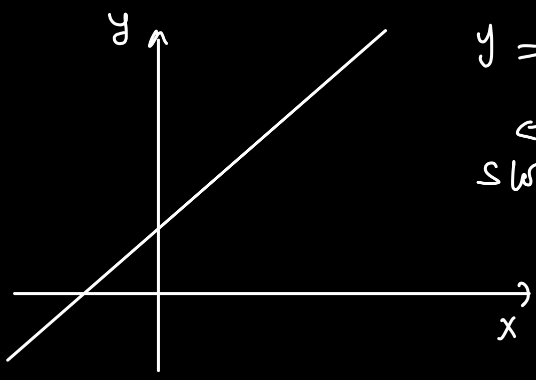
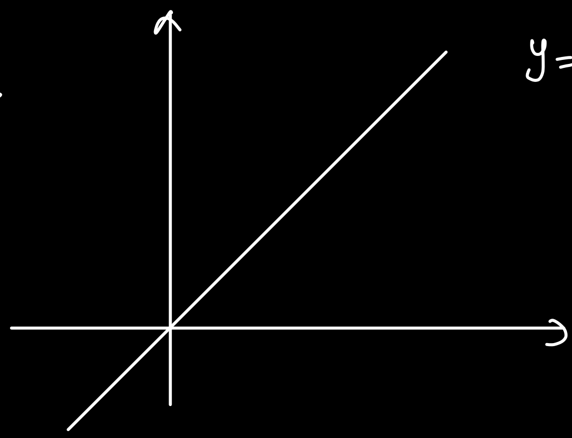
Graphs in kinematics

(i) Straight line



$y = mx + c$

m → slope

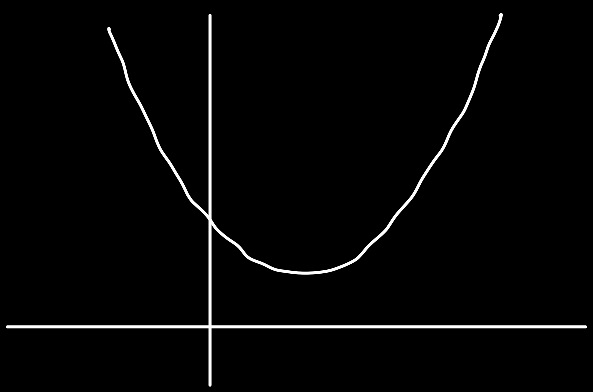


$y = mx$

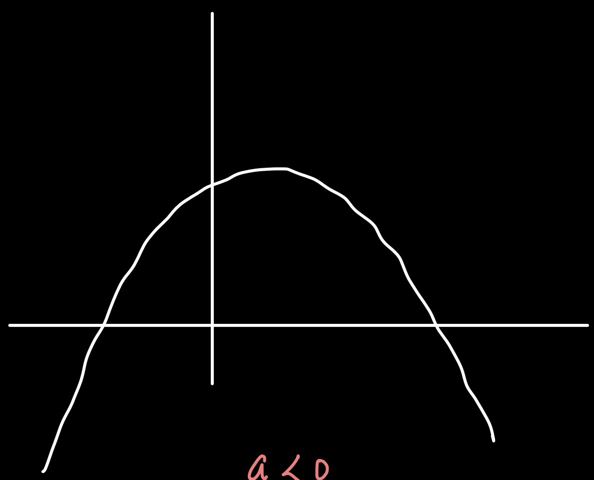
Line passing through origin

(ii) parabola

$y = ax^2 + bx + c$

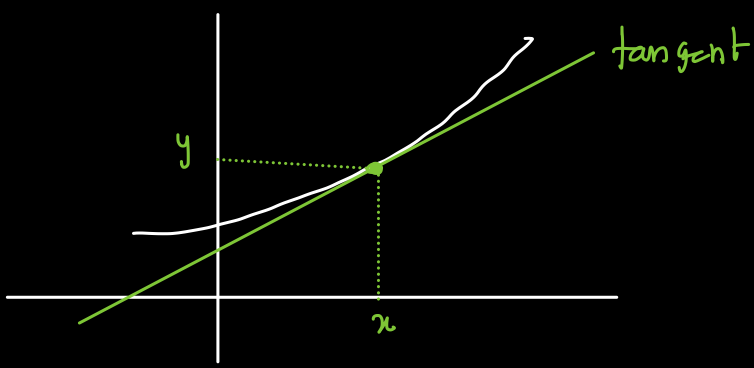


$a > 0$



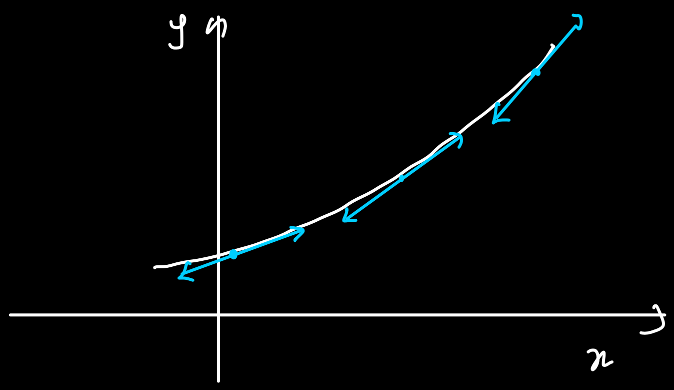
$a < 0$

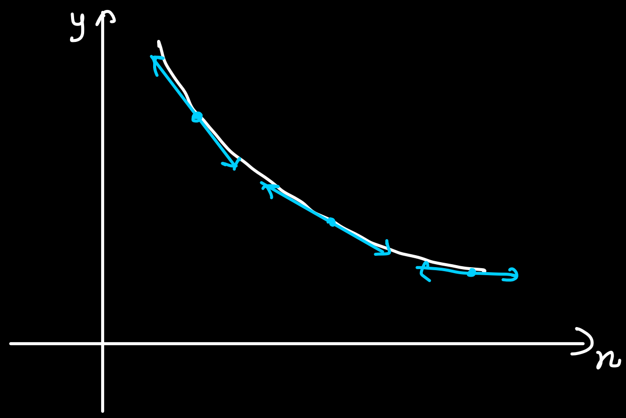
Properties related to differential Calculus



slope of tangent $= \dfrac{dy}{dx}$

Increasing / decreasing functions or graphs



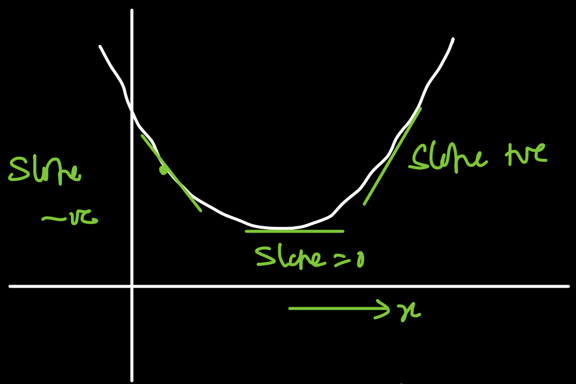


Slope of tangent at every point is +ve

$$\frac{dy}{dx} > 0$$

Slope of tangent at every point is −ve

$$\frac{dy}{dx} < 0$$

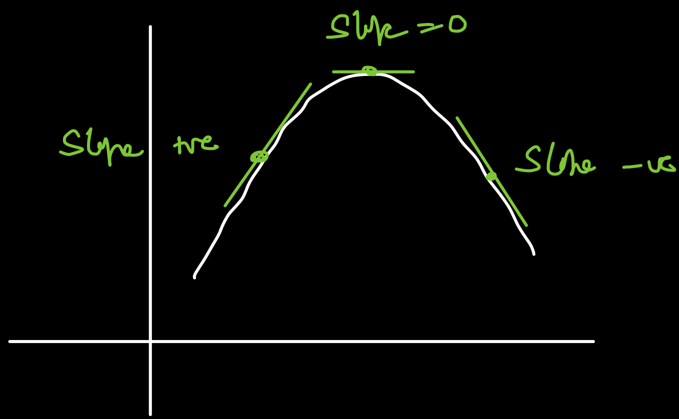


Concave up

$$\frac{d^2y}{dx^2} > 0$$

Slope $= \frac{dy}{dx} \longrightarrow$ inc

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) > 0$$



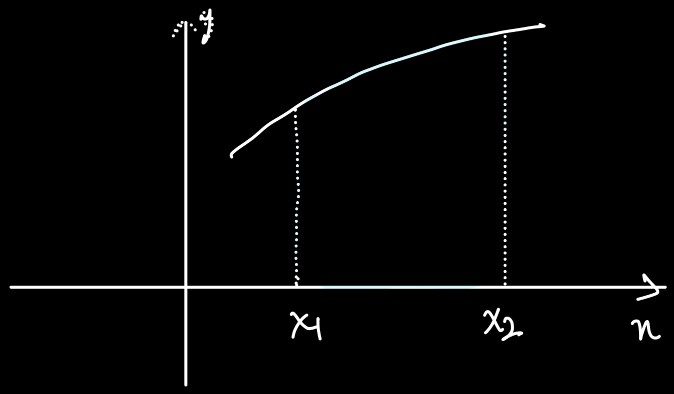
Concave down

$$\frac{d^2y}{dx^2} < 0$$

Slope $= \frac{dy}{dx}$ is dec

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) < 0$$

Properties related to integral Calculus

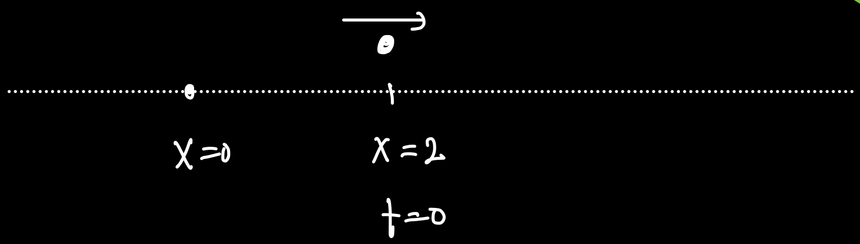


Area under curve $= \int_{x_1}^{x_2} y\,dx$

Standard graphs

position-time graph

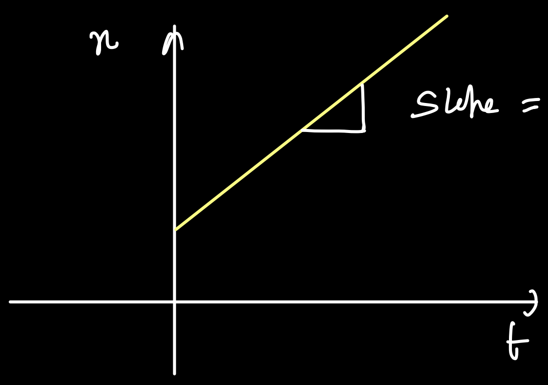
(i) uniform motion



position at time 't'

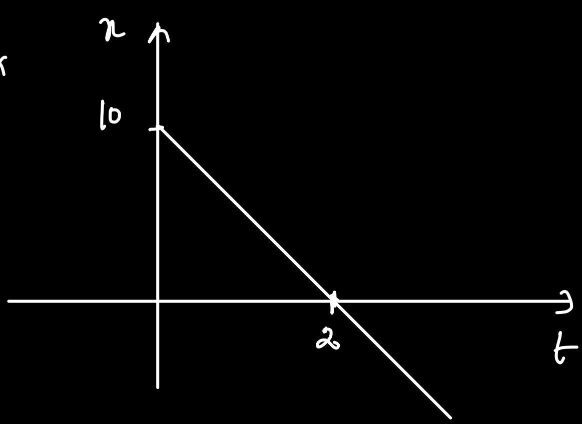
$x = 3t + 2$

$y = mx + c$



slope $= \dfrac{dx}{dt} =$ velocity

Ex



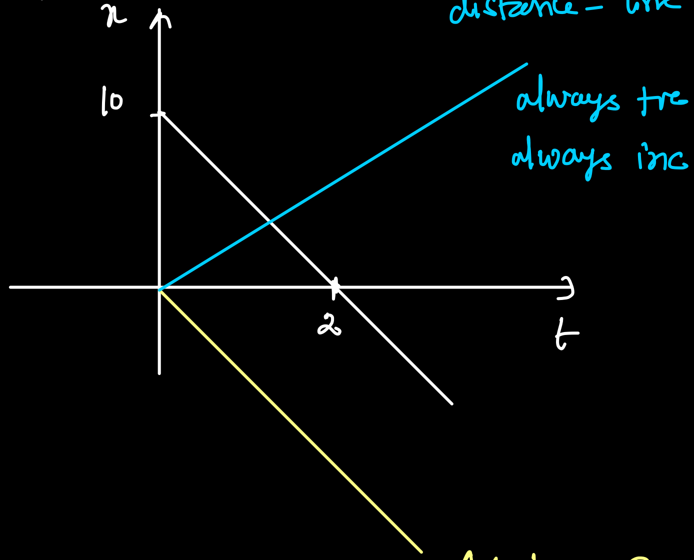
(i) velocity

(ii) Draw disp-time / distance-time graphs

(i) velocity = slope = $-5\ m/s$

$$x = -5t + 10$$

(ii)



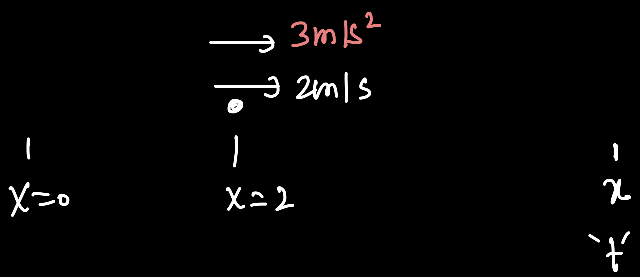
At time t

$$\Delta x = x - 10 = -5t$$

$\Delta x - t$ Same slope at $x-t$

but starts from origin

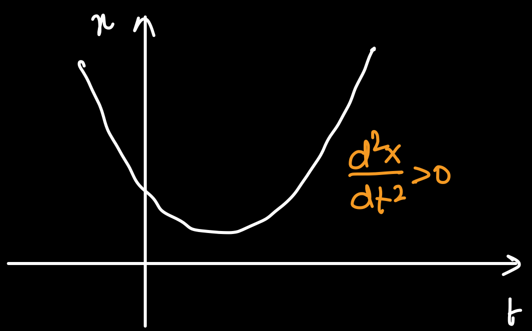
(iii) Motion with Constant acc

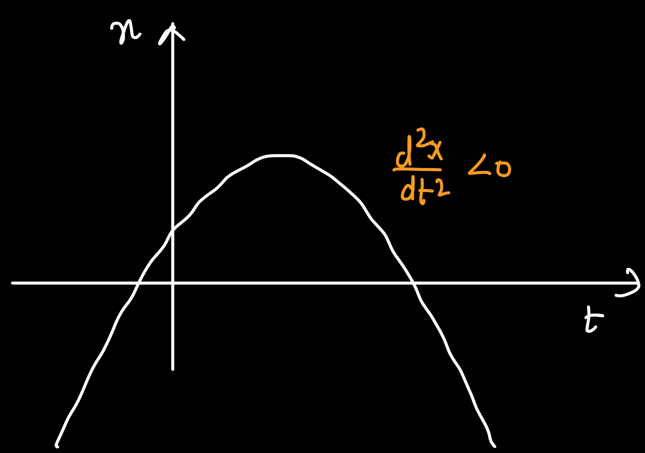


$$x = 2t + \frac{1}{2}(3)t^2 + 2$$

$$x = \frac{3}{2}t^2 + 2t + 2$$

$$y = ax^2 + bx + c$$





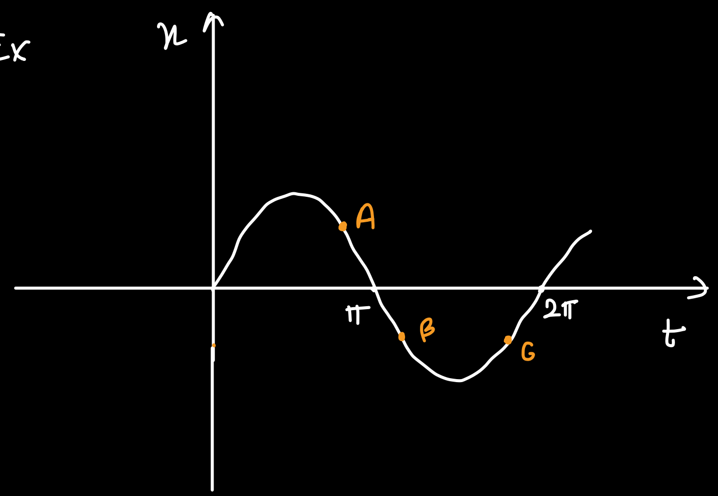
acc > 0

upward opening parabola

acc < 0

downward opening parabola

Ex



$X = \sin t$

(i) Indicate the sign of acc at A, B, C

(ii) Is acc zero at some point? Yes

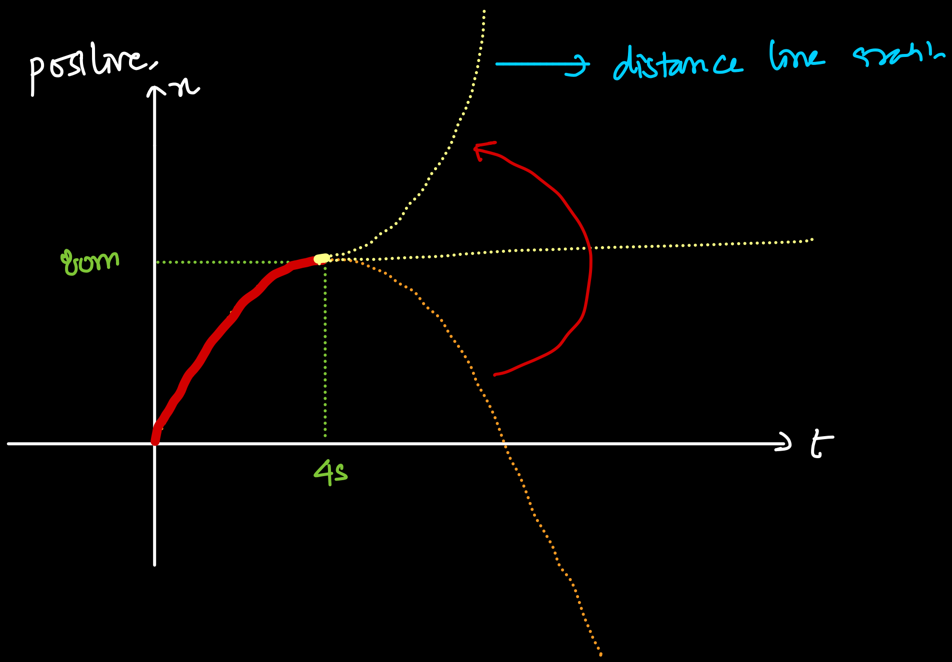
Concave down $\quad \frac{d^2x}{dt^2} < 0 \quad a < 0 \quad a_A < 0$

Concave up $\quad \frac{d^2x}{dt^2} > 0 \quad a > 0 \quad a_B, a_C > 0$

$$a = \frac{d^2x}{dt^2} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d}{dt}(\cos t) = -\sin t$$

Ex A particle is thrown up from $x=0$ with a velocity 40 m/s ($g = 10\ m/s^2$) sketch a rough distance time graph of the motion. Assume upward direction as positive.

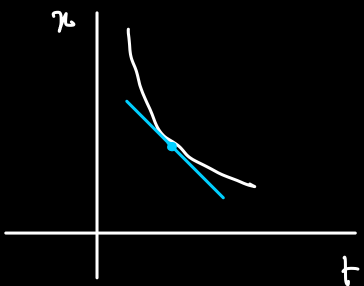
Sol:



Increasing / decreasing speeds

if direction of $\vec{a}$ & $\vec{v}$ are same — Speed inc

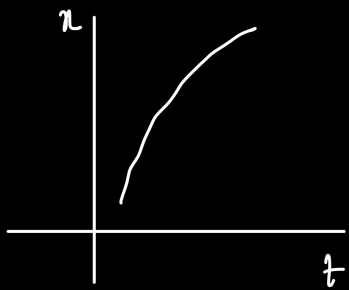
if direction of $\vec{a}$ & $\vec{v}$ are opp — Speed dec



$a > 0$

$v = \frac{dx}{dt} = < 0$

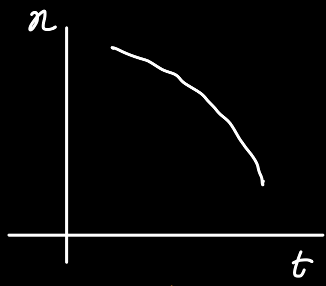
Slow down



$a < 0$

$v = \frac{dx}{dt} = > 0$

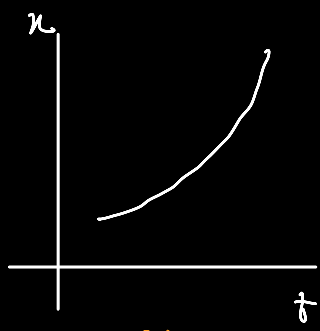
Slow down



$a < 0$

$v = \frac{dx}{dt} = < 0$

Speed up



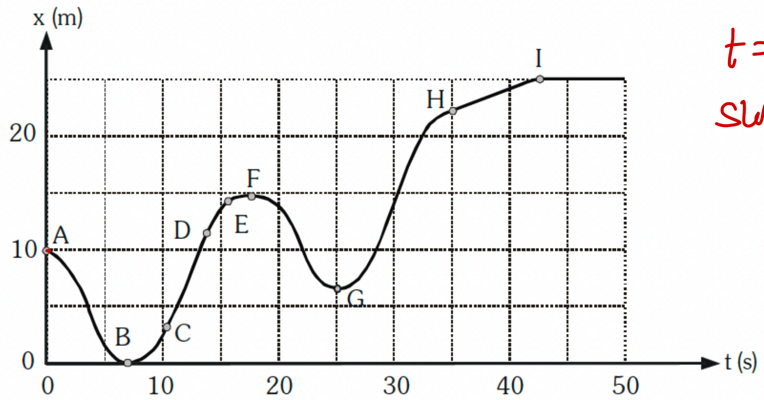
$a > 0$

$v = \frac{dx}{dt} = > 0$

Speed up

Paragraph for Question 4 to 6

An observer records position of a particle moving on a straight-line path at various instants of time. He starts his stopwatch when the particle is passing the point x = 10 m. With the help of these data he prepares the following graph, where position x is shown on the ordinate in meters and time t on the abscissa in seconds.



t=0 velocity is slope of tangent at t=0

. At the instant t = 0,

(A) the particle was moving in the negative x-direction and the observer started his stopwatch.

(B) the particle was moving in the positive x-direction and the observer started his stopwatch.

(C) the particle started its motion from origin with a negative velocity and the observer started his stopwatch.

(D) the particle started its motion with a positive velocity and the observer started his stopwatch.

. Speed of the particle

(A) first increases then decreases in time interval between points B and F

(B) first increases then decreases in time interval between points D and F

(C) always increases between points H and I.

(D) always decreases between points F and G.

. The particle is changing its direction of motion at the instant corresponding to points

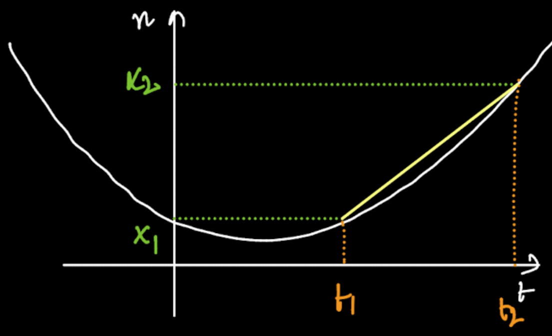
(A) A and E only.

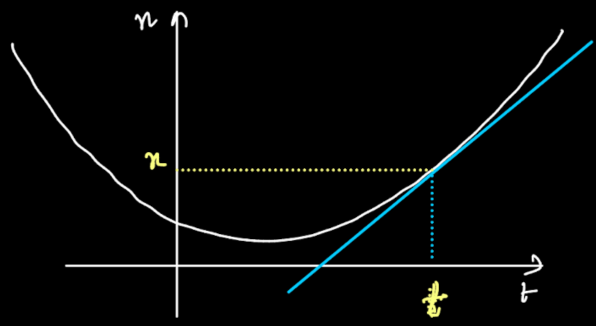
(B) B, F and G only.

(C) C, D, E and H only.

(D) C, D, E, H and I only.

Average / Instantaneous velocity

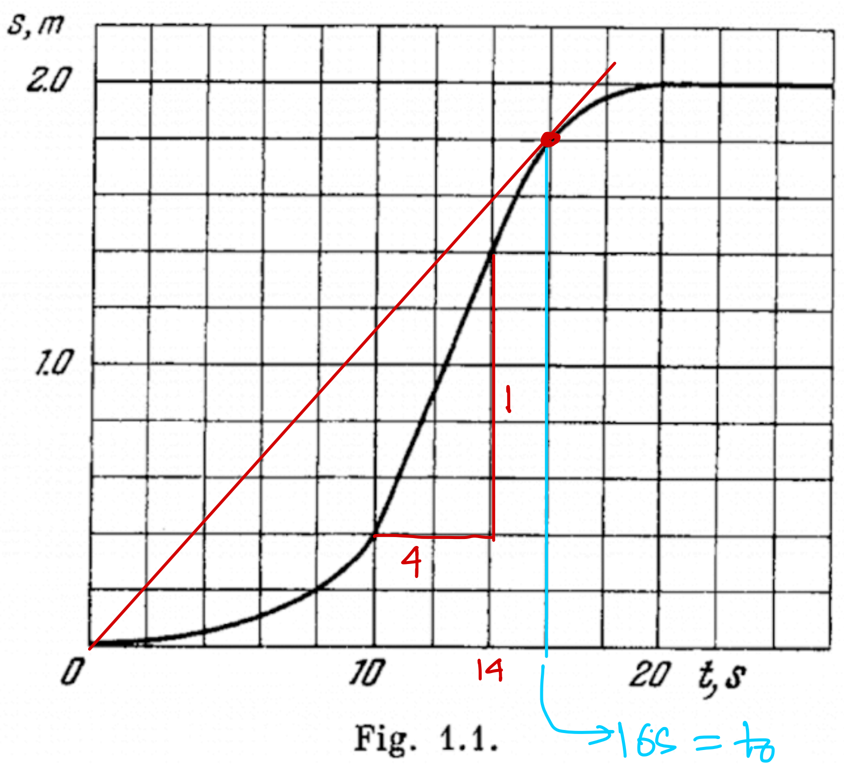




Slope of chord $= \dfrac{x_2 - x_1}{t_2 - t_1} = \vec{V}_{avg}$

Slope of tangent $= \dfrac{dx}{dt} = \vec{v}$

1.4. A point moves rectilinearly in one direction. Fig. 1.1 shows



$V_{max} = \frac{1}{4}$

$= 0.25$ m/s

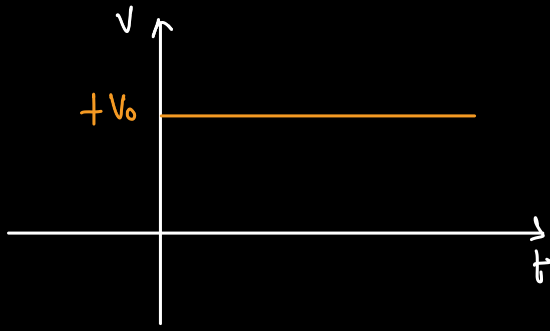
Fig. 1.1.

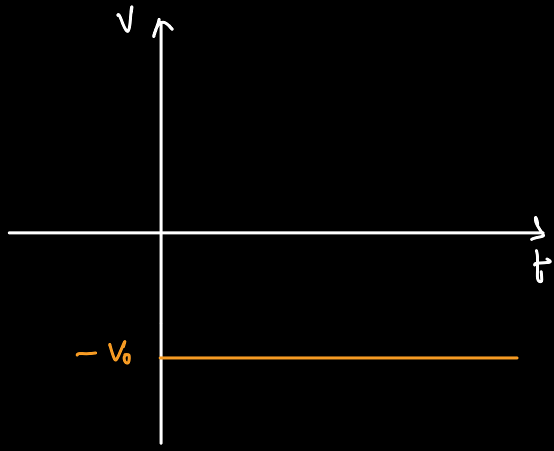
the distance s traversed by the point as a function of the time t. Using the plot find:

(a) the average velocity of the point during the time of motion;
(b) the maximum velocity;
(c) the time moment t_0 at which the instantaneous velocity is equal to the mean velocity averaged over the first t_0 seconds.

Velocity time graph

(i) uniform motion : Velocity is Constant

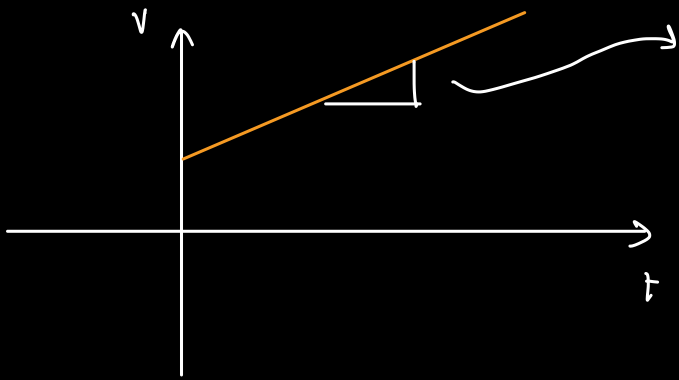




Slope of v-t graph $= \frac{dv}{dt} =$ accelaration

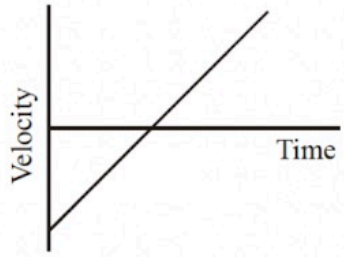
Area under v-t graph $= \int_{t_1}^{t_2} v\,dt =$ displacement

(ii) Motion with Constant acc



Slope $= \frac{dv}{dt} =$ acc $=$ constant

The graph below shows the velocity with respect to time of an object moving in a straight line. The positive direction is to the right and the negative direction is to the left. Which of the following statements best describes the motion of this object?

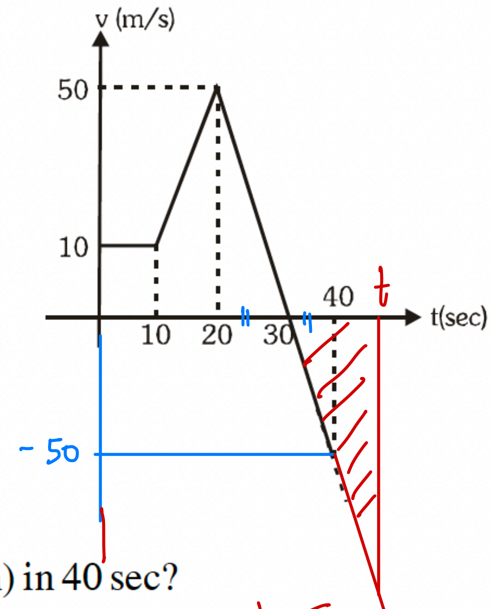


(A) The object starts at a location to the left of the origin and travels at a constant speed toward the right.

(B) The object starts at a location to the left of the origin at a slow speed and speeds up as it moves to the right.

(C) The object slows down as it moves to the left, stops, and starts moving to the right.

(D) The object slows down as it moves to the right, stops, and continues moving to the right.



What is the displacement (in m) in 40 sec?

(A) 450 (B) 500 (C) 400 (D) 600

What is the velocity (in m/s) at t = 12 sec?

(A) 19 (B) 18 (C) 20 (D) 21

For what time in seconds (t > 0) the displacement of the particle is zero?

(A) $2\sqrt{60}+30$ (B) $2\sqrt{63}+30$

(C) $2\sqrt{67}+30$ (D) $2\sqrt{65}+30$

Displacement = Area under graph

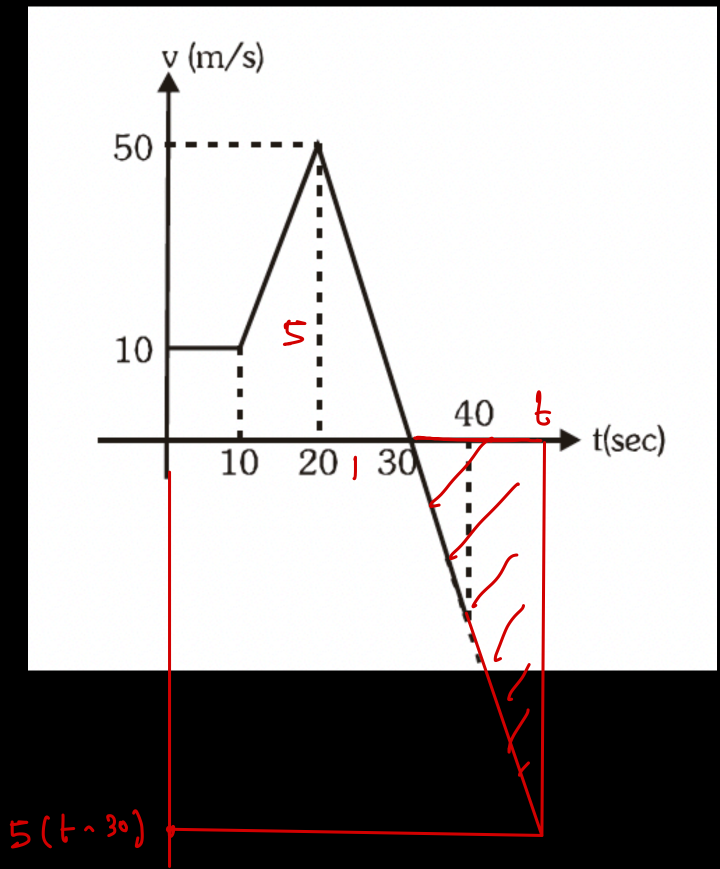
$= 10\times10 + \frac{1}{2}(10+50)\times10 + \frac{1}{2}\times50\times10 - \frac{1}{2}50\times10$

$= 400$

Add Area with proper Signs

$$\text{Distance} = 10\times 10 + \frac{1}{2}(10+50)\times 10 + \frac{1}{2}\times 50\times 10 + \left|-\frac{1}{2}50\times 10\right|$$

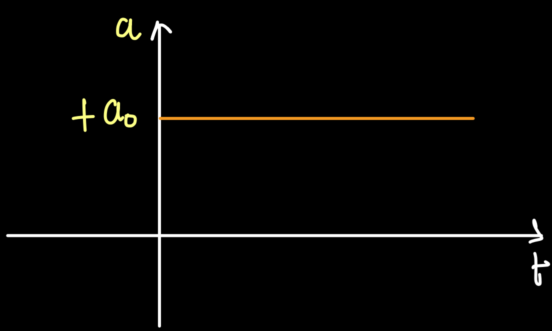
$$= 900\,m$$

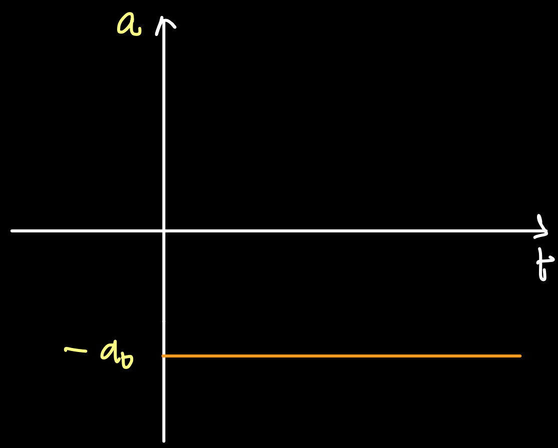


$$\Delta x = 10\times 10 + \frac{1}{2}(10+50)\times 10 + \frac{1}{2}\times 50\times 10 = \frac{1}{2}(t-30)(5(t-30))$$

acceleration – time graph

motion with constant acc



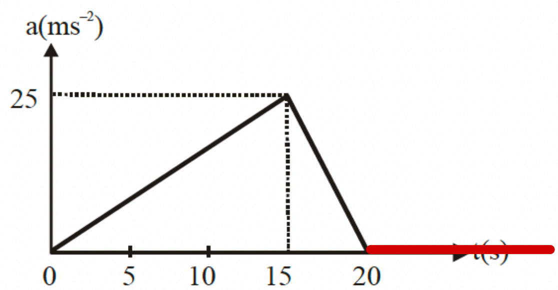


$$\frac{dv}{dt} = a$$

$$\int_{v_1}^{v_2} dv = \int_{t_1}^{t_2} a\,dt$$

$$\boxed{v_2 - v_1 = \text{Area under a-t graph}}$$

Figure shows acceleration versus time graph of a particle starting from rest and moving along a straight line. Which of the following statement(s) is/are correct ?



(A) The maximum velocity attained by the particle is 250 ms^{-1}

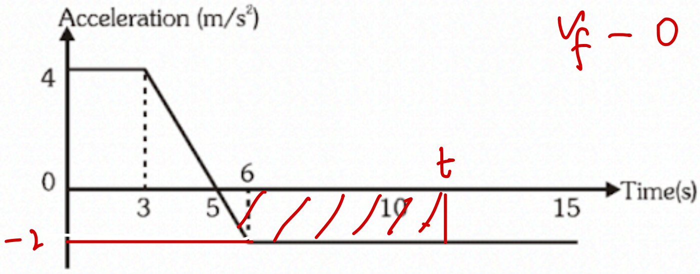
(B) The particle stops nowhere → for $t > 20s$

(C) The particle ~~eventually~~ moves with constant speed.

(D) The body gets retarded during the time interval 15 s < t ≤ 20 s .

$$v_f - \cancel{v_i}^{\,0} = \Delta V = \text{Area under a-t} = \frac{1}{2} \times 20 \times 25 = 250 \text{ m/s}$$

A particle starts from rest from the origin at the instant t = 0 s and moves along the x-axis. During first 15s of its motion, it is subjected to accleration, which varies according to the given graph. After the isntant t = 15 s acceleration vanishes.



$v_f - 0$ = Area under a-t graph

Its speed increases during the time interval or intervals

(A) 0 s to 3 s only
(B) 0 s to 5 s only
(C) 0 s to 3 s and 13.5 s to 15 s
(D) 0 s to 5 s and 13.5 s to 15 s

Its speed decreases during the time interval

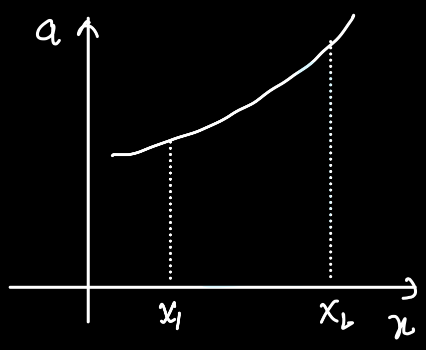
(A) 3 s to 5 s
(B) 3 s to 6 s
(C) 5 s to 6 s
(D) 5 s to 13.5 s

It reverses direction of motion at the instant

(A) t = 3 s
(B) t = 5 s
(C) t = 6 s
(D) t = 13.5 s

$$\frac{1}{2}(5+3)\times 4 = \frac{1}{2}(t-5+t-6)\times 2 \qquad t = 13.5\,s$$

Special graphs



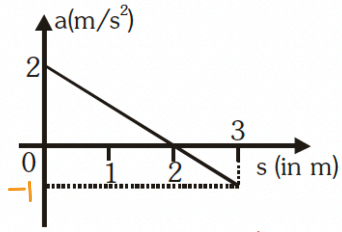
$$a = \frac{dv}{dt} = \frac{dv}{dx}\cdot\frac{dx}{dt}$$

$$a = v\frac{dv}{dx}$$

$$\boxed{a\,dx = v\,dv}$$

$$\text{Area under } a\text{-}x \text{ graph} = \int_{x_1}^{x_2} a\,dx = \int_{v_1}^{v_2} v\,dv = \boxed{\frac{v_2^2 - v_1^2}{2}}$$

The acceleration-displacement graph of a particle moving in x-direction is shown. If the initial speed of the particle is $\vec{u} = -3\hat{i}$ m/s, the velocity of the particle at s=3m is-

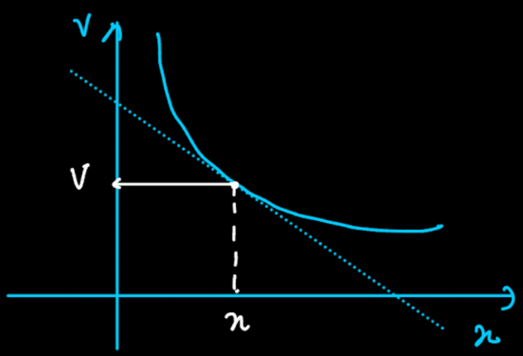


(A) $\sqrt{13}\,\hat{i}$ m/s (B) $\sqrt{14}\,\hat{i}$ m/s (C) $-2\sqrt{3}\,\hat{i}$ m/s (D) $2\sqrt{3}\,\hat{i}$ m/s

$$\text{Area} = \frac{1}{2}\times 2\times 2 - \frac{1}{2}\times 1\times 1 = 3/2 = \frac{v^2 - 3^2}{2}$$

$$V = \sqrt{12} = 2\sqrt{3}\text{ m/s}$$

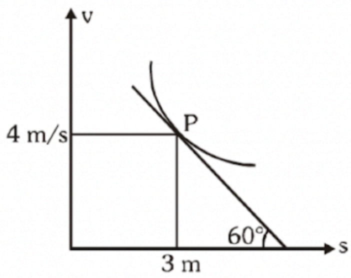
v- x graph



$$a = v\frac{dv}{dx}$$

Slope of v-x graph $\boxed{\dfrac{dv}{dx} = \dfrac{a}{v}}$

A particle is moving along a straight line whose velocity-displacement graph is as shown in figure : A tangent is drawn at point P on the graph. At the point P

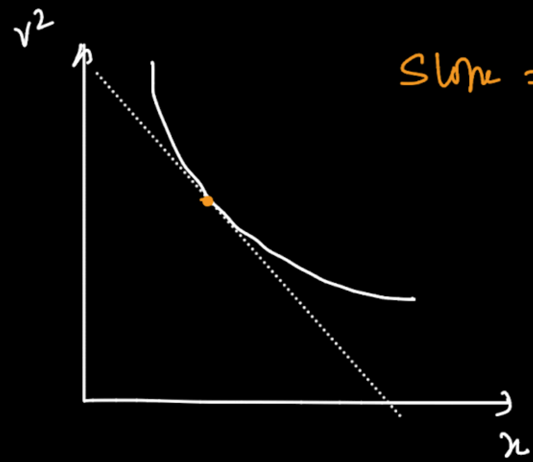


(A) the particle is speeding up
(B) numerical value of velocity and acceleration of the particle are equal
(C) numerical value of velocity is more than the numerical value of acceleration of the particle
(D) numerical value of acceleration is more than the numerical value of velocity of the particle

At P $\frac{dv}{dx} = -\tan 60^\circ = \frac{a}{v}$ a & v have opp Signs

$$\sqrt{3} = \left|\frac{a}{v}\right| > 1$$

$v^2 - x$ graph



Slope $= \frac{d}{dx}(v^2) = 2v\frac{dv}{dx} = 2a$