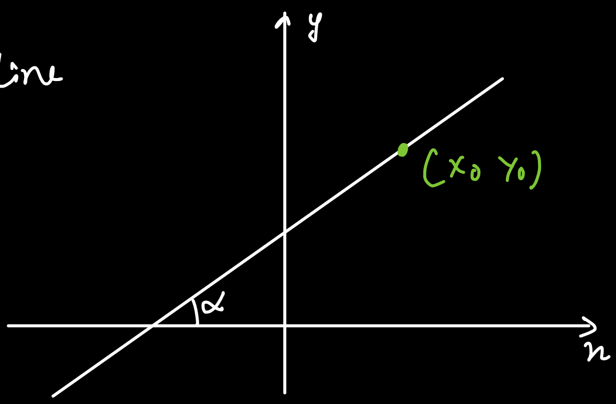
Graphs in Kinematics

Standard Graphs

(i) Straight line

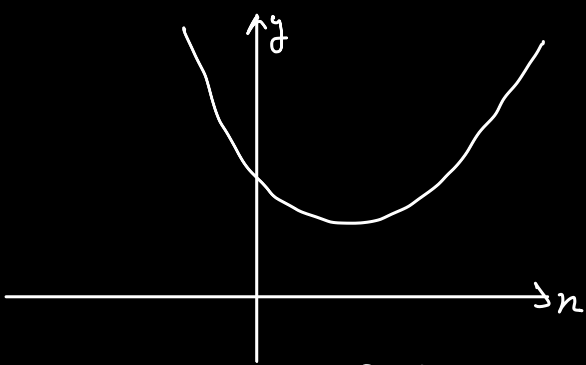


$$y = \underset{\tan\alpha}{\overset{\text{slope}}{m}}\,x + c$$

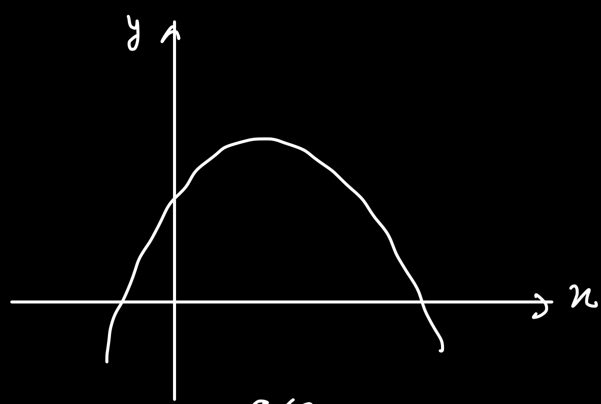
$$y - y_0 = m(x - x_0)$$

(ii) parabola

$$y = \overset{\text{coeff of } x^2}{a}\,x^2 + bx + c$$



$a > 0$

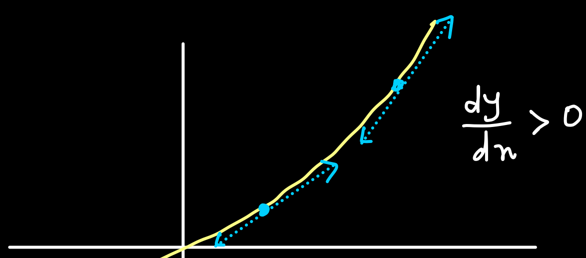


$a < 0$

Slope of the tangent to the curve

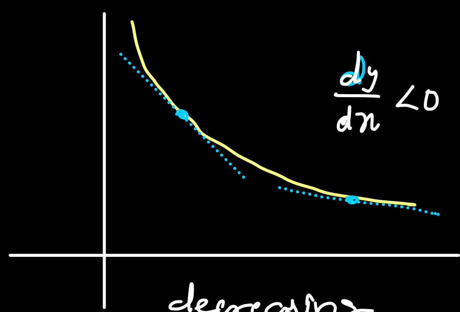
$$\text{slope} = \frac{dy}{dx}$$

Increasing / decreasing



$\frac{dy}{dx} > 0$

Increasing

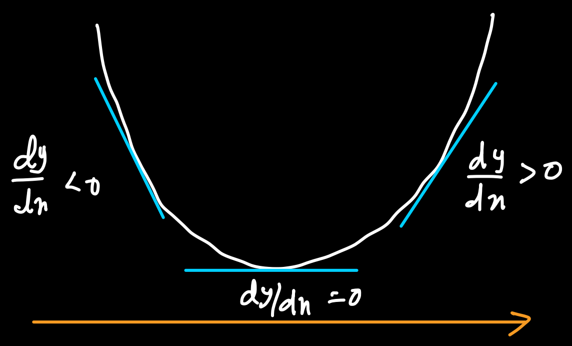


$\frac{dy}{dx} < 0$

decreasing

Concavity / Convexity

Concave up



$\frac{dy}{dx}$ increasing

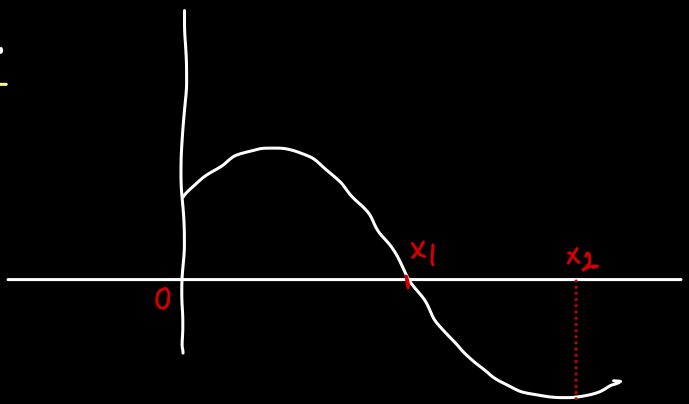
$$\frac{d}{dx}\left(\frac{dy}{dx}\right) > 0$$

$$\frac{d^2y}{dx^2} > 0$$

Concave down

$$\frac{d^2y}{dx^2} < 0$$

Area under curve

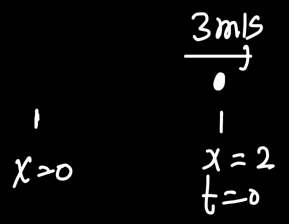


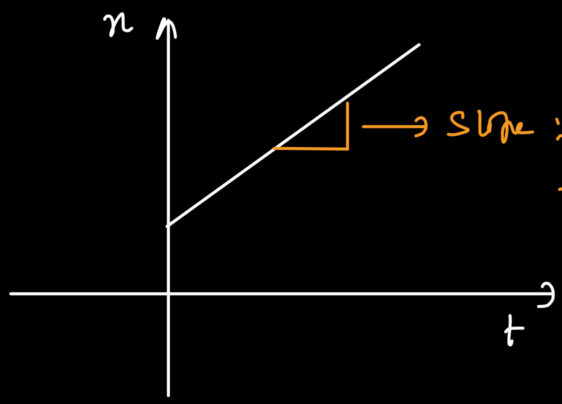
$$\int_0^{x_1} f(x)\,dx + \left| \int_{x_1}^{x_2} f(x)\,dx \right|$$

Standard graphs

(i) position - time graph

(i) uniform motion





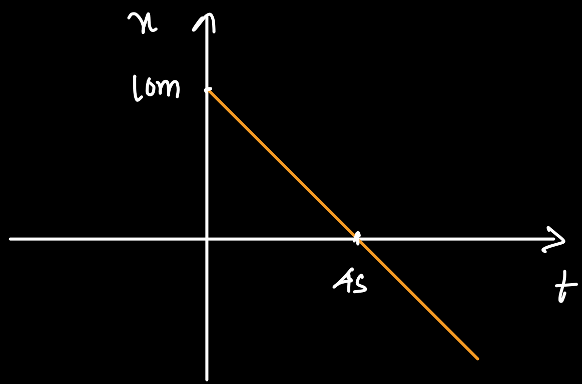
→ slope : $\frac{dx}{dt}$

velocity

$x = 3t + 2$

$y = mx + c$

Ex

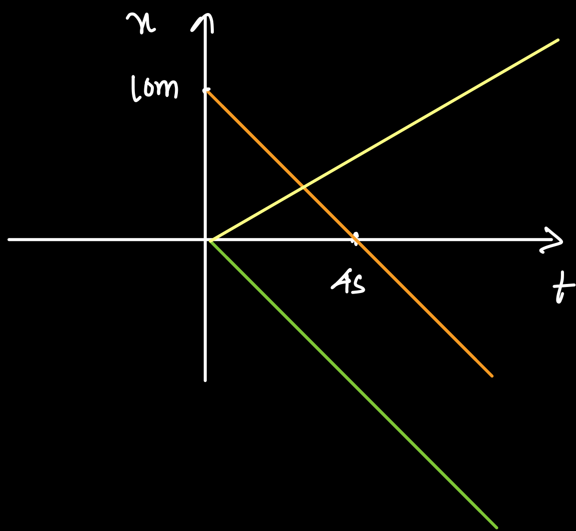


Draw disp- time & distance time graphs.

$x = -2.5t + 10$

displacement

$x - 10 = -2.5t$

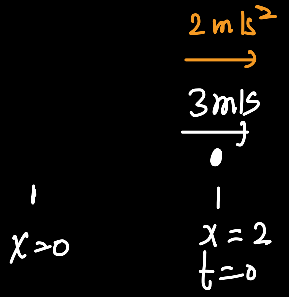


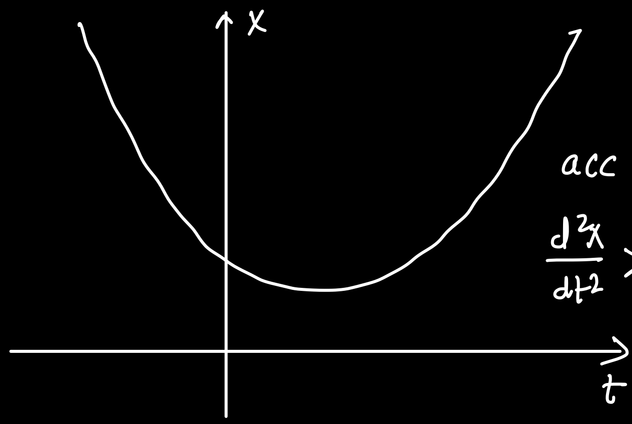
$d = |\Delta \vec{x}|$

disp-time same as x-t but starts from origin

Imp distance - time graph is always an increasing graph

(ii) motion with constant acc





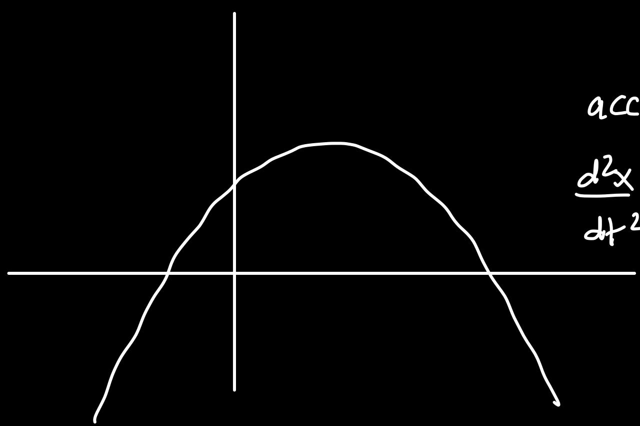
acc > 0

$\frac{d^2x}{dt^2} > 0$

$x = 3t + \frac{1}{2}(2)t^2 + 2$

$x = t^2 + 3t + 2$

$y = ax^2 + bx + c$



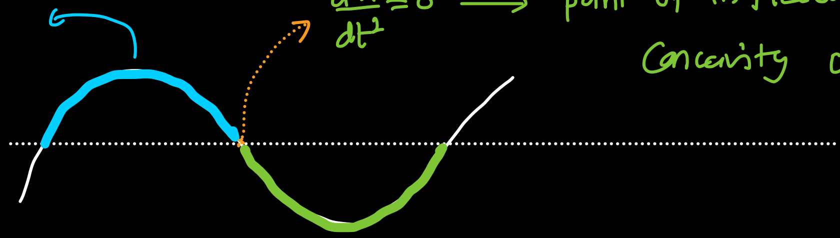
acc < 0

$\frac{d^2x}{dt^2} < 0$

Imp

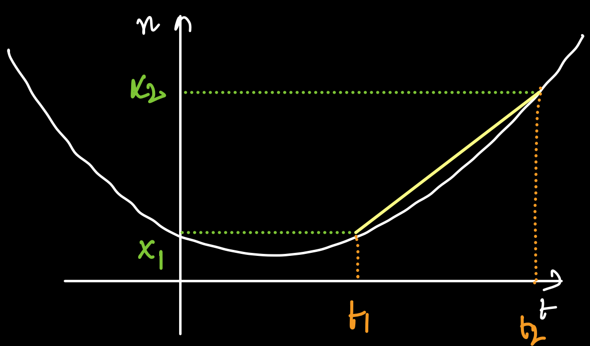
$\frac{d^2x}{dt^2} < 0$

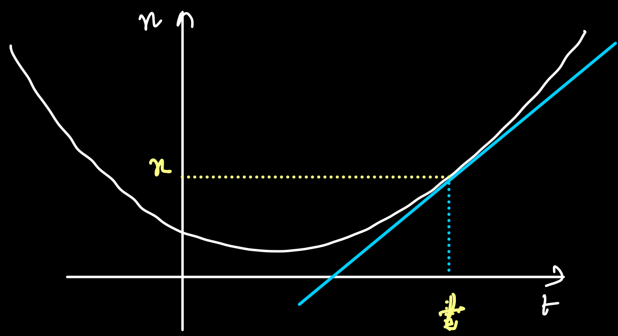
$\frac{d^2x}{dt^2} = 0 \longrightarrow$ point of inflection, point where concavity changes



$\frac{d^2x}{dt^2} > 0$

Average / Instantaneous velocity





Slope of chord = $\frac{x_2 - x_1}{t_2 - t_1} = \vec{v}_{avg}$

Slope of tangent = $\frac{dx}{dt} = \vec{v}$

1.4. A point moves rectilinearly in one direction. Fig. 1.1 shows

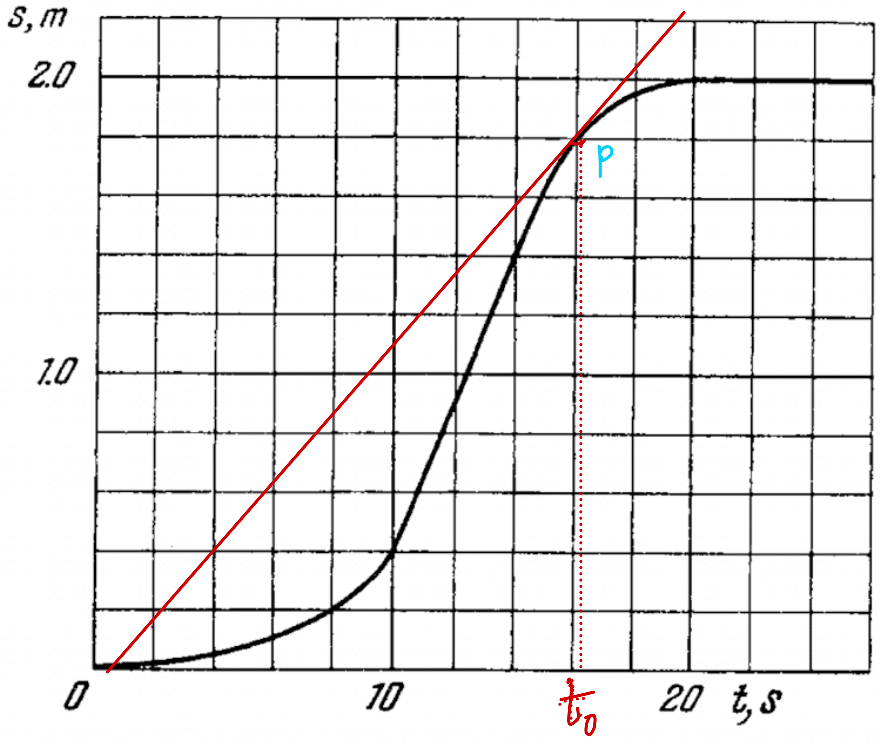


Fig. 1.1.

the distance s traversed by the point as a function of the time t. Using the plot find:

(a) the average velocity of the point during the time of motion;

(b) the maximum velocity;

(c) the time moment t_0 at which the instantaneous velocity is equal to the mean velocity averaged over the first t_0 seconds.

Draw a line from O to a point P which is tangent to the curve at P

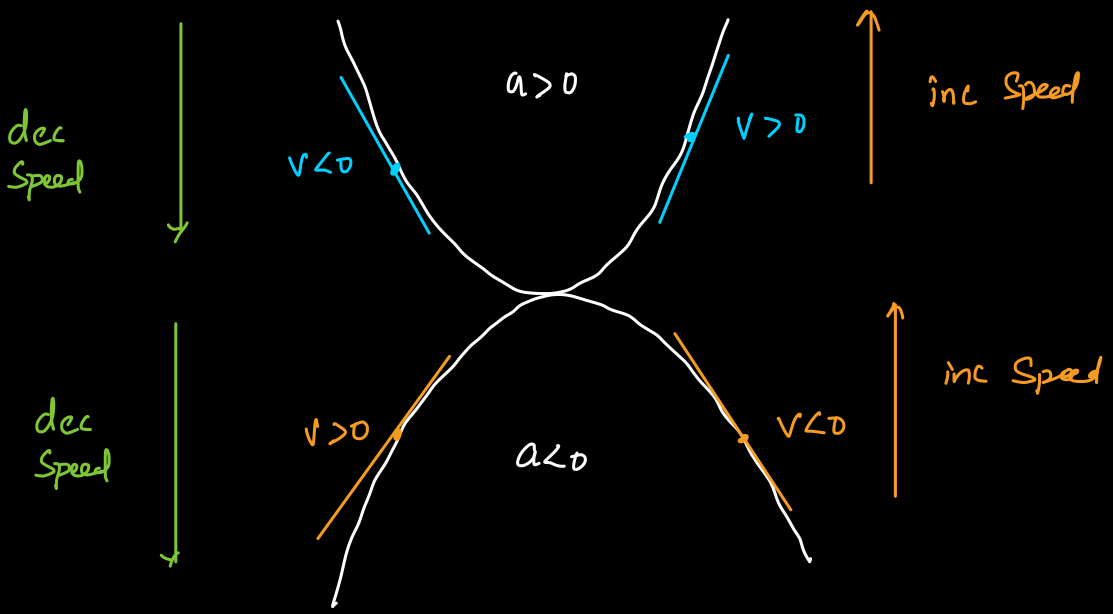
Increasing decreasing Speed

Increasing Speed — $\vec{a}$ & $\vec{v}$ are in same direction

$\vec{a}$	$\vec{v}$
+	+
−	−

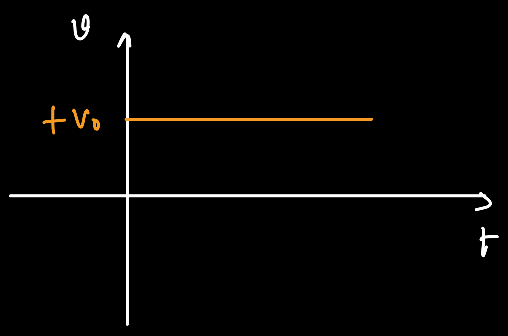
decreasing Speed — $\vec{a}$ & $\vec{v}$ are in opp direction

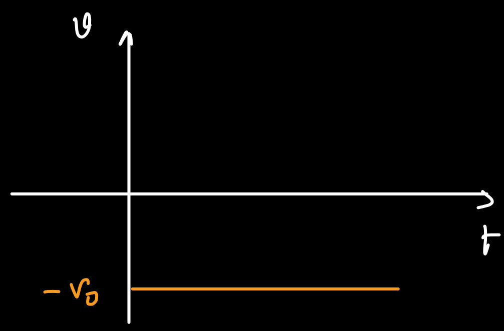
$\vec{a}$	$\vec{v}$
+	−
−	+



velocity time graph

(i) For uniform motion



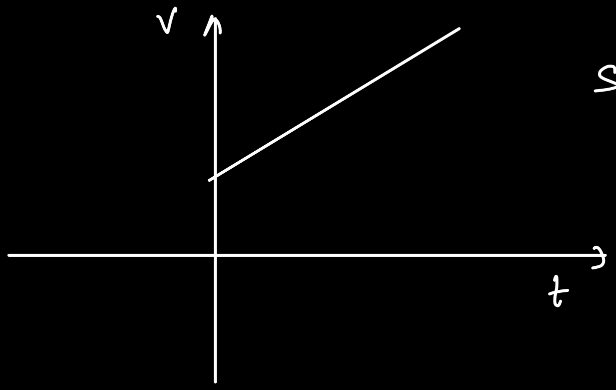


(ii) Motion with constant acc

slope = acc

$$v = at + u$$

$$y = mx + c$$

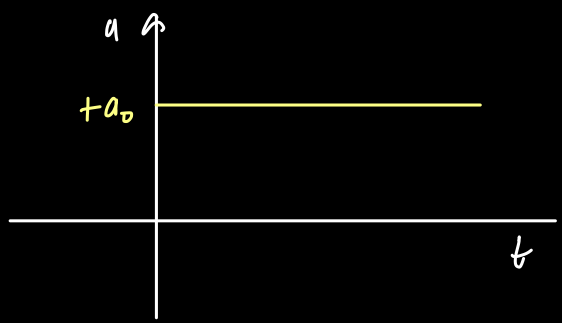


Slope $= \dfrac{dv}{dt}$ = acceleration

Area under curve $\Delta x = \int_{t_1}^{t_2} v\,dt$ = displacement

Acceleration time graph

motion with Constant acc

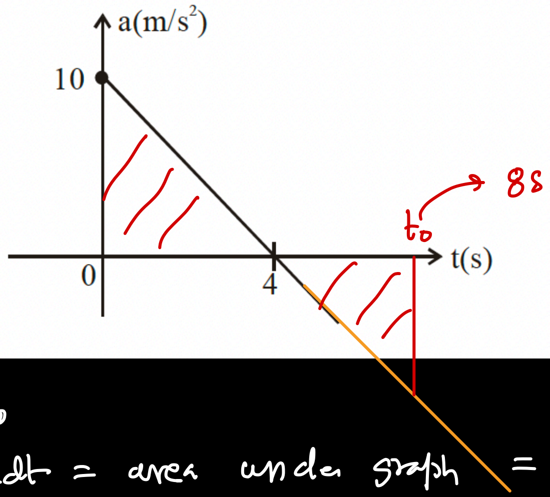


Area under curve

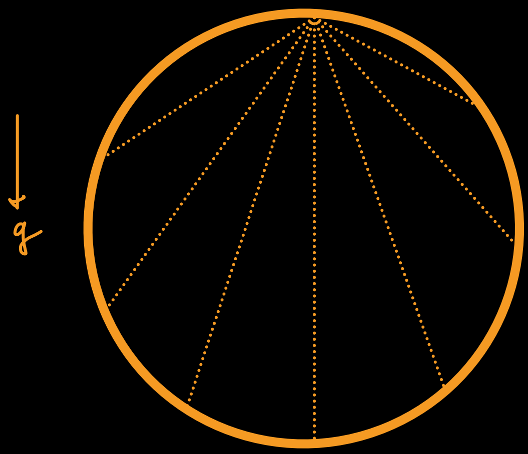
$\Delta v = \int_{t_1}^{t_2} a\,dt$

Change in velocity

The acceleration-time graph of a particle moving along a straight line is shown in figure. At what time (in sec) the particle acquires its initial velocity ?



$\Delta v = \int_0^{t_0} a\,dt$ = area under graph = 0



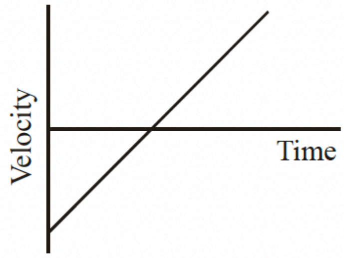
(i) Assume beads slide down without friction. find locus of beads

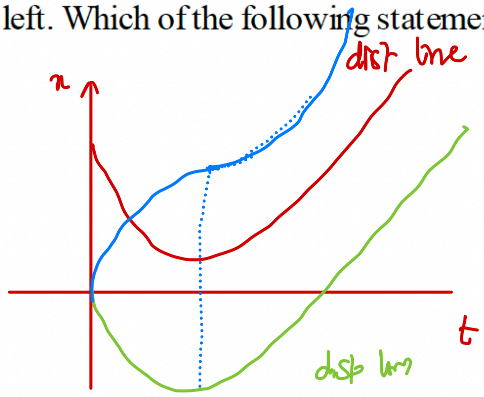
(ii) Assume friction coefficient between the beads & wires to be μ

find the locus of beads as they move

(iii) Assume $f = -kv$ then find the locus of beads position as they move

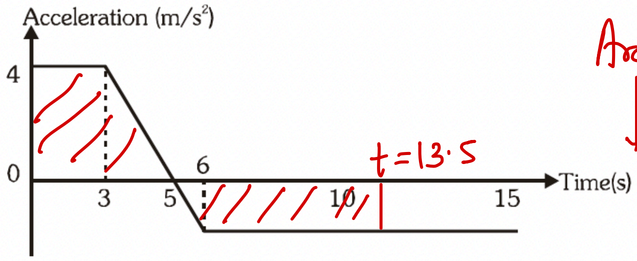
The graph below shows the velocity with respect to time of an object moving in a straight line. The positive direction is to the right and the negative direction is to the left. Which of the following statements best describes the motion of this object?





(A) The object starts at a location to the left of the origin and travels at a constant speed toward the right.
(B) The object starts at a location to the left of the origin at a slow speed and speeds up as it moves to the right.
(C) The object slows down as it moves to the left, stops, and starts moving to the right.
(D) The object slows down as it moves to the right, stops, and continues moving to the right.

A particle starts from rest from the origin at the instant t = 0 s and moves along the x-axis. During first 15s of its motion, it is subjected to accleration, which varies according to the given graph. After the isntant t = 15 s acceleration vanishes.



Area = $\Delta V = V_f - V_i$

inc   inc

. Its speed increases during the time interval or intervals

(A) 0 s to 3 s only
(B) 0 s to 5 s only
(C) 0 s to 3 s and 13.5 s to 15 s
(D) 0 s to 5 s and 13.5 s to 15 s

. Its speed decreases during the time interval

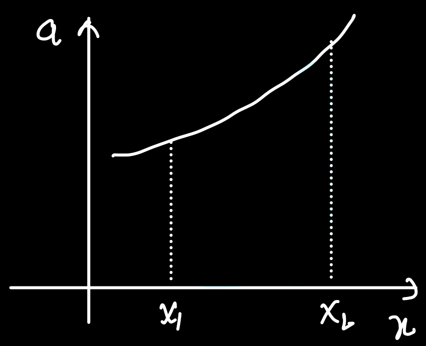
(A) 3 s to 5 s
(B) 3 s to 6 s
(C) 5 s to 6 s
(D) 5 s to 13.5 s

0. It reverses direction of motion at the instant

(A) t = 3 s
(B) t = 5 s
(C) t = 6 s
(D) t = 13.5 s

$$\frac{1}{2}(5+3)\times 4 = \left(\frac{2t-11}{2}\right)\times 2$$

Special graphs



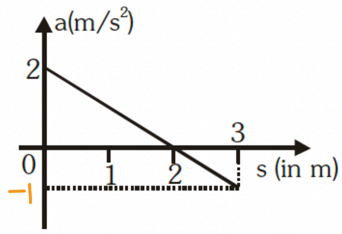
$$a = \frac{dv}{dt} = \frac{dv}{dx} \cdot \frac{dx}{dt}$$

$$a = v\frac{dv}{dx}$$

$$\boxed{a\,dx = v\,dv}$$

$$\text{Area under a-x graph} = \int_{x_1}^{x_2} a\,dx = \int_{v_1}^{v_2} v\,dv = \boxed{\frac{v_2^2 - v_1^2}{2}}$$

The acceleration-displacement graph of a particle moving in x-direction is shown. If the initial speed of the particle is $\vec{u} = -3\hat{i}$ m/s, the velocity of the particle at s=3m is-



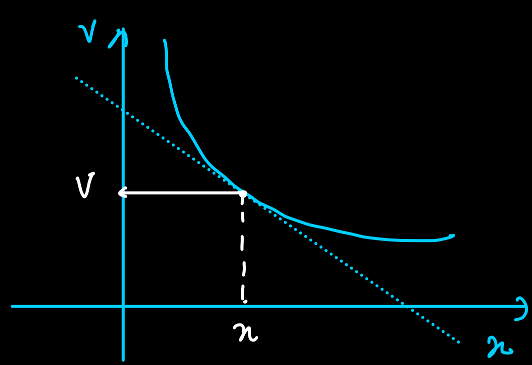
(A) $\sqrt{13}\,\hat{i}$ m/s (B) $\sqrt{14}\,\hat{i}$ m/s (C) $-2\sqrt{3}\,\hat{i}$ m/s (D) $2\sqrt{3}\,\hat{i}$ m/s

$$\text{Area} = \frac{1}{2}\times2\times2 - \frac{1}{2}\times1\times1 = 3/2 = \frac{v^2 - 3^2}{2}$$

$$v = \sqrt{12} = 2\sqrt{3}\text{ m/s}$$

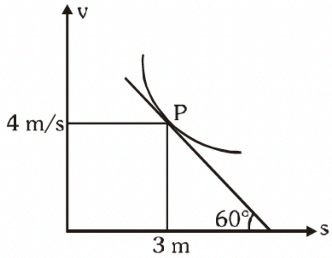
v- x graph

$$a = v\frac{dv}{dx}$$



$$\text{Slope of v-x graph } \boxed{\frac{dv}{dx} = \frac{a}{v}}$$

A particle is moving along a straight line whose velocity-displacement graph is as shown in figure : A tangent is drawn at point P on the graph. At the point P

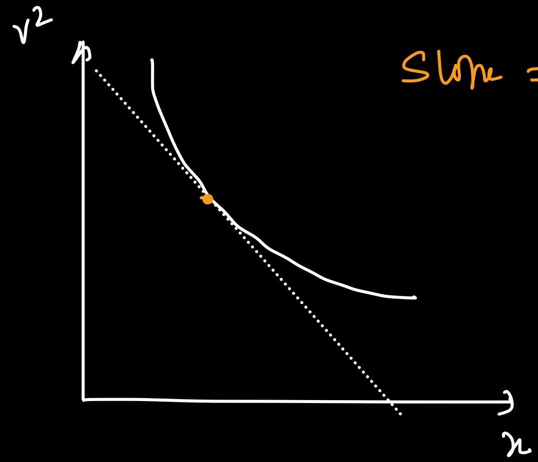


(A) the particle is speeding up
(B) numerical value of velocity and acceleration of the particle are equal
(C) numerical value of velocity is more than the numerical value of acceleration of the particle
(D) numerical value of acceleration is more than the numerical value of velocity of the particle

At P $\quad \frac{dv}{dx} = -\tan 60^\circ = \frac{a}{v}$ $\quad$ a & v have opp Signs

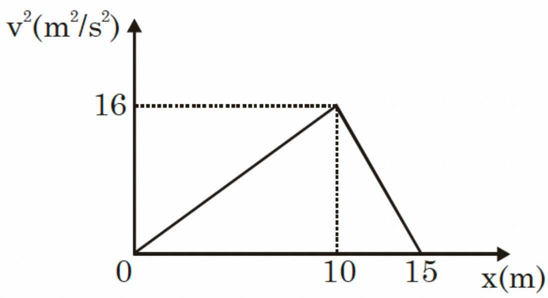
$$\sqrt{3} = \left|\frac{a}{v}\right| > 1$$

$v^2 - x$ graph



Slope $= \frac{d}{dx}(v^2) = 2v\frac{dv}{dx} = 2a$

A particle is moving on a straight line along x-direction. A graph between square of its velocity & position is drawn as shown below. Choose the **CORRECT** statement(s) :-



(A) Acceleration of the particle is 0.8 m/s² for 0 < x < 10.
(B) Acceleration of the particle is –3.2 m/s² for 10 < x < 15.
(C) Velocity of particle first increases linearly with position till 10m & then decreases linearly with position till 15 m.
(D) The particle's velocity increase linearly with time for first five seconds & decreases linearly with time till it stops.