y-axis and has a magnitude that is twice the magnitude of $\vec{A}$. If the magnitude of $\vec{A}$ is $\frac{8}{\sqrt{x}}$ then find x. Ans 5



$$8^2 = |A|^2 + (2|\bar{A}|)^2$$

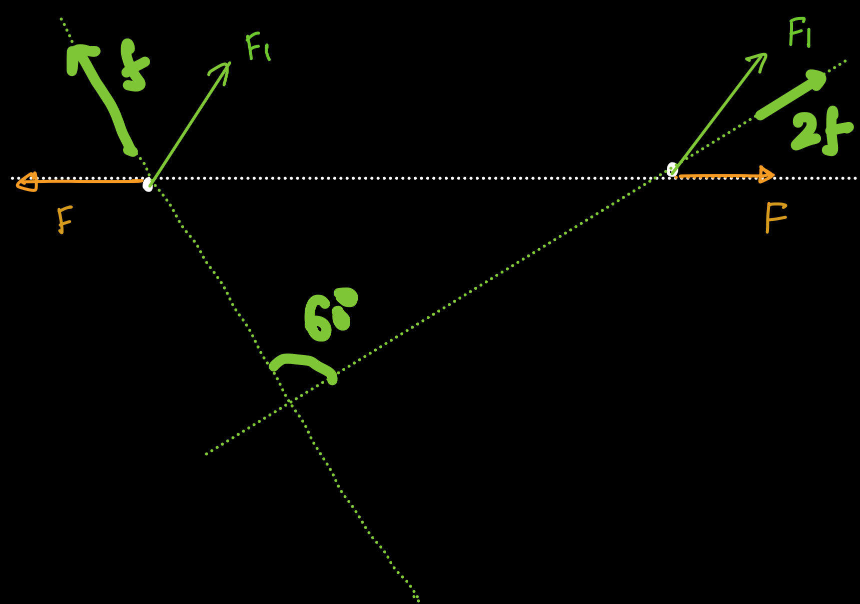
$$8^2 = 5|\bar{A}|^2$$

$$|\bar{A}| = \frac{8}{\sqrt{5}}$$

x

$$\tan(\theta - 90) = -\cot\theta = \frac{1}{2} \qquad \cot\theta = -\frac{1}{2}$$

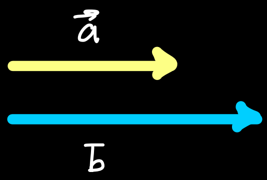
PROBLEM 7. ("Future Researchers - The Future of Science", 2018, 11) Two identical point charges of magnitude q are located in a uniform electric field of strength E_0. The electric forces acting on the charges differ by a factor of two and are directed at an angle of 60° to each other. Find the distance between the charges.



Special cases

$$|\vec{a}+\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}|\cos\theta$$

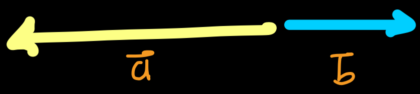
if $\theta = 0^\circ$



$|\vec{a}+\vec{b}| = |\vec{a}| + |\vec{b}|$

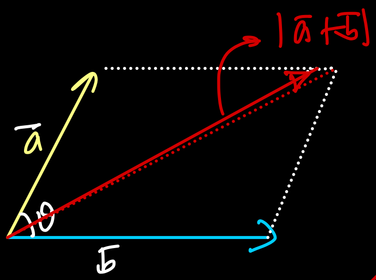
largest magnitude of vector sum

if $\theta = 180^\circ$



$|\vec{a}+\vec{b}| = |\vec{a}| - |\vec{b}|$

Smallest magnitude of vector sum



$|\vec{a}+\vec{b}|$

largest value $\quad |\vec{a}| + |\vec{b}|$

$\theta = 0^\circ$

Smallest value $\quad |\vec{a}| - |\vec{b}|$

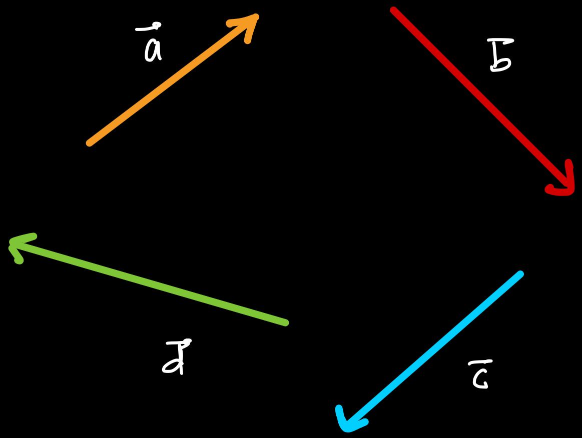
$\theta = 180^\circ$

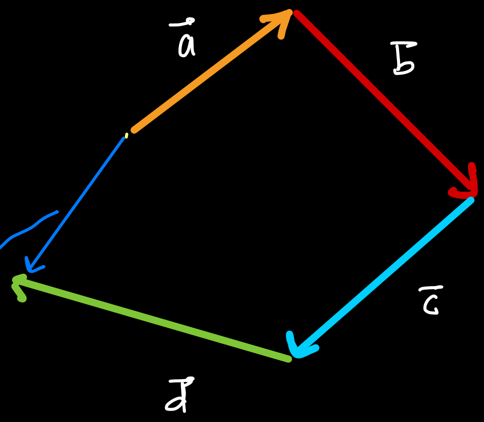
Which of the following forces cannot be a resultant of 5 N force and 7 N force?

(A) 2 N (B) 10 N ~~(C)~~ 14 N (D) 5 N

$|\vec{a}+\vec{b}|_{largest} = 12 \qquad |\vec{a}+\vec{b}|_{smallest} = 2$

Polygon law

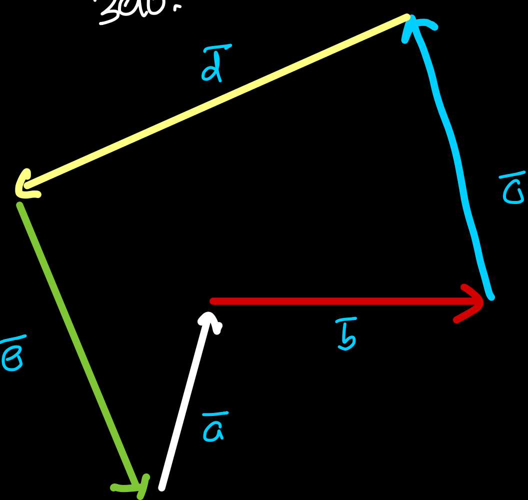




put all vectors head to tail

vector joining free tail to the free head is the vector sum of all vectors

If any number of vectors are head to tail so that they form a closed polygon. Then the resultant of vectors is zero.

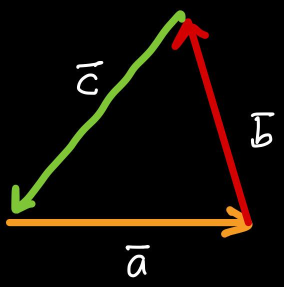


$$\bar{a} + \bar{b} + \bar{c} + \bar{d} + \bar{e} = 0$$

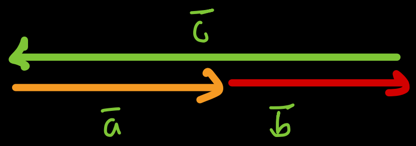
Which of the following sets of forces can not give zero resultant force :

(A) 1 N, 1 N, 1 N (B) 2 N, 3 N, 4 N (C) 2 N, 3 N, 5 N (D) 2 N, 3 N, 6 N

Resultant of three vectors is zero if they form a triangle

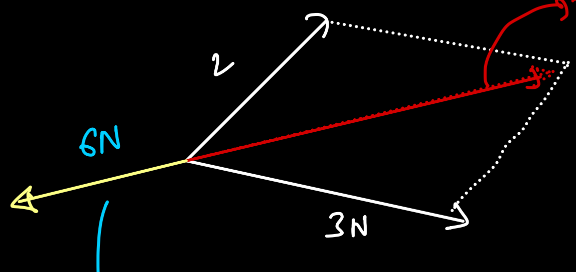


For three length to form a Δ, sum of two sides must be greater than 3rd side



resultant is also zero if sum of two is equal to 3rd

Another method



resultant can have length between 5 & 1

6N vector can't cancel the resultant of 2N & 3N since the resultant will have magnitude between 5N & 1N

The value of $|\vec{A}+\vec{B}-\vec{C}+\vec{D}|$ can be zero if :–

(A) $|\vec{A}|=5, |\vec{B}|=3, |\vec{C}|=4; |\vec{D}|=13$

(B) $|\vec{A}|=2\sqrt{2}, |\vec{B}|=2, |\vec{C}|=2; |\vec{D}|=5$

(C) $|\vec{A}|=2\sqrt{2}, |\vec{B}|=2, |\vec{C}|=2; |\vec{D}|=10$

(D) $|\vec{A}|=5, |\vec{B}|=4, |\vec{C}|=3; |\vec{D}|=8$

$$\left| \vec{A} + \vec{B} + (-\vec{C}) + \vec{D} \right|$$

@dhir_1

9, 17

$|\vec{A}|=5, |\vec{B}|=3, |\vec{C}|=4; |\vec{D}|=13$

8, 2

5, 11

$|\vec{A}|=5, |\vec{B}|=4, |\vec{C}|=3; |\vec{D}|=8$

1 – 9