lecture 3 — Work Energy theorem

Basic trigonometry

$$\cos(180-\theta) = -\cos\theta \quad \Big| \quad \sin(180-\theta) = \sin\theta$$

$$\cos 120^\circ = \cos(180-60^\circ) = -\cos 60^\circ = -\frac{1}{2}$$

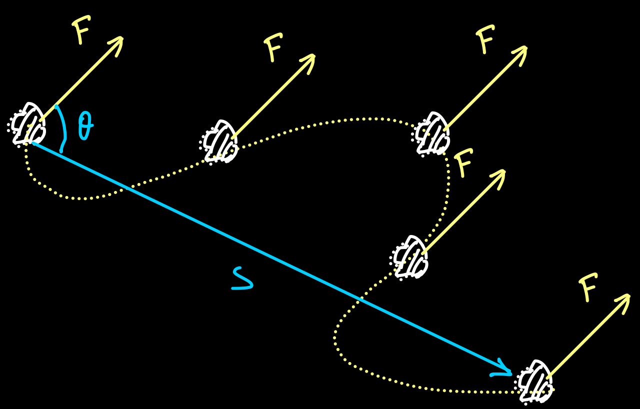
$$\cos 135^\circ = \cos(180-45^\circ) = -\cos 45^\circ = -\frac{1}{\sqrt{2}}$$

$$\cos 150^\circ = \cos(180-30^\circ) = -\cos 30^\circ = -\frac{\sqrt{3}}{2}$$

$$\cos 180^\circ = \cos(180^\circ-0^\circ) = -\cos 0^\circ = -1$$

supplementary angles

Work done by a Constant force



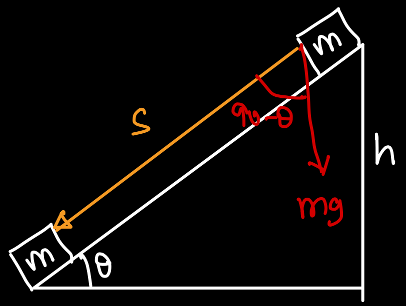
Work done by constant force F is defined as

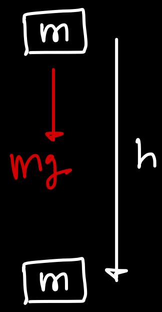
$$\boxed{W_F = FS\cos\theta}$$

Imp

Work done by a Constant force does not depend on the actual path of body but only on displacement.

Ex





Work done by gravity

$= mg\,s\,\cos(90-\theta)$

$= mg\,s\,\sin\theta \qquad \sin\theta = \frac{h}{s}$

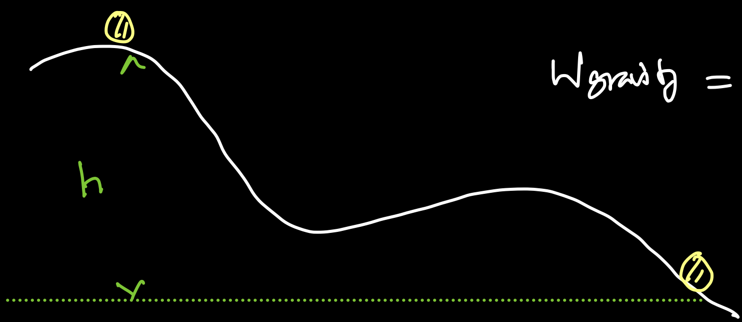
$= mgh$

Work done by gravity

$= mg \cdot h \cos 0^\circ$

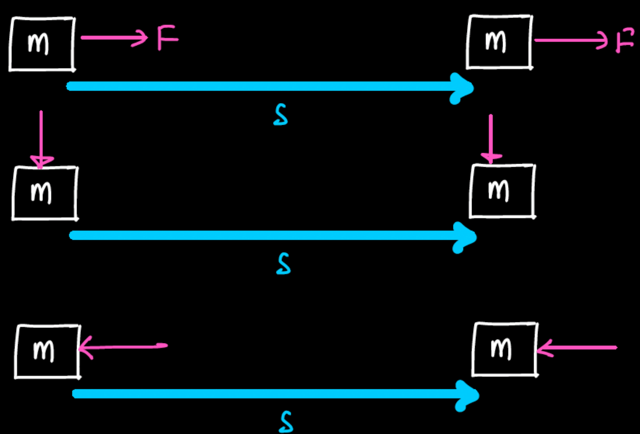
$= mgh$

In general work done by gravity only depends on vertical disp of the body only and not on the actual path



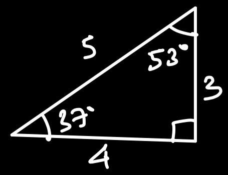
$W_{gravity} = +mgh$

Special cases

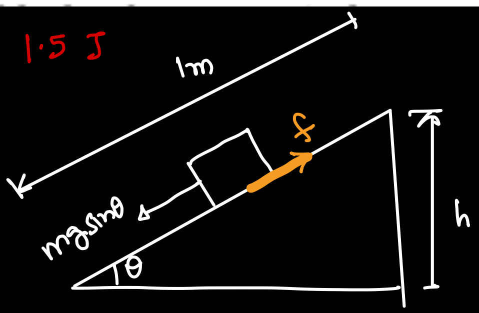


$\boxed{Fs\cos\theta}$	+ve
$Fs\cos 0^\circ$	$+Fs$
$Fs\cos 90^\circ$	0
$Fs\cos 180^\circ$	$-Fs$
	-ve

A block of mass 250 g slides down an incline of inclination 37° with a uniform speed. Find the work done against the friction as the block slides through 1·0 m.



1.5 J



Constant velocity

$mg\sin\theta = f$

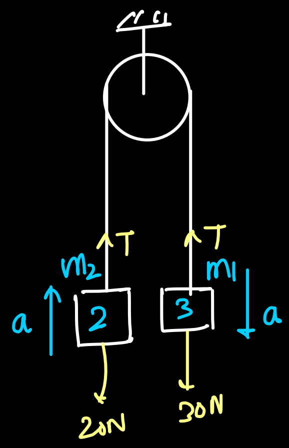
Work done by friction = Work done by gravity

$= mg\,(1\sin 37^\circ)$

$= 250 \times 10^{-3} \times 10 \times 3/5$

$= 1.5\ \text{J}$

The two blocks in an Atwood machine have masses 2·0 kg and 3·0 kg. Find the work done by gravity during the fourth second after the system is released from rest.



$30 - T = 3a$

$T - 20 = 2a$ add

$10 = 5a$

$a = 2\ \text{m/s}^2$

Displacement in nth second

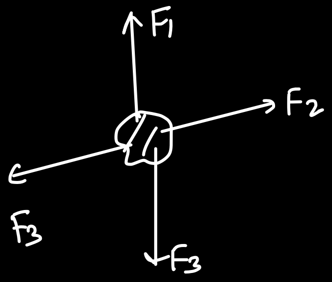
$s = u + \frac{1}{2}a(2n-1)$

$= 0 + \frac{1}{2}(2)(2\times 4-1)$

$= 7\ \text{m}$

Work done by gravity $= \underbrace{+\,3 \times 10 \times 7}_{+m_1 g \times h} \; \underbrace{-\,2 \times 10 \times 7}_{-m_2 \times g \times h_2} = 70\ \text{J}$

Work Energy theorem



$$\boxed{W_{F_1} + W_{F_2} + W_{F_3} + W_{F_4} = \text{Final KE} - \text{Initial KE}}$$

Total Work done by all forces is equal to change in kinetic Energy.

The workdone to accelerate a body from 30 ms^{-1} to 60 ms^{-1} is three times the work done to accelerate it from $\mathbf{10\ ms\ m}^{-1}$ to '$\mathbf{v}$'. The value of '$\mathbf{v}$'in $\mathbf{ms}^{-1}$ is

1. 30
2. $20\sqrt{2}$
3. $30\sqrt{3}$
4. $10\sqrt{10}$

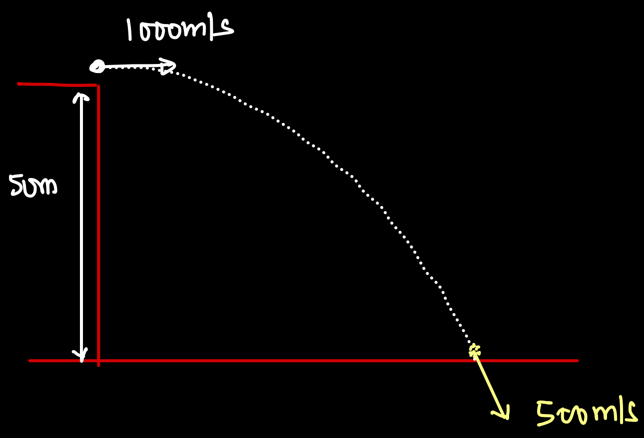
$$W_1 = \frac{1}{2}m(60)^2 - \frac{1}{2}m(30)^2$$

$$W_1 = 3W_2$$

$$W_2 = \frac{1}{2}m(v^2) - \frac{1}{2}m(10)^2$$

A bullet of mass $\mathbf{10gm}$ is fired horizontally with a velocity $1000\ \mathrm{ms}^{-1}$ from a riffle situated at a height 50 m above the ground. If the bullet reaches the ground with a velocity $500\ \mathrm{ms}^{-1}$, the work done against air resistance in the trajectory of the bullet is (in joule) $(g = 10\ \mathrm{ms}^{-2})$

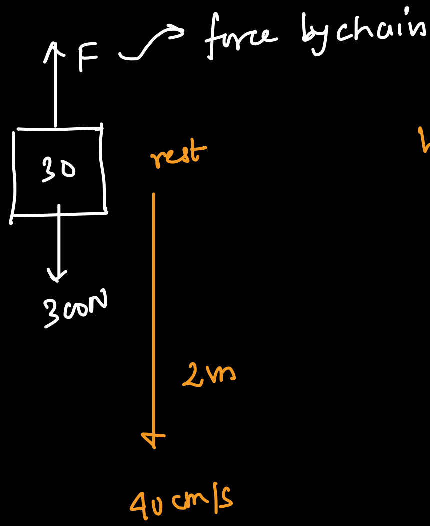
1. 5005 2) 3755 3) 3750 4) 17.5



$$W_{gravity} + W_{air\ resistance} = \frac{1}{2}10^{-2}(500)^2 - \frac{1}{2}\times 10^{-2}(1000)^2$$

$$W_{gravity} \rightarrow 10^{-2}\times 10\times 50$$

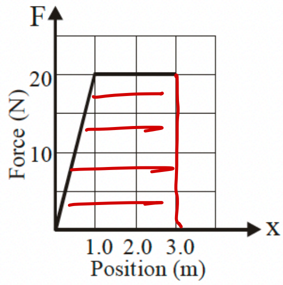
A block of mass 30·0 kg is being brought down by a chain. If the block acquires a speed of 40·0 cm/s in dropping down 2·00 m, find the work done by the chain during the process.



$W_{chain} + W_{gravity} = \frac{1}{2} \times 30 (0.4)^2 - 0$

$W_{chain} = -597.6$ J

The graph below shows how the force on a mass depends on the position of the mass. What is the change in the kinetic energy of the mass as it moves from x = 0.0 m to x = 3.0 m?



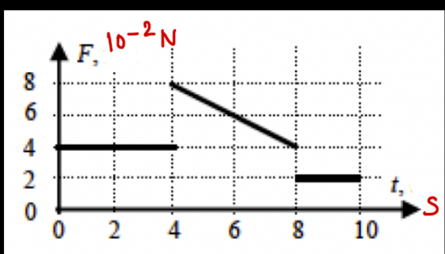
(A) 0.0 J (B) 20 J (C) 50 J (D) 60 J

Area under F - x graph = Work = $KE_f - KE_i$

$\frac{1}{2}(3+2) \times 20$ = Change in KE

PROBLEM 4. (MOSH, 2017, Grade 10)

A single unbalanced external force begins to act on a body of mass $m = 20$ g. The graph of its magnitude versus time is shown in the figure. Find the work done by this force in a reference frame where the initial velocity of the body is $v_0 = 2$ m/s. The force and velocity vectors always have the same direction.



Ans: 5.72 J

Area under F-t graph gives impulse.

impulse = change in linear momentum

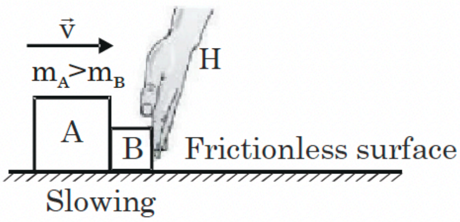
$$\left\{ 4\times 4 + \frac{1}{2}(8+4)\times 4 + 2\times 2 \right\} \times 10^{-2} = mv_f - mv_i$$

$$v_f = 2.4\,m/s$$

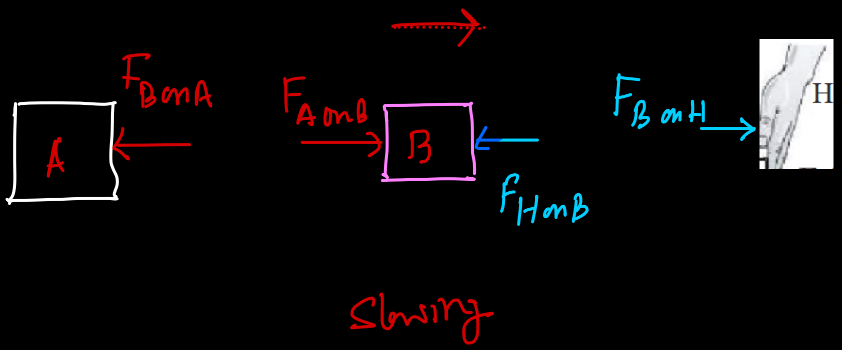
Work done by force = $\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = 5.72\,J$

Practice problems in laws of motion (Lecture 1 & 2)

Boxes A and B are sliding to the right across a frictionless table. The hand H is slowing them down. The mass of A is larger than the mass of B. Here $F_{A\text{ on }B}$ represents force on B due to A and same for other:



(A) $F_{B\text{ on }H} = F_{H\text{ on }B} = F_{A\text{ on }B} = F_{B\text{ on }A}$
(B) $F_{B\text{ on }H} = F_{H\text{ on }B} > F_{A\text{ on }B} = F_{B\text{ on }A}$ ✓
(C) $F_{B\text{ on }H} = F_{H\text{ on }B} < F_{A\text{ on }B} = F_{B\text{ on }A}$
(D) $F_{B\text{ on }H} = F_{H\text{ on }B} > F_{A\text{ on }B} > F_{B\text{ on }A}$



$$F_{H\,on\,B} > F_{A\,on\,B}$$

A particle is in equilibrium under the action of the six forces. If three of these forces are reversed, the particle still remains in the equilibrium. Which of the given statement is/are **CORRECT**?

(A) If any two of these six forces is removed it will be in equilibrium

(B) If any one from three reversed forces is removed system will not be in equilibrium

(C) If all these reversed forces are removed then particle will be in equilibrium

(D) If all the three (not reversed one) forces are removed then particle will be in equilibrium

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 + \vec{F}_5 + \vec{F}_6 = 0$$

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 - \vec{F}_4 - \vec{F}_5 - \vec{F}_6 = 0$$

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$$

$$\vec{F}_4 + \vec{F}_5 + \vec{F}_6 = 0$$

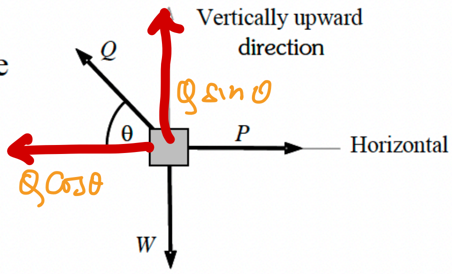
A block of weight W is kept in equilibrium by two forces P and Q as shown in the figure. Which of the following give the condition for the block to be in equilibrium?

(A) W = P tanθ

(B) W = P cotθ

(C) W = Q sinθ and P = Q cosθ

(D) P = Q sinθ and W = Q cosθ

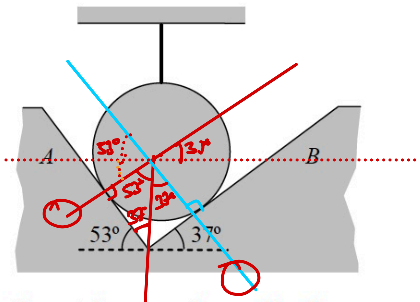


$$P = Q\cos\theta$$

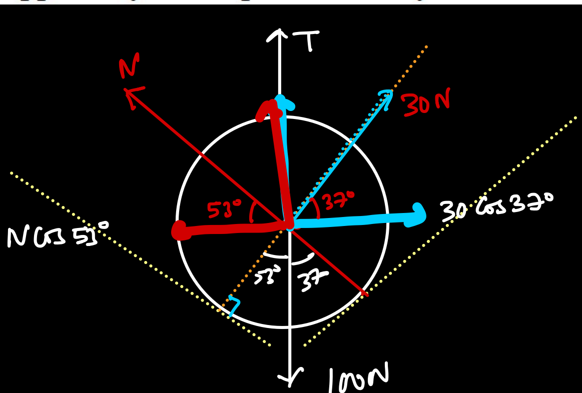
$$W = Q\sin\theta$$

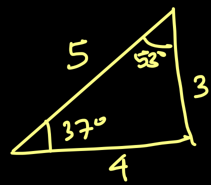
$$\frac{P}{W} = \cot\theta$$

In the system is shown a 10 kg horizontal cylinder suspended from string from the ceiling and supported by two fixed frictionless slopes A and B. If normal reaction applied by slope A on the cylinder is 30 N, which of the following conclusion can you make? [$g = 10\ m/s^2$]



(A) No tension force is applied by the string on the cylinder.
✓(B) Tension force applied by the string on the cylinder is 50 N.
(C) Normal reaction applied by the slope B on the cylinder is 60 N.
✓(D) Normal reaction applied by the slope B on the cylinder is 40 N.





$N \cos 53^\circ = 30 \cos 37^\circ$

$N \cdot \frac{3}{5} = 30 \cdot \frac{4}{5}$

$N = 40\ N$

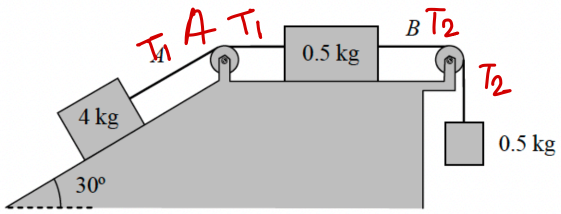
$T + N \sin 53^\circ + 30 \sin 37^\circ = 100$

$T + 40 \times \frac{4}{5} + 30 \times \frac{3}{5} = 100$

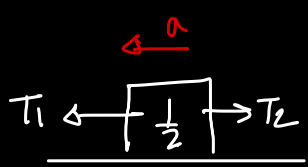
$T + 50 = 100$

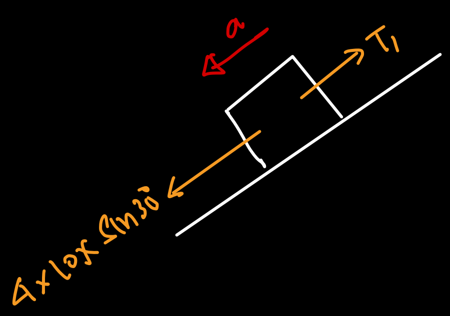
$T = 50\ N$

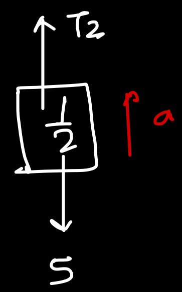
System shown in the figure is released from rest. Which of the following statements is/are true? [$g = 10\ m/s^2$]



(A) Magnitude of acceleration of each block is 3 m/s^2.
(B) Tension in the cord A is 6.5 N.
(C) Tension in the cord B is 8 N.
(D) Difference of magnitudes of tensions in both the cord is 1.5 N.







$$20 - T_1 = 4a$$

$$T_1 - T_2 = \frac{1}{2}a$$

$$T_2 - 5 = \frac{1}{2}a$$

add

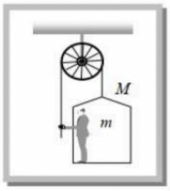
$$20 - 5 = 5a$$

$$a = 3 m/s^2$$

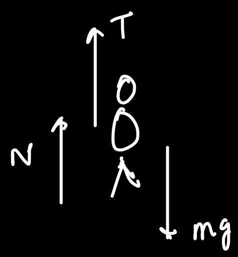
$$20 - T_1 = 4(3) \qquad T_1 = 8N$$

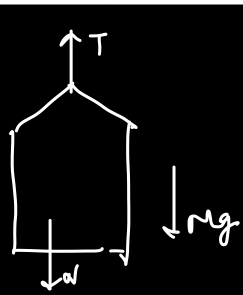
$$T_2 - 5 = \frac{1}{2} \times 3 \qquad T_2 = 6.5N$$

A man of mass stands on a crate of mass M. He pulls on a light rope passing over a smooth light pulley. The other end of the rope is attached to the crate. For the system to be in equilibrium, the force exerted by the man on the rope will be



(A) $(M-m)g$ (B) $\frac{1}{2}(M+m)g$ (C) Mg (D) mg

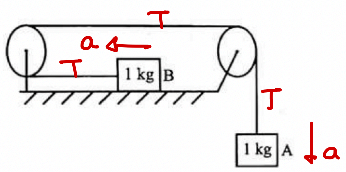




$N + T = mg$

$T = N + Mg$

Refer to the figure, the pulleys and the string are light. T is the tension in the string. Take $g = 10\,ms^{-2}$. Which of the following are correct? (All surfaces are smooth)



(A) The acceleration of the system is $5\,ms^{-2}$ (B) $T = 0N$

(C) The acceleration of the system is $10\,ms^{-2}$ (D) $T = 5N$

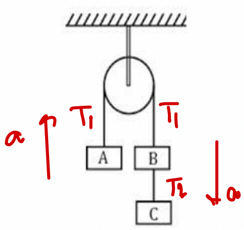
B $T = 1 \cdot a$

A $10 - T = 1 \cdot a$

$10 = 2a$

$a = 5\,m/s^2$

Three equal weights A, B and C each of mass 2kg are hanging on a string passing over a fixed frictionless pulley as shown in the figure. The tension in the string between B and C is



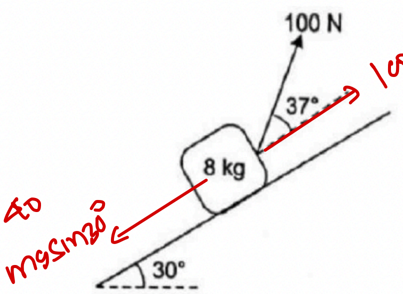
(A) Zero (B) 13 N (C) 3.3 N (D) 19.6 N

A $T_1 - 20 = 2a$

B $T_2 + 20 - T_1 = 2a$

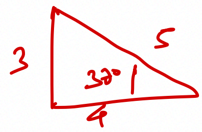
C $20 - T_2 = 2a$

A block of mass 8 kg is pulled up on a smooth fixed incline with the help of a 100 N force as shown in figure. Determine the acceleration of block (in m/s^2) (Take $g = 10\, ms^{-2}$, and neglect friction.)



$100 \cos 37°$

$\cancel{100}^{20} \times \frac{4}{\cancel{5}} = 80$



$a = \frac{80 - 40}{8} = \frac{40}{8} = 5\, m/s^2$