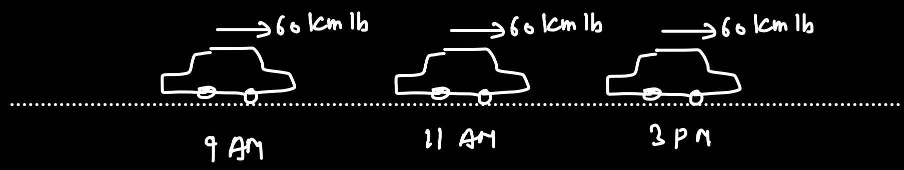
Lecture 3

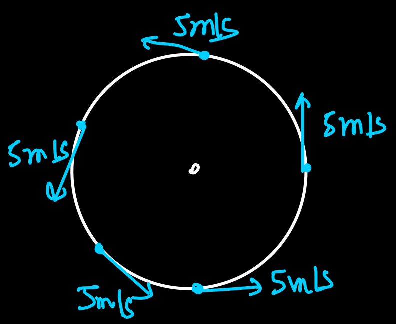
uniform motion (uniform velocity)

Object moving in a straight line at constant speed

No change in direction of motion and speed

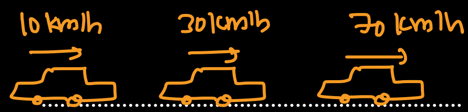


uniform speed ⟶ Speed is constant, object travels equal distances in equal intervals of time



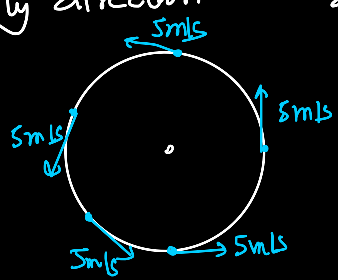
⟶ example of uniform speed but not uniform motion.

Non-uniform motion

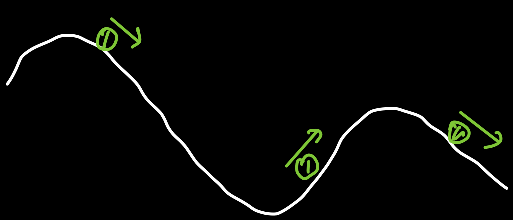


velocity of the body is changing

- ⟶ only speed changes
- ⟶ only direction changes



- ⟶ both speed & direction change

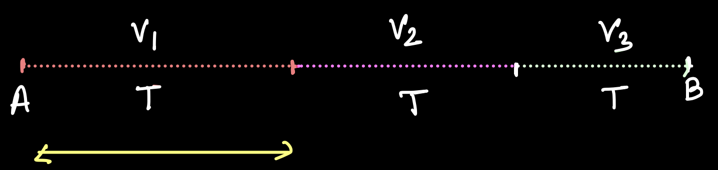


Roller Coaster

Calculation of average speeds

Average speed $V_{avg} = \dfrac{\text{Total distance}}{\text{Total time}} = \dfrac{d}{t}$

(i) Total time of travel is divided into equal time intervals



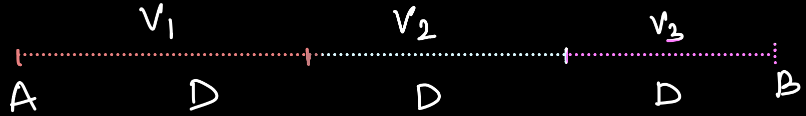
distance = speed × time

$$V_{avg} = \frac{d}{t} = \frac{v_1 T + v_2 T + v_3 T}{3T} = \frac{v_1 + v_2 + v_3}{3}$$

If total time is divided into 'n' equal time intervals

$$V_{avg} = \frac{v_1 + v_2 + v_3 + v_4 + \cdots + v_n}{n}$$

(ii) Total distance is divided into equal lengths



time = $\dfrac{\text{distance}}{\text{speed}}$

$$V_{avg} = \frac{\text{Total distance}}{\text{total time}} = \frac{3D}{\frac{D}{v_1} + \frac{D}{v_2} + \frac{D}{v_3}} = \frac{3}{\frac{1}{v_1} + \frac{1}{v_2} + \frac{1}{v_3}}$$

If total distance is divided into 'n' equal parts

$$V_{avg} = \frac{n}{\frac{1}{v_1} + \frac{1}{v_2} + \frac{1}{v_3} + \dots + \frac{1}{v_n}}$$

If a body covers first half of its journey with uniform speed v_1 and the second half of the journey with uniform speed v_2, then the average speed is

1) $v_1 + v_2$ 2) $\frac{2v_1v_2}{v_1 + v_2}$ ✓

3) $\frac{v_1v_2}{v_1 + v_2}$ 4) v_1v_2

For a train that travels from one station to another at a uniform speed of 40 kmh^{-1} and returns to final station at speed of 60 kmh^{-1}, then its average speed is

1) 98 km/hr 2) 0 km/hr

3) 50 km/hr 4) 48 km/hr ✓

40 km/h →

A d B

← 60 km/h

v_1 v_2

D D

$$V_{avg} = \frac{2}{\frac{1}{v_1} + \frac{1}{v_2}}$$

$$= \frac{2v_1v_2}{v_1+v_2}$$

$$V_{avg} = \frac{2d}{\frac{d}{40} + \frac{d}{60}} = \frac{2 \times 40 \times 60}{40 + 60}$$

$= 48$ km/h

If a car covers $2/5^{th}$ of the total distance with v_1 speed and $3/5^{th}$ distance with v_2 then average speed is

1) $\frac{1}{2}\sqrt{v_1v_2}$ 2) $\frac{v_1+v_2}{2}$ 3) $\frac{2v_1v_2}{v_1+v_2}$ 4) $\frac{5v_1v_2}{3v_1+2v_2}$ ✓

A moving car possesses average velocities of $5ms^{-1}$, $10ms^{-1}$ and $15ms^{-1}$ in the first, second, and third seconds respectively. What is the total distance covered by the car in these 3s?

1) 15m 2) 30m ✓ 3) 55m 4) 45m

v_1 v_2

$\frac{2}{5}D$ $\frac{3}{5}D$

D

5m/s 10m/s 15m/s

1s 1s 1s

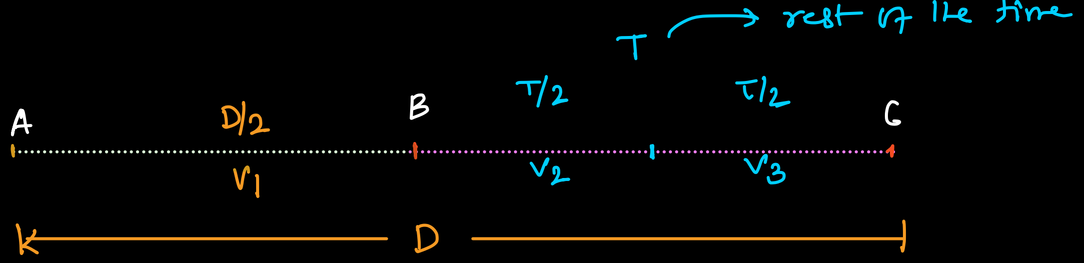
distance $= 5 \times 1 + 10 \times 1 + 15 \times 1$

$= 30$ m

$$\text{Avg speed} = \frac{D}{\frac{\frac{2}{5}D}{v_1} + \frac{\frac{3}{5}D}{v_2}}$$

$$= \frac{1}{\frac{2}{5v_1} + \frac{3}{5v_2}} = \frac{1}{\frac{2v_2+3v_1}{5v_1v_2}} = \frac{5v_1v_2}{3v_1+2v_2}$$

Ex A particle travels with a speed of v_1 for 1st half of the total distance of journey, 1 half of the rest of total time of journey it travels with v_2 and the next it half time it travels with v_3 find the avg speed of the motion



For BC
$$\frac{D}{2} = v_2 T/2 + v_3 T/2$$

$$\frac{D}{2} = \frac{v_2 T}{2} + \frac{v_3 T}{2}$$

$$D = v_2 T + v_3 T$$

$$D = (v_2 + v_3) T$$

$$T = \frac{D}{v_2 + v_3}$$

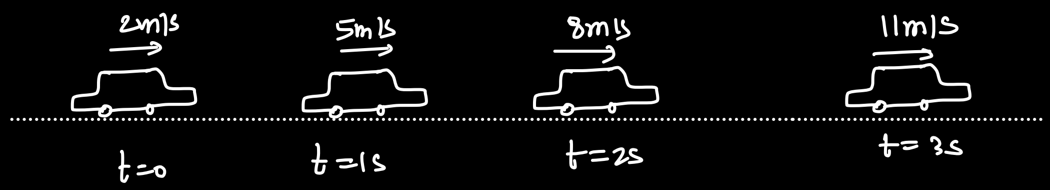
$$\text{Average Speed} = \frac{D}{\frac{D/2}{v_1} + T}$$

$$= \frac{D}{\frac{D}{2v_1} + \frac{D}{v_2+v_3}}$$

$$= \frac{1}{\frac{1}{2v_1} + \frac{1}{v_2+v_3}}$$

$$= \frac{2v_1(v_2+v_3)}{2v_1 + v_2 + v_3} \quad \text{Ans}$$

Acceleration

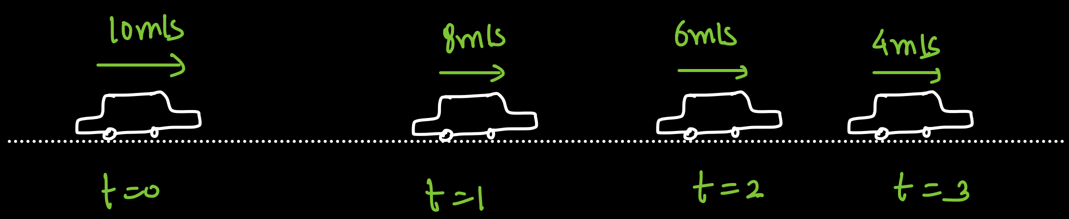


Speed is increasing

acceleration = rate at which speed is changing $= \frac{3\,m/s}{1s} = 3m/s^2$

Speed is inc by 3m/s every second

$$a = \frac{V_f - V_i}{t} = \frac{8-5}{1}$$



acceleration = rate at which speed is changing $= \frac{2m/s}{1s} = 2m/s^2$

Speed is dec by 2m/s every second

$$a = \frac{V_f - V_i}{t} = \frac{6-8}{1} = -2m/s^2$$

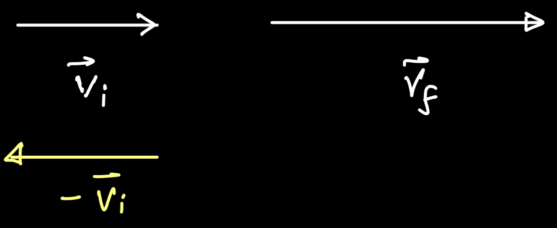
For motion with Constant acc

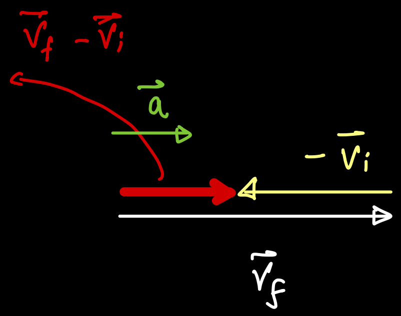
$$a = \frac{V_f - V_i}{t}$$

Acceleration as a vector

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{t}$$ $\longrightarrow$ motion with constant acc

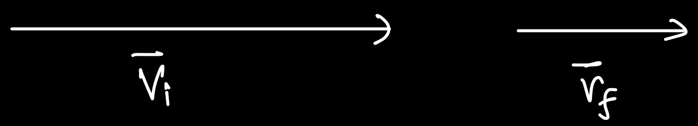
(i) particle is speeding up

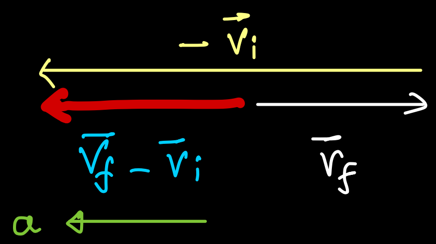




acc vector is in in direction of $\vec{v}_f - \vec{v}_i$ vector which is in the direction of velocity vector

(ii) particle is slowing

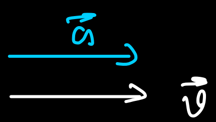




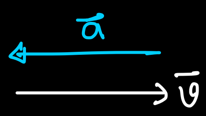
acc vector is in in direction of $\vec{v}_f - \vec{v}_i$ vector which is opposite to direction of velocity vector

$\boxed{\text{Summary}}$

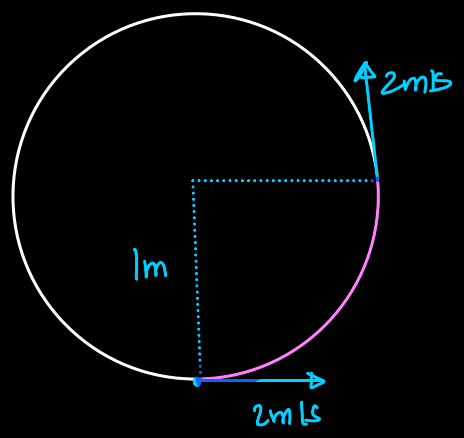
Speed up



Slow up

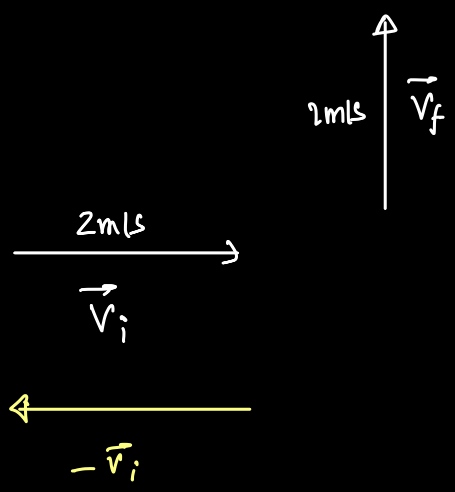


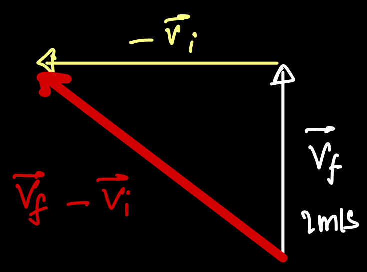
Acceleration in curved motion



Average acceleration

$$\boxed{\vec{a}_{avg} = \frac{\vec{v}_f - \vec{v}_i}{t}}$$





$$|\vec{v}_f - \vec{v}_i| = 2\sqrt{2} \text{ m/s}$$

$$t = \frac{d}{v} = \frac{\frac{\pi \overset{(1)}{R}}{2}}{2} = \frac{\pi}{4} \text{ s}$$

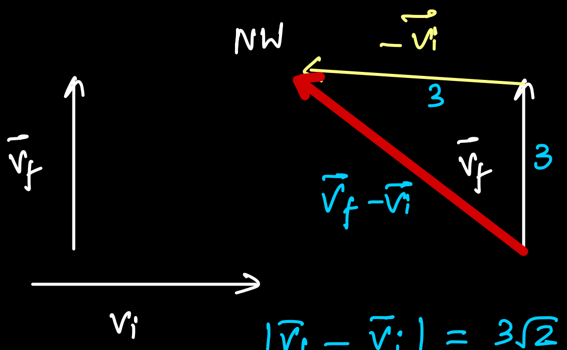
$$|\vec{a}_{avg}| = \frac{|\vec{v}_f - \vec{v}_i|}{t} = \frac{2\sqrt{2}}{\pi/4} = \frac{8\sqrt{2}}{\pi} \text{ m/s}^2$$

A body moves with a velocity of 3m/s due east and then turns due north to travel with the same velocity. If the total time of travel is 6s, the acceleration of the body is

1) $\sqrt{3}$ m/s^2 towards north west

2) $\frac{1}{\sqrt{2}}$ m/s^2 towards north west

3) $\sqrt{2}$ m/s^2 towards north east 4) all the above

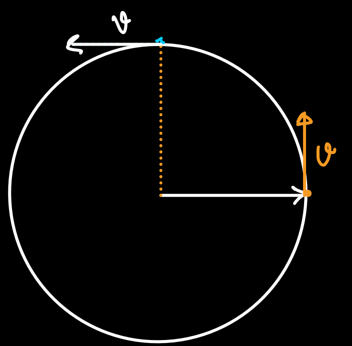


$|\vec v_f - \vec v_i| = 3\sqrt{2}$

$$|\vec a_{avg}| = \frac{|\vec v_f - \vec v_i|}{t} = \frac{3\sqrt{2}}{6} = \frac{1}{\sqrt{2}} \text{ m/s}^2 \quad \text{north west}$$

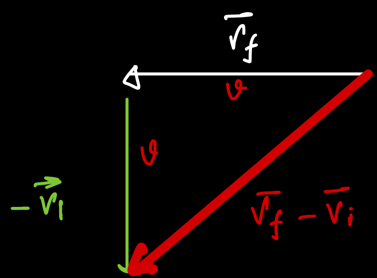
The length of a seconds hand in a watch is 1cm. Magnitude of change in velocity of its tip in 15 second is :-

(A) 0 cm/sec (B) $\frac{\pi}{30\sqrt{2}}$ cm/sec (C) $\frac{\pi}{30}$ cm/sec ✓(D) $\frac{\pi\sqrt{2}}{30}$ cm/sec



Tip moves complete circle in 60s

$$v = \frac{2\pi(1)}{60} = \frac{\pi}{30} \text{ m/s}$$



$$|\vec v_f - \vec v_i| = v\sqrt{2} = \frac{\pi\sqrt{2}}{30}$$

The initial and final velocities of an object are as shown in figure-(a). Which arrows shown in figure-(b) can represent change in velocity vector ?

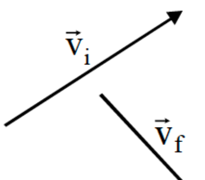


Figure-(a)

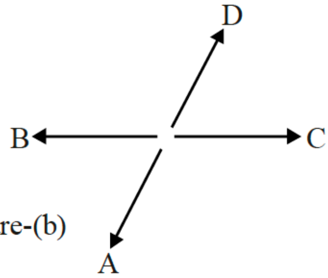
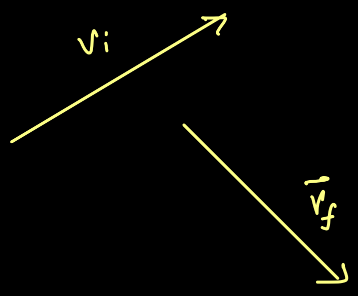
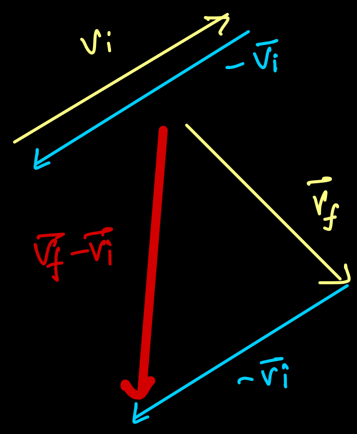


Figure-(b)


v_i
$\vec{v}_f$


v_i
$-\vec{v}_i$
$\vec{v}_f$
$\vec{v}_f - \vec{v}_i$
$-\vec{v}_i$