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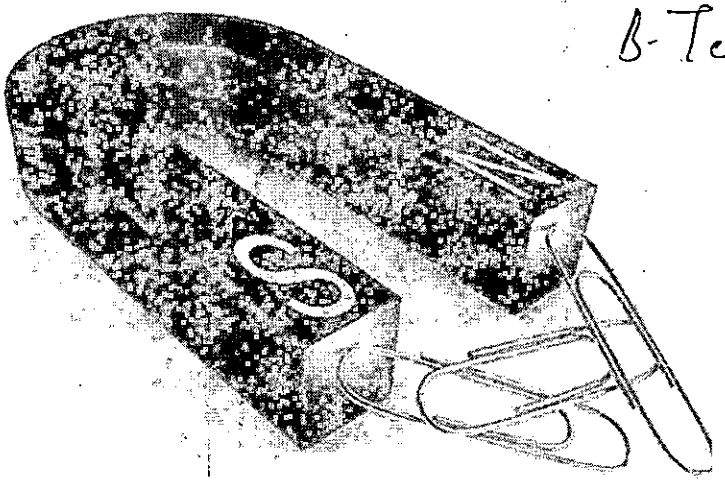

Electricity & Magnetism

for JEE (Main & Advanced)

by:
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Preface

My interaction with students, sincerely preparing for IIT-JEE motivated me to write this book on electricity & magnetism.

Electricity and Magnetism is also as important as Mechanics because in all the competitive examinations, this part is given same or even more weightage than mechanics. Like Mechanics I & II this book "Electricity and Magnetism is also designed to clear the concepts through numerical approach.

This book will help the students in building analytical and quantitative skills, addressing key misconceptions and developing confidence in problem solving.

I sincerely wish that this book will fulfill all the aspirations of the readers. Although utmost full care has been taken to make the book free from error but some errors inadvertently may creep in. Author and Publisher shall be highly obliged if suggestions regarding improvement and errors are pointed out by readers. I am indebted Neeraj Ji for providing me an opportunity to write a book of this magnitude.

I am indebted to my father Sh. Bhavesh Mishra, my mother Smt. Priyamvada Mishra, my wife Manjari, my sister Parul, my little kids Vrishank and Ira for giving their valuable time which I utilized during the writing of this book and people of Moradabad, who supported me throughout my career.

I am also thankful to **Mr. T. Kondala Rao, Mr. Abhishek Sinha (Ranchi), Mr. Sunil Manohar, Mr. S.P. Sharma, Mr. Sudhir Sharma** and **Mr. P. Narendra Reddy** for their valuable suggestions in improving the book.

In the last, I also pay my sincere thanks to all the esteemed members of **Shri Balaji Publications** in bringing out this book in the present form.

Anurag Mishra

GROW GREEN

Save Nature

How to face the challenge ?

Following are some doubts which arise in the mind of almost all the students but may face them by taking some care.

1. *I can not solve numerical because my concepts are not clear.* In fact numerical solving itself is an exercise to learn concepts.

2. *I can not study because I am in depression, I fell into it because I was not studying*! Depression is escape mechanism of people afraid of facing failures. Failure is integral part of learning.

3. *I understand everything in class but can not solve on my own.* WRITING work is vital. It is a multiple activity, initially idea comes in mind then we put into language to express it, we are focussed in hand eye coordination, eyes create visual impression on brain which is recorded there. WRITING WORKS ARE EMBOSSED ON BRAIN LIKE CARVINGS OF AJANTA CAVES.

4. *In exams my brain goes blank, but I can crack them at home.* Home attempt is your second attempt! you are contemplating about it while home back. You do not behave differently in exam you replicate your instincts. Once a fast bowler was bowling no balls. His coach placed a stump on crease, in fear of injury he got it right. CONCEPTUALIZATION, WRITING EQUATION, SOLVING, THEN PROBLEM GETS TO CONCLUSION!

5. *I am an average student.* It is a rationalization used by people afraid of hard work. In their reference frame Newton's first law applies "if I have a misconception I will continue with it unless pushed by an external agent even I will surround him in my web of misconception yielding zero resultant." AVERAGE IS NOT DUE TO CAPACITY LACUANE BUT DUE TO LACK OF DETERMINATION TO SHED INERTNESS.

6. A famous cliche "*I do not have luck in my favour*" PRINCIPLE OF CAUSALITY: CAUSE OF AN EVENT OCCURS IN TIME BEFORE OCCURRENCE OF THAT EVENT *i.e.,* cause occurs first then event occurs. SHINING OF LUCK IS NOT AN INSTANTANEOUS EVENT IT IS PRECEDED BY REL. ITLESS HARD WORK. Sow a seed of aspiration in mind, water it with passion, dedication it will bear fruit, luck can give you sweeter fruit.

Useful tips :

1. Do not take study as a burden actually its a skill like singing and dancing. It has to be honed by proper devotion and dedication.

2. Without strong sense of achievement you can't excel. Before entering the competitive field strong counselling by parents is must. Majority do not know what for they are here. No strategic planning, they behave like a tail ender batting in front of Steyn's bouncers.

3. Science is not a subject based on well laid down procedures or based on learning some facts, it involves very intuitive and exploratory approach. Unless their is desire and passion to learn you can not discover new ideas. It requires patience and hard work, whose fruits may be tangible later on.

4. Some students realize very late that they are studying for acquiring skills and honing them. Their is a feeling that they can ride at the back of instructor and achieve excellence. Study comes as torturous exercise enforced on them and their is some mechanism that can take this burden of them.

5. Science is not about gaining good marks, up to Xth by reading key points good marks are achieved but beyond that only those survive who have genuine interest in learning and exploring. Self study habit is must.

6. IF YOU WANT TO GAIN LEAD START EARLY. Majority of successful students try to finish major portion elementary part of syllabus before they enter Coaching Institute. Due to this their maturity level as compared to others is more they get ample time to adjust with the fast pace. They are less traumatized by the scientific matter handed over. For those who enter fresh must be counselled to not get bullied by early starters but work harder initially within first two months initial edge is neutralized.

7. Once a student lags behind due to some forced or unforced errors his mind begins to play rationalization remarks like I am an average student, my mind is not sharp enough, I have low IQ etc. These words are mechanisms used to avoid hard work. These words are relative terms a person who has early start may be intelligent relative to you.

   Intelligence means cumulative result of hard work of previous years, that hard work has eventually led to a development of instinct to crack things easily.

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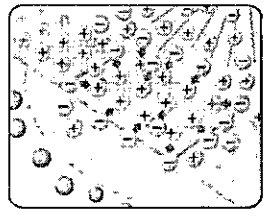

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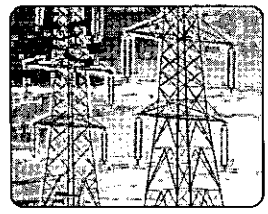

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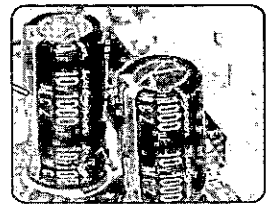

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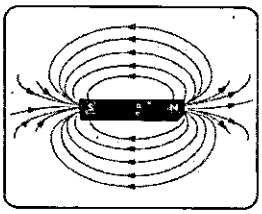

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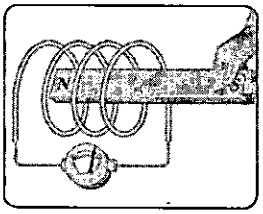

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1 ELECTROSTATICS

ELECTRIC CHARGE

In ancient time the Greeks noticed that amber, a yellow-brown material (fossilized pine tree resin) when rubbed with cloth or fur, it attracts small bits of straw, hair, etc. The word *electric* derives from the Greek word for amber (*electron*). By the mid-seventeenth century it was established that a substance, activated by rubbing, possessed some sort of "amber stuff" or *electricity*. A piece of hard rubber, a glass rod or a plastic comb or ruler after being vigorously rubbed with a towel can pick up small pieces of paper. From our knowledge of the laws of mechanics we conclude that there exists another force, *electric force*. The physical attribute responsible for such electric interaction is **electromagnetic charge** (electric and magnetic phenomena both are manifestations of charge). In this chapter we will study phenomena due to charges that are stationary, called **electrostatics,** or static electricity.

In this chapter, we focus on **the physics of electrostatics, the study of the interactions between electric charges that are at rest (or at most moving very slowly).**

Though the electric and magnetic forces are distinct, they are intimately related. That is why we speak of electromagnetism as a unified subject.

Electromagnetism is one of the parts of physics in which we distinguish between electrostatics and magnetostatics and study the two subjects separately. Then we study the remarkable connection between electricity and magnetism that becomes clear in moving systems.

Finally, we come to see how electricity and magnetism are two manifestations of the same basic phenomenon, electromagnetism.

Some of the basic characteristics of electrostatics are :

1. Physical quantity responsible for static electricity is electric charge.

2. There are two types of charges, one called **positive** and the other called **negative.**

3. Like charges repel, whereas unlike charges attract.

4. **The net amount of charge produced in any process is zero**. Whenever a certain amount of charge is produced on one body in a process, an equal amount of the opposite type of charge is produced on another body. The positive and negative charges are to be treated *algebraically*.

When a plastic ruler is rubbed with a paper towel, the plastic acquires a negative charge and the towel an equal amount of positive charge; the sum of these two charges is zero. If one object or one region of space acquires a positive charge, then an equal amount of negative charge will be established in the neighbouring areas or objects.

There are two types of charges, positive and negative. This fact can be easily verified by a simple experiment. When glass is rubbed with silk, the glass becomes positively charged and the silk negatively charged [Fig. 1.1 (a)]. Since the glass and silk have opposite charges, they attract one another. Two glass rods rubbed in this manner repel one another, since each rod has positive charge on it [Fig. 1.1(b)]. Similarly two silk cloths so rubbed repel each other because both cloths have negative charge [Fig. 1.1(c)].

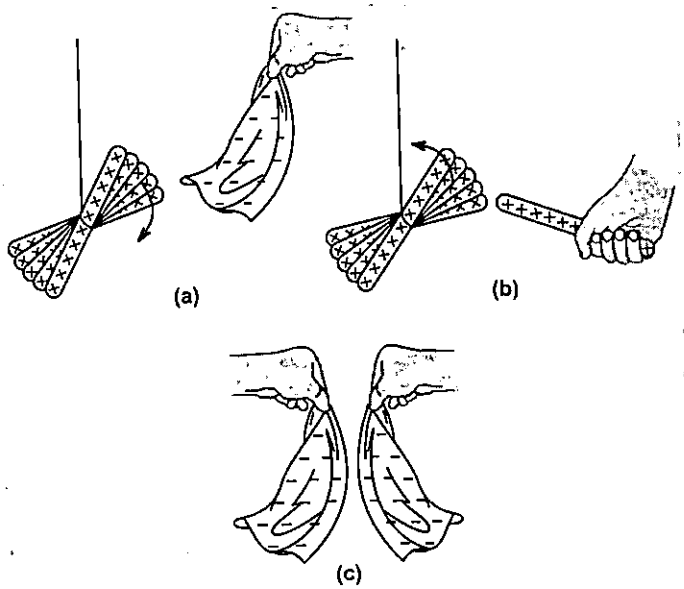


Fig. 1.1

The normal matter is made of atoms, and atoms contain positive and negative charge in equal amounts. An electron is negatively charged; it repels other electrons and it attracts protons which are positively charged.

WHAT IS CHARGE ?

Fig. 1.2 (a) to (d) show certain observations about a charged body

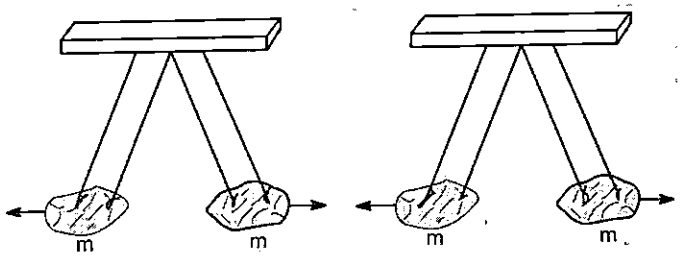


Fig. 1.2 (a) Two pieces of rubbed amber always repel each other, regardless of which way they face each other.

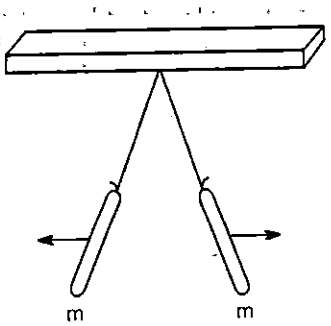


Fig. 1.2 (b)
Two glass rods rubbed with silk repel each other

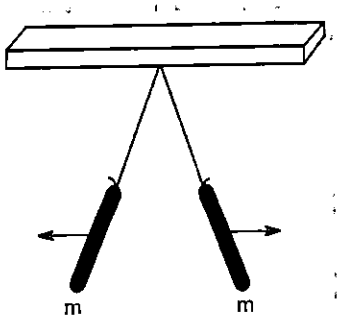


Fig. 1.2 (c)
Two rubber or amber rods rubbed with fur repel each other

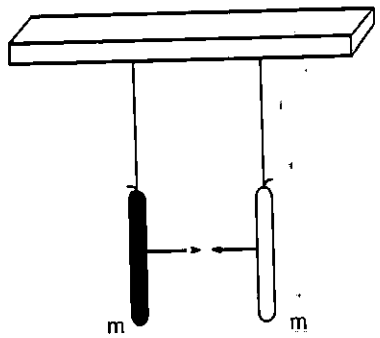


Fig. 1.2 (d)
An electrified glass rod attracts an electrified rubber or amber rod

Two different kinds of electrification properties are needed to explain all of these observations.

All electrical phenomena can be described by assuming only two different kinds of electrification properties. Dufay called the two types of electrification properties *vitreous electricity* (that similar to glass) and *resinous electricity* (that similar to amber). One never observes an electrified material that repels (or attracts) both an electrified glass rod and an electrified rubber rod; such an observation would imply a third type of electrified state or electrification property.

Furthermore, electrical forces are quite distinct from gravitational forces in several respects:

1. Electrical forces between electrified materials are quite apparent even with small pieces of electrified matter, whereas the gravitational force between such small masses is almost negligible and detectable only with the most sensitive types of equipment. Thus electrical forces evidently are intrinsically much stronger than gravitational forces (which are appreciable only when one or both of the masses in huge by laboratory standards).

2. Gravitation is always and only observed as an attractive force, and so we have need for only one kind of mass, positive mass. Electrical forces are observed to be *either* attractive or repulsive, hence the need for two types of electrification property, the vitreous and resinous electrifications of Dufay. The electric and gravitational forces are *similar* in two ways:

(i) Both are observed to be *central forces*. They act along the line connecting point like materials causing the force.

(ii) Both are *conservative forces*. The work done by the force around a closed path is zero (equivalently, the work done by the force along a path connecting any two points in space is independent of the path between the two points).

It was Benjamin Franklin who introduced the two properties of electrification :

A particle or mass is said to be **positively electrified** if it is repelled by a glass rod that has been freshly rubbed with silk.

All glass rods thus rubbed also have positive electrification, since they individually repel each other.

A particle or mass is said to be **negatively electrified** if it is repelled by rubber or amber that has been freshly rubbed with fur.

Hence the rubber or amber itself has a negative electrification property.

The names for the electrification properties are arbitrary. Dufay called them vitreous and resinous electricity; Franklin called them positive and negative electricity. One could have called the two electrification properties Ram and Shyam and *lean* and fat electricity, Jai and Veeru, haired and *bald* electricity, or even *male* and *female* electricity.

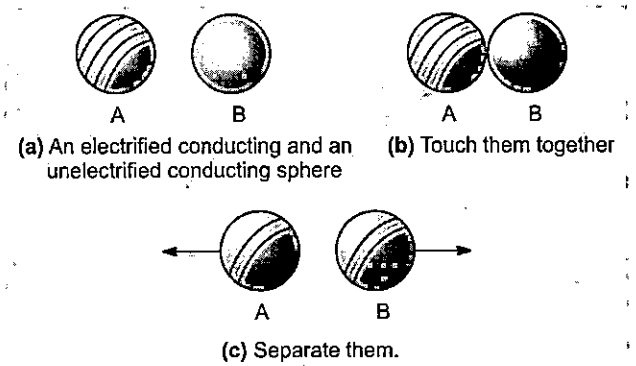
The two electrification properties are easily transferred through and shared among materials. Some materials easily let the electrification property move from one place to another; these materials are called conductors. With other materials the electrical property lacks mobility (at least over short time intervals); these are called insulators. Insulators also are called dielectrics.

We designate the electric charge property of matter symbolically by q (or Q), which may be either a positive or negative *scalar* according to Franklin's convention.

Mass itself quantifies the property we called inertia or resistance to a change in motion. Both concepts thus are defined operationally by experiment. The concept of mass more real and tangible than charge. Mass and charge both are abstractions used to describe the way things in nature behave in certain experiments. The terms mass and charge are our ways of describing the response of a simple or complex system to certain types or classes of experiments.

Thus, when we say an object has a **positive charge**, we mean the object has the electrification property that makes it repelled by a glass rod that has been freshly rubbed with silk Fig. 1.3 (a). When we say an object has a **negative charge**, we mean the object has the electrification property that makes it repelled by a rubber rod that has been freshly rubbed by fur Fig 1.3 (b). This is what we mean by the terms positive of negative charge; fundamental particles nonzero mass are the only kinds of fundamental particles that exhibit electric charge (*i.e.*, can have one or the other electrification property), but that not all particles with mass have nonzero total charge (*e.g.*, the neutron has zero total charge).

If an electrified conducting sphere and an identical but un- electrified conducting sphere [see Fig. 1.3 (a)] are brought into contact [Fig. 1.3 (b)], and then separated [Fig. 1.3 (c)]



(a) An electrified conducting and an unelectrified conducting sphere (b) Touch them together

(c) Separate them.

Fig. 1.3 Transferring Charge

We find that *both spheres* now *are electrified* and repel each other. We can measure the repulsive force that exists between the two spheres at a fixed separation. If two more identical conducting but unelectrified spheres are each now brought into contact with one of the identically electrified spheres, we find that the repulsive force between *any two* of the four electrified spheres, when separated by the same distance, is one-fourth what it was between the original two electrified spheres.

The condition of electrification thus is quantifiable as measured by the forces.

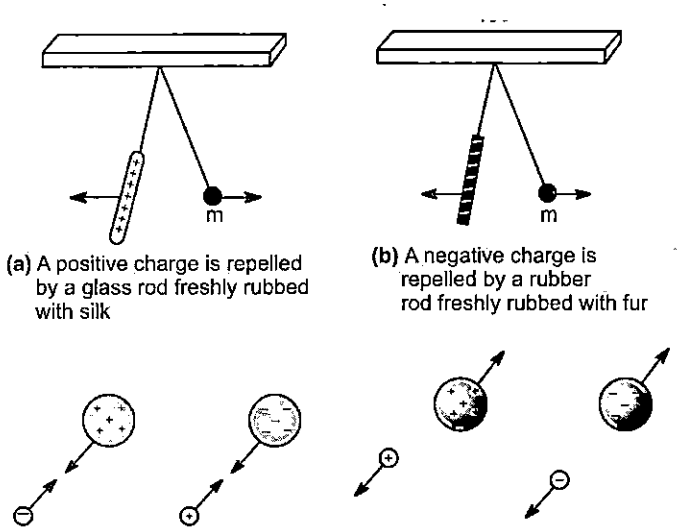
Combining materials having equal amounts of opposite electrification properties exactly cancels their total effectiveness.

The two electrification states or properties thus are quantifiable and behave algebraically and arithmetically as scalars.

Analogously to gravitation, we could call the electrification properties of matter the positive or negative *electrical masses*, but to avoid confusion with gravitation, the name used for the two electrification properties is **electrical charge.**

The word **charge** means to endow with electricity (or the electrification property).

Two particles with the same electrification property either both positive or both negative, will feel repulsive electrical forces of equal magnitude on each. We say the charges **like charges;** this does *not* mean that charges are of equal magnitude, only that they have the same type of electrification property. If the two particles have opposite electrification (one positive, one negative), we say the charges are **unlike charges;** these produce attractive electrical forces of equal magnitude on each other.



(a) A positive charge is repelled by a glass rod freshly rubbed with silk

(b) A negative charge is repelled by a rubber rod freshly rubbed with fur

(c) Unlike charges attract each other with forces of equal magnitude

(d) Like charges repel each other with forces of equal magnitude

Fig. 1.4

The electrical forces that two charged masses exert on each other are of equal magnitude and opposite in direction, *regardless* of the quantity of charge each has, in accordance with Newton's third law of motion.

These experimental results (and others like them) lead us to the following general rules:

1. When two bodies are rubbed together, it is not possible to create electric charge on only one of them. Either neither is charged or both are.

2. Two bodies rubbed together and thus charged exert *attractive* forces on one another.

3. When two bodies made of the same material are charged in the same way (say, by rubbing with the same kind of cloth), each exerts a *repulsive* force on the other.

4. The force exerted by a charged body on an uncharged body is always *attractive*.

Fig. 1.5 shows a simple model of an atom with negative electrons orbiting the nucleus.

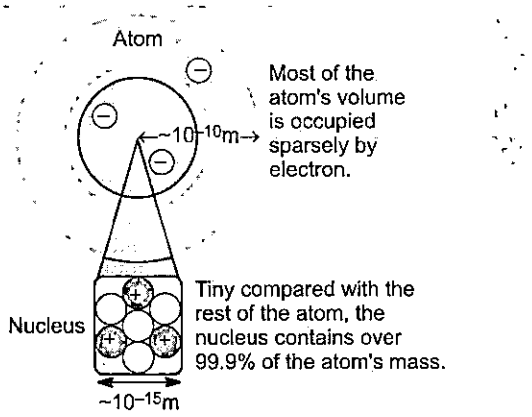


Fig. 1.5 The structure of an atom

The nucleus is positive due to the presence of positive protons. The nucleus also contains a neutral particle neutron whose properties will be discussed in later chapters. Other charge particles are observed in cosmic rays and nuclear decay, which are very short lived. The charges of electron and proton are identical in magnitude but opposite in sign. All the charges in nature are integral multiples of a basic unit of charge. All the charges are formed by combinations of electrons and protons.

The magnitude of this basic charge is

$$|q_e| = 1.60 \times 10^{-19}\,\text{C}$$

The symbol q is commonly used for charge. The SI unit of charge is **Coulomb** (C).

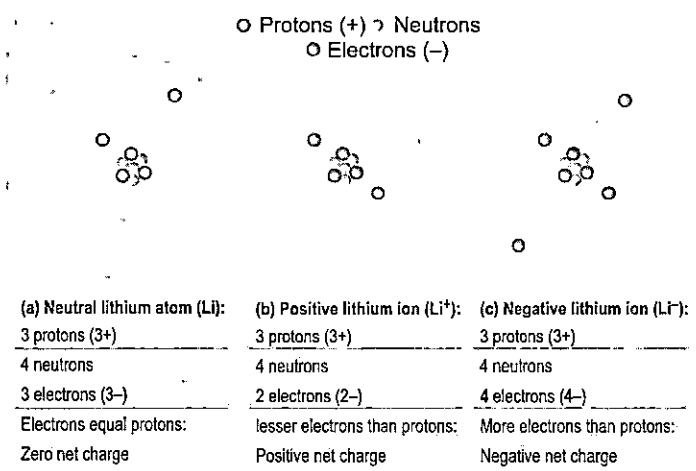


Fig. 1.6

The negative charge of the electron has (within experimental error) *exactly* the same magnitude as the positive charge of the proton. In a neutral atom the number of electrons equals the number of protons in the nucleus, and the net electric charge (the algebraic sum of all the charges) is exactly zero [Fig. 1.6(a)]. The number of protons or electrons in a neutral atom of an element is called the **atomic number** of the element. If one or more electrons are removed, the remaining positively charged structure is called a **positive ion** [Fig. 1.6(b)]. A **negative ion** is an atom that has *gained* one or more electrons [Fig. 1.6 (c)]. This gaining or losing of electrons is called **ionization.**

Following a convention established by Benjamin Franklin, we call the two kinds of charge **positive** and **negative.** When the two are present in a body in equal amounts, a body is electrically **neutral;** that is, it does not exhibit the behavior of charged bodies described in the preceding list. If an excess of either positive or negative charge is present, the body is **positively** or **negatively charged** and does exhibit such behavior. Which kind of charge we call positive and which negative is entirely arbitrary, but Franklin's convention has been universally adopted: When a glass rod is rubbed with a silk cloth, we say that the glass becomes *positively* charged and the silk becomes *negatively* charged. **Rubbing results in a net transfer of one kind of charge from one body to the other. This leaves one of the bodies with an excess of positive charge and the other body with an excess of negative charge. Implicit in this picture is an important point: *The charging process neither creates nor destroys charge; it only redistributes it.* This is one statement of the principle of conservation of charge.**

According to contemporary physicists most of heavy subatomic particles are actually composite systems made up of several varieties of smaller fundamental entities called quarks. These are supposed to have charges or $\pm\frac{1}{3}q_e$ and $\pm\frac{2}{3}q_e$. It is believed that quarks cannot ordinary exist in the free state, so the observable unit of charge is indeed q_e, the electronic charge.

CHARGING BY RUBBING

Most of us have the common experience that a plastic wrap clings to a container, a rubbed balloon on a shirt sticks to the wall, the cloths cling in a dryer.

Atoms are neutral and they have equal number of electrons and protons. But the outer electrons are the least strongly bound and they can be easily shed. The process of transfer is not entirely understood till date. Different materials have different affinities for electrons. When two substances are in close contact one of them may give up some of its loose electrons while the other may accept them. When a plastic sheet is pressed down onto a metal plate, electrons will be transferred from the donor plastic to the acceptor metal. The plastic which has lost electrons now contains a number of immobile positive ions on its surface and has become charged. The positive plastic attracts the negative metal and the two cling to each other.

When a hard rubber rod is stroked with a piece of fur, the rod draws off electrons, becoming negatively charged, and the donor fur becomes equally positively charged. *The rubbing mainly increases the area of contact between the bodies.*

A substance that can attract electrons form a material may serve as a donor in another situation. Glass rubbed with asbestos draws off electrons from the fibrous material, becoming negative, but if stroked with persistence against silk or flannel, the glass will emerge positively charged having lost electrons.

The table known as *triboelectric* sequence shows the behaviour of various materials.

When a negatively charged object (that has an excess of electrons) is placed in contact with a neutral body, some of these electrons are transferred to the neutral body, charging it negatively. Similarly, a positively charged body has a deficiency of electrons or an excess of positive ions. When placed in contact with a neutral body it attracts and draws off electrons becoming less positive, while the neutral body becomes positive due to loss of electrons! Only electrons are transferred but the system behaves exactly as though positive charge is being transferred from the charged one to the neutral one.

INSULATORS AND CONDUCTORS

Substances which do not allow the charges to move through them are called **insulators, nonconductors, dielectrics.** The electrons and ions in insulator are bound in place; they have limited mobility. They will move only when their mutual repulsion is great enough to overcome the tendency to be held in place by the host atoms.

The charges received by insulators are confined within two regions in which it was introduced.

When a charge is introduced anywhere within a **conductor** it *allows the charges to flow freely and redistribute.*

The conductors and insulators are distinguished by the relative mobility of charge within the material. In metal atoms the outermost electrons are weakly held, so a bulk sample contains a tremendous number of free electrons.

Pure water and dry table salt are insulators whereas molten salt and salty water are conductors. Air is good insulator even though it contains some 300 ions per cubic centimetre. If large negative charge builds up on an object, the mutual repulsion may propel them into the surrounding air. The air will have some of its own electrons removed from it, becoming ionized. The ionized space may create a temporary conductive pathway along which the bulk of the charge then flows. Collisions with the gas increase its

temperature and cause some of the atoms to emit light, known as **spark.**

When a charge is introduced on a conductor, under mutual repulsion they move until they are as distant from one another as possible. **Irrespective of the shape of the conductor excess charge always resides on its outer surface.** On a metal sphere the charges are uniformly distributed and at rest on the outer surface. With a non-spherical conductor the charge distribution will be non-uniform.

Concept : *insulator = Dielectric*

1. The use of the word insulator as a synonym for dielectric is based on Gray's demonstration. The insulator isolates the conductor from the out side world by preventing the flow of charge onto or off the conducting body, analogous to the way a thermal insulator isolates a body from the outside world by preventing the flow of heat into or out of it.

2. It is easy to see why an ***electrical conductor****—a body made of a substance that conducts electric charge—cannot be charged by rubbing in the ordinary way. Suppose, for example, that you hold a lump of copper in one hand and rub it against a piece of glass held in your other hand. Electric charge that is transferred to the copper surface where it rubs against the glass can flow away through the copper and then through your body (also a conductor) to the ground, thus dispersing so widely that no electric force can be detected. Indeed, Gray showed that conductors could be charged by rubbing, if they were supported on good dielectrics.*

3. The difference between dielectrics and conductors has to do with the mobility of the charge, which is a property of the particular substance. In conductors, at least one kind of charge can flow freely. In dielectrics, both kinds of charge are bound in place and can flow only with difficulty. Although no dielectric is perfect, there is a factor of roughly 10^{20} *between the rates at which charge flows through a conductor such as copper and through a dielectric such as glass.*

CHARGING BY CONTACT

Consider a negative conductor made to touch an uncharged metal body. Electrons are transferred onto the neutral body by their mutual repulsion, which depends on how densely packed the charges were initially. The charge flows similar to fluid flow from a filled container into a connecting container of arbitrary shape. The gravity driven flow continues until the liquid levels are the same, the pressures equalize and equilibrium is reached. Similarly if a total excess charge Q is placed on one of two identical metal spheres and those spheres are brought into contact and then separated, a charge $(1/2)Q$ will end upon each of them (Fig. 1.7).

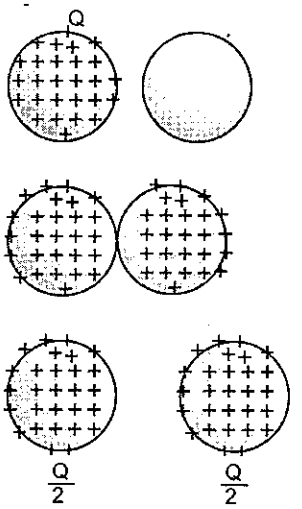


Fig. 1.7

An **electroscope** is an instrument that can be used for detecting charge (Fig. 1.8). It is made with gold foil leaves hung from a metal stem and is insulated from the air in a glass-walled container.

A positively charged glass is brought into contact with an electroscope [Fig. 1.8(a)]. The glass rod is an insulator, so it must be brought into contact with the electroscope to transfer charge to or from it. The electrons can move in metals; they are attracted to the top of the electroscope and some are transferred to the positive rod by touch, which leaves the electroscope positively charged.

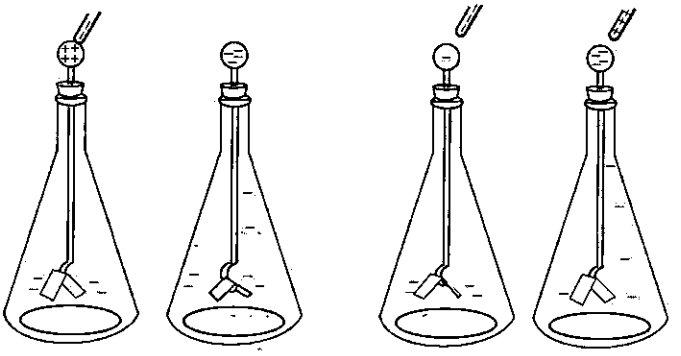


Fig. 1.8

The positively charged leaves of the electroscope separate against the force of gravity. Similarly the electroscope can be negatively charged by contact with a negatively charged object [Fig. 1.8(b)].

CHARGING BY INDUCTION

Consider a positively charged object brought close to

neutral metal rod, without any contact. The electrons in the metal can move within the metal towards the positively

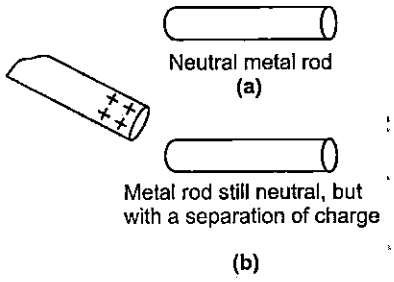


Fig. 1.9: Inducting a charge on an object connected to ground

charged object (electrostatic attraction) which leaves a positive charge at the opposite end. A charge is said to be **induced** at the two ends of the metal rod. Note that no net charge has been created in the rod. A charge redistribution has taken place. The net charge on the metal rod is still zero. If we break the rod in the middle we could have two charged objects, one charged positively and the other negatively. **Charge separation in electrically neutral materials caused by presence of another nearly charged object, is called electrical polarization.**

We take a fixed negatively charged object near an uncharged conductor. The uncharged conductor is polarized. The electrons move to the far end of the conductor, since they are repelled by the negatively charged object. Next we connect the conductor with conducting wire to the ground. Connecting a conductor to Earth by means of another conductor is called **grounding** the conductor, or **earthing.** The Earth is gigantic and can conduct, can easily give up or accept electrons; hence it acts like a reservoir of charge. The electrons in the conductor repelled by negatively charged object can escape to Earth. This leaves the conductor positively charged. If the wire is now removed

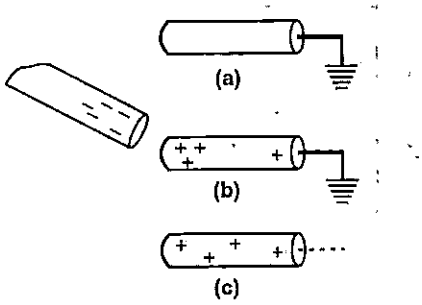


Fig. 1.10

the electrons cannot get back into the conductor from which they escaped earlier. So the conductor now has a positive charge and is no longer electrically neutral. The negatively charged object can be removed and the initially neutral conductor remains positively charged. This process is called **charging by induction.**

The Earth is a conductor because of the presence of metal ions and moisture. The Earth is large enough that for many purposes it may be thought of as a limitless reservoir of charge.

To ground a conductor means to provide a conducting path between it and the ground (or to another charge reservoir). A charged conductor and onto the Earth.

A buildup of even a relatively small amount of charge on a truck that delivers could be dangerous—a spark could trigger an explosion. To prevent such a charge buildup, the truck grounds its tank before starting to deliver petrol to the service station.

The third opening of modern electrical outlets is called *ground*. It is literally connected by a conducting wire to the ground, either through a metal rod driven into the Earth or through underground metal water pipes. The purpose of the ground connection is that it prevents static charges from building up on the conductor that is grounded.

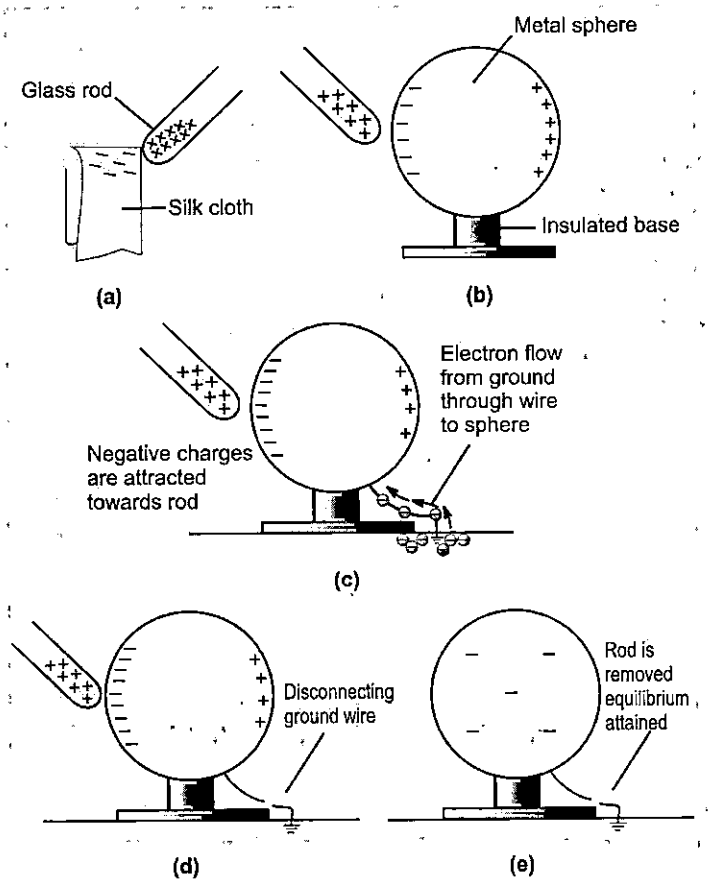


Fig. 1.11: The symbol ⏚ represents a connection to ground. Charging by induction. **(a)** A glass rod is charged by rubbing it with silk. **(b)** The positively charged glass rod is held near a metal sphere, but does not touch it. The sphere is polarized as free electrons within the sphere are attracted toward the glass rod. **(c)** When the sphere is grounded. electrons from the ground move onto the sphere, attracted there by positive charges on the sphere **(d)** The ground connection is broken without moving the glass rod. **(e)** Now the glass rod is removed. Charge spread over the metal surface as the like charges repel each other. The sphere is left with a net negative charge because of the excess electrons.

In case of insulator the electrons are not free for macroscopic charge separation. In this case local

polarization occurs only on local atomic or molecular scale. When a charged object is brought near an insulator the distribution of charge in atoms and molecules is shifted slightly. Opposite charge is attracted nearer the external charged object while two like charges are repelled. The small shift of charge position for each atom or molecule is balanced out by the same effect on all its nearest neighbours. But at the surface the cancellation is not complete. The excess of positive or negative charge over the near and far surfaces gives a resultant force of attraction towards the charged object.

The **polar molecules** like water have natural separation of charge, although they are neutral overall. Polar molecules show greater polarization effects than the molecules with uniform charge distribution.

POLARIZATION

An electrically neutral object may have regions of positive and negative charge within it, separated from one another. Such an object is **polarized.** A polarized object can experience an electric force even though its net charge is zero. A rubber rod charged negatively after being rubbed with fur attracts small bits of paper. So does a glass rod that is *positively* charged after being rubbed with silk [Fig. 1.12]. The bits of paper are electrically neutral, but a charged rod polarizes the paper—it attracts the unlike charge in the paper a bit closer and pushes the like charge in the paper a bit farther away [Fig. 1.12(c)]. The attraction between the rod and the unlike charge then becomes a little stronger than the repulsion between the rod and the like charge, since the electrical force gets weaker as the separation increases and the like charge is farther away. Thus, the net force on the paper is always attractive, regardless of the sign of charge on the rod.

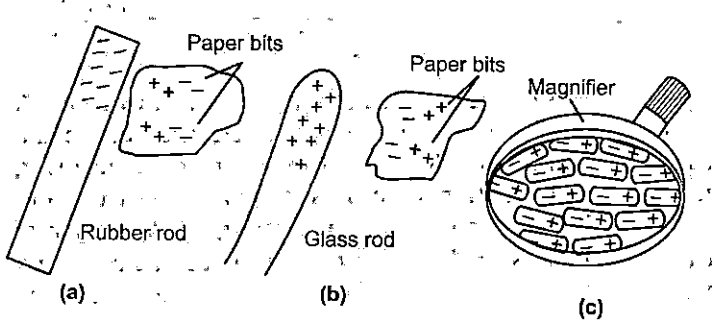


Fig. 1.12

In this case, we say that the paper is *polarized by induction*; the polarization of the paper is induced by the charge on the nearby rod. When the rod is moved away, the paper is no longer polarized. Some objects, including some molecules, are intrinsically polarized. An electrically neutral water molecule, for example, equal amounts of positive and negative charge (10 protons and 10 electrons), but the center of positive charge and the center of negative charge do not coincide. The electrons in the molecule are shared in such a way that the oxygen end of the molecule has a negative charge, while the hydrogen atoms are positive.

On a dry day, run a comb through your hair or rub the comb on a wool sweater., go to a sink and turn the water on so that a thin stream of water comes out. It does not matter if the stream breaks up into droplets near the bottom. Hold the charged comb near the stream of water. You should see that the water experiences a force due to the charge on the comb. Is the force attractive or repulsive? Does this mean that the water coming from the tap has a net charge? Explain your observations.

Ordinary tape has an adhesive that allows it to stick to paper and many other materials. Since the sticking force is electrical in nature,. If you have ever peeled a roll of tape too quickly and noticed that the strip of tape curls around and behaves strangely, the strip of tape has a net charge (and so does the tape left behind, but of opposite sign). Tape pulled *slowly* off a surface does not tend to have a net charge.

A similar phenomenon occurs on a dry day when you walk across a carpeted room wearing rubber-soled shoes. Charge is transferred between the carpet and your shoes and between your shoes and your body. Some of the charge you have accumulated may be unintentionally transferred from your fingertips to a doorknob or to a friend—accompanied by the sensation of a shock.

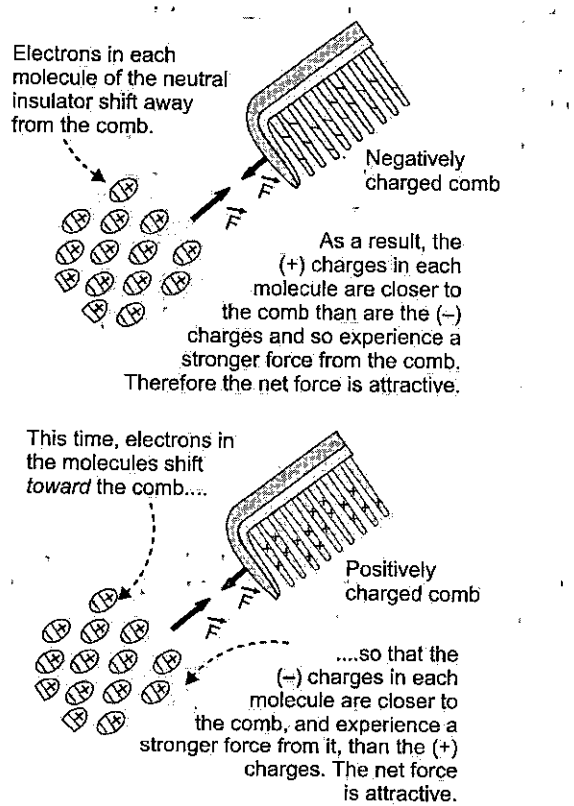


Fig. 1.13

PROPERTIES OF CHARGE

1. Charge: *It is the property associated with matter due to which it produces and experiences electrical and magnetic effects. The excess or deficiency of electrons in a body gives the*

concept of charge. So a negatively charged body means that the body has gained electrons while a positively charged body means that the body has lost some of its electrons.

2. $1\text{ coulomb} = 3\times10^9$ esu of charge

$$= \frac{1}{10}\text{ emu of charge}$$

3. Charge is transferable: If a charged body is put in contact with an uncharged body, transfer of electrons from one body to the other may take place. If the charged body is positive it will withdraw some electrons from the uncharged body and if negative will transfer some of its excess electrons to the uncharged body. Whole of the charge cannot be transferred by conduction from one body to the other. Actually the flow of charge stops when both acquire same potential.

4. Charge is always associated with mass: The particles such as *photon* or *neutrino* which have no (rest) mass can never have a charge. *The presence of charge itself is a convincing proof of existence of mass.* β-rays, canal rays or cosmic rays gets deflected by electric or magnetic field will be charged and hence will consist of particles. However if electric or magnetic field does not deflect a beam, the beam may consist of neutral particles such as atoms or neutrons or electromagnetic waves.

5. *In charging,* if electrons are removed from the body, the mass of the body will decrease and the body will become positively charged.On the other hand, if electrons are added to a body, the mass of the body will increase and the body will acquire a net negative charge.

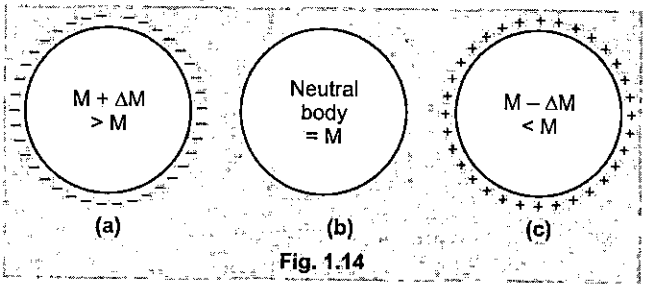


Fig. 1.14

6. Charge is quantised: When a physical quantity can have only discrete values rather than any value, the quantity is said to be quantised.

Millikan oil drop experiment have established that *the smallest charge that can exist in nature is the charge of an electron.* If the charge of an electron ($=1.6\times10^{-19}$C) is taken as the elementary unit, *i. e.*, quanta of charge, and is denoted by *e*; the charge on any body will be some integral multiple of *e*, *i. e.*,

$$q = \pm ne \qquad \text{with} \qquad n = 1, 2, \ldots$$

Charge on a body can never be $(2/3)e$, $14.5e$ or $10^{-5}e$, etc.

7. Charge is conserved: *In a isolated system, total charge does not change with time. In pair production* and *annihilation* neither mass nor energy is conserved separately but (mass + energy) is conserved. In pair production energy is converted into mass while in annihilation mass is converted into energy. In pair production presence of nucleus is a must to conserve momentum. In absence of nucleus both energy and momentum will not be conserved simultaneously and hence the process cannot take place.

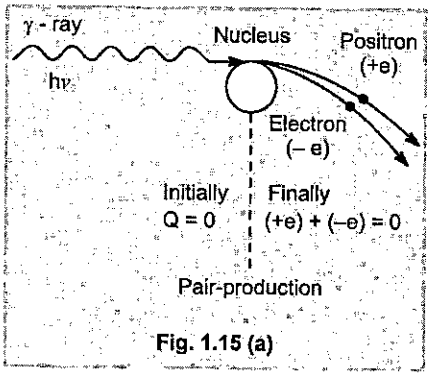


Fig. 1.15 (a)

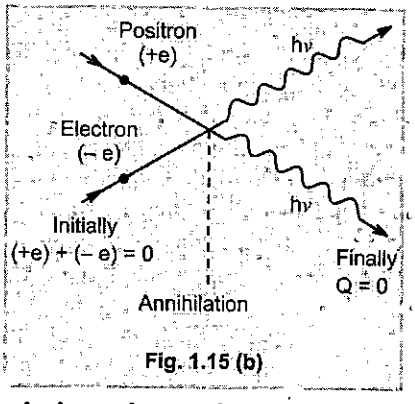


Fig. 1.15 (b)

8. Charge is invariant: This means that charge like *phase* is independent of frame or reference, *i. e., charge on a body does not change whatever be its speed. Charge density* or *mass* of a body depends on its speed and increases with increase in speed.

9. Accelerated charge radiates energy: A charged particle at rest produces only electric field in the space surrounding it. However, if the charged particle is in unaccelerated motion,it produces both electric and magnetic fields but does not radiate energy. *If the motion of charged particle is accelerated, it not only produces electric and magnetic fields but also radiates energy* in the space surrounding the charge in the form of electromagnetic waves.

10. *Similar charges repel each other while dissimilar attract. True test of electrification is repulsion and not attraction* as attraction may also take place between a charged and an uncharged body.

11. If a charged body is brought near a neutral body, the charged body will attract opposite charge and repel similar charge present in the neutral body. As a result of this one side of the neutral body becomes positively charged while the other negative. This process is called *Electrostatic induction*. Charging a body by induction is depicted in Fig. 1.16.

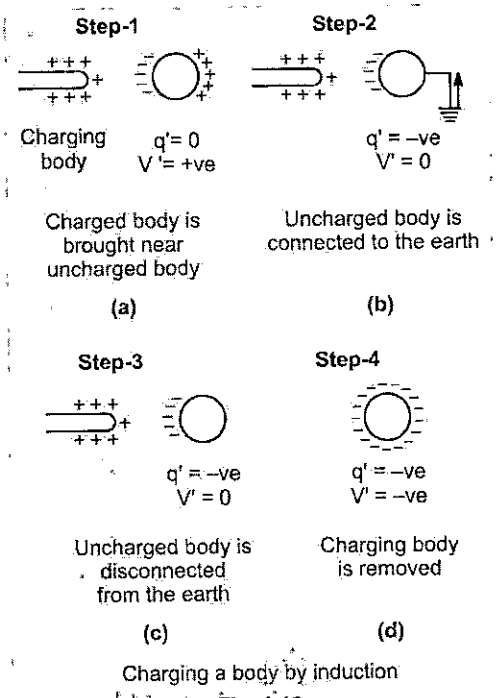


Fig. 1.16

Electroscope

An electroscope is built by attaching two pieces of aluminium foil to a conducting rod and mounting the assembly in an insulated container, as in Fig. 1.17 (a)

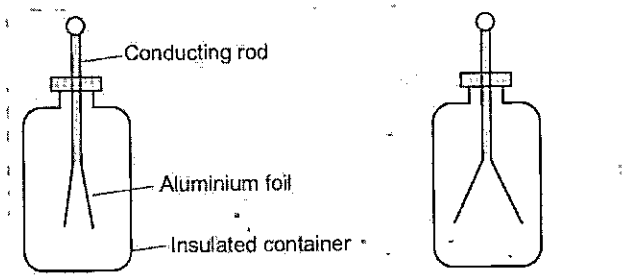


Fig. 1.17

Since there is initially no charge on the rod and foil, the two foil pieces will not exert any electrical forces on each other and they will both hang down vertically. If positive charge approaches the top of the rod, then negative charge will be attracted to the top of the rod (or equivalently, positive charge will appear to be repelled down the rod to the two foil pieces). Each of the foil pieces will become positively charged and they will repel each other. They will come to static equilibrium with the force of the earth's gravity by moving apart as in Fig. 1.17 (b).

The same will be true for an approaching negative charge except that each foil piece will now be negatively charged.

If the rod and foils are initially charged with positive charge, then this positive charge will be distributed over all the surfaces, including the foils. Each foil will therefore be positively charged, and the two foils will initially move apart as in Fig. 1.17(b). If one approaches the top of the rod with a positive charge, then more positive charge will be pushed to the foils, the electrical force will increase and the foils will move further apart. If one approaches the top of the rod with negative charge, some of the positive charge on the foils will be attracted to the top, and the charge on the foils will be reduced. Then the electrical force decreases, and the foils will move closer together. We see that this is a sensitive means of determining the sign of the charge on some materials, without transferring any of the charge from that material.

Charge can be detected and measured with the help of many instrument like *gold-leaf electroscope, electrometer, voltameter* or *ballistic-galvanometer* but here we refer a gold-leaf electroscope:

(a) If a charged body is brought near uncharged electroscope, charge on the disc of electroscope will be opposite to that of body while on leaves similar to that of body [Fig 1.18 (a)] and leaves will diverge.

(b) If an uncharged electroscope is touched by a charged body, disc and leaves both acquire charge similar to that of body and leaves will diverge [Fig. 1.18 (b)]

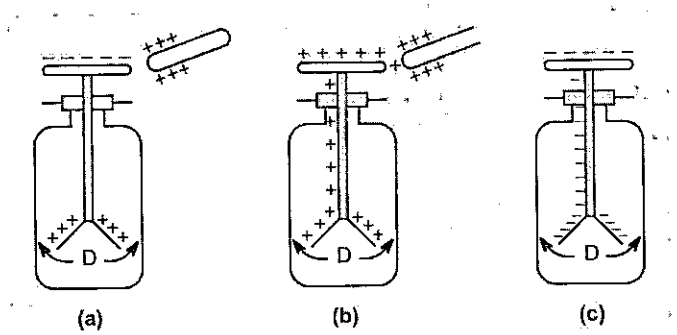


Fig. 1.18

(c) If electroscope is '*charged by induction*', disc and leaves both will acquire charge opposite to that of inducing body and leaves will diverge. In Fig. 1.18 (c) the electroscope is charged by induction using a positively charged body.

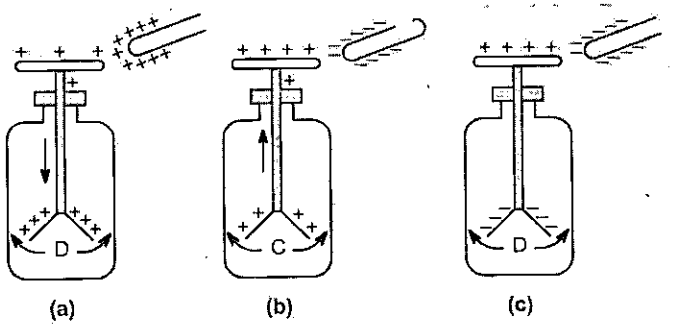


Fig. 1.19

(d) If a charged body is brought near a charged electroscope, the leaves will further diverge if the charge on

the body is similar to that on the electroscope and will usually converge if opposite.

(e) What happens if X-rays are incident on a charged electroscope. X-rays are electromagnetic radiation, photons of sufficient energy may ionize an atom. Due to ionisation of air by X-rays the electroscope will get discharged and hence its leaves will collapse. However, if the electroscope is evacuated, X-rays will cause photoelectric effect with gold and so the leaves will further diverge if it is positively charged (or uncharged) and will converge if it is negatively charged.

Conceptual Example 1. *Can ever the whole charge of a body be transferred to the other? If yes, how and if not, why?*

Solution. Yes; if the charged body is enclosed by a conducting body and connected to it, the whole charge will be transferred to the conducting body, as charge resides on the outer surface of a conductor.

Conceptual Example 2. *Can two similarly charged bodies ever attract each other?*

Solution: Yes; when the charge on one body (Q) is much greater than that on the other (q) and they are close enough to each other so that force of attraction between Q and induced charge on the other exceeds the force of repulsion between Q and q.

Note: If the charges are point, no induction will take place and, hence, two similar *'point charges'* can never attract each other.

COULOMB'S LAW

The French physicist Charles Coulomb (1736-1806) investigated electric forces using a torsional balance. This device has a beam suspended horizontally from a vertical fibre (Fig. 1.20). On one end of the beam is a sphere that can be charged by bringing it into contact with a charged object. The beam is balanced by a counter weight on the other end. When a charged sphere is brought near the sphere the electrostatic force causes the beam to rotate. By proper calibration the force necessary to cause a given rotation was known. From this experiment Coulomb concluded that :

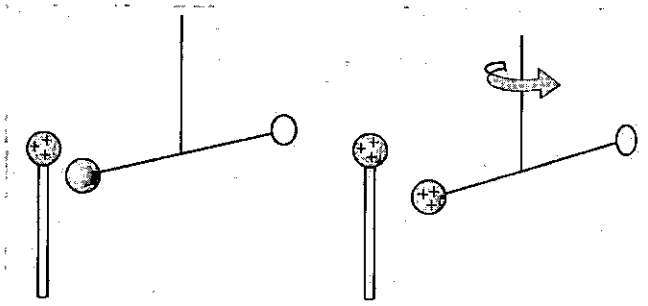


Fig. 1.20

(1) The magnitude of the force is proportional to the product of the magnitudes of the charges q_1 and q_2.

(2) The magnitude of the force is inversely proportional to the square of the distance, separating the charges.

(3) The direction of force is always along the line joining the two charges.

(4) The force is repulsive if the charges have the same sign and attractive if their signs are opposite.

Mathematically, Coulomb's law can be written as :

$$F_{12} = k\frac{q_1 q_2}{r^2}$$

where k is a proportionality constant.

In SI units k has the value

$$k = 8.988 \times 10^9 \text{ N m}^2/\text{C}^2$$
$$\simeq 9.0 \times 10^9 \text{ N m}^2/\text{C}^2$$

It is helpful to adopt a convention for subscript notation.

F_{12} = force on 1 due to 2

F_{21} = force on 2 due to 1

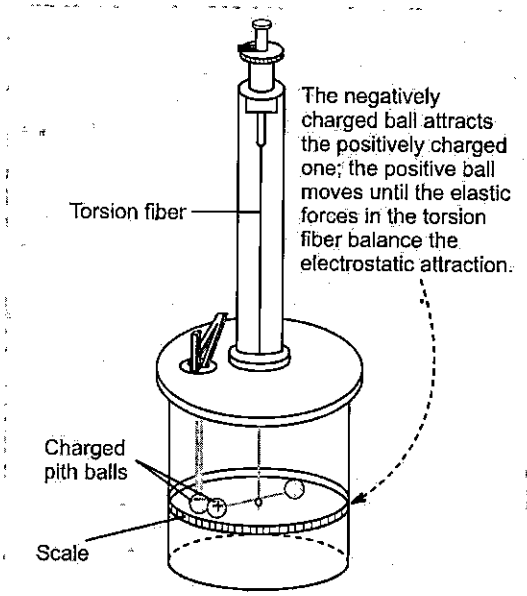


Fig. 1.21 (a) A torsion balance of the type used by coulomb to measure the electric force

The constant k is often written in terms of another constant ε_0.

$$k = \frac{1}{4\pi\varepsilon_0}$$

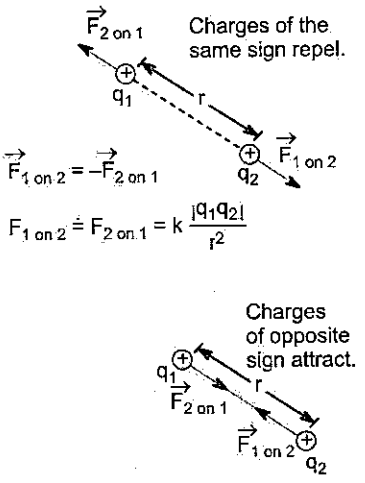


Fig. 1.21 (b) Interactions between point charges

The new constant ε_0 is called the permittivity of free space. Coulomb's law then can be written as :

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}$$

where $\varepsilon_0 = \frac{1}{4\pi k}$

$$= 8.85\times10^{-12}\,C^2/N\ m^2$$

Important Points

1. Permittivity of a given medium is measure of the fact have strongly a medium is influenced by external electric field. If an externally applied field has stronger polarising effect on medium then it has high permittivity.

2. If two charges are placed in any medium other than vacuum or air the force between two charges decreases due to polarisation of medium, therefore resultant force on a charge is gets reduced by a factor k known as dielectric constant of medium or relative permittvity of medium

$$\frac{\varepsilon}{\varepsilon_0} = k = \varepsilon_r$$

ε = permittivity of medium

ε_0 = permittivity of vacuum

ε_r = relative permittivity

Thus, $|\vec{F}| = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{q_1q_2}{r^2}$

For vacuum $\varepsilon_r = 1$

For air $\varepsilon_r \simeq 1$

For conductivity medium $\varepsilon_r = \infty$

For water $\varepsilon_r = 80$

3. Coulombic force between two charges is an action reaction pair, conservative in nature, central force. It acts along line joining two point charges:

4. Coulomb's law is valid only for point charges when the size of an object is very small as compared to their separation then they are considered as point charges.

5. The force between two points charges is independent of presence or absence of any other charges. Due to presence of surrounding medium resultant force changes because of polarisation of molecules of medium.

Force is a Vector, so in Vector form the Coulomb's Law is Written as :

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}\hat{r}_{12}$$

where $\hat{r}_{12}$ is a unit vector directed towards q_1 from q_2. This form of coulomb's law is illustrated in Fig. 1.22 for three different point charge distributions.

Note $\hat{r}_{12} = -\hat{r}_{21}$

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}\hat{r}_{12}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q_2q_1}{r^2}(-\hat{r}_{21})$$

$$= -\frac{1}{4\pi\varepsilon_0}\frac{q_2q_1}{r^2}\hat{r}_{21} = -\vec{F}_{21}$$

Remember conversion for $\hat{r}$.

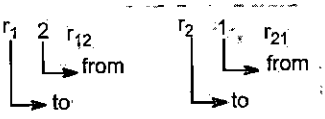


Fig. 1.22

The $\vec{F}_{12}$ or $\vec{F}_{21}$ are never shown with a minus sign.

When more than two charges are present, the net force on one of the charges is the vector sum of the individual Coulomb forces. For N charges, the total force on the charge labelled 1 due to the other $N-1$ charges is

$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \ldots + \vec{F}_{1N} = \sum_{i=2}^{N}\vec{F}_{1i}$$

This result is called **principal of superposition.**

Remember that the Coulomb's force equation is precisely applicable to objects whose size is much smaller than the distance between them. Ideally it is precise for **point charges,** whose spatial size is negligible as compared to other distances. In case of finite size objects, it is always not possible to identify value of r, particularly when there is nonuniform charge distribution. **For conducting spheres the charge is uniformly distributed, then r is the distance between centres.**

Coulomb's law describe the force between charges at rest. In this chapter our discussion is restricted to charges at rest, the study of which is called **electrostatics.** When charges are in motion additional forces come into play, which will be discussed in later chapters.

Concepts: 1. *To make the study of the distance dependence as simple as possible, consider the force exerted by point charge 1 on point charge 2. A point charge—**a finite charge located at a geometric point—is an idealization analogous to that of a point mass (a particle). If we consider two point charges. We can precisely define the vector $\mathbf{r}_{1\to2}$ that extends from body 1 to body 2. (We approximate this ideal situation when the distance between the two charged bodies is large compared with the size of either).***

2. *The forces that any two point like charges exert on each other are of equal magnitude but in opposite directions, in accordance with Newton's third law. The force on each charge has the same magnitude even if q and Q are of different magnitude, as shown in Fig. 1.23*

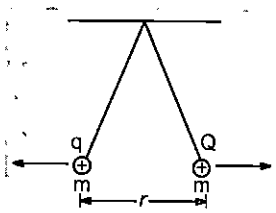


Fig. 1.23

The electrical force on each of two charges satisfies Newton's third law.

3. *If the electric force is likewise an inverse-square force, it should have the form*

$$\vec{\mathbf{F}}_{1\to2} \propto \frac{q_1 q_2}{r^2}\hat{\mathbf{r}}_{1\to2}$$

In this proportionality, the direction of $\mathbf{F}_{1\to2}$ takes care of itself if we attach to each q the proper sign of charge. For like charges (++ or − −), we have $\mathbf{F}_{1\to2} = \hat{\mathbf{r}}_{1\to2}$; for unlike charges (+ − or − +), $\mathbf{F}_{1\to2} = -\hat{\mathbf{r}}_{1\to2}$.

The SI Unit of Charge

We begin with Coulomb's law in the form of Proportionality

$$\vec{\mathbf{F}}_{1\to2} \propto \frac{q_1 q_2}{r^2}\hat{\mathbf{r}}_{1\to2}$$

As we did for Newton's law of gravitation, we would like to convert this proportionality into an equation. In the analogous gravitational proportionality,

$$\vec{\mathbf{F}}_{1\to2} \propto -\frac{m_1 m_2}{r^2}\hat{\mathbf{r}}_{1\to2}$$

the units of all the quantities are defined, and all that is necessary is to determine experimentally the numerical value of the proportionality constant G.

$$\vec{\mathbf{F}}_{1\to2} = -G\frac{m_1 m_2}{r^2}\hat{\mathbf{r}}_{1\to2}$$

Equation then determines the units of G, which in SI must be $\mathrm{N\text{-}m^2 / kg^2}$.

To express Coulomb's law as an equation, we must deal with an additional problem. When we developed Newton's law of gravitation, we had already defined mass as one of the basic dimensions—mass, length, and time—of SI and had established a unit of mass, the kilogram. Electric charge q, which appears in Proportionality cannot be expressed in terms of these basic dimensions. Consequently, we must expand our set of basic dimensions to include a fourth dimension, *electric charge*, for which we must then establish an SI unit. With this in mind, we proceed in two steps:

*1. As a preliminary to defining it, we give the unit of electric charge a name, the **coulomb** (C). This is , the quantities q_1 and q_2 that appear in our desired equation will be expressed in coulombs. With this in mind, we use a proportionality constant to rewrite Coulomb's law as an equation:*

$$\vec{\mathbf{F}}_{1\to2} = (\text{constant})\frac{q_1 q_2}{r^2}\hat{\mathbf{r}}_{1\to2},$$

From this equation, it follows that the SI units of the proportionality constant are $\mathrm{N\text{-}m^2 / C^2}$. *For reasons that will be clear later on, we write the constant in the form*

$$\text{constant} \equiv \frac{1}{4\pi\varepsilon_0}$$

*The quantity ε_0("epsilon-zero") that appears in this definition is called the **permittivity of free space**. We can now write Coulomb's law in its most requently used form,*

$$\vec{\mathbf{F}}_{1\to2} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{\mathbf{r}}_{1\to2}$$

2. *We define the value of the proportionality constant:*

$$\frac{1}{4\pi\varepsilon_0} \equiv 10^{-7} c^2 \mathrm{N\text{-}m^2 / C^2},$$

Where c is the speed of light in m/s. (The rationale for defining $1/4\pi\varepsilon_0$ in terms of the speed of light lies in the fundamental connection between electric charge and electro-magnetic radiation, Because c is defined rather than measured, it is an exact quantity: $c \equiv 2.997\ 92458 \times 10^8$ m/s. Thus, $1/4\pi\varepsilon_0$ is exact as well. For many purposes, it suffices to use the rounded value

$$\frac{1}{4\pi\varepsilon_0} = 8.99\times 10^9\ \mathrm{N.m^2 / C^2}$$

$$\approx 9\times 10^9 \mathrm{N\text{-}m^2}$$

If follows immediately that the value of ε_0, rounded to three significant figures, is

$$\varepsilon_0 = 8.85\times 10^{-12} \mathrm{C^2 / N\text{-}m^2}$$

With the value of ε_0, thus fixed and F and r expressed in SI units, the magnitude of the coulomb is determined by coulomb's law.

The units for quantifying the electric charge property of matter depend on the units used for measuring force and distance. In the SI system of units, the unit of charge is defined in terms of *charge flow* or **electric current.** The SI unit for electrical current is the *ampere* (A) and will be defined later more precisely. A current of one ampere means that one **coulomb**(C) of charge flows past a specified

region during one second. The coulomb turns out to be a very big charge unit; smaller divisions, such as millicoulombs (mC) $(=10^{-3}\,C)$, **microcoulombs** (μC) $(=10^{-6}\,C)$, and nanocoulombs. (N-C) $(=10^{-9}\,C)$ also are employed.

Concept: *We have some electric charges, $q_1, q_2, q_3 \cdots$ (referred as source charges); what force do they exert on another charge, Q (call it the **test charge**)?*

*The **principle of superposition** states that the interaction between any two charges is completely unaffected by the presence of others. This means that to determine the force on Q, we can first compute the force $\vec{F}_1$, due to q_1 alone (ignoring all the others); then we compute the force $\vec{F}_2$, due to q_2 alone; and so on. Finally, we take the vector sum of all these individual forces: $\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ldots$*

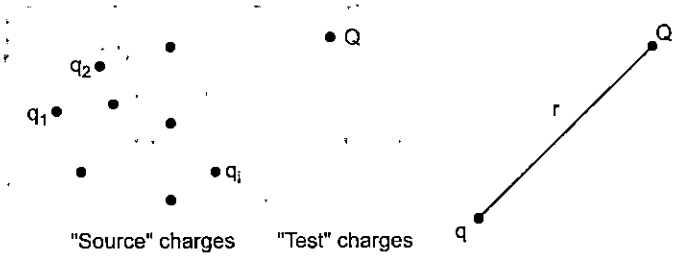


Fig. 1.24

Superposition is not a logical necessity, but an experimental fact.

The force on Q depend on the separation distance r between the charges, it also depends on both their velocities and on the acceleration of q. When charge q is moved Electromagnetic "news" travels at the speed of light, so what concerns Q is the position, velocity, and acceleration q had at some earlier time, when the electromagnetic interaction message.

*We shall consider the special case of **electrostatics** in which all the source charges are stationary (though the test charge may be moving).*

PROBLEM SOLVING TACTICS

1. While using Coulomb's law express the distance between charges in metre and the charge in coulomb.

2. Draw a free body diagram showing all the forces acting on the body. Determine the direction of force physically. Force is along the line joining two charges, show repulsive force for similar charges and attractive for unlike charges. In order to avoid confusion deal with charge magnitudes only (leaving out minus sign) to get the magnitude of each force; the direction of force is determined after examining the types of charges involved.

3. While dealing with several charges use double subscript notation. The first subscript refers to the particle that exerts the force. Add all the forces acting on one object as vectors.

4. Use consistent units; since we know k in standard SI units ($N\,m^2/C^2$), distances should be in meters and charges in coulombs. When the charge is given in μC or nC, be sure to change the units to coulombs: $1\,\mu C = 10^{-6}C$ and $1nC = 10^{-9}C$.

5. When finding the electric force on a single charge due to two or more other charges, find the force due to each of the other charges separately. The net force on a particular charge is the vector sum of the forces acting on that charge due to each of the other charges. Often it helps to separate the forces into x- and y- components, add the components separately, then find the resultant force from the net x- and net y-components.

6. If several charges lie along the same line, we do not have to worry about an intermediate charge "shielding" the charge located on one side from the charge on the other side. The electric force is long-range just as is gravity; the gravitational force on the Earth due to the Sun does not stop when the Moon passes between the two.

Example 1

Compare the magnitudes of the electric and gravitational forces between an electron and a proton by calculating the ratio of these forces.

Solution: Charge of electron, $-e = -1.6\times10^{-19}C$

Mass of electron, $m_e = 9.1\times10^{-31}kg$

Charge of proton, $+e = +1.6\times10^{-19}C$

Mass of proton $= 1.7\times10^{-27}\,kg$

Coulombic force, $F_E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{e^2}{r^2}$

Gravitational force, $F_G = G\dfrac{m_p m_e}{r^2}$

The ratio is $\dfrac{F_E}{F_G} = \dfrac{1}{4\pi\varepsilon_0 G}\dfrac{e^2}{m_p m_e}$

$$= \frac{(9.0\times10^9)(1.6\times10^{-19})^2}{(6.7\times10^{-11})(1.7\times10^{-27})(9.1\times10^{-31})}$$

$$= 2\times10^{39}$$

which shows that between an electron and proton, the gravitational force is negligible in comparison with the electric force.

Conceptual Example 1 : *Consider Fig. 1.25*

(a) What would happen if the rod were removed before the spheres are separated ? (b) Would the induced charges be equal in magnitude even if the spheres had different

sizes? Different conductors ? (c) Is there something misleading about Fig. 1.24 (c)?

Reasoning and Answer: (a) When a positively charged rod brought near A, the free electron in the metal are attracted to the rod, and some move to the left side of A. This movement leaves unbalanced positive charge on B. If the rod is removed before the spheres are separated, the excess electrons on sphere A would flow back to B.

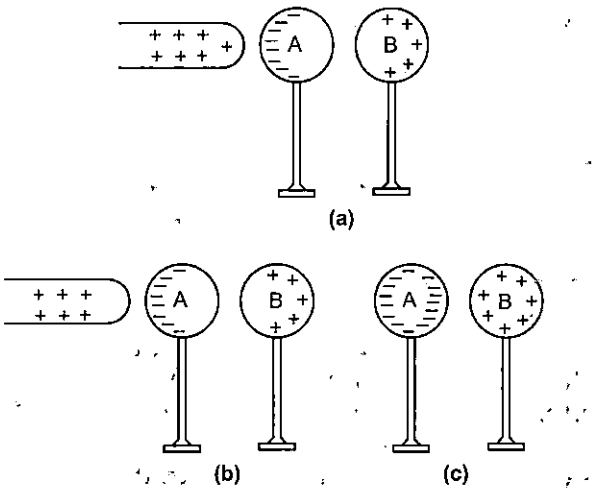


Fig. 1.25

(b) Yes, net charge is conserved. Before the rod is brought near A, both A and B were neutral. They will remain so even if they have different size or material.

(c) If the spheres A and B are close to each other, the facing side will have more concentration of charges, due to mutual attraction. The charge distribution will be non-uniform.

Conceptual Example 2 : *By considering the phenomena of attraction and repulsion, comment on the possible validity of each of the following expressions for the force between two point charges q_1 and q_2 separated by a distance r.*

(a) $\frac{q_1^2 q_2}{r^2}$ (b) $\frac{1}{4\pi\varepsilon_0}\frac{(q_1+q_2)}{r^2}$

Reasoning and Answer : (a) As q_1 occurs as square, so its sign would not matter for consideration of forces. Secondly, if q_1 and q_2 were interchanged, the force would not be the same.

(b) If the charges had equal and opposite signs, the force would be zero.

Conceptual Example 3 : *Two positive point charges $Q_1 = 20\,\mu C$ and $Q_2 = 10\,\mu C$ are separated by a distance a, which is larger in magnitude, the force that Q_1 exerts on Q_2 or the force that Q_2 exerts on Q_1?*

Reasoning and Answer : In accordance with Coulomb's law, the force on Q_1 exerted by Q_2 is

$$|F_{12}| = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{a^2}$$

The coulombic force is an action-reaction pair. The force on Q_2 exerted by Q_1 has same magnitude but direction is reversed. The equation is symmetric w.r.t. two charges.

$$|F_{21}| = \frac{1}{4\pi\varepsilon_0}\frac{Q_2 Q_1}{a^2}$$

Thus $|F_{12}| = |F_{21}|$

In accordance with Newton's third law, F_{12} and F_{21} must be equal in magnitude but opposite in direction.

Example 2

Consider three charges q_1, q_2 and q_3 each equal to q at the vertices an equilateral triangle of side l. What is the force on a charge Q placed at the centroid of the triangle?

Solution: Method 1 : The resultant of three equal coplanar vectors acting at a point is zero. These vectors form a closed polygon.

Hence the vector sum of the forces $\vec{\mathbf{F}}_1, \vec{\mathbf{F}}_2$ and $\vec{\mathbf{F}}_3$ is zero.

Method 2 : The forces acting on the charge Q are

$\vec{\mathbf{F}}_1$ = force on Q due to $q_1 = \frac{1}{4\pi\varepsilon_0}\frac{Qq_1}{AO^2}\vec{\mathbf{AO}}$

$\vec{\mathbf{F}}_2$ = force on Q due to $q_2 = \frac{1}{4\pi\varepsilon_0}\frac{Qq_2}{BO^2}\vec{\mathbf{BO}}$

$\vec{\mathbf{F}}_3$ = force on Q due to $q_3 = \frac{1}{4\pi\varepsilon_0}\frac{Qq_3}{CO^2}\vec{\mathbf{CO}}$

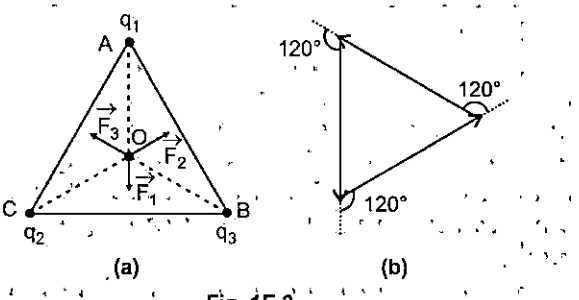


Fig. 1E.2

The resultant force is

$$\vec{\mathbf{F}}_R = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 + \vec{\mathbf{F}}_3$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Qq}{AO^2}(\vec{\mathbf{AO}} + \vec{\mathbf{BO}} + \vec{\mathbf{CO}})$$

as $|q_1| = |q_2| = |q_3|$ and $|\vec{\mathbf{AO}}| = |\vec{\mathbf{BO}}| = |\vec{\mathbf{CO}}|$

Also $\vec{\mathbf{AO}} + \vec{\mathbf{BO}} + \vec{\mathbf{CO}} = 0$,

because these are three equal vectors in a plane.

Method 3 : The resultant force $\Sigma\vec{\mathbf{F}}$ is the vector sum of individual forces

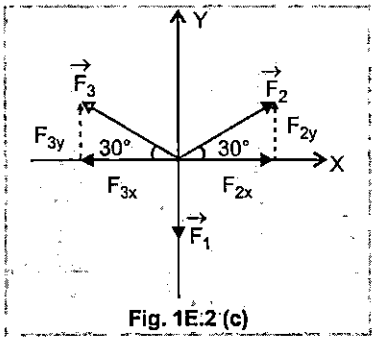


Fig. 1E.2 (c)

$$\sum \vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$$

or $$\sum F_x = F_{1x} + F_{2x} + F_{3x}$$
$$= 0 + F_2 \cos 30° - F_3 \cos 30° \quad ...(1)$$

and
$$\sum F_y = F_{1y} + F_{2y} + F_{3y} = -F_1 + F_2 \sin 30° + F_3 \sin 30° \quad ...(2)$$

As $|F_1| = |F_2| = |F_3| = |F|$ (say), the eqn. (1) and (2) become

$$\sum F_x = 0 \text{ and } \sum F_y = 0$$

Hence resultant force $\sum \vec{F} = 0$

Example 3

(a) Suppose you acquire a net charge of $-2.0\,\mu C$ while shuffling across a carpeted floor. Will you have a deficiency or excess of electrons? (b) How many missing or extra electrons will you have ?

Solution: (a) As 1 microcoulomb $= 1\mu C = 10^{-6}$C, the charge acquired is

$$q = -2 \times 10^{-6} C$$

The charge acquired is negative and electrons have negative charge. You have acquired an excess of electrons.

(b) The net charge is an integral multiple of electronic charge.

Thus, $$n = \frac{q}{e} = \frac{-2 \times 10^{-6}}{1.6 \times 10^{-19}}$$
$$= 1.3 \times 10^{13} \text{ electrons}$$

Conceptual Example 1 : *A rubber comb pulled through dry hair can acquire a negative charge. The charged comb can pick up small pieces of uncharged paper. Is this a violation of Coulomb's law because paper has no net charge.*

Reasoning and Answer : When the comb is brought near the paper, the paper gets polarized. The positive end of the paper is closer to the comb than the negative end. In accordance with the Coulomb's law the electric force varies inversely with the square of the distance. The attraction (F_1) between the comb and the positive end is greater than the repulsion (F_2) between the comb and the negative end. Hence the vector sum of the forces on the paper is toward the comb. Thus, this experiment supports Coulomb's law and illustrates clearly that the electrostatic forces get weaker with greater distance.

Example 4

The Bohr model of the electrically neutral hydrogen atom consists of an electron orbiting a single proton. The electron has a number of possible (or allowed) orbits around the central nucleus but assume the electron "orbits" the proton at its average distance of $r = 0.53 \times 10^{-10}$ m.

(a) Find the magnitude and direction of the electrical force on the electron in the hydrogen atom when it is in this orbit.

(b) Calculate the magnitude of the acceleration of the electron in this orbit, and compare this magnitude with that of the acceleration due to gravity ($g = 9.81\,m/s^2$).

Solution: The magnitude of the force on the electron is determined from Coulomb's law,

$$F = \frac{1}{4\pi\varepsilon_0} \frac{|q||Q|}{r^2} = \frac{1}{4\pi\varepsilon_0} \frac{e^2}{r^2}$$
$$= \frac{(9 \times 10^9)\,(1.602 \times 10^{-19})^2}{(5.29 \times 10^{-11})}$$
$$= 8.25 \times 10^{-8} \text{N}$$

The force on the electron is attractive and directed radially toward the proton because the two are unlike charges.

Remark :

(a) Actually both the electron and the nucleus orbit the centre of mass of the two-particle system. However, since the mass of a proton is 1836 times that of an electron, the centre of mass of the system is essentially coincident with the proton. Thus we say that the electron orbits the proton (thinking that the proton is relatively fixed).

(b) As the gravitational force is negligible in comparison to electrical force, from Newton's second law we have

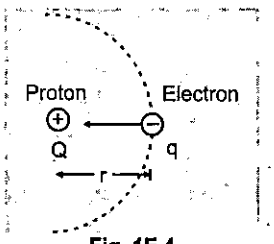


Fig. 1E.4

$$F_{elec.} = m_e a$$

or $$a = \frac{F_{elec.}}{m_e} = \frac{8.25 \times 10^{-8}}{9.11 \times 10^{-31}} = 9.06 \times 10^{22} \text{ m/s}$$

and $$\frac{a}{g} = \frac{9.06 \times 10^{22}}{9.81} = 9.24 \times 10^{21}$$

Thus the electron has a very large acceleration as compared to g.

Example 5

Two identical small charged spheres, each having a mass m, hang in equilibrium as shown in Fig. 1E.5(a). The length of each string is l and the angle is θ. Find the magnitude of the charge on each sphere.

Solution: Method 1: The forces acting on the sphere are tension in the string T; force of gravity, mg; repulsive electric force, F_e, as shown in the free body diagram of the sphere. The sphere is in equilibrium. The forces in the horizontal and vertical directions must separately add up to zero.

$$\Sigma F_x = T\sin\theta - F_e = 0 \quad ...(1)$$

$$\Sigma F_y = T\cos\theta - mg = 0 \quad ...(2)$$

From eqn. (2), $T = \dfrac{mg}{\cos\theta}$. Thus we can eliminate T from eqn. (1), from which we obtain

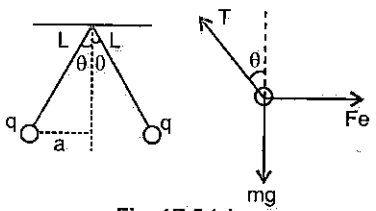


Fig. 1E.5 (a)

$$F_e = mg\tan\theta$$

$$\frac{kq^2}{r^2} = mg\tan\theta \quad ...(3)$$

where $k = \dfrac{1}{4\pi\varepsilon_0}$

and $r = 2L\sin\theta$.

The eqn. (3) now reduces to

$$\frac{1}{4\pi\varepsilon_0}\frac{q^2}{(2l\sin\theta)^2} = mg\tan\theta$$

$$q = \sqrt{16\pi\varepsilon_0 l^2 mg\tan\theta\sin^2\theta}$$

Method 2 : We can use Lami's theorem, if body is in equilibrium under the effect of three concurrent forces. Then

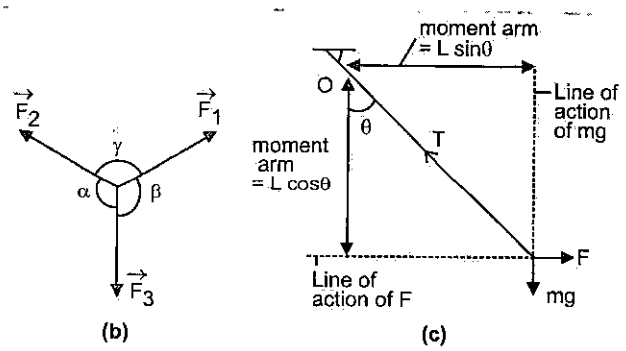


Fig. 1E.5

$$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$$

From the force diagram we see that

$$\frac{T}{\sin 90°} = \frac{F}{\sin(180°-\theta)} = \frac{mg}{\sin(90°+\theta)}$$

or, $F = T\sin\theta$

and $mg = T\cos\theta$

Hence $\tan\theta = \dfrac{F}{mg}$

Method 3 : When a body is in equilibrium under the action of certain forces, net torque about any point is zero [Fig. 1E.5(c)].

$$\Sigma\tau = 0$$

For simplicity we choose point of suspension as point of application. As the line of action of tension passes through it, the torque is zero. Taking anticlockwise torque to be positive, we get

$$\Sigma\tau = F(L\cos\theta) - mg(L\sin\theta)$$

$$\tan\theta = \frac{F}{mg}$$

Example 6

Two identical balls each having a density ρ are suspended from a common point by two insulating strings of equal length. Both the balls have equal mass and charge. In equilibrium each string makes an angle θ with the vertical. Now both the balls are immersed in a liquid. As a resultant the angle θ does not change. The density of liquid is σ. Find the dielectric constant of the liquid.

Solution: When the balls are in air, from previous problem,

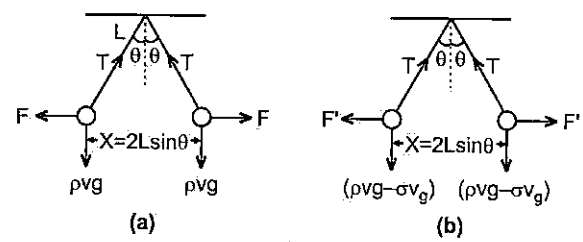


Fig. 1E.6

$$F = (\rho vg)\tan\theta \quad ...(1)$$

When the balls are suspended in a liquid, the Coulombic force is reduced to $F' = F/K$ and apparent weight = weight – upthrust, $W' = (\rho vg - \sigma vg)$. According to the problem, angle θ is unchanged.

$$F' = (\rho vg - \sigma vg)\tan\theta \quad ...(2)$$

From eqn. (1) and (2), we get

$$\frac{F}{F'} = K = \frac{\rho vg}{\rho vg - \sigma vg} = \frac{\rho}{\rho - \sigma}$$

Example 7

A small point mass m has a charge q, which is constrained to move inside a narrow frictionless cylinder. At the base of the cylinder is a point mass of charge Q having the same sign as q. Show that if the mass m is displaced by a small amount from its equilibrium position and released, it will exhibit simple harmonic motion with angular frequency $\omega = (2g/y_0)^{1/2}$ *where* y_0 *is the equilibrium position of charge q.*

Solution: In equilibrium position, gravitational force is balanced by Coulombic repulsive force

$$mg = \frac{Qq}{4\pi\varepsilon_0 y_0^2}$$

If charge q is displaced in positive y-direction, such that $y << y_0$ from Newton's second law,

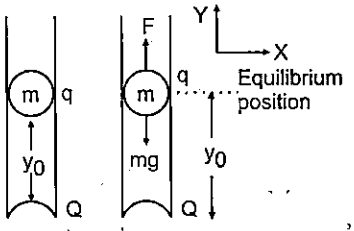


Fig. 1E.7

$$\frac{Qq}{4\pi\varepsilon_0(y_0+y)^2} - mg = ma$$

$$\frac{Qq}{4\pi\varepsilon_0 y_0^2}\left[\frac{1}{(1+y/y_0)^2}\right] - mg = ma$$

or $$mg\left[1-\frac{2y}{y_0}\right] - mg = ma \quad \text{or} \quad a = -\frac{2gy}{y_0}$$

$$\frac{d^2y}{dt^2} + \frac{2g}{y_0}y = 0$$

which is equation for SHM with $\omega = \sqrt{\frac{2g}{y_0}}$

Example 8

A small bead of mass m, charge −q is constrained to move along a frictionless wire. A positive charge Q lies at a distance L from the wire. Show that if the bead is displaced a distance x, where $x << L$, *and released, it will exhibit simple harmonic motion. Obtain an expression for the time period of simple harmonic motion.*

Solution: Component $F\cos\theta$ of attractive Coulombic force restores the charge q to its mean position O.

From Newton's second law,

$$-F\cos\theta(\hat{\mathbf{i}}) = ma$$

$$-\frac{Qq}{4\pi\varepsilon_0(x^2+L^2)}\frac{x}{\sqrt{x^2+L^2}}\hat{\mathbf{i}} = ma$$

$$a = -\frac{Qq}{4\pi\varepsilon_0 m}\frac{x}{(x^2+L^2)^{3/2}}\hat{\mathbf{i}}$$

For a small linear displacement, $x << L$, we can ignore x^2 in comparison to L^2.

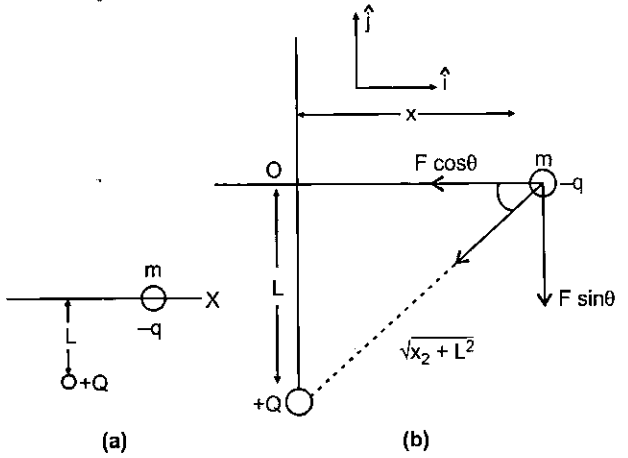


Fig. 1E.8

$$a \simeq \frac{Qq}{4\pi\varepsilon_0 mL^3}x\hat{\mathbf{i}}$$

which is equation of SHM with

$$\omega = \sqrt{\frac{Qq}{4\pi\varepsilon_0 mL^3}} = \frac{2\pi}{T} = 2\pi\upsilon$$

$$T = 2\pi\sqrt{\frac{4\pi\varepsilon_0 mL^3}{Qq}}$$

Example 9

Four identical charges are fixed at the corners of a square of side a. A fifth point charge −Q lies a distance z along the line perpendicular to the plane of the square. Show that for $z << a$ *the motion of −Q is simple harmonic. What would be the period of motion, if the mass of −Q is m? (Neglect gravitational force.)*

Solution: We consider two diagonally opposite charges. They exert attractive forces on charge $-Q$. The horizontal component of force F cancels, whereas vertical components add, similarly for other two diagonally opposite charges only vertical component remains.

$$\vec{\mathbf{F}}_R = 4F\cos\theta(-\hat{\mathbf{k}})$$

$$= -4\times\frac{1}{4\pi\varepsilon_0}\frac{Qq}{(z^2+a^2/2)}\times\frac{z}{(z^2+a^2/2)^{1/2}}(\hat{\mathbf{k}})$$

$$= -\frac{4Qqz}{4\pi\varepsilon_0(z^2+a^2/2)^{3/2}}(\hat{\mathbf{k}})$$

For $z << a$, we may neglect z^2 term in the denominator.

Thus, $\vec{\mathbf{F}}_R = -\frac{4Qqz\cdot 2^{3/2}}{4\pi\varepsilon_0 a^3}$

$$\vec{a}_R = -\frac{4Qq \cdot 2^{3/2}}{4\pi\varepsilon_0 a^3 m} z$$

Comparing acceleration with equation of SHM, $a = -\omega^2 z$.

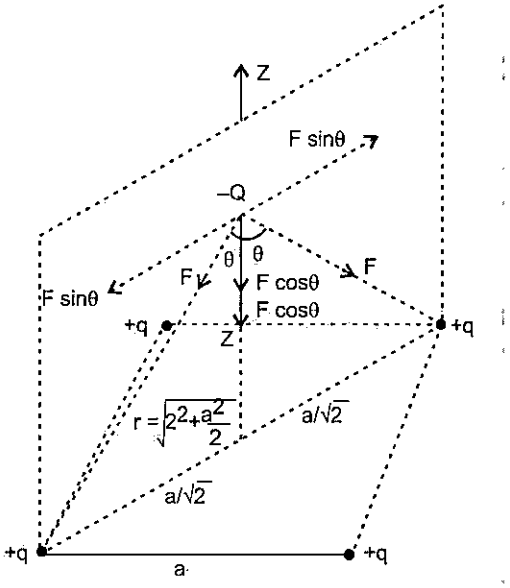


Fig. 1E.9

$$\omega = \sqrt{\frac{4Qq \cdot 2^{3/2}}{4\pi\varepsilon_0 a^3 m}} = \frac{2\pi}{T} = 2\pi\upsilon$$

$$T = 2\pi\sqrt{\frac{4\pi\varepsilon_0 a^3 m}{4Qq : 2^{3/2}}}$$

THE ELECTRIC FIELD

The charges can exert forces on each other without actually touching each other- *action-at-a-distance.* The concept of field helps to visualize the distribution of forces in space surrounding an object of mass m or a charge object q. Similar to gravitational force there is an electromagnetic force. Just as we envision a gravitational field, we envision an electromagnetic field.

A field of force exists in a region of space when an object placed at any point therein experiences a force. A gravitational field surrounds a mass m; similarly an electric field surrounds a charge q.

We define **the electric field ($\vec{E}$) at a point in space to be the electric force experienced by a positive test charge at that point divided by that charge.**

$$\vec{E} = \frac{\vec{F}}{q_0}$$

Note that $\vec{E}$ has the unit newton per coulomb. The electric field $\vec{E}$ is a vector whose direction is that of $\vec{F}$.

The test charge is of very small magnitude so that it does not disturb the charges whose field you are trying to measure. The experimentally measured electric field is given by

$$\vec{E} = \lim_{q \to 0} \frac{\vec{F}}{q}$$

By stipulating $q \to 0$, we ensure that the existing field we are trying to measure is not disturbed by the measuring process.

Concepts: **1.** *First a source charge produces electric field at the position of other charge (test charge) and this electric field exerts force on (test charge) charge. Electric field produced by static charge system is conservative. Electric field is also produced by time varying magnetic field but this field is not conservative and this field will be discussed later in electromagnetic induction chapter. Electrical force due to*

Source charge creates field	Test charge experiences field

2. $\vec{F}_E$ *is the force experienced by the charge* q_0 , $\vec{F}_E = q_0 \vec{E}$.

A positive charge (q) placed in an external field $\vec{E}$ *experiences the force* $(q\vec{E})$ *in the direction of the electric field. While the direction of force on a negative charge is opposite to that of the electric field.* $\vec{F}_E = q\vec{E}$ *is applicable for point charge . For finite size charged body (continuous charge distribution),* $\vec{F}_E = q\vec{E}$ *is applicable only when* $\vec{E}$ *for every charge element is identical, i.e., it is uniform field.*

If extended charged body is placed in a non-uniform electric field, then it is considered as made up of number of small charge elements behaving like point charge. Force on one charge element is calculated as $d\vec{F} = (dq)\vec{E}$

Net force on the body is given by vector superposition, *i.e.,* $\vec{F} = \int d\vec{F} = \int (dq)\vec{E}$.

3. *Electric field in a space can be variable with respect to position or at any position vary with respect to time. Different types of electric field are:*

(i) If electric field in the space is independent of position, then it is a uniform electric field.

(ii) If electric field in the space is a function of position, then it is a non-uniform electric field.

(iii) If electric field in the space is independent of time, then it is a constant electric field.

(iv) If electric field in the space is the function of time, then it is a variable electric field.

4. *Electric field due to a point charge in vector notation*

Electric force on test charge $+q_0$ *in the direction of position vector* $\vec{r}_{P/A}$ *is given by,*

$$\vec{\mathbf{r}}_{P/A} = \vec{\mathbf{r}}_{P/O} - \vec{\mathbf{r}}_{A/O}$$

$$\vec{\mathbf{F}}_E = \frac{1}{4\pi\varepsilon_0}\frac{(+q)(+q_0)}{r^3}(\vec{\mathbf{r}}_{P/A}); \quad (|\vec{\mathbf{r}}_{P/A}| = r)$$

$$\frac{\vec{\mathbf{F}}_E}{q_0} = \vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0}\frac{(+q)}{r^3}(\vec{\mathbf{r}}_{P/A})$$

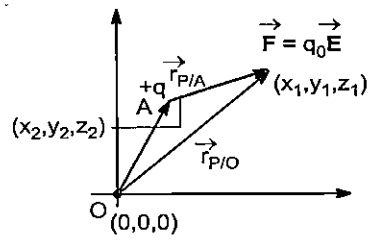


Fig. 1.25

$\vec{\mathbf{E}}$ *is the electric field intensity vector at position P due to a point charge placed at the position A. Proper sign of charge must be considered for* $\vec{\mathbf{E}}$.

$$|\vec{\mathbf{E}}| = E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2};$$

$$\vec{\mathbf{r}}_{P/A} = \vec{\mathbf{r}}_{P/O} - \vec{\mathbf{r}}_{A/O}$$

If $\vec{\mathbf{r}}_{P/O} = x_1\hat{\mathbf{i}} + y_1\hat{\mathbf{j}} + z_1\hat{\mathbf{k}}$

and $\vec{\mathbf{r}}_{A/O} = x_2\hat{\mathbf{i}} + y_2\hat{\mathbf{j}} + z_2\hat{\mathbf{k}}$

then

$$\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q\{(x_1-x_2)\hat{\mathbf{i}} + (y_1-y_2)\hat{\mathbf{j}} + (z_1-z_2)\hat{\mathbf{k}}\}}{\{(x_1-x_2)^2 + (y_1-y_2)^2 + (z_1-z_2)^2\}^{3/2}}$$

POINT CHARGE DISTRIBUTION

We place a test charge q_0 a distance r from a single point charge q whose field is to be measured. The force on test charge q_0 is

$$\vec{\mathbf{F}} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{\mathbf{r}}$$

Hence the electric field produced by q at position r is $E = F/q_0$

$$\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{\mathbf{r}}$$

where the unit vector $\hat{\mathbf{r}}$ points from q towards position of charge.

The electric field of a positive charge q points radially outward from q, as shown in Fig. 1.26. If q is negative, the field is directed inward toward q.

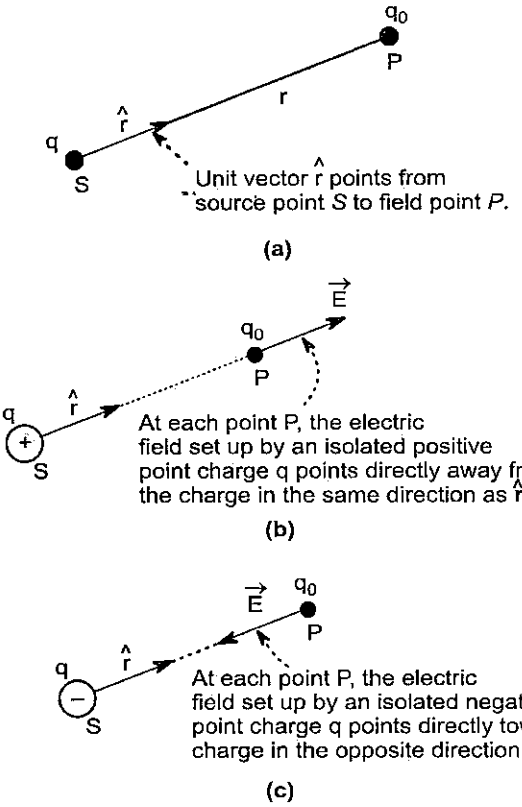


Fig. 1.26

Note that q_0 does not appear in the expression for electric field.

Concepts: *1. For our treatment of electrical phenomena., we will always assume that the presence of a test charge* q_{test} *in an electric field* $\vec{\mathbf{E}}$ *does not alter the distribution or motion of the charges creating the field.*

2. *It is important to realize that the charge q placed in the field need not be fixed or static but may accelerate in response to the electrical force it experiences. It is all the many other charges that create the field that are static, not necessarily the charge placed in the field of these static charges.*

3. *Just as the gravitational field depends on the shape and distribution of the masses producing the field (e.g., the shape and distribution of mass within the Earth), the electric field* $\vec{\mathbf{E}}$ *depends on the specific arrangement of the charges that create the field.*

4. $\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{\mathbf{r}}$ *(electric field of a point charge)*

By definition, the electric field of a point charge always points away from a positive charge (that is, in the same direction as $\hat{\mathbf{r}}$; but toward a negative charge) (that is, in the direction opposite $\hat{\mathbf{r}}$;).

But since $\vec{\mathbf{E}}$ can vary from point to point, it is not a single vector quantity but rather an infinite set of vector quantities, one associated with each point in space. This is an example of a ***vector field.*** *If we use a rectangular (x, y, z,) coordinate system, each component of $\vec{\mathbf{E}}$ at any point is in general a function of the coordinates (x, y, z,) of the point. We can represent the functions as E_x (x, y, z.) E_y (x, y, z.), and E_z (x, y, z,). One everyday example of a vector field is the velocity, $\vec{\mathbf{v}}$ of wind currents; the magnitude and direction of $\vec{\mathbf{v}}$ and hence its vector components, vary from point to point in the atmosphere.*

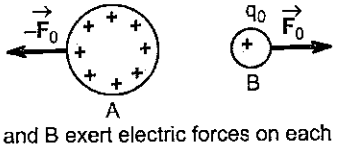
The Electric Field is a Property of Charge *q* Only

Knowing electric field at any location in space (irrespective of source) we can calculate the force F experienced by any point charge q placed at that location from.

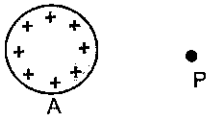
$$\vec{\mathbf{F}} = q\vec{\mathbf{E}}$$

Note that, when q is positive, force $\vec{\mathbf{F}}$ is in the same direction as $\vec{\mathbf{E}}$. When q is negative, the force is opposite to electric field.

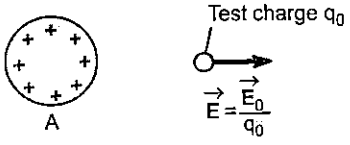
A charged body creates an electric field in the space around it.



(a) A and B exert electric forces on each other



(b) Removed body B and label its former position as P. Body A still creates its field at P.

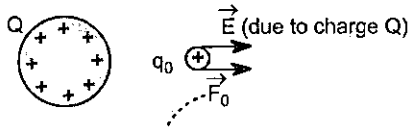


$\vec{E}$ is the force per unit charge exerted by A on a test charge at P.

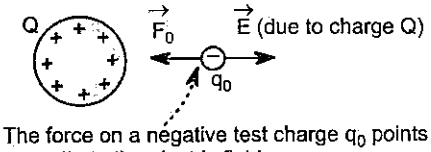
(c) Body A sets up an electric field $\vec{E}$ at point P. When a test charge is experiences force of already existing field.

Fig. 1.27

The force $\vec{\mathbf{F}}_0 = q_0 \vec{\mathbf{E}}$ exerted on a point charge q_0 placed in an electric field $\vec{\mathbf{E}}$.



The force on a positive test charge q_0 points in the direction of the electric field.



The force on a negative test charge q_0 points opposite to the electric field.

Fig. 1.28

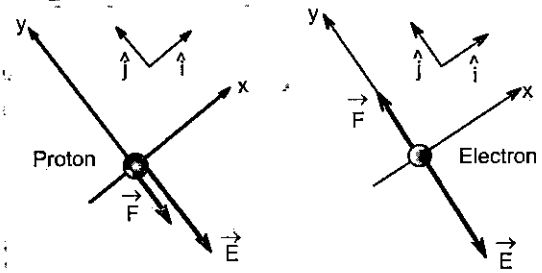


Fig. 1.29

PRINCIPLE OF SUPERPOSITION

Let us apply the concept of electric field to evaluating the force exerted on a test charge placed in the vicinity of a number of point source $q_1, q_2, \ldots, q_n$. Like gravitational forces, electric forces exerted simultaneously on the same body add vectorially. Each source charge exerts a force on q_t independent of the others. For example the force F_1 exert on q_t owing to the presence of q_1 is just the same as if all the other charges were absent. This statement, which is confirmed by experiment, is called the **superposition principle.** And $\mathbf{F}_1$ can be calculated by means of Coulomb's law

$$\vec{\mathbf{F}}_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_t}{r_{1\to t}^2}\hat{\mathbf{r}}_{1\to t}.$$

Thus the resultant force $\vec{\mathbf{F}}$ exerted on the test charge is given by the sum

$$\vec{\mathbf{F}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 + \vec{\mathbf{F}}_3 + \ldots + \vec{\mathbf{F}}_n$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_t}{r_{1\to t}^2}\hat{\mathbf{r}}_{1\to t} + \frac{q_2 q_t}{r_{2\to t}^2}\hat{\mathbf{r}}_{2\to t} + \frac{q_3 q_t}{r_{3\to t}^2}\hat{\mathbf{r}}_{3\to t} + \ldots + \frac{q_n q_t}{r_{n\to t}^2}\hat{\mathbf{r}}_{n\to t}\right)$$

The quantity q_t appears in every term, and we can factor it out to obtain the electric field E thus we have

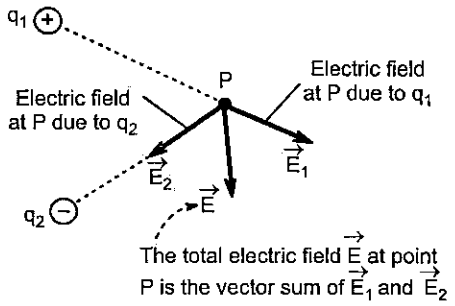


Fig. 1.30

$$\vec{\mathbf{E}}=\frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_{1\to t}^{\,2}}\hat{\mathbf{r}}_{1\to t}+\frac{q_2}{r_{2\to t}^{\,2}}\hat{\mathbf{r}}_{2\to t}+\frac{q_3}{r_{3\to t}^{\,2}}\hat{\mathbf{r}}_{3\to t}+\cdots\frac{q_n}{r_{n\to t}^{\,2}}\hat{\mathbf{r}}_{n\to t}\right)$$

or $$\vec{\mathbf{E}}=\frac{1}{4\pi\varepsilon_0}\sum_{j=1}^{n}\frac{q_j}{r_{j\to t}^{\,2}}\hat{\mathbf{r}}_{j\to t}.$$

In this sum, each term is the contribution of one source charge to the electric field. That is, the individual electic field contributions E add vectorially just like the individual force contributions:

$$E=\sum_{j=1}^{n}E$$

For a given array of source of source charges, the electric field depends on the location at which we wish to evaluate it. If we establish a coordinate system, every point in space can be specified by a vector $\hat{\mathbf{r}}$, and $\vec{\mathbf{E}}$ is a function of $\hat{\mathbf{r}}$:

$$\vec{\mathbf{E}}=\vec{\mathbf{E}}(\hat{\mathbf{r}})$$

Any vector that is a function is a function of a position vector in this way is called a **field vector** . We will consider another such vector, the *magnetic field B*, in later Chapter.

E is called the **electric field** of the source charges. Notice that it is a function of position **(r),** because the separation vectors r_i depend on the location of the field point **P** (Fig. 1.31). **But** it makes no reference to the test charge Q. The electric field is a vector quantity that varies **from point to point and is determined by the configuration of source charges; physically, E(r) is the force per unit charge that would be exerted on a test charge, if you were to place one at P.**

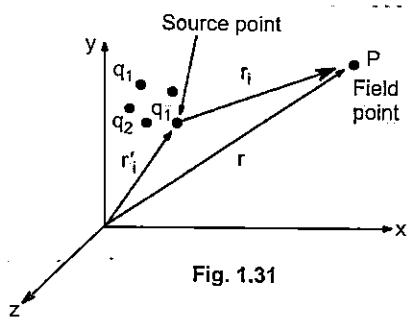


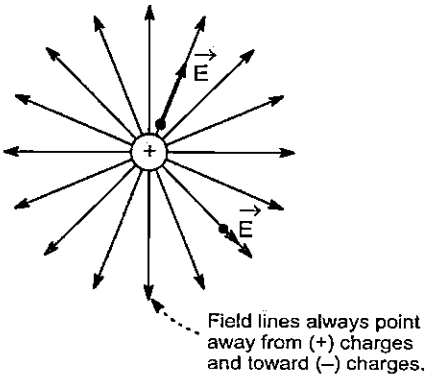
Fig. 1.31

What exactly is an electric field? it is alvised to students to think of the field as a "real" physical entity, filling the space in the neighborhood of any electric charge.

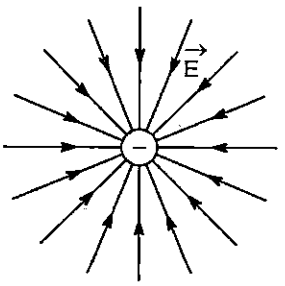
ELECTRIC FIELD LINES

The electric filed can be visualized by drawing lines to indicate its direction. At any given point, the field vector $\vec{\mathbf{E}}$ is tangent to the lines. Electric field lines are also called **lines of force** because they show the direction of the force exerted on a positive test charge. At any point near a positive charge, the electric field points radially away from the charge. Similarly, the electric field lines converge toward a point occupied by a negative charge.

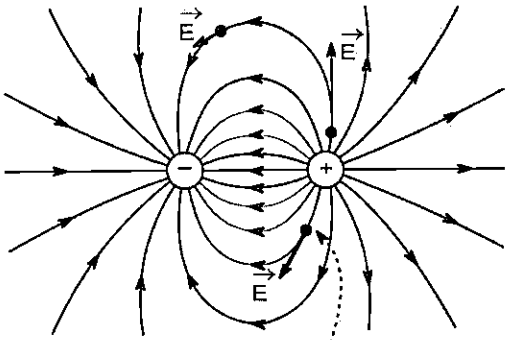
Consider a spherical surface of radius r with its centre at the charge. Its area is $4\pi r^2$. If N lines diverge from the point charge, the number of lines per unit area on a spherical surface a distance r away is $N/4\pi r^2$. Thus, as the distance increases, the density of the field lines (the number of lines per unit area) decreases as $1/r^2$, the same rate of decrease as E. We adopt the convention of drawing a fixed number of lines from a point charge, the number being proportional to the charge q. The field strength is indicated by the density of the lines.



(a) A single positive charge

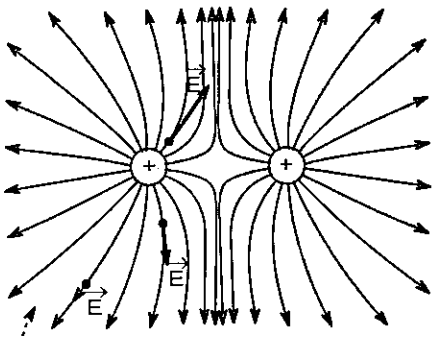


(b) A single negative charge



At each point in space, the electric field vector is *tangent* to the field line passing through that point.

(c) Two equal and opposite charges (a dipole)



Field lines are close together where the field is strong, farther apart where it is weaker.

(d) Two equal positive charges

Fig. 1.32

Concepts: *An electric field whose strength has the same magnitude and direction at all points is called a* ***uniform field.*** *The direction and density of field lines of a uniform filed are the same at all points (Fig. 1.33)*

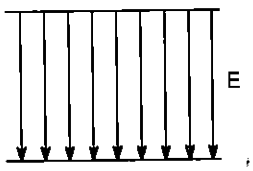
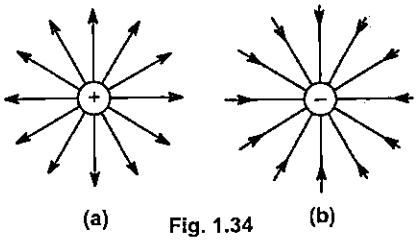


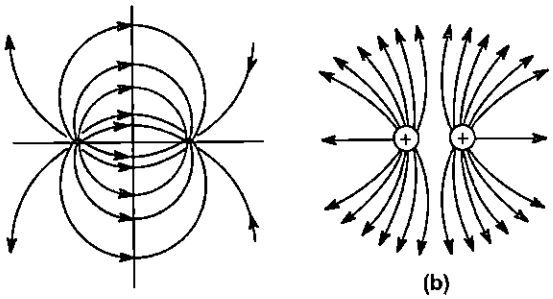
Fig. 1.33

Fig. 1.34 depicts the field lines of two insulated point charges, while Fig. 1.35 shows the field lines of two interacting point charges of equal magnitude. These fields are nonuniform. The strength of electric field of a point charge varies in inverse proportion to the squared distance from the charge (since the area of the sphere pierced by the field lines increases in proportion to the square of its radius).



(a) (b)

Fig. 1.34



(a) (b)

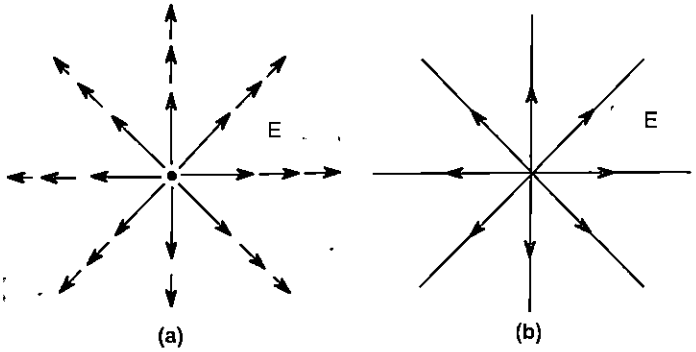
Fig. 1.35

Concept: *Concept of electric field lines is a qualitative, and perhaps more illuminating, intuitive approach. A single point charge q, situated at the origin has field*

$$\vec{\mathbf{E}}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{\mathbf{r}}.$$

To get a physical "feel" for this field, we sketch a few representative vectors, as in Fig. 1.36 (a). Because the field falls off like $1/r^2$*, the vectors get shorter as you go farther away from the origin; they always point radially outward. To represent this field, we concept up the arrows to form field lines. The magnitude of the field is indicated by the density of the field lines: it's strong near the center where the field lines are close together, and weak farther out, where they are relatively far apart.*

In truth, the field-line diagram is deceptive, when I draw it on a two-dimensional surface, for the density of lines passing through circle of radius r is the total number divided by the circumference $(n/2\pi r)$*, which goes like (1/r), not* $(1/r^2)$*. But if you imagine the model in three dimensions then the density of lines is the total number divided by the area of the sphere* $(n/4\pi r^2)$



(a) (b)

Fig. 1.36

2. The number N of the field lines emerging from a positive point charge (or terminating on a negative charge) can be found from the principle underlying the plotting of the lines. Their density should be proportional to the

magnitude of E at a given point of the field. At all points of the sphere of radius r, we have

$$E = (1/4\pi\varepsilon_0)(q/\varepsilon_r r^2).$$

Multiplying this expression by the area of the surface of the sphere (equal to $4\pi r^2$), we obtain the number of lines of the field created by the point charge.

$$N = (1/4\pi\varepsilon_0)(q/\varepsilon_r r^2)4\pi r^2$$
$$= q/\varepsilon_0\varepsilon_r$$

(for a negative sign, we must take $|q|$ instead of q in this formula).

3. At the interface between two media with different permitivities, the field strength, and hence the density of the field lines,change abruptly. Since there is no electric field inside a conductor, the lines of an external field terminate and start on the conductor surface. There are no field lines inside a conductor.

4. Actually the field lines do not exist. This is just a convenient method of graphical representation of a field.

5. Electric field lines should not be identified with the trajectories of motion of very light charged bodies in an electrostatic field (the so-called point charges). The tangent to such a trajectory at a certain point coincides in direction with the velocity of the body.

On the other hand, the tangent to a field line at any point coincides in direction with the force qE, *i.e.*, with the acceleration a. vectors a and v are directed along the same straight line only in rectilinear motion. In the general case, their directions do not coincide.

Concept: *Electric field lines are not the same as trajectories.* *It's a common misconception that if a charged particle of charge q is in motion where there is an electric field, the particle must move along an electric field line. Because* $\vec{E}$ *at any point is tangent to the field line that passes through that point, it is indeed true that the force* $\vec{F} = q\vec{E}$ *on the particle, and hence the particle's acceleration, are tangent to the field line. That when a particle moves on a curved path, its acceleration cannot be tangent to the path. So in general, the trajectory of a charged particle is not the same as a field line.*

6. The pattern of electric field lines can be observed, for example, in the following experiment. Wheat groats are poured in a vessel containing a liquid dielectric (castor oil or turpentine). If an electric field is created in the dielectric, wheat groats get electrolized and move and turn until they form chains coinciding with the electric field lines.

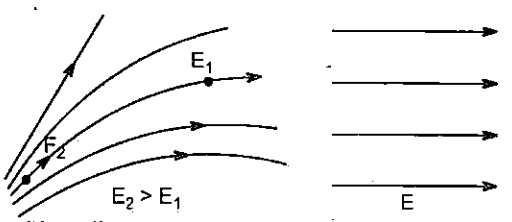


Fig. 1.37

7. If the lines of force are parallel and equally spaced then they represent a uniform electric field. Electric lines of force can never bend sharply because at bending point tangent can't be defined and so the direction of electric field is not defined. Number of lines of forces originating or terminating to any charge is proportional to the amount of that charge.

Fig. 1.38

Electric lines of force are always smooth continuous curve.

8. Lines of force exist only where there is an electric field, since field inside a conductor or at a neutral point is zero, hence there can not be any lines of force. Lines of force enter or leave a conducting surface normally.

9. They never form a closed loop when produced by a static charge because if they do so, then the work done round a closed path will not be zero which contradicts its conservative nature.

10. If a positive point charge is allowed to move very slowly under the influence of electric field then path followed by point charge is electric lines of force.

11. They contract longitudinally (lengthwise) producing attraction between opposite charges and expand laterally producing repulsion between similar charges.

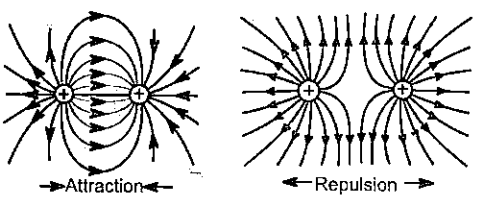


Fig. 1.39

The rules for drawing electric field lines can be summarized as follows :

1. Electric field lines begin on positive charges (or at infinity) and end on negative charges (or at infinity).

2. The lines are drawn symmetrically entering or leaving an isolated charge.

3. The number of lines leaving a positive charge or entering a negative charge is proportional to the magnitude of the charge.

4. The density of the lines (the number of lines per unit area perpendicular to the lines) at any point is proportional to the magnitude of the field at that point.

5. At large distance from a system of charges, the field lines are equally spaced and radial, as if they came from a single point charge equal to the net charge of the system.

6. Field lines do not cross. If two field lines crossed, that would indicate two directions for $\vec{E}$ at the point of intersection.

Illustration 1: The electric field lines for two conducting spheres are shown in Fig. 1.40.

The charge on a sphere is positive if more lines leave than enter and negative if more enter than leave. The ratio of the magnitudes of the charges equals the ratio of the net number of lines entering or leaving.

Since 11 electric field lines leave the large sphere on the left and 3 enter, the net number leaving is 8, so that charge on the large sphere is positive. For the small sphere on the right, 8 lines leave and none enter, so its charge is also positive. Since the net number of lines leaving each sphere is 8, the spheres carry equal positive charges. The charge on the small sphere creates an intense field at the nearby surface of the large sphere that causes a local accumulation of negative charge on the large sphere-indicated by the three entering field lines, but its total charge is positive.

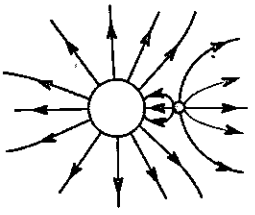


Fig. 1.40

Illustration 2 : Fig. 1.41 shows the sketch of field lines for two, point charges $2Q$ and $-Q$.

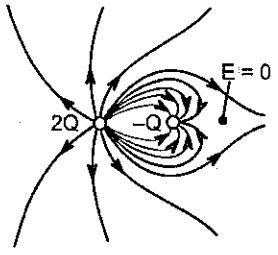


Fig. 1.41

The pattern of field lines can be deduced by considering the following points :

(a) *Symmetry* : For every point above the line joining the two charges there is an equivalent point below it. Therefore, the pattern must be symmetrical about the line joining the two charges.

(b) *Near field* : Very close to a charge, its field predominates. Therefore, the lines are radial and spherically symmetric.

(c) *Far field* : Far from the system of charges, the pattern should look like that of a single point charge of value $(2Q - Q) = +Q$, *i.e.*, the lines should be radially outward.

(d) *Null point* : There is one point at which $E = 0$. No lines should pass through this point.

(e) *Number of lines* : Twice as many lines leave $+2Q$ as enter $-Q$.

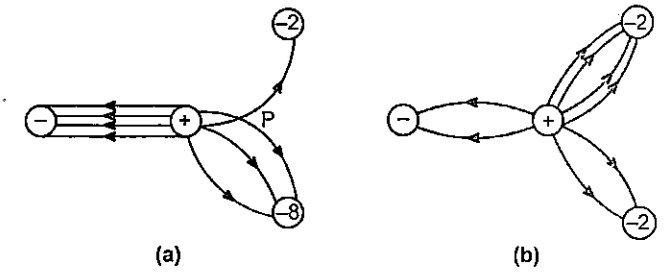


Fig. 1.42

Figs. 1.42(a) and (b) show the correctly and incorrectly drawn field lines for a collection of four charges $-e$, $4e$, $-2e$ and $+e$.

PROBLEM SOLVING TACTICS

Calculating the Total Electric Field from Multiple Charges

1. Locate each charge in Cartesian coordinate system. Locate the point P where we wish to compute the electric field.

2. For each charge draw an electric field vector at point P. The field vector is along the line joining the charge and point P. The field is directed away from charge q if it is positive; towards charge q if it is negative.

3. Determine the magnitude of each of the electric field contributions using

$$E = \frac{1}{4\pi\varepsilon_0}\frac{|q|}{r^2}$$

4. Determine sum of x- and y-components of each of the electric field vectors computed above.

5. Determine sum of x-components, ΣE_x and sum of y-components ΣE_y. The magnitude of the resultant field is.

$$|\vec{E}| = \sqrt{(\Sigma E_x)^2 + (\Sigma E_y)^2}$$

and angle of resultant field with the x-axis is

$$\tan\theta = \frac{E_y}{E_x}$$

Example 10

Calculate the electric field of a dipole at a point P located a distance x from the centre of the dipole along its axis as shown in Fig. 1E.10.

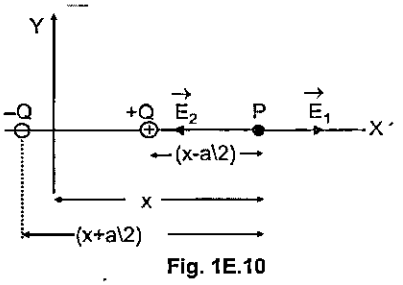


Fig. 1E.10

Solution: The positive charge creates a field $\vec{E}_1$ at P directed away from it, as shown in Fig. 1E.10. The negative charge creates a field $\vec{E}_2$ at P directed towards it, as shown in Fig. 1E.10. The resultant field at P is the vector sum of these fields.

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

$$\vec{E}_1 = \frac{1}{4\pi\varepsilon_0}\frac{|Q|}{(x-a/2)^2}\hat{i}$$

$$\vec{E}_2 = \frac{1}{4\pi\varepsilon_0}\frac{|Q|}{(x+a/2)^2}(-\hat{i})$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\left[\frac{|Q|}{(x-a/2)^2} - \frac{|Q|}{(x+a/2)^2}\right]\hat{i}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{|Q|}{x^2}\left[\left(1-\frac{a}{2x}\right)^{-2} - \left(1+\frac{a}{2x}\right)^{-2}\right]\hat{i}$$

If $x >> a$ we can use binomial expansion to simplify the above expression,

$$(1+x)^n \simeq 1+nx \qquad \text{for} \qquad x \ll 1$$

Thus,

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{|Q|}{x^2}\left\{\left[1+(-2)\left(-\frac{a}{2x}\right)\right] - \left[1+(-2)\left(\frac{a}{2x}\right)\right]\right\}\hat{i}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{|Q|}{x^2}\frac{2a}{x}\hat{i}$$

As magnitude of dipole moment vector $|\vec{P}| = |Q|a$, therefore the field is

$$\vec{E} \simeq \frac{1}{4\pi\varepsilon_0}\frac{2p}{x^3}\hat{i}$$

Example 11

Calculate the resultant electric field caused by an electric dipole at a point P in the plane that is the perpendicular bisector of the line connecting the two charges as shown in Fig. 1E.11.

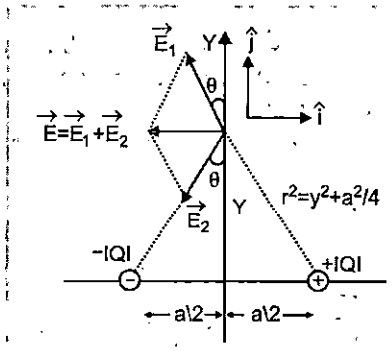


Fig. 1E.11

Solution: Since the charges are of same magnitude and the point P is equidistant from both the charges, the electric field of each of the two charges (as if they created field alone) at the point P has the same magnitude.

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{|Q|}{r^2}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{|Q|}{(y^2+a^2/4)}$$

$$\vec{E}_1 = \frac{1}{4\pi\varepsilon_0}\frac{|Q|}{(y^2+a^2/4)}[-(\sin\theta)\hat{i} + (\cos\theta)\hat{j}]$$

$$\vec{E}_2 = \frac{1}{4\pi\varepsilon_0}\frac{|Q|}{(y^2+a^2/4)}[(-\sin\theta)\hat{i} - (\cos\theta)\hat{j}]$$

$$\vec{E}_R = \vec{E}_1 + \vec{E}_2$$

$$= \left[\frac{-2}{4\pi\varepsilon_0}\frac{|Q|}{(y^2+a^2/4)}\sin\theta\right]\hat{i}$$

From Fig. 1E.17,

$$\sin\theta = \frac{a/2}{(y^2+a^2/4)^{1/2}}$$

also $\theta|\vec{P}| = Q\cdot a$

$$\vec{E}_R = \frac{-1}{4\pi\varepsilon_0}\frac{P}{(y^2+a^2/4)^{3/2}}\hat{i}$$

For $y >> a$, the a^2 term in the denominator in the expression for $\vec{E}_R$ may be neglected.

Therefore $$\vec{E}_R \simeq \frac{-1}{4\pi\varepsilon_0}\frac{P}{y^3}\hat{i}$$

Example 12

Three charges $-q, +2q$ and $-q$ are arranged on a line as shown in the Fig. 1E.12. Calculate the electric field at a distance $r > a$ on the line.

Solution: The field at point P is superposition of fields $\vec{E}_1, \vec{E}_2, \vec{E}_3$ due to each charge.

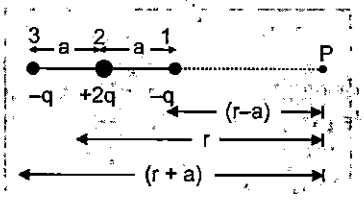


Fig. 1E.12

$$\vec{E}_1 = -\frac{|q|}{4\pi\varepsilon_0(r-a)^2}$$

$$\vec{E}_2 = +\frac{|2q|}{4\pi\varepsilon_0 r^2}$$

$$\vec{E}_3 = -\frac{|q|}{4\pi\varepsilon_0(r+a)^2}$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

$$= \frac{q}{4\pi\varepsilon_0}\left[-\frac{1}{(r-a)^2} + \frac{2}{r^2} - \frac{1}{(r+a)^2}\right]$$

$$= \frac{q}{4\pi\varepsilon_0 r^2}\left[-\left\{1-\left(\frac{a}{r}\right)^2\right\}^{-2} + 2 - \left\{1-\left(\frac{a}{r}\right)^2\right\}^{-2}\right]$$

If $r >> a$, we can use binomial approximation

$$(1+\alpha)^n \simeq 1+n\alpha \quad \text{for } \alpha << 1$$

Therefore, $$E = \frac{q}{4\pi\varepsilon_0 r^2}\left[\left(-1-\frac{2a}{r}\right)+2+\left(-1+\frac{2a}{r}\right)\right]$$

$$= \frac{6a^2 q}{4\pi\varepsilon_0 r^4}$$

The charges in this problem may be considered as two dipoles placed close together. Such on arrangement of charges is called an **electric quadrupole**.

Example 13

A particle of mass m and charge q is released at rest in a uniform field of magnitude E. The uniform field is created between two parallel plates of charge density $+\sigma$ and $-\sigma$ respectively. The particle accelerates towards the other plate a distance d away. Determine the speed at which it strikes the opposite plate.

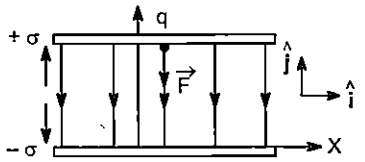


Fig. 1E.13

Solution: The applied electric field is

$$\vec{\mathbf{E}} = -E_0\hat{\mathbf{j}}$$

The force experienced by the charge q,

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}} = -qE_0\hat{\mathbf{j}}$$

The force is constant, and so the acceleration is constant as well

$$\vec{\mathbf{a}} = \frac{\vec{\mathbf{F}}}{m} = -\frac{qE_0}{m}\hat{\mathbf{j}}$$

Due to constant acceleration the particle moves in y-direction; the problem is analogous to motion of a mass released from rest in a gravitational field.

From equations of motion,

$$v_y = v_{y_0} + a_y t$$

$$= 0 - \frac{qE_0}{m}t \qquad \ldots(1)$$

and $$y = y_0 + v_{y_0}t + \frac{1}{2}a_y t^2$$

$$0 = d + 0 - \frac{1}{2}\frac{qE_0}{m}t^2 \qquad \ldots(2)$$

Particle starts at $y_0 = d$ and impact occurs at $y = 0$.

From eqn. (2), $$t = \left(\frac{2dm}{qE_0}\right)^{1/2}$$

From eqn. (1), $$v_y = -\frac{qE_0}{m}\left(\frac{2dm}{qE_0}\right)^{1/2}$$

Method 2. We can use work-energy theorem.

$$W = \Delta KE$$

They only force acting on the charge is the electric force

$$\vec{\mathbf{F}} = -qE_0\hat{\mathbf{j}}$$

$$W = \vec{\mathbf{F}}\cdot\Delta\vec{\mathbf{r}}$$

$$\Delta\vec{\mathbf{r}} = -d\hat{\mathbf{j}}$$

$$W = (-qE_0\hat{\mathbf{j}})\cdot(-d\hat{\mathbf{j}}) = qE_0 d$$

$$qE_0 d = \left(\frac{1}{2}mv_{y_f}^2 - \frac{1}{2}mv_{y_i}^2\right) = \frac{1}{2}mv_{y_f}^2$$

$$v_y = -\left(2d\frac{qE_0}{m}\right)$$

CONTINUOUS CHARGE DISTRIBUTION

We have learned earlier that charge is quantized (the discret nature of charge). Let us calculate the number of electrons or protons per coulomb, n .

$$n = \frac{1}{1.60\times10^{-19}}$$

$$= 6.25\times10^{18} \text{ particle/C}$$

This is a very large number. Suppose we have 1μC of charge spread over a distance of 1 km. The distance between adjacent charge will come out to be only 1.60×10^{-7} mm; so we can ignore the discrete nature of charge. We can assume charge distributions as they were continuous. There are infinite number of ways in which we can spread a continuous charge distribution over a region of space. Mainly three types of charge distributions will be used. We define three different **charge densities.**

Symbol		Definition	SI units
(lambda) λ	=	Charge per unit length	C/m
(sigma) σ	=	Charge per unit area	C/m^2
(rho) ρ	=	Charge per unit volume	C/m^3

If a total charge q is distributed along a line of length l, over a surface area A or throughout a value V, we can calculate charge densities from.

$$\lambda = \frac{q}{l}$$

$$\sigma = \frac{q}{A}$$

$$\rho = \frac{q}{V}$$

PROBLEM SOLVING TACTICS

Calculating the Total Electric Field From Continuous Charge Distributions

Concepts: *Use the principle of superposition whenever you need to calculate the electric field due to a charge distribution.*

1. *Make a drawing that clearly shows the locations of the charges and your choice of coordinate axes.*

2. *On your drawing, indicate the position of the field point (the point at which you want to calculate the electric field $\vec{E}$). Some-times the field point will be at some arbitrary position along a line. For example, you may be asked to find $\vec{E}$ at any point on the x-axis.*

3. *Take advantage of any symmetries in the charge distribution. For example, if a positive charge and a negative charge of equal magnitude are placed symmetrically with respect to the field point, they produce electric fields of the same magnitude but with mirror-image directions. Exploiting these symmetries will simplify your calculations.*

In working out the directions of $\vec{E}$ vectors, be careful to distinguish between the source point and field point. The field produced by a point charge always points from source point to field point if the charge is positive.

(i) Identify the type of charge distribution and compute the charge density λ, σ or ρ.

(ii) Divide the charge distribution into infinitesimal charges dq. each of which will act as a tiny point charge.

(iii) The amount of charge dq. i.e., within a small element dl, dA or dV is

$dq = \lambda\, dl$ (charge distributed in length)

$dq = \sigma\, dA$ (charge distributed over a surface)

$dq = \rho\, dV$(charge distributed throughout a volume)

(iv) Draw at point P the dE vector produced by the charge dq. The magnitude of dE is

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}$$

Vector dE is along radial line joining dq to P, dE is directed away for positive charge dq, while directed towards dq for negative dq.

(v) Resolve the dE vector into its components. Identify any special symmetry features to show whether any component vanishes. Write an expression for the component(s) of the field that are not cancelled by other components.

(vi) Write the distance r and any trigonometric factors in terms of given coordinates and parameters.

(vii) The electric field is obtained by summing over all the infinitesimal contributions.

$$\vec{E} = \int d\vec{E} = \int \frac{dq}{4\pi\varepsilon_0 r^2}$$

(viii). Perform the indicated integration over limit or integration that include all the source charges.

Example 14

A charged wire of length L lies along x-axis in such a way that its linear charge density is given by

$$\lambda = ax^2$$

where a is constant. Determine the total charge on the wire.

Solution: We consider a different element *dx* on the wire a distance *x* from origin. Charge on this differential element.

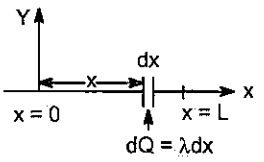


Fig. 1E.14

$$dq = \lambda\, dx$$

or $$q = \int_0^L \lambda dx = \int_0^L ax^2\, dx$$

$$= \left[\frac{ax^3}{3}\right]_0^L = \frac{aL^3}{3}$$

Example 15

Charge is distributed throughout a spherical region of space in such a manner that its volume charge density is given by

$$\rho = ar^2, \qquad 0 \le r \le R$$

where a is a constant. Find the total charge within the sphere.

Solution: Since charge distribution is volumetric we choose a small volume element *dV* within the sphere. Because the charge distribution is spherically symmetric, we choose a thin spherical shell of radius *r* and thickness *dr*. We can think of entire spherical charge to be made up of concentric shells. Area of sphere is $4\pi r^2$.

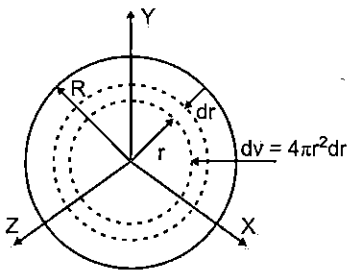


Fig. 1E.15

Therefore, $dV = 4\pi r^2 dr$

$$q = \int \rho\, dV = \int_0^R (ar^2)(4\pi r^2 dr) = \frac{4}{5}\pi a r^5$$

Example 16

A ring of radius R has a total charge Q distributed uniformly along its circumference,as shown in Fig. 1E.16(a). Calculate the electric field of the ring at a point P along the axis of the ring, a distance z from the plane of the ring.

Solution: Linear charge density λ of the ring,

$$\lambda = \frac{Q}{2\pi R}$$

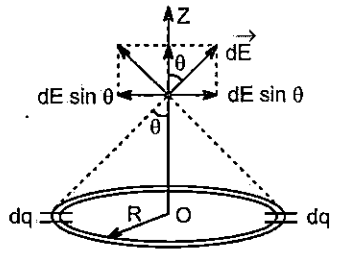


Fig. 1E.16 (a)

We consider a differential element ds on the ring, which has charge $dq = \lambda\, ds$. This point-like charge creates a field at the axial point P. The magnitude of electric field $d\vec{\mathbf{E}}$ due to charge dq,

$$d\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{x\,ds}{r^2}$$

A corresponding element dq on the opposite side will create field of same magnitude. Note that the components of the two differential fields perpendicular to the axis will cancel, but the components of the field along axis add. Similarly we can imagine the ring to consist of a large number of such paired differential fields, whose contribution to the field is along axis only.

$$d\vec{\mathbf{E}}_R = \left(\frac{1}{4\pi\varepsilon_0}\frac{\lambda\, ds}{r^2}\cos\theta\right)\hat{\mathbf{k}}$$

$$\vec{\mathbf{E}}_R = \int d\vec{\mathbf{E}}_R = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\cos\theta}{r^2}\int_{\text{ring}} ds\,\hat{\mathbf{k}}$$

$$= \left(\frac{1}{4\pi\varepsilon_0}\frac{\lambda\cos\theta}{r^2}2\pi R\right)\hat{\mathbf{k}}$$

From Fig. 22(a),

$$\cos\theta = \frac{z}{(z^2+R^2)^{1/2}}$$

and $$r^2 = z^2 + R^2$$

Therefore $$\vec{\mathbf{E}}_R = \frac{1}{4\pi\varepsilon_0}\frac{Qz}{(z^2+R^2)^{3/2}}\hat{\mathbf{k}}$$

For positively charged ring, and for $z > 0$, the field points along $+\hat{\mathbf{k}}$; for $z < 0$ the field points along $-\hat{\mathbf{k}}$. In both the cases it is directed away from the positively charged ring.

Similarly for a negatively charged ring, the field points towards the ring.

For $z = 0, E_R = 0$, this conclusion may be arrived at from symmetry considerations.

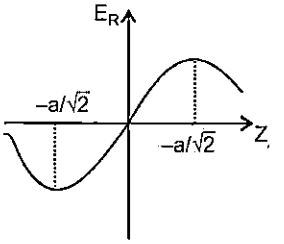


Fig. 1E.16 (b)

For $z >> R$ we can neglect R^2 compared to z^2 in the denominator of eqn. (1).

$$\vec{\mathbf{E}}_R = \frac{1}{4\pi\varepsilon_0}\frac{Q}{z^2}\hat{\mathbf{k}}$$

which is field of a point-like charge.

Variation of electric field on the axis is shown in Fig. 1E.16 (b) with maximum magnitude of field at $z = \pm a/\sqrt{2}$.

IMPORTANT GRAPHS

(i) Field of a point charge

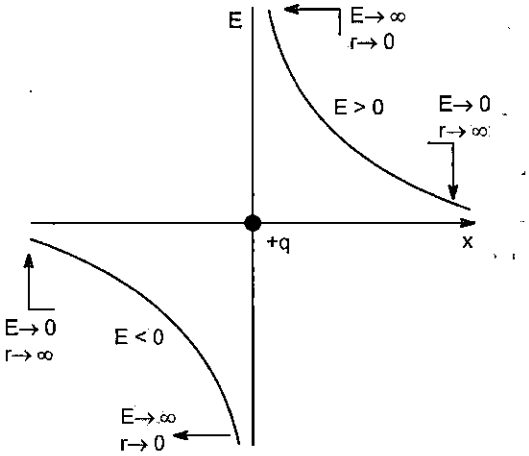


Fig. 1.43 (a)

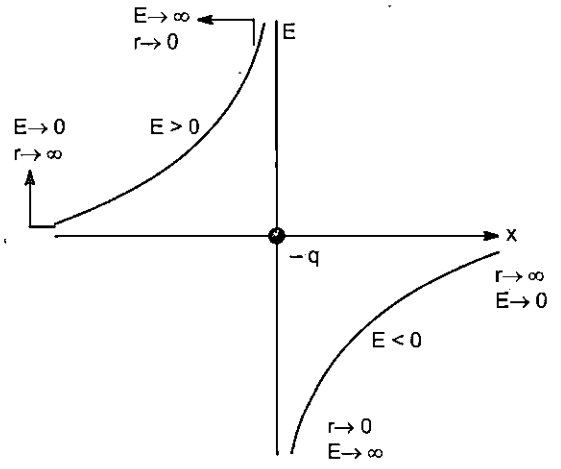


Fig. 1.43 (b)

Check 1. Direction of field on x axis for a positive charge ($q > 0$) field in the region $-\infty < x < 0$ is along negative x direction therefore $E < 0$

For a positive charge ($q > 0$) field in the region $0 < x < \infty$ therefore $E > 0$

Similarly for a negative charge ($q < 0$) field in the region $-\infty < x < 0$ is along positive x direction therefore $E > 0$ and along negative x direction in region $\infty < x < 0$ therefore $E < 0$.

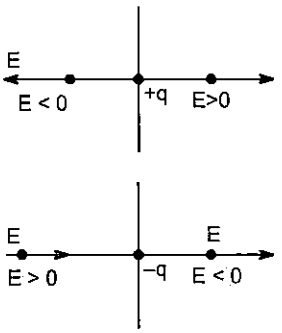


Fig. 144

Check 2. For $x \longrightarrow 0$; $E \longrightarrow \infty$ as $E = \dfrac{Kq}{r^2}$

Check 3. For $x \longrightarrow \infty$; $E \longrightarrow 0$, as $E = \dfrac{Kq}{r^2}$

(ii) Two positive charges are placed at $x = -a$ and $x = +a$. Figure shows graph of electric field on axis due to given charges.

Check 1. In region I field due to left charge dominates. Resultant field will have same nature as that of a single point charge

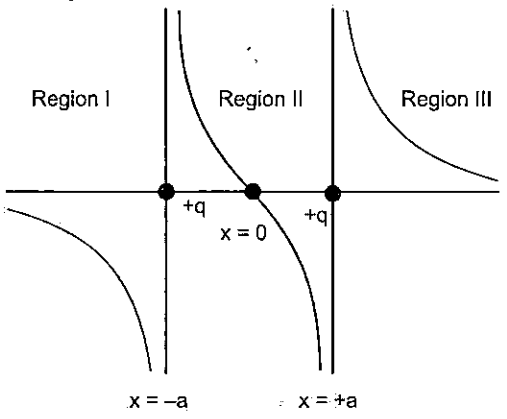


Fig. 1.45

Similarly in region III field due to right charge dominates.

Check 2. In region II from $-a < x < 0$ field of left charge dominates and $0 < x < a$ field of right charge dominates. At $x = 0$ resultant field is zero.

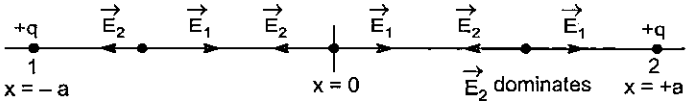


Fig. 1.46

Check 3. For points very close to charges field tends to infinity and at large distance field tends to zero.

Concepts: *If we place a point charge on axis so that it is in equilibrium, determine its location.*

At $x = 0$, resultant field is zero from graph.
Is equilibrium stable or unstable ?
Does it depend on sign of charge ?

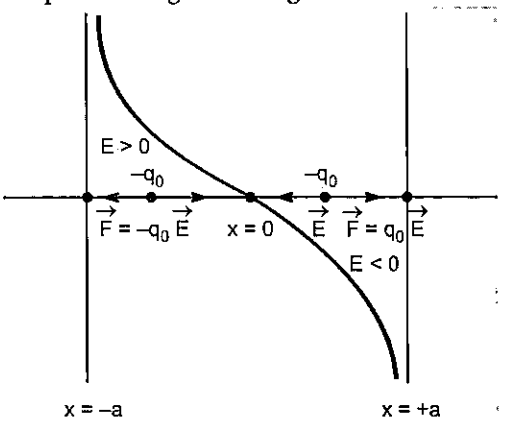


Fig. 1.47

Charge	Position	Field	Force	Nature of equilibrium
$q < 0$	$x < 0$	$E > 0$	$F > 0$	Unstable
	$x > 0$	$E < 0$	$F > 0$	

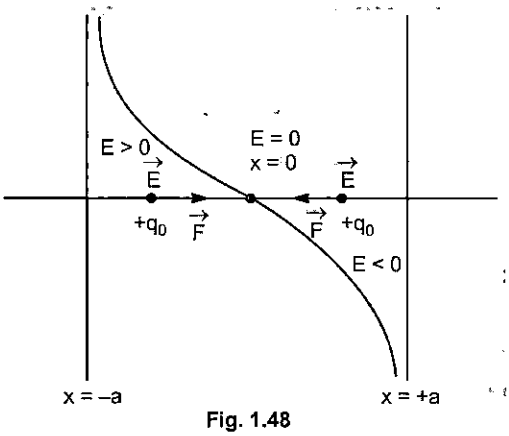


Fig. 1.48

Charge	Position	Field	Force	Nature of equilibrium
$q > 0$	$x < 0$	$E > 0$	$F > 0$	Stable
	$x > 0$	$E < 0$	$F < 0$	

(iii) Two Positive point charge placed at $x = +a$ and $x = -a$. Draw variation of E along y-axis:

$$E_y = E_{\text{rsultant}} = 2E\cos\theta = \frac{2Kqy}{(y^2 + a^2)^{3/2}}$$

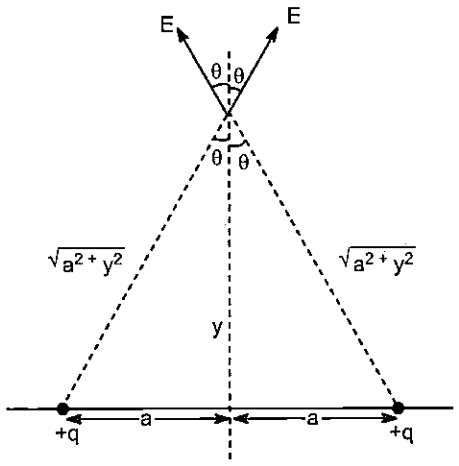


Fig. 1.49

$$\frac{dE_y}{dy} = 0$$

Gives $y = \pm\frac{a}{\sqrt{2}}$

and $E_{y_{max}} = \sqrt{\frac{8}{27}}\frac{Kq}{a^2}$

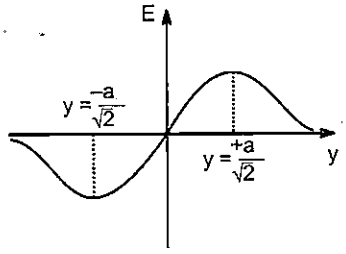


Fig. 1.50

If a positive test charge q_0 is placed on y-axis, plot variation its acceleration with y. Plot the same variation for a negative point charge.

(iv) Four positive charges are placed on the vertex of a square as shown in Fig. 1.51. Field on z-axis is given by

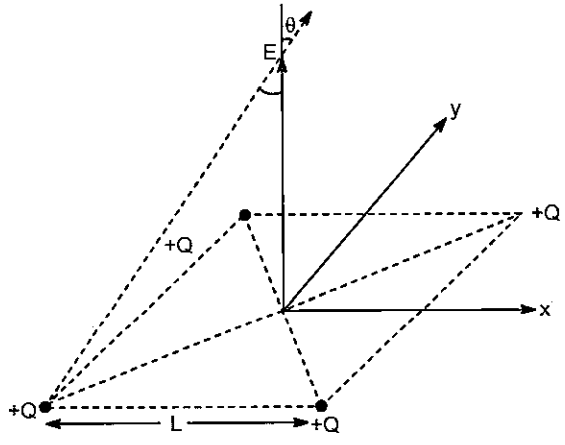


Fig. 1.51

$E_z = 4E\cos\theta$ where E is field due to single charge.

$$E_z = \frac{4Kqz}{\left[\left(\frac{L}{\sqrt{2}}\right)^2 + z^2\right]^{3/2}}$$

Can you plot variation E_z with the help of case (iii)

(v) Field on the axis of a charged ring:

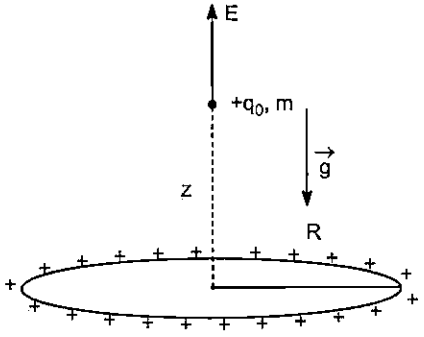


Fig. 1.52

$$E_z = \frac{Qz}{4\pi\varepsilon_0(R^2+z^2)^{3/2}}$$

$$E_{z_{max}} = \frac{Q}{6\sqrt{3}\pi\varepsilon_0 R^2}$$

at $z = \pm\frac{R}{\sqrt{2}}$

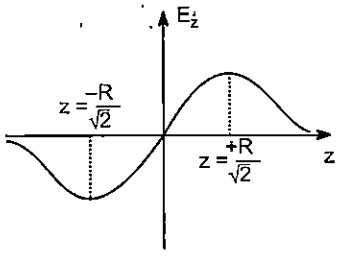


Fig. 1.53

Concept: *Suppose a possible charge q_0 is placed on z-axis, with gravitational force along $z < 0$. Charge may be in equilibrium. Discuss nature of equilibrium if equilibrium occurs at*

(i) $z < \frac{R}{\sqrt{2}}$

(ii) $z > \frac{R}{\sqrt{2}}$

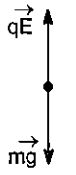


Fig. 1.54

(i) If charge is placed at $z < \frac{R}{\sqrt{2}}$

If charge is displaced up magnitude of field increases, charge will be pushed up.

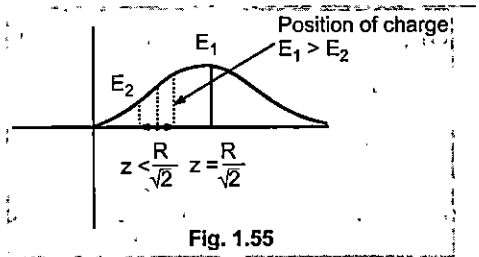
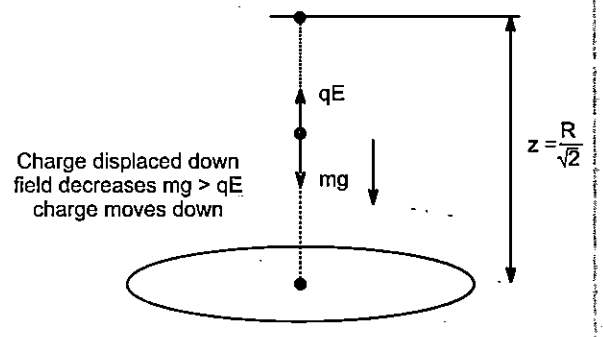
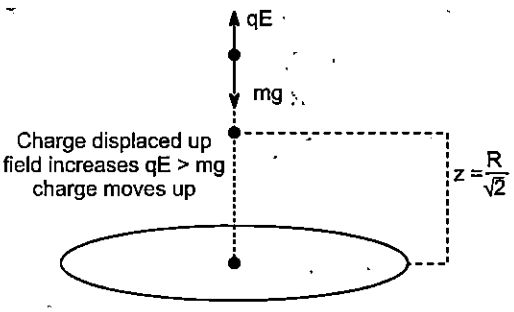


Fig. 1.55

If charge is displaced down magnitude of field decreases, charge will move down.



(a)



(b)

Fig. 1.56

Nature of equilibrium is unstable equilibrium

If charge is placed at $Z > \frac{R}{\sqrt{2}}$ and it is in equilibrium.

When charge is displaced up field decreases gravitational force will bring charge down. When charge is displaced up field increases, electric pushes charge upward.

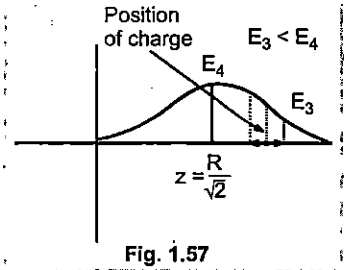


Fig. 1.57

Fig. 1.58 shows *E versus x* graph for two point charges +Q and −4Q placed at $x = +a$ and $x = -a$ respectively.

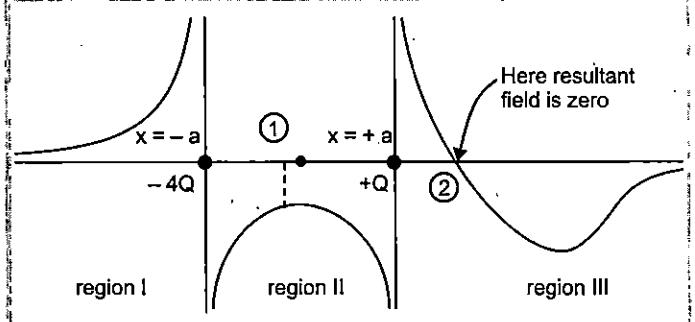


Fig. 1.58

1. Determine the position (2) where resultant field is zero.

2. Place a positive test charge at position (2). Will it be in equilibrium at this position.

Is this equilibrium stable or unstable?

3. Place a negative test charge at position (2). Is its equilibrium stable on unstable?

4. Is there any position in region II where a test charge can be in equilibrium?

5. Is there any position in region I where any test charge can be in equilibrium.

Example 17

Show that a uniform electric field $\vec{E}$ exerts a torque τ about the midpoint of the dipole that is given by

$$\vec{\tau} = \vec{p} \times \vec{E}$$

where $\vec{p}$ is th dipole moment vector.

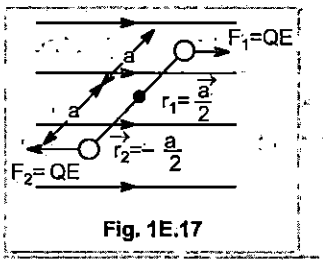


Fig. 1E.17

Solution: Each charge of the dipole experiences an electric force.

The force on the positive charge,

$$\vec{F}_1 = +Q\vec{E}$$

and

$$\vec{F}_2 = -Q\vec{E}$$

Torque of a force is given by

$$\vec{\tau} = \vec{r} \times \vec{F}$$

Where $\vec{r}$ is the position vector locating the point of application of the force w.r.t. the point about which the torque is taken. According to the problem we have to take torque about mid-point.

Thus, $\vec{r}_1 = \frac{\vec{a}}{2}$, $\vec{r}_2 = -\frac{\vec{a}}{2}$

$$\vec{\tau}_1 = \frac{\vec{a}}{2} \times \vec{F}_1 = \frac{\vec{a}}{2} \times (Q\vec{E})$$

$$\vec{\tau}_2 = -\frac{\vec{a}}{2} \times \vec{F}_2$$

$$= -\frac{\vec{a}}{2} \times (-Q\vec{E}) = \frac{\vec{a}}{2} \times (Q\vec{E})$$

Total torque,

$$\vec{\tau} = \vec{\tau}_1 + \vec{\tau}_2$$

$$= \frac{\vec{a}}{2} \times (Q\vec{E}) + \frac{\vec{a}}{2} \times (Q\vec{E})$$

$$= Q\vec{a} \times \vec{E} = \vec{p} \times \vec{E}$$

Example 18

Find the electric field at a distance z along the axis of a uniformly charged circular disc of radius R and charge Q.

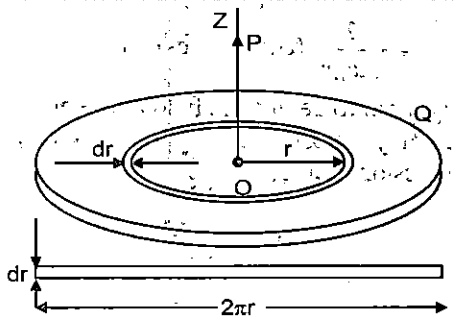


Fig. 1E.18 (a)

Solution: The surface charge density of disc,

$$\sigma = \frac{Q}{\pi R^2}$$

We consider a differential ring of radius r and thickness dr on the disc.

If we cut the ring and unwrap the ring into a strip, the differential area dt of the strip, $dA = 2\pi r dr$

The charge dq on this strip $= \sigma dA = (2\pi r dr)\sigma$.

The electric field of this ring at point P is along the axis (as obtained in Example 16)

$$d\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{z(2\pi r dr)\sigma}{(z^2 + r^2)^{3/2}} \hat{k}$$

Now we integrate it over the entire disc.

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} z(2\pi\sigma) \int_0^R \frac{r\,dr}{(z^2 + r^2)^{3/2}} \hat{k}$$

$$= \left[\frac{1}{4\pi\varepsilon_0} (z \cdot 2\pi\sigma) \frac{-1}{(z^2 + r^2)^{1/2}} \right]_0^R \hat{k}$$

$$= \frac{1}{4\pi\varepsilon_0} 2\pi\sigma \left[1 - \frac{z}{(z^2 + R^2)^{1/2}} \right] \hat{k}$$

* Note that for $Q > 0$, the field is along the axis directed away from the disc; for $Q < 0$ it is directed along the axis towards the disc.
* From binomial expansion,

$$(1+\alpha)^n \simeq 1 + n\alpha \quad \text{for} \quad \alpha << 1$$

$$\frac{z}{\sqrt{z^2 + R^2}} = \frac{z}{z(1 + R^2/z^2)^{1/2}}$$

$$= \left(1 + \frac{R^2}{z^2}\right)^{-1/2} \simeq 1 - \frac{R^2}{2z^2} + \ldots$$

$$E_2 \simeq \frac{1}{4\pi\varepsilon_0} 2\pi\sigma \left(1 - 1 + \frac{R^2}{2z^2} + \ldots\right)$$

$$= \frac{Q}{4\pi\varepsilon_0 z^2}$$

where $Q = \sigma\pi R^2$, the total charge on the disc.

* We can find the electric field of the uniformly charged infinite sheet by letting the radius R of the disc approach infinity.

$$E = \lim_{R\to\infty} \frac{1}{4\pi\varepsilon_0} 2\pi\sigma \left[1 - \frac{z}{(z^2 + R^2)^{1/2}} \right]$$

$$= \frac{\sigma}{2\varepsilon_0}$$

Field is independent of distance from sheet, therefore a uniform electric field is created. If the charge on the sheet is positive, the field is directed perpendicularly away from the sheet, as shown in Fig. 1E.18(b).

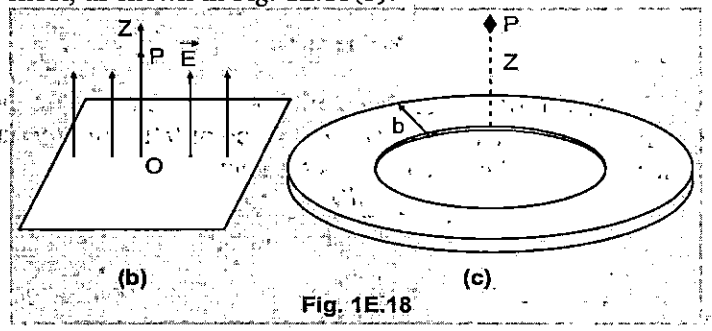


Fig. 1E.18

* Consider an annulus disc of inner radius a and outer radius b [as shown in Fig. 1E.18(c)]. We can determine electric field on its axis by integrating expression for $d\vec{E}$ in limits $r = a$ to $r = b$.

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} z(2\pi\sigma) \int_a^b \frac{r\,dr}{(z^2 + r^2)^{3/2}} \hat{k}$$

$$= \frac{1}{4\pi\varepsilon_0} \left[(z \cdot 2\pi\sigma) \frac{-1}{(z^2 + r^2)^{1/2}} \right]_a^b \hat{k}$$

$$= \frac{\sigma z}{2\varepsilon_0} \left[\frac{1}{(z^2 + a^2)^{1/2}} - \frac{1}{(z^2 + b^2)^{1/2}} \right]$$

* Consider an infinite sheet charged with uniform charge density σ with a circular hole of radius R. We

can determine field on z-axis by integrating expression for $d\vec{\mathbf{E}}$ in limits $r = R$ to $r = \infty$.

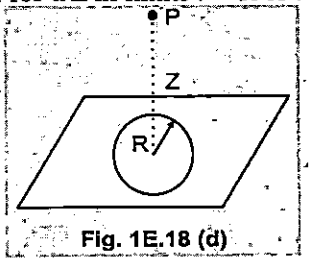


Fig. 1E.18 (d)

$$\vec{\mathbf{E}} = \frac{\sigma z}{2\varepsilon_0}\int_R^\infty \frac{r\,dr}{(z^2+r^2)^{3/2}}\hat{\mathbf{k}}$$

$$= \frac{\sigma z}{2\varepsilon_0}\left[\frac{-1}{(z^2+r^2)^{1/2}}\right]_R^\infty \hat{\mathbf{k}}$$

$$= \frac{\sigma z}{2\varepsilon_0}\left[\frac{1}{(z^2+R^2)^{1/2}}\right]$$

Example 19

Calculate the electric field intensity generated by a straight infinitely long wire, uniformly charged with a linear density λ, at a point distant r_0 from the wire.

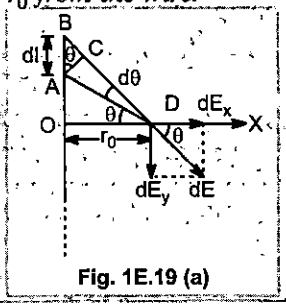


Fig. 1E.19 (a)

Solution: We consider a differential element dl on the string, carrying a charge $dQ = \lambda\,dl$. At point D the differential electric field dE created by this element,

$$dE = \frac{dQ}{4\pi\varepsilon_0 r^2} = \frac{\lambda\,dl}{4\pi\varepsilon_0 r^2}$$

From ΔAOD, $\quad r = \dfrac{r_0}{\cos\theta}$,

$$AC = r\,d\theta = \frac{r_0\,d\theta}{\cos\theta}$$

From ΔABC, $\quad dl = \dfrac{AC}{\cos\theta} = \dfrac{r_0\,d\theta}{\cos^2\theta}$

Alternatively, $\quad l = r_0\tan\theta$

$$dl = r_0\sec^2\theta\,d\theta$$

So, $\quad dE = \dfrac{\lambda\,d\theta}{4\pi\varepsilon_0 r_0}$

Field dE has components dE_x, dE_y given by

$$dE_x = \frac{\lambda\cos\theta\,d\theta}{4\pi\varepsilon_0 r_0}$$

$$dE_y = \frac{\lambda\sin\theta\,d\theta}{4\pi\varepsilon_0 r_0}$$

On integrating expression for dE_x and dE_y in limits $\theta = -\pi/2$ to $\theta = +\pi/2$ we obtain E_x and E_y. Note that as the length of wire increases, the angle θ increases; for a very long wire, it approaches $\pi/2$.

$$E_x = \int_{-\pi/2}^{+\pi/2}\frac{\lambda\cos\theta\,d\theta}{4\pi\varepsilon_0 r_0} = \frac{\lambda}{2\pi\varepsilon_0 r_0}$$

$$E_y = \int_{-\pi/2}^{+\pi/2}\frac{\lambda\sin\theta\,d\theta}{4\pi\varepsilon_0 r_0} = 0$$

Thus $\quad E = E_x = \dfrac{\lambda}{2\pi\varepsilon_0 r_0}$

* If the wire has finite length and the angle subtended by ends of wire at a point are θ_1 and θ_2, the limits of integration would change.

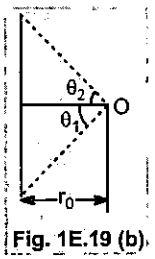


Fig. 1E.19 (b)

$$E_x = \int_{-\theta_1}^{+\theta_2}\frac{\lambda\cos\theta\,d\theta}{4\pi\varepsilon_0 r_0}$$

$$= \frac{\lambda}{4\pi\varepsilon_0 r_0}(\sin\theta_1 + \sin\theta_2)$$

$$E_y = \int_{-\theta_1}^{+\theta_2}\frac{\lambda\sin\theta\,d\theta}{4\pi\varepsilon_0 r_0}$$

$$= \frac{\lambda}{4\pi\varepsilon_0 r_0}(\cos\theta_1 - \cos\theta_2)$$

* If we wish to determine field at the end of a long wire, we may substitute $\theta_1 = 0$ and $\theta_2 = \pi/2$ in the expressions for E_x and E_y.

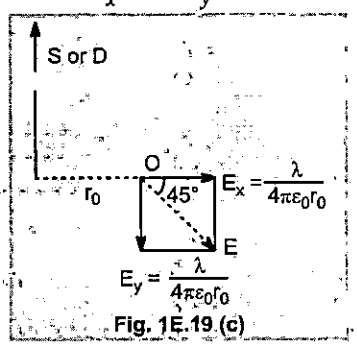


Fig. 1E.19 (c)

$$E_x = \frac{\lambda}{4\pi\varepsilon_0 r_0}\left[\sin(0) + \sin\left(\frac{\pi}{2}\right)\right] = \frac{\lambda}{4\pi\varepsilon_0 r_0}$$

$$E_y = \frac{\lambda}{4\pi\varepsilon_0 r_0}\left[\cos(0) + \cos\left(\frac{\pi}{2}\right)\right] = \frac{\lambda}{4\pi\varepsilon_0 r_0}$$

Magnitude of resultant field $\vec{\mathbf{E}}$:

$$|\vec{\mathbf{E}}| = \sqrt{E_x^2 + E_y^2} = \frac{\sqrt{2}\lambda}{4\pi\varepsilon_0 r_0}$$

$\vec{\mathbf{E}}$ makes an angle θ with the x-axis, where

$$\tan\theta = \frac{|E_y|}{|E_x|} = 1$$

or $\theta = 45°$

* If point lies on perpendicular bisector,

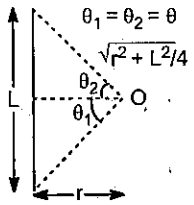


Fig. 1E.19 (d)

$$E_x = \frac{\lambda}{4\pi\varepsilon_0 r}[\sin\theta + \sin\theta]$$

$$= \frac{2\lambda\sin\theta}{4\pi\varepsilon_0 r} = \frac{2\lambda}{4\pi\varepsilon_0 r}\left[\frac{L/2}{\sqrt{r^2 + L^2/4}}\right]$$

$$= \frac{\lambda L}{4\pi\varepsilon_0\sqrt{r^2 + L^2/4}}$$

$$E_y = \frac{\lambda}{4\pi\varepsilon_0 r}[\cos\theta - \cos\theta] = 0$$

Thus $$\vec{\mathbf{E}} = E_x\hat{\mathbf{i}} = \frac{\lambda L}{4\pi\varepsilon_0\sqrt{r^2 + L^2/4}}\hat{\mathbf{i}}$$

Example 20

A segment of a charged wire of length l, charge density λ_2, and an infinitely long charged wire, charge density λ_1, lie in a plane at right angles to each other. The separation between the wires is r_0. Determine the force of interaction between the wires.

Solution: Electric field near a long wire is given by the expression :

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

The second wire lies in the non-uniform field of first wire. Each element of second wire experiences different magnitude of field. Therefore we consider a differential element dx, charge $dQ = \lambda_2 dx$, at a distance x from the long wire. The force acting on this element dF is

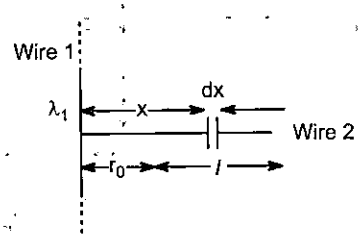


Fig. 1E.20

$$dF = EdQ = \left(\frac{\lambda_1}{2\pi\varepsilon_0 x}\right)\lambda_2 dx$$

The force acting on each element depends on x, the separation between wire 1 and 2.

Integrating the expression for dF in the limits $x = r_0$ to $x = r_0 + l$, we obtain

$$F = \int_{r_0}^{r_0+l}\frac{\lambda_1\lambda_2}{2\pi\varepsilon_0}\frac{dx}{x} = \frac{\lambda_1\lambda_2}{2\pi\varepsilon_0}\ln\left(1 + \frac{l}{r_0}\right)$$

Example 21

A uniformly charged wire, linear charge density λ, is laid in the form of a semicircle of radius R. Find the electric field generated by the semicircle at the centre.

Solution: We consider a differential element dl on the ring, that subtends an angle $d\theta$ at the centre of the ring.

$$dl = R\,d\theta$$

This element creates a field dE which makes an angle θ at the centre as shown in Fig. 1E.21. For each differential element in the upper half of the ring, there corresponds a symmetrically placed charge in the lower half plane. The y-components of field due to these symmetric elements cancel out, and x-components remain.

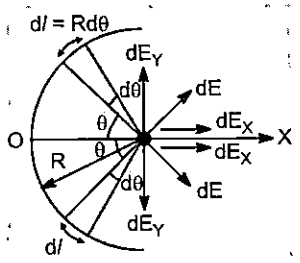


Fig. 1E.21 (a)

$$dE_x = dE\cos\theta = \frac{dQ}{4\pi\varepsilon_0 R^2}\cos\theta$$

$$= \frac{\lambda(R\,d\theta)\cos\theta}{4\pi\varepsilon_0 R^2}$$

On integrating the expression for dE_x w.r.t. angle θ, in limits $\theta = -\pi/2$ to $\theta = +\pi/2$, we obtain

$$E = \int_{-\pi/2}^{+\pi/2}\frac{\lambda R}{4\pi\varepsilon_0 R^2}\cos\theta\,d\theta = \frac{\lambda}{2\pi\varepsilon_0 R}$$

In terms of charge,

$$\lambda = \frac{Q}{\pi R}\ ;\ E = \frac{Q}{2\pi^2\varepsilon_0 R^2}$$

* If we consider the wire in the form of an arc as shown in the figure, the symmetry consideration does not apply in this case. We will integrate dE_x as well as dE_y in limits $\theta = -\theta_1$ to $\theta = +\theta_2$.

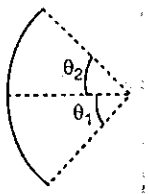


Fig. 1E.21 (b)

$$E_x = \int_{-\theta_1}^{+\theta_2}\frac{\lambda R}{4\pi\varepsilon_0 R^2}\cos\theta\,d\theta$$

$$= \frac{\lambda}{4\pi\varepsilon_0 R}(\sin\theta_1 + \sin\theta_2)$$

$$E_y = \int_{-\theta_1}^{+\theta_2}\frac{\lambda R}{4\pi\varepsilon_0 R^2}\sin\theta\,d\theta$$

$$= \frac{\lambda}{4\pi\varepsilon_0 R}(\cos\theta_1 - \cos\theta_2)$$

For a symmetrical arc, $\theta_1 = \theta_2$. Thus E_y vanishes and $$E_x = \frac{\lambda\sin\theta}{2\pi\varepsilon_0 R}$$

Example 22

A long wire with a uniform charge density λ is bent in two configurations shown in Figs. 1E.22(a) and (b). Determine the electric field intensity at point O.

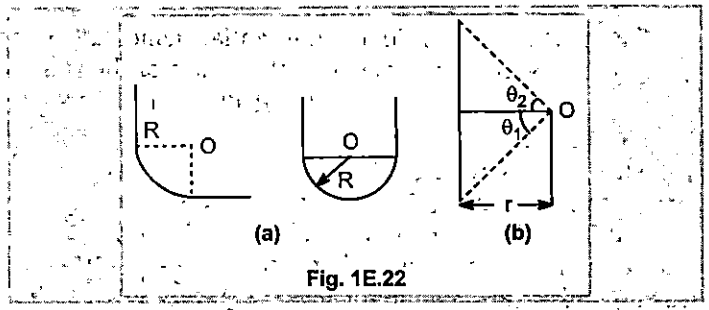


Fig. 1E.22

Solution: (a) We may divide the given configuration into three parts, two straight wires and an arc.

Field at a distance r from a wire is given by the expression

$$E_x = \frac{\lambda}{4\pi\varepsilon_0 r}(\sin\theta_1 + \sin\theta_2)$$

$$E_y = \frac{\lambda}{4\pi\varepsilon_0 r}(\cos\theta_1 - \cos\theta_2)$$

Field at the centre of arc-shaped wire is given by

$$E_x = \frac{\lambda}{4\pi\varepsilon_0 r}(\sin\theta_1 + \sin\theta_2)$$

$$E_y = \frac{\lambda}{4\pi\varepsilon_0 r}(\cos\theta_1 - \cos\theta_2)$$

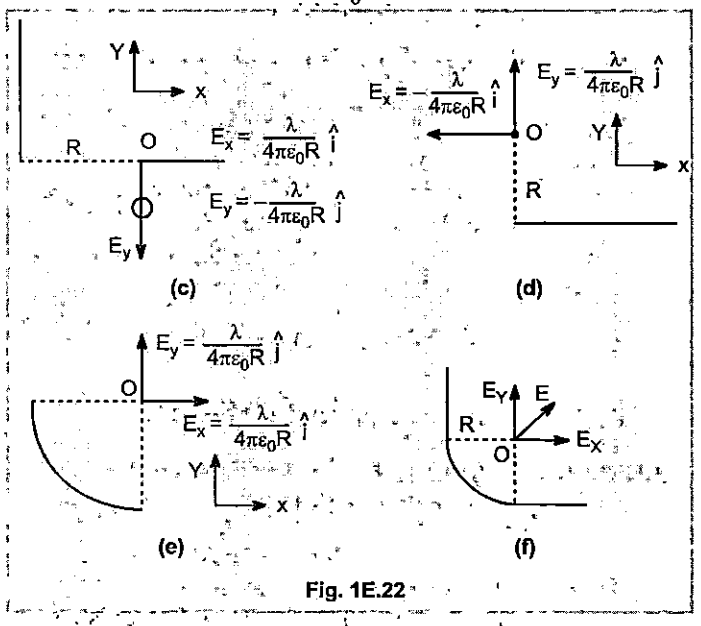


Fig. 1E.22

Resultant field is superposition of fields due to each part.

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

For vertical wire the field $\vec{E}_1$ is as shown in Fig. 1E.22(c). Here $\theta_1 = 0$ and $\theta_2 = \pi/2$.

Therefore

$$\vec{E}_1 = \left(\frac{\lambda}{4\pi\varepsilon_0 r}\right)\hat{i} + \left(-\frac{\lambda}{4\pi\varepsilon_0 r}\right)\hat{j}$$

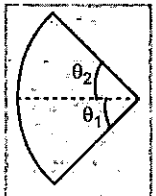


Fig. 1E.22 (g)

For horizontal wire,

$$\vec{E}_2 = \left(-\frac{\lambda}{4\pi\varepsilon_0 r}\right)\hat{i} + \left(\frac{\lambda}{4\pi\varepsilon_0 r}\right)\hat{j}$$

For arc-shaped wire, $\theta_1 = 0, \theta_2 = \pi/2$

$$\vec{E}_3 = \left(\frac{\lambda}{4\pi\varepsilon_0 R}\right)\hat{i} + \left(\frac{\lambda}{4\pi\varepsilon_0 R}\right)\hat{j}$$

Thus, $$\vec{E} = \left(\frac{\lambda}{4\pi\varepsilon_0 R}\right)\hat{i} + \left(\frac{\lambda}{4\pi\varepsilon_0 R}\right)\hat{j}$$

$$|\vec{E}| = \left[\left(\frac{\lambda}{4\pi\varepsilon_0 R}\right)^2 + \left(\frac{\lambda}{4\pi\varepsilon_0 R}\right)^2\right]^{1/2} = \frac{\sqrt{2}\lambda}{4\pi\varepsilon_0 R}$$

Here, $E_x = E_y = \frac{\lambda}{4\pi\varepsilon_0 R}$, hence the resultant field will make an angle of 45° with the x-axis.

(b) Field due to segment 1,

$$\vec{E}_{x_1} = \frac{\lambda}{4\pi\varepsilon_0 R}\hat{i}$$

$$\vec{E}_{y_1} = -\frac{\lambda}{4\pi\varepsilon_0 R}\hat{j}$$

$$\vec{E}_1 = \frac{\lambda}{4\pi\varepsilon_0 R}[\hat{i} - \hat{j}]$$

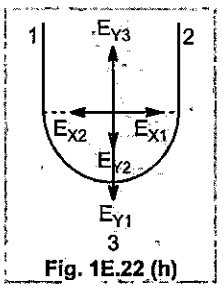


Fig. 1E.22 (h)

Field due to segment 2,

$$\vec{E}_{x_2} = -\frac{\lambda}{4\pi\varepsilon_0 R}\hat{i}$$

$$\vec{E}_{y_2} = -\frac{\lambda}{4\pi\varepsilon_0 R}\hat{j}$$

$$\vec{E}_2 = -\frac{\lambda}{4\pi\varepsilon_0 R}[\hat{i} + \hat{j}]$$

Field due to segment 3,

$$\vec{E}_{x_3} = 0; \quad \vec{E}_{y_3} = \frac{\lambda}{2\pi\varepsilon_0 R}\hat{j}$$

$$\vec{E}_3 = \frac{\lambda}{2\pi\varepsilon_0 R}\hat{j}$$

From principle of superposition of electric field.

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

$$= \frac{\lambda}{4\pi\varepsilon_0 R}[\hat{i} - \hat{j}] - \frac{\lambda}{4\pi\varepsilon_0 R}[\hat{i} + \hat{j}] + \frac{\lambda}{2\pi\varepsilon_0 R}\hat{j} = 0$$

Example 23

A thin charged ring of radius R, charge Q and a long, uniformly charged wire, charge density λ, oriented along the axis of the ring with one of its ends coinciding with the centre of the ring, form a system. Find the force of interaction between the ring and the thread.

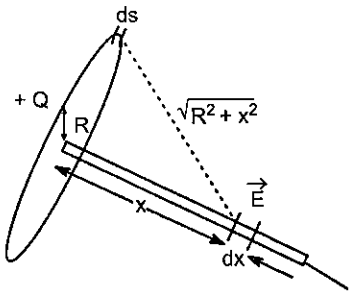


Fig. 1E.23 (a)

Solution: Proceeding similar to Example 20, we consider a differential element dx on the wire, charge $dQ = \lambda dx$. Field due to ring on its axis is given by the expression

$$|E| = \frac{Qx}{4\pi\varepsilon_0 (R^2 + x^2)^{3/2}}$$

Force, dF, on the differential element is

$$dF = Edq = E\lambda dx = \frac{1}{4\pi\varepsilon_0} \frac{Qx}{(R^2 + x^2)^{3/2}} \lambda dx$$

$$F = \frac{Q\lambda}{4\pi\varepsilon_0} \int_0^\infty \frac{x\,dx}{(R^2 + x^2)^{3/2}}$$

$$= -\frac{Q\lambda}{4\pi\varepsilon_0} \left[\frac{1}{\sqrt{R^2 + x^2}} \right]_0^\infty = \frac{Q\lambda}{4\pi\varepsilon_0 R}$$

Method 2 : Consider two symmetrically placed differential charge elements. Force on the elements due to radial field component cancels out. Force on an element dq due to $E_{\|}$.

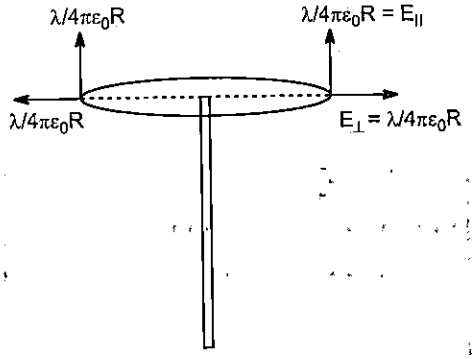


Fig. 1E.23 (b)

$$dF = \frac{\lambda dq}{4\pi\varepsilon_0 R} \Rightarrow F = \frac{\lambda}{4\pi\varepsilon_0 R} \int dq = \frac{\lambda Q}{4\pi\varepsilon_0 R}$$

Example 24

A small freely oriented dipole with dipole moment p lies at the centre of a uniformly charged hemisphere, charge density σ and radius R. Determine the potential energy of the dipole and the period of small oscillations about an axis perpendicular to the axis of the hemisphere. The moment of inertia of the dipole about the rotation axis is I.

Solution: As the dipole is a point dipole, it may be considered to be kept at the centre. An electric dipole experiences torque in an electric field given by the expression :

$$\vec{\tau} = \vec{\mathbf{p}} \times \vec{\mathbf{E}}$$

Potential energy of dipole is given by the expression

$$U = -\vec{\mathbf{p}} \cdot \vec{\mathbf{E}}$$

First we will calculate the electric field at the centre of the hemisphere.

We can imagine the sphere to consist of large number of differential rings. Fig. 1E.24(a) shows one such ring, whose thickness is $ds = R\,d\theta$, and radius $r = R\sin\theta$. The charge dQ on the ring is $dQ = (2\pi R\sin\theta)R\,d\theta \cdot \sigma$.

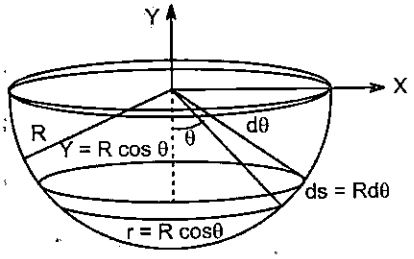


Fig. 1E.24 (a)

Field on the axis of ring is given by the expression

$$\vec{\mathbf{E}} = \frac{Qx}{4\pi\varepsilon_0 (R^2 + x^2)^{3/2}} \hat{\mathbf{i}}$$

so, $$d\vec{\mathbf{E}} = \frac{dQy}{4\pi\varepsilon_0 (y^2 + r^2)^{3/2}} (\hat{\mathbf{j}})$$

$$= \frac{\sigma(2\pi R\sin\theta)R\,d\theta \times R\cos\theta}{4\pi\varepsilon_0 [(R\cos\theta)^2 + (R\sin\theta)^2]^{3/2}} (\hat{\mathbf{j}})$$

$$= \frac{\sigma\sin\theta\cos\theta\,d\theta}{2\varepsilon_0} (\hat{\mathbf{j}})$$

On integrating the expression for $d\vec{\mathbf{E}}$ w.r.t. θ from $\theta_1 = 0$ (the farthest ring) to $\theta_2 = \pi/2$ (closest ring), we obtain $\vec{\mathbf{E}}$.

$$\vec{\mathbf{E}} = \int_0^{\pi/2} \frac{\sigma\sin\theta\cos\theta\,d\theta}{2\varepsilon_0} \hat{\mathbf{j}} = \frac{\sigma}{4\varepsilon_0} \hat{\mathbf{j}}$$

Thus torque experienced by dipole has magnitude

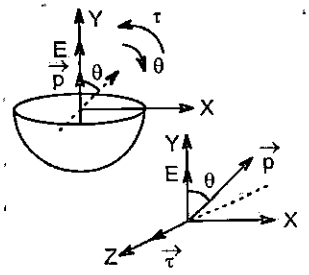


Fig. 1E.24 (b)

$$\tau = p\left(\frac{\sigma}{4\varepsilon_0}\right)\sin\theta$$

For small angular displacement,

$$\tau = -\left(\frac{\sigma}{4\varepsilon_0}\right)\theta = I\alpha$$

or $$\alpha = -\left(\frac{\sigma}{4\pi\varepsilon_0 I}\right)\theta$$

On comparing the above expression with standard expression for SHM,

$$\alpha = -\omega^2\theta$$

we obtain

$$\omega = \sqrt{\frac{\sigma}{4\pi\varepsilon_0 I}} = \frac{2\pi}{T}$$

$$T = 2\pi\sqrt{\frac{4\pi\varepsilon_0 I}{\sigma}}$$

Potential energy of dipole $= -pE\cos\theta$

$$= -p\left(\frac{\sigma}{4\pi\varepsilon_0}\right)\cos\theta$$

Example 25

A small ball of mass 2×10^{-3} kg having a charge of 1 μC is suspended by a string of length 0.8 m. Another identical ball having the same charge is kept at the point of suspension. Determine the minimum horizontal velocity which should be imparted to the lower ball so that it can make complete revolution.

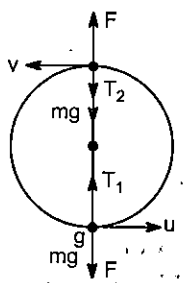


Fig. 1E.25

Solution: If the ball has to just complete the circle then the tension must vanish at the topmost point, *i.e.*, $T_2 = 0$.

From Newton's second law,

$$T_2 + mg - \frac{q^2}{4\pi\varepsilon_0 l^2} = \frac{mv^2}{l} \quad ...(1)$$

At the topmost point $T_2 = 0$

$$mg - \frac{q^2}{4\pi\varepsilon_0 l^2} = \frac{mv^2}{l} \quad ...(2)$$

From energy conservation,

Energy at lowest point = Energy at topmost point

$$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mg\,2l \quad ...(3)$$

$$v^2 = u^2 - 4gl \quad ...(4)$$

From Eq. (2),

$$v^2 = gl - \frac{q^2}{4\pi\varepsilon_0 ml} \quad ...(5)$$

From eqns. (4) and (5),

$$u = \sqrt{5gl - \frac{q^2}{4\pi\varepsilon_0 ml}}$$

$$= \left(\frac{275}{8}\right)^{1/2} = 5.86 \text{ m/s}$$

Example 26

A positive charge q is at $x = a$ and a second negative charge q is at $x = -a$ (see Fig. 1E.26). (a) Find the electric field on the x-axis in the region defined by $x < a, x > a$ and $-a < x < a$ and sketch E_x versus x. (b) Find the limiting form of field for $x >> a$.

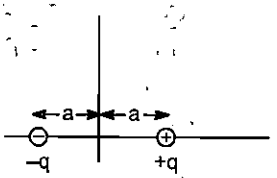


Fig. 1E.26 (a)

Solution: We calculate the field using

$$\vec{\mathbf{E}} = \sum_i \frac{1}{4\pi\varepsilon_0}\frac{q_i}{r_i^2}\hat{\mathbf{r}}_i$$

in the region $x < a$,

$$\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0}\frac{q}{(x-a)^2}(+\hat{\mathbf{i}}) + \frac{1}{4\pi\varepsilon_0}\frac{q}{(x+a)^2}(-\hat{\mathbf{i}})$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(x-a)^2} - \frac{1}{(x+a)^2}\right]\hat{\mathbf{i}}$$

In the region $-a < x < a$

$$\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0}\frac{1}{(x-a)^2}(-\hat{\mathbf{i}}) + \frac{1}{4\pi\varepsilon_0}\frac{q}{(x+a)^2}(-\hat{\mathbf{i}})$$

$$= -\frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(x-a)^2} + \frac{1}{(x+a)^2}\right]\hat{\mathbf{i}}$$

In the region $x > a$

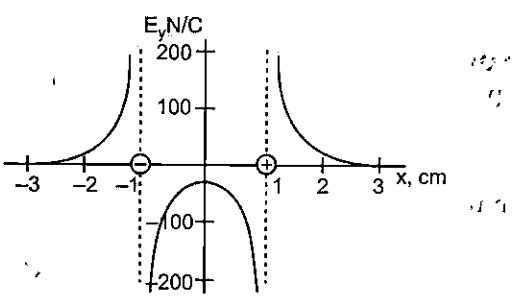


Fig. 1E.26 (b)

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{(x-a)^2}(+\hat{i}) + \frac{1}{4\pi\varepsilon_0}\frac{q}{(x+a)^2}(-\hat{i})$$
$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(x-a)^2} - \frac{1}{(x+a)^2}\right]\hat{i}$$

In the region $-a < x < a$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{1}{(x-a)^2}(-\hat{i}) + \frac{1}{4\pi\varepsilon_0}\frac{q}{(x+a)^2}(-\hat{i})$$
$$= -\frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(x-a)^2} + \frac{1}{(x+a)^2}\right]\hat{i}$$

In the region $x > a$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{(x-a)^2}\hat{i} + \frac{1}{4\pi\varepsilon_0}\frac{(-q)}{(x+a)^2}\hat{i}$$
$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(x-a)^2} - \frac{1}{(x+a)^2}\right]\hat{i}$$
$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{(x+a)^2-(x-a)^2}{(x+a)^2(x-a)^2}\right]\hat{i}$$
$$= \frac{q}{4\pi\varepsilon_0}\frac{4ax}{(x^2-a^2)}\hat{i}$$

For $x >> a$ we can neglect a^2 compared with x^2 in the denominator.

$$E \simeq \frac{4qa}{4\pi\varepsilon_0 x^3}\hat{i}$$

Example 27

A positive charge $q_1 = +8nC$ is at the origin, and a second positive charge $q_2 = +12nC$ is on the x-axis at $a = 4$ m [see Fig. 1E.27 (a)]. Find the electric field at points defined by $x < 0$, $0 < x < a$ and $x > a$. Draw a graph of E_x versus x for this system.

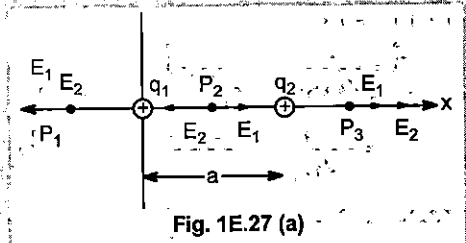


Fig. 1E.27 (a)

Solution: The resultant field at any position is superposition of field due to charges q_1 and q_2. We apply

$$\sum\vec{E} = \sum\frac{1}{4\pi\varepsilon_0}\frac{q_i}{r_i^2}\hat{r}_1$$

In the region $x < 0$,

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{x^2}(-\hat{i}) + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{(a+x)^2}(-\hat{i})$$
$$= 9\times10^9\times10^{-9}\left[-\frac{8}{x^2} - \frac{12}{(4+x)^2}\right]\hat{i}$$

In the region $0 < x < a$,

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{x^2}\hat{i} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{(4-x)^2}(-\hat{i})$$
$$= 9\times10^9\times10^{-9}\left[\frac{8}{x^2} - \frac{12}{(4-x)^2}\right]\hat{i}$$

In the region $x > a$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{x^2}\hat{i} + \frac{q_2}{(x-a)^2}\hat{i}$$
$$= 9\times10^9\times10^{-9}\left[\frac{8}{x^2} + \frac{12}{(x-4)^2}\right]\hat{i}$$

The electric field in the region $x < a$ is in negative x-direction because the field of both the positive charges is in the negative x-direction. Near the charges q_1 and q_2 field is undefined (it tends to infinity). In the region $0 < x < a$, near charge q_1, the field is in positive x-direction and near charge q_2, the field is in negative x-direction. As we move

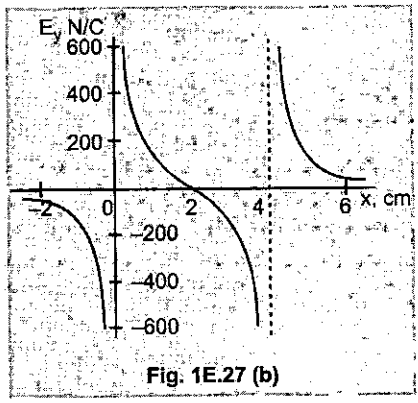


Fig. 1E.27 (b)

from q_1 to q_2 the electric field contribution due to q_1 decreases while q_2 increases ($E \propto 1/r^2$). There is one point between the charges where the net electric field is zero. At this point, a test charge experiences no net force. Beyond this point, near q_2, electric field of q_2 is larger and net field points in the negative x-direction. In the region $x > a$, field due to both the charges is in the same direction, along positive x.

Example 28

A charge 10^{-9} coulomb is located at origin in free space and another charge Q at (2,0,0). If the x-component of the electric field at (3,1,1) is zero, calculate the value of Q. Is the y-component zero at (3,1,1)?

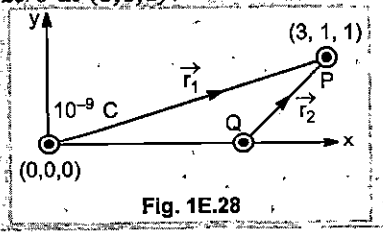


Fig. 1E.28

Solution: Electric field due to a point charge q_i at position $\vec{r}_i$ in vector form is expressed as

$$\vec{E}_i = \frac{1}{4\pi\varepsilon_0}\frac{q}{r_i^3}\vec{r}_i$$

$$\vec{r}_1 = (3-0)\hat{i} + (1-0)\hat{j} + (1-0)\hat{k}$$
$$= 3\hat{i} + \hat{j} + \hat{k}$$

with $|\vec{r}_1| = \sqrt{(3^2+1^2+1^2)} = \sqrt{11}$ m

$$\vec{r}_2 = (3-2)\hat{i} + (1-0)\hat{j} + (1-0)\hat{k}$$
$$= \hat{i} + \hat{j} + \hat{k}$$

with $|\vec{r}_2| = \sqrt{(1^2+1^2+1^2)} = \sqrt{3}$ m

Thus electric fields due to charges are

$$\vec{E}_1 = \frac{1}{4\pi\varepsilon_0}\frac{10^{-9}}{(11)^{3/2}}[3\hat{i} + \hat{j} + \hat{k}] \text{ and}$$

$$\vec{E}_2 = \frac{1}{4\pi\varepsilon_0}\frac{Q}{(3)^{3/2}}[\hat{i} + \hat{j} + \hat{k}]$$

Resultant electric field at point P is

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\left(\frac{3\times10^{-9}}{11\sqrt{11}} + \frac{Q}{3\sqrt{3}}\right)\hat{i} + \left(\frac{10^{-9}}{11\sqrt{11}} + \frac{Q}{3\sqrt{3}}\right)\hat{j} + \left(\frac{10^{-9}}{11\sqrt{11}} + \frac{Q}{3\sqrt{3}}\right)\hat{k}\right]$$

According to given condition

$$E_x = 0$$

i.e.,
$$\frac{1}{4\pi\varepsilon_0}\left[\frac{3\times10^{-9}}{11\sqrt{11}} + \frac{Q}{3\sqrt{3}}\right] = 0$$

so that

$$Q = -\left[\frac{3}{11}\right]^{3/2} \times 3\times10^{-9}$$
$$= -4.27\times10^{-10} \text{ coulomb}$$

and for this value of Q

$$E_r = \frac{1}{4\pi\varepsilon_0}\left[\frac{10^{-9}}{11\sqrt{11}} - \frac{(3/11)^{3/2}\times3\times10^{-9}}{3\sqrt{3}}\right]$$
$$= -\frac{1}{4\pi\varepsilon_0}\frac{2\times10^{-9}}{11\sqrt{11}} \neq 0$$

i.e., E_r is not zero.

Example 29

Equal charges are placed at the corners of a cube of side a. What force acts on any one of the charges?

Solution: Method 1 : We will determine electric field at the position of one of the charges, for example, charge at the 7th corner.

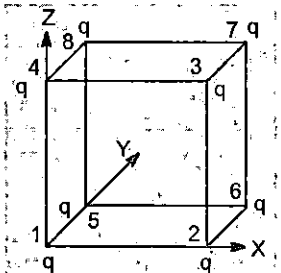


Fig. 1E.29 (a)

$$\vec{E}_1 = \frac{q}{4\pi\varepsilon_0(\sqrt{3}a)^2}\left(\frac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}}\right)$$
$$= \frac{q}{4\pi\varepsilon_0 3\sqrt{3}a^2}(\hat{i}+\hat{j}+\hat{k})$$

$$\vec{E}_2 = \frac{q}{4\pi\varepsilon_0(\sqrt{2}a)^2}\left(\frac{\hat{j}+\hat{k}}{\sqrt{2}}\right)$$
$$= \frac{q}{4\pi\varepsilon_0(2\sqrt{2})a^2}(\hat{j}+\hat{k})$$

$$\vec{E}_3 = \frac{q}{4\pi\varepsilon_0 a^2}\hat{j}$$

$$\vec{E}_4 = \frac{q}{4\pi\varepsilon_0(\sqrt{2}a)^2}\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$$
$$= \frac{q}{4\pi\varepsilon_0(2\sqrt{2})a^2}(\hat{i}+\hat{j})$$

$$\vec{E}_5 = \frac{q}{4\pi\varepsilon_0(\sqrt{2}a)^2}\left(\frac{\hat{i}+\hat{k}}{\sqrt{2}}\right)$$
$$= \frac{q}{4\pi\varepsilon_0(2\sqrt{2}a^2)}(\hat{i}+\hat{k})$$

$$\vec{E}_6 = \frac{1}{4\pi\varepsilon_0}\frac{q}{a^2}\hat{k}$$

$$\vec{E}_8 = \frac{1}{4\pi\varepsilon_0}\frac{q}{a^2}\hat{i}$$

The resultant electric field is

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4 + \vec{E}_5 + \vec{E}_6 + \vec{E}_8$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{a^2}\left[\frac{\hat{i}+\hat{j}+\hat{k}}{3\sqrt{3}} + \frac{\hat{j}+\hat{k}}{2\sqrt{2}} + \frac{\hat{i}+\hat{j}}{2\sqrt{2}} + \hat{j} + \frac{\hat{i}+\hat{k}}{2\sqrt{2}} + \hat{k} + \hat{i}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{a^2}\left[\frac{1}{3\sqrt{3}} + \frac{1}{\sqrt{2}} + 1\right][\hat{i}+\hat{j}+\hat{k}]$$

$$|\vec{E}| = \frac{q}{4\pi\varepsilon_0 a^2}\left[\frac{1}{3} + \sqrt{\frac{3}{2}} + \sqrt{3}\right]$$

The resultant force is

$$F = q|\vec{E}|$$
$$= \frac{q^2}{4\pi\varepsilon_0 a^2}\left[\frac{1}{3} + \sqrt{\frac{3}{2}} + \sqrt{3}\right]$$

Method 2 : The three charges at points denoted by 8, 3 and 1 and the three at 7, 5 and 2 are symmetrically placed w.r.t. the diagonal 4-6. By symmetry the force on the charge at 4 is outwards in the direction of diagonal 4-6 from the opposite corner. The component of the force due to the charge at *B* along 4-6 is

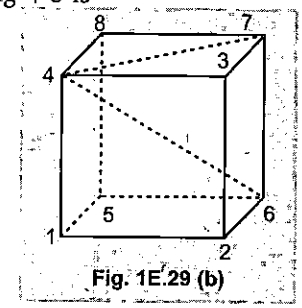


Fig. 1E.29 (b)

$$\frac{q^2}{4\pi\varepsilon_0 a^2}\frac{1}{\sqrt{3}}$$

The component due to charge at 7 is

$$\frac{q^2}{4\pi\varepsilon_0(\sqrt{2}\,a)^2}\times\frac{\sqrt{2}}{\sqrt{3}}$$

and the force due to charge at 6 is

$$\frac{q^2}{4\pi\varepsilon_0(\sqrt{3}a)^2}$$

The total force is therefore

$$\frac{q^2}{4\pi\varepsilon_0 a^2}\left[\frac{3}{\sqrt{3}}+\frac{3}{2}\times\frac{\sqrt{2}}{\sqrt{3}}+\frac{1}{3}\right]$$

$$=\frac{q^2}{4\pi\varepsilon_0 a^2}\left[\frac{1}{3}+\sqrt{\frac{3}{2}}+\sqrt{3}\right]$$

Example 30

A uniform electric field E exists between the plates of a capacitor. The plate length is l and the separation of the plates is d.

(a) An electron and a proton start from the negative plate and positive plate respectively and go to the opposite plates. Which of them wins this race?

(b) An electron and a proton start from the midpoint of the separation of plates at one end of the plates. Which of the two will have greater deviation when they start with the :

(i) same initial velocity

(ii) same initial kinetic energy

(iii) same initial momentum ?

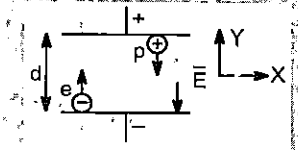


Fig. 1E.30 (a)

Solution: (a) In the coordinate system shown in Fig. 1E.30 (a), there is no force along *x*-axis. The accelerations of the electron and the proton along *y*-axis are

$$a_e=\frac{-eE}{m_e}\quad\Rightarrow\quad a_p=\frac{eE}{m_p}$$

From $S=ut+\frac{1}{2}at^2$ on substituting $S=d, u=0$, we get

$$t_e=\sqrt{\frac{2S}{a_e}}\quad\text{and}\quad t_p=\sqrt{\frac{2S}{a_p}}$$

or $$t_e=\sqrt{\frac{2dm_e}{eE}}\quad\text{and}\quad t_p=\sqrt{\frac{2dm_p}{eE}}$$

As $m_e<m_p, t_e<t_p \Rightarrow$ Electron takes less time to cross over than the proton.

(b) When the proton moves parallel to the plates, it is attracted towards the negative plate.

Time taken by proton to cross over , $t=\frac{l}{V_x}$.

During this time, deflection along vertical direction,

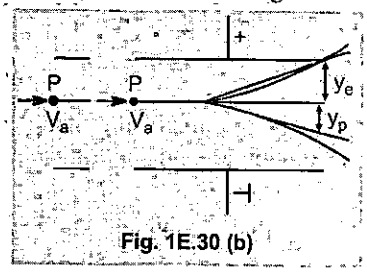


Fig. 1E.30 (b)

$$y=\frac{1}{2}at^2\quad\text{or}\quad y=\frac{1}{2}\frac{eE}{m}\left[\frac{l}{V_0}\right]^2$$

(i) Thus $$y_p=\frac{1}{2}\frac{eE}{m_p}\frac{l^2}{V_x^2}$$

Similarly for electrons,

$$y_e=\frac{1}{2}\frac{eE}{me}\frac{l^2}{Vx^2}$$

As $m_p>m_e, y_p<y_e$, note that the electron will be deflected in the opposite direction.

(ii) Also $$y=\frac{1}{2}\frac{eEl^2}{2\left[\frac{m}{2}v_x^2\right]}=\frac{1}{4}\frac{eEl^2}{K}$$

where $K=\frac{1}{2}mv_x^2$ or initial kinetic energy

Hence $$y_p=y_e$$

(iii) Also, as $K=\frac{p^2}{2m}$, where $p=$ momentum,

$$y=\frac{eEl^2}{4\left(\frac{p^2}{2m}\right)}=\frac{eEl^2\cdot m}{2p^2}$$

$$\Rightarrow \quad y_p = \frac{eEl^2 m_p}{2p^2}$$

$$y_e = \frac{eEl^2 m_e}{2p^2}$$

Thus for $m_p > m_e$, $y_p > y_e$

Example 31

A point charge $Q_1 = -125\,\mu C$ is fixed at the centre of an insulated disc of mass 1 kg. The disc rests on a rough horizontal plane. Another charge $Q_2 = 125\,\mu C$ is fixed vertically above the centre of the disc at a height of $h = 1\,m$. After the disc is displaced slightly in the horizontal direction (friction is sufficient to prevent slipping). Find the period of oscillation of disc.

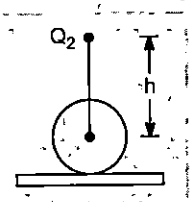


Fig. 1E.31 (a)

Solution: Let the radius of the disc be R. If the disc is displaced x, the corresponding angular displacement $\theta = x/R$

The restoring torque τ about the point of contact of disc with ground,

$$\tau_p = (F \sin\theta) R$$

$$(F \sin\theta) R = I\alpha = \left[\frac{MR^2}{2} + MR^2\right]\alpha$$

where $$F = \frac{Q^2 x}{4\pi\varepsilon_0 (h^2 + x^2)}$$

and $$\sin\theta = \frac{R}{\sqrt{h^2 + x^2}}$$

Hence $$\frac{Q^2 xR}{4\pi\varepsilon_0 (h^2 + x^2)^{3/2}} = \left[\frac{MR^2}{2} + MR^2\right]\alpha$$

or $$\alpha = \frac{Q^2 x}{6\pi\varepsilon_0 MR(h^2 + x^2)^{3/2}}$$

For $x << l$,

$$\alpha \simeq -\frac{Q^2 x}{6\pi\varepsilon_0 MRh^3} = -\frac{Q^2 \theta R}{6\pi\varepsilon_0 MRh^3}$$

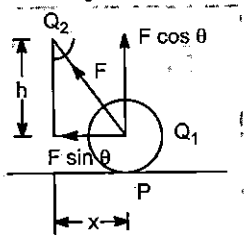


Fig. 1E.31 (b)

Negative sign is being introduced because angular acceleration and angular displacement are opposite to each other.

Thus, $$\alpha = -\frac{Q^2\theta}{6\pi\varepsilon_0 Mh^3}$$

Hence $$\omega = \sqrt{\frac{Q^2}{6\pi\varepsilon_0 Mh^3}}$$

or $$T = 2\pi\sqrt{\frac{6\pi\varepsilon_0 Mh^3}{Q^2}} = 2\pi\frac{h}{Q}\sqrt{6\pi\varepsilon_0 Mh}$$

Example 32

A particle of charge q and mass m moves rectilinearly under the action of an electric field $E = A - Bx$ where B is a positive constant and x is a distance from the point where the particle was initially at rest. Calculate :

(i) distance traveled by the particle till it comes to rest and

(ii) acceleration at that moment.

Solution: Method 1. Force acting on the charged particle as function of x is

$$F = qE = q(A - Bx)$$

From Newton's second law, we have

$$F = ma = m\frac{dv}{dt}$$

$$\therefore \quad \frac{dv}{dt} = \frac{dv}{dx}\cdot\frac{dx}{dt} = \frac{q}{m}(A - Bx)$$

or $$v\frac{dv}{dx} = \frac{q}{m}(A - Bx)$$

or $$\int_0^v v\,dv = \frac{q}{m}\int_0^x (A - Bx)\,dx$$

or $$\frac{v^2}{2} = \frac{q}{m}\left(Ax - \frac{Bx^2}{2}\right)$$

When the particle comes to rest momentarily,

$$v = 0, \quad i.e., \quad Ax - \frac{Bx^2}{2} = 0$$

i.e., $$x = 0, \frac{2A}{B}$$

The two values of x correspond to initial position and desired position,

Acceleration at this instant,

$$a = \frac{q}{m}\left(A - B\cdot\frac{2A}{B}\right) = -\frac{qA}{m}$$

Method 2 : The particle will come to rest when its kinetic energy is zero, *i.e.,*

$$\text{K.E.} = -\int_0^x F\,dx = -\int_0^x q(A - Bx)\,dx = 0$$

$$\therefore \quad Ax = \frac{Bx^2}{2},$$

i.e., $x = 0, x = \frac{2A}{B}$

and acceleration $= \frac{F}{m} = \frac{q}{m}\left(A - B\cdot\frac{2A}{B}\right) = -\frac{qA}{m}$

Example 33

A non-conducting ring of mass m and radius R, the charge per unit length λ is as shown in Fig. 1E.33(a). It is then placed on a rough non-conducting horizontal plane. At time $t = 0$, a uniform electric field $\vec{E} = E_0\hat{i}$ is switched on and the ring starts rolling without sliding. Determine the friction force (magnitude and direction) acting on the ring.

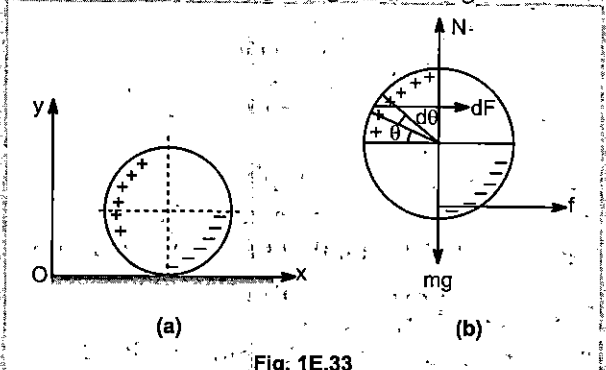


Fig. 1E.33

Solution: Consider a differential element subtending an angle $d\theta$ at the centre and at angle θ as shown in Fig. 1E.33(b).

$$dF = \lambda R\, d\theta\, E_0$$

A force of same magnitude but in opposite direction acts on a corresponding element in the region of negative charge.

∴ Equation for pure rolling motion is

$$\int_0^{\pi/2} 2\lambda R d\theta\, E_0 R\sin\theta - fR = mR^2\alpha$$

or $2\lambda R^2 E_0 - fR = mR^2\alpha$ ...(1)

and $f = ma$ ...(2)

and $a = R\alpha$ ...(3)

Solving eqns. (1), (2) and (3), we get

$f = \lambda R E_0$ along positive x-axis.

Example 34

An infinitely long conducting wire of charge density +λ and a point charge –Q are at a distance from each other. In which of the three regions (I, II or III) are there points that (a) lie on the line passing through point charge perpendicular to the conductor and (b) at which the field is zero?

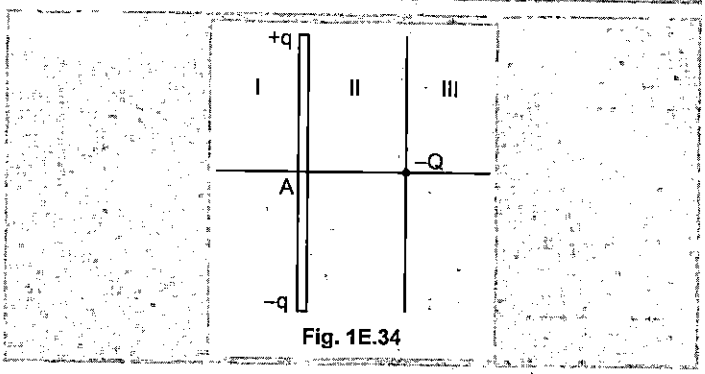


Fig. 1E.34

Solution: In the region II the electric field of wire and point charge point in the same direction, positive x-axis. So no point can exist where the field is zero. Now we take a point to the right of the point charge, at a distance x from it. Resultant field at this point is

$$\vec{E}_R = \frac{\lambda}{2\pi\varepsilon_0(x+a)}\hat{i} + \frac{Q}{4\pi\varepsilon_0 x^2}(-\hat{i})$$

Resultant field is zero if

$$\frac{\lambda}{(a+x)} = \frac{Q}{2x^2}$$

On solving the quadratic in x, we have

$$x = \frac{Q}{4\lambda} \pm \sqrt{\frac{Q^2}{16\lambda^2} + \frac{aQ}{2\lambda}}$$

The negative sign infront of the radical has no meaning because it would mean that the point is to the left of point charge, where field of wire and point charge are added, the magnitude of the two fields are zero.

Now we check the region I, take a point to the left of wire at a distance x from it. The resultant field is

$$\vec{E}_R = \frac{\lambda}{2\pi\varepsilon_0 x}(-\hat{i}) + \frac{Q}{4\pi\varepsilon_0 (a+x)^2}\hat{i}$$

The two fields point in the opposite directions, so resultant field can be zero if,

$$\frac{\lambda}{2\pi\varepsilon_0 x} = \frac{Q}{4\pi\varepsilon_0 (a+x)^2}$$

or $x^2 + \left(2a - \frac{Q}{2\lambda}\right)x + a^2 = 0$

or $x = \frac{1}{2}\left(\frac{Q}{2\lambda} - 2a\right) \pm \sqrt{\frac{1}{4}\left(\frac{Q}{2\lambda} - 2a\right)^2 - a^2}$

If the discriminant of the quadratic equation is real, we have two points where the field is zero. Discriminant is positive for $Q \geq 8a\lambda$.

Example 35

Two point-like charges are positioned at points 1 and 2. The field intensity to the right of the charge Q_2 on the line that passes through the two charges varies according to a law that is represented schematically in Fig. 1E.35. The field intensity is assumed to be positive if its direction coincides with the positive direction on the x-axis. The distance between the charges is l.

(a) Find the sign of each charge

(b) Find the ratio of the absolute value of the charges $\left|\frac{Q_1}{Q_2}\right|$

(c) Find the value of b, where the field intensity is maximum.

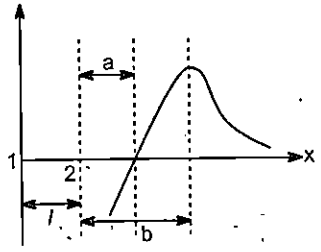


Fig. 1E.35

Solution: (a) The charge Q_2 is negative and the charge Q_1 is positive.

(b) Since the electric field is zero at a distance 'a' from the point 2, therefore

$$\frac{Q_1}{(l+a)^2}-\frac{Q_2}{a^2}=0 \Rightarrow \frac{Q_1}{Q_2}=\left(\frac{l+a}{a}\right)^2$$

(c) For all point $x > a$,

$$E=\frac{Q_1}{(l+x)^2}-\frac{Q_2}{a^2}$$

For max E, $\quad \frac{dE}{dx}=0=-\frac{2Q_1}{(l+x)^3}+\frac{2Q_2}{x^3}$

Thus, $\quad x=\frac{1}{(Q_1/Q_2)^{1/3}-1}=b$

Example 36

A thin nonconducting ring of radius R has a linear charge density $\lambda=\lambda_0\cos\phi$ where λ_0 is a constant, ϕ is the azimuthal angle. Find the magnitude of the electric field strength on the axis of the ring as a function of the distance x from its centre. Investigate the obtained function at $x >> R$.

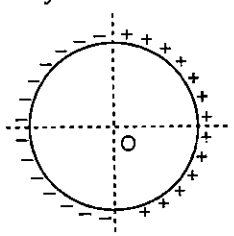


Fig. 1E.36 (a)

Solution: Azimuthal angle is the angle with diameter. Since, λ is cosine function of ϕ,

$$\lambda=\lambda_0\cos\phi$$

So, first and fourth quadrants are positively charged but second and third quadrants are negatively charged.

Let the ring plane coincide with y-z plane [Fig. 1E.36(b)]. We consider a small element dl on the ring.

Here $dl = Rd\theta$, where R is the radius of the ring.

From Fig. 1E.36 (b),

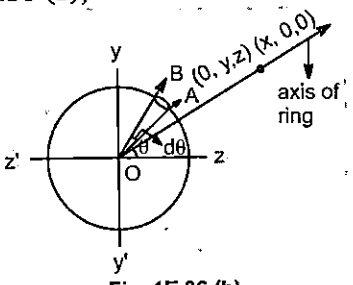


Fig. 1E.36 (b)

$y = R\sin\theta$ and $z = R\cos\theta$.

The electric charge on the considered element is

$$dq=\lambda dl=\lambda_0\cos\theta\,(Rd\theta)=\lambda_0 R\cos\theta\,d\theta$$

The axis of the ring is X-axis.

The electric field at point P due to considered element is

$$d\vec{\mathbf{E}}=\frac{dq(\vec{\mathbf{r}}_p-\vec{\mathbf{r}}_A)}{4\pi\varepsilon_0|\vec{\mathbf{r}}_p-\vec{\mathbf{r}}_A|^3}$$

or $$d\vec{\mathbf{E}}=\frac{(\lambda_0 R\cos\theta\,d\theta)(x\hat{\mathbf{i}}-y\hat{\mathbf{j}}-z\hat{\mathbf{k}})}{4\pi\varepsilon_0|x\hat{\mathbf{i}}-y\hat{\mathbf{j}}-z\hat{\mathbf{k}}|^3}$$

or $$d\vec{\mathbf{E}}=\frac{\lambda_0 R\cos\theta\,d\theta}{4\pi\varepsilon_0}\frac{(x\hat{\mathbf{i}}-R\sin\theta\hat{\mathbf{j}}-R\cos\theta\hat{\mathbf{k}})}{(x^2+y^2+z^2)^{3/2}}$$

$$=\frac{\lambda_0 R\cos\theta\,d\theta}{4\pi\varepsilon_0}\frac{(x\hat{\mathbf{i}}-R\sin\theta\hat{\mathbf{j}}-R\cos\theta\hat{\mathbf{k}})}{(x^2+R^2\sin^2\theta+R^2\cos^2\theta)^{3/2}}$$

$$=\frac{\lambda_0 R\cos\theta\,d\theta}{4\pi\varepsilon_0}\frac{(x\hat{\mathbf{i}}-R\sin\theta\hat{\mathbf{j}}-R\cos\theta\hat{\mathbf{k}})}{(x^2+R^2)^{3/2}}$$

$$\therefore\quad d\vec{\mathbf{E}}=\frac{\lambda_0 R}{4\pi\varepsilon_0(R^2+x^2)^{3/2}}(x\cos\theta\,d\theta\hat{\mathbf{i}}-R\sin\theta\cos\theta d\theta\hat{\mathbf{j}}$$

$$-R\cos^2\theta\,d\theta\hat{\mathbf{k}})$$

$$\therefore\quad dE_x=\frac{\lambda_0 Rx\cos\theta\,d\theta}{4\pi\varepsilon_0(R^2+x^2)^{3/2}},$$

$$dE_y=\frac{-\lambda_0 R^2\sin\theta\cos\theta\,d\theta}{4\pi\varepsilon_0(R^2+x^2)^{3/2}},$$

and $$dE_z=\frac{-\lambda_0 R^2\cos^2\theta\,d\theta}{4\pi\varepsilon_0(R^2+x^2)^{3/2}}$$

$$E_x=\int dE_x=\frac{-\lambda_0 Rx}{4\pi\varepsilon_0(R^2+x^2)^{3/2}}\int_0^{2\pi}\cos\theta\,d\theta$$

After integrating, $E_x = 0$ and

$$E_y = \int dE_y = \frac{-\lambda_0 R^2}{4\pi\varepsilon_0 (R^2+x^2)^{3/2}} \int_0^{2\pi} \frac{\sin 2\theta}{2} d\theta$$

$$= \frac{-\lambda_0 R^2}{8\pi\varepsilon_0 (R^2+x^2)^{3/2}} \left[\frac{-\cos 2\theta}{2}\right]_0^{2\pi} = 0$$

$$E_y = 0$$

Similarly,

$$E_z = \int dE_z = \frac{-\lambda_0}{4\pi\varepsilon_0 (R^2+x^2)^{3/2}} \int_0^{2\pi} \cos^2\theta \, d\theta$$

$$= \frac{-\lambda_0 R^2}{4\varepsilon_0 (R^2+x^2)^{3/2}}$$

$$\therefore \quad \vec{\mathbf{E}} = E_x \hat{\mathbf{i}} + E_y \hat{\mathbf{j}} + E_z \hat{\mathbf{k}}$$

$$\therefore \quad |\vec{\mathbf{E}}| = E = |E_z \hat{\mathbf{k}}| \qquad (\because E_x = 0, E_y = 0)$$

$$= \frac{\lambda_0 R^2}{4\varepsilon_0 (R^2+x^2)^{3/2}}$$

For $x >> R$, $R^2 + x^2 \approx x^2$

$$\therefore \quad E = \frac{\lambda_0 R^2}{4\varepsilon_0 x^3} = \frac{P}{4\pi\varepsilon_0 x^3}$$

where $P = \lambda_0 \pi R^2$.

Example 37

Two mutually perpendicular infinitely long lines of charge having charge per unit length as λ_1 and λ_2 are located in air as shown in Fig. 1E.37. Show that the force of interaction between them is $\frac{\lambda_1\lambda_2}{2\varepsilon_0}$.

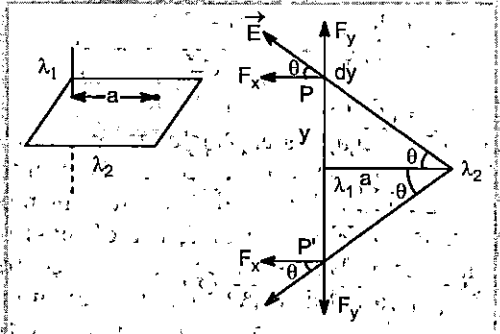


Fig. 1E.37

Solution: Consider a differential element of length dy located at P on the line of charge carrying charge density λ_1.

Electric field at P due to other line of charge,

$$E = \frac{\lambda_2}{2\pi\varepsilon_0\sqrt{a^2+y^2}}$$

Consider a symmetrical element at point P'. The forces on symmetrical elements along the wire cancel, whereas in perpendicular direction they add.

$$\therefore \quad F = \int_{-\infty}^{\infty} \frac{\lambda_2}{2\pi\varepsilon_0\sqrt{a^2+y^2}} (\lambda_1 \, dy) \cos\theta$$

But $y = a\tan\theta$

and $\therefore \quad dy = a\sec^2\theta \, d\theta.$

$$\therefore \quad F = \int_{-\pi/2}^{\pi/2} \frac{\lambda_1\lambda_2}{2\pi\varepsilon_0} d\theta = \frac{\lambda_1\lambda_2}{2\varepsilon_0}$$

Example 38

A square frame of side a, made of a charged wire of uniform charge density λ, lies in the yz-plane with centre at the origin. Determine the net electric field at distance r along the axis of the frame.

Solution: We consider a differential element dy on the side AB of the square. The charge on this element is $dq = \lambda\, dy$. The electric field due to this charge element is

$$dE = \frac{\lambda\, dx}{4\pi\varepsilon_0 (a^2+y^2+r^2)}$$

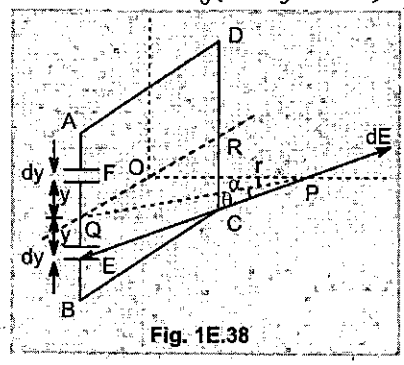


Fig. 1E.38

Field is along the line EP. We resolve this electric field along parallel and perpendicular to QP. From symmetry we can see that the field due to a symmetrically placed element has similar field components. The perpendicular components cancel while the components along QP add. Thus the resultant electric field due to side AB is

$$E = \int dE\cos\theta$$

$$= \int_0^a \frac{2\lambda\, dy}{4\pi\varepsilon_0 (a^2+r^2+y^2)} \frac{(r^2+a^2)^{1/2}}{(a^2+r^2+y^2)^{1/2}}$$

Substituting $a^2 + r^2 = b^2$ and $x = b\tan\theta$, we get

$$E = \frac{\lambda}{2\pi\varepsilon_0 b} \int_0^{\sin^{-1}(a/\sqrt{a^2+b^2})} \cos\theta \, d\theta$$

$$= \frac{\lambda}{2\pi\varepsilon_0\sqrt{a^2+r^2}} \cdot \frac{a}{\sqrt{2a^2+r^2}}$$

In a similar manner the electric field due to CD will be along RP and will have the same magnitude as that of AB. Fields due to AB and CD are at an angle α with OP, so their resultant is

$$E' = \frac{\lambda a\, 2\alpha}{2\pi\varepsilon_0\sqrt{a^2+r^2}} \frac{\alpha}{\sqrt{2a^2+r^2}}$$

$$= \frac{a\lambda r}{\pi\varepsilon_0 (a^2+r^2)\sqrt{r^2+2a^2}}$$

The field due to sides AD and BC has the same magnitude, E', and direction as E'. Hence the resultant field at P is in the direction OP and has the magnitude

$$E = \frac{2ar\lambda}{\pi\varepsilon_0 (a^2 + r^2)\sqrt{r^2 + 2a^2}}$$

Example 39

Two balls of charge q_1 and q_2 initially have a velocity of the same magnitude and direction. After a uniform electric field has been applied for a certain time interval, the direction of first ball changes by 60° and the velocity magnitude is reduced by half. The direction of velocity of the second ball changes thereby 90°. In what ratio will the velocity of the second ball change? Determine the magnitude of the charge-to-mass ratio of the second ball if it is equal to α_1 for the first ball. Ignore the electrostatic interaction between the balls.

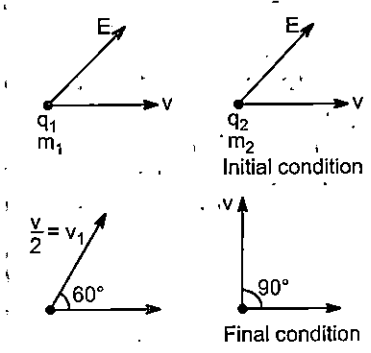


Fig. 1E.39

Solution: Let the electric field on each ball be given by

$$E = E_x\hat{\mathbf{i}} + E_y\hat{\mathbf{j}}$$

From impulse-momentum equation, we have

Impulse = Change in momentum

Let the final velocities of the balls be v_1 and v_2. Noting that $v_1 = v/2$, we have

$$q_1(E_x\hat{\mathbf{i}} + E_y\hat{\mathbf{j}})\Delta t = m_1\left(\frac{v}{2}\cos 60°\,\hat{\mathbf{i}} + \frac{v}{2}\sin 60°\,\hat{\mathbf{j}}\right) - m_1 v\,\hat{\mathbf{i}} \quad ...(1)$$

$$q_2(E_x\hat{\mathbf{i}} + E_y\hat{\mathbf{j}})\Delta t = m_2(v_2\cos 90°\,\hat{\mathbf{i}} + v_2\sin 90°\,\hat{\mathbf{j}}) - m_2 v\,\hat{\mathbf{i}} \quad ...(2)$$

On comparing the x and y-components on both sides of eqn. (1), we get

$$\frac{q_1}{m_1}E_x\Delta t = -\frac{3}{4}v$$

and

$$\frac{q_1}{m_1}E_y\Delta t = \frac{\sqrt{3}}{4}v \quad ...(3)$$

Similarly, for eqn. (2), we get

$$\frac{q_2}{m_2}E_x\Delta t = -v$$

and

$$\frac{q_2}{m_2}E_y\Delta t = v_2 \quad ...(4)$$

From eqns. (3) and (4), by dividing the equations expressing x-components, we get

$$\frac{q_1/m_1}{q_2/m_2} = \frac{3}{4} \quad ...(5)$$

or

$$\frac{q_2}{m_2} = \frac{4}{3}\frac{q_1}{m_1} = \frac{4}{3}\alpha_1$$

Also

$$\frac{q_1/m_1}{q_2/m_2} = \frac{\sqrt{3}v}{4v_2}$$

or

$$\frac{\sqrt{3}v}{4v_2} = \frac{3}{4}$$

or

$$v_2 = \frac{v}{\sqrt{3}}$$

Example 40

Two small balls having the same mass and charge and located on the same vertical at heights h_1 and h_2 are thrown in the same direction along the horizontal at the same velocity v. The first ball touches the ground at a horizontal distance R from the initial vertical position. At what height h_2 will the second ball be at this instant? Neglect any frictional resistance of air and the effect of any induced charges on the ground.

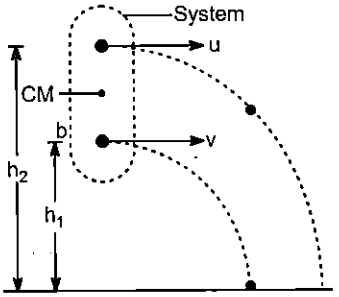


Fig. 1E.40

Solution: Our system includes the two balls. The coulombic force between the two balls is an internal force for the system. Internal forces do not affect the motion of the centre of mass. The motion of centre of mass takes place only under the influence of gravity. The centre of mass moves along a parabolic trajectory. Since the initial velocity of the two balls is horizontal, the time taken to travel distance x is x/v and the vertical height fallen by the centre of mass in this time is

$$y = \frac{1}{2}g\left(\frac{x}{v}\right)^2$$

Position of centre of mass at this moment from the ground is

$$h = \frac{h_1 + h_2}{2} - \frac{1}{2}g\left(\frac{x}{v}\right)^2$$

when the first ball touches the ground at a distance $x = R$, the height of the centre of mass from the ground is

$$h = \frac{h_1 + h_2}{2} - \frac{1}{2}g\left(\frac{R}{v}\right)^2$$

As the masses of the balls are equal, the second ball will be at a height $h_2 = 2h$ at this instant. Therefore

$$h_2 = h_1 + h_2 - g\left(\frac{R}{v}\right)^2$$

FLUX

The idea of flux and field was mainly derived from fluid dynamics. The flow of a fluid as represented by its velocity field is depicted *via* streamlines. Fig. 1.59 shows a moving fluid flowing perpendicularly through a rectangular wire frame surrounding an area A. Volume flux (Av) is the volume of fluid passing through a point per unit time. When the frame is tilted up at angle θ, some of the flow misses the frame. The frame's effective area now corresponds to $A_\perp = A\cos\theta$ with as many flow lines passing through the tilted A as through $A_\perp$.

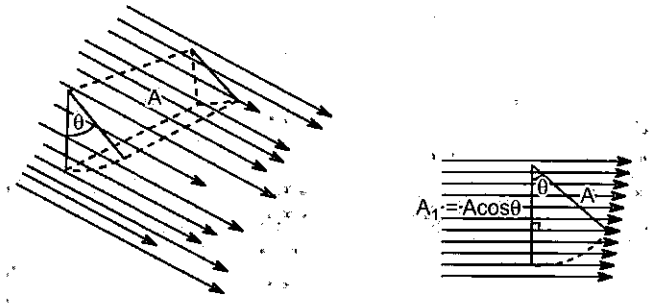


Fig. 1.59

The amount of fluid passing per second through area A and A' must be same even though $Av \neq A'v$, which means that we have to define flux in such a way that the flux through either area, perpendicular or not, should be same. For the tilted area A' the flux is determined by either the component of the velocity perpendicular to A' times v, *i.e.*, $v_\perp A'$,or by v times the component of A' perpendicular to v, *i.e.*, $vA'_\perp$. Since $v_\perp = v\cos\theta$ and $A'_\perp = A'\cos\theta$,

$$\text{Flux} = (v_\perp)A' = (v)(A'_\perp)$$
$$= (v\cos\theta)A' = (v)(A'\cos\theta)$$

Consider a bent pipe as shown in Fig. 1.59 (i). Fluid enters and leaves the tube, *i.e.*, flux is in and out of the tube. In order to distinguish between flux in and out; we define area vector $\vec{\mathbf{A}}$.

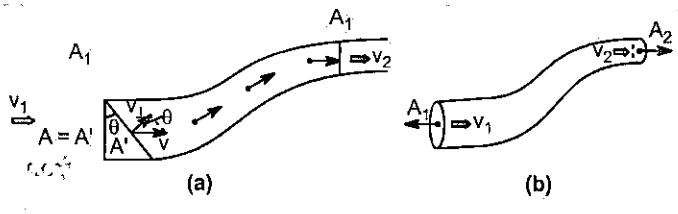


Fig. 1.59 (i)

$\vec{A}$ is the area vector; its magnitude is the area A (units of m^2 in SI) and its direction is normal to the surface and pointing outward.

$$\text{Volume flux} = vA\cos\theta = \vec{v}\cdot\vec{\mathbf{A}}$$

At the input face angle between area vector and velocity vector is 180°.

$$\text{Volume flux} = v_1A_1\cos 180° = -v_1A_1$$

At the output face angle between area vector and velocity vector is 0°.

$$\text{Flux} = v_2A_2\cos 0° = +v_2A_2$$

Flux through curved surface of tube is zero because the angle between v and A is everywhere 90°.

From the equation of continuity the volume flux through both end surfaces is equal in magnitude.

The net fluid flux (into and out of the closed area) summed over all the surfaces equals zero.

Now we will apply these ideas to an electric field.

ELECTRIC FLUX

If we visualize electric field lines in three dimensions, surrounding a charge distribution, the number of lines per unit area that pass perpendicular through a surface is proportional to the electric field strength. The electric flux ϕ_E is a measure of the number of electric field lines passing perpendicularly through a surface. Consider an imaginary area A placed in an electric field as shown in Fig. 1.60. The small area element ΔA_i is small so that field at this location can be assumed to be uniform. We define electric flux $\Delta\phi_E$ through ΔA_i as

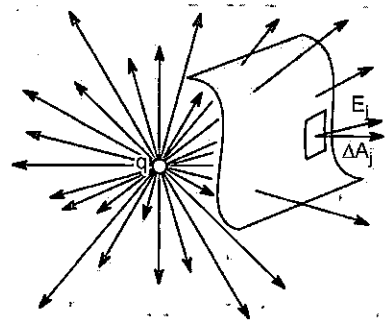


Fig. 1.60

$$\Delta\phi_{Ei} = E_i\Delta A_i\cos\theta$$
$$= (E_{\perp i})\Delta A_i$$
$$= E_i(\Delta A_{\perp i})$$
$$= E_i\cdot\Delta A_1$$

Since flux is scalar, the total flux through the entire surface is sum of flux through all the area elements constituting A.

If we consider infinite area elements on the surface, the size of each element ΔA_i grows to zero, $\Delta A_i \to 0$ and the sum is replaced by an integral.

Thus, $$\phi_E = \oint \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}}$$

The integral on the right hand side is known as a **surface integral**.

AREA VECTOR

We define a vector that describes not only the area of a surface element, but its orientation as well. Fig 1.61(a) shows an element of area dA. It may be a part of a larger curved surface, but if $d\vec{\mathbf{A}}$ itself is sufficiently small it is indistinguishable from a planar surface element. We draw a vector $d\vec{\mathbf{A}}$ according to the following procedure:

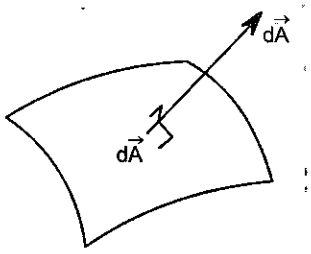


Fig. 1.61 (a)

1. The direction $d\hat{\mathbf{A}}$ is taken normal to the surface element. Because the direction of the normal to a plane is determined by the orientation of the plane, we specify the orientation of the plane when we specify the direction of the normal.

2. The magnitude $d\vec{\mathbf{A}}$ is equal to the magnitude of the area.

The vector $d\vec{\mathbf{A}} \equiv dA\ d\hat{\mathbf{A}}$ then expresses both the area of the element and its orientation. This procedure for defining $d\hat{\mathbf{A}}$ is incomplete because it does not distinguish between the vector $d\vec{\mathbf{A}}$ shown and its negative.

3. When the area element is a part of a closed surface, the outward sense determines the direction of the unit normal vector $d\hat{\mathbf{A}}$ [Fig. 1.61(b)].

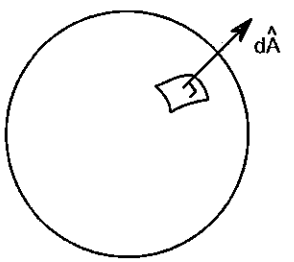


Fig. 1.61 (b)

Now consider the dot product $\vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$. The electric field lines penetrate $d\vec{\mathbf{A}}$ in some arbitrary direction, and $\vec{\mathbf{E}}$ is tangent to the electric field lines (all of which are essentially parallel over the infinitesimal area element). Let θ be the angle between $\vec{\mathbf{E}}$ and $d\vec{\mathbf{A}}$. As Fig. 1.61 (c) shows, the same electric lines that

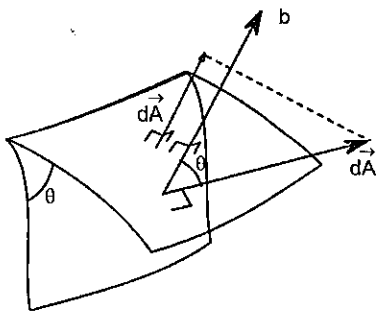


Fig. 1.61 (c)

penetrate $d\vec{\mathbf{A}}$ also penetrate an element of area $dA' = dA \cos\theta$ that lies normal to $\vec{\mathbf{E}}$. This element satisfies the condition for using equation

$$d\Phi = E\ dA \cos\theta \qquad \ldots(i)$$

We write this in the more convenient and more general form

$$d\Phi = \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} \qquad \ldots(ii)$$

The *total* flux Φ penetrating the Gaussian surface is found by integrating Equation (i) over the whole surface:

$$\Phi = \int_{\text{closed surface}} d\Phi = \int_{\text{closed surface}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} \qquad \ldots(iii)$$

$\vec{\mathbf{E}}$ can vary over a surface, only in certain symmetry cases $\vec{\mathbf{E}}$ turns out to be a constant on the surface.

If we consider a spherical surface surrounding a point charge, the field on the surface of the sphere is constant in magnitude, but it varies in direction for different point (Fig. 1.62).

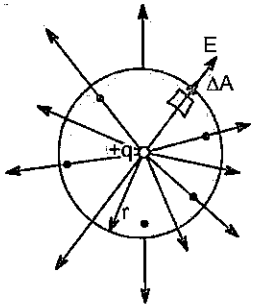


Fig. 1.62

Electric flux through an area element can be positive, negative or zero depending on whether the angle between the electric field vector and the area vector is acute, obtuse or right angle, as shown in Fig. 1.63.

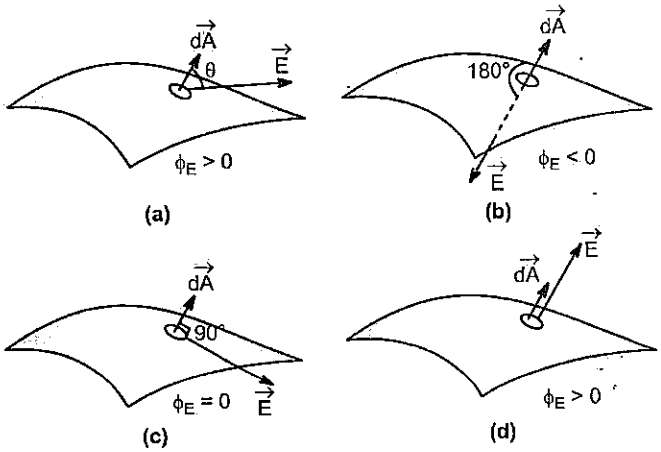


Fig. 1.63

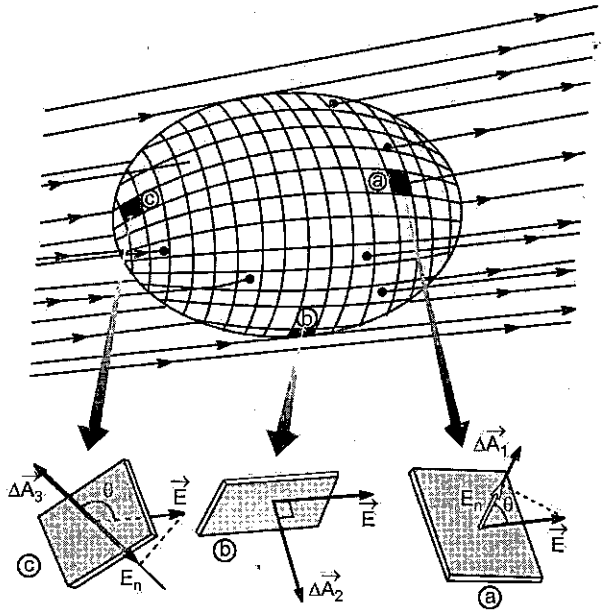
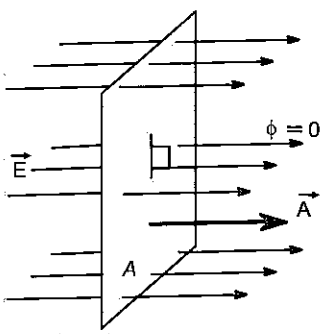
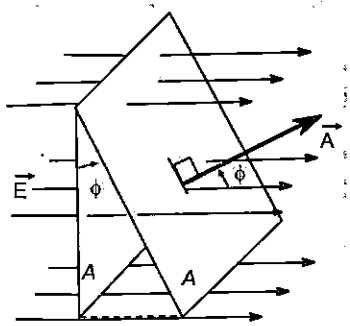


Fig. 1.64: A closed surface in an electric field. The area vectors are, by convention, normal to the surface and point out-ward. The flux through an area element can be positive [element (a)], zero [element (b)], or negative [element (c)].

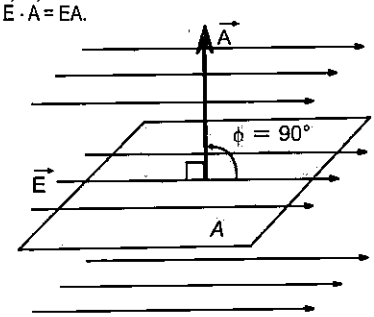
Concept: *A flat surface in a uniform electric field. The electric flux Φ_E through the surface equals the scalar product of the electric field $\vec{E}$ and the area vector $\vec{A}$.*



(a) $\vec{E}$ and $\vec{A}$ are parallel (the angle between $\vec{E}$ and $\vec{A}$ is $\phi = 0$).
The flux $\Phi_E = \vec{E} \cdot \vec{A} = EA$.



(b) The angle between $\vec{E}$ and $\vec{A}$ is ϕ.
The flux $\Phi_E = \vec{E} \cdot \vec{A} = EA \cos\phi$.



(c) $\vec{E}$ and $\vec{A}$ are perpendicular (the angle between $\vec{E}$ and $\vec{A}$ is $\phi = 90°$)
The flux $\Phi_E = \vec{E} \cdot \vec{A} = EA \cos 90° = 0$.

Fig. 1.65

Example 41

Find the flux of the electric field through each of the five surfaces of the inclined plane as shown in Fig. 1E.41. What is the total flux through the entire closed surface?

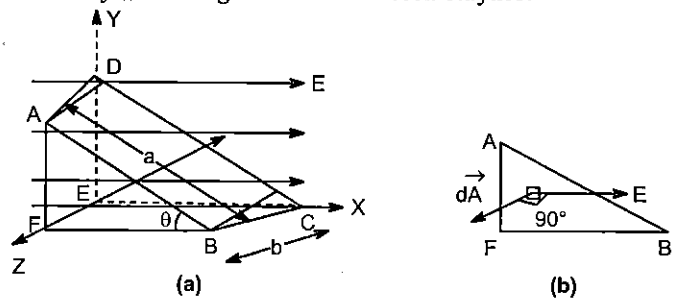


Fig. 1E.41

Solution: Note that flux through the faces *ABF, CDE* and *BCEF* is zero. Area vector of face *ABF* points in the positive z-direction, area vector of *CDE* points in the negative z-direction and area vector of *BCEF* points in the negative y-direction. In all the three cases, field E is normal to area vector.

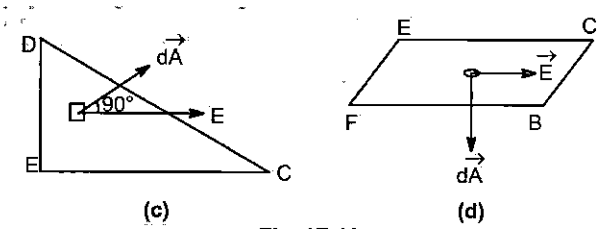
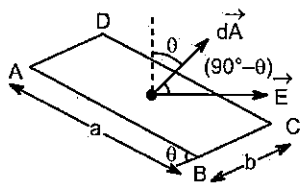
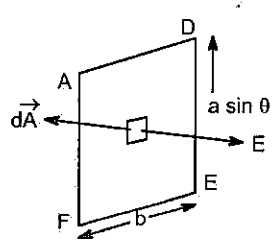


Fig. 1E.41

Flux through face *ABCD*; Magnitude of area vector of face *ABCD* = ab



(e)



(f)

Fig. 1E.41

$$\phi_E = \vec{E} \cdot \vec{A}$$
$$= E(ab)\cos(90°-\theta) = Eab\sin\theta$$

Flux through face *ADEF*;

Magnitude of area vector of face

$$ADEF = (a\sin\theta)b$$
$$= ab\sin\theta$$

$$\phi_E = E\cos 180°(ab\sin\theta)$$
$$= -Eab\sin\theta$$
$$(\phi_E)_{ABF} = 0$$
$$(\phi_E)_{CDE} = 0$$
$$(\phi_E)_{BCEF} = 0$$
$$(\phi_E)_{ABCD} = +Eab\sin\theta$$
$$(\phi_E)_{ADEF} = -Eab\sin\theta$$

Flux is a scalar quantity, therefore total flux is algebraic sum of flux through each surface.

$$\phi_{total} = (\phi_E)_{ABF} + (\phi_E)_{CDE} + (\phi_E)_{BCEF} + (\phi_E)_{ABCD} + (\phi_E)_{ADEF}$$
$$= 0 + 0 + 0 + Eab\sin\theta - Eab\sin\theta$$
$$= 0$$

* Note that the contribution to the flux for a closed surface is positive for the surface where the field is directed out and negative for the surface where the field is directed into the surface.
* The net flux for this closed surface can also be seen to be zero from examination of the field lines. If the field is uniform, the number of lines that enter the closed surface equals the number of lines that come out.
* **The flux of a constant vector through any closed surface is zero.**

Example 42

Consider a cube of edge a, kept in a uniform electric field of magnitude E, directed along x-axis as shown in Fig. 1E.42.

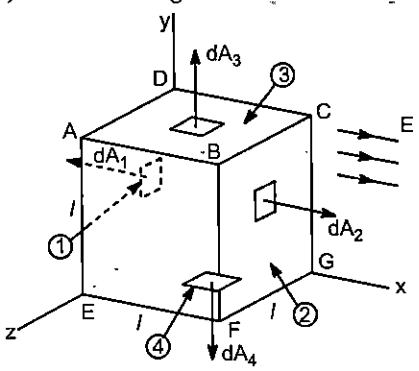


Fig. 1E.42

Solution: The net flux is algebraic sum of the flux through all the faces of the cube.

Note that flux through faces *ABFE, BCGF, ADHE, CDHG* is zero because *E* is normal to area vector on these faces.

Flux through *EFGH*,

$$(\phi_E)_{EFGH} = E(a^2)\cos 180°$$
$$= -Ea^2$$

Area vector and electric field vector are opposite to each other. Flux through *ABCD*,

$$(\phi_E)_{ABCD} = E(a^2)\cos 0°$$
$$= +Ea^2$$

Area vector and electric field vector are parallel to each other. Net Flux over all the six faces is

$$\phi_E = (\phi_E)_{ABFE} + (\phi_E)_{BCGF} + (\phi_E)_{ADHE} + (\phi_E)_{CDHG} + (\phi_E)_{ABCD} + (\phi_E)_{EFGH}$$
$$= 0 + 0 + 0 + 0 + (-Ea^2) + Ea^2$$
$$= 0$$

Concepts: ***A cube of side*** *L is placed in a region of uniform electric field* $\vec{E}$. *Find the electric flux through each face of the cube and the total flux through the cube when the cube is turned by an angle* θ, *as in Fig. 1.66.*

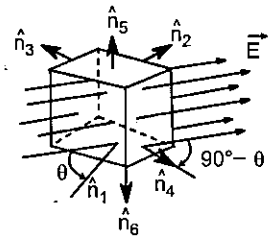


Fig. 1.66

The unit vectors for each face ($\hat{n}_1$ *through* $\hat{n}_6$) *are shown in the figure; the direction of each unit vector is outward from the closed surface of the cube.*

The fluxes through faces 1 and 3 are negative, since $\vec{E}$ *is directed into those faces; the field is directed out of faces 2 and 4, so the fluxes through those faces are positive. We find*

$$\Phi_{E1} = \vec{E}\cdot\hat{n}_1 A = EL^2\cos(180° - \theta) = -EL^2\cos\theta$$

$$\Phi_{E2} = \vec{E}\cdot\hat{n}_2 A = +EL^2\cos\theta$$

$$\Phi_{E3} = \vec{E}\cdot\hat{n}_3 A = EL^2\cos(90° + \theta) = -EL^2\sin\theta$$

$$\Phi_{E4} = \vec{E}\cdot\hat{n}_4 A = EL^2\cos(90° - \theta) = +EL^2\sin\theta$$

$$\Phi_{E5} = \Phi_{E6} = EL^2\cos 90° = 0$$

The total flux $\Phi_E = \Phi_{E1} + \Phi_{E2} + \Phi_{E3} + \Phi_{E4} + \Phi_{E5} + \Phi_{E6}$ *through the surface of the cube is again zero.*

Example 43

Consider a cylindrical surface of radius R, length l, in a uniform electric field E. Compute the electric flux if the axis of the cylinder is parallel to the field direction.

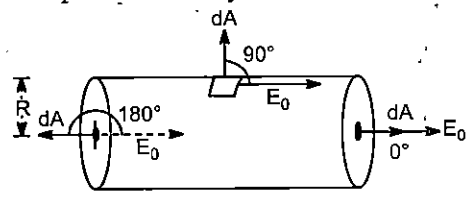


Fig. 1E.43

Solution: We can divide the entire curved surface into three parts, right and left plane faces and curved portion of its surface. Hence the surface integral consists of the sum of the three terms :

$$\phi_E = \oint E \cdot dA$$
$$= \int_{\text{left end}} E \cdot dA + \int_{\text{right end}} E \cdot dA + \int_{\text{curved}} E \cdot dA$$

All the area elements on the left end and electric field E are at an angle of 180°

$$(\phi_E)_{\text{left end}} = \int_{\text{left end}} E \cdot dA$$
$$= \int_{\text{left end}} EdA\cos 180°$$
$$= -E\int_{\text{left end}} dA = -E\pi R^2$$

Note that E is constant over the entire plane surface of left end; therefore we take it out from the integral.

Similarly, all the area elements on the right end are parallel to electric field E, *i.e.*, $\theta = 0°$.

$$(\phi_E)_{\text{right end}} = \int_{\text{right end}} E \cdot dA$$
$$= \int_{\text{right end}} EdA\cos 0°$$
$$= +E\int_{\text{right end}} dA = E\pi R^2$$

Finally at every point on the curved surface the area vectors are perpendicular to the direction of the electric field. Thus,

$$(\phi_E)_{\text{curved}} = \int_{\text{curved surface}} E \cdot dA$$
$$= \int_{\text{curved surface}} E\,dA(\cos 90°) = 0$$

$$\text{Total flux} = (\phi_E)_{\text{right end}} + (\phi_E)_{\text{left end}} + (\phi_E)_{\text{curved suface}}$$
$$= (+E\pi R^2) + (-E\pi R^2) + 0 = 0$$

Example 44

Consider the radial field of a point charge q. Compute the flux through a spherical surface that encloses this charge, with the charge at its centre.

Solution: From symmetry, the electric field strength is constant in magnitude on the surface. The words *from symmetry* imply that all the points on the surface of the sphere are equidistant from point charge. From any point on the surface the charge q looks the same. At every point on the spherical surface the electric field $\vec{\mathbf{E}}$ is parallel to the area vector $d\vec{\mathbf{A}}$.

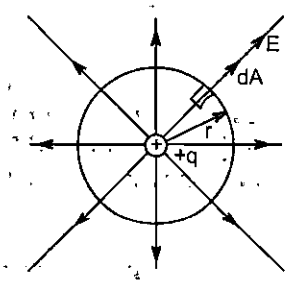


Fig. 1E.44

$$\phi_E = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$$
$$= \oint EdA(\cos 0°)$$
$$= E\oint dA = E \cdot 4\pi r^2$$

The summation of dA is the surface area of sphere, $4\pi r^2$

Point Charge Inside a Non-spherical Surface

Concepts: *Let us surround the sphere of radius R by a surface of irregular shape, as in Fig. 1.67 (a). Consider a small element of area dA on the irregular surface; we note that this area is larger than the corresponding element on a spherical surface at the same distance from q. If a normal to dA makes an angle ϕ with a radial line from q, the electric flux through the spherical surface element is equal to the flux E dA cos ϕ through the corresponding irregular surface element.*

We can divide the entire irregular surface into elements dA, compute the electric flux E dAcos ϕ for each, and sum the results by integrating, Each of the area elements projects onto a corresponding sherical surface element. Thus the total electric flux through the irregular surface, must be the same as the total flux through a sphere.

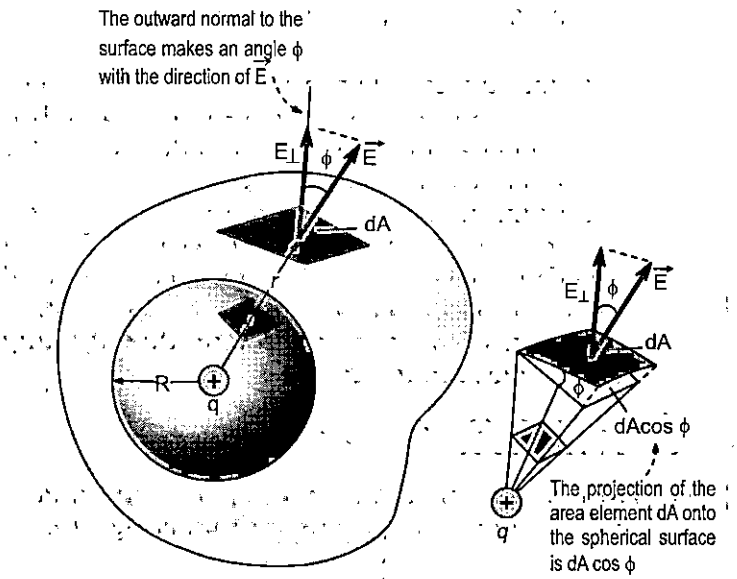


Fig. 1.67

More About Field Lines

The flux for a surface is proportional to the number of field lines that cross the surface. For the case of a spherical

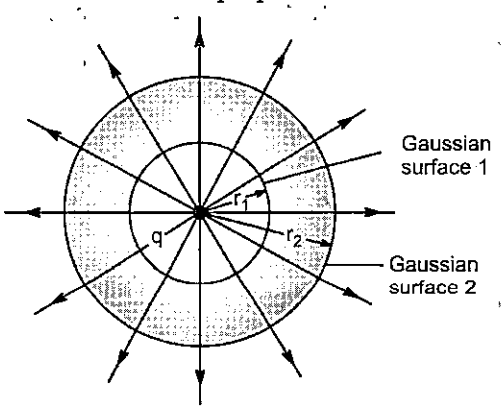


Fig. 1.68

surface centered at a point charge, we can now demonstrate this with Gauss's law. Fig. 1.68 shows two such surfaces and the field lines around the point charge q. Since the lines emanate from a positive charge, terminate on a negative charge, and are continuous in between, the number of lines that cross each sphere is the same. By Gauss's law, the flux ϕ_E for each sphere is the same because the enclosed charge $\Sigma q = q$ is the same. Thus the flux for each surface is proportional to the number of lines that cross the surface.

When field lines were introduced, we stated that the density of the lines (or their spacing) indicates the field magnitude. Again, this can now be demonstrated for the case of a point charge. In Fig. 1.68, the radius of the larger sphere is twice that of the smaller sphere: $r_2 = 2r_1$. Thus the area of the larger sphere is 4 times that of the smaller sphere: $A_2 = 4\pi r_2^2 = 4\pi(2r_1)^2 = 4(4\pi r_1^2) = 4A_1$. If N is the number of lines crossing a sphere of area, A, then the density of lines is N/A. Since the number of lines crossing each sphere is the same, the density of lines at the larger sphere is one-fourth that at the smaller sphere. Also, since the field is an inverse-square field, E at the larger sphere is one-fourth that at the smaller sphere. Thus E is proportional to the density of lines.

SOLID ANGLE

Solid angle is a generalization of the plane angle. To construct a solid angle, we start with a surface S and join all the points on the periphery such as A, B, C, D etc. with the given point O. We then say that a *solid angle* is formed at O and that the surface S has subtended the solid angle. The solid angle is formed by the lines joining the points on the periphery with O. The whole Fig looks like a cone.

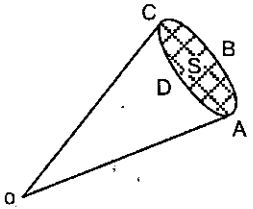


Fig. 1.69

In order to measure a solid angle at the point O (Fig. 1.70), we draw a sphere of any radius r with O as the centre and measure the area S of the part of the sphere intercepted by the cone. The solid angle Ω is then defined as

$$\Omega = A / r^2$$

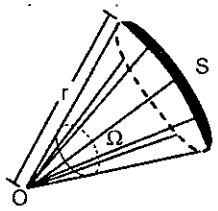


Fig. 1.70

Note that this definition makes the solid angle a dimensionless quantity. It is independent of the radius of the sphere drawn.

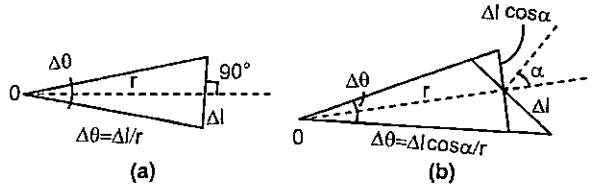


Fig. 1.71

Solid Angle (General Definition)

Fig. 1.72 shows a differential area element $d\vec{\mathbf{A}}$ at a position vector $\vec{\mathbf{r}}$ from point O, that subtends solid angle dω at O.

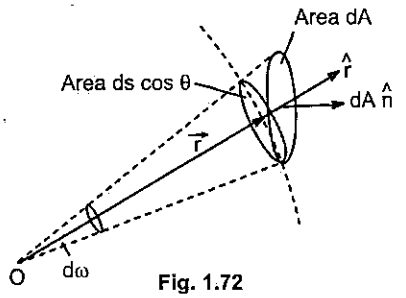


Fig. 1.72

$$d\omega = \frac{d\vec{\mathbf{A}} \cdot \hat{\mathbf{r}}}{r^2} = \frac{dA}{r^2}\,\hat{\mathbf{n}} \cdot \hat{\mathbf{r}}$$

$$d\omega = \frac{dA\cos\theta}{r^2}$$

As $\hat{\mathbf{n}} \cdot \hat{\mathbf{r}} = \cos\theta.$

Here vector $d\vec{\mathbf{A}}$ has magnitude ds and direction outward normal represented by unit vector $\hat{\mathbf{n}}$. Angle between $\hat{\mathbf{r}}$ and $\hat{\mathbf{n}}$ is θ. Total solid angle subtended by a finite area can be obtained from

$$\omega = \int \frac{dA\cos\theta}{r^2}$$

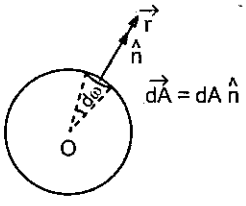


Fig. 1.73

Solid angle subtended by a spherical surface at its centre is

$$\omega = \frac{(4\pi r^2)\cos\theta}{r^2} = 4\pi \text{ steradian}$$

Any closed surface subtends 4π steradian at any inside point and zero at outside point.

Complete sphere subtends a solid angle

$$\Omega = \frac{A}{r^2}\,\frac{4\pi r^2}{r^2} = 4\pi$$

At the centre. Also, any closed surface subtends a solid angle 4π at any internal point.

Let us first consider the field of a single point charge q. We enclose this charge by an arbitrary closed surface S (Fig. 1.74) and find the flux of **E** through the area element $d\mathbf{S}$:

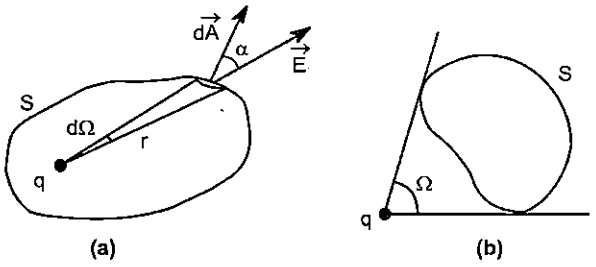


Fig. 1.74

$$d\Phi = EdA\cos\alpha$$
$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}dS\cos\alpha$$
$$= \frac{q}{4\pi\varepsilon_0}d\Omega \qquad ...(1)$$

Where $d\Omega$ is the solid angle resting on the area element dA and having the vertex at the point where the charge q is located.

It should be noted that for a more complicated shape of a closed surface, the angles α may be greater than $\pi/2$ and hence cos α and $d\Omega$ generally assume either positive or negative values. Thus, $d\Omega$ is an algebraic quantity : if $d\Omega$ rests on the inner side of the surface S, $d\Omega > 0$, while if it rests on the outer side, $d\Omega < 0$.

In particular, this leads to the following conclusion: if the charge q is located outside a closed surface S, the flux of E through this surface is equal to zero. In order to prove this, it is sufficient to draw through the charge q a conical surface tangent to the closed surface S. Then the integration of eqn. (1) over the surface S is equivalent to the integration over Ω [Fig. 1.74 (b)]: the outer side of the surface S will be seen from the point q at an angle $\Omega > 0$, while the inner side, at an angle $-\Omega$ (the two angles being equal in magnitude). The sum is equal to zero, and $\Phi = 0$.

Example 45

A very thin disc is uniformly charged with surface charge density $\sigma > 0$. Find the electric field intensity E on the axis of this disc at the point from which the disc is seen at an angle Ω.

Solution. It is clear from symmetry considerations that on the disc axis vector $\vec{\mathbf{E}}$ must coincide with the direction of this axis [Fig. 1E.45]. Hence, it is sufficient to find the component dE_z from the charge of the area element dA at the point A and then integrate the obtained expression over the entire surface of the disc. It can be easily seen that

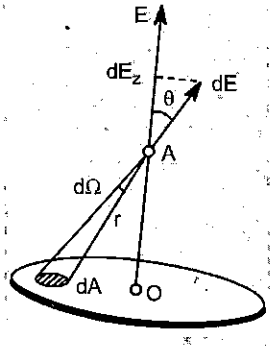


Fig. 1E.45

$$dE_z = \frac{1}{4\pi\varepsilon_0}\frac{\sigma dA}{r^2}\cos\theta \qquad ...(1)$$

In our case $(dA\cos\theta)/r^2 = d\Omega$ is the solid angle at which the area element dA is seen from the point A, and eqn. (1) can written as

$$dE_z = \frac{1}{4\pi\varepsilon_0}\sigma d\Omega.$$

Hence, the required quantity is

$$E = \frac{1}{4\pi\varepsilon_0}\sigma\Omega.$$

It should be noted that at large distances from the dice, $\Omega = A/r^2$, where S is the area of the disc and $E = q/4\pi\varepsilon_0 r^2$ just as the field of the point charge $q = \sigma A$. In the immediate vicinity of the point O, the solid angle $\Omega = 2\pi$ and $e = \sigma/2\varepsilon_0$.

DEVELOPING GAUSS'S LAW FROM COULOMB'S LAW

We present Coulomb's law as the result of experiment, and used it to write the field produced by a point charge as

$$E = \frac{q}{4\pi\varepsilon_0 r^2}\hat{r}$$

We now give Gauss's law this same experimental foundation by developing Gauss's law from Coulomb's law and the principle of superposition. The above expression for the field produced by a point charge is the form of Coulomb's law we shall use.

Flux for an Arbitrary Surface, Charged Particle Outside

Consider the flux for the gaussian surface shown in Fig. 1.75. The field is due to a charged particle, and the surface is bounded by four flat sides and two spherical caps. Each flat side is aligned radially with the particle, and each cap is a patch of a sphere centered at the particle. Therefore the flux for the sides is zero because $\vec{\mathbf{E}}$ is perpendicular to $d\vec{\mathbf{A}}$ at each point on the sides.

The flux ϕ_{E_1} for cap 1 is negative (assuming q is positive) because the direction of $\vec{\mathbf{E}}$ is opposite $d\vec{\mathbf{A}}$ at each point on cap 1: $\int \vec{\mathbf{E}}.d\vec{\mathbf{A}} = -\int \vec{\mathbf{E}}d\vec{\mathbf{A}}$. Further, E is the same at each point on the surface and can be factored out of the flux integral: $E = q/4\pi\varepsilon_0 r_1^2$. Thus

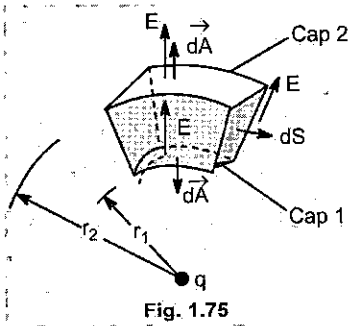


Fig. 1.75

$$\phi_{E_1} = -\frac{q}{4\pi\varepsilon_0 r_1^2}\int d\vec{\mathbf{A}}$$
$$= -\frac{q}{4\pi\varepsilon_0 r_1^2} - \Delta A_1$$

where ΔA_1 is the area of cap 1. The flux ϕ_{E_2} for cap 2 is calculated in a similar fashion, except that ϕ_{E_2} is positive because $\vec{\mathbf{E}}$ is in the same direction as $d\vec{\mathbf{A}}$ at each point on cap 2:

$$\phi_{E_2} = \frac{q}{4\pi\varepsilon_0 r_2^2}\Delta A_2$$

where ΔA_2 is the area of cap 2. Since the two spherical caps are bounded by flat sides that are aligned radially, the ratio of their areas is equal to the ratio of their radii squared: $\Delta A_2/\Delta A_1 = r_2^2/r_1^2$,

or $\Delta A_2 = (r_2^2/r_1^2)\Delta A_1$. Substituting this result into the equation for ϕ_{E_2} gives

$$\phi_{E_2} = \frac{q}{4\pi\varepsilon_0 r_2^2}\frac{r_2^2}{r_1^2}\Delta A_1$$

$$= \frac{q}{4\pi\varepsilon_0 r_1^2}\Delta A_1 = -\phi_{E_1}$$

The net flux for the closed surface is

$$\phi_E = \phi_{E_2} + \phi_{E_1}$$
$$= -\phi_{E_1} + \phi_{E_1} = 0$$

The net flux is zero because the flux for cap 1 is the negative of the flux for cap 2.

Concepts: *Now we introduce another point: A surface of any shape can be constructed from an infinite number of infinitesimal spherical caps and flat sides. Fig. 1.76(a) shows a cross section of an arbitrarily shaped, closed surface with a particle of charge q outside the enclosed volume.*

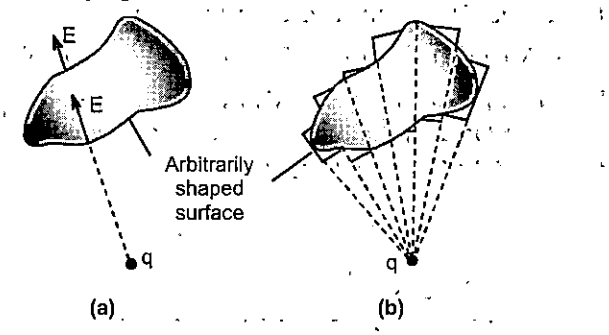


Fig. 1.76

Fig. 1.76(b) shows this same surface with a superimposed approximation to the surface that consists of a number of spherical caps centered at the particle and flat sides aligned with the particle. You can see that the surface can be viewed as the limit of an infinite number of infinitesimal spherical caps and flat sides. Since the flux for the caps cancels in pairs and the flux for the sides is zero, for the arbitrarily shaped surface in Fig. 1.76 (a), the flux due to the charged particle outside the enclosed volume is zero.

$$\phi_E = 0$$

(arbitrarily shaped closed surface, q outside)

Flux for an Arbitrary Surface, Charged Particle Inside

Now consider the flux due to a point charge q at the center of a spherical gaussian surface of radius r (Fig. 1.77). At each point on the surface, $\vec{\mathbf{E}}$ is parallel to $d\vec{\mathbf{A}}$ ($\vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = E\,dA$) and E has the same value and can be factored from the integral ($E = q/4\pi\varepsilon_0 r^2$). Thus

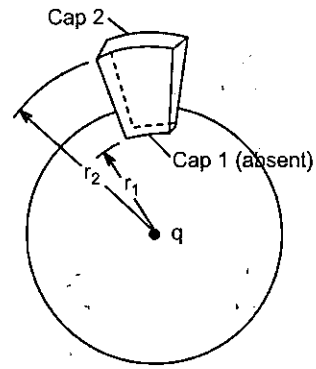


Fig. 1.77

$$\phi_E = \oint \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}}$$

$$= \oint E\,dA = E\oint dA = \frac{q}{4\pi\varepsilon_0 r^2}4\pi r^2 = \frac{q}{\varepsilon_0}$$

Since ϕ_E does not contain r, the flux is the same for a sphere of any radius.

We can use this result to find the flux for the gaussian surface shown in Fig. 1.77. This surface is mostly a sphere centered at the particle, except that a cap of area ΔS_1 is cut out and replaced with a raised cap of area ΔS_2. The volume directly beneath the raised cap is enclosed by flat sides (aligned with the particle) so that the surface is a closed surface. The flux for this surface is the same as for a sphere because the flux for the flat sides is zero and the flux for cap 2 is equal to the flux missing because of the absence of cap 1; the argument is similar to the previous discussion. Thus $\phi_E = q/\varepsilon_0$.

Concepts: *Further, any arbitrarily shaped surface may be regarded as the limit of an infinite number of infinitesimal spherical caps and flat sides. Fig. 1.78 (a) shows the cross section of an arbitrarily shaped closed surface with a charged particle inside, and Fig. 1.78 (b) shows a number of spherical caps and flat sides centered at the particle. As before, the arbitrarily shaped surface can be viewed as the limit of an infinite number of infinitesimal spherical caps and flat sides. Therefore, for a closed surface with any shape, the flux due to a particle of charge q inside the enclosed volume is q/ε_0:*

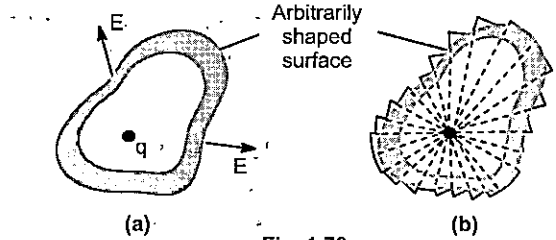


Fig. 1.78

$$\phi_E = \frac{q}{\varepsilon_0}$$

(arbitrarily shaped closed surface, q inside)

Flux for an Arbitrary Surface, Charged Particles Inside and Outside

So far we have considered the electric flux due to a single charged particle. Suppose there is more than one particle to consider. To be specific, consider the surface in Fig. 1.79, where the flux is due to the three charged particles. From the principle of superposition, the field is the vector sum of the individual contributions to the field: $\vec{\mathbf{E}} = \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2 + \vec{\mathbf{E}}_3$. The flux is

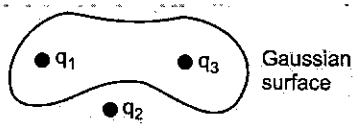


Fig. 1.79

$$\phi_E = \oint \vec{\mathbf{E}}.d\vec{\mathbf{A}} = \oint (\vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2 + \vec{\mathbf{E}}_3).d\vec{\mathbf{A}}$$

Since the integral of a sum is the sum of the integrals, we may write this as

$$\phi_E = \oint \vec{\mathbf{E}}_1.d\vec{\mathbf{A}} + \oint \vec{\mathbf{E}}_2\, d\vec{\mathbf{A}} + \oint \vec{\mathbf{E}}_3.d\vec{\mathbf{A}}$$

Particles 1 and 3 are inside the surface so that their contributions to the flux are q_1/ε_0 and q_3/ε_0, respectively. Particle 2 is outside the surface so that its contribution to the flux is zero.

$$\phi_E = \frac{q_1}{\varepsilon_0} + 0 + \frac{q_3}{\varepsilon_0}$$

Generally, for any number of charged particles, the flux for an arbitrarily shaped closed surface is

$$\phi_E = \frac{\Sigma q_i}{\varepsilon_0}$$

or
$$\oint \vec{\mathbf{E}}.d\vec{\mathbf{A}} = \frac{\Sigma q_i}{\varepsilon_0}$$

Concepts: *which is Gauss's law. Our development shows clearly that the field E in the flux integral is the field due to all charged particles, both inside and outside the enclosed volume, but the charges included in the sum Σq_i are only the charges of the particles that are inside the enclosed volume.*

STATEMENT OF GAUSS'S LAW

The Gauss's law can be stated as :

The electric flux for any closed surface is equal to the net charge enclosed by the surface divided by ε_0.

In equation form,

$$\phi_E = \frac{\text{net charge enclosed}}{\varepsilon_0}$$

or
$$\oint E \cdot dA = \frac{\text{net charge enclosed}}{\varepsilon_0}$$

Concepts: **1.** *If several charges are enclosed by a surface, the electric field follows a principle of superposition. The flux of each field through the surface add algebraically.*

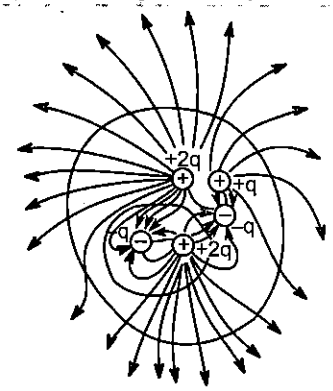


Fig. 1.80

$$\oint \vec{\mathbf{E}}.d\vec{\mathbf{A}} = \oint \vec{\mathbf{E}}_1.d\vec{\mathbf{A}} + \oint \vec{\mathbf{E}}_2.d\vec{\mathbf{A}}$$

$$\oint \vec{\mathbf{E}}.d\vec{\mathbf{A}} = \oint \vec{\mathbf{E}}_1.d\vec{\mathbf{A}} + \oint \vec{\mathbf{E}}_2.d\vec{\mathbf{A}} + \oint \vec{\mathbf{E}}_3.d\vec{\mathbf{A}} + \ldots$$

$$= \frac{Q_1}{\varepsilon_0} + \frac{Q_2}{\varepsilon_0} + \frac{Q_3}{\varepsilon_0} + \ldots$$

$$= \frac{Q_{net}}{\varepsilon_0}$$

As shown in Fig. 1.80, the positive charges provide an outwardly directed flux; negative charges draw the flux inward, and the difference, in or out of the surface, is the net flux associated with the charge distribution.

2. *Note that the electric field in the expression $\oint E \cdot dA$ is the resultant field on the Gaussian surface, whereas Q_{net} is the charge enclosed by the Gaussian surface. Consider the two Gaussian surfaces A_1 and A_2 as shown in Fig. 1.81.*

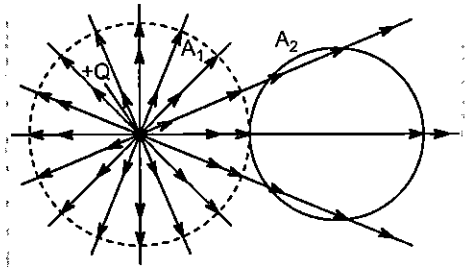


Fig. 1.81

Charge Q lies at the centre of the Gaussian surface A_1. For surface A_1 the net flux through A_1 is Q/ε_0. For surface A_2, charge Q is outside A_2, so that the net flux through A_2 is zero. Note that the field lines that enter the Gaussian surface (net flux in) also leave it (net flux out).

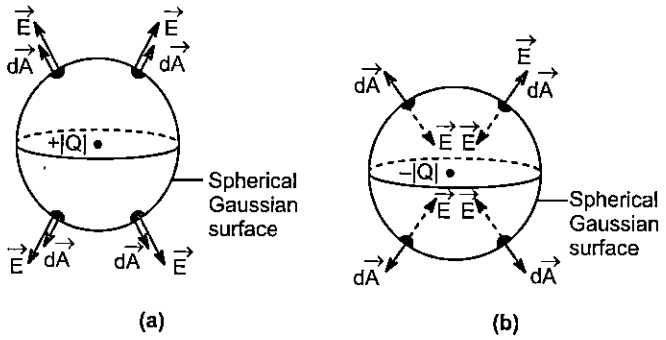


Fig. 1.82

Definition of a Gaussian Surface

While applying Gauss's law we are interested in evaluating the integral.

$$\phi_E = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$$

The closed surface for which the flux is calculated is generally an imaginary or hypothetical surface, called a **Gaussian surface**. Whenever we apply Gauss's law we may devise a surface of any size and shape as our Gaussian surface. But selecting a proper size and shape for a Gaussian surface is a key factor for determining flux.

Concepts: ***1.*** *Consider a point charge q at the origin, see figure the flux of* $\vec{\mathbf{E}}$ *through a sphere of radius r is*

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \int \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r^2}\, r\right) \cdot (r^2 \sin\theta\, d\theta\, d\phi\, \hat{\mathbf{r}}) = \frac{1}{\varepsilon_0} q \qquad \ldots(1)$$

Notice that the radius of the sphere cancels out, as the surface area goes up as r^2*, the field goes down as* $1/r^2$*, and so the product is constant. In terms of the field-line the same number of field lines passes through any sphere centred at the origin, regardless of its size. For an arbitrary surface, whatever its shape, would trap the same number of field lines. Evidently the flux through any surface enclosing the charge is* q/ε_0.

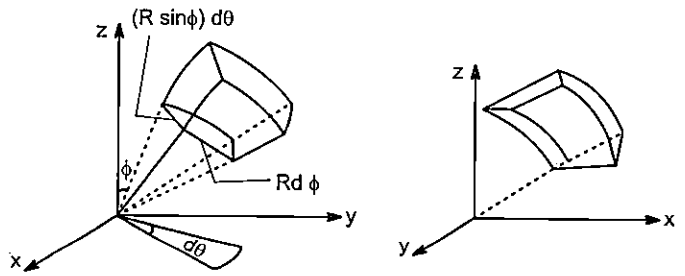


Fig. 1.83

Volume element in spherical coordinates $dV = R^2 \sin\phi\, dR\, d\theta\, d\phi$

2. *The origin, for a collection of charges. According to the principle of superposition, the total field is the (vector) sum of all the individual fields:*

$$\vec{\mathbf{E}} = \sum_{i=1}^{n} \vec{\mathbf{E}}_i$$

The flux through a surface that encloses them all, then is

$$\oint \vec{\mathbf{E}} . d\vec{\mathbf{A}} = \sum_{i=l}^{n} (\oint \vec{\mathbf{E}}i \, . d\vec{\mathbf{A}}) = \sum_{i=1}^{n} \left(\frac{1}{e_0}\, qi\right)$$

For any closed surface, then,

$$\oint_s \vec{\mathbf{E}} . d\vec{\mathbf{A}} = \frac{1}{\varepsilon_0} Q_{enc}$$

Where Q_{enc} *is the total charge enclosed within the surface. This is the quantitative statement of Gauss's law.*

3. *Notice that it all depends on the* $1/r^2$ *character of Coulomb's law; without* ***that the crucial cancellation of the r' s,*** *and the total flux of* $\vec{\mathbf{E}}$ *would depend on the surface chosen, not merely on the total charge enclosed.*

4. ***Other*** $1/r^2$ *forces (I Newton's law of universal gravitation) will obey "Gauss's laws" in a modified way, and the applications we develop here will have anologue in gravitation.*

5. *A Gauss's law is an integral equation, but we can readily turn it into a differential one which is beyond scope of this book we are just quoting the result.*

$$\nabla . \mathbf{E} = \frac{1}{\varepsilon_0} p$$

Equation carries the same message as integral equation, it is ***Gauss's law in differential form.***

6. *When we should apply Gauss's law for calculation of electric field*

Gauss's law is always true, but it is not always useful. If q had not been uniform (or, is even not spherically symmetrical), or some other arbitrary shape for Gaussian surface not it would still have been true that he the flux of $\vec{\mathbf{E}}$ *is* $(1/\varepsilon_0)q$*, but now we are certain that* $\vec{\mathbf{E}}$ *was in the same direction as* $d\vec{\mathbf{A}}$ *and constant in magnitude over the surface, and without that we can not take* $|\vec{\mathbf{E}}|$ ***out of the integral. Symmetry is crucial to this*** *application of Gauss's law there are only three kinds of symmetry that work :*

(i) *Spherical symmetry. Make your Gaussian surface a concentric sphere.*

(ii) *Cylindrical symmetry. Make your Gaussian surface a coaxial cylinder.*

(iii) *Plane symmetry. Use a Gaussian "pillbox," Although (ii) and (iii) technically require infinitely long cylinders, and planes extending to infinity in all directions, to get approximate answers for "long" cylinders or "large" plane surfaces, at points far from the edges. We can use them.*

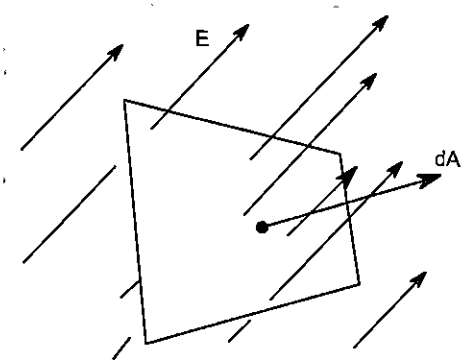


Fig. 1.84

7. *Consider a plane surface the flux of* $\mathbf{E}$ *through a surface* S, *placed in an electric field*

$$\Phi_E \equiv \int_S \mathbf{E} \cdot d\vec{\mathbf{A}}, \qquad ...(1)$$

Is a measure of the "number of field lines" passing through S. *We can only draw a representative sample of the field lines—the total number would be infinite. But for a given case the flux is proportional to the number of lines drawn, because the field strength, remember, is proportional to the density of field lines (the number per unit area), and hence* $\mathbf{E}.d\vec{\mathbf{A}}$ *is proprtional to the number of lines passing through the infinitesimal area* $d\vec{\mathbf{A}}$. *(The dot product gives the component of* $\vec{\mathbf{A}}$ *along the direction of* $\mathbf{E}$, *thus the area in the plane perpendicular to* $\mathbf{E}$ *included in the density of field lines is the number per unit area.)*

This suggests that the flux through any closed surface is a measure of the total charge inside. Because field lines that originate on a positive charge must either pass out through the surface or else terminate on a negative charge inside [Fig. 1.85(a)], a charge outside the surface will contribute nothing to the total flux be cause its field lines pass in one side and exit the other [Fig. 1.85(b)].

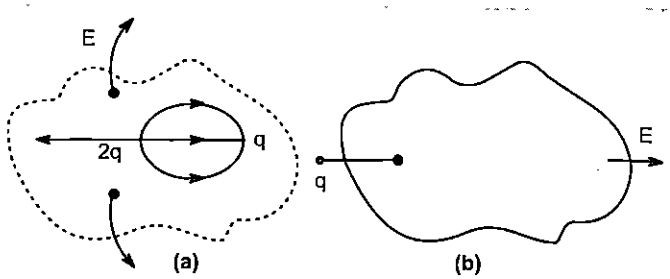


Fig 1.85

8. *From Fig. 1.86 we see that the number of lines that pass through two arbitrary closed surfaces enclosing a point charge are the same. Since the concept of flux, rather than lines, is used in Gauss's law, one must show that the flux through two arbitrary surfaces is the same. Consider a cone of field lines that emerge from a point charge Q, as shown in Fig. 1.86. The size of the cone is specified by the solid angle* Ω *which is defined by*

$$\Omega = \frac{A_n}{r^2} = \frac{A\cos\theta}{r^2}$$

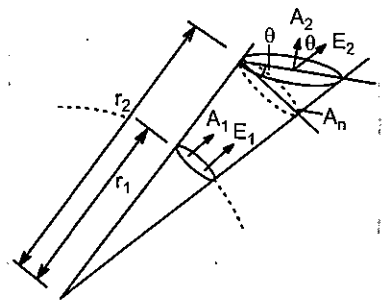


Fig. 1.86

where $A_n = A\cos\theta$ *is the projection of A perpendicular to the axis of the cone. The unit of solid angle is the steradian. A closed surface subtends a solid angle of* 4π *at any internal point (consider the special case of a sphere whose surface area is* $4\pi r^2$*). In Fig. 1.86 the cone intercepts an area* A_1 *of a spherical surface of radius* r_1 *and an area* A_2 *of an arbitrary surface. The areas can be of any shape. From Coulomb's law* $(E \propto 1/r^2)$ *the ratio of the field strengths at the two areas is*

$$\frac{E_2}{E_1} = \frac{r_1^2}{r_2^2} \qquad ...(1)$$

The solid angle of the cone may be expressed in terms of either area :

$$\Omega = \frac{A_1}{r_1^2} = \frac{A_2\cos\theta}{r_2^2} \qquad ...(2)$$

The flux through each surface is $\phi_1 = E_1A_1$ *and* $\phi_2 = E_2A_2\cos\theta$. *Using (1) and (2) we see that* $\phi_1 = \phi_2$. *That is, the flux within a given solid angle is constant, independent of the shape or orientation of the surface.*

Problem Solving Tactics for Gauss's Law:

1. Try to find out the symmetry of the electric field produced by the charge distribution :

Charge distributions	Gaussian surface	Electric Field
Point charge	Spherical	Radial
Spherical charge	Spherical	Radial
distribution	Cylindrical	Radial
Line of charge	Cylindrical	Normal to Sur-
Planar charge		face

2. The Gaussian surface should surrounded all or a portion of the charge in such a manner that the electric field is either constant over all parts of the surface or is perpendicular to the surface area vector.

3. Evaluate the flux through the Gaussian surface.

4. Compute the net charge enclosed within the surface.

5. Apply Gauss's law.

$$\oint E \cdot ds = \frac{Q_{\text{net enclosed}}}{\varepsilon_0}$$

6. ***The relevant concepts:*** Gauss's law is most useful in situations where the charge distribution has spherical or cylindrical symmetry or is distributed uniformly over a plane. In these situations we determine the direction of $\vec{\mathbf{E}}$ from the symmetry of the charge distribution. If we are given the charge distribution, we can use Gauss's law to find the magnitude of $\vec{\mathbf{E}}$. Alternatively, if we are given the field, we can use Gauss's law to determine the details of the charge distribution.

7. The Gaussian surface does not have to be a real physical surface, such as a surface of a solid body. Often the appropriate surface is an imaginary geometric surface: it may be in empty space, embedded in a solid body, or both.

8. If $\vec{\mathbf{E}}$ is *perpendicular* (normal) at every point to a surface with area A, if it points *outward* from the interior of the surface, and if it also has the same *magnitude* at every point on the surface, then $E_\perp = E = \text{constant}$, and $\int E_\perp \, dA$ over that surface is equal to EA. If instead $\vec{\mathbf{E}}$ is perendicular and *inward*, then $E_\perp = -E$ and $\int E_\perp \, dA = -EA$.

9. If $\vec{\mathbf{E}}$ is *tangent* to a surface at every point, then $E_\perp = 0$ and the integral over that surface is zero.

10. If $\vec{\mathbf{E}} = \mathbf{0}$ at every point on a surface, the integral is zero.

11. In the integral $\oint E_\perp \, dA$, $E_\perp$ is always the perpendicular component of the *total* electric field at each point on the closed Gaussian surface. In general, this field may be caused partly by charges within the surface and partly by charges outside it. Even when there is *no* charge within the surface, the field at points on the Gaussian surface is not necessarily zero. In that case, however, the *integral* over the Gaussian surface—that is, the total electric flux through the Gaussian surface—is always zero.

12. For a continuous distribution, the **charge density** is usually the most convenient way to describe how much charge is present. There are three kinds of charge densities:

- If the charge is spread throughout a volume, the relevant charge density is the charge per unit *volume* (symbol ρ).
- If the charge is spread over a two-dimensional surface, then the charge density is the charge per unit *area* (symbol σ).
- If the charge is spread over a one-dimensional line or curve, the appropriate charge density is the charge per unit *length* (symbol λ).

COULOMB'S LAW FROM GAUSS'S LAW

We choose an imaginary sphere (Gaussian surface) of radius r centred on the charge q. Due to symmetry, E must have the same magnitude at any point on the surface, and E points radially outward (or inward for a negative charge) parallel to dA (or antiparallel to dA for negative charge). Hence we write the integral in Gauss's law as

$$\oint \mathbf{E} \cdot dA = \oint E dA = E \oint dA = E(4\pi r^2)$$

$$Q_{\text{enclosed}} = q$$

Thus $$E(4\pi r^2) = \frac{q}{\varepsilon_0}$$

or $$E = \frac{q}{4\pi\varepsilon_0 r^2}$$

From the definition of the electric field, the force on a point charge q_0 located at a distance r from the charge q is $F = q_0 E$. Therefore,

$$F = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2}$$

Which is **Coulomb's law.**

Now let us do the reverse and derive Gauss's law from Coulomb's law. Consider a point charge q surrounded by a spherical surface. From Coulomb's law, electric field of a point charge at a distance r from the charge is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

Thus we have

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \oint \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} dA$$

$$= \frac{q}{4\pi\varepsilon_0 r^2}(4\pi r^2)$$

$$= \frac{q}{\varepsilon_0}$$

which is Gauss's law with $Q_{\text{enclosed}} = q$

Note that Gauss's law will take a more complicated form in terms of the constant $k = \frac{1}{4\pi\varepsilon_0}$ that we used in Coulomb's law.

Coulomb's law	**Gauss's law**
$E = \frac{kq}{r^2}$	$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = 4\pi kQ$
$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$	$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q}{\varepsilon_0}$

Since Coulomb's law can be derived from Gauss's law, the Gauss's law is considered more general, and the normal convention is to use ε_0 rather than k.

Concept : *What happens if point charge is in an arbitrary surface?*

By superposition, it suffices to consider the case of a single point charge, q, at an arbitrary location inside a closed surface S. At a point P of S (see Fig. 1.87), the field due to q is given by Coulomb's law as

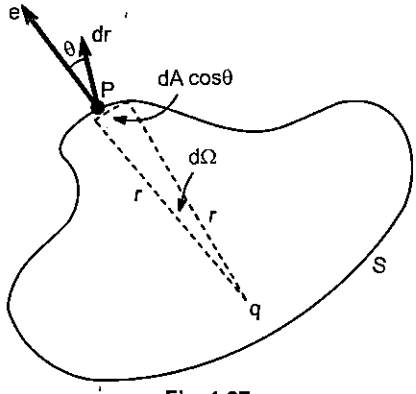


Fig. 1.87

$$\vec{E}=\frac{q}{4\pi\varepsilon_0 r^2}\mathbf{e}$$

The flux through S is then

$$d\phi=\vec{E}\cdot d\vec{A}=\frac{q}{4\pi\varepsilon_0 r^2}\mathbf{e}.d\vec{A}$$

$$=\frac{q}{4\pi\varepsilon_0}\frac{d\vec{A}\cos\theta}{r^2}=\frac{q}{4\pi\varepsilon_0}d\Omega$$

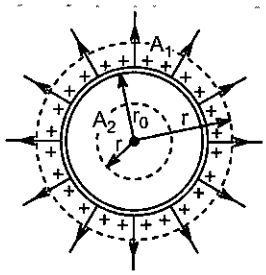
Where $d\Omega$ is the infinitesimal solid angle subtended by dS at the location of q. At that location, the total solid angle subtended by the entire closed surface S is 4π steradians. Thus,

$$\phi=\int_s d\phi=\frac{q}{4\pi\varepsilon_0}\int_s d\Omega=\frac{q}{4\pi\varepsilon_0}(4\pi)=\frac{q}{\varepsilon_0}$$

Which is Gauss' law.

Example 46

A thin spherical shell of radius R possesses a total net charge Q that is uniformly distributed on it (see Fig. 1E.46). Determine the electric field at points (a) outside the shell, and (b) inside the shell. (c) What is the result if the conductor were a solid sphere ?



Cross-sectional drawing of a thin spherical shell of radius R carrying a net charge Q uniformly distributed. A_1 and A_2 represent two Gaussian surface we use to determine $\vec{E}$

Fig. 1E.46

Solution:

Concept: *The field outside a uniformly charged solid sphere of radius R and total charge q.*

$\vec{E}$ certainly points radially outward, as does $d\vec{A}$, so we can drop the dot product, in $\vec{E}.d\phi_E = EdA$

Suppose, say, that it points due east, at the "equator". But the orientation of the equator is perfectly arbitrary, so there is no natural "north-south" axis—any argument that show that $\vec{E}$ points east could just as well be used to show it points west, or north, or any other direction. The only unique direction on a sphere is radial.

(a) Due to symmetry of charge distribution the field must be symmetric. The field must be directed radially outward (inward for a negative charge). First we want to find E outside the spherical shell, so we choose our Gaussian surface (A_1) to be a sphere of radius r ($r > R$) concentric with the shell as shown in Fig. 1E.46.

$$\phi_E=\oint\vec{E}\cdot d\vec{A}=E\int dA=E(4\pi r^2)$$

The electric field has the same magnitude at all points on the surface, and E is perpendicular to the Gaussian surface; therefore $\vec{E}\cdot\vec{dA}=EdA\cos 0°=EdA$,

From Gauss's law,

$$E(4\pi r^2)=Q_{\text{enclosed}}=Q$$

$$E=\frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} \qquad [r>R]$$

* The field outside a uniform shell of charge is same as if all the charge were concentrated at the centre of the point charge.

(b) Inside the shell the field must be symmetric. We consider a Gaussian sphere (A_2) concentric with the shell.

In this case $Q_{\text{enclosed}} = 0$, we have

$$\oint\vec{E}\cdot\vec{dA}=E(4\pi r^2)=0$$

Hence $E = 0$ $(r < R)$

Inside a spherical shell electric field is zero.

Let us summarise our results :

$$E(r>R)=\frac{q}{4\pi\varepsilon_0 r^2}$$

$$E(r=R)=\frac{q}{4\pi\varepsilon_0 R^2}$$

$$E(r<R)=0$$

(c) A conductor has all the charge distributed on its surface, in a very thin layer. Therefore a Gaussian surface inside the sphere will not enclose any charge. But in case of insulator, the charge can be distributed inside (volumetric charge distribution). Above results apply to a uniformly charged **solid spherical conductor.**

Example 47

A charge Q is distributed uniformly throughout a spherical volume of radius R. Find the electric field at a point (a) outside the sphere (b) inside the sphere.

Solution:

Concept: *Consider a sphere of radius* **R**, *which is uniformly charged throughout its volume. We now state by symmetry arguments, that at a point,* **P**, *outside the sphere, at a distance r from the center of the sphere (a) the field must point in the radial direction away from the center of the sphere 1E.47 and (b) the magnitude of the field depends only on r and not on any other coordinate of the point P.*

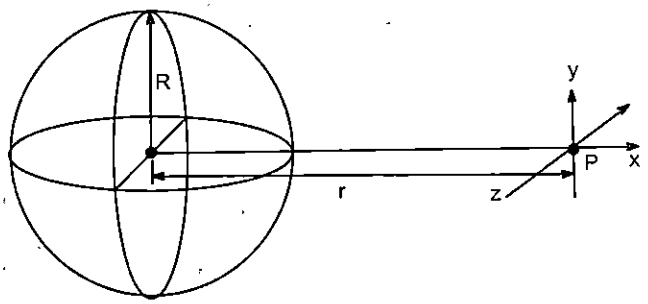


Fig. 1E.47 (a)

(a) *At point P we draw a coordinate system x, y, z as shown. At this point the ± y directions are indistinguishable from each other since the sphere appears identical from both directions. Thus if there is a field in one of those directions there should also be a field in the other, and therefore there cannot be a field in either direction. The same symmetry argument is applicable to the ± z directions. The only possible direction for the field is therefore the x direction. Since the field must point away from positive charge, the field must be radially away from the center of the sphere.*

(b) *All points at the same distance r from the center of the sphere are equivalent since the sphere appears identical to each point. Thus they must all have the same magnitude for the electric field, and the field cannot vary with any coordinate other than r.*

By observing symmetry of the electric field produced by the spherical charge distribution we choose a spherical Gaussian of radius r. E must have the same magnitude at all points on the surface of the sphere. Note that E is parallel to area vector $\vec{\mathbf{dA}}$ at all points.

$$\vec{\mathbf{E}} \cdot \vec{\mathbf{dA}} = E dA$$

$$\phi_E = \oint \vec{\mathbf{E}} \cdot \vec{\mathbf{dA}}$$

$$= E \oint dA = E \cdot 4\pi r^2$$

Gauss's law is

$$\int \vec{\mathbf{E}} \cdot \vec{\mathbf{dA}} = \frac{Q_{\text{net enclosed}}}{\varepsilon_0}$$

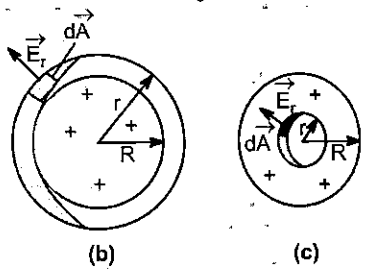


Fig. 1E.47

For $r \geq R$, the charge enclosed in the Gaussian surface is the total charge Q.

$$E \cdot 4\pi r^2 = \frac{Q}{\varepsilon_0}$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} \quad (r \geq R)$$

For $r < R$, the amount of charge enclosed within the Gaussian surface is just that fraction of charge that lies from centre to radius r. Let ρ be the volume charge density of the charge distribution.

$$\rho = \frac{Q}{(4/3)\pi r^3}$$

Then the charge within the Gaussian sphere of radius r is

$$(4/3)\pi r^3 \rho = \frac{Q r^3}{R^3}.$$

Hence Gauss's law becomes

$$E(4\pi r^2) = \frac{\frac{r^3}{R^3}Q}{\varepsilon_0}$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^3}r \qquad (r \leq R)$$

* Note that electric field varies linearly with r until $r = R$

* Note that at the surface $r = R$, boundary between charged sphere and empty region outside, the two solutions $E(r \geq R)$ and $E(r \leq R)$ agree, because the electric field lines start and end only on charges. Imagine a shrinking Gaussian sphere starting from $r > R$, as the sphere shrinks past $r = R$, initially few charges are excluded and the electric field decreases gradually. It is a general rule that the *electric field must be continuous in the absence of a continuous charge distribution.* In the case of a surface charge distribution, charges are spread out in a nearly zero thickness layer, therefore E jumps abruptly.

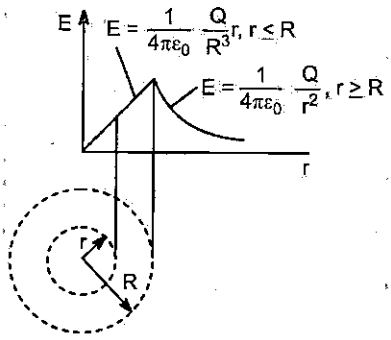


Fig. 1E.47 (d)

Example 48

Find the electric field at distance y from an infinite line of charge if the linear charge density is λ.

Solution: For a linear distribution the electric lines of force are radial, *i.e.*, perpendicular to the line of charge. In this case we choose a cylindrical Gaussian surface. The strength of the field at the surface of the cylinder is same at all points, as all the points are at a constant distance y from the line of charge.

$$\phi_E = \oint E \cdot dA$$

$$= \int_{\text{left cap}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} + \int_{\text{right cap}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} + \int_{\text{curved surface}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$$

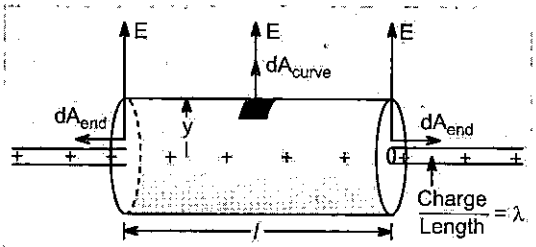


Fig. 1E.48

On both the end caps the radial $\vec{\mathbf{E}}$ is perpendicular to the axial area vector $d\vec{\mathbf{A}}$. Hence in the first two integrals $\vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = 0$. On the curved surface $\vec{\mathbf{E}}$ is always parallel to the area vector, so that $\vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = EdA$

$$\phi_E = 0 + 0 + E\int dA = E \cdot 2\pi rl$$

Net charge enclosed by Gaussian surface

$$Q_{\text{net enclosed}} = \lambda l$$

Hence Gauss's law becomes

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_{\text{net enclosed}}}{\varepsilon_0}$$

$$E(2\pi rl) = \varepsilon_0(\lambda l)$$

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

Concept: *Consider a long wire carrying a uniform charge per unit length of λ.*

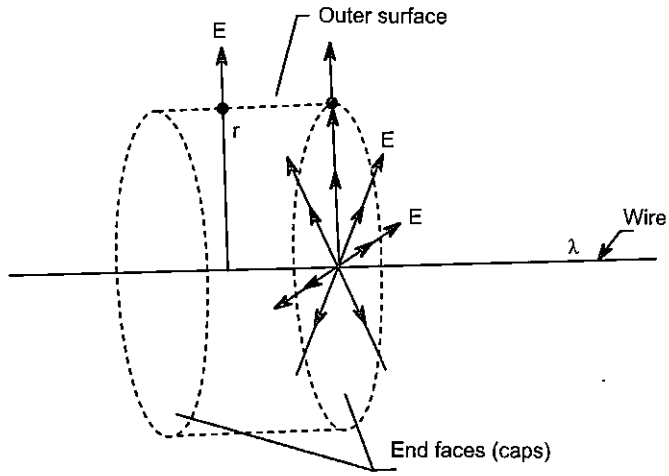


Fig. 1.88

Because of the symmetry we know that the field will be the same at all points at the same perpendicular distance r from the axis of the wire. We also know from symmetry that the field cannot have a component along the direction parallel to the wire since there is no difference between the direction to the right or the left. Similarly, the field cannot have a component that circulates around the wire since there is no difference between the directions of circulation. Therefore, for any point, the direction of the field must be along the line radiating out perpendicularly from the axis of the wire to that point. This direction is shown for several points in (Fig. 1.88). We choose as our Gaussian surface a cylinder of radius r and length L, with axis along the wire as in the Fig. 1.88.

The electric field at any point is the sum of the electric field contributions from the charge all along the wire. Coulomb's law tells us that the strongest contributions come from the charge on nearby parts of the wire, with contributions falling of as $1/r^2$ for faraway points. When concerned only with points near the wire, and far from either end, an approximately correct answer is obtained by assuming the wire is infinitely long.

The field lines either start or stop on the wire (depending on whether the charge is positive or negative). Then what do the field lines do? The only possibility is that they move radially outward (or inward) from the wire. Fig. 1.89 (a) shows sketches of the field lines for positive and negative charges, respectively. The wire looks the same from all sides, so a field line could not start to curl around as in Fig. 1.89 (b) : how would it determine which way to go? Also, the field lines cannot go along the wire as in Fig. 1.89 (c) : again, how could the lines decide whether to go right or left? The wire looks exactly the same in both directions

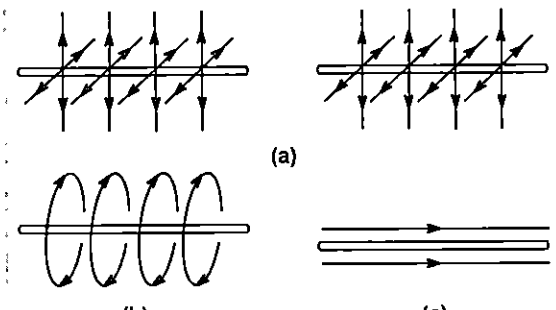


Fig. 1.89

What if the line segment in tis example were not infinitely long?

If the line charge in this example were of finite length, A finite line charge does not possess sufficient symmetry to make use of Gauss's law because the magnitude of the electric field is no longer constant over the surface of the gaussian cylinder : the field near the ends of the line would be different from that far from the ends. $\vec{E}$ *is not perpendicular to the cylindrical surface at all points: the field vectors near the ends would have a component parallel to the line.*

Example 49

Find the electric field strength of an infinite nonconducting plane carrying a uniform charge density σ.

Solution:

Concept: *Consider a large planar plate carrying a uniform charge per unit area of* σ, *as in Fig. 1E.49 (a)*

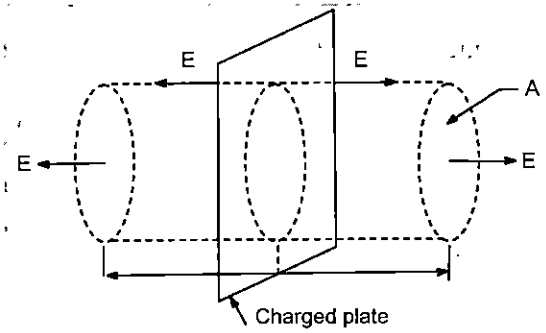


Fig. 1E.49 (a)

Because of the symmetry we know that the field will be the same at all points at the same distance x from the plate as long as we are far from the edge of the plate. We also know from symmetry that the field cannot be along the direction parallel to the plate since there is no difference between any parallel direction. Therefore, the field must be along the direction perpendicular to the plane. We choose as our Gaussian surface the "pillbox" cylinder of base area A and length 2x perpendicular to the plate, as in the Fig. 1E.49(a). The closed surface has two flat caps of area A, one on each side of the plate, and a cylindrical surface.

On the cylindrical surface the field is parallel to the surface and therefore $\vec{E} \perp \vec{A}$ *(cos* $\theta = 0$*), and the flux through this surface is zero. On the caps the field is perpendicular to the surface at every point so that* $\vec{E}$ *is parallel to* $\vec{A}$ *(*$\cos\theta = 1$*). The field is directed away from the positive charge on the plate and is therefore the left on the left plate and to the right on the right plate. In each case the field will be in the same direction as* $\vec{A}$*, since we always choose* $\vec{A}$ *to point from the inside to the outside on the Gaussian surface. The flux through each cap will therefore equal EA, and the total flux through the closed surface will equal 2EA. Field of an infinite plane is given by*

$$\vec{E} = \frac{\sigma}{2\varepsilon_0}\hat{n}$$

Where $\hat{n}$ *is a unit vector pointing away from the surface.*

The field of an infinite plane is independent of how far away you are. But it should vary as $1/r^2$ *in accordance with Coulomb's law. We can resolve this as you move farther and, farther away from the plane, more and more charge comes into your "field of view" (a cone shape extending out from your eye), and this compensates for the diminishing influence of any particular charge element on sheet. The electric field of a sphere like* $1/r^2$*; the electric field of an infinite line like* $1/r$*; and the electric field of an infinite plane does not decrease off at all.*

We choose a small, closed cylinder whose axis is perpendicular to the plane, as our Gaussian surface as shown in Fig. 1E.49 (b). From symmetry we expect E to be directed perpendicular to the plane on both sides as shown, and to be uniform over the end caps of the cylinder.

$$\phi_E = \oint \vec{E}\cdot d\vec{A}$$

$$= \int_{\text{left cap}} \vec{E}\cdot d\vec{A} + \int_{\text{right cap}} \vec{E}\cdot d\vec{A} + \int_{\text{curve}} \vec{E}\cdot d\vec{A}$$

$\vec{E}$ is perpendicular to $d\vec{A}$ at every point on the curved surface; therefore $\vec{E}\cdot d\vec{A} = 0$. On both end caps $\vec{E}$ and $d\vec{A}$ are parallel.

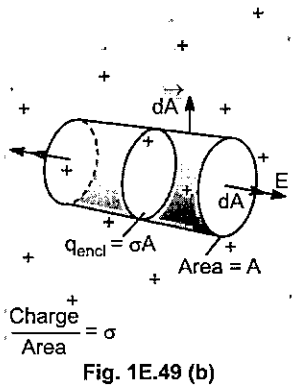


Fig. 1E.49 (b)

$$\phi_E = EA + EA + 0 = 2EA$$

The charged enclosed by the Gaussian surface

$$Q_{\text{net enclosed}} = \sigma A$$

From Gauss's law

$$\oint \vec{\mathbf{E}} \cdot \vec{d\mathbf{A}} = \frac{Q_{\text{net enclosed}}}{\varepsilon_0}$$

$$2EA = \frac{\sigma A}{\varepsilon_0}$$

$$E = \frac{\sigma}{2\varepsilon_0}$$

Note that field is uniform, independent of distance from plane.

The direct use of Gauss's law to compute electric fields is limited to cases of spherical, cylindrical, and planar symmetry, we can find field for combinations of objects possessing such symmetry, invoking the principle of superposition, we could find the field in the vicinity of two uniformly charged parallel cylinders, or a sphere near an infinite charged plane.

Two infinite parallel planes carry equal but opposite uniform charge densities $\pm\sigma$ [Fig. 1E.49 (c)] now we calculate the field in each of the three regions: (i) to the left of both, (ii) between them, (iii) to the right of both.

The left plate being positively produces a field $(1/2\varepsilon_0)\sigma$ which points away from it [Fig. 1E.49 (d)] to the lift in region (i) and to the right in regions (ii) and (iii). The right plate, being negatively charged, porduces a field $(1/2\varepsilon_0)\sigma$, which points toward it to the right in regions (i) and (ii) and to the left in region (iii). The two fields cancel in regions (i) and (iii); they add in region (ii). The field is $(1/\varepsilon_0)\sigma$, and points to the right, between the planes; elsewhere it is zero.

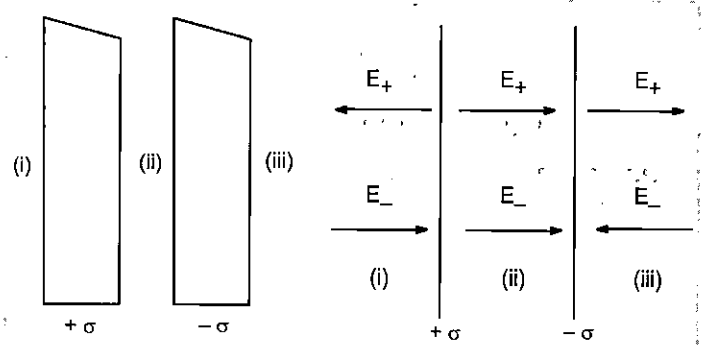


Fig. 1E.49

Gauss's Law is Consequence of the $\frac{1}{r^2}$ Nature of Coulomb's Law

We consider a point charge, q_1 isolated in space, as shown in Fig. 1.90. Field lines have been drawn to show the spherical symmetry of the electric field. We assume a gaussian spherial surface of radius r centered at the charge q. Since the field strength E is equal in magnitude everywhere on the surface and is radially ouward in direction, the total flux, ϕ, is just EA, where A is the area of the spherical surface. Then from Gauss' law, we get $E = [1/(4\pi\varepsilon_0)](q/r^2)$, which together with the radial direction of E is Coulomb's law.

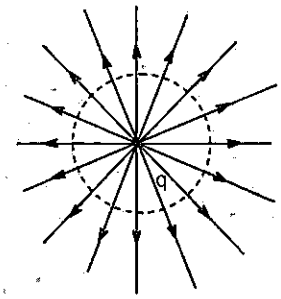


Fig. 1.90

Note that the flux, f is not dependent on the radius of the spherical surface, r. This is of course a consequence of the $1/r^2$ nature of Coulomb's law and the fact that A increases as r^2. Furthermore, for any shape surface enclosing q, ϕ_E would still be the same since the total number of lines of force passing through every surface enclosing the charge is the same.

Let us consider that electric field due to a point charge varies as $1/r^3$, *i.e.*, $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^3}$ due to a point charge.

Now to calculate the flux through a sphere of radius 'r' with a point charge 'q' at its centre.

$$\phi = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{r^3}\oint dS$$

where $\oint dS$ = surface area of sphere

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{r^3} \times 4\pi r^2$$

or $$\phi = \frac{q}{\varepsilon_0 r}$$

This means that the flux will depend on the radius of the sphere which contradicts the Gauss's law, therefore for Gauss's law to hold Coulomb's law must be an inverse square law.

Flux, Linked with the Curved Surface of a Hemispherical Gaussion Surface Placed in a Uniform Electric Field

The flux linked with the hemispherical surface will be zero as it does not enclose any charge *i.e.*,

$$\phi = \phi_{\text{curved}} + \phi_{\text{plane}} = 0 \quad \ldots (1)$$

Where ϕ_{curved} and ϕ_{plane} are flux linked to the curved and the plane surfaces respectively as the electric field is perpendicular to the base and uniform, hence,

$$\phi_{\text{plane}} = E \times \pi R^2 \cos 180°$$

$$\phi_{\text{plane}} = -\pi R^2 E$$

Hence from equation (1) we get.

Flux linked with the curved surface

$$\phi_{\text{plane}} = -\phi_{\text{curved}} = \pi R^2 E$$

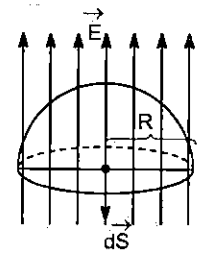


Fig. 1.91

What happens if a point charge q is placed at the centre of the hemisphere, then with hemisphere, the flux linked will be $q/2\varepsilon_0$.

Electric Flux through a Circle of Radius *R* due to an Infinitely Long Wise Placed along its Axis

Consider a ring element of radius x and thickness dx. Area of the ring,

$$dA = 2\pi x\, dx$$

Component of electric field perpendicular to the ring contributes to net flux of shown in Fig. 1.92.

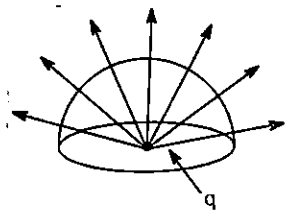


Fig. 1.92

Hence total flux linked with the ring element.

$$d\phi = 2\pi x\, dx \frac{\lambda}{4\pi\varepsilon_0 x}$$

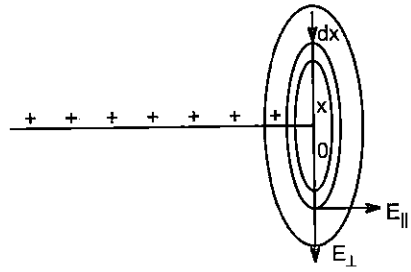


Fig. 1.93

Electric field due to semi infinite wire $E_{\parallel} = \dfrac{\lambda}{4\pi\varepsilon_0 x}$

Net flux through the complete ring area,

$$\phi_{net} = \oint d\phi = \frac{\lambda}{2\varepsilon_0}\int_0^R dx$$

$$= \frac{\lambda R}{2\varepsilon_0}$$

Example 50

A point charge q is placed on the apex of a cone of semi-vertex angle θ. Show that the electric flux through the base of the cone is $\dfrac{q(1-\cos\theta)}{2\varepsilon_0}$.

Solution: Consider a Gaussian sphere with its centre at the apex and radius the slant length of the cone. The flux through the whole sphere is q/ε_0. Therefore, the flux through the base of the cone,

$$\phi_E = \left(\frac{A}{A_0}\right)\cdot\frac{q}{\varepsilon_0}$$

Here, A_0 = area of whole sphere = $4\pi R^2$

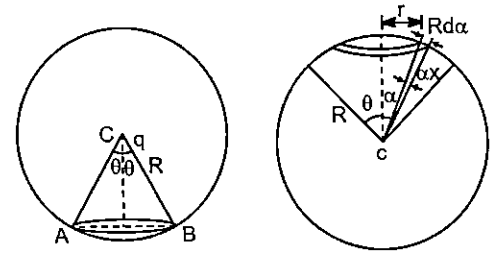
and A = area of sphere below the base of the cone.

Consider a differential ring of radius r and thickness dr.

$$dA = (2\pi r)R\, d\alpha$$

$$= (2\pi R\sin\alpha)R\, d\alpha \quad \text{as } r = R\sin\alpha$$

$$= (2\pi R^2)\sin\alpha\, d\alpha$$



(a) (b)

Fig. 1E.50

$$\therefore \quad A = \int_0^{\theta}(2\pi R^2)\sin\alpha d\alpha$$

$$\therefore \quad A = 2\pi R^2(1-\cos\theta)$$

The desired flux is

$$\phi_E = \left(\frac{A}{A_0}\right)\cdot\frac{q}{\varepsilon_0}$$

$$= \frac{(2\pi R^2)(1-\cos\theta)}{(4\pi R^2)}\cdot\frac{q}{\varepsilon_0}$$

$$= \frac{q(1-\cos\theta)}{2\varepsilon_0}$$

Flux Through the Plane Surface of a Cylinder due to a Charge *q* Placed at its Geometrical Centre

Divide the plane cap of cylinder (as shown in Fig. 1.94) into differential rings.

Electric field at any point P of an differential ring between 'r' and $(r + dr)$ is:

$$\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r^2 + l^2/4)} \text{ along } \overrightarrow{\mathbf{OP}}$$

Flux through the differential,

$$d\phi = E\cos\theta \times 2\pi r dr$$

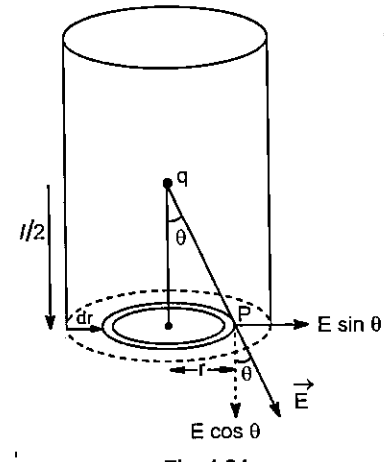


Fig. 1.94

Putting the value of E, we get,

$$d\phi = \frac{q\,2\pi r dr\cos\theta}{4\pi\varepsilon_0(r^2+l^2/4)}$$

$$= \frac{qlrdr}{4\varepsilon_0(r^2+l^2/4)^{3/2}}$$

Total flux,

$$\phi = \int_0^a \frac{qlrdr}{4\varepsilon_0(r^2+l^2/4)^{3/2}}$$

$$= \frac{q}{2\varepsilon_0}\left\{1-\frac{1/2}{(a^2+l^2/4)^{1/2}}\right\}$$

Alternate method: We can solve this problem by concept of solid angle.

Electric flux through the base is the flux linked with solid angle,

$$\omega = 2\pi(1-\cos\alpha)$$

$$\phi = \frac{q}{4\pi\varepsilon_0}\omega$$

$$= \frac{q}{4\pi\varepsilon_0}2\pi(1-\cos\alpha),$$

$$\phi = \frac{q}{2\varepsilon_0}\left(1-\frac{l/2}{\sqrt{a^2+l^2/4}}\right)$$

(where a is the radius of circular base)

Concentric Charged Spheres

Consider three concentric charged spherical shells as shown in Fig. 1.95. Shell have charge q_1, q_2, q_3 as shown in Fig. 1.95. We will apply Gauss's law on the surfaces s_1, s_2, s_3.

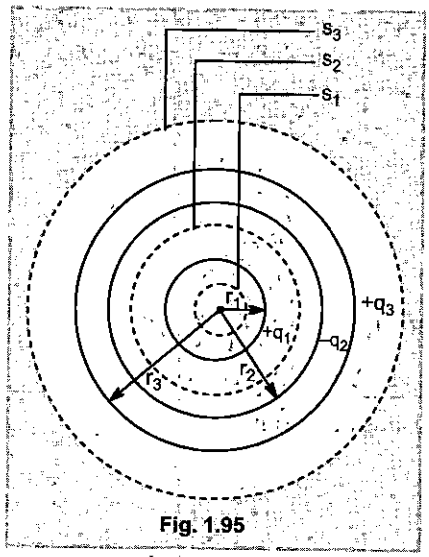


Fig. 1.95

At $r < r_1$, $E = 0$

Because Gaussian surface S_1 will enclose no charge.

At $r = r_1$ $\quad E = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1^2}$

At $r_1 < r < r_2$, $\quad E = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r^2}$

Since Gaussian surface S_2 encloses a total charge of q_1

At $r_2 \le r < r_3$, $\quad E = \frac{1}{4\pi\varepsilon_0}\frac{q_1-q_2}{r^2}$

Since Gaussian surface S_3 encloses a total charge of q_1+q_2.

At $r_3 \le r < \infty$ $\quad E = \frac{1}{4\pi\varepsilon_0}\frac{(q_1-q_2+q_3)}{r^2}$

Electric Field Due to a Uniform Charge Distribution on the Surface of Very Long Cylindrical Shell

We draw a Gaussian surface in the form of a cylinder of radius r and length L as in Fig. 1.96 (a). As in the field must always be in the radial direction perpendicular to the axis of the cylinder. Therefore the field is parallel to the surface on the two caps of the cylinder and those two surfaces do not contribute to the flux entering or leaving the surface.

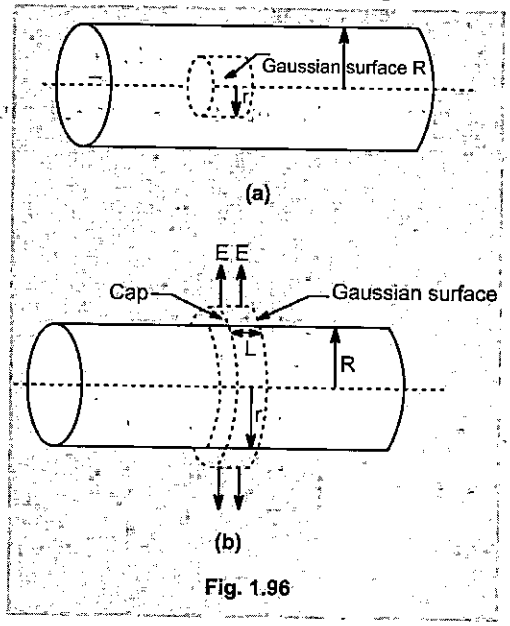


Fig. 1.96

On the outer surface of the cylinder, the field is perpendicular to the surface (A is parallel to E), and the flux equals $EA = E(2\pi rL)$. By Gauss' Law, this equals Q_{in}/ε_0.

Electric field inside the cylinder ($r < R$); $E = 0$ because enclosed charge is zero.

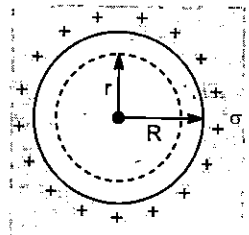


Fig. 1.97

Concept: $\oint \vec{E}\cdot d\vec{A} = 0$ *does not imply that field is zero. If electric field is proved to be cylindrically symmetric or spherically symmetric then* $\vec{E}$ *must be zero.* $\oint \vec{E}\cdot d\vec{A} = 0$ *also implies that* $\vec{E} \perp \vec{A}$ *or summation of* $\vec{E}\cdot d\vec{A}$ *over entire surface is zero not* $\vec{E}$ *to be zero individually.*

Electric field outside the cylinder;

Using Gauss theorem,

$$\oint \vec{E}.d\vec{S} = \frac{1}{\varepsilon_0} q_{enclosed}$$

or $$E\oint dS = \frac{1}{\varepsilon_0} q_{enclosed}$$

$$E.2\pi rl = \frac{1}{\varepsilon_0} q_{enclosed}$$

or $$E = \frac{q_{enclosed}/l}{2\pi\varepsilon_0 r}$$

$$E = \frac{\lambda_{enclosed}}{2\pi\varepsilon_0 r} = \frac{\sigma R}{\varepsilon_0 r}$$

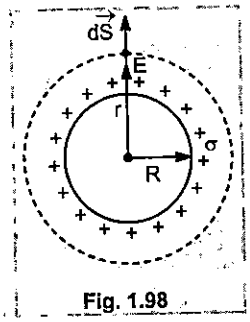


Fig. 1.98

where $\lambda_{enclosed} = \sigma 2\pi R$

If the total charge is assumed to be distributed linearly cylinder on a line, then linear charge density of the cylindrical shell is given by,

$$\lambda = \frac{2\pi Rl\sigma}{l} = 2\pi R\sigma$$

Hence electric field outside the surface can also be written as,

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

for $r \geq R$

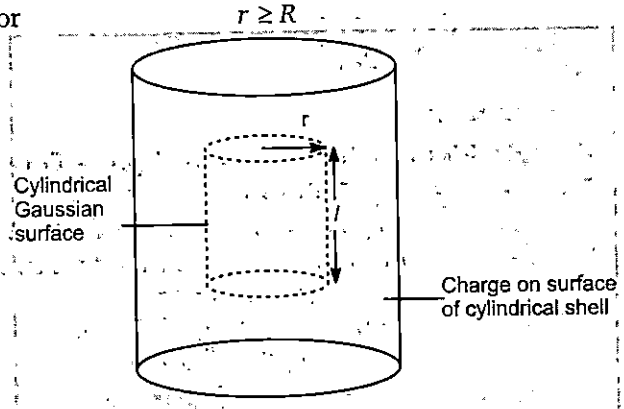


Fig. 1.99

Infinitely Long Coaxial Cylindrical Shells

Fig. 1.100 shows cylindrical shells having charge density $+\lambda_1$ and $-\lambda_2$. Using results of previous article we get

At a point $r > r_2$,

$$E = \frac{(\lambda_1 - \lambda_2)}{2\pi\varepsilon_0 r}$$

At a point $r_1 < r < r_2$,

$$E = \frac{\lambda_1}{2\pi\varepsilon_0 r}$$

At a point $r < r_1$,

$$E = 0$$

uniform volumetric

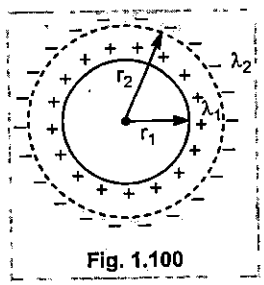


Fig. 1.100

Electric Field Due to the Cylindrical Charge Distribution

Consider a cylinder where charge is distributed over the volume with density ρ

Electric field inside the surface $r < R$

Let us consider a cylindrical Gaussian surface of length l and radius 'r' as shown in Fig. 1.101. Using Gauss's law,

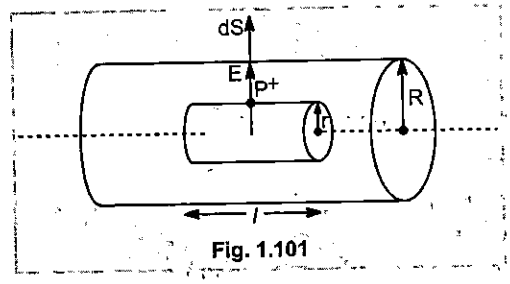


Fig. 1.101

$$\oint \vec{E}.d\vec{S} = \frac{q_{enclosed}}{\varepsilon_0}$$

$$\Rightarrow \quad \oint E\,dS\cos 0 = \frac{q_{enclosed}}{\varepsilon_0}$$

For plane faces the flux $\int \vec{E}.d\vec{S} = 0$ because $\vec{E}$ and $d\vec{A}$ are perpendicular.

$$E\oint dS = \frac{q_{enclosed}}{\varepsilon_0} \quad \Rightarrow E2\pi rl = \frac{q_{enclosed}}{\varepsilon_0}$$

$$E = \frac{q_{enclosed}/l}{2\pi\varepsilon_0 r} = \frac{\lambda_{enclosed}}{2\pi\varepsilon_0 r};$$

$$(q_{enclosed}/l = \lambda_{enclosed})$$

This behaves like line charge of linear charge density $\lambda_{enclosed}$ which is the charge enclosed by unit length of gaussian surface,

$$q_{enclosed} = \pi r^2 l\rho;$$

$$\lambda_{enclosed} = \frac{q_{enclosed}}{l} = \pi r^2\rho$$

Thus, $$E = \frac{\rho r}{2\varepsilon_0}$$

In vector form, $$\vec{E} = \left(\frac{\rho}{2\varepsilon_0}\right)\vec{r}$$

Electric field outside the surface ($r > R$)

Using Gauss theorem;

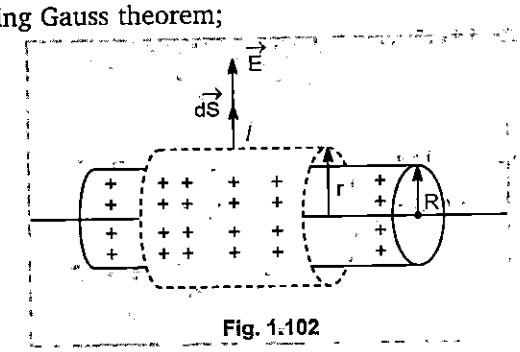


Fig. 1.102

$$\oint \vec{E}.d\vec{S} = \frac{q}{\varepsilon_0}$$

$$E\,2\pi r l = \frac{q_{enclosed}}{\varepsilon_0}$$

$$\Rightarrow \quad E = \frac{q_{enclosed}/l}{2\pi\varepsilon_0 r} = \frac{\lambda_{enclosed}}{2\pi\varepsilon_0 r}$$

$$E = \frac{\rho R^2}{2r\varepsilon_0}$$

$$(q_{enclosed} = \pi r^2 l\rho;\ \lambda_{enclosed} = \frac{q_{enclosed}}{l} = \pi r^2\rho)$$

Example 51

A sphere of radius R has a uniform volume density ρ. A spherical cavity of radius b whose centre lies at $\vec{r} = \vec{a}$ is removed from the sphere.
Find the electric field at any point inside the spherical cavity.

Solution: The field within the cavity or outside is the superposition of the field due to the original uncut sphere, plus the field due to a sphere of the size of the cavity but with a uniform negative charge density. The effective charge distribution is composed of an uniformly charged sphere of radius r, charge density ρ, superposed on it, a charge density −ρ filling the cavity. Electric field $\vec{E}_1$ caused by the charge distribution +ρ at a point $\vec{r}$, inside the spherical cavity is found from Gauss's law.

$$\phi_E = -\oint \vec{E}\cdot d\vec{A}$$

$$= 4\pi r^2 E_1 = \frac{\left(\frac{4}{3}\pi r^3\rho\right)}{\varepsilon_0}$$

$$\vec{E}_1 = \frac{\rho r}{3\varepsilon_0}\hat{r} = \frac{\rho\,\vec{r}}{3\varepsilon_0}$$

where $\hat{r}$ is a unit vector in radial direction.

Similarly, the electric field $\vec{E}_2$ formed by the charge density −ρ inside the cavity is

$$\vec{E}_2 = \frac{(-\rho)\vec{s}}{3\varepsilon_0} \quad \Rightarrow \quad \vec{s} = \vec{r} - \vec{a}$$

The $\vec{s}$ is radius vector from cavity centre to the point P.

$$\vec{E}_2 = \frac{-\rho(\vec{r}-\vec{a})}{3\varepsilon_0}$$

The resultant electric field inside the cavity is therefore given by the superposition of $\vec{E}_1$ and $\vec{E}_2$.

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{\rho\vec{r}}{3\varepsilon_0} + \left[\frac{-\rho(\vec{r}-\vec{a})}{3\varepsilon_0}\right]$$

$$= +\frac{\rho\,\vec{a}}{3\varepsilon_0} = \text{constant}$$

Cylindrical Cavity

As in case of a spherical cavity. We can assume that a cylinder with the same charge density but of opposite nature is placed inside the a solid cylinder one so as to form the cavity. It is super position of two oppositely charged cylinders.

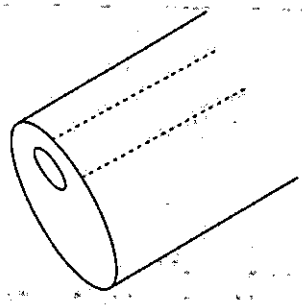


Fig. 1.103

Taking cross-sectional view,

$$\vec{E}_{net} \text{ at point } A = \vec{E}_1 + \vec{E}_2$$

$$= \frac{\rho\cdot\vec{r}_1}{2\varepsilon_0} + \frac{(-\rho)\vec{r}_2}{2\varepsilon_0}$$

$$= \frac{\rho}{2\varepsilon_0}(\vec{r}_1 - \vec{r}_2)$$

$$\vec{E}_{net} = \frac{\rho}{2\varepsilon_0}\vec{l}$$

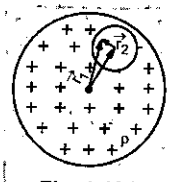


Fig. 1.104

Concept:

Electric Field in the Common Space Due to the Two Intersecting Spheres

Two intersecting spheres having the same charge density but opposite nature of charge are shown in the Fig. 1.105.

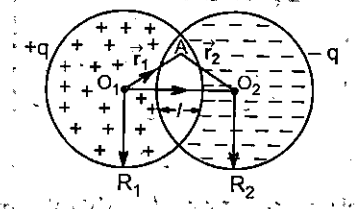


Fig. 1.105

To Calculate E at point A within the common space we apply principle of super position, let radius vectors of the point A with respect to the centres of the two spheres O_1 and O_2 be $\vec{r}_1$ and $\vec{r}_2$.

$$\vec{E}_{net} = \vec{E}_1 + \vec{E}_2$$

$$= \rho\frac{\vec{r}_1}{3\varepsilon_0} + (-\rho)\frac{\vec{r}_2}{3\varepsilon_0}$$

$$= \frac{\rho}{3\varepsilon_0}(\vec{r}_1 - \vec{r}_2) = \frac{\rho\,\vec{l}}{3\varepsilon_0}$$

Hence in the common space of intersection, the electric field is uniform and parallel to the vector $\vec{1}$, *field is independent of the radii of the two spheres. Where* $\vec{1}$ *is the position vector of* O_2 *with respect to* O_1.

Example 52

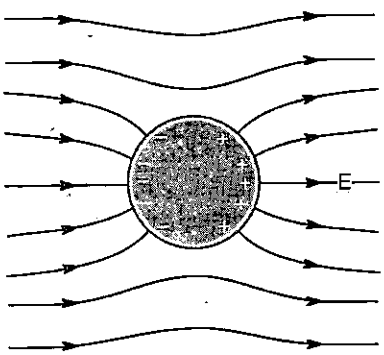
(a) Show that the electric field just outside the surface of any conductor of arbitrary shape is given by

$$E = \frac{\sigma}{\varepsilon_0}$$

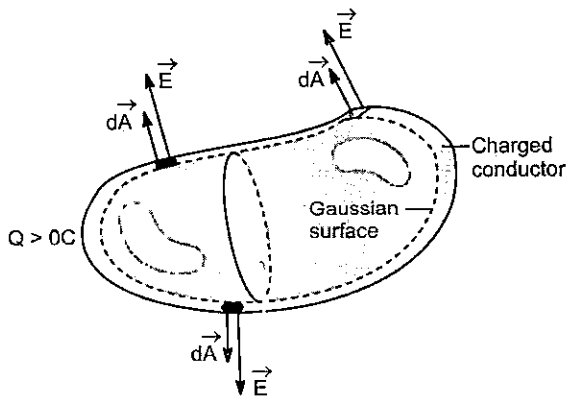
Where σ *is the surface charge density on the conductor's surface at that point.*

(b) Show that electrostatic field inside a conductor is zero.

Solution: 1. When we place an ideal conductor in an electric field E, the free electrons experience a force in the opposite direction of the field and migrate to one side of the conductor as shown in Fig. 1E.52 (a). The accumulation of electrons leaves one side positively charged and the other negative. This charged distribution creates an electric field in a direction opposite to the applied field. The redistribution of charge takes place till net field inside the conductor is zero. Therefore, in electrostatic equilibrium, **the electric field inside an ideal conductor is zero.**



(a)



(b)

Fig. 1E.52

2. If we place a Gaussian surface, an infinitesimal distance below the surface, the electric field is zero at every point on this Gaussian surface because it is inside the conductor [Fig. 1E.52 (b)]. Gauss's law then implies that the net charge contained within the Gaussian surface is zero. **In electrostatic equilibrium, excess charge on an ideal isolated conductor must reside on the conductor's surface.** No free charge can exist anywhere within the electrostatic conductor.

3. In electrostatics the electric field vector at the surface of a conductor must be perpendicular to the surface [Fig. 1E.52 (c)]. If the electric field is at an angle to the surface there will be a component parallel to the surface. This tangential component of field would cause the conduction electrons in the conductor to move. The situation will not be electrostatic.

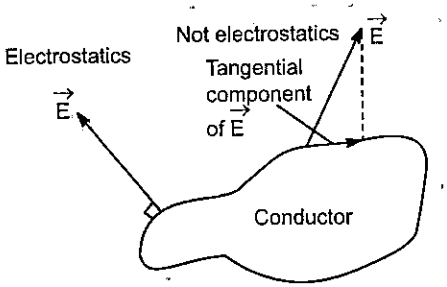


Fig. 1E.52 (c)

4. We choose as our Gaussian surface a small cylindrical box, one end of which is inside the conductor and the other end just outside the conductor.

The electric field is zero inside a conductor and is perpendicular to the surface just outside it. Therefore flux passes only through the outside end cap of the cylindrical box.

From Gauss's law

$$\oint E \cdot dA = \frac{Q_{\text{net enclosed}}}{\varepsilon_0}$$

$$EA = \frac{\sigma A}{\varepsilon_0}; \quad E = \frac{\sigma}{\varepsilon_0}$$

5. We consider a cylindrical box-shaped Gaussian surface on each side of an arbitrary surface carrying a surface charge density σ. As shown in Fig. 1E.52 (d), E_{n_2} and E_{n_1} represent normal component of field on either side of the surface.

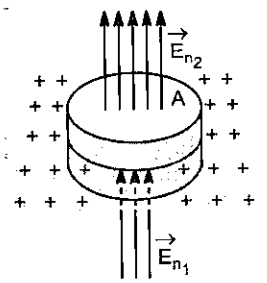


Fig. 1E.52 (d)

From Gauss's law,

$$E_{n_2}A - E_{n_1}A = \frac{\sigma A}{\varepsilon_0}$$

$$E_{n_2} - E_{n_1} = \frac{\sigma}{\varepsilon_0}$$

Which shows that there is a discontinuity in the electric field at the surface. This is the general result for a surface charge distribution.

$$E_{out} - E_{in} = \frac{\sigma}{\varepsilon_0}$$

6. An arbitrarily shaped conductor carrying a charge on its surface [Fig. 1E.52 (e)]. The charge in the vicinity of point P near the surface looks like an infinite plane of charge, giving an electric field of magnitude $\sigma/2\varepsilon_0$ pointing away from the surface both inside and outside the surface. Inside the conductor this field points down from point P. Since the net field inside the conductor is zero, the rest of the charge must produce a field of magnitude $\sigma/2\varepsilon_0$ in the upward direction. The field due to this charge is the same inside the surface as outside the surface. Inside the surface, the field shown in Fig. 1E.52 (e) cancel, but outside at point P they add to give $E_n = \sigma/\varepsilon_0$.

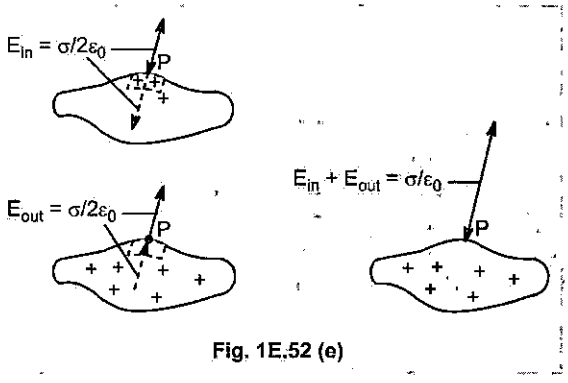


Fig. 1E.52 (e)

The field outside a large plane nonconductor is $E = \frac{\sigma}{2\varepsilon_0}$ whereas outside a conductor is $E = \frac{\sigma}{\varepsilon_0}$. For a conductor, the charge lies at the surface and all the electric field lines leave on one side of the surface. For an infinite nonconducting plane the lines leave both sides. For a thin flat conducting plane, the charge would accumulate on both surfaces and the field will result from both sides. If σ' is the total charge density, each face of the plane would have surface charge $\sigma = \frac{\sigma'}{2}$. Hence $E = \frac{\sigma'/2}{\varepsilon_0} = \frac{\sigma'}{2\varepsilon_0}$ which is same as for a nonconducting plane. Generally, σ implies charge density on one face of a conducting plane.

(i) Consider an empty cavity inside a conductor [Fig. 1E.52 (f)]. Imagine a Gaussian surface just inside the conductor above the cavity. The $\vec{E}$ must be zero everywhere on this surface since it is inside the conductor; therefore there can be no net charge at the surface of the cavity.

(ii) Consider a conductor which carries a net charge $+Q$ and contains a cavity, inside of which also resides a point charge $+q$ [Fig. 1E.52 (g)].

A Gaussian surface just inside the conductor surrounding the cavity must contain zero net charge also

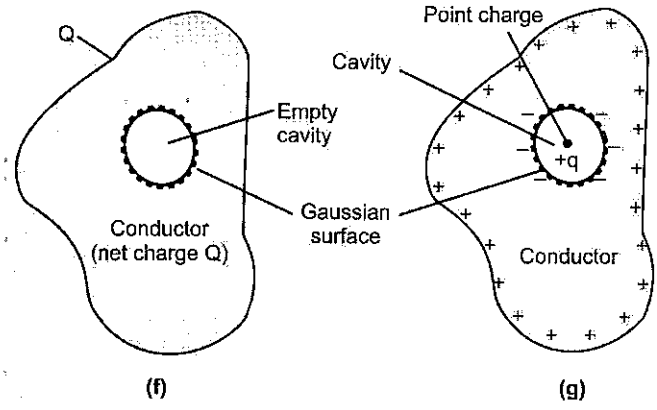


Fig. 1E.52

$E = 0$ in a conductor. Therefore, a net charge of $-q$ must exist on the cavity surface. The conductor carreis a net charge $+Q$, so its outer surface must carry a charge equal to $Q + q$. In other words, we may say that no net charge resides inside the surface of an electrostatic conductor, it appears on the surface of the conductor.

(iii) Fig. 1E.52 (h) shows a positive point charge q at the centre of a spherical cavity inside a spherical conductor. Since the net charge must be zero within any surface drawn within the conductor, there must be a negative charge $-q$

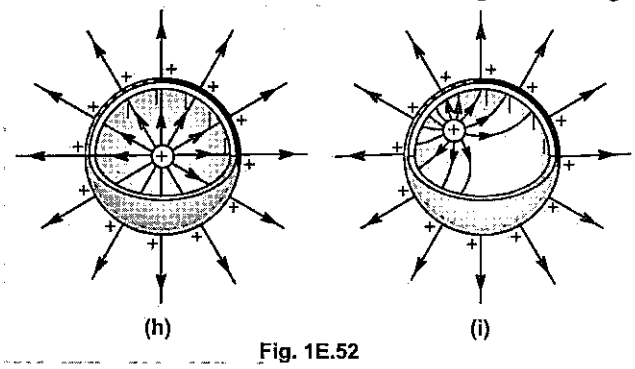


Fig. 1E.52

induced in the inside surface. In Fig. 1E.52 (i), the point charge has been moved so that it is no longer at the centre of the cavity. The field lines in the cavity are altered, and the surface charge density of the induced negative charge on the inner surface is no longer uniform. However, the positive surface charge density on the outside surface is not disturbed— it is still uniform— because it is shielded from the cavity by the conductor.

Concept: *Where the tips of two positively charged wires are inserted in a container of insulating liquid, and some grass seeds are floated on the liquid. The grass seeds are electrically neutral insulators, but the electric field of the two charged wires causes polarization of the grass seeds; there is a slight shifting of the positive and negative charges within the molecules of each seed, like that shown in Fig. 1.106.*

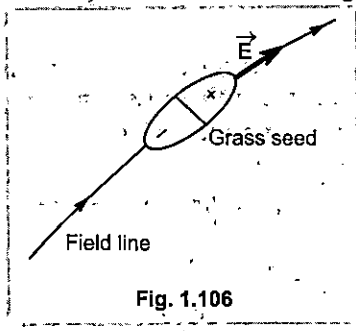


Fig. 1.106

The positively charged end of each grass seed is pulled in the direction of $\vec{E}$ *and the negatively charge end is pulled opposite* $\vec{E}$ *Hence the long axis of each grass seed tends to orient parallel to the electric field, in the direction of the field line that passes through the position of the seed.*

Example 53

A tiny mass m with charge q is attached to an infinite sheet of charge with surface charge density σ by means of an insulating massless cord of length l. Neglect gravitational effects.

(a) Specify the signs of the charge q and σ such that the cord is taut.

(b) Show that if the mass is pulled slightly in a direction parallel to the plane and then released, the mass executes simple harmonic motion with frequency ν, where

$$\nu = \frac{1}{2\pi}\left(\frac{q\sigma}{2\varepsilon_0 ml}\right)^{1/2}$$

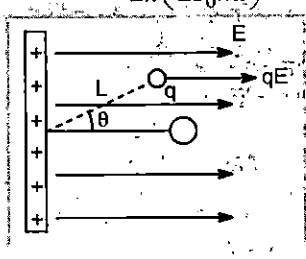


Fig. 1E.53

Solution: The cord will be taut when charge q is positive; the electric force of the plane repels the charge q.

Electric field near an infinite plane is $E = \frac{\sigma}{2\varepsilon_0}$

Force exerted on the charge q is $F = \left(\frac{\sigma}{2\varepsilon_0}\right)q$.

Restoring torque when string is displaced parallel to sheet,

$$\tau = -qEl\sin\theta = I\alpha$$

For small angular displacement $\sin\theta \simeq \theta$,

$$-qEl\theta = I\alpha$$

or $$\alpha = -\frac{qEl}{I}\theta$$

Since angular acceleration is proportional to θ, the mass executes SHM.

On comparing expression for α with equation of SHM,

$$\alpha = -\omega^2\theta$$

we obtain $$\omega = \sqrt{\frac{qEl}{I}} = \sqrt{\frac{q\sigma}{2\varepsilon_0 ml}}$$

where $$E = \frac{\sigma}{2\varepsilon_0}$$

and $$I = ml^2$$

or $$\omega = 2\pi\nu = \sqrt{\frac{q\sigma}{2\varepsilon_0 ml}}$$

or $$\nu = \frac{1}{2\pi}\sqrt{\frac{q\sigma}{2\varepsilon_0 ml}}$$

Example 54

Two thin concentric spherical shells of radii R_1 *and* R_2 $(R_1 < R_2)$ *contain uniform charge distribution* σ_1 *and* σ_2 *(surface charge density) respectively. Determine the electric field for (a)* $r < R_1$, *(b)* $R_1 < r < R_2$, *(c)* $r > R_2$, *(d) Under what condition will* $E = 0$ *for* $r > R_2$? *(e) Under what condition will* $E = 0$ *for* $R_1 < r < R_2$?

Solution: As we have learned for a spherical Gaussian surface and radial field, the electric flux,

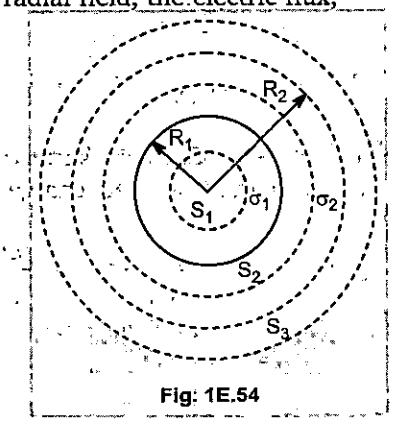


Fig: 1E.54

$$\phi_E = E \cdot 4\pi r^2$$

(a) $\boldsymbol{E(r < R_1)}$: Consider the Gaussian surface S_1,

$$E \cdot 4\pi r^2 = Q_{\text{enclosed}} = 0$$

$$E = 0$$

(b) $\boldsymbol{E(R_1 < r < R_2)}$: Consider the Gaussian surface S_2,

$$E \cdot 4\pi r^2 = Q_{\text{enclosed}}$$

$$= \sigma_1 \times 4\pi R_1^2$$

$$E = \frac{\sigma_1 R_1^2}{r^2}$$

(c) $\boldsymbol{E(r > R_2)}$: Consider the Gaussian surface S_3,

$$E \cdot 4\pi r^2 = Q_{\text{enclosed}} = \sigma_1 \cdot 4\pi R_1^2 + \sigma_2 \cdot 4\pi R_2^2$$

$$E = \frac{\sigma_1 R_1^2 + \sigma_2 R_2^2}{r^2}$$

(d) From (c), $\boldsymbol{E(r > R_2)}$ is zero for

$$\sigma_1 R_1^2 + \sigma_2 R_2^2 = 0 \quad \text{or} \quad \frac{\sigma_1}{\sigma_2} = -\frac{R_2^2}{R_1^2}$$

(e) From (b), $\boldsymbol{E(R_1 < r < R_2)}$ is zero for

$$\sigma_1 = 0$$

Example 55

A point charge $+Q$ is placed at the centre of an uncharged spherical conducting shell of inner radius a and outer radius b as shown in Fig. 1E.55 (a).

(a) Find the electric field for $r < a$.

(b) What is the magnitude and sign of the induced charge q' on the inner shell surface?

(c) What is the electric field for $a < r < b$?

(d) What is the electric field at points $r > b$?

(e) What is the surface charge on the outer surface of the conductor?

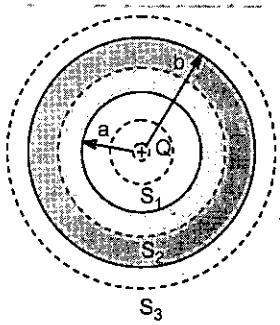


Fig. 1E.55 (a)

Solution: (a) Consider a Gaussian surface of radius $r < R$ inside the cavity, centred on the charge Q. From Gauss's law.

$$\phi_E = \oint E \cdot dA = E(4\pi r^2) = \frac{+Q}{\varepsilon_0}$$

from which we find the electric field to be

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

This result is the same as that of a point charge in vacuum.

(b) Consider a Gaussian surface inside the conducting material, s_2. We do not know if there is a charge on the inside surface of the conductor or not. We assume that the charge is q', if q' is zero the result of Gauss's law will show it. Because the Gaussian surface is inside the conductor, the electric field is zero. From Gauss's law,

$$\oint \vec{\mathbf{E}} \cdot \vec{dA} = E(4\pi r^2)\frac{Q_{\text{enclosed}}}{\varepsilon_0} = 0$$

$$\frac{Q_{\text{enclosed}}}{\varepsilon_0} = \frac{Q + q'}{\varepsilon_0} = 0$$

which implies $q' = -Q$.

There is a charge on the inside surface of the conductor. The total charge induced on the inside surface of the cavity is the negative of the charge placed at its centre.

(c) **The field inside a conductor in electrostatic equilibrium is always zero.**

(d) For $E(r > b)$ consider a Gaussian surface s_3, from Gauss's law we have

$$\phi_E = \oint E \cdot dA = \frac{Q_{\text{enclosed}}}{\varepsilon_0}$$

$$E(4\pi r^2) = \frac{+Q}{\varepsilon_0}$$

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

It was stated in the problem that the conducting sphere has no net charge. Consequently the total charge inside our Gaussian surface s_3 is sum of charge $+Q$ and induced charges $-Q$ on the inner surface of conductor and $+Q$ on the surface. Once more we can see that field outside the sphere is same as for a point charge. The conducting sphere has no shielding effect at all.

However, such a conducting shield does prevent electrostatic fields from charges outside the shell from entering it.

(e) The conducting shell has no net charge, yet there is a surface charge $-Q$ on its surface. Because the net charge on the shell is zero and no charge can reside inside a

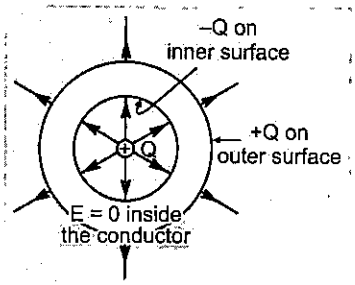


Fig. 1E.55 (b)

conductor, there must be $+Q$ on the outer surface of the conductor [Fig. 1E.55 (b)].

Example 56

Consider two concentric conducting spheres. The outer sphere is hollow and initially has a charge $-7Q$ on it. The inner sphere is solid and has a charge $+2Q$ on it.

(a) How much charge is on the outer surface and inner surface of the outer sphere.

(b) If a wire is connected between the inner and outer spheres, after electrostatic equilibrium is established, how much total charge is on the outside sphere? How much charge is on the outer surface and inner surface of outer sphere? Does the electric field at the surface of the inside sphere change when the wire is connected.

(c) We return to original condition in (a). We now connect the outer sphere to ground with a wire and then disconnect it. How much total charge will be on the outer sphere? How much charge will be on the inner surface and outer surface of the outer sphere?

Solution: (a) The charges on the inner sphere induce equal magnitude of charge, but opposite in sign, on the inner surface of outer sphere. Sum of all the induced charges is always zero. Therefore, an equal amount of charge must come on the outer surface. Thus outer and inner surface of outer sphere have charges $-5Q$ and $-2Q$ respectively.

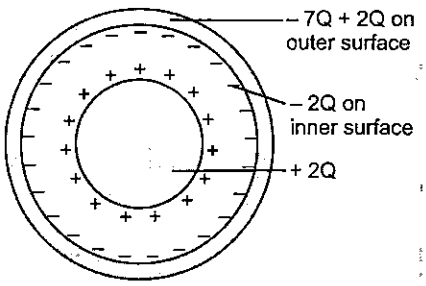


Fig. 1E.56 (a)

(b) When outer and inner spheres are connected by a wire, the entire charge is transferred to the outer surface from inner sphere. In electrostatic equilibrium charge does not reside inside a conductor.

Total charge on outer surface of outer sphere is $-5Q$.

Total charge on inner surface is 0.

The electric field at the surface of the inside sphere goes to zero after connection. Consider a Gaussian surface just on the surface of inner sphere.

$$\phi_E = E(4\pi r^2) = \frac{Q_{\text{enclosed}}}{\varepsilon_0} = 0,$$

as $Q_{\text{enclosed}} = 0.$

Thus, we have $E = 0$

(c) When the outer sphere is grounded the charge on the surface is transferred to ground, thus charge is reduced to zero. The final charge distribution is shown in Fig. 1E.56 (b).

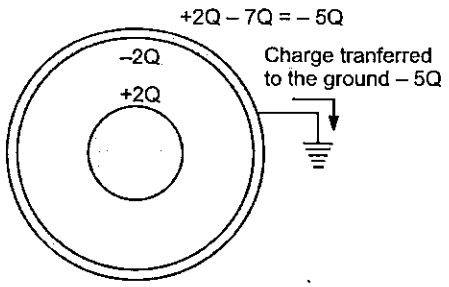


Fig. 1E.56 (b)

Example 57

Consider two concentric spheres of radii a and b (a < b), respectively. A point charge q is placed at the origin, r = 0, and the region between concentric spheres contains volume charge density $\rho = \frac{c}{r}$, where c is a constant and r is the radial distance as shown in Fig. 1E.57. Find the value of c for which the electric field in the region between the spheres is constant (i.e., r independent).

Solution: The electric field of given charge distribution is radial and spherically symmetric.

We consider a Gaussian surface of radius r. First we will determine the net charge enclosed in this Gaussian surface. Consider a spherical shell of radius r, thickness dr, the volume of this shell,

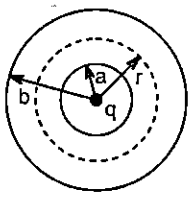


Fig. 1E.57

$$dv = 4\pi r^2 dr$$

Charge enclosed in the thickness of shell,

$$dq = (4\pi r^2 dr)\frac{c}{r}$$

Total charge enclosed,

$$Q_{\text{enclosed}} = 4\pi\int_a^r \frac{c}{r} r^2 dr = 2\pi c(r^2 - a^2)$$

From Gauss's law,

$$\phi_E = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = E_1 4\pi r^2 = \frac{Q_{\text{enclosed}}}{\varepsilon_0}$$

Thus we have

$$E_1 4\pi r^2 = \frac{2\pi c(r^2 - a^2)}{\varepsilon_0}$$

or $$\vec{\mathbf{E}}_1 = \frac{c}{2\varepsilon_0}\left(1 - \frac{a^2}{r^2}\right) \quad \text{for} \quad a < r < b$$

The field due to point charge $\vec{\mathbf{E}}_2 = \frac{q}{4\pi\varepsilon_0 r^2}$

The resultant electric field $\vec{\mathbf{E}} = \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2$

$$= \frac{c}{2\varepsilon_0}\left(1 - \frac{a^2}{r^2}\right) + \frac{q}{4\pi\varepsilon_0 r^2} = \frac{c}{2\varepsilon_0} + \frac{q - 2\pi c a^2}{4\pi\varepsilon_0 r^2}$$

E is constant in the region $a < r < b$ for $c = \frac{q}{2\pi a^2}$.

Example 58

Consider three concentric metallic spheres. Sphere I is solid whereas II and III are hollow, as shown in the figure. A negative charge $-Q_0$ is placed on sphere I and a positive charge $+Q_0$ is placed on sphere III. Find charge on all the surfaces and plot E versus r.

Solution: Sphere I induces charge $-Q_0$ and $+Q_0$ on the inner and outer surface of sphere II. Sphere II induces charges $-Q_0$ and $+Q_0$ on the inner and outer surface of sphere III. The sphere III already has charge $+Q_0$ on its outer surface (charges reside on outer surface of a conductor). Therefore, net charge on the outer sphere is zero.

1. Gaussian surface S_1,
   $E(r < R_1) = 0$ $\quad (Q_{enclosed} = 0)$
2. Gaussian surface S_2,
   $E(R_1 < r < R_2) = -\dfrac{Q}{4\pi\varepsilon_0 r^2}$ $\quad (Q_{enclosed} = -Q)$
3. Gaussian surface S_3,
   $E(R_2 < r < R_3) = 0$ $\quad (Q_{enclosed} = 0)$

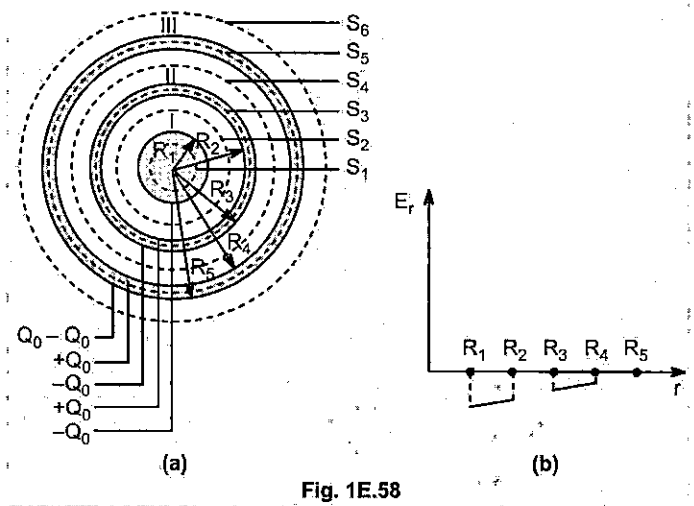


Fig. 1E.58

4. Gaussian surface S_4,
   $E(R_3 < r < R_4) = -\dfrac{Q}{4\pi\varepsilon_0 r^2}$ $\quad (Q_{enclosed} = -Q)$
5. Gaussian surface S_5,
   $E(R_4 < r < R_5) = 0$ $\quad (Q_{enclosed} = 0)$
6. Gaussian surface S_6,
   $E(r < R_6) = 0$ $\quad (Q_{enclosed} = 0)$

Example 59

The intensity of an electric field depends only on the coordinates x and y as follows :

$$\vec{\mathbf{E}} = \frac{a(x\hat{\mathbf{i}} + y\hat{\mathbf{j}})}{x^2 + y^2}$$

where a is a constant and $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ are the unit vectors of the x and y-axes. Find the charge within a sphere of radius R with the centre at the origin.

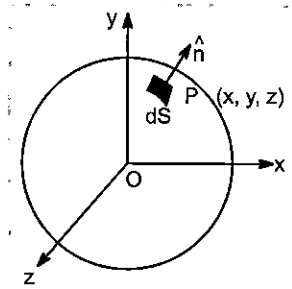


Fig. 1E.59

Solution: A unit vector perpendicular to the sphere radially outwards of any point of the sphere is given by

$$\hat{\mathbf{n}} = \frac{x}{\sqrt{x^2+y^2+z^2}}\hat{\mathbf{i}} + \frac{y}{\sqrt{x^2+y^2+z^2}}\hat{\mathbf{j}} + \frac{z}{\sqrt{x^2+y^2+z^2}}\hat{k}$$

$$= \frac{x}{R}\hat{\mathbf{i}} + \frac{y}{R}\hat{\mathbf{j}} + \frac{z}{R}\hat{\mathbf{k}} \qquad (\text{as } x^2+y^2+z^2 = R^2)$$

The electric flux through a differential area dA at point P on the sphere,

$$d\phi_E = \vec{\mathbf{E}} \cdot \hat{\mathbf{n}} dA$$

$$= \left\{\frac{ax^2}{R(x^2+y^2)} + \frac{ay^2}{R(x^2+y^2)}\right\} dA$$

$$= \left(\frac{a}{R}\right) dA$$

Note that $d\phi_E$ is independent of the coordinates x, y and z. Therefore, total flux passing through the sphere,

$$\phi_E = \int d\phi_E = \frac{a}{R}\int dS$$

$$= \left(\frac{a}{R}\right)(4\pi R^2) = 4\pi aR$$

From Gauss's law, we have $\phi_E = \dfrac{q_{enclosed}}{\varepsilon_0}$

or $\quad (4\pi aR) = \dfrac{q_{enclosed}}{\varepsilon_0}$

or $\quad q_{enclosed} = 4\pi\varepsilon_0 aR$

Example 60

A thin cylindrical shell of radius R_1 is surrounded by a second concentric cylindrical shell of radius R_2. The inner shell has total charge +Q and the outer shell –Q. Assuming the length L of the shells is much greater than R_1 and R_2, determine the electric field as a function of r (the perpendicular distance from the common axis of the cylinders) for (a) $r < R_1$, (b) $R_1 < r < R_2$, and (c) $r > R_2$. (d) What is the kinetic energy of an electron if it moves between (and concentric with) the shells in a circular orbit of radius $(R_1 + R_2)/2$?

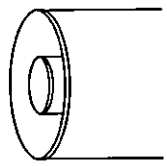


Fig. 1E.60

Solution: The electric field of the given charge distribution is expected to be radial. The electric flux through a cylindrical Gaussian surface of radius r and length l is

$$\phi_E = \int E \cdot dA = (E\, 2\pi rl)$$

(a) In the region $r < R_1$,

$$\varepsilon_0 E(2\pi rl) = Q_{\text{enclosed}} = 0$$

or $$E = 0$$

(b) In the region $R_1 < r < R_2$,

$$\varepsilon_0 E(2\pi rl) = Q_{\text{enclosed}} = -\frac{Q}{L}l$$

$$E = -\frac{Q}{2\pi\varepsilon_0 rL}$$

(c) In the region $r > R_2$,

$$\varepsilon_0 E(2\pi rl) = Q_{\text{enclosed}} = 0$$

$$E = 0$$

(d) The electric field exists only between concentric shells and it is radially inward. The electric force on the electron provides centripetal force for the circular motion of the electron. Thus we have

$$\frac{eQ}{2\pi\varepsilon_0\left(\frac{R_1+R_2}{2}\right)L} = \frac{mv^2}{\frac{R_1+R_2}{2}}$$

Hence kinetic energy of electron is

$$\text{KE} = \frac{1}{2}mv^2 = \frac{eQ}{4\pi\varepsilon_0 L}$$

Example 61

A point charge q is placed at one corner of a cube of edge a. What is the flux through each of the cube faces?

Solution: In order to completely enclose the charge q, eight cubes are required, each having a corner at A. The flux linked with each cube (due to a charge q at the corner) will be $(q/8\varepsilon_0)$.

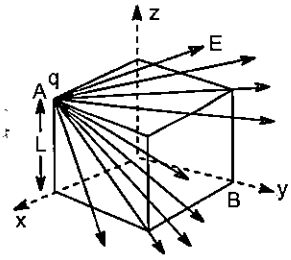


Fig. 1E.61

For the faces passing through the edge A, electric field E at a face will be parallel to area of face and so flux for these three faces will be zero.

The cube has six faces and the flux linked with three faces (through A) is zero, so the flux linked with the remaining three faces will be $(q/8\varepsilon_0)$. Now as the remaining three faces are identical, so the flux linked with each of the three faces passing through B will be

$$= \frac{1}{3} \times \left[\frac{1}{8}\left(\frac{q}{\varepsilon_0}\right)\right] = \frac{1}{24}\frac{q}{\varepsilon_0}$$

Example 62

A cube of side l has one corner at the origin of coordinates and extends along the positive x, y and z-axis. Suppose the electric field in this region is given by $E = (a + by)\hat{\mathbf{j}}$. Determine the charge inside the cube.

Solution: The electric flux through the faces *abfe, bcgf, cdgh, adhe* is zero because the area vector is normal to electric field.

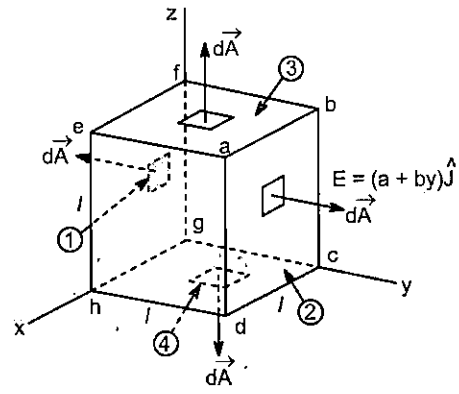


Fig. 1E.62

Flux through face *efgh*,

$$\phi_2 = \int \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$$

$$= a(\hat{\mathbf{j}}) \cdot l^2(-\hat{\mathbf{j}}) = -al^2$$

As the field at the face *efgh* (that lies in the *yz*-plane, $y = 0$) is $E = a\hat{\mathbf{j}}$ and area vector is $l^2(-\hat{\mathbf{j}})$ (direction outward normal)

Flux through face *abcd*

$$\phi = (a + bl)\hat{\mathbf{j}} \cdot l^2\hat{\mathbf{j}}$$

$$= (al^2 + bl^3)$$

Net flux through the cube

$$= \phi_1 + \phi_2 = bl^3.$$

From Gauss's law,

$$Q_{\text{enclosed}} = \varepsilon_0\phi_E = \varepsilon_0 bl^3$$

ELECTRIC POTENTIAL ENERGY AND ELECTRIC POTENTIAL

When a free positive charge is released in an electric field, as shown in Fig. 1.107, the electric force will do work on the charge and accelerate it toward the negative plate. It gains kinetic energy. The process is analogus to gravity.

It is as if the charge is going down an electric hill where its electric potential energy is converted to kinetic energy. In accordance with the conservation of energy, decrease in electric potential energy is transformed into kinetic energy. Note that the greatest potential energy of charge q is at point A, near the positive plate; so $U_B - U_A < 0$, whereas for a negative charge its potential energy is greatest near the negative plate.

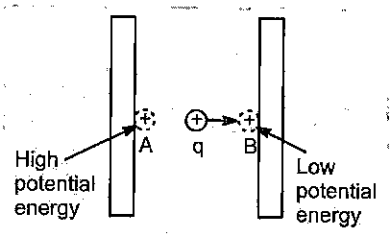


Fig. 1.107

The electric force caused by static charges is a conservative force. The conservative force has the following properties :

1. Work done by a conservative force is independent of the path followed by a particle as it moves from an initial position to a final position.

2. Work done by a conservative force is zero around any closed path.

3. The work done by a conservative force on a particle is the negative of the charge in the potential energy associated with the conservative force.

$$W_{\text{consrevative}} = -\Delta \text{PE}$$

When a positive charge is accelerated in an electric field, the work done by field results in loss in potential energy, or a negative ΔPE.There must be a minus sign in front of ΔPE to make W positive.

Work done by an electric force on a charge q as it moves from an initial to a final location is

$$\int_i^f \vec{\mathbf{F}}_{\text{elec.}} \cdot d\vec{\mathbf{r}} = -\Delta \text{PE} = -(\text{PE}_f - \text{PE}_i) \qquad ...(1)$$

Since the electric force is conservative, the result of integral is independent of path.

$$\vec{\mathbf{F}}_{\text{elec.}} = q\vec{\mathbf{E}}$$

$$q\int_i^f \vec{\mathbf{E}} \cdot d\vec{\mathbf{r}} = -(\text{PE}_f - \text{PE}_i)$$

$$\int_i^f \vec{\mathbf{E}} \cdot d\vec{\mathbf{r}} = -\left(\frac{\text{PE}_f}{q} - \frac{\text{PE}}{q}\right) \qquad ...(2)$$

The right hand side of equation (2) represents change in potential energy per unit charge. Now we define a scalar quantity **electric potential** V at a point in space to be the potential energy per unit charge at the point.

$$V = \frac{\text{PE}_{\text{of}\,q}}{q}$$

The SI unit of electric potential is joule/coulomb which is called **volt** (V) = J/C.

The electric potential energy of a charge q in an electric field where potential has the value is

$$\text{PE}_{\text{of}\,q} = qV$$

The eqn. (2) can now be written in terms of potential difference

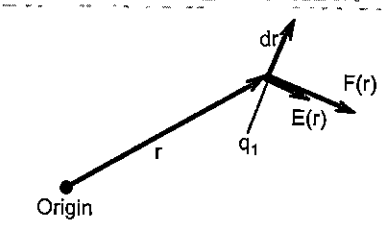
$$\text{N}-\int_i^f \vec{\mathbf{E}} \cdot d\vec{\mathbf{r}} = (V_f - V_i)$$

Thus potential difference between two points is

$$V_f - V_i = \frac{\text{PE}_f - \text{PE}_i}{q} = -\frac{W_{fi}}{q}$$

Potential difference is change in potential energy per unit charge, is equal to the negative of the work done by the electric field to move the charge from initial point to final point.

Concept:



A test charge q1 lies in an electric field E (r) at the point r. The test charge experiences the electric force F(r) as it moves through the infinitestimal displacement dr (perhaps under the influence of some other force or forces, not shown. in addition to the electric force). The work done by the electric force is dW = F(r). dr, and the potential energy change is

$$dU = -dW = -F(r).dr$$

Fig. 1.108

The Relation between Potential and Field

Suppose a test charge experiences an infinitesimal displacement in the direction of the electric field vector $\vec{\mathbf{E}}$ *. In this case, we can write in the scalar form*

$$dV = -E(r)\,dr \quad \text{For } d\mathbf{r} \;||\; E(r) \qquad ...(1)$$

The electric field at $\mathbf{r}$ *then has magnitude*

$$E(r) = -\frac{dV(r)}{dr} \qquad ...(2)$$

We say that the electric field is the negative of the ***gradient*** *of the electric potential the change in V per unit displacement. At any location* $\mathbf{r}$*, we wish to express the potential difference dV across a displacement dr that is not necessarily parallel to E. In (Fig. 1.109), the angle between E and dr is θ. We have*

$$dV = -E\cos\theta\,dr$$

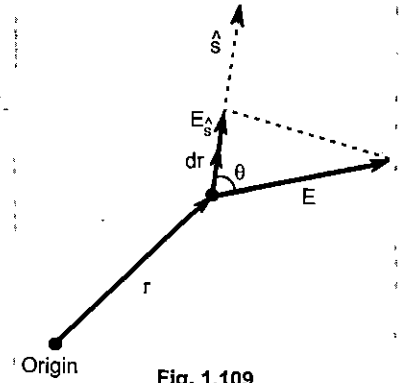


Fig. 1.109

Now, E cos θ is just the component of E along dr. Let us give this component the name $E_{\hat{s}}$. We can then rewrite in the generalized form

$$dV = -E_{\hat{s}}\, dr \qquad ...(3)$$

Rearranging this equation to express the field component in terms of the potential difference, we obtain

$$E_{\hat{s}} = -\frac{dV}{dr} \qquad ...(4)$$

Which is a generalization of the one-dimensional eqn. (2). The change of potential per unit displacement in a given direction $\hat{\mathbf{s}}$ is equal to the negative of the component of the electric field in that direction.

Interpreting Electric Potential Energy

When a particle moves from point a to point b, the work done on it by the electric field is $W_{a\to b} = U_a - U_b$. Thus the potential-energy difference $U_a - U_b$ equals the work that is done by the electric force when the particle moves from a to b. When U_a is greater that U_b, the field does positive work on the particle as it "falls" from a point of higher potential energy (a) to a point of lower potential energy (b).

Consider how much work we would have to do to move the particle slowly (so as not to give it any kinetic energy), we need to exert an additional external force $\vec{\mathbf{F}}_{ext}$ that is equal and opposite to the electric-field force and does positive work. The potentialenergy differene $U_a - U_b$ is then defined as the work that must be done by an external force to move the particle slowly from b to a against the electric force.

Total Energy of a Charged Particle

When a particle having charge q and mass m moves from a point where the potential is V_i to a point where the potential is V_f, its potential energy changes by an amount

$$\Delta U = q(V_f - V_i) = q\Delta V$$

Suppose the electric force is the only force acting on the particle. Then its total energy E must be conserved. Hence its kinetic energy K must change in such a way that $\Delta K = -\Delta U$. If the initial kinetic energy is zero that is, if the particle is initially at rest we can write

$$\tfrac{1}{2}mv^2 = -q\Delta V$$

The Electron-Volt

When an electron "falls" or, more precisely, is accelerated through a potential difference $|\Delta V| = 1$ V, it acquires kinetic energy and loses potential energy of magnitude

$$|\Delta K| = |-\Delta U| = |q\Delta V| = e \times 1\,\text{V} \qquad ...(1)$$

Where e is the quantum of charge, the magnitude of the charge on the electron. We insert the value of e into this equation to obtain

$$e \times 1\,\text{V} = 1.60\times10^{-19}\,\text{C} \times 1\,\text{V} = 1.60\times10^{-19}\,\text{J} \qquad ...(2)$$

This quantity makes a convenient unit of energy for the study of electrons and other microscopic charged particles because such particles are often made to pass through potential differences measured in volts. The new unit is given the name **electron-volt (eV):**

$$1\ \text{eV} \equiv e\,\text{J} \approx 1.60\times10^{-19}\,\text{J} \qquad ...(3)$$

Thus, for example, the energy change of an electron that has passed through a potential difference of 20,000 V is simply 20,000 eV (or 20 keV). It is simpler to express energy directly in terms of the potential difference than to multiply the charge of the electron by the potential difference in volts and thus express the energy in joules.

Since potential energy and charge are scalars, potential is also a scalar.

Concept:

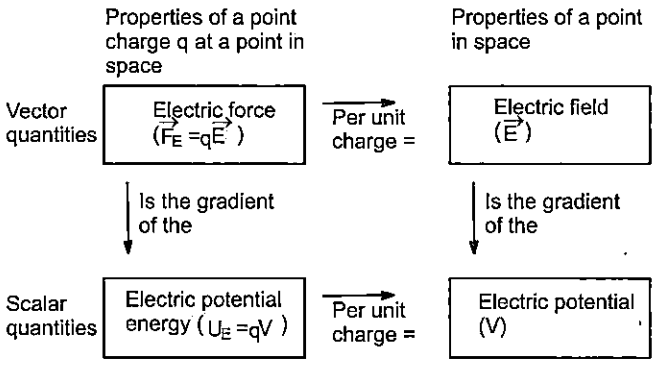


Fig. 1.110

Potential Difference in a Uniform Field

Consider a pair of large, parallel, charged sheets (Fig. 1.111). The electric field is directed from the positively charged plate to negatively charged plate. The electric field in the vector form can be written as

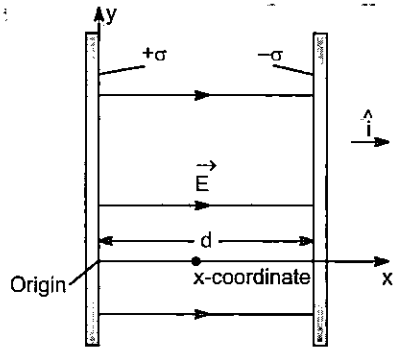


Fig. 1.111

$$\vec{\mathbf{E}} = E\hat{\mathbf{i}},$$

according to coordinate system shown in the figure. To calculate the electric potential at point x, we begin with the equation

$$V_f - V_i = \int_i^f -\vec{\mathbf{E}}\cdot d\vec{\mathbf{r}} \qquad ...(1)$$

$$V(x) - V(0) = -\int_0^x \vec{\mathbf{E}}\cdot d\vec{\mathbf{r}} \qquad ...(2)$$

For a small differential change in position vector $d\vec{\mathbf{r}} = dx\,\hat{\mathbf{i}}$

$$V(x) - V(0) = -\int_0^x (E\hat{\mathbf{i}}) \cdot (dx\,\hat{\mathbf{i}})$$

$$V(x) - V(0) = -E\int_0^x dx$$

$$V(x) - V(0) = -Ex \qquad \ldots(3)$$

The quantity $-Ex$ is negative, since E is the magnitude of field and the coordinate x is positive. Hence we conclude that location x has lower potential than the origin. The equation (3) indicates that the electric potential decreases with x in the same direction as the uniform field.

The direction of the electric field is from regions with higher values of the electric potential to regions of lower potential irrespective of the charge configuration.

At the position of the negatively charged plate,

$$V(d) - V(0) = -Ed \qquad \ldots(4)$$

which is the potential difference between the locations $x = d$ and $x = 0$ m.

In the gravitational situation near the surface of the Earth, we often choose the zero for the gravitational potential energy to be at ground level. In electric case we can choose any point in space and assign it zero potential.

The definition of potential is based on the reference position we choose for potential energy. The selection of reference point is simply a matter of convenience. When dealing with electric circuits, a convenient referenced position is the Earth or "ground". Only a change in potential energy has significance, and correspondingly only a change in potential or *potential difference* has physical significance. So remember there is no sign board in space which says, "This is the absolute zero of potential."

Choice 1 : If we choose point $x = d$ to be the reference point (zero potential), the eqn. (4) becomes

$$V(0\,\text{m}) = Ed$$

and eqn. (3) becomes

$$V(x) - Ed = -Ex$$

$$V(x) = Ed - Ex$$

The potential $V(x)$ *versus* x is graphed in Fig. 1.112 (a).

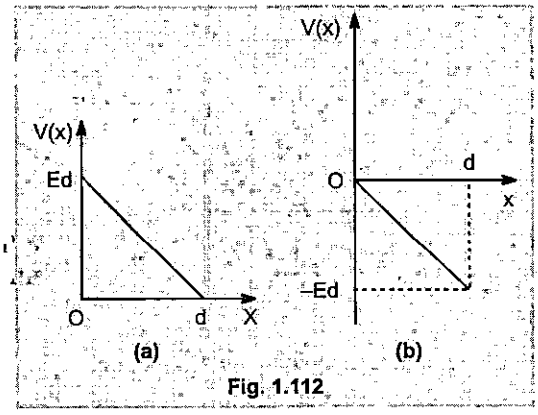


Fig. 1.112

Choice 2 : If we choose $x = 0$ to be the reference point, then $V(0) = 0$ V and eqn. (4) becomes

$$V(d) - V(0) = -Ed$$

$$V(d) = -Ed.$$

Eqn. (3) becomes

$$V(x) - 0\,V = -Ex$$

$$V(x) = -Ex$$

The potential function is graphed in Fig. 1.112 (b). The difference in potential is same irrespective of choice of reference position. The charge in electric potential is same in both the cases.

The Electric Potential of a Point-like Charge

The electric field at a distance r from a point charge Q is given by the equation

$$\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{\mathbf{r}}$$

Where $\hat{r}$ is a unit vector in radial direction. The field is directed radially outward from the charge if the charge is positive and toward the charge if the charge is negative. We wish to find the potential difference between two points at $\vec{\mathbf{r}}_f$ and $\vec{\mathbf{r}}_i$ shown in Fig. 1.113 (a).

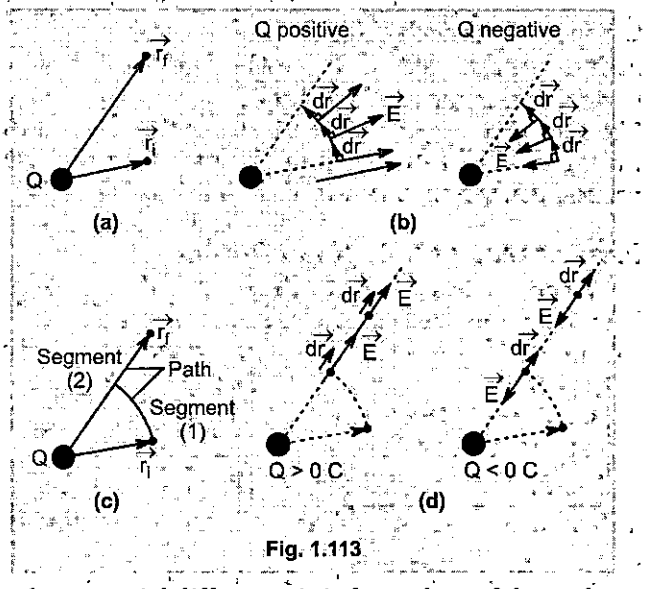


Fig. 1.113

The potential difference is independent of the path used from initial to final point. We choose a path consisting of an arc of a circle and a radial segment out to the tip of $\vec{\mathbf{r}}_f$ as shown in Fig. 1.113 (b).

At every point along the circular segment of the path, the scalar product $\vec{\mathbf{E}} \cdot d\vec{\mathbf{r}}$ is zero because E is perpendicular to the displacement vector $d\vec{\mathbf{r}}$ along this segment. Along the second segment the electric field is alo the path parallel to $d\vec{\mathbf{r}}$ if Q is positive and antiparallel to $d\vec{\mathbf{r}}$ if Q is negative.

$$V(r_f) - V(r_i) = -\frac{1}{4\pi\varepsilon_0} Q\int_i^f \frac{\hat{\mathbf{r}}}{\mathbf{r}^2} \cdot dr\,\hat{\mathbf{r}} \qquad \ldots(1)$$

where $d\vec{\mathbf{r}} = dr\,\hat{\mathbf{r}}$

Thus for a positive charge $\vec{\mathbf{E}}$ and $d\vec{\mathbf{r}}$ are parallel; eqn. (1) becomes

$$V(r_f) - V(r_i) = -\frac{1}{4\pi\varepsilon_0} Q \int_i^f \frac{dr}{r^2}$$

$$= -\frac{1}{4\pi\varepsilon_0} Q \left(-\frac{1}{r}\right)_{r_i}^{r_f}$$

$$= \frac{1}{4\pi\varepsilon_0} Q \left(\frac{1}{r_f} - \frac{1}{r_i}\right) \quad ...(2)$$

For the electric potential of a point charge, it is convenient to choose the reference position to be at infinity. In other words, V is taken to be zero at points far removed from the charge distribution ($V_\infty = 0$).

The eqn. (2) then becomes

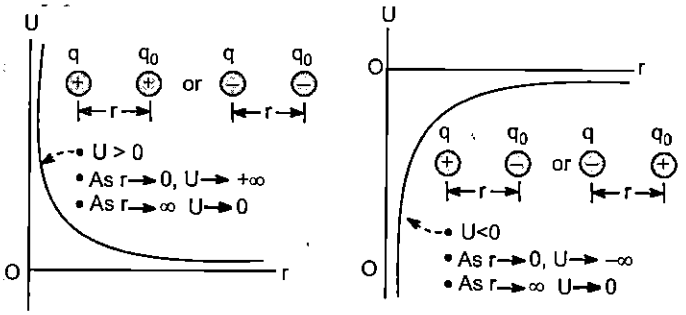
$$0V - V(r_i) = 0V - \frac{1}{4\pi\varepsilon_0} \frac{Q}{r_i}$$

or $$V(r_i) = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r_i}$$

Thus the electric potential of a point charge Q at a distance r from the charge is

$$V = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r}$$

Concepts: *The potential energy is positive if the charges q and q_0 have the same sign [Fig. 1.114(a)] and negative if they have opposite signs [Fig. 1.114(b)].*



(a) q and q_0 have the same sign. **(b)** q and q_0 have the opposite sign.

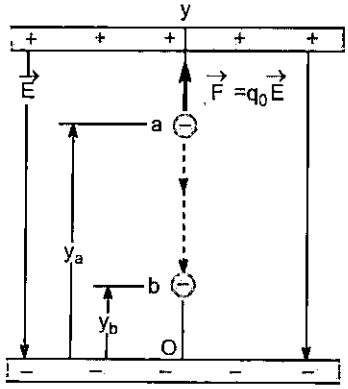
Fig. 1.114

Electric potential energy vs. electric force. *Be careful not to confuse.*

Potential energy is always defined relative to some reference point where $U = 0$. U is zero when q and q_0 are infinitely far apart and $r = \infty$. Therefore U represents the work that would be done on the test charge q_0 by the field of q if q_0 moved from an initial distance r to infinity.

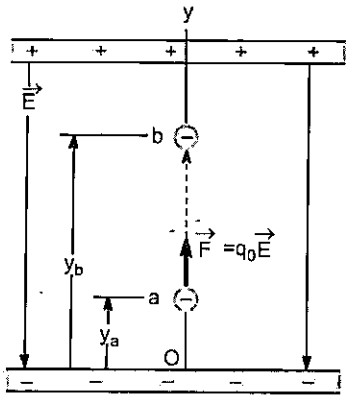
If q and q_0 have the same sign, the interaction is repulsive, this work is positive, and U is positive at any finite separation[Fig. 1.114(a)]. If the charges have opposite signs, the interaction is attractive, the work done is negative, and U is negative [Fig. 1.114(b)].

The potential energy U is a consequence of the interaction between these two bodies. If the distance between the charges is changed from r_a to r_b, the change in potential energy is the same whether q is held fixed and q_0 is moved or q_0 is held fixed and q is moved. For this reason, we never use the phrase "the electric potential energy of a point charge."



Negative charge moves in the direction of $\vec{E}$
- Field does negative work on charge
- U increases

Fig. 1.115



Negative charge moves opposite $\vec{E}$
- Field does positive work on charge
- U decreases

Fig. 116

Concept: *If you move in the direction of $\vec{\mathbf{E}}$, electric potential V decreases; if you move in the direction opposite $\vec{\mathbf{E}}$, increases.*

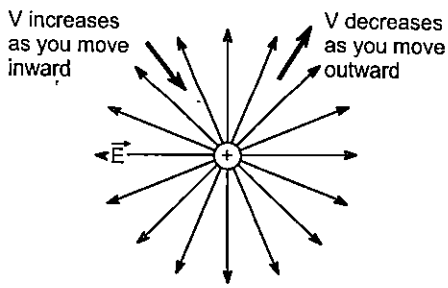


Fig. 1.117 (a) A positive point charge

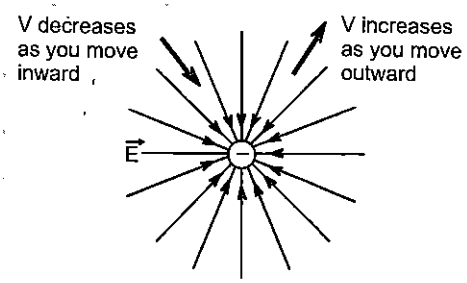


Fig. 1.117 (b) A negative point charge

Concept: *Graph of potential of a point charge*

Notice that as $r \to \infty$ m, the electric potential of the point charge Q approaches 0 V, as we chose.

The electric potential at all points surrounding a positive point charge ($Q > 0$ C) is positive. If the point charge Q is negative, the potential is negative at all points.

In the $x-y$ plane, where $r=(x^2+y^2)^{1/2}$, the electric potentials of positive and negative pointlike charges have rather spectacular graphs (see Fig. 1.118 and 1.119). These potentials approach zero for large r and diverge as $r \to 0$ m.

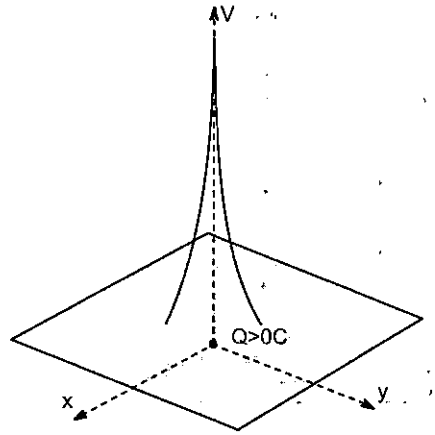


Fig. 1.118: A graph of the electric potential V of a positive point like charge at the origin in the x-y-plane.

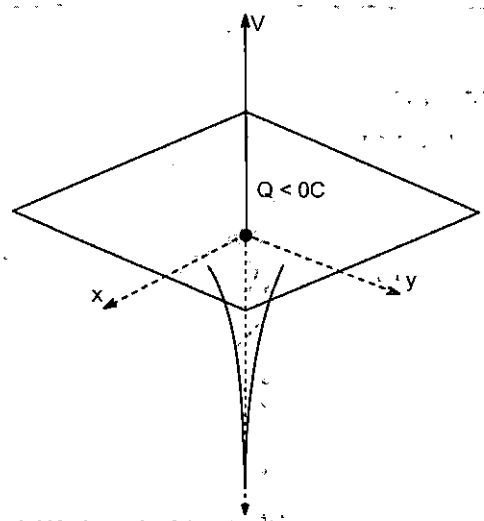


Fig. 1.119: A graph of the electric potential V of a negative point like charge at the origin in the x-y-plane.

The Electric Potential of a Collection of Point-like Charges

For a collection of point charges the total electric potential at a point P is the algebraic sum of the potentials of each charge.

Electric potential is scalar quantity, therefore it adds algebraically. Potential is positive due to a positive point charge and negative due to a negative point charge. Consequently it is certainly possible that at some point in space a positive potential due to one charge can be cancelled by a negative potential due to another charge. If a positive test charge is brought to that zero potential point from infinity, in the process no net work will be done by or against the field.

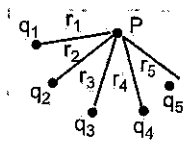


Fig. 1.120

There can be a resultant field at a point even though the potential at that point is zero, and conversely, there can be a non-zero potential at a point where the net field is zero.

$$V = \sum_{i=1}^{n} V_i = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{Q_i}{r_i}$$

The Electric Potential of Continuous Charge Distribution of Finite Size

To compute the electric potential of continuous charge distribution we break up the distribution into a large number of small charge elements dq as shown in Fig. 1.121. The contribution to the potential due to one of these differential elements is

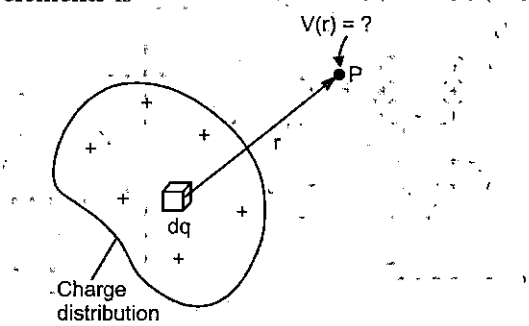


Fig. 1.121

$$dV = \frac{1}{4\pi\varepsilon_0} \frac{dq}{r}$$

where $dq = \lambda\, ds$ (for linear charge distribution)

$dq = \sigma\, dA$ (for surface charge distribution)

$dq = \rho\, dv$ (for volumetric charge distribution)

The total electric potential at point P is obtained by integrating over the entire charge distribution.

$$V = \frac{1}{4\pi\varepsilon_0} \int \frac{dq}{r} \qquad (V = 0 \text{ at } r = \infty)$$

Note that we have assumed $V = 0$ at infinity.

Concept: *To find the total electric potential at the point P, we then sum these scalar contributions from all the differential charge elements. The sum is a continuous one, however: an integration over the charge distribution, be it a line, surface, or volume:*

$$V=\frac{1}{4\pi\varepsilon_0}\int_{\text{finite charge distribution}}\frac{dQ}{r}$$

This is a scalar integration because V is a scalar quantity.

There is only one condition attached to above equation, but it is an important one: the charge distribution must be of finite size.

If the charge distribution extends off to infinity, you cannot use above equation. Above equation was formulated by considering a superposition of point-like differential charge elements. For point-like charges, the electric potential was chosen conveniently to be zero at infinity. If there is charge at infinity (as there would be for a charge distribution of infinite extent), this choice does not make sense.

If the charge distribution does extend to infinity [e.g., infinite sheets of charge, infinite linear (line) charge distributions], we must revert to the defining above equation to calculate the electric potential difference between two points.

The electric potential is the same at all points in any region in which $E=0$. Such a region is called a ***field-free region****. The value of the potential in a field-free region is determined by, (1) the choice of the reference point $\mathbf{r}_0$ at which $V=0$ and (2) the distribution of charge outside the region.*

Computing the Electric Field from the Electric Potential

The problems based on the calculation of electric potential due to a continuous charge distribution must have convinced you that it is often easier to compute the electric potential than the electric field. In both the cases in integration over a charge distribution is required. However, in the case of electric field the integration involves vectors.

We can use the eqn. $V_f - V_i = -\int_i^f \vec{\mathbf{E}}\cdot d\vec{\mathbf{r}}$ to determine the difference in potential between two points if the electric field is known. By inverting this equation we can write the electric field in terms of the potential.

$$dV = -\vec{\mathbf{E}}\cdot d\vec{\mathbf{r}} = -E_r dr$$

where dV is the potential difference between two points a distance dr apart and E_r is the component of the electric field in the direction of the infinitesimal displacement dr. Then we can write

$$E_r = -\frac{dV}{dr}$$

Therefore the component of the electric field in any direction is equal to the negative of the rate of change of the electric potential with displacement in that direction.

If the potential function V depends on all three coordinates : $V = V(x, y, z)$, the components of E can be expressed as

$$E_x = -\frac{\partial V}{\partial x};\qquad E_y = -\frac{\partial V}{\partial y};\qquad E_z = -\frac{\partial V}{\partial z}$$

Here $\frac{\partial V}{\partial x}$ is the "partial derivative" of V w.r.t. x, with y and z held constant.

The Relation Between *V* and *E* in Three Dimensions (Optional)

We begin with the equation

$$dV = -\vec{\mathbf{E}}\cdot d\vec{\mathbf{r}}$$

In general, the potential function V depends on x-, y- and z-coordinates.

$$dV = \frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}dz$$

$$\vec{\mathbf{E}} = E_x\hat{\mathbf{i}} + E_y\hat{\mathbf{j}} + E_z\hat{\mathbf{k}}$$

and $$d\vec{\mathbf{r}} = dx\,\hat{\mathbf{i}} + dy\,\hat{\mathbf{j}} + dz\,\hat{\mathbf{k}}$$

$$\vec{\mathbf{E}}\cdot d\vec{\mathbf{r}} = (E_i\,\hat{\mathbf{i}} + E_y\hat{\mathbf{j}} + E_z\hat{\mathbf{k}})\cdot(dx\,\hat{\mathbf{i}} + dy\,\hat{\mathbf{j}} + dz\,\hat{\mathbf{k}})$$
$$= E_x dx + E_y dy + E_z dz$$

Now using the expression for dV, we have

$$\frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}dz = -(E_x dx + E_y dy + E_z dz)$$

Equating the coefficients of dx, dy and dz on both sides of this equation, we have

$$E_x = -\frac{\partial V}{\partial x} \quad\Rightarrow\quad E_y = -\frac{\partial V}{\partial y}$$

$$E_z = -\frac{\partial V}{\partial z}$$

as $$\vec{\mathbf{E}} = E_x\hat{\mathbf{i}} + E_y\hat{\mathbf{j}} + E_z\hat{\mathbf{k}}$$
$$= -\left(\frac{\partial V}{\partial x}\hat{\mathbf{i}} + \frac{\partial V}{\partial y}\hat{\mathbf{j}} + \frac{\partial V}{\partial z}\hat{\mathbf{k}}\right)$$
$$= -\left(\frac{\partial}{\partial x}\hat{\mathbf{i}} + \frac{\partial}{\partial y}\hat{\mathbf{j}} + \frac{\partial}{\partial z}\hat{\mathbf{k}}\right)V(x, y, z)$$
$$= -\nabla V(x, y, z)$$

where ∇ is the *gradient operator*, or del.

Concept: *In terms of unit vectors we can write* $\vec{\mathbf{E}}$ *as*

$$\vec{\mathbf{E}} = -\left(\hat{\mathbf{i}}\frac{\partial V}{\partial x} + \hat{\mathbf{j}}\frac{\partial V}{\partial y} + \hat{\mathbf{k}}\frac{\partial V}{\partial z}\right)\quad(\vec{\mathbf{E}}\text{ in terms of }V)\qquad ...(1)$$

In vector notation the following operation is called the ***gradient*** *of the function f:*

$$\vec{\nabla} f = \left(\hat{\mathbf{i}}\frac{\partial}{\partial x} + \hat{\mathbf{j}}\frac{\partial}{\partial y} + \hat{\mathbf{k}}\frac{\partial}{\partial z}\right)f$$

The operator denoted by the symbol $\vec{\nabla}$ is called "grad" or "del." Thus in vector notation,

$$\vec{\mathbf{E}} = -\vec{\nabla} V \qquad ...(2)$$

This is read "$\vec{\mathbf{E}}$ is the negative of the gradient of V" or "$\vec{\mathbf{E}}$ equals negative grad V." The quantity $\vec{\nabla} V$ is called the potential gradient.

As at each point the direction of $\vec{\mathbf{E}}$ is the direction in which V decreases most rapidly and is always perpendicular to the equipotential surface through the point.

Equation (2) doesn't depend on the particular choice of the zero point for V. If we were to change the zero point, the effect would be to change V at every point by the same amount; the derivatives of V would be the same.

If $\vec{\mathbf{E}}$ is radial with respect to a point or an axis and r is the distance from the point or the axis, the relationship corresponding to eqn. (2) is

$$E_r = -\frac{\partial V}{\partial r} \qquad \text{(radial electric field)}$$

Often we can compute the electric field caused by a charge distribution in first calculating the potential and then taking its gradient to find the field. If we know $\vec{\mathbf{E}}$ as a function of position, we can calculate V using $dV = -\vec{\mathbf{E}}.d\vec{\mathbf{r}}$ and if we know V as a function of position, we can calculate $\vec{\mathbf{E}}$ using $\vec{\mathbf{E}} = -\vec{\nabla} V$. Deriving V from $\vec{\mathbf{E}}$ requires integration, and deriving $\vec{\mathbf{E}}$ from V requires differentiation.

The potential at a radial distance r from a point charge q is $V = q / 4\pi\varepsilon_0 r$. Find the vector electric field from this expression for V.

From eqn. $E_r = -\frac{\partial V}{\partial r}$ *we get*

$$E_r = -\frac{\partial V}{\partial r} = -\frac{\partial}{\partial r}\left(\frac{1}{4\pi\varepsilon_0}\frac{q}{r}\right) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

So the vector electric field is

$$\vec{\mathbf{E}} = \hat{\mathbf{r}} E_r = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{\mathbf{r}}$$

An alternative approach is to ignore the radial symmetry, write the radial distance as $r = \sqrt{x^2+y^2+z^2}$, and take the derivatives of V with respect to x, y, and z, we find

$$\frac{\partial V}{\partial x} = \frac{\partial}{\partial x}\left(\frac{1}{4\pi\varepsilon_0}\frac{q}{\sqrt{x^2+y^2+z^2}}\right)$$

$$= -\frac{1}{4\pi\varepsilon_0}\frac{qx}{(x^2+y^2+z^2)^{3/2}} = -\frac{qx}{4\pi\varepsilon_0 r^3}$$

and similarly

$$\frac{\partial V}{\partial y} = -\frac{qy}{4\pi\varepsilon_0 r^3}$$

$$\frac{\partial V}{\partial z} = -\frac{qz}{4\pi\varepsilon_0 r^3}$$

From eqn. (1), the electric field is

$$\vec{\mathbf{E}} = -\left[\hat{\mathbf{i}}\left(-\frac{qx}{4\pi\varepsilon_0 r^3}\right) + \hat{\mathbf{j}}\left(-\frac{qy}{4\pi\varepsilon_0 r^3}\right) + \hat{\mathbf{k}}\left(-\frac{qz}{4\pi\varepsilon_0 r^3}\right)\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\left(\frac{x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}}{r}\right) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{\mathbf{r}}$$

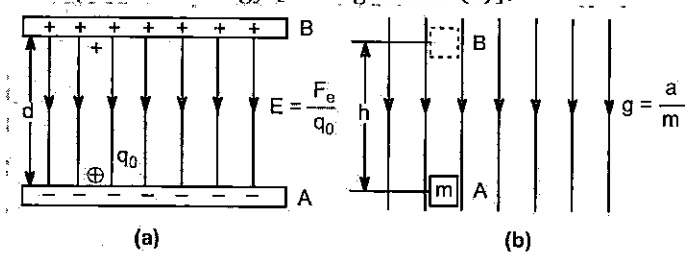
This approach gives us the same answer, but with a bit more effort. Clearly it's best exploit the symmetry of the charge distribution whenever possible.

Conservation of Energy Under the Influence of Electric Forces

The potential difference between any two points was defined as the change in potential energy per unit charge.

$$V_f - V_i = \frac{\Delta U_{\text{elec.}}}{q_0} = \frac{W_{\text{ext.}}}{q_0}$$

☛The change in electrical potential energy is also equal to the work done by an external force in moving a charge in an electric field. Moving a positive charge q_0 against the electric field requires positive work and increases the potential energy [see 1.122 (a)]. In moving a mass *m* against the gravitational field requires positive work and increases the gravitational potential energy [see Fig. 1.122 (b)].



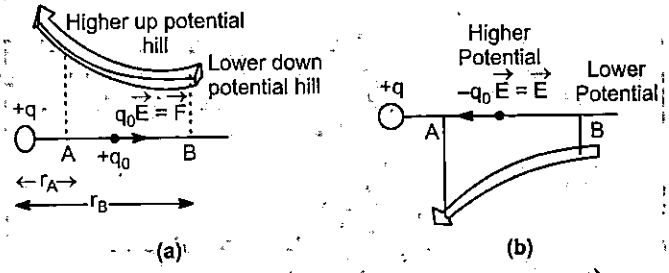
Potential energy changes in uniform electric and gravitational fields: (a) Moving a positive charge q_0 against the electric field requires positive work and increase the electric potential energy. (b) Moving a mass *m* against the gravitational field requires positive work and increases the gravitational potential energy.

Fig. 1.122

☛In the case of gravitational field we defined the zero potential energy point as the Earth's surface for convenience. However, zero point can also be defined at infinite distance from the Earth. The same is true about electrical potential energy or electrical potential.

We may take the potential as zero at the negative plate of a pair of charged parallel plates or sometimes it is more convenient to take the zero point at infinity. In any case the potential difference between two points is unaffected.

Suppose for one choice of zero point (often referred to as reference level/point) point A is at a potential of 100V and point B at 200 V. With a different zero point, point A might have potential 1000 V, but the potential at B would then be 1200 V. If A is −500 V, the potential at B would then be −400 V. The difference $V_B - V_A$ would always be + 100 V, regardless of the point where we assign the zero reference point. The concept of unique potential of a point is meaningless, only *changes in V*, which is called *voltage*, are meaningful [see Fig. 1.123 (a) and (b)]. Moving a positive test charge $+q_0$ against the electric field requires positive



The positive test charge $+q_0$ experiences a repulsive force $\vec{F}$ due to the positive point charge +q. As a result, work done by this force when the test charge moves from A to B. Consiquently, the electric potential is higher (up hill) at A and lower (down hill) at B.

Fig. 1.123

work and increases the electrical potential energy, consequently potential at point A is higher than at A. When it is released from point A the positive test charge accelerates back to point A, gaining kinetic energy at the expense of the electric potential energy.

A negative charge placed at point B will move toward the point A, gaining kinetic energy and losing electrical potential energy. Note that a negative test charge moves from a point B at lower potential to a point A at higher potential, $\Delta V = V_A - V_B > 0$.

ACCELERATION OF CHARGED PARTICLES UNDER THE INFLUENCE OF ELECTRICAL FORCES

The classical work energy (CWE) theorem provides a simple way to analyse the dynamics of such motions.

If the charged particle moves only under the influence of the conservative electrical force, the work done by non-conservative forces is zero. Thus from the CWE theorem

$$W_{\text{noncon}} = \Delta KE + \Delta PE \quad ...(1)$$

is zero, and theorem simplifies to

$$0\text{ J} = \Delta KE + \Delta PE$$

$$0\text{ J} = \Delta(KE + PE) \quad ...(2)$$

In other words, the total mechanical energy of the charged particle is conserved throughout its motion under the influence of the static electrical force.

Consider a charged particle q in space, initially at a point where it has kinetic energy KE_i and electric potential energy $PE_i = qV_i$. When the particle has moved under the influence of the electrical force to another position where the electric potential is V_f, the potential energy of the charge is $PE_f = qV_f$ and its kinetic energy is KE_f. Conservation of mechanical energy, implies that the change in the kinetic energy ΔKE is

$$0\text{ J} = \Delta KE = (qV_f - qV_i)$$

$$0\text{ J} = \Delta KE + q\Delta V$$

or $$\Delta KE = -q\Delta V \quad ...(3)$$

Equation (3) indicates that a positive charge in an electric field, moving under the influence of only the electrical force, will *increase* its kinetic energy (while decreasing its electric potential energy) if it moves to regions of *lower* electric potential (since then $V_f < V_i$ and $\Delta V < 0$V, and so $\Delta KE > 0$ J). The total mechanical energy is constant according to the CWE theorem. Correspondingly, a *negative* charge moves to regions of higher electric potential to increase its kinetic energy (and decrease its electric potential energy) (since then $V_f > V_i$ and $\Delta V > 0$V but $q < 0$C, giving $\Delta KE > 0$J).

Suppose an electron enters a region with a uniform field between two parallel uniformly charged plates. The electric field is directed from a high potential region to a low potential region. The electric field is thus antiparallel to the initial direction of velocity. The electric force is given by

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}}$$

For a negatively charged electron ($-e$) the electric force is antiparallel to the electric field. We can expect that the electron will gain speed and emerge with a higher velocity. From conservation of energy, We have

$$\text{KE}_f + \text{PE}_f = \text{KE}_i + \text{PE}_i$$

$$\frac{1}{2}mv_f^2 + qV_f = \frac{1}{2}mv_i^2 + qV_i$$

or $$\frac{1}{2}mv_f^2 = \frac{1}{2}mv_i^2 + q(V_i - V_f)$$

$$= \frac{1}{2}mv_i^2 + (-e)(V_i - V_f)$$

$$= \frac{1}{2}mv_i^2 + e(V_f - V_i)$$

We may also apply CWE theorem

$$W_{\text{non-cons.}} = \Delta\text{KE} + \Delta\text{PE}$$

The electrical force is conservative; hence

$$0 = \Delta\text{KE} + (qV_f - qV_i) \quad \text{or} \quad \Delta\text{KE} = -q\Delta V$$

or $$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = -(-e)(V_f - V_i)$$

or $$\frac{mv_f^2}{2} = \frac{mv_i^2}{2} + e(V_f - V_i)$$

☛ If an electron accelerates through a potential difference of exactly one volt, the change in kinetic energy in joule is

$$\Delta\text{KE} = -q\Delta V$$

$$= -(-1.602\times10^{-19})\ (1)$$

$$= 1.602\times10^{-19}\text{ J}$$

By definition **one electron volt** is the change in potential energy (equal to gain in kinetic energy) when the electron moves through a potential difference of 1 volt.

$$1 \text{ eV} = 1.602 \times 10^{-19} \text{J}$$

If a particle has charge $q = ne$ where n is an integer.

$$\Delta \text{KE} = -q\,\Delta V = -ne\,\Delta V$$

$$\Delta \text{KE}_{\text{in eV}} = -n\,\Delta V$$

If a proton and an electron ($n = \pm 1$) are accelerated by electrical forces alone between two points, the absolute value of their kinetic energy expressed in eV is numerical equal to the absolute value of the difference in electric potential between the two points.

$|\Delta \text{KE}_{\text{in eV}}| = |\Delta V|$ for electron and proton.

The Potential Energy of a Collection of Charges:

If a positive point charge q_1 is fixed in space and a second positive charge q_2 is brought near if from a very large distance ($r = \infty$) to a distance r_{12}, the work done is positive. The system of two charges gains electrical potential energy

$$\Delta U = q_2 \Delta V$$

$$= q_2\left(\frac{q_1}{4\pi\varepsilon_0 r_{12}} - \frac{q_1}{4\pi\varepsilon_0(\infty)}\right)$$

$$= \frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}}$$

The electric potential caused by q_1 and q_2 where a third charge is to be placed is

$$V' = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_{13}} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_{23}}$$

If q_3 is now brought in from infinity in the presence of other two charges, it has potential energy

$$\Delta U = q_3 V'$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{r_{13}} + \frac{1}{4\pi\varepsilon_0}\frac{q_2 q_3}{r_{23}}$$

So the total energy of the three charges in place is

$$U_{\text{total}} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}} + \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{r_{13}} + \frac{1}{4\pi\varepsilon_0}\frac{q_2 q_3}{r_{23}}$$

For a configuration of many charges

$$U_{\text{total}} = U_{12} + U_{23} + U_{13} + \ldots$$

If the total final energy of such a collection of charges is negative, electrical forces did positive work in assembling the charges and according to CWE we will need to do positive work to separate them to infinite distance apart with zero change in their kinetic energy.

Equipotential Volumes and Surfaces

The locus of points or regions for which the electric potential has a constant value are called **equipotential regions** or **equipotentials.** Such equipotentials can be surfaces, volumes or lines.

The potential difference between two points separated by differential distance $d\vec{r}$ is

$$dV = -\vec{E}\cdot d\vec{r}.$$

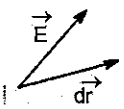


Fig. 1.124

Because the potential is constant on an equipotential surface the change in V when a test charge is moved parallel to surface is , $dV = -\vec{E}\cdot d\vec{l} = 0$. As $\vec{E}\cdot d\vec{l} = 0$, $\vec{E}$ must be perpendicular to $d\vec{l}$. Thus the electric field lines emanating from an equipotential surface must be perpendicular to the surface.

Fig. 1.125 (a) shows the equipotential surfaces that surround a point charge or a uniformly charged spherical conductor. The radially directed electric

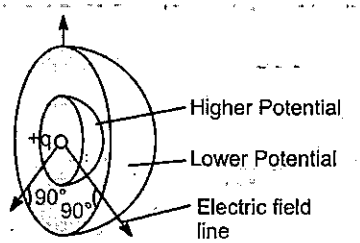


Fig. 1.125 (a)

field of a point charge is perpendicular to the equipotential surface and points in the direction of decreasing potential. Fig. 1.125 (b) shows equipotenital surfaces and electric field lines outside a nonspherical conductor. The electric field lines are always perpendicular to the equipotential surfaces.

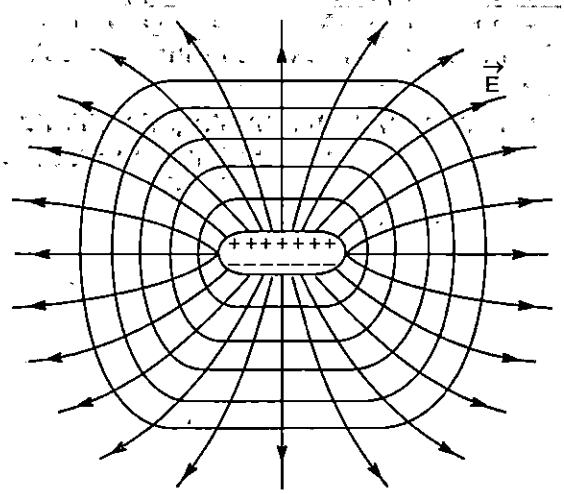


Fig. 1.125 (b)

For a given electric field pattern, to sketch

equipotentials, pick a point in the region of interest and move so that your path is always perpendicular to the next field line. Keep your curve as smooth as possible because sharp kinks and loops are not allowed. To draw a higher potential value, move opposite the direction of the electric field and repeat it process. In Fig. 1.125 (c) shown,

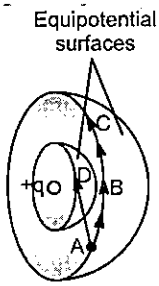
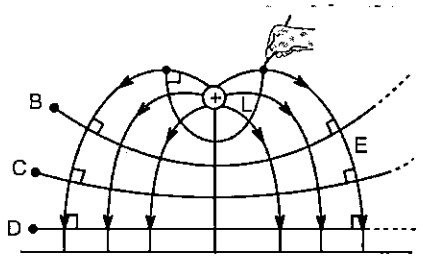


Fig. 1.125 (c)

$$V_A = V_B = V_C \quad \text{and} \quad V_A < V_D$$

☛In order to sketch an electric field from equipotentials start at a convenient point and trace a curve that crosses each equipotential at a right angle. Repeat the process number of times to reveal the field pattern. Do not forget to add arrows to field lines in the sense from high to low potential.



(a)

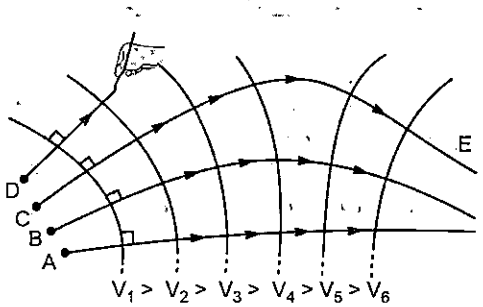


Fig. 1.126 (b)

☛Fig. 1.127 shows a number of equipotentials. The approximate magnitude of electric field is

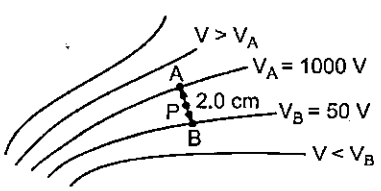


Fig. 1.127

$$|E| = \frac{\Delta V}{\Delta x} = \frac{1000 - 950}{2.0 \times 10^{-2}}$$

$$= 2.5 \times 10^3 \text{ V/m}.$$

The electric field points in the direction from A and B (from higher potential to lower).

☛From the definition of potential difference, we have

$$V_B - V_A = \frac{(w_{AB})_{\text{external agent}}}{q_0}$$

In Fig. 1.129 (c) shown, points A, B and C lie on the same equipotential surface, so $V_A = V_B = V_C$. The electric force does no work as a charge moves on a path that lies on an equipotential, $w_{ABC} = 0$. However work is done by the electric force when a charge moves from one equipotential to other, as long the path AD.

EQUIPOTENTIALS AND ELECTRIC FIELD LINES

The relation between electric potential and electric field is

$$dV = -\vec{\mathbf{E}}(r).\, dr$$

If, dr is parallel to E the change in potential per unit distance will have its maximum value. This happens when dr lies along the electric field line. Indeed, the field line is made up of an infinite number of infinitesimal displacements of this kind. The field line is thus the "*path of steepest descent.*" (Here "descent' implies decrease in V.)

If we choose dr perpendicular to E. In that case, the dot product is equal to zero and there is *no* change of potential over the displacement; $dV = 0$. As Fig. 1.128. Shows, the directions satisfying this requirement define the plane that is

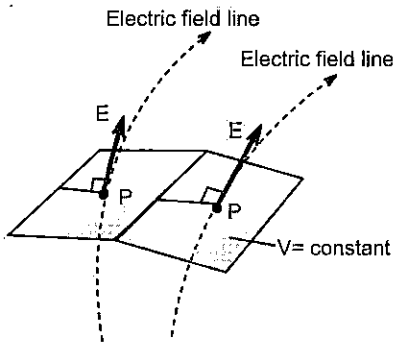


Fig. 1.128

normal to E at P. The infinitesimal area element centerd on E is a part of this plane. Because any displacement over the element leads to zero change in potential, *the potential is constant over this element.*

Adjacent to this area element and sharing a common edge with it is another infinitesimal area element. The same argument can be made with respect to the field line that passes through this second element. By combining together many such adjoining elements, we can construct a finite curved surface over which the potential is constant. This surface is called an **equipotential surface** or simply an **equipotential.**

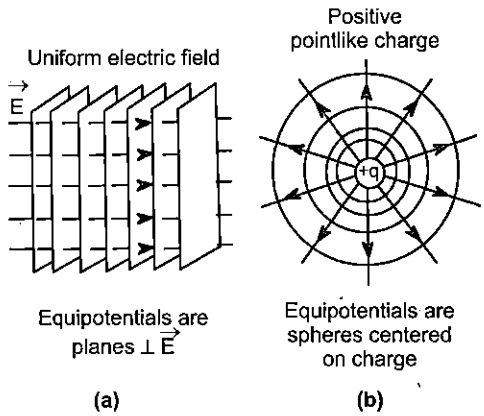
Like electric field lines, equipotential surfaces obey:

Concept 1. *Two equipotential surfaces on which the potentials are different cannot touch or cross one another.*

2. *Either a closed equipotential surface contains charge or else the electric field is zero everywhere in the space enclosed by the surface.*

In the absence of charge within the closed surface, all field lines within it must originate and terminate outside it. If the electric field within the surface is not zero everywhere, such field lines must exist. Imagine a field line that penetrates the surface inward at one point and outward at another. Because $E\,dr$ has the same sign everywhere along this line, integrating along this line from the entrance point to the exit point will yield a nonzero potential difference between the two points, which contradicts the statement that the surface is an equipotential.

Concept: *Equipotential surfaces need not be physical surfaces (such as that of a conductor). Any imaginary surface over which the electric field is everywhere perpendicular to it is called an equipotential surface. Various equipotential surfaces along with the associated electric fields are shown in (Fig.1.129) for a number of different charge distributions. The surfaces are chosen and drawn so that the potential difference between successive surfaces is a constant value.*



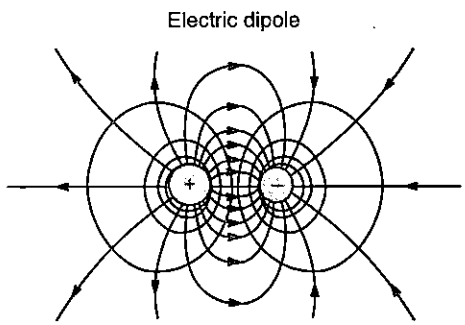


Fig. 1.129

Any equipotential volume is enclosed within a surface that iteslf must be an equipotential surface. The converse, however, is not true: an equipotential surface need not surround an equipotential volume.

Concept: *In equilibrium, the surface of a conductor of arbitrary shape is an equipotential. Suppose there is a potential difference between any two points on the surface. Then an electric field must exist between them. This field will drive a flow of free charge until the field decreases to zero and the surface becomes an equipotential. The electric field just outside a charged conducting body is directed normal to the surface. Because the surface is everywhere normal to the field, the surface must conincide with an equipotential. In equilibrium, the electric field with in a conducting body of arbitrary shape is everywhere zero.*

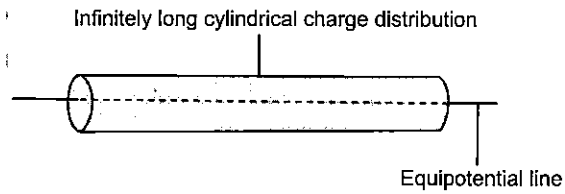


Fig. 1.130 An equipotential line exists along the axis of a cylindrical charge distribution.

Since a conductor is an equipotential volume, the surface of a conductor is an equipotential surface and, therefore, any electric field at the surface of the conductor must be perpendicular to the surface for any static distributions of charge.

The only charge distribution that can produce a true equipotential line in space is a uniformly charged infinite cylindrical shell or cylindrical volume, as shown in Fig. (1.130). An equipotential line exists along the axis of the cylinder; in this situation, the electric field is zero along the axis as well.

Concept: *On an equipotential surface, if we move in any direction on the surface (see Fig. 1.131) the value of the potential is unchanged: That is,*

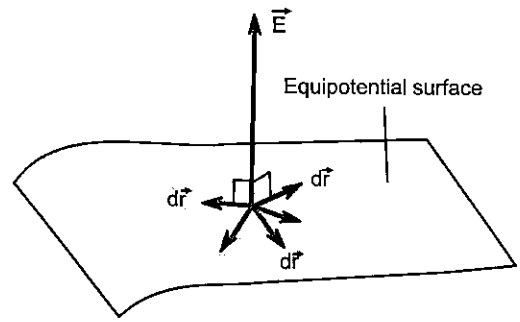


Fig. 1.131 If you move in any direction on an equipotential surface, the potential is unchanged. The electric field must be perpendicular to an equipotential surface

$$dV = -\vec{\mathbf{E}} \cdot d\vec{\mathbf{r}}$$

$$0\,\mathrm{V} = -\vec{\mathbf{E}} \cdot d\vec{\mathbf{r}}$$

For any $d\vec{\mathbf{r}}$ confined to the surface. Therefore any electric field $\vec{\mathbf{E}}$ must have a perpendicular equipotential surface associated with it at every point. The surface of a conductor in electrostatics (i.e., with electric charges at fixed locations on its surface) exemplifies an equipotential surface.

Equipotential surfaces are closely spaced where electric field intensity is large and widely spaced where electric field intensity is small.

Concept:1. *Inside an equipotential volume, V is constant, hence change in potential is zero,*

i.e., $dV = -\vec{\mathbf{E}} \cdot d\vec{\mathbf{r}} = 0$

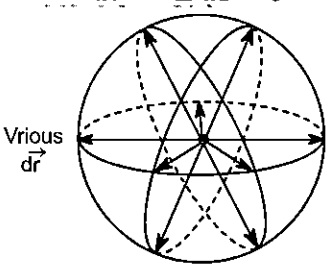


Fig. 1.132

regardless of the direction of the differential change in the position vector $d\vec{\mathbf{r}}$. The only way $\vec{\mathbf{E}} \cdot d\vec{\mathbf{r}}$ can be zero for every $d\vec{\mathbf{r}}$ is that $\vec{\mathbf{E}}$ itself has zero value. Hence ***the electric field must be zero within an equipotential volume.***

2. *Since $\vec{\mathbf{E}} = 0$ inside a conductor, this means that the inside of a conductor is an equipotential volume.The same is true if the conductor is hollow and there is no charge at all inside the hollow (situation is different where the total charge sums to zero).*

The electric field free region can be created by surrounding the region with a conductor, without charge within the region. Inside such metal cavities there is no electric field. Such a procedure is called ***electrostatic shielding.*** *Sensitive electronic instruments are shielded from external electrical influence by enclosing them in metal boxes.*

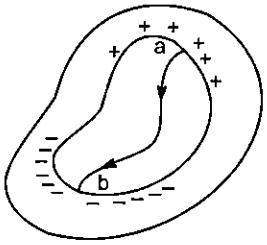


Fig. 1.133

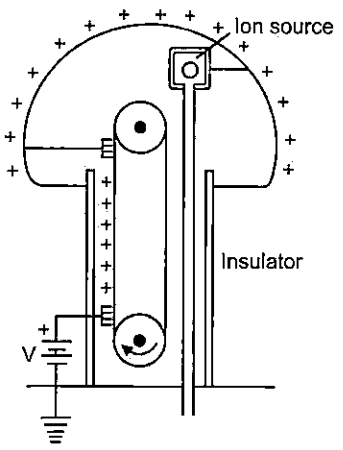
3. The charge density σ is zero at each point on the cavity, surface, if there are no charged objects inside the cavity.

Consider an arbitrary field line that originates at a positive charge on the cavity surface. If the path of the line integral, $\vec{\mathbf{E}} \cdot d\vec{\mathbf{r}}$, is taken along the field line, then the line integral cannot be zero because $E > 0$ and E is parallel to dl at each point on the path. This predicts that $V_b \neq V_a$ inside the cavity. As $V_b = V_a$, both points are on the conductor's surface. Hence the prediction that $V_b \neq V_a$, which is based on the supposition that $E \neq 0$ inside the cavity and that $\sigma \neq 0$ on the cavity surface is false. Thus we conclude that $E = 0$ and $\sigma = 0$ at each point on the cavity surface.

THE VAN DE GRAAFF GENERATOR

The Van de Graaff generator is a device that produces intense electric field (building up high voltages of a few million volt).

Consider a small conductor carrying a positive charge q kept inside the cavity of a large conductor. The electric field lines that leave the positive charge q must end on the inner surface of the large conductor irrespective of charge on the outer surface. The potential on the inner conductor must be higher because electric field lines begin from it and end on the larger conductor. If the two conductors are now connected by a conducting wire, all the charges originally on the smaller conductor will flow to the larger one. The positive charge transferred to the larger sphere resides completely on the outside surface of the larger conductor. This process can be repeated indefinitely to produce large potential on the outer conductor.



A Van de Graaff generator

Fig. 1.134

Consider a shell of radius R and charge Q enclosing a smaller sphere of radius r and charge q. The potential of the two spheres are

$$V(R) = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{R} + \frac{q}{R}\right)$$

and

$$V(r) = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{R} + \frac{q}{r}\right)$$

The potential difference between the two inner and outer spheres is

$$V(r) - V(R) = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r} - \frac{1}{R}\right]$$

Thus for positive q, whatever be the magnitude and sign of Q, the small sphere is at a higher potential than the shell. When an electric contact is provided the charge would flow from the small sphere to the shell.

Lightning Rods

- Suppose two spherical conductors of radii R_1 and R_2 are connected by a long conducting wire. Due to the conducting wire the system is electrically a single conductor. Since the surface of a conductor is equipotential, both the conductors must have same potential. The charge will flow from one to the other until their potentials are equal.

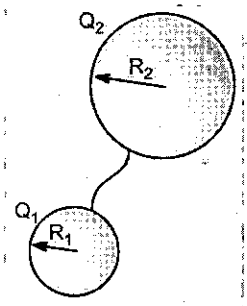


Fig. 1.135

Since the two spheres are separated by a large distance, the charge distribution on them is essentially uniform and spherically symmetric. The equality of potentials implies that

$$\frac{1}{4\pi\varepsilon_0}\frac{Q_1}{R_1} = \frac{1}{4\pi\varepsilon_0}\frac{Q_2}{R_2}$$

or $$\frac{Q_1}{Q_2} = \frac{R_1}{R_2} \quad ...(1)$$

We infer that $Q \propto R$, the charge on each sphere in equilibrium is directly proportional to the radius, *i.e.*, the larger sphere shares the larger fraction of charge.

- For a uniform surface charge density $\sigma C/m^2$, the total charge $Q = 4\pi R^2\sigma$, so eqn. (1) becomes

$$\frac{\sigma_1}{\sigma_2} = \frac{R_2}{R_1} \quad ...(2)$$

So we infer that $\sigma \propto \frac{1}{R}$; the surface charge density on each sphere is inversely proportional to the radius.Now we can make a qualitative statement regarding charge distribution on a conductor of irregular shape. *The regions with smallest radii of curvature have the greatest surface charge densities.*

- Close to surface of a conductor the field strength is $E = \sigma/\varepsilon_0$. From eqn. (2) we infer that the field is greatest at the sharp points of the conductor. This is the basic principle of working of a lightning rod. Charged clouds in the vicinity attract or repel electrons in the conducting rod. Due to sharp tip of the rod the electric field near it will easily reach the critical magnitude of 3×10^6 N/C at which air ceases to be insulator and becomes a conductor. So an electrical discharge initiates near the tip of the rod rather than near other objects such as a building structure or a tall tree.

The insulating medium, such as air, that surrounds the conductor has a property **dielectric strength.** It is the maximum magnitude E_{max} of the electric field that can exist in the material without electrical breakdown. When an insulator breaks down, its constituent molecules ionize and the material begins to conduct. In a gas such as air, visible light is emitted (the gas glows) as the electrons recombine with the ionized molecules, a phenomenon called a **corona discharge.**

Example 63

The radius of the dome of a Van de Graaff generator is $r = 0.13$ m. Assume that the maximum magnitudes for the charge (Q_{max}) and potential (V_{max}) of the dome are determined by the electric strength of the surrounding air ($E_{max} = 3\times10^6$ V/m). Find Q_{max} and V_{max}.

Solution: Considering the dome as an isolated spherical conductor, we have

$$E = \frac{|Q|}{4\pi\varepsilon_0 r^2} = \frac{|V|}{r}$$

Therefore the maximum charge Q_{max} and the maximum potential V_{max} for a conducting sphere surrounding by an insulator with dielectric strength E_{max} are

$$Q_{max} = E_{max}4\pi\varepsilon_0 r^2 = \frac{(3\times10^6)(0.13)^2}{9\times10^9} = 6\mu C$$

and $$V_{max} = E_{max}r = (3\times10^6)(0.13) \approx 400 \text{ kV}$$

Example 64

Find the electric potential at a point P located a distance z along the axis of a uniformly charged ring of radius R and charge Q. Draw the graphs of V as a function of z, for positive Q as well as negative Q.

Solution: We consider a differential element dQ on the ring; it is at a distance r from point P, where $r = (z^2 + R^2)^{1/2}$.

All the elements on the ring are at the same distance from the point P, and the potential is scalar; we can sum it over the ring of charge.

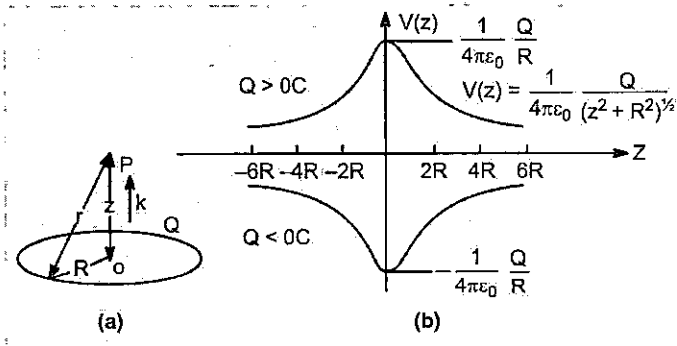


Fig. 1E.64

$$V = \frac{1}{4\pi\varepsilon_0}\int\frac{dQ}{r}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Q}{(z^2+R^2)^{1/2}}$$

For positive charge ($Q > 0$), $V > 0$ for all z; if $Q < 0$, then $V < 0$ for all z.

Example 65

Find the electric potential at a point P located on the axis of a uniformly charged circular disc of radius R and charge Q at a distance z from the plane of the disc. Draw the graph of V as a function of z, for positive as well as negative Q.

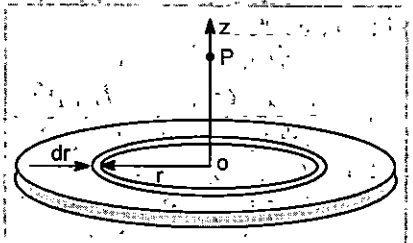


Fig. 1E.65(a)

Solution: Surface charge density of disc,

$$\sigma = \frac{Q}{\pi R^2}$$

We consider a differential ring of radius r and thickness dr on the disc.

Area of differential ring $= 2\pi r\,dr$.

The charge dQ on the differential ring,

$$dQ = \sigma(2\pi r)dr$$

Potential due to differential ring at point P,

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dQ}{(z^2+r^2)^{1/2}}$$

Now we substitute for dQ and integrate the expression for dV over the entire disc.

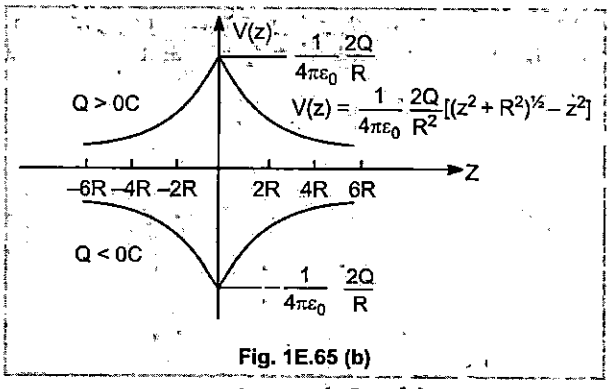


Fig. 1E.65 (b)

$$V = \frac{1}{4\pi\varepsilon_0}\int_0^R \frac{(\sigma 2\pi r\,dr)}{(z^2+r^2)^{1/2}}$$

$$= \frac{1}{4\pi\varepsilon_0}2\pi r[(z^2+R^2)^{1/2} - z]$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{2Q}{R^2}[(z^2+R^2)^{1/2} - z]$$

Example 66

Determine the electric field at point P on the axis of (a) a circular ring of charge, (b) a uniformly charged disc, from the expression for electric potential.

Solution: (a) Potential on the axis of ring is given by the expression

$$V(x) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{(x^2+R^2)^{1/2}}$$

$$E_x = -\frac{\partial V}{\partial x}$$

$$= -\frac{Q}{4\pi\varepsilon_0}\left(-\frac{1}{2}\right)(x^2+R^2)^{-3/2}(2x)$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Qx}{(x^2+R^2)^{3/2}}$$

$$E_y = 0; \quad E_z = 0$$

(b) The electric potential on the axis of the disc is given by the expression

$$V(x) = \frac{\sigma}{2\varepsilon_0}(\sqrt{x^2+R^2} - x)$$

$$E(x) = -\frac{\partial V}{\partial x}$$

$$= \frac{d}{dx}\left[\frac{\sigma}{2\varepsilon_0}(\sqrt{x^2+R^2} - x)\right]$$

$$= \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{x}{\sqrt{x^2+R^2}}\right)$$

Example 67

(a) Find the electric potential at a point P located a distance r from a sphere of radius R that has a total charge Q distributed uniformly throughout its volume. Consider the cases :
(i) $r > R$; (ii) $r = R$; (iii) $r < R$
(b) If the spherical charge distribution is on a conducting sphere, what is the potential within the sphere in the three cases :
(i) $r > R$; (ii) $r = R$; (iii) $r < R$
(c) Draw a graph of V as a function or r

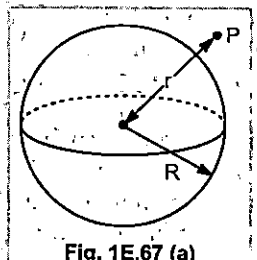


Fig. 1E.67 (a)

Solution: (a) Electric field due to a spherical charge distribution is given by

$$\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{\mathbf{r}} \qquad [\text{for } r > R]$$

Here $\hat{\mathbf{r}}$ is a unit vector in radial direction. We can find the electric potential at r from the relation

$$V(r_f) - V(r_i) = -\int_i^f \vec{\mathbf{E}}\cdot d\vec{\mathbf{r}}$$

$$V(r) - V(\infty) = -\frac{1}{4\pi\varepsilon_0} Q\int_\infty^r \frac{\hat{\mathbf{r}}}{r^2}\cdot dr\,\hat{\mathbf{r}}$$

$$= -\frac{1}{4\pi\varepsilon_0} Q\left(-\frac{1}{r}\right)_\infty^r = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

Conventionally, zero potential is chosen to be $r = \infty$; so $V(\infty) = 0$.

$$V(r) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

The potential on the surface can be found by setting $r = R$.

$$V(R) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$$

- Note that the expression for $V(r)$ is same as the potential of a point charge Q located at the centre of the sphere. So for distance $r \geq R$ the spherically symmetric charge distribution behaves as if its total charge is concentrated at the centre of the sphere.
- This result holds for a uniformly charged conducting sphere (charge of a conductor resides on the outer surface of a conductor) or an insulating with uniform charge distribution either on its surface or distributed volumetrically, as long as the charge distribution is spherically symmetric.

(b) The electric field inside a nonconducting spherical charge distribution is given by the expression :

$$\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^3} r\,\hat{\mathbf{r}}$$

$$V(r_f) - V(r_i) = -\int_i^f \vec{\mathbf{E}}\cdot d\vec{\mathbf{r}}$$

$$V(r) - V(R) = -\frac{1}{4\pi\varepsilon_0}\frac{Q}{R^3}\int_R^r r\hat{\mathbf{r}}\cdot dr\,\hat{\mathbf{r}}$$

$$= -\frac{1}{4\pi\varepsilon_0}\frac{Q}{R^3}\int_R^r r\,dr$$

$$= -\frac{1}{4\pi\varepsilon_0}\frac{Q}{R^3}\left(\frac{r^2}{2}\right)_R^r$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^3}\left(\frac{R^2}{2} - \frac{r^2}{2}\right)$$

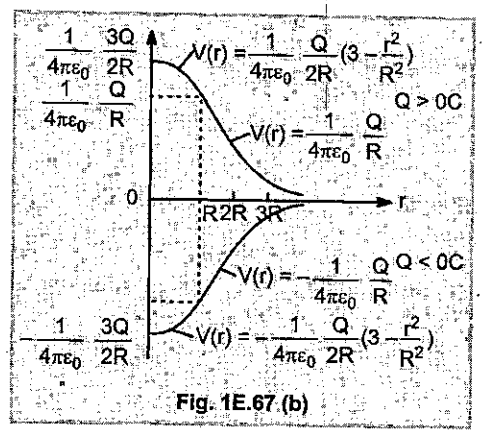


Fig. 1E.67 (b)

since $V(R) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$

$$V(r) - \frac{1}{4\pi\varepsilon_0}\frac{Q}{R} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^3}\left(\frac{R^2}{2} - \frac{r^2}{2}\right)$$

$$V(r) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{2R}\left(3 - \frac{r^2}{R^2}\right) \text{ (for } r < R)$$

- Note that at $r = 0$, at the centre of spherical insulator.

$$V(0) = \frac{1}{4\pi\varepsilon_0}\frac{3Q}{2R}$$

- At $r = R$, at the surface of insulator

$$V(R) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$$

(c) The static charge on a conductor resides entirely on its surface whether it is a hollow or solid conductor. The potential is constant within the sphere and equal to the value of the potential on its surface.

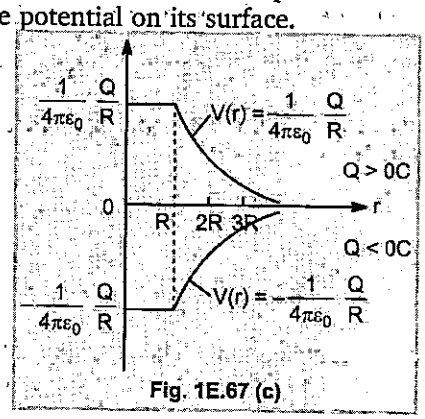


Fig. 1E.67 (c)

$$V(r) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R} \quad (r < R)$$

For outside points the potential function is similar to a spherical insulator.

Example 68

Find the potential at a distance r from a very long line of charge with linear charge density (charge per unit length) λ.

Solution: The electric field at a distance r from a long straightline charge (Fig.1E.68) has only a radial component, given by

$$E_r = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{r}$$

Since the field has only a radial component, the scalar product $\vec{\mathbf{E}}\cdot d\vec{\mathbf{l}}$ is equal to $E_r dr$. Hence the potential of any point a with respect to any other point b, at radial distances r_a and r_b from the line of charge, is

$$V_a - V_b = \int_a^b \vec{\mathbf{E}}\cdot d\vec{\mathbf{l}}$$

$$= \int E_r dr = \frac{\lambda}{2\pi\varepsilon_0}\int_{r_a}^{r_b}\frac{dr}{r}$$

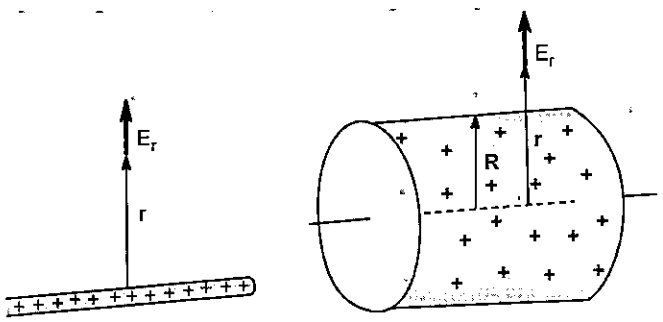
$$= \frac{\lambda}{2\pi\varepsilon_0}\ln\frac{r_b}{r_a}$$

If we take point b at infinity and $V_b = 0$, we find that V_a is infinite

$$V_a = \frac{\lambda}{2\pi\varepsilon_0}\ln\frac{\infty}{r_a} = \infty$$

This shows that if we try to define V to be zero at infinity, then V must be infinite at *any* finite distance from the line charge. This is *not* a useful way to define V for this problem! The difficulty is that the charge distribution itself extends to infinity.

To get around this difficulty, remember that we can define V to be zero at any point we set $V_b = 0$ at point b at an



(a) (b)

Fig. 1E.68

arbitrary radial distance r_0. Then the potential $V = V_a$ at point a at a radial distance r is given by $V - 0 = (\lambda / 2\pi\varepsilon_0)\ln(r_0 / r)$, or

$$V = \frac{\lambda}{2\pi\varepsilon_0}\ln\frac{r_0}{r}$$

The expression for E_r also applies outside a long charged conducting cylinder with charge per unit length λ [Fig. 1E.68 (b)]. Hence our result also gives the potential for such a cylinder, but only for values of r equal to or greater than the radius R of the cylinder. If we choose r_0 to be the cylinder radius R, so that $V = 0$ when $r = R$, then at any point for which $r > R$,

$$V = \frac{\lambda}{2\pi\varepsilon_0}\ln\frac{R}{r}$$

Inside the cylinder, $\vec{\mathbf{E}} = \mathbf{0}$, and V has same value (zero) as on the cylinder's surface.

Example 69

A linear charge distribution, linear charge density λ lies along a straight line of length L along x-axis. Compute the electric potential at a point along the line located at a distance d from one end of the distribution.

Solution: We consider the left end of the charge distribution to be at the origin [Fig. 1E.69 (a)]. Consider a small element dx on the line of charge at a distance x from origin. Charge on it is $dq = \lambda dx$; it is at a distance $r = d + L - x$ from point P. We wish to determine the electric potential at point P by using by equation

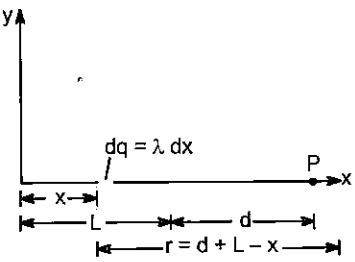


Fig. 1E.69 (a)

$$V = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r} \qquad \ldots(1)$$

therefore, $$V = \frac{\lambda}{4\pi\varepsilon_0}\int_0^L \frac{dx}{(d+L)-x}$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\{-\ln[(d+L)-x]\}_0^L$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\ln\left(\frac{d+L}{d}\right) \qquad \ldots(2)$$

Suppose the rod of length L and linear charge density λ is kept as shown in the figure [Fig. 1E.69 (b)]. The point P where the electric potential is to be determined lies on the y-axis.

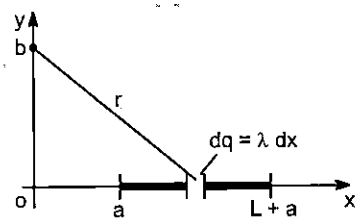


Fig. 1E.69 (b)

$$V = \int_a^{L+a}\frac{\lambda dx}{4\pi\varepsilon_0\sqrt{b^2+x^2}} \qquad \ldots(3)$$

Note the limits of the integral; when x is at one end of the rod, $x = a$; at the other end, $x = L + a$.

We will use the integral result listed in tables.

$$\int\frac{dx}{\sqrt{b^2+x^2}} = \ln(x + \sqrt{b^2+x^2})$$

Using this to evaluate V, we obtain

$$V = \frac{\lambda}{4\pi\varepsilon_0} \ln\left[\frac{(L+a)+\sqrt{b^2+(L+a)^2}}{a+\sqrt{b^2+a^2}}\right] \quad ...(4)$$

☛When $b = 0$, $L << a$.

From eqn. (4)

$$V = \frac{\lambda}{4\pi\varepsilon_0} \ln\frac{2(L+a)}{2a}$$

$$= \frac{\lambda}{4\pi\varepsilon_0} \ln\left(1+\frac{L}{a}\right)$$

Now $\frac{L}{a} << 1$, and so we can use the expansion

$$\ln(1+z) = z - \frac{z^2}{2} + \frac{z^2}{3} - ... = z$$

For small z, then we have

$$V = \frac{\lambda L}{4\pi\varepsilon_0 a}.$$

But $\lambda L = Q$, the total charge on the rod, and a is the distance of the point P from the far end of the rod. At such large distance the rod appears to be a point charge.

We have $$V = \frac{Q}{4\pi\varepsilon_0 a}$$

If $$a = 0, L << b.$$

This is the same case as previous one with the result

$$V = \frac{Q}{4\pi\varepsilon_0 b}$$

Potential at a Point on the Edge (Rim) of a Uniformly Charged Disc

We have to calculate the potential at the point P, let us consider an arc of radius 'r' and thickness dr. Disc can be considered as made of large number of elemental arcs which are coplanar and concentric and its common centre is at P. It makes an angle 2α at the point P. Charge on this elemental arc is given by,

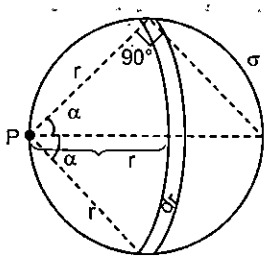


Fig. 1.136

$$dq = 2r\alpha\, dr\, \sigma$$

where σ = charge density

Potential 'dV' at the point P due to this elemental arc;

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\frac{2r\sigma\alpha\, dr}{r}$$

$$dV = \frac{1}{4\pi\varepsilon_0} 2\alpha\sigma\, dr = \left(\frac{\sigma\alpha}{2\pi\varepsilon_0}\right) dr$$

From the figure $r = 2a\cos\alpha$

(a is the radius of disc)

$$dr = -2a\sin\alpha\, d\alpha$$

Hence total potential due to the entire disc is given by;

$$V = \frac{\sigma}{2\pi\varepsilon_0}\int_{\pi/2}^{0} -2a\,\alpha\sin\alpha\, d\alpha$$

$$= \frac{\sigma a}{\pi\varepsilon_0}$$

Comparing it with the potential at the centre of the disc $\left(V = \frac{\sigma a}{2\varepsilon_0}\right)$, we find that potential falls off from centre to the edge of the disc.

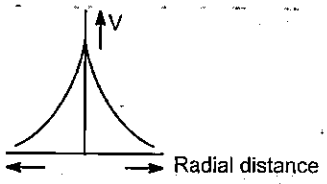


Fig. 1.137

Potential Due to a Uniformly Charged Wire

Since the formula due to a point charge can not be directly applied, for whole of the wire, hence let us break the wire into very small elements (point charges) one such element is of length dy and is at a distance 'y' from the reference point O.

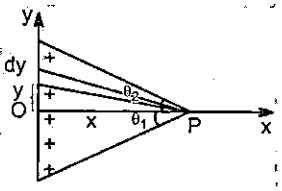


Fig. 1.138

Potential due to the small element;

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{\sqrt{(y^2+x^2)}}$$

$$\Rightarrow \quad dV = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dy}{\sqrt{(y^2+x^2)}} \quad (\text{where } dq = \lambda dy)$$

From the figure;

$$\tan\theta = \frac{y}{x}$$

or $$y = x\tan\theta$$

or $$dy = x\sec^2\theta\, d\theta$$

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{\lambda x\sec^2\theta\, d\theta}{x\sec\theta}$$

$$= \frac{1}{4\pi\varepsilon_0}\lambda\sec\theta\, d\theta$$

or $$V = \frac{\lambda}{4\pi\varepsilon_0}\int_{-\theta_1}^{\theta_2}\sec\theta\, d\theta$$

$$= \frac{\lambda}{4\pi\varepsilon_0}[\log_e(\sec\theta+\tan\theta)]_{-\theta_1}^{\theta_2}$$

or $$V = \frac{\lambda}{4\pi\varepsilon_0}\left[\log_e\left(\frac{\sec\theta_2+\tan\theta_2}{\sec\theta_1-\tan\theta_1}\right)\right]$$

Example 70

Two point charges Q_1 and Q_2 lie along a line, at a distance from each other. Figure shows the potential variation along the line of charges. At which points 1, 2 and 3 is the electric field zero? What are the signs of the charges Q_1 and Q_2 and which of the two charges is greater in magnitude?

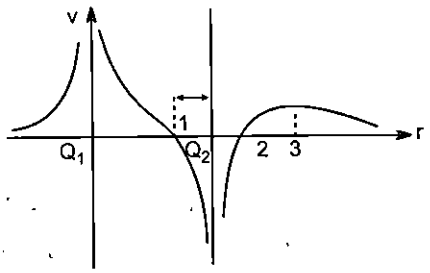


Fig. 1E.70

Solution: The electric field vector is zero at point 3. As $-\frac{dV}{dr} = E_r$, the negative of slope of *V vs r curve represents component of electric field along r. Slope of curve is zero only at 3.*

Near positive charge net potential is positive and negative near a negative charge. Thus charge Q_1 is positive and Q_2 negative. From the graph it can be seen that net potential due to the two charges is positive in the region left of charge Q_1. Therefore magnitude of potential due to charge Q_1 is greater than due to Q_2. Therefore absolute value of charge Q_1 is greater than that of Q_2. Secondly, the point 1 where potential due to two charges is zero, is nearer to charge Q_2 thereby implying that Q_1 has greater absolute value.

Example 71

Fig. 1E.71 shows potential function variation between two point charges in Figs. (a)-(d). The magnitudes of the two charges are equal. Determine the signs of the charges in each case. If Fig. 1E.71 represents variation of electric field, what are the signs of charges?

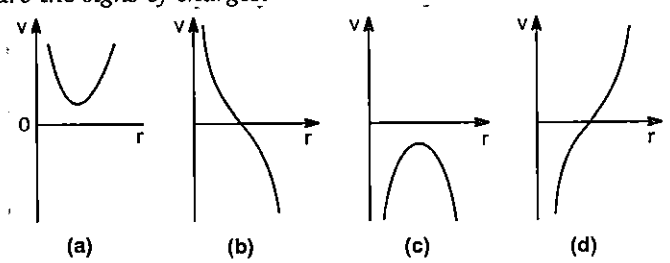


Fig. 1E.71

Solution: The net potential at any point near point charges is the algebraic sum of potentials due to each charge. If the two charges have same sign, the net potential at a distance r from one of the charges is

$$V = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r} + \frac{1}{a-r}\right]$$

where a is the separation between charges. The sign of potential will coincide with the sign of charge, hence in Fig. 1E.87 (a) both the charges are positive and in Fig.1E.87 (c) both the charges are negative.

In the case of unlike charges, the potential midway between them is zero, positive near positive charge and negative near negative charge. In Fig. 1E.71 (b) positive charge is on the left and in Fig. 1E.71 (d) negative charge is on the left.

Example 72

Fig. 1E.72 (a) shows the variation of electric potential along the x-axis. For each of the intervals shown determine the x-component of the electric field and plot E_x vs x.

Solution: From the equation, $-\frac{dV}{dx} = E_x$, we see that negative of slope of *V vs x* graph gives component of field along *x*-axis.

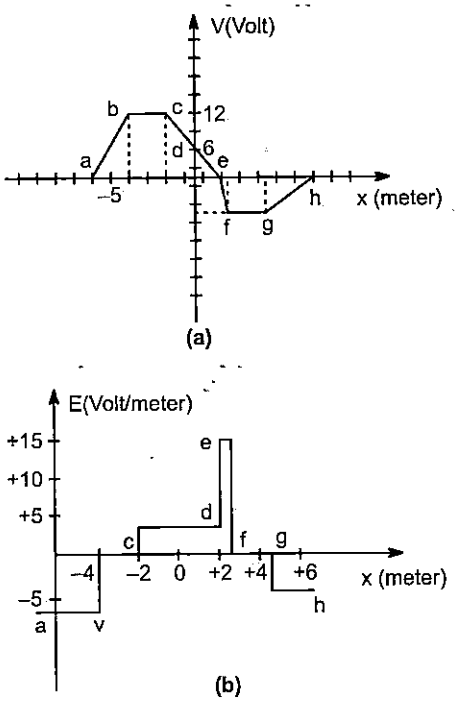


Fig. 1E.72

Example 73

Two fixed charges $-2Q$ and Q are located at the points with co-ordinates $(-3a, 0)$ and $(3a, 0)$ respectively in the x-y plane. (a) Show that all the points in the x-y plane where the electric potential due to the two charges is zero lie on a circle.

Find its radius and the location of its centre. (b) Give the expression for the potential $V_{(x)}$ at a general point on the x-axis and sketch the function $V_{(x)}$ on the whole x-axis. (c) If a particle of charge +q starts from rest at the centre of the circle, show by a short qualitative argument that the particle eventually crosses the circle. Find its speed when it does so.

Solution: (a) Consider a point P at a distance r_1 from $-2Q$ and r_2 from Q. Potential at P is

$$V_p = V_1 + V_2 = \frac{1}{4\pi\varepsilon_0}\left[\frac{-2Q}{r_1} + \frac{Q}{r_2}\right]$$

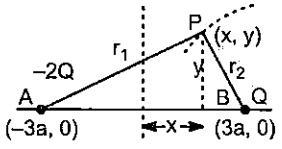


Fig. 1E.73 (a)

Condition of the problem states

$$V_p = 0 \quad i.e., \quad r_1 = 2r_2$$

As $r_1^2 = [(x+3a)^2 + y^2]$

and $r_2^2 = [(3a-x)^2 + y^2]$

so $4[(3a-x)^2 + y^2] = [(x+3a)^2 + y^2]$

or $3x^2 - 30ax + 27a^2 + 3y^2 = 0$

or $(x-5a)^2 + (y-0)^2 = (4a)^2$

This is the equation of a circle in the x-y plane. So all points in the x-y plane where potential due to the two given charges is zero lie on a circle of radius $4a$ and centre $(5a, 0)$.

(b) For a point on x-axis, $y = 0$, so

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{-2Q}{|(x+3a)|} + \frac{Q}{|(x-3a)|}\right]$$

The value of V for different values of x in a tabular form is given below :

$x \rightarrow$	$-\infty$	$-3a$	0	a	$3a$	$5a$	$9a$	$12a$	$15a$
$V \rightarrow$	0	$-\infty$	$-\frac{Q}{3a}\times\frac{1}{4\pi\varepsilon_0}$	0	∞	$\frac{Q}{4a}\times\frac{1}{4\pi\varepsilon_0}$	0	$-\frac{Q}{45}\times\frac{1}{4\pi\varepsilon_0}$	$-\frac{Q}{36}\times\frac{1}{4\pi\varepsilon_0}$

The sketch of V *vs* x curve is as shown in Fig. 1E.73 (b).

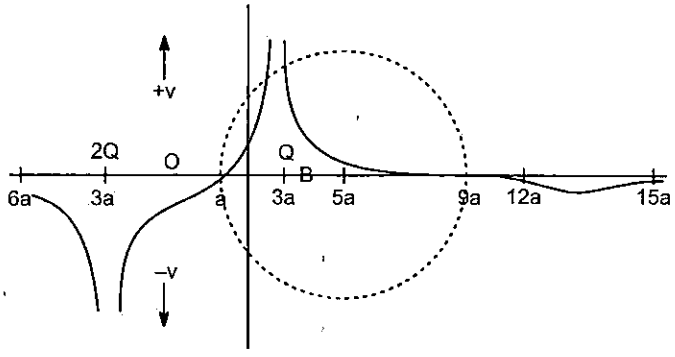


Fig. 1E.73 (b)

(c) The potential at the centre of the circle ($x = 5a$) is $(Q/16\pi\varepsilon_0 a)$ while on the circle ($x = 9a$) is zero. A positive charge moves from higher potential to lower, so if the charge $+q$ starts moving, it will cross the circle with whole of its potential energy being converted into kinetic energy, *i.e.*,

$$0 + \frac{1}{4\pi\varepsilon_0}\frac{qQ}{4a} = \frac{1}{2}mv^2 + 0$$

so that $$v = \sqrt{\frac{4}{4\pi\varepsilon_0}\times\frac{qQ}{2ma}}$$

Example 74

Two equal point charges are fixed at $x = -a$ and $x = +a$ on the x-axis. Another point charge Q is placed at the origin. Find the change in electrical energy of Q (approximately) when it is displaced by a small distance along the x-axis.

Solution: Initially, the potential energy of Q is

$$U_1 = \frac{1}{4\pi\varepsilon_0}\cdot\left[\frac{Qq}{a} + \frac{Qq}{a}\right] = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2Qq}{a}$$

When Q is displaced by a small amount x to the right, the potential energy of Q becomes

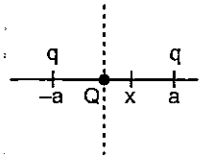


Fig. 1E.74

$$U_2 = \frac{1}{4\pi\varepsilon_0}\left[\frac{Qq}{(a+x)} + \frac{Qq}{(a-x)}\right] = \frac{1}{4\pi\varepsilon_0}\frac{2Qqa}{(a^2-x^2)} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2Qq}{a}\left(1-\frac{x^2}{a^2}\right)^{-1}$$

Expanding binomially and nelgecting nigher powers of $\frac{x^2}{a^2}$ as $x \ll a$,

$$U_2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2Qq}{a}\left(1+\frac{x^2}{a^2}\right)$$

∴ The change in elecrostatic potential energy of the system,

$$dU = U_2 - U_1 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2Qq}{a}\cdot\frac{x^2}{a^2}$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{2Qqx^2}{a^3}$$

Example 75

A proton of mass m approaches from a very large distance towards a free proton with a velocity v_0 along the straight line joining their centres. Find the closest distance of approach between the two protons.

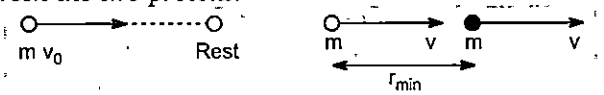


Fig. 1E.75

Solution: From momentum conservation, we have

$$mv_0 = 2mv \quad \text{or} \quad v = v_0/2 \qquad ...(1)$$

From energy conservation, we have

$$\frac{1}{2}mv_0^2 = 2\left(\frac{1}{2}mv^2\right) + \frac{e^2}{4\pi\varepsilon_0 r_{\text{min.}}} \qquad ...(2)$$

At the closest distance of approach, both the protons have same velocity.

On solbing eqns. (1) and (2) for $r_{\text{min.}}$ we get

$$r_{\text{min}} = \frac{e^2}{\pi\varepsilon_0 mv_0^2}$$

Example 76

A particle of positive charge Q is assumed to have fixed position at P. A second particle of mass m and negative charge $-q$ moves at a constant speed in a circle of radius r_1, centred at P. Derive an expression for the work W that must be doen by an external agent on the second particle in order to increase the radius of the circle of motion, centred at P_1 to r_2. Express W in terms of quantities chosen from among m, r_1, r_2, q, Q and ε_0 only.

Solution: In any orbit the negative charge q has kinetic energy due to orbital velocity and electric potential energy due to interaction with the nucleus.

The Coulombic force of attraction is centripetal force for orbital motion.

$$\frac{Qq}{4\pi\varepsilon_0 r_1^2} = \frac{mv_1^2}{r_1}$$

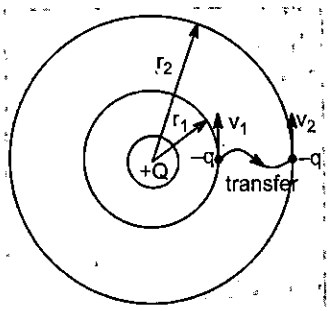


Fig. 1E.76

Kinetic energy K of charge q is

$$K = \frac{1}{2}mv_1^2 = \frac{Qq}{8\pi\varepsilon_0 r_1}$$

Kinetic energy is always positive.

Electrical potential energy of two charge sytem is

$$U_1 = -\frac{Qq}{4\pi\varepsilon_0 r_1}$$

Total energy E_1 in initial orbit is

$$E_1 = K_1 + U_1$$

$$= \frac{Qq}{8\pi\varepsilon_0 r_1} - \frac{Qq}{4\pi\varepsilon_0 r_1} = -\frac{Qq}{8\pi\varepsilon_0 r_1}$$

Total energy of charge $-q$ is negative, it implies that it is bound to charge Q, energy is required to move it out from the influence of Q.

Similarly in the circular orbit of radius r_2, total energy is

$$E_2 = -\frac{Qq}{8\pi\varepsilon_0 r_2}$$

From work-energy theorem, the required work done is

$$W_2 = E_2 - E_1 = \frac{Qq}{8\pi\varepsilon_0}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

Example 77

Consider Earth to be a ball of radius R and mass M, let the charge of Earth be Q. (a) What must be the maximal mass m of an object carrying an electric charge equal to that of a proton and moving in the electric field of the Earth so that the object may escape Earth's gravitational pull and fly off into outer space ? (b) What can be the maximal charge $Q_{\text{max.}}$ carried by a dust particle (an object) and how can such a charge be imparted to the object ?

Solution: (a) Let the object be launched from the Earth's surface with an initial velocity $v_0 = 0$. Then from law of conservation of energy we have

$$-\frac{GMm}{R} + \frac{QQ_P}{4\pi\varepsilon_0 R} = 0 \qquad ...(1)$$

or $$m = \frac{QQ_P}{4\pi\varepsilon_0 GM}$$

(b) We consider the dust particle to be a small metal ball of radius r that acquires its charge from Earth, with which it

is in direct contact. The electric charge flows to the ball from Earth until the potential of dust particle and Earth become equal, when charge flow stops.

$$V = \frac{Q_{max.}}{4\pi\varepsilon_0 r} = \frac{Q}{4\pi\varepsilon_0 r}$$

or $$Q_{max.} = \frac{Qr}{R} \quad ...(2)$$

Let ρ be the density of the metal ball, then its mass is

$$m = \frac{4}{3}\pi R^3 \rho$$

Substituting the values of the maximal charge $Q_{max.}$ and mass of the ball m into eqn. (1), we get

$$-G\frac{M(4/3)\pi r^3\rho}{R} + \frac{QrQ}{4\pi\varepsilon_0 R^2} = 0$$

On solving for r, we get $r = \frac{Q}{4\pi}\sqrt{\frac{3}{\varepsilon_0 G\rho MR}}$

On substituting the expression for r in eqn. (2), we get

$$Q_{max} = \left(\frac{Q}{4\pi}\sqrt{\frac{3}{\varepsilon_0 G\rho MR}}\right)\times\frac{Q}{R}$$

$$= \frac{Q^2}{4\pi R}\sqrt{\frac{3}{\varepsilon_0 G\rho MR}}$$

Example 78

A spherical ball of radius R is uniformly charged with charge density ρ. Determine the electrostatic energy of the sphere.

Solution: We can consider the sphere to be composed of differential spherical shells of radius r and thickness dr. The charge on such a shell is

$$dq = \rho 4\pi r^2 dr$$

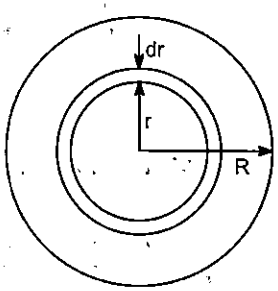


Fig. 1E.78

The work done required to assemble such a shell on a sphere of charge q and radius r from infinity against the electrostatic repulsion is electrostatic energy of charged sphere and shell.

The charge on the sphere of radius r is

$$q = \rho\times\frac{4}{3}\pi r^3$$

Electric potential of sphere is

$$V = \frac{q}{4\pi\varepsilon_0 r} = \frac{\rho\times(4/3)\pi r^3}{4\pi\varepsilon_0 r}$$

Work done required to assemble the shell on the sphere is

$$dW = V\,dq = \frac{\rho\times(4/3)\pi r^3}{4\pi\varepsilon_0 r}\times\rho\,4\pi r^2 dr$$

Hence total work done is

$$U = W = \int dw$$

$$= \int_0^R \frac{\rho\times(4/3)\pi r^3}{4\pi\varepsilon_0 r}\cdot\rho\,4\pi r^2\,dr = \frac{4\pi\rho^2 R^5}{15\varepsilon_0}$$

If total charge on sphere is $Q = \rho\left(\frac{4}{3}\pi R^3\right)$, then

$$U = \frac{3Q^2}{20\pi\varepsilon_0 R}$$

Example 79

Four point particles each of mass m and charge q, are initially held in a plane at the four corners of the square of side l_0. If the particles are simultaneously released, they fly apart. Determine the velocity and acceleration of each particle as it moves away. What will be the final velocity of each particle as they separate infinitely apart?

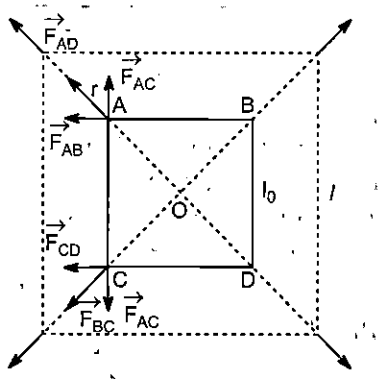


Fig. 1E.79

The charges move, their kinetic energy is gained from the loss in electrostatic potential energy. When the charges are at distance r from O, their total kinetic energy is $4\times\frac{1}{2}mv^2 = 2mv^2$, where v is the velocity of charge at distance r.

The total potential energy of the system at that instant is

$$U_{Tot.} = U_{AB} + U_{AC} + U_{AD} + U_{BC} + U_{BD} + U_{CD}$$

$$= \frac{1}{4\pi\varepsilon_0}Q^2\left(4\times\frac{1}{l} + 2\times\frac{1}{2r}\right)$$

$$= \frac{Q^2}{4\pi\varepsilon_0 r}(2\sqrt{2}+1) \quad (l^2 = 2r^2)$$

There are 6 independent pairs of charges. The total potential energy of the system is obtained by adding the potential energy of all the 6 pairs. The change in potential energy of the system as the charges move from r_0 to r is therefore

$$\Delta U = \frac{Q^2(2\sqrt{2}+1)}{4\pi\varepsilon_0}\left(\frac{1}{r_0} - \frac{1}{r}\right)$$

Equating it to total kinetic energy of the particles, we get

$$v^2 = \frac{Q^2(1+2\sqrt{2})}{8\pi\varepsilon_0 m}\left(\frac{1}{r_0} - \frac{1}{r}\right)$$

Differentiating the above expression, we get te required acceleration.

Example 80

A circular ring of radius R with uniform positive charge density λ per unit length is located in the Y-Z plane with its centre at the origin O. A particle of mass m and positive charge q is projected from the point P(R√3, 0, 0) on the positive X-axis directly towards O, with initial velocity v. Find the smallest (non-zero) value of the speed v such that the particle does not return to P.

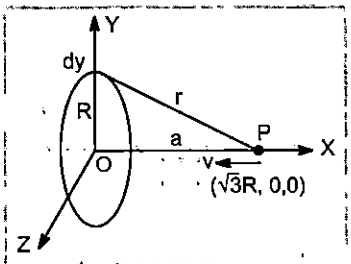


Fig. 1E.80

Solution: The situation is shown in Fig. 1E.80.

Total potential at the centre of a ring is given by

$$V = \frac{\lambda R}{2\varepsilon_0\sqrt{(a^2+R^2)}}$$

$$= \frac{\lambda R}{2\varepsilon_0\sqrt{[(\sqrt{3}R)^2+R^2]}} = \frac{\lambda}{4\varepsilon_0}$$

Potential energy at $P = \lambda q/4\varepsilon_0$

Energy of particle at $P = \frac{1}{2}mv^2$

$\therefore$ Total energy at $P' = \frac{\lambda q}{4\varepsilon_0} + \frac{1}{2}mv^2$

The potential energy at centre $= (\lambda q/2\varepsilon_0)$

The particle will not return to P, when

$$\frac{\lambda q}{4\varepsilon_0} + \frac{1}{2}mv^2 = \frac{\lambda q}{2\varepsilon_0}$$

$$\frac{1}{2}mv^2 = \frac{\lambda q}{2\varepsilon_0} - \frac{\lambda q}{4\varepsilon_0} = \frac{\lambda q}{4\varepsilon_0}$$

$$v^2 = \frac{\lambda q}{2\varepsilon_0 m} \quad \text{or} \quad v = \sqrt{\left(\frac{\lambda q}{2\varepsilon_0 m}\right)}$$

Example 81

A non-conducting disc of radius 'a' and uniform positive surface charge density σ is placed on the ground, with its axis vertical. A particle of mass m and posiitive charge q is dropped, along the axis of the disc, from a height H with zero initial velocity. The particle has $q/m = 4\varepsilon_0 g/\sigma$:

(a) Find the value of H if the particle just reaches the disc.

(b) Sketch the potential energy of the particle as a function of its height and find its equilibrium position.

Solution: (a) Potential due to disc is given by

$$V = \frac{1}{4\pi\varepsilon_0}\cdot 2\pi\sigma[\sqrt{y^2+a^2} - y]$$

Applying the law of conservation of energy, between initial and final points, we have

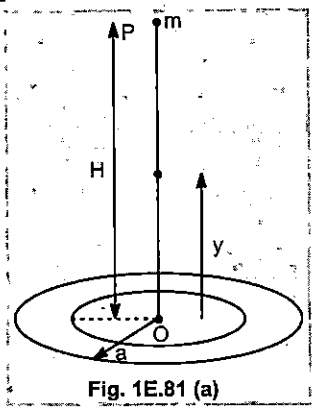


Fig. 1E.81 (a)

$$GPE_i + EPE_i = GPE_f + EPE_f$$

$$mgH + \frac{1}{4\pi\varepsilon_0}\cdot\frac{2\pi\sigma q}{1}[\sqrt{H^2+a^2} - H] = \frac{1}{4\pi\varepsilon_0}\cdot 2\pi\sigma qa$$

On substituting,

$$\frac{q}{m} = \frac{4\varepsilon_0 g}{\sigma}$$

i.e., $$q = \frac{4\varepsilon_0 mg}{\sigma}$$

or $$mgH = \frac{\sigma}{2\varepsilon_0}\cdot\frac{4\varepsilon_0 mg}{\sigma}[H - \sqrt{H^2+a^2}] + \frac{\sigma}{2\varepsilon_0}\cdot\frac{4\varepsilon_0 mga}{\sigma}$$

or $$mg(H-2a) = 2mg[H - \sqrt{H^2+a^2}]$$

or $$H = \frac{4a}{3}$$

(b) Potential energy of particle at height y,

$$U = \frac{\sigma q}{2\varepsilon_0}[\sqrt{y^2+a^2} - y] + mgy$$

Substituting for q,

$$U = \frac{\sigma}{2\varepsilon_0}\cdot\frac{4\varepsilon_0 mg}{\sigma}[\sqrt{y^2+a^2} - y] + mgy$$

$$= 2mg[\sqrt{y^2+a^2} - y] + mgy.$$

For equilibrium,

$$\frac{dU}{dy} = 0$$

$$2mg\left[\frac{1\cdot 2y}{2\sqrt{y^2+a^2}} - 1\right] + mg = 0$$

$$mg\left[\frac{2y}{\sqrt{y^2+a^2}} - 1\right] = 0$$

$$2y = \sqrt{y^2+a^2}$$

$\therefore$ $$y = a/\sqrt{3}$$

$$U_{min} = mg\left[2\frac{a^2}{3} + a^2 - \frac{2a}{\sqrt{3}} + \frac{a}{\sqrt{3}}\right]$$

$$= \sqrt{3}\,mga$$

At $y = 0, \quad U = 2mga$

At $y = \frac{4a}{3},$

$$U = 2mg\left[\frac{16a^2}{9} + a^2 - \frac{4a}{3}\right] + mg \times \frac{4a}{3}$$

$$= 2mga$$

The variation of U with y is shown in Fig. 1E.81 (b).

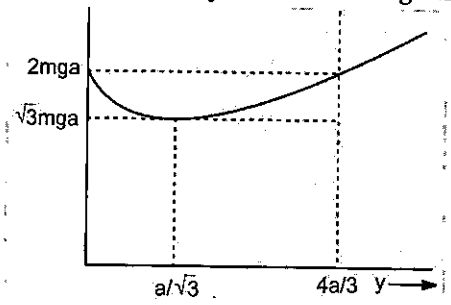


Fig. 1E.81 (b)

Example 82

A small positively charged ball of mass m is suspended by an insulating thread of negligible mass. Another positively charged small ball is moved very slowly from a large distance until it is in the original position of the first ball. As a result, the first ball rises by h. How much work has been done?

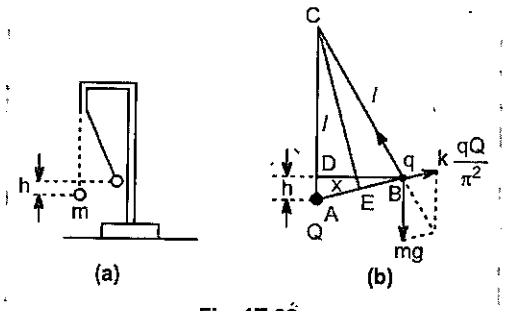


Fig. 1E.82

Solution: Using the notation in Fig. 1E.82, the equilibrium condition for the first ball is

$$s\frac{mg}{F} = \frac{l}{x}$$

where $F = kqQ/x^2$ is the Coulomb force acting on the first ball and x is the distance between the balls carrying charges q and Q.

It is clear that the triangles ABD and CAE are similar, and that consequently

$$\frac{x}{2} : l = h : x$$

From the three equations above we can calculate the separation of the charges and the electrostatic energy of the system :

$$x = k\frac{qQ}{2mgh}$$

and $$E_{electro} = k\frac{qQ}{x}$$

$$= 2mgh.$$

The work done is the sum of the changes in electrical and gravitational potential energy.

$$W = 2mgh + mgh = 3mgh$$

Note that the work done does not depend on either the magnitudes of the charges or the length of the thread.

Example 83

A straight infinitely long cylinder of radius R_0 is uniformly charged with charge density σ. The cylinder serves as a source of electrons, with the velocity vector of emitted electrons perpendicular to its surface. What must be the electron velocity to ensure that the electron can move away from the axis of the cylinder to a distance greater than r ?

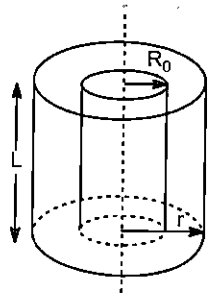


Fig. 1E.83

Solution: First we will determine electric field at a distance r from the cylinder. Consider a coaxial cylinder as a Gaussian surface, then we have

$$\varepsilon_0(2\pi rL)E = (2\pi R_0 L)\sigma$$

or $$E = \frac{\sigma R_0}{\varepsilon_0 r} \qquad ...(1)$$

If we apply Newton's second law we find that

$$m_e\frac{d^2r}{dt^2} = -e\frac{R_0\sigma}{\varepsilon_0 r} \qquad ...(2)$$

where m_e is the mass of electron and e is the electron charge, but this differential equation is difficult to solve. So we will apply law of energy conservation,

$$\frac{1}{2}m_e v_0^2 - eV_0 = -eV$$

where V_0 is the potential of cylinder and V is the potential at a distance r from the cylinder's axis.Employing the relationship $E = -\frac{dV}{dr}$, we have

$$\frac{R_0\sigma}{\varepsilon_0 r} = -\frac{dV}{dr} \qquad ...(3)$$

On integration we get

$$V = -\frac{R_0\sigma}{\varepsilon_0}\ln r + C \qquad ...(4)$$

where C is constant of integration. Also

$$V_0 = -\frac{R_0\sigma}{\varepsilon_0}\ln R_0 + C \qquad ...(5)$$

From eqn. (3), (4) and (5), we get

$$v_0 = \sqrt{\frac{2eR_0\sigma\ln(r/R_0)}{\varepsilon_0 m_e}}$$

Example 84

Show that the configuration of an electrostatic field shown in Fig. 1E.84 is not possible.

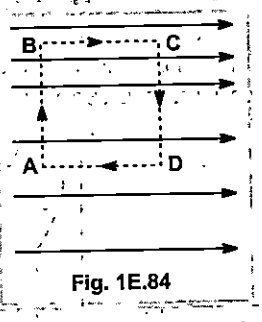


Fig. 1E.84

Solution: We take a closed path *ABCDA* in the electric field. The line integral over this path should be zero,

$$\oint \vec{\mathbf{E}}\cdot d\vec{\mathbf{l}} = 0$$

Along the path *AB* and *CD* the line integral of electric field is zero (since $\vec{\mathbf{E}} \perp d\vec{\mathbf{l}}$)

$$\int_A^B \vec{\mathbf{E}}\cdot d\vec{\mathbf{l}} = \int_C^D \vec{\mathbf{E}}\cdot d\vec{\mathbf{l}} = 0$$

Thus, for $\oint \vec{\mathbf{E}}\cdot d\vec{\mathbf{l}} = 0, \int_B^C \vec{\mathbf{E}}\cdot d\vec{\mathbf{l}}$ should be equal to $-\int_D^A \vec{\mathbf{E}}\cdot d\vec{\mathbf{l}}$

But since lines are closer along *BC*, *i.e.*,

$$|\vec{\mathbf{E}}_{BC}| > |\vec{\mathbf{E}}_{DA}|$$

hence $$\int_B^C \vec{\mathbf{E}}\cdot d\vec{\mathbf{l}} \neq -\int_D^A \vec{\mathbf{E}}\cdot d\vec{\mathbf{l}}$$

Example 85

An infinite plane of charge density σ is in the yz-plane at $x = 0$. (a) Find the potential function for $-\infty < x < 0$ and $0 < x < +\infty$. Can we choose the potential to be zero at $x = \infty$? Why or why not? (b) Now a point charge q is kept on the x-axis at $x = a$. Find the potential at some point P a distance r from the point charge for $x > 0$.

Solution: (a) For an infinite plane of charge density σ in the *yz*-plane, the electric field for positive x is given by

$$\vec{\mathbf{E}} = \frac{\sigma}{2\pi\varepsilon_0}\hat{\mathbf{i}}$$

The potential is then

$$dV = -\vec{\mathbf{E}}\cdot dl = -\left(\frac{\sigma}{2\pi\varepsilon_0}\hat{\mathbf{i}}\right)\cdot(dx\,\hat{\mathbf{i}} + dy\,\hat{\mathbf{j}} + dz\,\hat{\mathbf{k}})$$

$$= -\frac{\sigma}{2\pi\varepsilon_0}dx$$

On integrating we obtain

$$V = V_0 - \frac{\sigma}{2\pi\varepsilon_0}x \qquad ...(1)$$

where the arbitrary constant V_0 is the potential at $x = 0$. The electric field for negative x is

$$\vec{\mathbf{E}} = -\frac{\sigma}{2\varepsilon_0}\hat{\mathbf{i}}$$

So $$dV = -\vec{\mathbf{E}}\cdot d\vec{\mathbf{l}} = +\frac{\sigma}{2\varepsilon_0}dx$$

On integrating we get

$$V = V_0 + \frac{\sigma}{2\varepsilon_0}x \qquad ...(2)$$

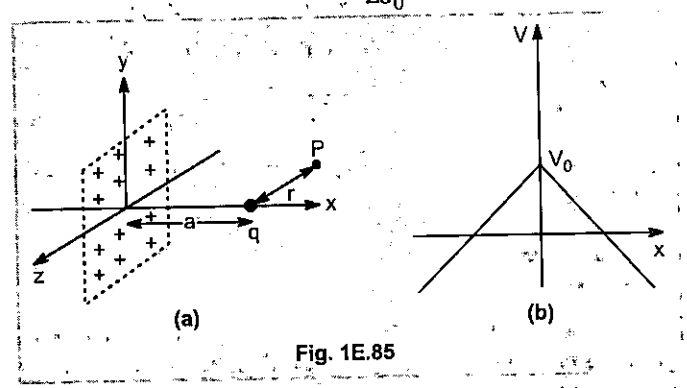


Fig. 1E.85

For either positive or negative x the potential can be written

$$V = V_0 - \frac{\sigma}{2\varepsilon_0}|x|$$

Note that the potential decreases with distance from plane and approaches $-\infty$ as x approaches $+\infty$. Therefore we cannot choose the potential to be zero at $x = \infty$. Similarly the potential again decreases with distance from the plane and approaches $-\infty$ as x approaches $-\infty$. Note that potential is continuous at $x = 0$ but electric field is not.

(b) The potential due to plane is given by [eqn. (1)]

$$V_{\text{plane}} = V_{01} - \frac{\sigma}{2\pi\varepsilon_0}x, \text{ where } V_{01} = \text{constant}$$

Potential due to point charge is

$$V_{\text{point}} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} + V_{02}$$

where V_{02} = constant

While determining the potential due to a point charge we had assigned potential at infinity to be zero, here we cannot do so.

Total potential at point P

$$V = V_{plane} + V_{point}$$
$$= V_{01} - \frac{\sigma}{2\pi\varepsilon_0}x + \frac{1}{4\pi\varepsilon_0}\frac{q}{r} + V_{02}$$

In other to determine constant $V_{01} + V_{02}$, we choose $V = 0$ at the origin, *i. e.*, at $x = 0$ and $r = a$, we get

$$V = 0 = V_{01} - \frac{\sigma}{2\pi\varepsilon_0}(0) + \frac{1}{4\pi\varepsilon_0}\frac{q}{a} + V_{02}$$

Hence,

$$V_{01} + V_{02} = -\frac{1}{4\pi\varepsilon_0}\frac{q}{a}$$

So the general expression for V reduces to

$$V = -\frac{\sigma}{2\pi\varepsilon_0}x + \frac{kq}{r} - \frac{kq}{a}$$

In rectangular coordinates

$$r = [(x-a)^2 + y^2 + z^2]^{1/2}$$

So, $$V = -\frac{\sigma}{2\pi\varepsilon_0}x + \frac{1}{4\pi\varepsilon_0}\frac{q}{[(x-a)^2 + y^2 + z^2)]^{1/2}} - \frac{1}{4\pi\varepsilon_0}\frac{q}{a}$$

Similarly we can show that for $x < 0$

$$V = \frac{\sigma}{2\pi\varepsilon_0}x + \frac{1}{4\pi\varepsilon_0}\frac{q}{[(x-a)^2 + y^2 + z^2]^{1/2}} - \frac{1}{4\pi\varepsilon_0}\frac{q}{a}$$

Students are advised to read example 86 to 91 after reading conductors.

Example 86

A hollow, uncharged spherical conductor has inner radius a and outer radius b. A positive point charge $+q$ is in the cavity at the centre of the sphere. Find the potential $V(r)$ everywhere, assuming that $V = 0$ at $r = \infty$

Solution: As potential $dV = -\vec{\mathbf{E}}\cdot d\vec{\mathbf{l}} = -E_r dr$. Inside the cavity, $E_r = kq/r^2$ where $k = 1/(4\pi\varepsilon_0)$, so V is of the form $V = kq/r + V_0$. The conductor is an equipotential volume, so V is constant for $a \le r \le b$. The field lines inside the cavity must end on the inner surface of the cavity, so this surface has an induced charge of $-q$. Since the shell is uncharged, a positive charge $+q$ is on the outer surface. The three charges q at the centre, $-q$ on the inner surface, and $+q$ on the outer surface produce a field $E_r = kq/r^2$ for $r > b$, so the potential for $r > b$ is $V = kq/r$.

- Outside the shell, $V(r)$ is the same as that due to a point charge q at the origin. Choosing $V = 0$ at $r = \infty$, we have

$$V(r) = \frac{kq}{r}, \quad r \ge b$$

- At $r = b$, the potential is kq/b. V remains at this constant value throughout the spherical shell.

$$V(r) = \frac{kq}{b}, \quad a \le r \le b$$

- Inside the cavity, V is the same as that due to a point charge q at the origin, but the arbitrary constant cannot be set equal to zero, because V has already been chosen to be zero at $r = \infty$.

$$V(r) = \frac{kq}{r} + V_0, \qquad r \le a$$

The constant V_0 is determined by the condition that V is continuous at $r = a$, *i. e.*, V must be kq/b at $r = a$.

$$V(a) = \frac{kq}{a} + V_0 = \frac{kq}{b}$$

or $$V_0 = \frac{kq}{b} - \frac{kq}{a}$$

Use this value of V_0 to find $V(r)$ for $r \le a$.

$$V(r) = \frac{kq}{r} + \frac{kq}{b} - \frac{kq}{a}, \qquad r \le a$$

- Fig. 1E.86 shows the electric potential as a function of the distance from the centre of the cavity.

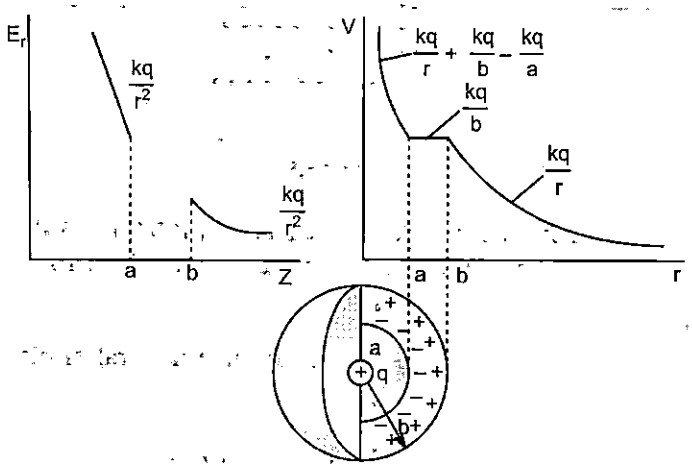


Fig. 1E.86

Example 87

Consider two concentric spherical metal shells of radii a and b, where $b > a$. The outer shell has charge Q but the inner shell is grounded. This means that the inner shell is at zero potential and that electric field lines leave the outer shell and go to infinity but other electric field lines leave the outer shell and end on the inner shell. Find the charge on the inner shell.

Solution: When an object is connected to earth (grounded), its potential is reduced to zero. Let q' be the charge on A after it is earthed.

The charge q' on A induces $-q'$ on inner surface of B and $+q'$ on outer surface of B, in equilibrium the charge distribution is as shown in Fig. 1E.87.

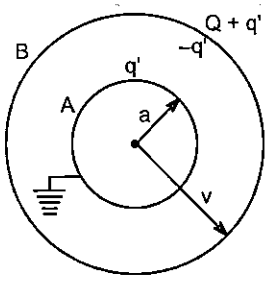


Fig. 1E.87

Potential of inner sphere = potential due to charge on A + potential due to charge on B.

$$V_A = \frac{q'}{4\pi\varepsilon_0 a} - \frac{q'}{4\pi\varepsilon_0 b} + \frac{Q+q'}{4\pi\varepsilon_0 b}$$

$$= V_{\text{Earth}} = 0$$

or $$q' = -Q\left(\frac{a}{b}\right)$$

This implies that a charge $+Q\left(\frac{a}{b}\right)$ has been transferred to earth leaving negative charge on A.

Example 88

Three conducting spherical shells have radii a, b and c such that $a < b < c$. Initially, the inner shell is uncharged, the middle shell has a positive charge Q and the outer shell has a negative charge $-Q$. (a) Find the electric potential of the three shells. (b) If the inner and outer shells are now connected by a wire that is insulated as it passes through the middle shell, what is the electric potential of each of the three shells, and what is the final charge on each shell?

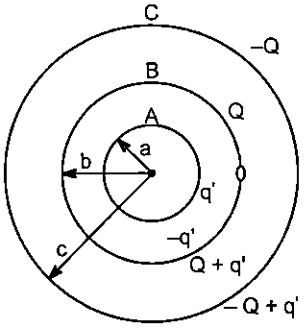


Fig. 1E.88

Solution: Potential of shell A,

$$V_A = V_{\text{due to charge on } A} + V_{\text{due to charge on } B} + V_{\text{due to charge on } C}$$

$$= 0 + \frac{Q}{4\pi\varepsilon_0 b} - \frac{Q}{4\pi\varepsilon_0 c}$$

Potential of shell B,

$$V_B = V_{\text{due to charge on } A} + V_{\text{due to charge on } B} + V_{\text{due to charge on } C}$$

$$= 0 + \frac{Q}{4\pi\varepsilon_0 b} - \frac{Q}{4\pi\varepsilon_0 c}$$

Potential of shell C,

$$V_C = V_{\text{due to charge on } A} + V_{\text{due to charge on } B} + V_{\text{due to charge on } C}$$

$$= 0 + \frac{Q}{4\pi\varepsilon_0 c} - \frac{Q}{4\pi\varepsilon_0 c} = 0$$

(b) Let the charge on inner shell be q', after inner and outer shells are connected by a conducting wire. Then final charge distribution is shown in figure. The wire connection equalises the potential of the two shells.

$$V_A = V_{\text{due to charge on } A} + V_{\text{due to charge on } B} + V_{\text{due to charge on } C}$$

$$= \frac{q'}{4\pi\varepsilon_0 a} + \frac{Q}{4\pi\varepsilon_0 b} - \frac{Q}{4\pi\varepsilon_0 c} \quad ...(1)$$

Similarly,

$$V_C = V_{\text{due to charge on } A} + V_{\text{due to charge on } B} + V_{\text{due to charge on } C}$$

$$= \frac{q'}{4\pi\varepsilon_0 c} + \frac{Q}{4\pi\varepsilon_0 c} - \frac{Q}{4\pi\varepsilon_0 c} \quad ...(2)$$

On equating expression for V_A and V_B, we have

$$\frac{q'}{4\pi\varepsilon_0 a} + \frac{Q}{4\pi\varepsilon_0 b} = \frac{q'}{4\pi\varepsilon_0 c} + \frac{Q}{4\pi\varepsilon_0 c}$$

or $$q' = -\frac{Qa}{b}\left[\frac{c-b}{c-a}\right]$$

Charge on B, $Q_b = Q - q' + q' = Q$ [shell B is located and sum of induced charges is zero]

Charge on C,

$$Q_c = -Q + q' = -Q + \frac{Qa}{b}\left[\frac{c-b}{c-a}\right]$$

$$= \frac{Qc}{b}\left[\frac{a-b}{c-a}\right]$$

If charge q' appears on the inner shell and equal magnitude, opposite sign charge must appear on the outer shell in accordance with the law of conservation of charge.

Example 89

Three concentric conducting shells of radii a, b and c are shown in Fig. 1E.89. Charge on the shell of radius b is Q. If the key K is closed, find the charges on the innermost and outermost shells and ratio of charge densities of the shells. Given that

$$a : b : c = 1 : 2 : 3$$

Solution: After closing the key the innermost and outermost shells will be at the same potential. Let the charge on the outer shell be q and that on the inner shell be $-q$, the total charge on inner and outer shells is zero.

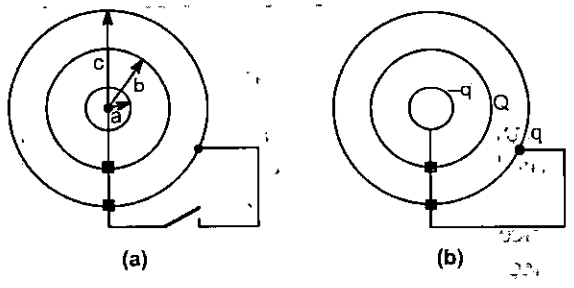


Fig. 1E.89

Potential on innermost shell,

V_a = sum of potentials due to $-q, Q$ and q

$$= -\frac{q}{a} + \frac{Q}{b} + \frac{q}{c}$$

Similarly, potential on the outermost shell,

$$V_c = -\frac{q}{c} + \frac{Q}{c} + \frac{q}{c}$$

As $V_a = V_c$, we have

$$-\frac{q}{a} + \frac{Q}{b} + \frac{q}{c} = -\frac{q}{c} + \frac{Q}{c} + \frac{q}{c} \quad ...(1)$$

From the given conditions,

$$c = 3a, b = 2a.$$

Eqn. (1) now becomes

$$-\frac{q}{a} + \frac{Q}{2a} = -\frac{q}{3a} + \frac{Q}{3a} \quad ...(2)$$

or $$q = \frac{Q}{4}$$

Thus charge on outermost shell $= \frac{Q}{4}$

charge on innermost shell $= -\frac{Q}{4}$

$$\sigma_a = \frac{1}{4\pi a^2}\left(-\frac{Q}{4}\right)$$

$$\sigma_b = \frac{+Q}{4\pi b^2} = \frac{Q}{4\pi(4a^2)}$$

$$\sigma_c = \frac{1}{4\pi c^2}\left(\frac{Q}{4}\right) = \frac{+Q}{4\pi(9a^2)}$$

$$\sigma_a : \sigma_b : \sigma_c = -9 : 9 : 1$$

Example 90

Fig. 1E.90 (a) shows three concentric spherical conductors A, B and C with radii R, 2R and 4R respectively. A and C are connected by a conducting wire and B is uniformly charged (charge = +Q). Find (a) charges on conductors A and C, (b) potentials of A and B.

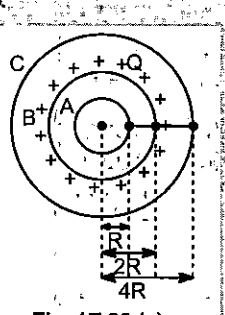


Fig. 1E.90 (a)

Solution: (a) Let the charges on A and C be q_1 and q_2 respectively. From conservation of charge, we have

$$q_1 + q_2 = 0$$

Hence $$q_1 = -q_2$$

Since A and C are connected by a conducting wire, so they have same potential.

$$V_A = \text{potential of } A$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q_1}{R} + \frac{1}{4\pi\varepsilon_0}\frac{Q}{2R} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{4R}$$

$$V_C = \text{potential of } C$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q_1}{4R} + \frac{1}{4\pi\varepsilon_0}\frac{Q}{4R} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{4R}$$

Equalising the potentials of A and C, we get

$$V_A = V_C$$

$$\frac{q_1}{R} + \frac{Q}{2R} + \frac{q_2}{4R} = \frac{q_1}{4R} + \frac{Q}{4R} + \frac{q_2}{4R}$$

or $$4q_1 + 2Q = q_1 + Q$$

$$q_1 = -Q/3$$

Hence $$q_2 = Q/3$$

(b) $$V_A = \frac{1}{4\pi\varepsilon_0 R}\left[\frac{-Q}{3} + \frac{Q}{2} + \frac{Q}{12}\right]$$

$$= \frac{Q}{16\pi\varepsilon_0 R}$$

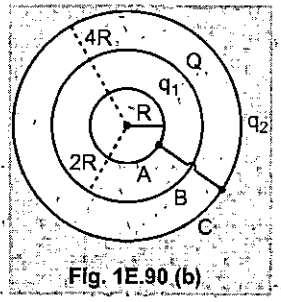


Fig. 1E.90 (b)

$$V_B = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{2R} + \frac{1}{4\pi\varepsilon_0}\frac{Q}{2R} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{4R}$$

$$= \frac{1}{8\pi\varepsilon_0 R}\left[\frac{-Q}{3} + Q + \frac{Q}{6}\right] = \frac{5Q}{48\pi\varepsilon_0 R}$$

Example 91

Two concentric shells of radii R and 2R are shown in Fig. 1E.91. Initially a charge q is imparted to the inner shell. Now key K_1 is closed and opened and then key K_2 is closed and opened. After the keys K_1 and K_2 are alternately closed n times each, find the potential difference between the shells. Note that finally the key K_2 remains closed.

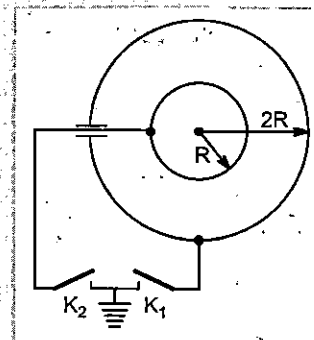


Fig. 1E.91

Solution: When K_1 is closed first time, outer sphere is earthed and the potential on it becomes zero.

Let the charge on it be q_1'.

V_1' = Potential due to charge on inner sphere and that due to charge on outer sphere

$$V_1' = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{2R} + \frac{q_1'}{2R}\right] = 0$$

or $$q'_1 = -q$$

When K_2 is closed first time, the potential V_2' on inner sphere becomes zero as it is earthed. Let the new charge on inner sphere be q'_2.

$$0 = \frac{1}{4\pi\varepsilon_0}\frac{q'_2}{R} + \frac{1}{4\pi\varepsilon_0}\frac{(-q)}{2R}$$

$$\Rightarrow \quad q'_2 = \frac{q}{2}$$

Now when K_1 will be closed second time, charge on outer sphere will be $-q'_2$, *i.e.*, $-q/2$

After one event involving closure and opening of K_1 and K_2, charge is reduced to half its initial value.

Similarly, when K_1 will be closed nth time, charge on outer sphere will be $-\frac{q}{2^{n-1}}$ as each time charge will be reduced to half the previous value.

After closing K_2 nth time, charge on inner shell will be negative of half the charge on outer shell, *i.e.*, $(+q/2^n)$ and potential on it will be zero.

For potential of outer shell,

$$V_0 = \frac{1}{4\pi\varepsilon_0}\frac{(+q/2^n)}{2R} + \frac{1}{4\pi\varepsilon_0}\frac{(-q/2^{n-1})}{2R}$$

$$V_0 = \frac{-q[-1+2]}{4\pi\varepsilon_0 2^{n+1}R} = \frac{-q}{4\pi\varepsilon_0 2^{n+1}R}$$

Potential difference

$$= V_0 - V_i = \frac{-q}{4\pi\varepsilon_0 2^{n+1}R} - 0$$

$$= \frac{-q}{4\pi\varepsilon_0 2^{n+1}R}$$

POTENTIAL ENERGY OF A DIPOLE IN A UNIFORM FIELD

Fig. 1.139 shows a dipole oriented at an angle θ with respect to a uniform electric field directed along the x-axis. The x-coordinates of the positive and negative point charges are $x_0 + a\cos\theta$ and $x_0 - a\cos\theta$ respectively. As the electric field is uniform the potential function can be described as

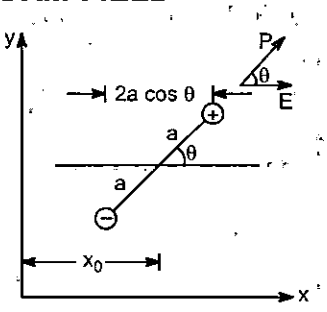


Fig. 1.139

$$V(x) = -Ex + V_0$$

where V_0 is the potential at points on the yz-plane ($x = 0$). V_0 can be assigned any arbitrary value.

Therefore the potential energy U_+ of the positive point charge is

$$U_+ = q[-E(x_0 + a\cos\theta) + V_0]$$

and the potential energy U_- of the negative point charge is

$$U_- = -q[-E(x_0 - a\cos\theta) + V_0]$$

Thus the potential energy of the dipole is

$$U = U_+ + U_-$$

$$= q[-E(x_0 + a\cos\theta) + V_0] - q[-E(x_0 - a\cos\theta) + V_0]$$

$$= -2aqE\cos\theta = -pE\cos\theta = -\vec{\mathbf{p}}\cdot\vec{\mathbf{E}}$$

where $\vec{\mathbf{p}}$ is the dipole moment.

The potential energy of a dipole in a uniform field is independent of its position x_0. Instead it depends on the orientation of the dipole with respect to the field direction.

When $\vec{\mathbf{p}}$ is parallel to $\vec{\mathbf{E}}$, the potential energy is

$$PE_{\text{dipole parallel}} = -pE\cos 0° = -pE \qquad [\text{as } \cos 0° = 1]$$

When $\vec{\mathbf{p}}$ is antiparallel to $\vec{\mathbf{E}}$, the potential energy is

$$PE_{\text{dipole antiparallel}} = -pE\cos 180° = +pE \qquad [\text{as } \cos 180° = -1]$$

When $\vec{\mathbf{p}}$ is perpendicular to $\vec{\mathbf{E}}$, the potential energy is

$$PE_{\text{dipole perpendicular}} = -pE\cos 90° = 0$$

Hence the potential energy is maximum when $\vec{\mathbf{p}}$ is antiparallel to $\vec{\mathbf{E}}$ and is minimum when $\vec{\mathbf{p}}$ is parallel to $\vec{\mathbf{E}}$.

If the dipole is free to rotate, the dipole will lower its electrical potential energy by rotating to the parallel orientation. The antiparallel orientation is unstable because it has the maximum potential energy.

Thus when a dipole is placed in an electric field, it will tend to orient itself so that $\vec{\mathbf{p}}$ is parallel to $\vec{\mathbf{E}}$, the molecular dipoles behave exactly in the similar fashion.

Method 2 : A uniform external field exerts no net force on a dipole because the electric force on the positive and negative charges constituting the dipole balances. But it does exert a torque that tends to rotate the dipole into the direction of the field. The torque is

$$\vec{\tau} = \vec{\mathbf{p}} \times \vec{\mathbf{E}}$$

When the dipole rotates through $d\theta$, the electric field does work.

$$dW = -\tau\, d\theta = -pE\sin\theta\, d\theta$$

The minus sign has been introduced because the torque tends to decrease θ. The torque of field tries to orient the dipole in a lower potential energy configuration.

Decrease in potential energy of dipole when torque of field changes the angle θ from θ_1 to θ_2 is

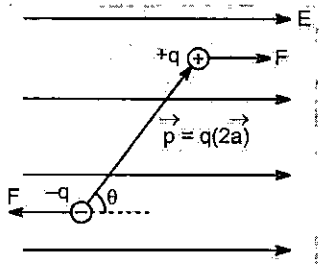


Fig. 1.140

$$U = \int dU = -\int dW = +\int_{\theta_1}^{\theta_2} pE \sin\theta \, d\theta$$

$$= pE(\cos\theta_1 - \cos\theta_2)$$

If we choose $U = 0$ when p is perpendicular to $E(\theta = 90°)$, then

$$U = -pE\cos\theta = -pE$$

Application : In microwave ovens, microwaves have an oscillating electric field that can cause the electric dipoles to oscillate. The microwaves in home ovens are tuned to the natural frequency of the vibration of water molecules. The water molecules in food oscillate in resonance with the electric field thereby absorbing large amounts of energy. This accounts for the rapid cooking times.

The Electric Potential and Electric Field of a Dipole

The dipole produces an electric potential at a point in space that is the algebraic sum of the electric potential at the point in question caused by each of the two charges. Letting V_+ and V_- represent the contributions to the potential due to the positive and negative charge respectively, we have

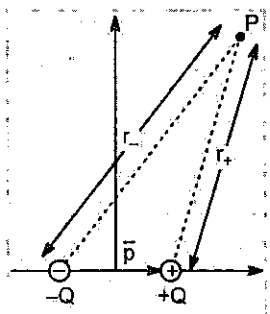


Fig. 1.141

$$V = V_+ + V_-$$

$$= \frac{q}{4\pi\varepsilon_0 r_+} + \frac{-q}{4\pi\varepsilon_0 r_-}$$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_+} - \frac{1}{r_-}\right)$$

$$= \frac{q}{4\pi\varepsilon_0}\frac{r_- - r_+}{r_+ r_-} \qquad ...(1)$$

If $r_1 >> a$ then from Fig. 1.141 $r_+ \simeq r_- \simeq r$ and from Fig. 1.142, $r_- - r_+ \simeq 2a\cos\theta$. Therefore

$$V \simeq \frac{2aq\cos\theta}{4\pi\varepsilon_0 r^2} = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2} \qquad ...(2)$$

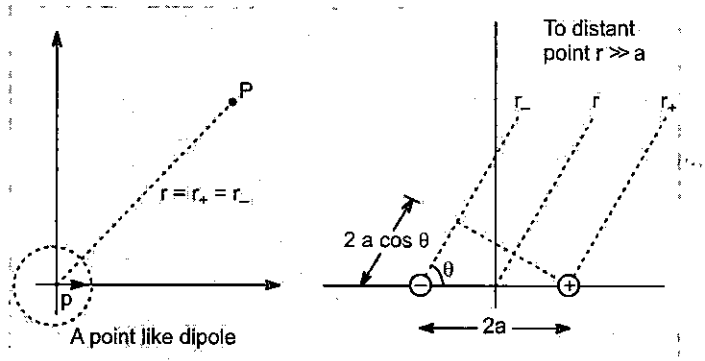
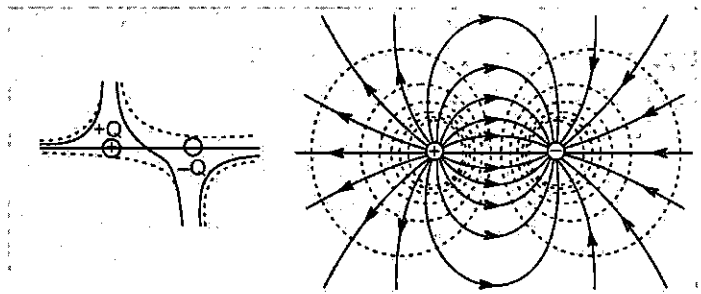


Fig. 1.142

Using a unit vector $\hat{\mathbf{r}}$ that points away from the origin toward point P, we have

$$V = \frac{p \cdot \hat{\mathbf{r}}}{4\pi\varepsilon_0 r^2} \qquad ...(3)$$

The potential is zero in the equatorial plane of the dipole, where $\theta = 90°$. The potential is positive at $(0 \le \theta \le 90°)$ and negative for $90° < \theta \le 180°$. At a fixed point r, the potential has maximum value at $\theta = 0°$, *i.e.*, along the axis of the dipole closer to the positive charge, the potential has its maximum negative value when $\theta = 180°$, along the axis of the dipole closer to negative charge.



The dashed lines are the potentials due to each of two equal and opposite charges. The solid lines show the total potential.

A two-dimensional view of the equipotentials (dashed lines) and the field lines (solid lines) for two equal and opposite charges.

Fig. 1.143

To calculate the electric field of the dipole at a point that is at a great distance from the dipole itself, we use the equation for potential with the cartesian coordinate system shown in Fig. 1.144, with the dipole moment $\vec{\mathbf{p}}$ oriented parallel to $\hat{\mathbf{k}}$ along the z-axis and with x-y plane, the equatorial plane of the dipole. Then from geometry,

$$r = (x^2 + y^2 + z^2)^{1/2}$$

and $$\cos\theta = \frac{z}{r} = \frac{z}{(x^2 + y^2 + z^2)^{1/2}}$$

Substituting for r and $\cos\theta$ in eqn. (2) we have

$$V = \frac{p}{4\pi\varepsilon_0}\frac{z}{(x^2 + y^2 + z^2)^{3/2}}$$

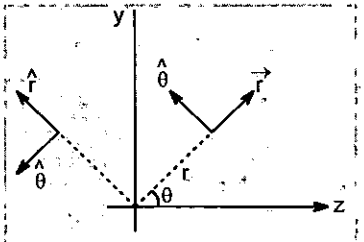


Fig. 1.144 (a)

To find the cartesian components of the electric field, we use

$$E_x = -\frac{\partial V}{\partial x}, \quad E_y = -\frac{\partial V}{\partial y}, \quad E_z = -\frac{\partial V}{\partial z}$$

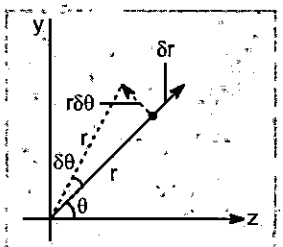


Fig. 1.144 (b)

After appropriate differentiation, we obtain

$$E_x = \frac{p}{4\pi\varepsilon_0}\frac{3zx}{(x^2+y^2+z^2)^{5/2}}$$

$$E_y = \frac{p}{4\pi\varepsilon_0}\frac{3zy}{(x^2+y^2+z^2)^{5/2}}$$

$$E_z = -\frac{p}{4\pi\varepsilon_0}\left[\frac{1}{(x^2+y^2+z^2)^{3/2}} - \frac{3z^2}{(x^2+y^2+z^2)^{5/2}}\right]$$

We can find the field of an electric dipole using the potential in the polar form. There is a slight complication, the eqn. (2) is expressed in polar coordinates r and θ. The polar unit vectors $\hat{r}$ and $\hat{\theta}$ change their orientation in space depending on the location of the point in question. Hence the electric field will have two component radial, E_r, along the radius vector; and azimuthal, E_θ, perpendicular to radius vector given by

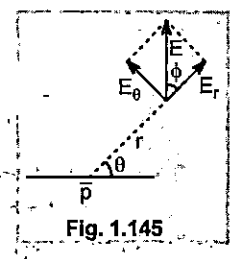


Fig. 1.145

$$E_r = -\frac{\partial V}{\partial r}$$

and $$E_\theta = -\frac{1}{r}\frac{\partial V}{\partial \theta}$$

Consider a differential displacement in radial direction, then

$$\partial V = -E_r \partial r$$

or $$E_r = -\frac{\partial V}{\partial r}$$

Similarly consider a small angular displacement $\partial\theta$ perpendicular to r, then

$$\partial V = -E_\theta (r\partial\theta)$$

or $$E_\theta = -\frac{1}{r}\frac{\partial V}{\partial \theta}$$

$$E_r = -\frac{\partial V}{\partial r}$$

$$= -\frac{\partial}{\partial r}\left[\frac{p\cos\theta}{4\pi\varepsilon_0 r^2}\right] = \frac{1}{4\pi\varepsilon_0}\frac{2p\cos\theta}{r^3}$$

and $$E_\theta = -\frac{1}{r}\frac{\partial V}{\partial \theta}$$

$$= -\frac{1}{r}\frac{\partial}{\partial\theta}\left[\frac{p\cos\theta}{4\pi\varepsilon_0 r^2}\right] = \frac{1}{4\pi\varepsilon_0}\frac{p\sin\theta}{r^3}$$

The magnitude of resultant field is

$$E = \sqrt{E_r^2 + E_\theta^2}$$

$$= \frac{p}{4\pi\varepsilon_0 r^3}\sqrt{1+3\cos^2\theta}$$

and $$\tan\phi = \frac{E_\theta}{E_r} = \frac{1}{2}\tan\theta$$

Remark :

The del operator in the polar coordinates has the form

$$\nabla = \frac{\partial}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial}{\partial\theta}\hat{\theta}$$

where at any point $\hat{r}$ is the unit vector in the direction of increasing r and $\hat{\theta}$ is a unit vector in the direction of increasing θ. So $E = -\nabla V$

$$= -\left[\frac{\partial}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial}{\partial\theta}\hat{\theta}\right]\frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}$$

$$= -\frac{p}{4\pi\varepsilon_0}\left[(\cos\theta)\left(-\frac{2}{r^2}\right)\hat{r} - \frac{\sin\theta}{r^3}\hat{\theta}\right]$$

$$= \frac{p}{4\pi\varepsilon_0}\left[\frac{2\cos\theta}{r^3}\hat{r} + \frac{\sin\theta}{r^3}\hat{\theta}\right]$$

ELECTRIC FIELD AND POTENTIAL DUE TO A DIPOLE

(a) Potential at the Axial Position

Potential due to charge $+q$ at P, $= \frac{1}{4\pi\varepsilon_0}\frac{+q}{(r-l)}$

Potential due to charge $-q$ at P, $= \frac{1}{4\pi\varepsilon_0}\frac{-q}{(r+l)}$

Potential at P due to the electric dipole,

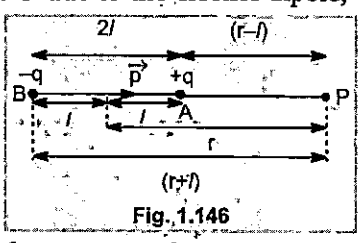


Fig. 1.146

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-l)} - \frac{1}{4\pi\varepsilon_0}\frac{q}{(r+l)}$$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{(r-l)} - \frac{1}{(r+l)}\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q2l}{(r^2-l^2)}$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p}{(r^2-l^2)} \quad (p = 2ql)$$

If $r >>> l$; $V = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^2}$

(b) Electric Field at the Axial or end-on Position (Outside Dipole)

The electric field E at the point on the axis of the dipole is the vector sum of the fields E_1 due to $+q$ and E_2 due to $-q$.

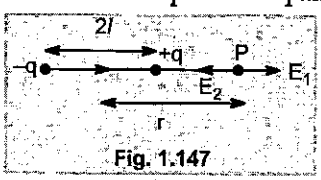


Fig. 1.147

Where $E_1 = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-l)^2}$ Along $\rightarrow x > 0$

$E_2 = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r+l)^2}$ Along $\leftarrow x < 0$

From the Fig.1.147; $E = E_1 - E_2$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{(r-l)^2} - \frac{q}{(r+l)^2}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q4lr}{(r^2-l^2)^2}$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{2pr}{(r^2-l^2)^2}$$

If $r >>> l$; $E = \frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3}$

The direction of electric field is in the direction of dipole moment.

EQUATORIAL POSITION (BROAD SIDE-ON POSITION)

(a) Potential

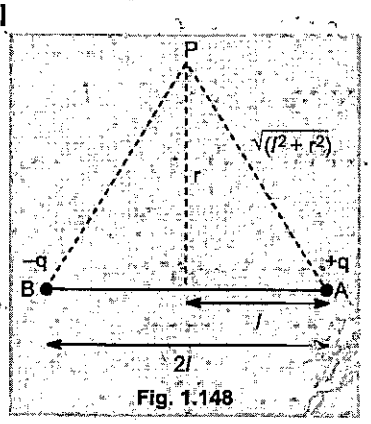


Fig. 1.148

Potential at P due to $+q$ $= \frac{1}{4\pi\varepsilon_0}\frac{q}{\sqrt{(l^2+r^2}}$

Potential at P due to the $-q$, $= \frac{1}{4\pi\varepsilon_0}\frac{-q}{\sqrt{(l^2+r^2)}}$

Potential at P due to dipole

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{\sqrt{(r^2+l^2)}} - \frac{1}{4\pi\varepsilon_0}\frac{q}{\sqrt{(l^2+r^2)}} \Rightarrow V = 0$$

(b) Electric Field

If E_1 and E_2 are the fields at the point P due to the individual charges $+q$ and $-q$ respectively. Resolving E_1 and E_2 as shown in figure, then the resultant field is given by,

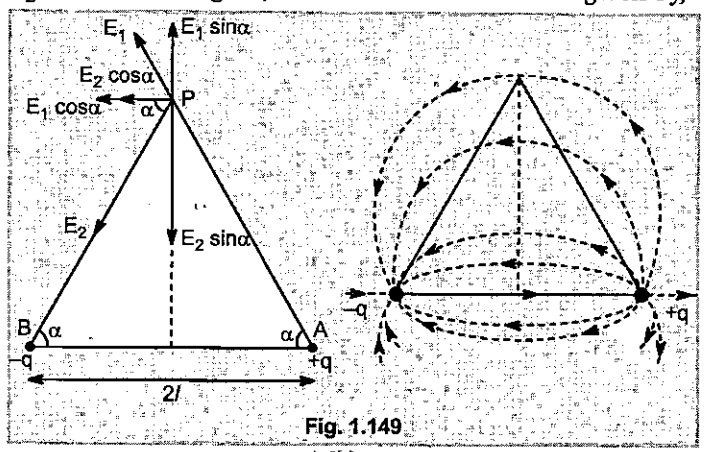


Fig. 1.149

$$E = E_1\cos\alpha + E_2\cos\alpha$$

Where

$$E_1 = E_2 = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r^2+l^2)}$$

and $\cos\alpha = \frac{l}{\sqrt{(r^2+l^2)}}$

Therefore $E = \frac{1}{4\pi\varepsilon_0}\frac{q2l}{(r^2+l^2)^{3/2}}$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{p}{(r^2+l^2)^{3/2}}$$

If $r >>> l$; $E = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}$

Direction of electric field is parallel to the axis of dipole and in the direction opposite to the dipole moment.

Example 92

Three point charges $+q$, $-2q$, $+q$ are arranged on the vertices of an equilateral triangle as shown in the Fig. 1E.92 (a).

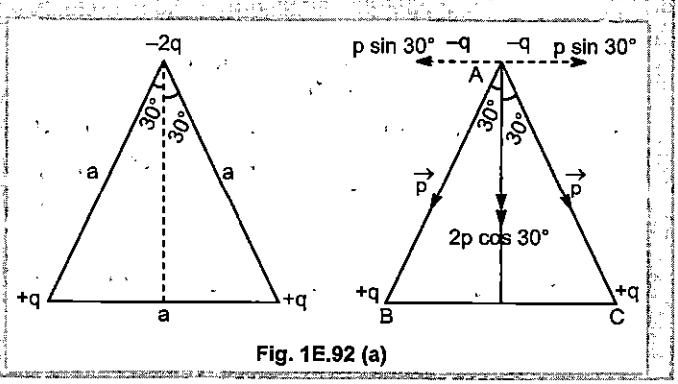


Fig. 1E.92 (a)

Solution: Since dipole moment is a vector quantity, hence in case of two or more than two dipoles, the resultant dipole moment will be the vector sum of the dipole moments of individual dipoles.

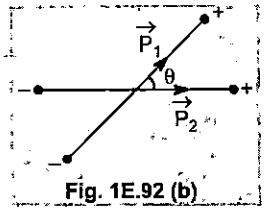


Fig. 1E.92 (b)

$$\vec{\mathbf{p}} = \vec{\mathbf{p}}_1 + \vec{\mathbf{p}}_2 + \vec{\mathbf{p}}_3 + \ldots\ldots$$

In this case, $\vec{\mathbf{p}}_{net} = \vec{\mathbf{p}}_1 + \vec{\mathbf{p}}_2$

$$|\vec{\mathbf{p}}_{net}| = \sqrt{p_1^2 + p_2^2 + 2p_1p_2\cos\theta}$$

Arrangement of the charges is equivalent to two dipoles having dipole moments 'p' each as shown above.

Net dipole moment,

$p_{net} = p\cos 30° + p\cos 30°$

$p_{net} = 2p\cos 30° = p\sqrt{3}$

Net Force on a Dipole in a Nonuniform Field

Suppose an electric dipole with dipole moment $\vec{\mathbf{p}}$ is placed in a nonuniform electric field $\vec{\mathbf{E}} = E\hat{\mathbf{i}}$ that points along x-axis. Let E depend only on x. The electric field at the position of negative charge is E and at the position of positive charge $(E + \Delta E)$. Net force acting on the dipole is then

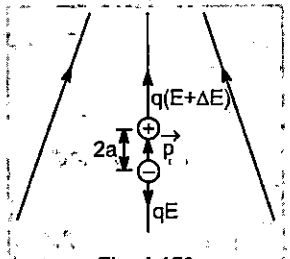


Fig. 1.150

$$F = q(E + \Delta E) - qE = q\,\Delta E = q\left[\frac{\Delta E}{\Delta x}2a\right]$$

$$= 2aq\frac{dE}{dx}\left[\text{as } \frac{\Delta E}{\Delta x} \simeq \frac{dE}{dx}\right] = p\frac{dE}{dx}$$

where $\frac{dE}{dx}$ is the gradient of the field in the x-direction.

Electric Force Between Two Electric Dipoles

Suppose that two dipoles are alligned along the x-axis and separated by a distance r.

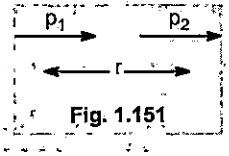


Fig. 1.151

The electric field due to first dipole on the second is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3}$$

The gradient of electric field on the axis is

$$\frac{dE}{dr} = -\frac{1}{4\pi\varepsilon_0}\frac{6p_1}{r^4}$$

The force of interaction between the two dipoles is

$$F = p_2\frac{dE}{dr} = -\frac{1}{4\pi\varepsilon_0}\frac{6p_1p_2}{r^4}$$

The negative sign shows that the force is attractive.

Method 2 : The second dipole lies in the electric field of first dipole, hence its potential energy is

$$U = -\vec{\mathbf{p}}_2\cdot\vec{\mathbf{E}}_1 = -\frac{2p_1p_2}{4\pi\varepsilon_0 r^3}$$

As the electric field is conservative, the force of interaction is

$$F = -\frac{dU}{dr} = -\frac{2p_1p_2}{4\pi\varepsilon_0}\frac{d}{dr}\left(\frac{1}{r^3}\right)$$

$$= -\frac{6p_1p_2}{4\pi\varepsilon_0 r^4}$$

Torque Due to Interaction of Dipoles

If a dipole of dipole moment $\vec{\mathbf{p}}_1$ is perpendicular to $\vec{\mathbf{p}}_2$, *i.e.*, $\vec{\mathbf{p}}_1 \perp \vec{\mathbf{p}}_2$, the torque experienced by it is

$$\vec{\tau}_1 = \vec{\mathbf{p}}_1 \times \vec{\mathbf{E}}$$

Here $\vec{\mathbf{p}}_1 = p_1\hat{\mathbf{j}}$

and $\vec{\mathbf{E}} = E\hat{\mathbf{i}}$

Thus $\vec{\tau}_1 = p_1E(\hat{\mathbf{j}}\times\hat{\mathbf{i}}) = -p_1E\,\hat{\mathbf{k}}$

$$= \frac{-1}{4\pi\varepsilon_0}\frac{2p_1p_2}{d^3}\hat{\mathbf{k}}$$

In order to determine the direction of torque experienced by dipole $\vec{\mathbf{p}}_1$, point your thumb in $(-\hat{\mathbf{k}})$ direction, your fingers will curl in the sense of rotation.

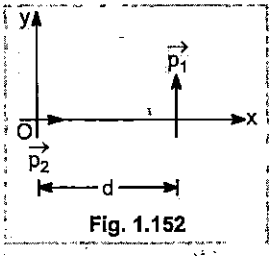


Fig. 1.152

Thus, $\vec{\mathbf{p}}_1$ will experience a clockwise torque of magnitude $\frac{1}{4\pi\varepsilon_0}\frac{2p_1p_2}{d^3}$.

The electric field at dipole of dipole moment $\vec{\mathbf{p}}_2$ due to dipole of dipole moment $\vec{\mathbf{p}}_1$ is $\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0}\frac{p_1}{d^3}\hat{\mathbf{j}}$.

Torque on $\vec{\mathbf{p}}_2$, $\vec{\tau}_2 = \vec{\mathbf{p}}_2\times\vec{\mathbf{E}} = p_2E(\hat{\mathbf{i}}\times\hat{\mathbf{j}}) = -\frac{1}{4\pi\varepsilon_0}\frac{p_1p_2}{d^3}\hat{\mathbf{k}}$

This is again in clockwise direction. Net clockwise torque of magnitude $\frac{3p_1p_2}{4\pi\varepsilon_0 d^3}$ acts on the system of two dipoles. It is left as an exercise to determine the force of interaction between dipoles in this orientation.

Example 93

Two electric dipoles with dipole moments p_1 and p_2 are in line with one another. (a) Find the potential energy of one in the presence of the other (their interaction energy) (b) Show that the electrostatic potential energy of the two dipoles in a plane as shown in Fig. 1E.93 (b) is given by

$$U = \frac{1}{4\pi\varepsilon_0}\frac{p_1 p_2}{r^3}[\cos(\theta_1 - \theta_2) - 3\cos\theta_1 \cos\theta_2]$$

Assume r is much larger than the length of each dipole. The vector dipole moments p_1 and p_2 point from the negative charge toward the positive charge of dipole.

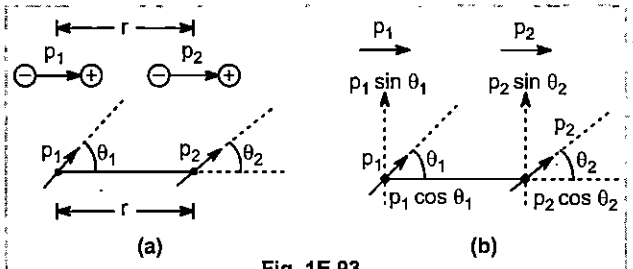


Fig. 1E.93

Solution: (a) The electric field at the axial point of a dipole is given by

$$E \simeq \frac{2p}{4\pi\varepsilon_0 r^3}$$

Potential energy of second dipole is

$$U = -\vec{\mathbf{p}}_2 \cdot \vec{\mathbf{E}}_1 = -(p_2\hat{\mathbf{i}}) \cdot \left(\frac{2p_1}{4\pi\varepsilon_0 r^3}\hat{\mathbf{i}}\right)$$

$$= -\frac{1}{2\pi\varepsilon_0}\frac{p_1 p_2}{r^3}$$

(b) The electric field due to first dipole on the second dipole can be expressed as

$$\vec{\mathbf{E}}_1 = \frac{2p_1\cos\theta_1}{4\pi\varepsilon_0 r^3}\hat{\mathbf{i}} + \frac{p_1\sin\theta_1}{4\pi\varepsilon_0 r^3}\hat{\mathbf{j}}$$

and $$\vec{\mathbf{p}}_2 = p_2\cos\theta_2\,\hat{\mathbf{i}} + p_2\sin\theta_2\,\hat{\mathbf{j}}$$

Potential energy of interaction

$$U = -\vec{p}_2 \cdot \vec{E}_1$$

$$= -\frac{2p_1 p_2\cos\theta_1\cos\theta_2}{4\pi\varepsilon_0 r^3} + \frac{p_1 p_2\sin\theta_1\sin\theta_2}{4\pi\varepsilon_0 r^3}$$

$$= \frac{p_1 p_2}{4\pi\varepsilon_0 r^3}[-2\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2]$$

$$= \frac{p_1 p_2}{4\pi\varepsilon_0 r^3}[\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2 - 3\cos\theta_1\cos\theta_2]$$

$$= \frac{p_1 p_2}{4\pi\varepsilon_0 r^3}[\cos(\theta_1 - \theta_2) - 3\cos\theta_1\cos\theta_2]$$

Example 94

Fig. 1E.94 (a) shows an assembly of two charged rods connected by nonconducting massless rods. The system is free to rotate about the x-axis. The system is given an anticlockwise small angular displacement. (a) Show that the motion is simple harmonic and determine its time period. (b) What is the work done required to turn the system through an angle (i) π/2 (ii) π ?

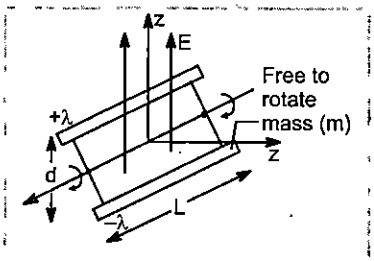


Fig. 1E.94 (a)

Solution: (a) The figure shows a side view of the system, it can be represented as an electric dipole of dipole moment

$$|\vec{\mathbf{p}}| = (\lambda L)\,d$$

The time period of oscillation of dipole is

$$T = 2\pi\sqrt{\frac{I}{pE}}$$

$$= 2\pi\sqrt{\frac{md^2/4 + md^2/4}{(\lambda L d)E}}$$

$$= 2\pi\sqrt{\frac{md}{2\lambda LE}}$$

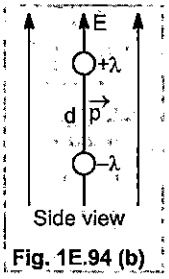


Fig. 1E.94 (b)

(b) Work done required to turn the dipole from initial angle θ_i to θ_f is

$$W = pE(\cos\theta_i - \cos\theta_f)$$

(i) For $\theta_i = 0,\ \theta_f = \pi/2,\quad W = (\lambda L d)E[1 - 0]$

$$= \lambda L d E$$

(ii) For $\theta_i = 0,\ \theta_f = \pi,\quad W = \lambda L d E(\cos 0 - \cos\pi) = 2\lambda L d E$

Example 95

Fig. 1E.95 (a) shows a charged rod, bent in the form of an arc of a circle. The charge distribution on the rod is shown in Fig. 1E.95 (a). The assembly is kept in a uniform electric field. (a) Show that for small angular displacement the system will perform SHM. Determine its time period. (b) Considering the size of the system to be very small, determine the work done required to turn it through 180°. Neglect any displacement of centre of charge.

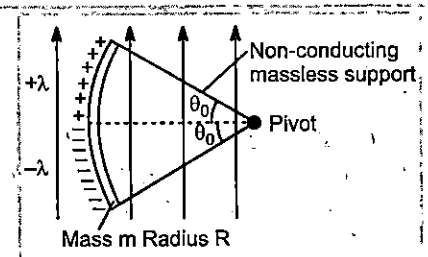


Fig. 1E.95 (a)

Solution: (a) We consider two differential elements on the rod as shown in Fig. 1E.95 (b). These elements constitute an electric dipole whose dipole moment is

$$|d\vec{\mathbf{p}}| = (\lambda R\, d\theta)(2R\sin\theta)$$

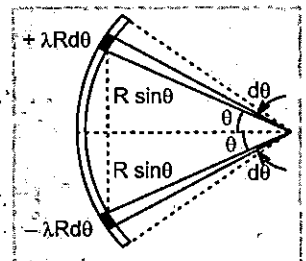


Fig. 1E.95 (b)

Net dipole moment of the system is

$$|\vec{\mathbf{p}}| = \int_0^{2\theta_0} (\lambda R\, d\theta)\, 2R\sin\theta$$

$$= 2\lambda R^2(1-\cos 2\theta_0) = 4\lambda R^2 \sin^2\theta_0$$

Time period of oscillation of a dipole in a uniform magnetic field is

$$T = 2\pi\sqrt{\frac{I}{pE}}$$

$$= 2\pi\sqrt{\frac{mR^2}{4\lambda R^2\sin^2\theta_0 E}}$$

$$= \pi\sqrt{\frac{m}{\lambda\sin^2\theta_0 E}}$$

(b) Work done required to rotate the dipole through 180° is

$$W = pE(\cos\theta_i - \cos\theta_f)$$

$$= pE(\cos 0 - \cos 180°)$$

$$= 2pE = 2(4\lambda R^2\sin^2\theta_0)E$$

$$= 8\lambda R^2\sin^2\theta_0 E$$

Example 96

A dipole with an electric moment P is located at a distance r from a long thread charged uniformly with a linear density λ. Find the force $\vec{\mathbf{F}}$ acting on the dipole if the vector $\vec{\mathbf{p}}$ is oriented:

(a) along the thread

(b) along the radius vector $\vec{\mathbf{r}}$

(c) at right angles to the thread and the radius vector $\vec{\mathbf{r}}$.

Solution: (a) In this case, the resultant force on the dipole is zero as shown in Fig. 1E.96 (a)

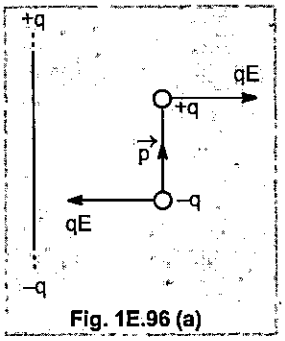


Fig. 1E.96 (a)

(b) The electric field at the position of negative charge $(-q)$,

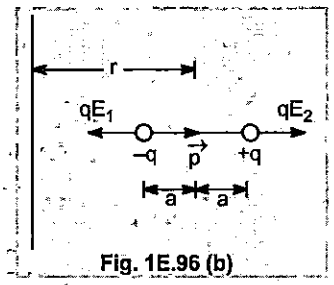


Fig. 1E.96 (b)

$$E_1 = \frac{\lambda}{2\pi\varepsilon_0(r-a)}$$

The electric field at the position of positive charge

$$E_2 = \frac{\lambda}{2\pi\varepsilon_0(r+a)}$$

Resultant force on the dipole is

$$F = qE_1 - qE_2$$

$$= \frac{q\lambda}{2\pi\varepsilon_0}\left[\frac{1}{r-a} + \frac{1}{r+a}\right]$$

$$= \frac{q\lambda}{2\pi\varepsilon_0}\frac{2a}{(r^2-a^{2)}}$$

For $r >> a$, $F \simeq \frac{(2aq)\lambda}{2\pi\varepsilon_0 r^2} = \frac{p\lambda}{2\pi\varepsilon_0 r^2}$

Alternatively, the electric force on an electric dipole in a nonuniform field is given by

$$F = p\frac{dE}{dr}$$

$$= p\frac{d}{dr}\left(\frac{\lambda}{2\pi\varepsilon_0 r}\right)$$

$$= -\frac{p\lambda}{2\pi\varepsilon_0 r^2}$$

or $\vec{\mathbf{F}} = -\frac{\lambda\vec{\mathbf{p}}}{2\pi\varepsilon_0 r^2}$ [force is attractive]

(c) The magnitude of electric field at the position of two charges is

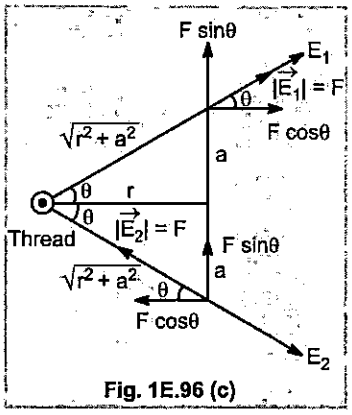


Fig. 1E.96 (c)

$$|\vec{E_1}|=|\vec{E_2}|=\frac{\lambda}{2\pi\varepsilon_0\sqrt{r^2+a^2}}$$

and force on the charges is

$$|\vec{F_1}|=|\vec{F_2}|=F=\frac{q\lambda}{2\pi\varepsilon_0\sqrt{r^2+a^2}}$$

As shown in Fig. 1E.96 (c), resultant force on the dipole is

$$F_R=2F\sin\theta$$
$$=\frac{2q\lambda}{2\pi\varepsilon_0\sqrt{r^2+a^2}}\frac{a}{\sqrt{r^2+a^2}}$$

For $r >> a$ and $2aq = p$, $F_R \simeq \frac{p\lambda}{2\pi\varepsilon_0 r^2}$

Resultant force is parallel to dipole moment.

$$\vec{F_R}=\frac{\lambda\vec{p}}{2\pi\varepsilon_0 r^2}$$

Example 97

A thin non-conducting ring of radius R has a linear charge density $\lambda = \lambda_0 \cos\theta$, where λ_0 is the value of λ at $\theta = 0$. Find the net electric dipole moment for this charge distribution.

Solution: Consider two differential charge elements at *A* and *B* as shown in Fig. 1E.97 (b). Dipole moment of this pair

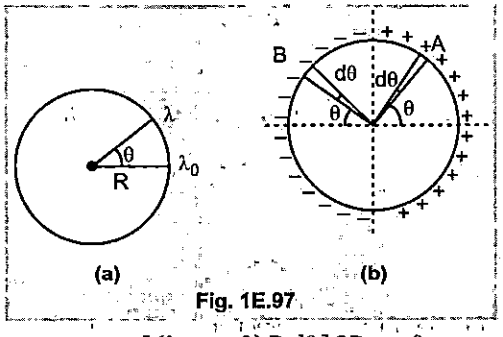


Fig. 1E.97

$$=[(\lambda_0\cos\theta)R\,d\theta]\,2R\cos\theta$$
$$=2\lambda_0R^2\cos^2\theta\,d\theta$$

∴ Dipole moment of the charge distribution

$$=2\lambda_0R^2\int_{-\pi/2}^{+\pi/2}\cos^2\theta\,d\theta=\pi R^2\lambda_0$$

Example 98

In Fig. 1E.98 shown, an electric dipole lies at a distance x from the centre of the axis of a charged ring of radius R with charge Q uniformly distributed over it.

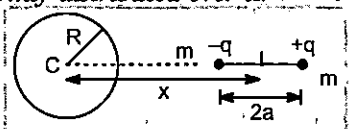


Fig. 1E.98

(a) Find the net force acting on the dipole.
(b) What is the work done in rotating the dipole through 180°?
(c) The dipole is slightly rotated about its equilibrium position. Find the time period of oscillation. Assume that the dipole is linearly restrained.

Solution: (a) Field at a distance x on the axis of a ring is given by

$$E=\frac{1}{4\pi\varepsilon_0}\frac{Qx}{(R^2+x^2)^{3/2}}$$

The net force on the dipole is $F=p\frac{dE}{dx}$

$$=2aq\frac{1}{4\pi\varepsilon_0}Q\frac{d}{dx}\left[\frac{x}{(R^2+x^2)^{3/2}}\right]$$
$$=\frac{aQq}{2\pi\varepsilon_0}\frac{R^2-2x^2}{(R^2+x^2)^{5/2}}$$

(b) Work done in rotating a dipole is equal to change in its potential energy

$$W=\Delta U$$
$$U_i=-\vec{p}\cdot\vec{E}=-pE\cos0=-pE$$
$$U_f=-\vec{p}\cdot\vec{E}=-pE\cos180°=+pE$$
$$W=2pE$$
$$W=\frac{2(2a)}{4\pi\varepsilon_0}\frac{Qx}{(R^2+x^2)^{3/2}}$$
$$=\frac{aQqx}{\pi\varepsilon_0(R^2+x^2)^{3/2}}$$

(c) Restoring torque $\tau=-pE\sin\theta\approx-pE\theta$ (for small θ)

$$I\frac{d^2\theta}{dt^2}=2(ma)^2\frac{d^2\theta}{dt^2}$$

or $$\frac{d^2\theta}{dt^2}=-\frac{pE}{2ma^2}\theta \quad \text{or} \quad T=2\pi\sqrt{\frac{2ma^2}{pE}}$$
$$=2\pi\sqrt{\frac{2ma^2}{2aq}\frac{(R^2+x^2)^{3/2}4\pi\varepsilon_0}{Qx}}$$
$$=\sqrt{\frac{16\pi^2\varepsilon_0 ma}{qQx}(R^2+x^2)^{3/2}}$$

Example 99

An infinite dielectric sheet having charge density σ has a hole of radius R in it. An electron is released on the axis of the hole at a distance $\sqrt{3}R$ from the centre. Find the speed with which it crosses the plane of the sheet.

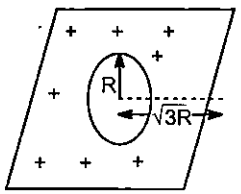


Fig. 1E.99 (a)

Solution: Electric field due to infinite dielectric sheet

$$E_1 = \frac{\sigma}{2\varepsilon_0}$$

Electric field at the axis of a disc of radius R,

$$E_2 = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{\sqrt{x^2+R^2}}\right]$$

The infinite charge sheet with a circular hole can be considered as superposition of an infinite sheet and charged disc of charge density $-\sigma$. From principle of superposition,

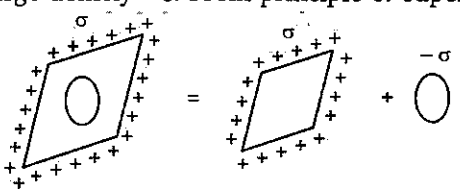


Fig. 1E.99 (b)

Resultant electric field

$$E = E_1 - E_2 = \frac{\sigma}{2\varepsilon_0}\frac{x}{\sqrt{x^2+R^2}}$$

Force on the electron,

$$F = -\frac{\sigma e x}{2\varepsilon_0\sqrt{x^2+R^2}}$$

$$mv\frac{dv}{dx} = -\frac{\sigma e x}{2\varepsilon_0\sqrt{x^2+R^2}}$$

$$m\int_0^v v\,dv = -\frac{\sigma e}{2\varepsilon_0}\int_{\sqrt{3R}}^{0}\frac{x}{\sqrt{x^2+R^2}}dx$$

$$m\frac{v^2}{2} = -\frac{\sigma e}{2\varepsilon_0}\left[\sqrt{x^2+R^2}\right]_{\sqrt{3R}}^{0}$$

$$v = \sqrt{\frac{\sigma eR}{m\varepsilon_0}}$$

Example 100

A charge Q is uniformly distributed on the surface of an angular disc of inner radius R and outer radius ηR. A charged particle of mass m and charge $-q$ is released from rest at a point on the axis of the ring far from the centre of the ring.
(a) With what velocity will it reach the centre of the ring ?
(b) What will be the period of oscillations if the charged particle is released from a point on the axis at a small distance from the centre of the ring?

Solution: (a) Potential at the centre of ring O is

$$V_0 = \int\frac{k\,dq}{r}$$

where $$dq = \frac{Q\,2\pi r\,dr}{\pi R^2(\eta^2-1)}$$

$$V_0 = \frac{2kQ}{(\eta+1)R}$$

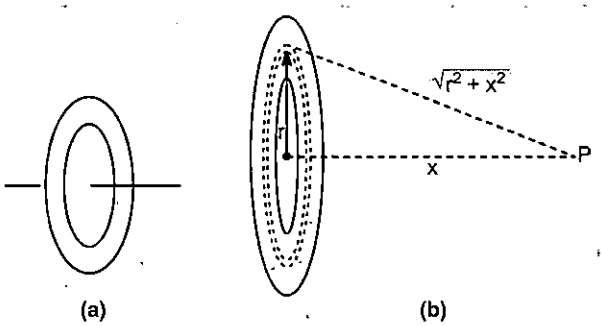


Fig. 1E.100

From energy conservation, we have

$$\frac{1}{2}mv^2 - qV_0 = 0$$

Thus $$v = \sqrt{\frac{1}{\pi\varepsilon_0}\frac{Qq}{mR(\eta+1)}}$$

(b) Potential at the point P,

$$V = k\int\frac{dq}{(r^2+x^2)^{1/2}} = \frac{2kQ\pi}{\pi R^2(\eta^2-1)}\int_R^{\eta R}\frac{r\,dr}{(r^2+x^2)^{1/2}}$$

$$= \frac{2kQ}{R^2(\eta^2-1)}[(\eta^2R^2+x^2)^{1/2} - (R^2+x^2)^{1/2}]$$

$$V = \frac{2kQ}{R^2(\eta^2-1)}(n-1)\left[R - \frac{x^2}{2R\eta}\right]$$

The energy $U = -qV$

$$F = \frac{-dU}{dx} = \frac{d}{dx}\left[\frac{2kqQ}{R^2(\eta+1)}\left(R - \frac{x^2}{2R\eta}\right)\right]$$

$$= \frac{-d}{dx}\left[\frac{2kQq}{R^3\eta(\eta+1)}x^2\right] = \frac{-kQq}{R^3\eta(\eta+1)}2x$$

$$F = -\left(\frac{2kQq}{R^3\eta(\eta+1)}\right)x$$

$$a = -\left(\frac{2kQq}{R^3\eta(\eta+1)m}\right)x$$

Comparing with SHM equation, $a = -\omega^2 x$,

$$\omega^2 = \frac{2kQq}{R^3\eta(\eta+1)m}$$

$$T = 2\pi\sqrt{\frac{R^2\eta(\eta+1)m}{2kQq}}$$

Example 101

Two concentric rings are placed in a gravity free region in yz-plane, one of radius R carries a charge +Q and the second of radius 4R and charge –8Q distributed uniformly over it. Find the minimum velocity with which a point charge of mass m and charge –q should be projected from a point at a distance 3R from the centre of the ring on its axis so that it will reach to the centre of the rings.

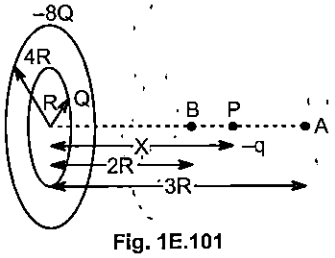


Fig. 1E.101

Solution: Note that at a point distant $3R$ charge $-q$ will have repulsive force but on points close to the centre it will have an attraction force. First we have to find out a point where the electric field is zero because, beyond that, the charge will have attraction force and will be attracted to the centre of the rings. Electric field at a point P distant x from the centre.

$$E_p = \frac{1}{4\pi\varepsilon_0}\frac{Qx}{(R^2+x^2)^{3/2}} - \frac{1}{4\pi\varepsilon_0}\frac{8Qx}{(16R^2+x^2)^{3/2}} = 0$$

On solving for x, we get

$$x = 2R$$

We have to give the charge enough KE so that it could reach this point because for $x < 2R$ the field will be attractive.

Potential at

$$A = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{(9R^2+R^2)^{1/2}} - \frac{8Q}{(9R^2+16R^2)^{1/2}}\right]$$

$$= -\frac{1}{4\pi\varepsilon_0}\left(\frac{16-\sqrt{10}}{10}\right)\frac{Q}{R}$$

Potential at point B

$$\frac{1}{4\pi\varepsilon_0}\frac{Q}{(4R^2+R^2)^{1/2}} - \frac{1}{4\pi\varepsilon_0}\frac{8Q}{(4R^2+16R^2)^{1/2}}$$

$$= -\frac{1}{4\pi\varepsilon_0}\frac{3}{\sqrt{5}}\frac{Q}{R}$$

From conservation of energy, we have

Now $$\frac{1}{2}mv^2 = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{R}\left[\frac{3}{\sqrt{5}} - \frac{16-\sqrt{10}}{10}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Qq}{R}\left[\frac{6\sqrt{5}+\sqrt{10}-16}{10}\right]$$

or $$v_{\min} = \left[\frac{2}{m}\cdot\frac{1}{4\pi\varepsilon_0}\frac{Qq}{R}\frac{\{6\sqrt{5}+\sqrt{10}-16\}}{10}\right]^{1/2}$$

Example 102

A small, charged bead can slide on a circular frictionless, insulating circular wire frame. A point-like electric dipole is fixed at the centre of the circle with the dipole's axis lying in the plane of the circle. Initially the bead is on the plane of symmetry of the dipole, as shown in Fig. 1E.102.
How does the bead move after it is released? Find the normal force exerted by the string on the bead. Where will the bead first stop after being released? How would the bead move in the absence of the wire frame? Ignore the effect of gravity, assuming that the electric forces are much greater than the gravitational ones.

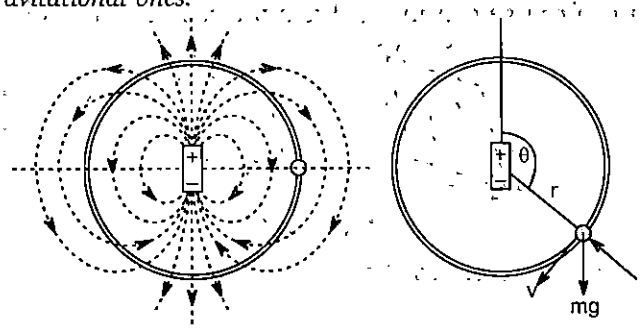


Fig. 1E.102

Solution: First of all, we apply the law of conservation of energy for a particle of mass m and charge Q:

$$\frac{1}{2}mv^2 + QK\frac{\cos\theta}{r^2} = \frac{1}{2}mv_0^2 + \frac{QK\cos(\pi/2)}{r^2} = 0$$

where $\frac{K\cos\theta}{r^2}$ is potential due to electric dipole on the bead.

We can then express the velocity of the bead at angle θ as

$$v = \sqrt{-2\frac{QK}{mr^2}\cos\theta}, \left(\frac{\pi}{2} \le \theta \le \pi\right) \quad ...(1)$$

The circular motion of the bead requires a centripetal force. The effect of the radial component of the electric field of dipole can be calculated as minus the derivative of the electric potential with respect to r.

$$E_r = -\frac{\partial V}{\partial r} = 2\frac{K\cos\theta}{r^3} \quad ...(2)$$

Using eqn. (1) for the velocity, we notice that QE_r is just equal to $-mv^2/r$, the required centripetal force. Thus the wire frame does not need to exert any force on the bead to sustain circular motion. If the string were not there, the bead would move along a circular path until it reached the point opposite its starting position. The bead would stop there, and then repeatedly retrace its path executing a periodic motion.

Example 103

Consider a metal sphere, of radius R that is cut in two along a plane whose minimum distance from the sphere's centre is h. Sphere is uniformly charged by a total electric charge Q. What force is necessary to hold the two parts of the sphere together ?

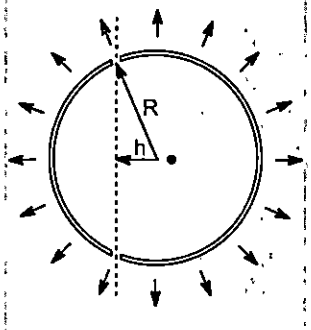


Fig. 1E.103

Solution: At the surface of the charged sphere, whether it consists of a single piece or two pieces close together, the electric field strength from Gauss's law is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}.$$

The electric charge per unit surface area is

$$\sigma = \frac{Q}{4\pi R^2}.$$

This electric field exerts a force $\Delta F = \frac{1}{2}E\,\Delta Q$ on the charge $\Delta Q = \sigma\,\Delta A$ which resides on a surface area ΔA, as illustrated in Fig. 1E.103. The reason for the factor of $\frac{1}{2}$ is that the electric field strength is E at the outer surface of the sphere and zero inside; its average value is therefore $E/2$.

The force per unit area exerted by the charges on the pieces of the sphere is therefore

$$\frac{\Delta F}{\Delta A} = \frac{Q^2}{32\pi^2\varepsilon_0 R^4} = p.$$

The required force can be considered to be analogous to the force with which a liquid at pressure p would push apart the two pieces of the sphere. As this force is also the product of p and the cross-sectional area of the intersection of the plane and sphere, *i.e.*, $p\pi(R^2-h^2)$, it follows that the two parts of the sphere can be held together by a force

$$F = \frac{Q^2}{32\pi\varepsilon_0 R^4}(R^2-h^2).$$

Example 104

Four point charges $+8\mu C, -1\mu C, -1\mu C$ and $+8\mu C$ are fixed at the points $-\sqrt{27/2}$ m, $-\sqrt{3/2}$ m, $+\sqrt{3/2}$ m and $+\sqrt{27/2}$ m respectively on the y-axis. A particle of mass 6×10^{-4} kg and of charge $+0.1\ \mu C$ moves along the $-x$ direction. Its speed at $x = +\infty$ is v_0. Find the least value of v_0 for which the particle will cross the origin. Find also the kinetic energy of the particle at the origin. Assume that space is gravity free. Given $1/(4\pi\varepsilon_0) = 9\times10^9\ Nm^2/C^2$.

Solution: The electric field at point P distant x from O,

$$\frac{1}{4\pi\varepsilon_0}\left[\frac{2\times8\times10^{-6}x}{\left(x^2+\frac{27}{2}\right)^{3/2}} - \frac{2\times1\times10^{-6}x}{\left(x^2+\frac{3}{2}\right)^{3/2}}\right]$$

which is zero at $x = \infty$, $x = 0$ and a point P at $x = x_0$.

$$\frac{8x_0}{\left(x_0^2+\frac{27}{2}\right)^{3/2}} = \frac{x_0}{\left(x_0^2+\frac{3}{2}\right)^{3/2}}$$

i.e., $$x_0 = \sqrt{5/2}\ \text{m}$$

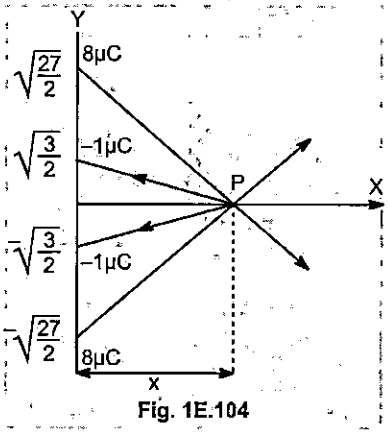


Fig. 1E.104

For $x > x_0$ electric force is repulsive and for $x < x_0$ electric force is attractive.

The particle moving towards origin will cross it, if it has sufficient energy to cross the point $x_0 = \sqrt{5/2}$ m, *i.e.*, its velocity v at this point ≥ 0.

For this to happen, $E_\infty \geq U_{x_0}$

where $$E_\infty = \frac{1}{2}mv_0^2,$$

and $$U_{x_0} = qV$$

where V is potential at $x = x_0$.

$$\frac{1}{2}mv_0^2 \geq \frac{1}{4\pi\varepsilon_0}\left(\frac{2\times 8\times 0.1\times 10^{-12}}{\sqrt{\frac{27}{2}+\frac{5}{2}}}-\frac{2\times 1\times 0.1\times 10^{-12}}{\sqrt{\frac{3}{2}+\frac{5}{2}}}\right)$$

$$v_0^2 \geq \frac{2\times 9\times 10^9}{6\times 10^{-4}}[4\times 10^{-13}-1\times 10^{-13}]$$

$$\geq 9$$

$\therefore$ Minimum value of $v_0 = 3$ m/s

Now, applying conservation of energy, taking $v_0 = 3$ m/s, the kinetic energy at the origin,

$$U_\infty + K_\infty = U_0 + K_0$$

$$0+\frac{1}{2}mv_0^2 = \frac{1}{4\pi\varepsilon_0}\left[\frac{2\times 8\times 0.1\times 10^{-12}}{\sqrt{27/2}}-\frac{2\times 1\times 0.1\times 10^{-12}}{\sqrt{3/2}}\right]+K_0$$

$$K_0 = \frac{1}{2}\times 6\times 10^{-4}\times 3^2 - \frac{9\times 10^9\times 10^{-12}}{3\sqrt{3/2}}$$

$$= 27\times 10^{-4} - 24.5\times 10^{-4} = 2.5\times 10^{-4}\ \text{J}$$

THE ENERGY OF A POINT CHARGE DISTRIBUTION

We now wish to find out, how much work would it take to assemble an entire collection of point charges? Imagine bringing in the charges, one by one, from far away. [Fig. 1.153]. The first charge, q_1, takes no work, since there is no field against which work can be performed. Now bring in q_2.

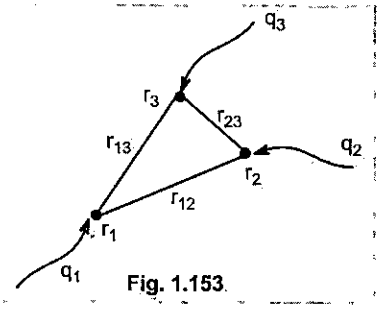


Fig. 1.153

This will require work $W_1 = q_2V_1(r_2)$, where V_1 is the potential due to q_1 and r_2 is the place we're putting q_2.

$$W_2 = \frac{1}{4\pi\varepsilon_0}q_2\left(\frac{q_1}{r_{12}}\right)$$

Where r_{12} is the distance between q_1 and q_2 once they are in position. Now bring in q_3; this requires work $q_3V_{12}(r_3)$, where V_{12} is the potential due to charges q_1 and q_2, namely, $(1/4\pi\varepsilon_0)(q_1/r_{13} + q_2/r_{23})$. Thus

$$W_3 = \frac{1}{4\pi\varepsilon_0}q_3\left(\frac{q_1}{r_{13}}+\frac{q_2}{r_{23}}\right)$$

Similarly, the extra work to bring in q_4 will be

$$W_4 = \frac{1}{4\pi\varepsilon_0}q_4\left(\frac{q_1}{r_{14}}+\frac{q_2}{r_{24}}+\frac{q_3}{r_{34}}\right)$$

The total work necessary to assemble the first four charges, then, is

$$W = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1q_2}{r_{12}}+\frac{q_1q_3}{r_{13}}+\frac{q_1q_4}{r_{14}}+\frac{q_2q_3}{r_{23}}+\frac{q_2q_4}{r_{24}}+\frac{q_3q_4}{r_{34}}\right)$$

Thus in general we can say that

$$W = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\sum_{\substack{j=1\\ j>i}}^{n}\frac{q_iq_j}{r_{ij}} \qquad ...(1)$$

The stipulation $j > i$ is just to remind you not to count the same pair twice. Also we can write it as

$$W = \frac{1}{8\pi\varepsilon_0}\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}\frac{q_iq_j}{r_{ij}} \qquad ...(2)$$

Now, we take q_i out of summation.

$$W = \frac{1}{2}\sum_{i=1}^{n}q_i\left(\sum_{\substack{j=1\\ j\neq i}}^{n}\frac{1}{4\pi\varepsilon_0}\frac{q_j}{r_{ij}}\right)$$

The term in parentheses is the potential at point $\vec{\mathbf{r}}_i$ (the position of q_i) due to all the other charges. Thus,

$$W = \frac{1}{2}\sum_{i=1}^{n}q_iV(\vec{\mathbf{r}}_i) \qquad ...(3)$$

It is also the amount of work you would get back out if you disassemble the system, it represents energy stored in the configuration (potential energy).

For a continuous distribution of charges over a region R, the calculus generalization of our formula is obviously

$$U = \frac{1}{2}\int_R V\,dq = \frac{1}{2}\int_R \rho V\,dV$$

where ρ is the charge density and $dV = dx\,dy\,dz$ the element of volume.

The electric potential energy of an isolated metal sphere of radius R with total charge Q can be obtained as below.

Recalling that the spherical surface is an equipotential with $V = Q/(4\pi\varepsilon_0R)$, we have

$$U = \frac{1}{2}V\int_{\text{surface}}dq = \frac{1}{2}VQ = \frac{Q^2}{8\pi\varepsilon_0R}$$

More directly, we can suppose the charge brought in from infinity in tiny increments dq. If charge q has already been assembled, the work needed to bring in the next dq is $dU = V(q)dq = (q/4\pi\varepsilon_0R)dq$. Integrating from $q = 0$ to $q = Q$ gives $U = Q^2/8\pi\varepsilon_0R$, as before.

The Energy of a Continuous Charge Distribution

For a volume charge density ρ, eqn. (3) becomes

$$W = \frac{1}{2}\int \rho V\,dV$$

It can be proved: [advanced level] that

$$W = \frac{\varepsilon_0}{2}\int_{\text{all space}}E^2dV$$

The corresponding integrals for line and surface charges would be $\int \lambda V\,dl$ and $\int \sigma V\,da$, respectively.

Example 105

Find the energy of a uniformly charged spherical shell of total charge q and radius R.

Solution: Inside the sphere $\vec{\mathbf{E}} = 0$; outside.

$$\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{\mathbf{r}}$$

so $$E^2 = \frac{q^2}{(4\pi\varepsilon_0)^2 r^4}$$

Therefore,

$$W_{\text{total}} = \frac{\varepsilon_0}{2(4\pi\varepsilon_0)^2}\int_{\text{outside}}\left(\frac{q^2}{r^4}\right)(r^2\sin\theta\, dr\, d\theta\, d\phi)$$

$$= \frac{1}{32\pi^2\varepsilon_0}q^2 4\pi\int_R^\infty \frac{1}{r^2}dr = \frac{1}{8\pi\varepsilon_0}\frac{q^2}{R}$$

Concept: *Electrostatic Energy*

(i) Equation $U = \frac{\varepsilon_0}{2}\int_{\text{all space}} E^2 dV$ *clearly implies that the energy of a stationary charge distribution is always positive. whereas the energy of two equal but opposite charges a distance r apart would be* $-(1/4\pi\varepsilon_0)(q^2/r)$.

Both equations are correct, but they pertain to slightly different situations. Equation of total energy U does not take into account the work necessary to make the point charges earlier. We simply found the work required to bring them together. The energy of a point charge is in fact infinite.

$$U = \frac{\varepsilon_0}{2(4\pi\varepsilon_0)^2}\int\left(\frac{q^2}{r^4}\right)(r^2\sin\theta\, dr\, d\theta\, d\phi)$$

$$= \frac{q^2}{8\pi\varepsilon_0}\int_0^\infty \frac{1}{r^2}dr = \infty$$

Equation for U tells you the total energy stored in a charge configuration, but eqn. $U = \frac{1}{2}\sum_{i=1}^{n} q_i V(\vec{\mathbf{r}}_i)$ *is more appropriate while dealing with point charges, because we leave out that portion of the total energy that is attributable to the assembly of the point charges themselves.*

In practice, after all, the point charges are given to us all we do is to assemble them. In eqn. $U = \frac{1}{2}\sum_{i=1}^{n} q_i V(\vec{\mathbf{r}}_i)$ *and* $W = \frac{\varepsilon_0}{2}\int E^2 dV$.

In the former, $V(\vec{\mathbf{r}}_i)$ *represents the potential due to all the other charges but not* q_i. *whereas in the latter,* $V(\vec{\mathbf{r}})$ *is the full potential.*

(ii) Where is the energy stored ?

It is useful to regard the energy as being stored in the field, with a density

$\frac{\varepsilon_0}{2}E^2$ *= energy per unit volume*

But in electrostatics one could just as well say it is stored in the charge with a density $\frac{1}{2}\rho V$.

ENERGY OF ELECTRIC FIELD

For a system of three point charges, the energy of interaction is expressed as

$$U = U_{12} + U_{13} + U_{23}$$

For a group of charges,

$$U = \frac{1}{2}\sum_{i=1}^{n} W_i = \frac{1}{2}\sum q_i V_i$$

where q_i is the ith charge of system and V_i is the potential at the position of ith charge created by all the remaining charges. Consider a system of four similar point charges located at the vertices of a tetrahedron with an edge a. The total number of interacting pairs is six, and for each pair energy of intersection is $q^2/4\pi\varepsilon_0 a$. Thus energy of intersection of all point charges of the system is

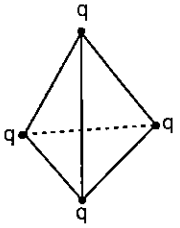


Fig. 1.154 (a)

$$U = \frac{6q^2}{4\pi\varepsilon_0 a}$$

Alternatively, the potential at the location of one of the charges due to other charges is $V = 3q/4\pi\varepsilon_0 a$. Hence

$$U = \frac{1}{2}\sum_{i=1}^{4} q_i V_i$$

$$= \frac{1}{2}4qV = \frac{6q^2}{4\pi\varepsilon_0 a}$$

- For a continuous charge distribution the summation becomes integration

$$U = \frac{1}{2}\int V\, dq$$

- Consider a system of two small balls having charges q_1 and q_2 respectively. If the separation between the balls is considerably larger than the dimensions of the ball so that q_1 and q_2 can be assumed point charges. The energy of the system is

$$U = \frac{1}{2}(q_1 V_1 + q_2 V_2)$$

where V_1 and V_2 are the potentials created by charge q_2 or q_1 at the position of the other charge.

Another energy involved in the system is intrinsic energy of the charged balls. Energy required to assemble differential charges dq by bringing them from infinity to assemble a charged ball. Thus total energy of system is

$$U = U_1 + U_2 + U_{12}$$

where U_1 = intrinsic energy of ball 1, U_2 = intrinsic energy of ball 2

U_{12} = interaction energy of balls 1 and 2

- Energy density of electric field, *i.e.,* energy per unit volume in a region where field E exists, is given by

$$u_E = \frac{1}{2}\varepsilon_0 E^2$$

- Consider a spherical shell having charge Q and radius R. Let the instantaneous charge on the shell be q. Work done by an external agent in slowly bringing a charge from infinity and assembling on the surface of the shell is

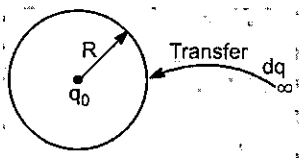


Fig. 1.154 (b)

$$dW = V\,dq = \frac{q}{4\pi\varepsilon_0 R}dq$$

Net work done in charging the shell is

$$W = \int_0^Q \frac{q}{4\pi\varepsilon_0 R}dq = \frac{Q^2}{4\pi\varepsilon_0 R}$$

This work done is electrical potential energy or self energy of the charged sphere.

Example 106

A spherical shell of radius R_1 with uniform charge q is expanded to a radius R_2. Find the work performed by the electric forces in this process.

Solution: The initial self potential energy of the spherical shell is

$$U_i = \frac{q^2}{8\pi\varepsilon_0 R_1}$$

The final self potential energy of the spherical shell is

$$U_f = \frac{q^2}{8\pi\varepsilon_0 R_2}$$

$$\therefore \quad \Delta U = U_f - U_i = \frac{q^2}{8\pi\varepsilon_0}\left(\frac{1}{R_2} - \frac{1}{R_1}\right)$$

Work done by electrical forces is

$$W = -\Delta U = \frac{q^2}{8\pi\varepsilon_0}\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

Example 107

A spherical shell of radius R_1 with a uniform charge q has a point charge q_0 at its centre. Find the work performed by the electric forces during the shell expansion from radius R_1 to radius R_2.

Solution: The electrical potential energy of the system is :

U = self potential energy of shell + interaction energy of shell and point charge

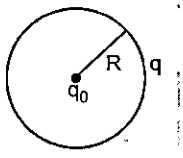


Fig. 1E.107

$$= \frac{q^2}{8\pi\varepsilon_0 R} + \frac{qq_0}{4\pi\varepsilon_0 R}$$

Initial potential energy of system is

$$U_i = \frac{q^2}{8\pi\varepsilon_0 R_1} + \frac{qq_0}{4\pi\varepsilon_0 R_1}$$

Final potential energy of system is

$$U_f = \frac{q^2}{8\pi\varepsilon_0 R_2} + \frac{qq_0}{4\pi\varepsilon_0 R_2}$$

From work-energy theorem,

$$W = -\Delta U = -(U_f - U_i)$$

$$W = U_i - U_f \qquad ...(1)$$

On substituting the values of U_i and U_f in eqn. (1), we get

$$A = \frac{q\left(q_0 + \frac{q}{2}\right)}{4\pi\varepsilon_0}\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

Example 108

A spherical shell is uniformly charged with the surface density σ. Using the energy conservation law, find the magnitude of the electric force acting on a unit area of the shell.

Solution: Electrical field is conservative in nature. For a conservative force,

$$F = \frac{-\partial U}{\partial r}$$

Here $U = \frac{q}{8\pi\varepsilon_0 r}$

or $F = \frac{-\partial U}{\partial r} = \frac{-\partial}{\partial r}\left(\frac{q}{8\pi\varepsilon_0 r}\right)$

or $F = \frac{-q}{8\pi\varepsilon_0}\frac{\partial(r^{-1})}{\partial r} = \frac{q^2}{8\pi\varepsilon_0 r^2}$

Force per unit area $= \frac{F}{4\pi r^2} = \frac{q^2}{8\pi\varepsilon_0 r^2 \times 4\pi r^2}$

$$= \frac{\sigma^2(4\pi r^2)^2}{8\pi\varepsilon_0 r^2 \times 4\pi r^2} = \frac{\sigma^2}{2\varepsilon_0}$$

Example 109

A point charge q is located at the centre O of a spherical uncharged conducting layer provided with a small orifice [Fig. 1E.109 (a)]. The inside and outside radii of the layer are equal to a and b respectively. What amount of work has to be performed to slowly transfer the charge q from the point O through the orifice and into infinity?

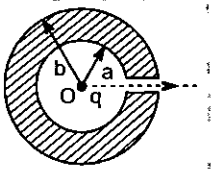


Fig. 1E.109 (a)

Solution: When point charge q is placed at the centre, then $-q$ and $+q$ charges are induced at the inner and outer surfaces of the conducting layer [shown in Fig. 1E. 109 (b)]

When the point charge is at the centre, the layer behaves as two concentric spheres A and B having respective charges $-q$ and $+q$.

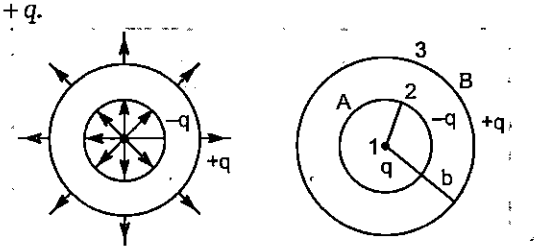


Fig. 1E.109 (b)

The equivalent system is shown in Fig. 1E.109 (b).

The total electrical energy of the system is

$$U_i = U_{12} + U_{23} + U_2 + U_3$$

Here U_{12} = Electrical interaction energy of point charge on hollow sphere A

$$= \frac{q(-q)}{4\pi\varepsilon_0 a} = \frac{-q^2}{4\pi\varepsilon_0 a}$$

U_{23} = Electrical interaction energy between hollow spheres A and B

= (Electric potential due to B on A) × charge on A

$$= \frac{q(-q)}{4\pi\varepsilon_0 b} = \frac{-q^2}{4\pi\varepsilon_0 b}$$

Similarly, $$U_{13} = \frac{q^2}{4\pi\varepsilon_0 b}$$

U_2 = Self potential energy of hollow sphere A

$$= \frac{-(q)^2}{8\pi\varepsilon_0 Q} = \frac{q^2}{8\pi\varepsilon_0 a}$$

Similarly, $$U_3 = \frac{q^2}{8\pi\varepsilon_0 b}$$

Hence $$U_i = \frac{-q^2}{4\pi\varepsilon_0 a} - \frac{q^2}{4\pi\varepsilon_0 b} + \frac{q^2}{4\pi\varepsilon_0 b} + \frac{q^2}{8\pi\varepsilon_0 a} + \frac{q^2}{8\pi\varepsilon_0 b}$$

$$= \frac{-q^2}{8\pi\varepsilon_0 a} + \frac{q^2}{8\pi\varepsilon_0 b} = \frac{q^2}{8\pi\varepsilon_0}\left(\frac{1}{b} - \frac{1}{a}\right)$$

When the point charge q is shifted to infinity, induced charges disappear.

Total electric potential energy of system is $U_f = 0$

Work done by external agent is

$$W_{\text{external}} = \Delta U = U_f - U_i$$

$$= 0 - \frac{q^2}{8\pi\varepsilon_0}\left(\frac{1}{b} - \frac{1}{a}\right) = \frac{q^2}{8\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$$

ENERGY FOR A CONTINUOUS DISTRIBUTION OF CHARGE

Energy Stored Inside a Uniformly Charged Sphere Charge

Let us consider a spherical shell of radius 'x' and thickness dx, energy stored in the differential shell.

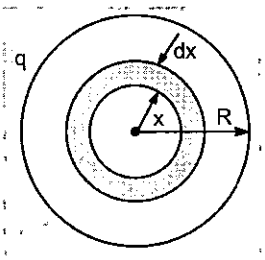


Fig. 1.155

dU = energy density in shell × volume due to thickness of the wall of the shell.

$$= \frac{1}{2}\varepsilon_0 E^2 \times 4\pi x^2 dx$$

Where inside is given by

$$E = \frac{1}{4\pi\varepsilon_0}\frac{qx}{R^3}$$

Hence $$dU = \frac{1}{2}\varepsilon_0 \frac{1}{(4\pi\varepsilon_0)^2}\frac{q^2x^2}{R^6} \times 4\pi x^2 dx$$

$$U = \frac{q^2}{8\pi\varepsilon_0 R^6}\int_0^R x^4 dx$$

$$U_{\text{inside}} = \frac{q^2}{40\pi\varepsilon_0 R}$$

Note that U_{inside} is energy stored in the space inside the sphere.

Similarly, energy stored outside the surface in an elemental shell of radius 'x' and thickness $dx (x \geq R)$ can also be obtained by integration.

$$dU = \frac{1}{2}\varepsilon_0 E^2 \times 4\pi x^2 dx$$

$$= \frac{1}{2}\varepsilon_0\left(\frac{1}{4\pi\varepsilon_0}\frac{q}{x^2}\right)^2 \times 4\pi x^2 dx$$

Note that electric field outside is given by $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{x^2}$

$$U = \int dU = \frac{q^2}{8\pi\varepsilon_0}\int_R^\infty \frac{1}{x^2}dx$$

$$U_{\text{outside}} = \frac{q^2}{8\pi\varepsilon_0 r}$$

Note that U_{outside} is the energy stored in the space outside the sphere.

Total self energy of the sphere,

$$U = U_{\text{inside}} + U_{\text{outside}} = \frac{3}{5}\left(\frac{q^2}{4\pi\varepsilon_0 r}\right)$$

also note that

$$U_{\text{outside}} = 5U_{\text{inside}}$$

Alternate method

We can follow conventional method.

Let us consider that the charges are being brought from infinity to form the sphere.

Let at any time charge q has already been brought to form a sphere of radius 'x' and further a very small charge dq is brought from infinity and assembled on the surface of sphere which increases its radius by 'dx'

Electrical potential energy of this charge 'dq' is given by,

$dU = dq \times$ potential at the position of charge dq.

$$dU = dq \times dV = dq\left(\frac{1}{4\pi\varepsilon_0}\frac{q}{x}\right)$$

$$dU = \frac{1}{4\pi\varepsilon_0}\frac{1}{x}qdq$$

If 'Q' is the total charge to be brought to form the complete sphere of radius R. Then the charge density ρ is given by,

$$\rho = \frac{Q}{\frac{4}{3}\pi R^3}$$

$$dU = \frac{1}{4\pi\varepsilon_0 x}\frac{4}{3}\pi x^3\rho(4\pi x^2\rho)dx$$

$$\Rightarrow \quad dU = \frac{4\pi\rho^2}{3\varepsilon_0}x^4dx$$

Total electrical potential energy,

$$U = \int dU = \int_0^R \frac{4\pi\rho^2}{3\varepsilon_0}x^4dx$$

$$\Rightarrow \quad U = \frac{3}{5}\left(\frac{q^2}{4\pi\varepsilon_0 R}\right)$$

Self and Interaction Energy of Two Spheres Placed at a Sufficient Distance Apart

Total energy of the system = *self energies of the spheres + interaction energy.*

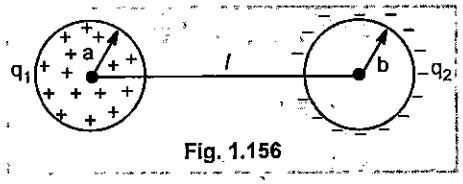


Fig. 1.156

For interaction energy, we can treat the two spheres as point charges in this case.

Hence total energy,

$$= \frac{3}{5}\left(\frac{q_1^2}{4\pi\varepsilon_0 a}\right) + \left(\frac{q_2^2}{8\pi\varepsilon_0 b}\right) + \frac{1}{4\pi\varepsilon_0}\frac{(-q_1q_2)}{l}$$

Concept: *Self or intrinsic energy is always positive. The interaction energy may be positive or negative.*

Example 110

If a small piece of radius b is removed from a charged spherical shell of radius a ($>> b$), calculate electric intensity at the mid-point of the aperture, assuming the density of charge to be σ.

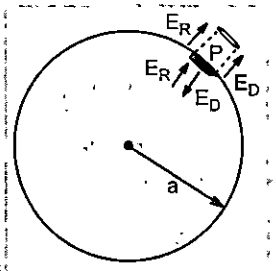


Fig. 1E.110

Solution: Consider the shell to be made up of a disc of radius b and the remainder. If E_D and E_R are the intensities due to disc and the remainder respectively at P, then for a charged spherical shell (or conductor).

$$E_{out} = \frac{\sigma}{\varepsilon_0} \quad \text{and} \quad E_{in} = 0 \qquad ...(1)$$

Now as for outside the shell both E_D and E_R will be directed outwards while inside E_R will be outwards while E_D inwards so that :

$$E_{out} = E_R + E_D \text{ and } E_{in} = E_R - E_D \qquad ... (2)$$

And hence equating eqn. (1) and (2),

$$E_R + E_D = \frac{\sigma}{\varepsilon_0} \quad \text{and} \quad E_R - E_D = 0$$

Solving these for E_R and E_D :

$$E_R = E_D = \frac{\sigma}{2\varepsilon_0}$$

i.e., field at the aperture will be $(\sigma / 2\varepsilon_0)$ directed outwards.

Note: As intensity on the disc (element) the to remainder is $(\sigma/2\varepsilon_0)$, electric force on it will be,

$$dF = dqE = (\sigma ds)\left[\frac{\sigma}{2\varepsilon_0}\right] = \left[\frac{\sigma^2}{2\varepsilon_0}\right]ds$$

So force per unit area on a charged conductor due to its own charge

$$\frac{dF}{ds} = \frac{\sigma^2}{2\varepsilon_0} = \frac{1}{2}\varepsilon_0 E^2 \qquad \left[\text{as for a conductor } E = \frac{\sigma}{\varepsilon_0}\right]$$

This force is called 'mechanical force' or electrostatic pressure.

Concepts:

1. *The reference point O. There is an ambiguity in the definition of potential the choice of reference point O for calculating potential difference was arbitrary. Changing reference points amounts to adding a constant C to the potential.*

$$V'(\vec{\mathbf{r}}) = -\int_{O'}^{r} \vec{\mathbf{E}}.d\vec{\mathbf{l}}$$

$$= -\int_{O'}^{O} \vec{\mathbf{E}}.d\vec{\mathbf{l}} - \int_{O}^{r} \vec{\mathbf{E}}.d\vec{\mathbf{l}} = C + V(\vec{\mathbf{r}})$$

where C is the line integral of E from the old reference point O to the new one O'. Note that adding a constant to V will not affect the potential difference between two points :

$$V'(b) - V'(a) = V(b) - V(a)$$

Since the C's cancel out. The potential difference is independent of O because it can be written as the line integral of $\vec{\mathbf{E}}$ from a to b with no reference to O. Nor does the choice of reference frame affect the gradient of V :

$$\nabla V' = \nabla V,$$

Since the derivative of a constant is zero. That's why all such potentials differing only in their choice of reference point, correspond to the same field $\vec{\mathbf{E}}$.

Evidently potential of a point has no real physical significance, for at any given point we can assign its value by a suitable choice of reference point.

Just as sea level is reference point for all the calculation of altitude above surface of earth we have reference for potential. There is a common reference point to use for O in electrostatics— analogous to sea level for altitude— and that is a point infinitely far from the charge. Ordinarily, then, we "set the zero of potential at infinity". (Since V (O) = 0, Choosing a reference point is equivalent to selecting a place where V is to be zero.)

There is one special circumstance in which this convention fails; when the charge distribution itself extends to infinity. For instance, the field of a uniformly charged plane is $(\sigma/2\varepsilon_0)\hat{\mathbf{n}}$. If we put $O = \infty$, then the potential at height z above the plane becomes

$$V(z) = -\int_{\infty}^{z} \frac{1}{2\varepsilon_0}\sigma\, dz$$

$$= -\frac{1}{2\varepsilon_0}\sigma(z - \infty)$$

The solution of this problem is simply to choose some other reference point (in this problem you might use the origin). In "real life" there is no charge distribution that goes on forever, and we can always use infinity as our reference point.

2. *The potential inside and outside a spherical shell of radius R, which carries a uniform surface charge. Set the reference point at infinity is given,*

For points outside the sphere (r > R).

$$V(r) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

The potential inside the sphere (r < R)

$$V(r) = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$$

Notice that the potential is not zero inside the shell, even though the field is, V is a constant in this region. In problems of this type you must always know the reference point : that's where the potential is zero. You may feel suppose that you could calculate the potential inside the sphere on the basis of the field there alone, but this is false. The potential inside the sphere depends on the charges present outside as well. If a second uniformly charged shell is placed at radius R' > R, the potential inside R would change, even though the field would still be zero. Gauss's law guarantees that charge placed exterior to a given point at larger r produces no net field at that point, provided it is spherically or cylindrically symmetric. But there is no such rule for potential, when infinity is used as the reference point.

3. *Setting the reference point at infinity, the potential of a point charge q at the origin is*

$$V(r) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

Notice the sign of V; presumably the conventional minus sign in the definition of V was chosen precisely in order to make the potential of a positive charge come out positive.

From the superposition principle, then, the potential of a collection of charges is

$$V(\vec{\mathbf{r}}) = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i}$$

or, for a continuous distribution,

$$V(\vec{\mathbf{r}}) = \frac{1}{4\pi\varepsilon_0}\int \frac{1}{r}\, dq$$

for a volume charge, it's

$$V(\vec{\mathbf{r}}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(\vec{\mathbf{r}})}{r}\, dV$$

The potential of line and surface charges are

$$\frac{1}{4\pi\varepsilon_0}\int \frac{\lambda(\vec{\mathbf{r}'})}{r}\, dl'$$

and

$$\frac{1}{4\pi\varepsilon_0}\int \frac{\sigma(\vec{\mathbf{r}'})}{r}\, da'$$

Everything in this section is predicated on the assumption that the reference point is at infinity. Remember that we got that equation from the potential of a point charge at the origin, $(1/4\pi\varepsilon_0)(q/r)$, which is valid only when $O = \infty$. If you try to apply these formulas to problems in which the charge itself extends to infinity it will not be applicable.

Conceptual Example 1.

On the impossibility of stable equilibrium of a charge in an electric field. Suppose that we have in vacuum a system of fixed point charges in equilibrium. Let us consider one of these charges, *e.g.* a charge q. Can its equilibrium be stable ?

Solution: In order to answer this question, let us envelop the charge q by a small closed surface S [Fig. 1.157 (a)]. For the sake of definiteness, we assume that $q > 0$. For the equilibrium of this charge to be stable, it is necessary that the field E created by all the remaining charges of the system at all the points of the surface S be directed towards the charge q. Only in this case any small displacement of the charge q from the equilibrium position will give rise to a restoring force and the equilibrium state will actually be stable. But such a configuration of the field $\vec{\mathbf{E}}$ around the charge q is in contradiction to the Gauss theorem; the flux of E through the surface S is negative, while in accordance with the Gauss theorem it must be equal to zero since it is created by charges lying outside the surface S. On the other hand, the fact that $\vec{\mathbf{E}}$ is equal to zero indicates that at some points of the surface S vector $\vec{\mathbf{E}}$ is directed inside it and at some other points it is directed outside.

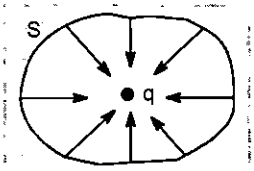


Fig. 1.157 (a)

Hence, it follows that in any electrostatic field a charge cannot be in stable equilibrium.

Conceptual Example 2.

The electric potential cannot assume a minimum (or a maximum) value inside a charge-free region.

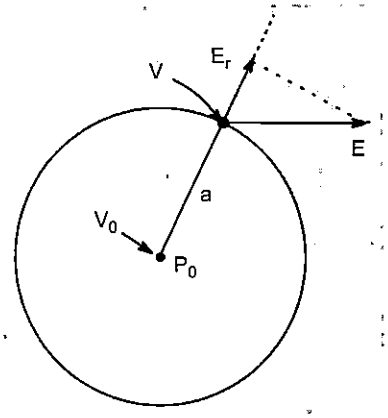


Fig. 1.157 (b)

Solution: Suppose that the potential has a local minimum value V_0, at the interior point P_0. Then [Fig. 1.157 (b)] we could enclose P_0 in a gaussian sphere of so small a radius, a, that (i) the sphere lies entirely within the charge-free region and (ii) $V \geq V_0$ at every point of the spherical surface. Gauss's law, applied to this sphere, would give

$$0 = \int_S \vec{\mathbf{E}}.d\vec{\mathbf{S}} = \int_S E_r dS \qquad \ldots (1)$$

where E_r is the normal (radial) component of the field at the surface of the sphere. But, by definition of the electric potential,

$$E_r = -\frac{dV}{dr} \qquad \ldots (2)$$

Allow the derivative in eqn. (2) to be approximated by

$$E_r = -\frac{dV}{dr} = -\frac{V - V_0}{a} \qquad \ldots (3)$$

and eqn. (1) becomes

$$0 = \int_S (V - V_0)\, dS \qquad \ldots (4)$$

But eqn. (4) is impossible: $V - V_0$ is non-negative at each point of S and so its integral over S must be positive. This contradiction establishes the desired result.

The implication is very significant: No charge placed in an electrostatic field can be in stable equilibrium, since that requires being at a minimum of potential energy. Note that unstable equilibrium does not demand a potential energy maximum, but only a saddle point.

PROPERTIES OF CONDUCTOR

A conductor is a material in which the electrons at the outer periphery of an atom have no great affinity for any particular individual atom; they are not bound or tied to individual atoms. These so-called conduction electrons are essentially free to move readily and quickly in response to electric fields.

We described how a piece of paper can be polarized by nearby charges. The polarization is the paper's response to an externally applied electric field. The separation of charge in the paper produces an electric field of its own. The net electric field at any point–whether inside or outside the paper–is the sum of the applied field and the field due to the separated charges in the paper.

How much charge separation occurs depends on both the strength of the applied field and properties of the atoms and molecules that make up the paper. Some materials are more easily polarized than others. The *most* easily polarized materials are conductors because they contain highly mobile charges that can move freely through the entire volume of the material.

In this article we restrict our attention to a conductor in which the mobile charges are at rest in equilibrium, a situation called **electrostatic equilibrium.**

1. When excess charge is placed on a solid conductor and is at rest, it resides entirely on the surface, not in the interior of the material. (By excess we mean charges other than the ions and free electrons that make up the neutral conductor.) In an electrostatic situation the electric field $\vec{\mathbf{E}}$ at every point in the interior of a conducting material is zero. If $\vec{\mathbf{E}}$ were *not* zero, the excess charges would move. Suppose

we construct a Gaussian surface inside the conductor, such as surface A in Fig. 1.158. Because $\vec{E} = 0$ everywhere on this surface, Gauss's law requires that the net charge inside the surface is zero. Now imagine the surface encloses a region so small region that we may consider it as a point P; then the charge at that point must be zero. We can do this anywhere inside the conductor, so *there can be no excess charge at any point within a solid conductor; any excess charge must reside on the conductor's surface.*

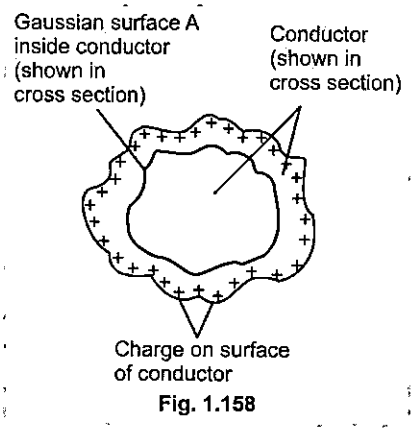


Fig. 1.158

2. $\rho = 0$ inside conductor. This follows from Gauss's law: $\vec{\nabla} \cdot \vec{E} = \rho/\varepsilon_0$. If $\vec{E} = 0$, so also is ρ. There is still charge present, but the *net* charge density in the interior is zero. *i.e.,* positive and negative charges exist in same magnitude.

A *perfect* conductor would be a material containing an *unlimited* supply of completely free charges. In real life there are no perfect conductors, but many substances come close.

3. $\vec{E} = 0$ inside a conductor. Because if there were any field, those free charges would move, and it wouldn't be electrostatics any more. When you put a conductor into an external electric field $\vec{E}_0$ (Fig. 1.159). Due to force of electric field, field will drive any free positive charges to the right, and negative ones to the left. Note that only the negative charges–electrons move but when they shift the right side is left with a net positive charge. When they come to the edge of the material, the charges pile up: plus on the right side, minus on the left.

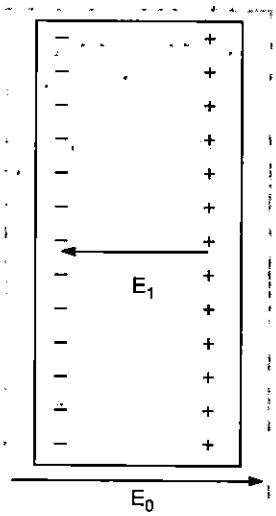


Fig. 1.159

Now, these **induced charges** produce a field of their own, $\vec{E}_1$, which is in the *opposite direction* to $\vec{E}_0$. It means that the field of the induced charges *tends to cancel off the original field.* Charge will continue to flow until this cancellation is complete, and the resultant field inside the conductor is precisely zero. Outside the conductor the field is *not* zero, for here E_0 and E_1 do not cancel. The whole process is practically instantaneous, it takes place with speed of light.

4. Surface of a conductor (in static equilibrium) is always an equipotential surface irrespective of the charge on the surface or of nearby charges.

A conductor is an equipotential. For if a and b are any two points within (or at the surface of) a given conductor,

$$V(a) - V(b) = -\int_a^b \vec{E} \cdot d\vec{l} = 0 \text{ and hence } V(a) = V(b).$$

Concept: *When a solid conductor in equilibrium carries a net charge, the charge resides on the conductor's outer surface, the electric field just outside the conductor is perpendicular to the surface and the field inside is zero.*

Consider two points a and b on the surface of a charged conductor. Along a surface path connecting these points, $\vec{E}$ is always perpendicular to the displacement $d\vec{l}$; therefore $\vec{E} \cdot d\vec{l} = 0$. Thus we conclude that the potential difference between a and b is necessarily zero:

$$V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{l} = 0$$

The surface of any charged conductor in electrostatic equilibrium is an equipotential surface. Because the electric field is zero inside the conductor, the electric potential is constant everywhere inside the conductor and equal to its value at the surface.

Because of the constant value of the potential, no work is required to move a test charge from the interior of a charged conductor to its surface.

The potential everywhere inside the conductor, including the surface, has the same value, which may or may not be zero, depending on where the zero of potential is defined.

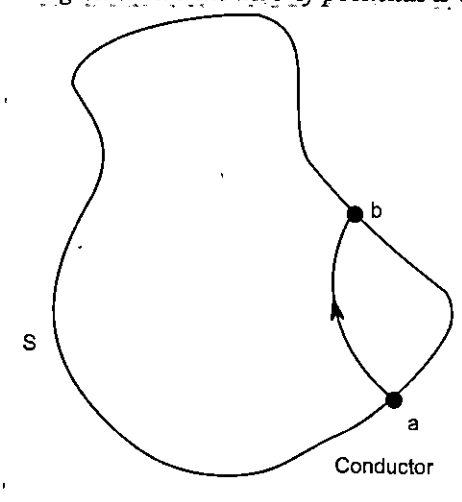


Fig. 1.160

Consider the conductor shown in Fig. 1.160 with surface S, and consider two points on the surface, a and b. We can use eqn. along any path leading from a to b to obtain $\Delta V = V_b - V_a$, including the path shown through the conductor.

For the path chosen, which is wholly in the conductor. E is zero everywhere along the path. Therefore, $\Delta V = 0 \rightarrow V_b = V_a$. Since this is true for all points a and b on the surface, the surface must be an equipotential. (Indeed, the whole conductor is an equipotential. by the same argument.)

5. Since the electric field $\vec{E}$ is always perpendicular to an equipotential surface. When all charges are at rest, the electric field just outside a conductor must be perpendicular to the surface at every point (Fig. 1.161) We know that $\vec{E} = 0$ everywhere inside the conductor; otherwise, charges would move. At any point just inside the surface the component of $\vec{E}$ tangent to the surface is zero. The tangential component of $\vec{E}$ is also zero just *outside* the surface a charge could move

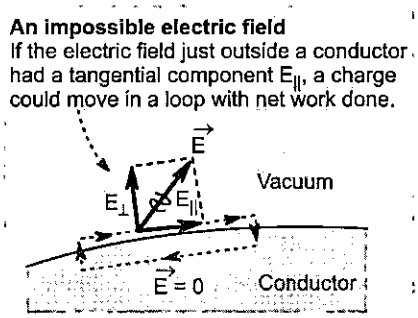


Fig. 1.161

around a rectangular path partly inside and partly outside and return to its starting point with a net amount of work having been done on it. This would violate the conservative nature of electrostatic fields, so the tangential component of $\vec{E}$ just outside the surface must be zero at every point on the surface. Thus $\vec{E}$ perpendicular to the surface at each point, proving our statement.

6. In an electrostatic situation, if a conductor contains a cavity and if no charge is present inside the cavity, then there can be no net charge *anywhere* on the surface of the cavity. *Every point in the cavity is at the same potential.* In Fig. 1.162 the conducting surface *A* of the cavity is an equipotential surface. Suppose point *P* in the cavity is at a different potential; then we can construct a different equipotential surface *B* including point *P*.

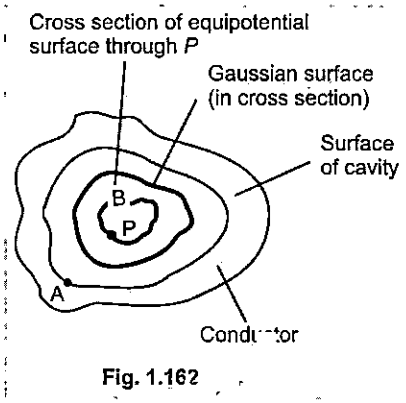


Fig. 1.162

Now consider a Gaussian surface, shown in Fig. 1.162 between the two equipotential surfaces. The field at every point between the equipotentials is from *A* toward *B*, or from *B* toward *A*, depending on which equipotential surface is at higher potential. In either case the flux through this Gaussian surface is certainly not zero. Then in accordance with Gauss's law the charge enclosed by the Gaussian surface cannot be zero. This contradicts our initial assumption that there is *no* charge in the cavity. So the potential at *P* must be same as that at the cavity wall.

The entire region of the cavity must therefore be at the same potential. But for this to be true, *the electric field inside the cavity must be zero everywhere.*

7. Suppose we place a small body with a charge *q* inside a cavity within a conductor (Fig. 1.163). consider a Gaussian surface in the material of conductor the conductor is uncharged and is insulated from the charge *q*. Again $\vec{E} = 0$ everywhere on surface *A*, so according to Gauss's law the *total* charge inside this surface must be zero. Therefore there must be a charge $-q$ distributed on the surface of the cavity. This charge appears due to induction of charge inside cavity. The *total* charge on the conductor must remain zero, so a charge $+q$ must appear either on its outer surface or inside the material. But in an electrostatic situation there can't be any excess charge within the material of a conductor. So we conclude that the charge $+q$ must appear on the outer surface. By the same reasoning, if the conductor originally had a charge q_C, then the total charge on the outer surface must be $q_C + q$ after the charge q is inserted into the cavity.

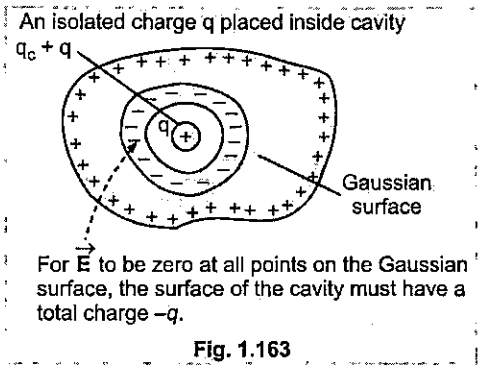


Fig. 1.163

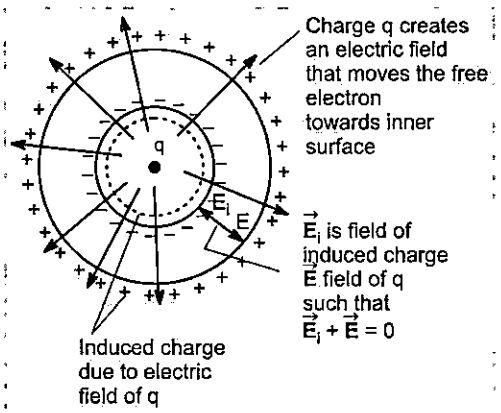


Fig. 1.164

Concept: *In an electrostatic situation, the electric field inside a conductor is zero. This means that the interior of a conductor is an example of an equipotential volume.*

The same is true if the conductor is hollow and there are no charge inside the hollow. To show this, imagine a Gaussian surface within the conductor surrounding the cavity, as shown in Fig. 1.165.

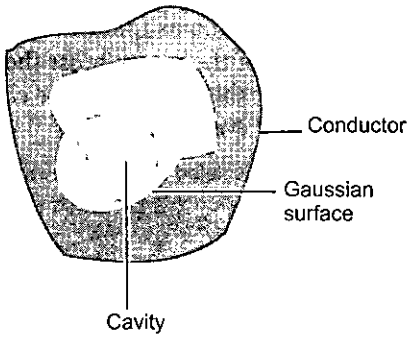


Fig. 1.165. Gaussian surface surrounding a hollow in a conductor.

Since the static electric field is zero within the conductor, the left-hand side of Gauss's law is zero, implying there is zero.

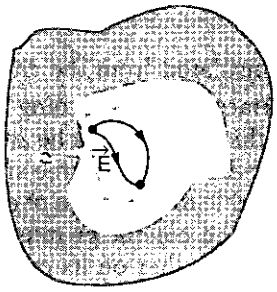


Fig. 1.166. Electric field lines from a positive to a negative charge

Total charge within the Gaussian surface. We imagine some positive charge to be on one side of the inside surface of the hollow conductor and some negative charge on the other, as shown in Fig. 1.166 so the total charge is zero, consistent with the prediction of Gauss's law. If such charge separation exists on the inside surface of the cavity, there will be electric field lines within the hollow from the positive to the negative charges.

If we use equation $dV = -\vec{\mathbf{E}} \cdot d\vec{\mathbf{r}}$ *and integrate the electric field along one of these field lines from a positive charge to a negative charge, the result will* not be zero *since* $d\vec{\mathbf{r}}$ *is parallel to* $\vec{\mathbf{E}}$ *over the whole path. This implies that there is a potential difference.*

Concept: *The charge on a conductor flows to the surface, because of their mutual repulsion, the charges naturally spread out as much as possible, but this is true regardless of the size or shape of the conductor. The charge on a conductor will seek the configuration that minimizes its potential energy. The electrostatic energy of a solid object (with specified shape and total charge) is a minimum when that charge is spread over the surface.*

For instance, the energy of a sphere is $(1/8\pi\varepsilon_0)(q^2/R)$ *if the charge is uniformly distributed over the surface, but it is greater,* $(3/20\pi\varepsilon_0)(q^2/R)$ *, if the charge is uniformly distributed throughout the volume.*

Concept: ***The electric field due to charges on the outer surface of conductor is zero for all the points inside the conductor separately.***

Consider a charged conductor having charge $+q_1$ *and* Q *is kept inside the cavity. Lets call charge* Q *inside cavity as A, the induced charge* $-Q$ *on the surface of the cavity as B and the charge on the surface of the conductor* $Q + q_1$ *as C.*

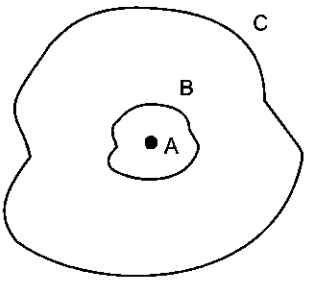


Fig. 1.167

Now field inside the conductor is, $\vec{\mathbf{E}}_{net}$ *and* $\vec{\mathbf{E}}_A, \vec{\mathbf{E}}_B$ *and* $\vec{\mathbf{E}}_C$ *are fields due to charge A, B and C inside the conductor.*

and $\vec{\mathbf{E}}_{net} = \vec{\mathbf{E}}_A + \vec{\mathbf{E}}_B + \vec{\mathbf{E}}_C = 0$

Now the electric field due to charges on the outer surface of conductor is zero for all the points inside the conductor separately and the $\vec{\mathbf{E}}_B + \vec{\mathbf{E}}_A$ *is zero separately.*

Concept: *Consider a solid metal conducting sphere of radius R and total positive charge Q as shown in Fig. 1.168 (a). The electric field outside the sphere is* k_eQ/r^2 *and points radially outward. Because the field outside of a spherically symmetric charge distribution is identical to that of a point charge, we expect the potential to also be that of a point charge,* k_eQ/r*. At the surface of the conducting sphere in Fig. 1.168 (a), the potential must be* k_eQ/R*.*

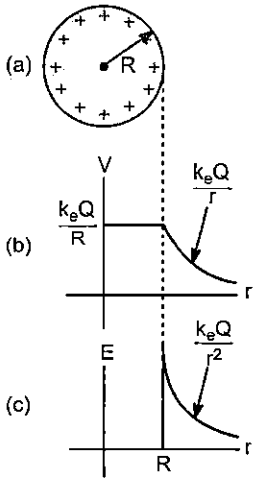


Fig. 1.168

Because the entire sphere must be at the same potential, the potential at any point within the sphere must also be k_eQ/R. Fig. 1.168 (b) is a plot of the electric potential as a function of r, and Fig. 1.168 (c) shows how the electric field varies with r.

Concept: *(a) A conducting box (a Faraday cage) immersed in a uniform electric field. The field of the induced charges on the box combines with the uniform field to give zero total field inside the box.*

(b) Electrostatic shielding can protect you from a dangerous electric discharge.

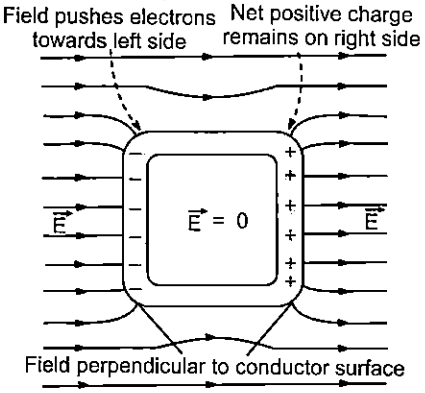


Fig. 1.169

Concept: *(a) A Charged conducting ball suspended by an insulating thread outside a conducting container on an insulating stand.*

(b) The ball is lowered into the container, and the lid is put on.

(c) The ball is touched to the inner surface of the container.

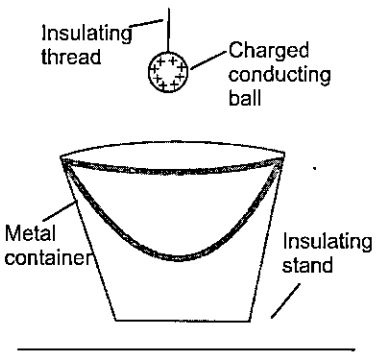


Fig. 1.170 (a)

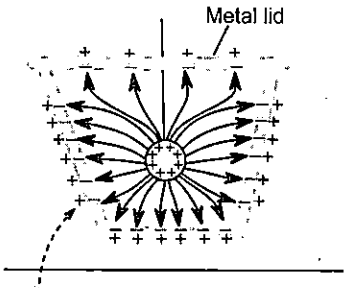


Fig. 1.170 (b)

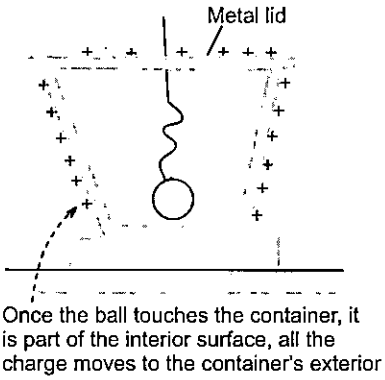


Fig. 1.170 (c)

Conceptual Example 3: Field lines for a thin spherical shell:

A thin metallic spherical shell of radius R carries a total charge Q, which is positive. The charge is spread out evenly over the shell's outside surface. We wish to sketch the electric field lines in two different views of the situation: (a) The spherical shell is very small and you are looking at it from distant points; (b) you are looking at the field inside the shell's cavity.

Solution: (a) A tiny spherical shell located far away cannot be distinguished from a point charge. The sphere looks like a point when seen from a large distance and the field lines look just like those emanating from a positive point charge Fig. 1.171 (a)

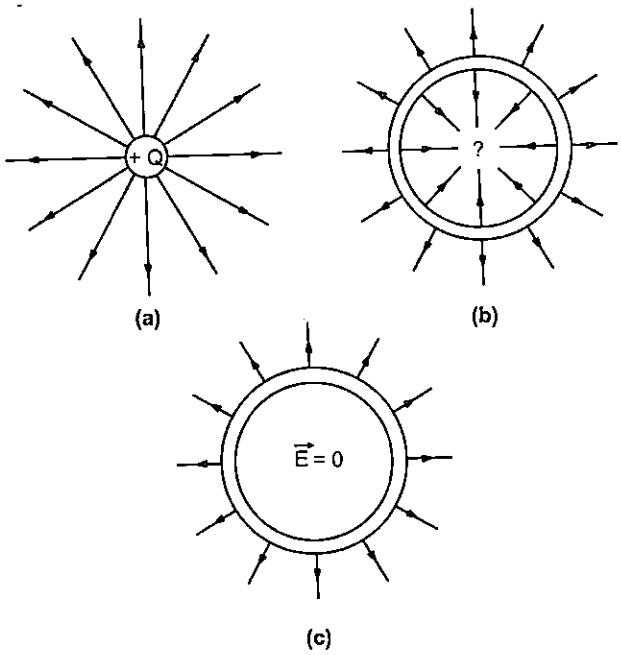


Fig. 1.171

(b) Field lines begin on the positive charges on the shell surface. Some go outward, representing the electric field outside the shell, while others may *perhaps* go inward, representing the field inside the shell. Any field lines inside must start evenly spaced on the shell and point directly toward the center of the shell [Fig. 1.171 (b)]; the lines cannot deviate from the radial direction due to the symmetry of the sphere.

The lines can only end at the center if a negative point charge is found there-but there is no point charge. If the lines do not end, they would cross at the center. The cannot be right since the field must have a *unique* direction at every point-field lines never cross. conclusion: *there are no field lines inside the shell* [Fig. 1.171 (c)].

We conclude that the electric field *inside* a spherical shell of charge is zero. *The electric field pattern outside a spherical shell is the same as if the charge were all condensed into a point charge at the center of the sphere.*

Conceptual example 4: Spherical conductor in a uniform applied field:

Two oppositely charged parallel plates produce a uniform electric field between them [Fig. 1.172 (a)]. An uncharged metal sphere is placed between the plates. Assume that the sphere is small enough that it does not affect the charge distribution on the plates. Sketch the electric field lines between the plates once electrostatic equilibrium is reached.

Electrons in the metal sphere are attracted to the positive plate, leaving the surface near the positive plate with a negative surface charge. The other side will have a positive surface charge. The electric field is changed by these surface charges, so that it is no longer uniform.

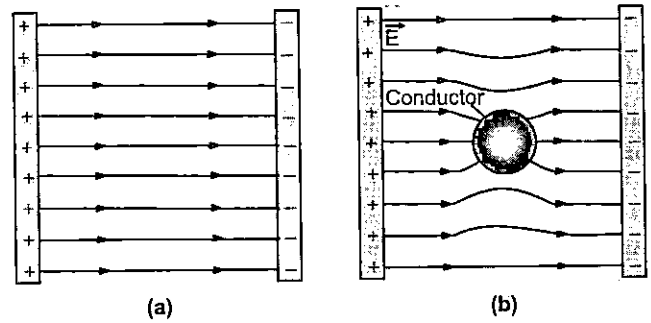


Fig. 1.172

Solution: There are no field lines inside the metal sphere. The field lines cannot "go around" tangentially to the sphere, since then there would be a field component parallel to the sphere's surface. As there is charge on the sphere's surface, some field lines must start on the positive side and others end on the negative side. The field lines must intersect the sphere perpendicular to the surface. Fig. 1.172 (b) shows a field line for the sphere.

Conceptual Example 5. A uniform electric field E exists over a very large region of space. A flat conducting plate, oriented so that its faces are normal to the field, is inserted into the field (Fig. 1.173). Find the charge per unit area on each surface, at locations not near the edges of the plate.

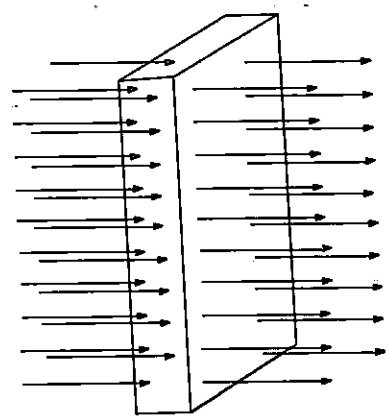


Fig. 1.173

Solution: Because the plate is a conductor, the field must be zero inside it. But the presence of the finite plate cannot affect the field at indefinitely distant points. That is, the uniform field can be distorted only within the general vicinity of the plate. Thus the electric field lines, which were continuous in the absence of the plate, must terminate on the left surface of the plate and resume at the right surface. This requires that there be a uniform distribution σ- of *negative* charge on the left surface, on which the uniform electric field lines that fill the left side of the space can terminate. Similarly, there must be a uniform distribution of *positive* charge σ_+ on the right surface, at which uniform electric field lines that fill the right side of the space can originate. Because the electric field has the same value E everywhere outside the plate, you must have $|\sigma_-|=|\sigma_+|\equiv\sigma$. And $\sigma=\varepsilon_0 E$. So the surface charges per unit area are

$$\sigma = -\varepsilon_0 E$$

and $$\sigma_+ = \varepsilon_0 E.$$

Because of the symmetry of the situation, the presence of the plate does not distort the electric field outside it at all, even at points quite close to its surface. Why would this not be true for a conducting body of arbitrary shape ?

For the conducting bodies of arbitrary surface, the surface of any such body can be pieced together by using small element of spherical surfaces of varying radii, as shown in Fig. 1.174.

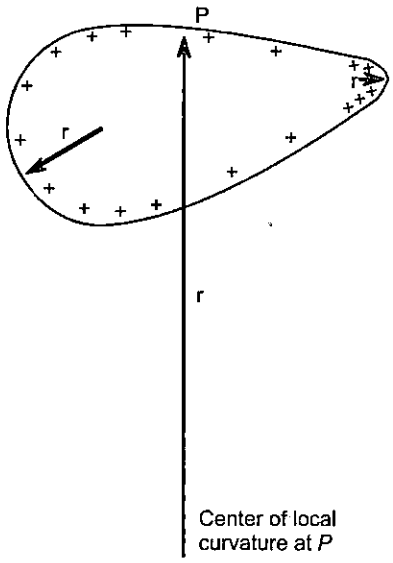


Fig. 1.174: A body of arbitrary shape has a surface of varying curvature. The electric field at any point just outside the surface is directly proportional to the local curvature. Consequently, the surface charge must be densest in regions of high curvature.

At any point just outside the surface, the field is inversely proportional to the radius of the local spherical element. The reciprocal of this radius is called the **curvature;** sharply curved parts of the surface have large curvatures. Thus the field is directly proportional to the curvature. Consequently, the field outside an irregular body is greatest where the curvature is greatest.

Concept: *If is a charge +q placed near an uncharged conductor Fig. 1.175, the two will attract one another. Because q will pull free electron over to the near side the charge moves around in such a way as to cancel off the field of q for points inside the conductor, where the total field must be zero. Since the negative induced charge is closer to q, there is a net force of attraction.*

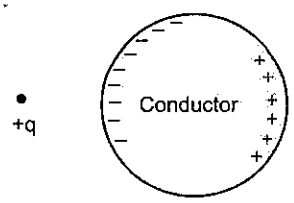


Fig. 1.175

Concept: If there is some cavity in the conductor, and within that cavity there is some charge, then the field in the cavity will not be zero. But the cavity and its contents are electrically isolated form the outside world by the surrounding conductor (Fig. 1.176). ***No external fields penetrate the conductor; they are canceled at the outer surface by the induced charge there. Similarly, the field due to charges within the cavity is cancelled off , for all exterior points, by the induced charge on the inner surface.*** *The total charge induced on the cavity wall is equal and opposite to the charge inside which can be proved by for if we surrounding the cavity with a Gaussian surface, all points of which are in the conductor (Fig. 1.176). As there is no field in the material of conductor* $\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = 0$, *and hence (by Gauss's law) the net enclosed charge must be zero. But* $Q_{\text{enc}} = q + q_{\text{induced}}$, *So* $q_{\text{induced}} = -q$.

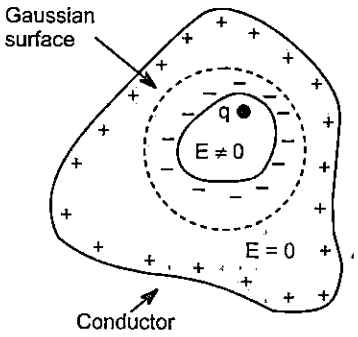


Fig. 1.176

Concept: *An uncharged spherical conductor centered at the origin has a cavity of some arbitrary shape carved out of it (Fig. 1.177). Somewhere within the cavity is a charge q. What is the field outside the sphere ?*

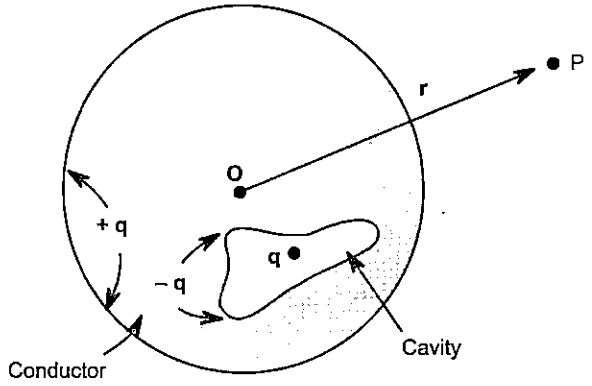


Fig. 1.177

The outside field is

$$\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{\mathbf{r}}$$

The charge +q induces an opposite charge –q on the wall of the cavity, which distributes itself in such a way that its field cancels that of q, for all points exterior to the cavity.

When the cavity is not empty but contains a certain electric charge q (or several charges). Suppose also that the entire external space is filled by a conducting medium. In equilibrium, the field in this medium is equal to zero, which means that the medium is electrically neutral and contains no excess charges.

Since $\vec{E} = 0$ inside the conductor, the field flux through a closed surface surrounding the cavity is also equal to zero. According to the Gauss theorem, this means that the algebraic sum of the charges within this closed surface is equal to zero as well. Thus, the algebraic sum of the charges induced on the cavity surface is equal in magnitude and opposite in sign to the algebraic sum of the charges inside the cavity. In equilibrium the charges induced on the surface of the cavity are arranged so as to compensate completely, in the space outside the cavity, the field created by the charges located inside the cavity.

Since the conducting medium is electrically neutral everywhere, it does not influence the electric field in any way. Therefore, if we remove the medium, leaving only a conducting shell around the cavity, the field will not be changed anywhere, and will remain equal to zero beyond this shell.

Thus, the field of the charges surrounded by a conducting shell and of the charges induced on the surface of the cavity (on the inner surface of the shell) is equal to zero in the entire outer space.

1. ***A closed conducting shell divides the entire space into the inner and outer parts which are completely independent of one another in respect of electric fields.***

2. Any arbitrary displacement of charges inside the shell does not introduce any change in the field of the outer space, and hence the charge distribution on the outer surface of the shell remains unchanged.

3. The field inside the cavity (if it contains charges) and to the distribution of charges induced on the cavity walls. They will also remain unchanged upon the displacement of charges outside the shell.

4. A point charge q is within an electrically neutral shell whose outer surface has spherical shape (Fig. 1.177). The potential at the point P lying outside the shell at a distance r from the centre O of the outer surface is given by

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}.$$

The field at the point P is determined only by charges induced on the outer spherical surface since, the field of the point charge q and of the charge induced on the inner surface of the sphere is equal to zero everywhere outside the cavity. Next, in view of symmetry, the charge on the outer surface of the shell is distributed uniformly.

An infinite conducting plane is a special case of a closed conducting shell. The space on one side of this plane is electrically independent of the space on its other side.

We shall repeatedly use this property of a closed conducting shell.

Concept: *There are actually three fields involved here, $\vec{E}_q$, $\vec{E}_{induced}$, and $\vec{E}_{leftover}$. The sum of the three is zero inside the conductor but the first two alone cancel, while the third is independently zero there.*

There exists a way of distributing –q over the inner surface so as to cancel the field of q at all exterior points.

CHARGE DISTRIBUTION ON A CONDUCTING SHEET

Case (i) Sheet is isolated

A large conducting sheet with on external charges near it, has net charge Q on it. Let charge on two surfaces are Q and $Q - x$.

Concept: *Electric field in material of conductor is zero. Consider a point P inside conductor.*

$$\vec{E}_1 + \vec{E}_2 = 0$$

i.e., $$\frac{x}{2\varepsilon_0 A} - \frac{Q-x}{2\varepsilon_0 A} = 0 \quad or \quad x = Q/2$$

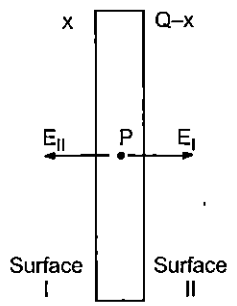


Fig. 1.178

Thus in electrostatic condition charge distributes equally on both the surfaces of conductor.

Case(ii) Conducting sheet with charge Q and placed in an external field.

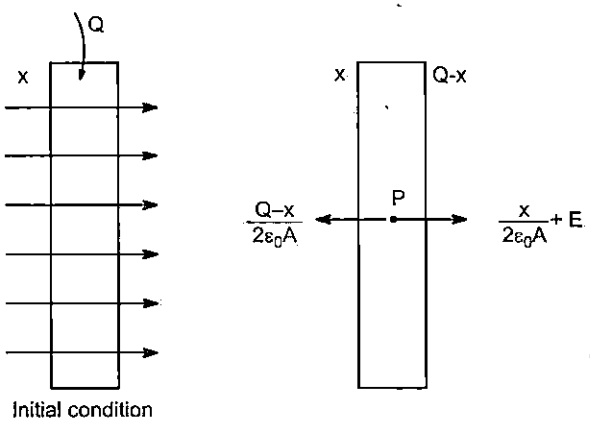


Fig. 1.179

We assume that charge on the two surfaces are x and $Q-x$.

Electric field at P is zero.

$$E_P = 0$$

$$\frac{x}{2\varepsilon_0 A} + E = \frac{Q-x}{2\varepsilon_0 A}$$

or $$x = Q/2 - \varepsilon_0 AE$$

and $$Q - x = \frac{Q}{2} + \varepsilon_0 AE$$

Concept: *Even if conducting sheet was uncharged the charge distribution would have occurred on the surface of sheet. In electrostatic condition the charge distribution is as shown in Fig. 1.180.*

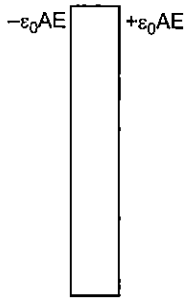


Fig. 1.180

If two uncharged metallic conducting plates are placed in a uniform electric field E at a finite distance from each other, the charge distribution will as shown in Fig. 1.181.

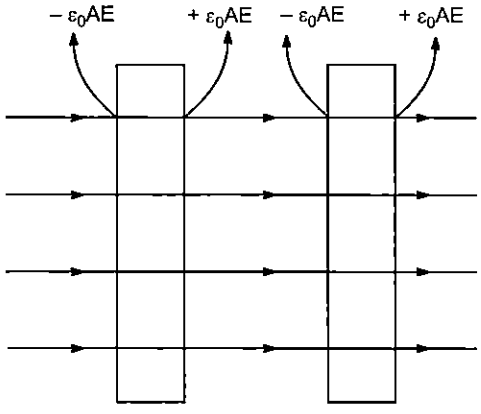


Fig. 1.181

Conceptual example: Electric Field and Potential Due to a Induced Charges:

A metal sphere of radius R is placed at a distance r from the point charge $+q$. There is a point M in the sphere at a distance x from $+q$. Find the electric field and potential at point M due to the induced charges on the surface of sphere.

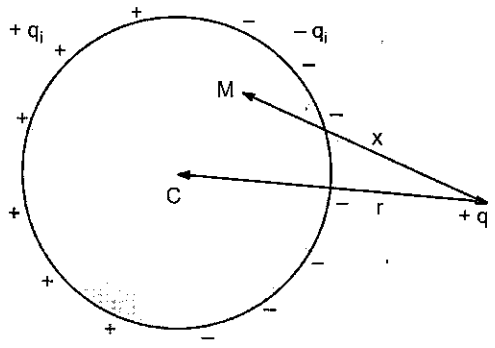


Fig. 1.182

Inside a net electric field is always zero. Thus at point M electric field due to induced charges is cancelled due to material of a conductor the electric field of the point charge $+q$. Hence electric field at point M due to induced charges on sphere can be given as

$$E_M = \frac{Kq}{x^2} \text{ (directed toward point charge } +q)$$

Material of a conductor is an equipotential region. Potential at centre of sphere, it is only due to the charge $+q$ potential due to induced charges, at centre will be zero. Net magnitude of induced charges is zero, secondly all the induced charges are equidistant from centre. Thus net potential at the centre of sphere can be given as

$$V_C = \frac{K_q}{r}$$

Sphere being equipotential at point M the potential must be equal to that at point C. Note that at M potential due to induced charges will be non-zero as all induced charges are not symmetrically from point M. Thus net potential at point M can be gives as:

$$V_M = \frac{Kq}{r} = \frac{Kq}{x} + V_i$$

Potential due to induced charges is zero as centre C is equidistant from all induced charges. Volume of solid conductor is equipotential volume.

$$\therefore \quad V_C = V_M = \frac{Kq}{r} + \frac{KQ}{R}$$

Also at M

$$V_M = V_{\text{due to q}} + V_{\text{due to induced charges}} + V_{\text{due to Q}}$$

$$\frac{Kq}{r} + \frac{KQ}{R} = \frac{Kq}{x} + \frac{KQ}{R} + V_{\text{due to induced charges}}$$

$$V_{\text{induced charge}} = \frac{Kq}{r} - \frac{Kq}{x}$$

Where V_i is the potential at M due to induced charges. Thus,

$$V_i = \frac{Kq}{r} - \frac{Kq}{x}$$

Conceptual example: In side a conducting hollow sphere of inner radius R_1 and outer radius R_2, a point charge q is placed at a distance x from the centre as shown. The electric potential at centre due to this system is due to q,

induced charge $-q$ on inner surface and induced charge $+q$ on outer surface.

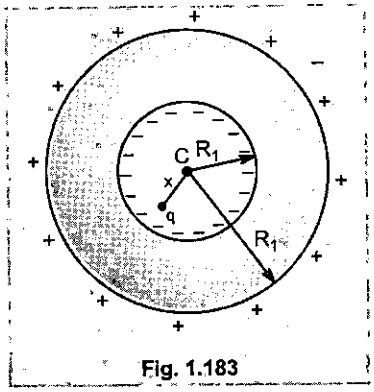


Fig. 1.183

Thus,

$$V_C = \frac{Kq}{r} - \frac{Kq}{R_1} + \frac{Kq}{R_2}$$

Electric field and potential at a distance r from the centre outside the shell will only be due to the charge on outer surface because outside cavity the field due all the cavity charges is always zero. As induced charge on inner surface of cavity always nullifies the effect of point charge inside it. Thus,

$$E_{out} = \frac{Kq}{r^2} \quad \text{and} \quad V_P = \frac{Kq}{r}$$

EARTHING OF A CONDUCTOR

Earthing means connecting a conductor with earth.

Earth is infinite resource and sink of charge so potential of earth will not change. The potential of earth is assumed to be "zero". So after earthing the charge on conductors vary so that potential of conductor becomes zero.

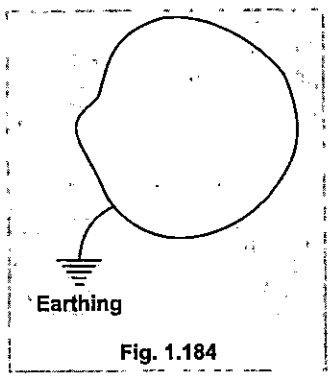


Fig. 1.184

Conceptual Example: Consider concentric spherical shells of negligible thickness. Initial charges on these shells are Q_1 and Q_2. Now inner shell is earthed.

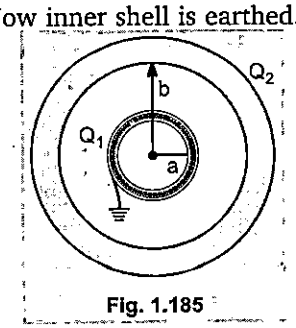


Fig. 1.185

Solution: Find charge on the inner shell.

Let the charge induced be q_1

$$\therefore \quad \frac{kq_1}{a} + \frac{kQ_2}{b} = 0$$

$$q_1 = -Q_2 \frac{a}{b} \text{ [new charge after earthing]}$$

This means that the inner shell is at zero potential and that electric field lines leave the outer shell and go to infinity but other electric field lines leave the outer shell and end on the inner shell.

This implies that a charge $+Q_2 \frac{a}{b}$ has been transferred to earth negative charge on A.

Also note that due to charge q_1 induction takes place an outer shell. But if thickness of outer shell is ignored the potential due to these charges will be zero inside.

Conceptual Example: A metal sphere A of radius a is charged to potential V. What will be its potential if it is enclosed by a spherical conducting shell B of radius b and the two are connected by a wire ?

Solution: If the charge on sphere of radius a is q,

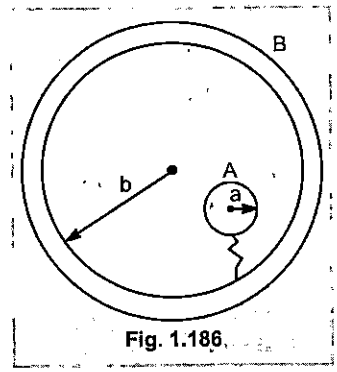


Fig. 1.186

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{a}$$

i.e., $$q = (4\pi\varepsilon_0 a)V$$

Now, when sphere A is enclosed by spherical conductor B and the two are connected by a wire, charge will reside on outer surface of B and so the potential of B will be,

$$V_B = \frac{1}{4\pi\varepsilon_0}\frac{q}{b}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{4\pi\varepsilon_0 a}{b}V$$

$$= \frac{a}{b}V$$

Now as sphere A is inside B so its potential,

$$V_A = V_B = \frac{a}{b}(V) \qquad [V \text{ as } a < b]$$

Earthing of Charged or Uncharged Metal Bodies

Earth is assumed to be a very large conducting sphere. If some charge Q is given to earth, its potential becomes

$$V_e = \frac{KQ}{R_e}$$

As R_e is very large V_e comes out to be negligibly small. Thus for point size bodies whose dimensions are negligible compared to earth we can assume that earth is at zero potential.

Consider a solid uncharged conducting sphere shown in Fig. 1.187. A point charge q is placed in front of the sphere centre at a distance x as shown. Here due to q, the potential at sphere is

$$V = \frac{Kq}{x}$$

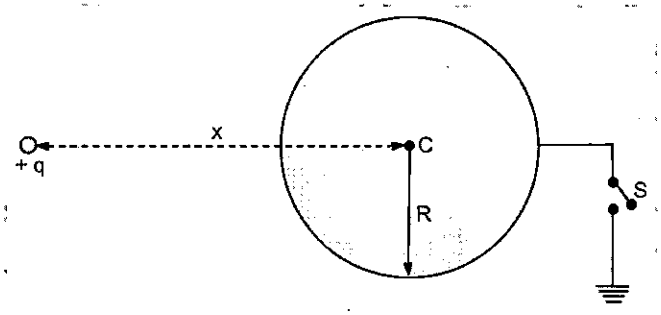


Fig. 1.187

The charge $+q$ will induce charges on sphere but potential due to induced charges on sphere is zero. If we close the switch S, earth supplies a charge q_e on to the sphere to render it zero potential. Thus the final potential on sphere can be taken as

$$V = \frac{Kq}{x} + \frac{Kq_e}{R} = 0$$

or $$q_e = -\frac{qR}{x}$$

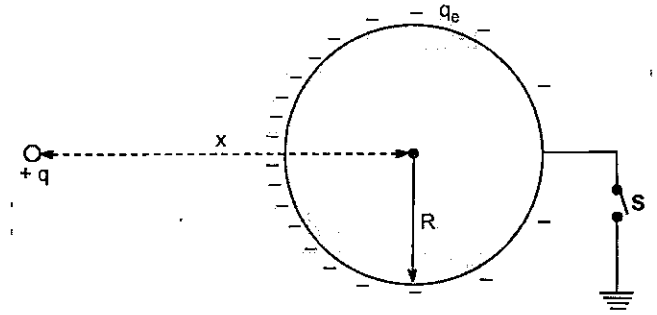


Fig. 1.188

Earth has supplied a negative charge to on nullify positive potential on it due to q.

Charge on a System of Concentric Shells with one of the Shells Grounded

Fig. 1.189 shows three concentric spherical shells of radii a, b and c having charges q_1, q_2 and q_3 on these respectively. Find the final potential of the three shells if switch S is closed.

When switch S is closed, charge flow from middle shell and earth till the final potential of middle shell becomes zero. Let the final charge on middle shell after closing the switch becomes q_e.

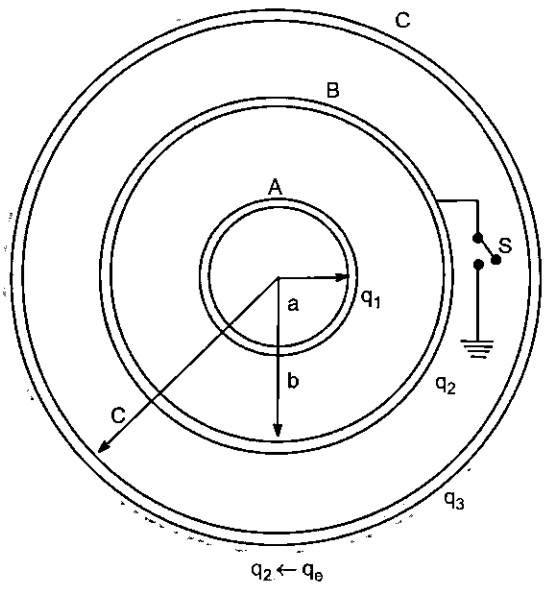


Fig. 1.189

$$V_B = \frac{Kq_1}{b} + \frac{Kq_e}{b} + \frac{Kq_3}{c} = 0$$

Here potential of B

= Potential of shell B due to shell A

+ potential of shell B due to its own charge q_e

+ potential of shell B due to shell C, as B is inside C its potential is equal to that on surface of C.

On solving the above equation, we get

$$q_e = -\left(q_1 + \frac{b}{c}q_3\right)$$

Now we can write the final potential of the three shells as

$$V_A = \frac{Kq_1}{a} + \frac{Kq_e}{b} + \frac{Kq_3}{c} \quad \Rightarrow \quad V_B = 0$$

$$V_C = \frac{K(q_1 + q_e + q_3)}{c}$$

Example 111

There are 4 concentric shells A, B, C and D of radius a, 2a, 3a, 4a respectively. Shells B and D are given charges + q and –q respectively. Shell C is now earthed. Find the potential difference $V_A - V_C$.

Solution: Let shell C acquires charge q' which will be such that final potential of C is zero

$$V_C = \frac{Kq}{3a} + \frac{Kq'}{3a} + \left(\frac{-Kq}{4a}\right) = 0$$

$$\frac{Kq}{3a} + \frac{Kq'}{3a} = \frac{Kq}{4a} \quad \Rightarrow \quad q' = 3q\left(\frac{1}{4} - \frac{1}{3}\right)$$

$$q' = -\frac{q}{4}$$

As $$V_C = 0$$

$$V_A - V_C = V_A$$

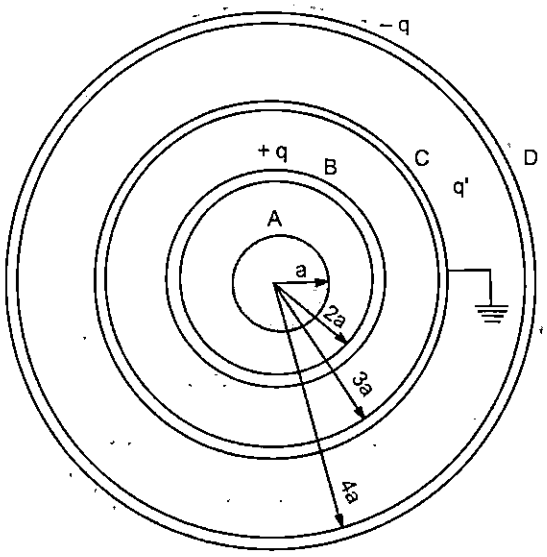


Fig. 1E.111

Now calculating V_A, we get

$$V_A = \frac{Kq}{2a} - \frac{K(q/4)}{3a} - \frac{Kq}{4a}$$

$$V_A = \frac{Kq}{6a} \quad \text{or} \quad V_A - V_C = \frac{Kq}{6a}$$

Example 112

Figure shows three thin concentric spherical shells with initial charges as shown in the figure shell A and C are connected by wire and such that it does not touch B and shell B is earthed. Determine the finally charges be q_A, q_B and q_C.

Solution: First equation holds for conservation of charge on A and C.

$$q_A + q_C = 2Q \qquad ...(1)$$

Second equation holds for zero potential of earthed surface

$$\frac{Kq_B}{2K} + \frac{Kq_C}{2K} + \frac{Kq_A}{3K} = 0$$

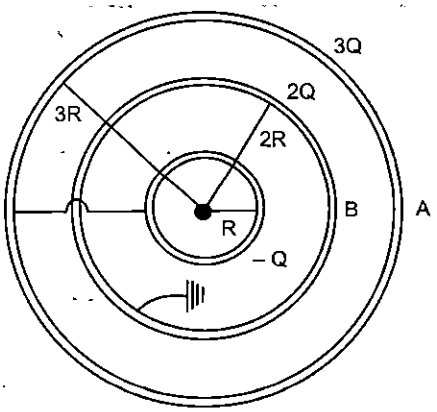


Fig. 1E.112

$$\Rightarrow \quad \frac{q_B}{2} + \frac{q_C}{2} + \frac{q_A}{3} = 0 \qquad ...(2)$$

Also third equation holds for potential of A and C being equal.

$$\frac{Kq_C}{R} + \frac{Kq_2}{2R} + \frac{Kq_A}{3R} = \frac{Kq_A}{3R} + \frac{Kq_B}{3R} + \frac{Kq_C}{3R}$$

$$\Rightarrow \quad q_C + \frac{q_B}{2} + \frac{q_A}{3} = \frac{q_A}{3} + \frac{q_B}{3} + \frac{q_C}{3} \qquad ...(3)$$

Now an solving for q_A, q_B and q_C

we get $q_C = Q/2$

$$q_A = 3Q/2; \; q_B = \frac{-3Q}{2}$$

Example 113

A solid conducting sphere having a charge Q is surrounded by an uncharged concentric conducting hollow spherical shell. Let the potential difference between the surface of the solid sphere and that of the outer surface of hollow shell be V. What will be the new potential difference between the same two surfaces if the shell given a charge –3Q?

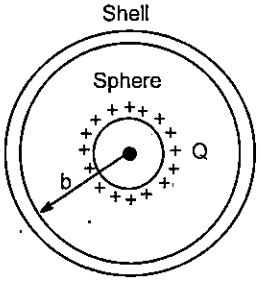


Fig. 1E.113

Solution: In case of a charged conducting sphere,

$$V_{in} = V_C = V_S = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$$

and $$V_{out} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

So if a and b are the radii of sphere and spherical shell respectively, potential at their surfaces will be,

$$V_{sphere} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{a}$$

and $$V_{shell} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{b}$$

And so according to given problem

$$V = V_{sphere} - V_{shell} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{a} - \frac{1}{b}\right] \qquad ...(1)$$

Now when the shell is given a charge $(-3Q)$ the potential at its surface and also inside will change by $V_0 = \frac{1}{4\pi\varepsilon_0}\left[\frac{-3Q}{b}\right]$

So that now

$$V'_{sphere} = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{a}\right] + V_0$$

and $V'_{shell} = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{b}\right] + V_0$

And hence

$V'_{sphere} - V'_{shell} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{a} - \frac{1}{b}\right] = V$ [from eq. (1)]

Example 114

Fig. shows three conducting plates with an initial charge distribution. Find charge on all surfaces in electrostatic equilibrium of plates shown in Fig. 1E.114.

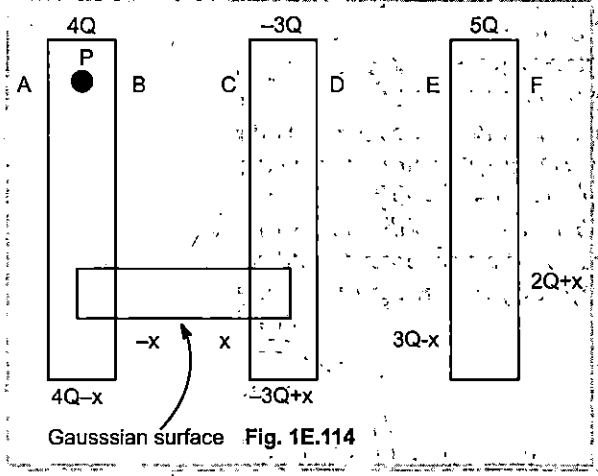


Fig. 1E.114

Solution: Applying Gauss' law on opposite faces.

$$\oint \vec{E} \cdot d\vec{s} = 0$$

as through two sides inside material of conductor $E = 0$

as part of conductor and through lateral surface $\vec{E} \cdot d\vec{s} = 0$

$$q_{enclosed} = 0$$

Thus facing surface have equal and opposite charges.

Using this we can also prove that outer faces of the two last plates have equal charges.

Now consider a point of initial a plate, the resultant field in material of plate is zero.

$$\frac{(4Q - x)/\text{Area}}{2\varepsilon_0} - \frac{(2Q + x)/\text{Area}}{2\varepsilon_0} = 0$$

$$4Q - x - 2Q - x = 0$$

or $2x = 2Q$

$\Rightarrow$ $x = Q$

Example 115

Figure shows three metallic plates with charges –Q, + 3Q and Q respectively. Determine the final charges on all the surfaces.

Solution: We assume that the charges on surfaces 2, 3 and 5 are q_1, q_2 and q_3 in equilibrium. Following conservation of charge, we see that surfaces 1, 4 and 6 have charges $-Q, -q_1, 3Q - q_2$ and $Q - q_3$ respectively. The electric field inside a metallic plate is zero; therefore fields at points P, Q and R are zero.

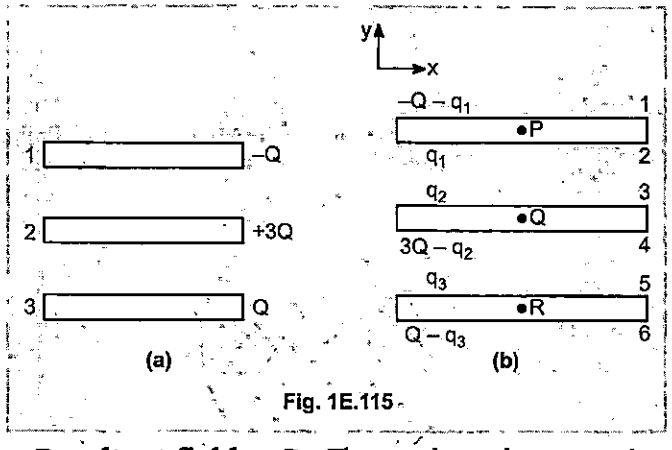


Fig. 1E.115

Resultant field at P : Thus we have three equations, as follows. Considering upward field as positive, we get

$$\frac{(Q + q_1)}{2\varepsilon_0} + \frac{q_1}{2\varepsilon_0} + \frac{q_2}{2\varepsilon_0} + \frac{3Q - q_2}{2\varepsilon_0} + \frac{q_3}{2\varepsilon_0} + \frac{Q - Q_3}{2\varepsilon_0} = 0 \quad ..(1)$$

or $$\frac{Q + q_1}{2\varepsilon_0} + \frac{q_1}{2\varepsilon_0} + \frac{3Q}{2\varepsilon_0} + \frac{Q}{2\varepsilon_0} = 0$$

or $$\frac{q_1}{\varepsilon_0} = -\frac{5Q}{2\varepsilon_0} \quad \text{or} \quad q_1 = -\frac{5}{2}Q$$

Resultant field at Q :

$$\frac{Q + q_1}{2\varepsilon_0} - \frac{q_1}{2\varepsilon_0} - \frac{q_2}{2\varepsilon_0} + \frac{3Q - q_2}{2\varepsilon_0} + \frac{q_3}{2\varepsilon_0} + \frac{Q - q_3}{2\varepsilon_0} = 0$$

or $$\frac{Q}{2\varepsilon_0} + \frac{3Q}{2\varepsilon_0} - \frac{2q_2}{2\varepsilon_0} + \frac{Q}{2\varepsilon_0} = 0$$

or $$\frac{q_2}{\varepsilon_0} = \frac{5Q}{2\varepsilon_0}$$

or $$q_2 = +\frac{5}{2}Q$$

Resultant field at R :

$$\frac{Q + q_1}{2\varepsilon_0} - \frac{q_1}{2\varepsilon_0} - \frac{q_2}{2\varepsilon_0} - \frac{(3Q - q_2)}{2\varepsilon_0} - \frac{q_3}{2\varepsilon_0} + \frac{Q - q_3}{2\varepsilon_0} = 0$$

or $$\frac{Q}{2\varepsilon_0} - \frac{3Q}{2\varepsilon_0} + \frac{Q}{2\varepsilon_0} - \frac{2q_3}{2\varepsilon_0} = 0$$

or $$\frac{q_3}{\varepsilon_0} = -\frac{Q}{2\varepsilon_0}$$

or $$q_3 = -\frac{Q}{2}$$

Thus the final charge distribution on all the faces is :

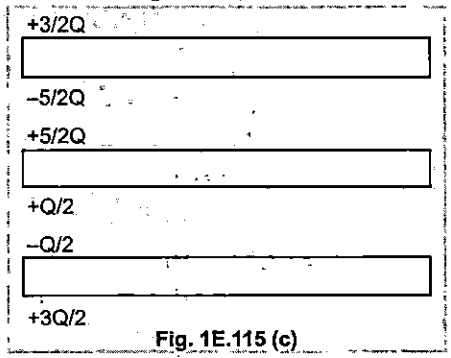


Fig. 1E.115 (c)

Example 116

Figure shows three conducting spherical shells A, B and C with charges $-q_1, +q/2, +q$ respectively. Determine the potential difference between points A and C.

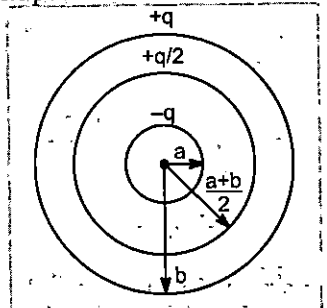


Fig. 1E.116

Solution: Potential at $A, (V_A) =$. Potential due to charge on sphere A + Potential due to charge on sphere C + Potential due to charge on sphere B

$$= -\frac{1}{4\pi\varepsilon_0}\frac{q}{a} + \frac{1}{4\pi\varepsilon_0}\frac{q}{b} + \frac{q/2}{4\pi\varepsilon_0[(a+b)/2]}$$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{b} - \frac{1}{a} + \frac{1}{(a+b)}\right)$$

Potential at $C, (V_C) =$ Potential due to charge on sphere A + Potential due to charge on sphere C + Potential due to charge on sphere B

$$= -\frac{1}{4\pi\varepsilon_0}\frac{q}{b} + \frac{1}{4\pi\varepsilon_0}\frac{q}{2b} + \frac{1}{4\pi\varepsilon_0}\frac{q}{b}$$

$$= \frac{q}{8\pi\varepsilon_0 b}$$

Potential difference $= V_A - V_C$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{b} - \frac{1}{a} + \frac{1}{(a+b)}\right) - \frac{q}{8\pi\varepsilon_0 b}$$

$$= \frac{q}{8\pi\varepsilon_0}\left(\frac{a^2 - 2b^2 + ab}{ab(a+b)}\right)$$

Example 117

A parallel plate capacitor has plates of area A, separated by a small distance d. One plate has a total charge Q and the other Q'. Neglecting edge effects, calculate the charge per unit area on each of the four metal surfaces and the electric field close to each surface.

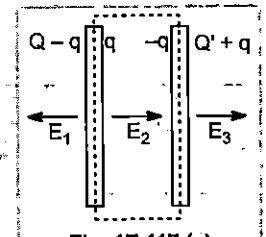


Fig. 1E.117 (a)

Solution: We assume that the inside surface of the left plate has a total charge q. From conservation of charge, we see that the outer surface of the left plate is $Q - q$. Consider the Gaussian surface shown in Fig. 1E.117(a); net flux through it is zero, thereby implying that the net charge enclosed in it zero. Now we can state that the facing surfaces of a charged capacitor have same magnitude of charge. The charge distribution on the right plate is shown Fig. 1E.117(b).

Let us call the electric fields outside the capacitor E_1 and E_3, and the field between the plates E_2, as shown in Fig. 1E.117 (b).

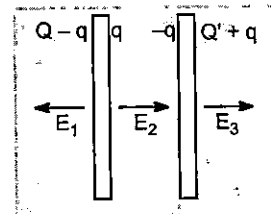


Fig. 1E.117 (b)

If a charge q is uniformly distributed over a flat metal sheet of area A, the electric field in the direction away from the sheet is $q/\varepsilon_0 A$. Thus we have

$$E_1 = \frac{Q-q}{\varepsilon_0 A}$$

$$E_2 = \frac{q}{\varepsilon_0 A}$$

$$E_3 = \frac{Q'+q}{\varepsilon_0 A}$$

In order to find the value of q, we need to consider the total energy of the system, which will be minimised at equilibrium. The energy per unit volume of an electric field E is $\frac{1}{2}\varepsilon_0 E^2$.

The volume occupied by the field E_2 is Ad. In practice, the fields are not uniform because the field lines begin to diverge at large distance from the plates. We can assume that the fields are uniform with a large but finite volume v. The total energy U of the system can be written as

$$U = \frac{1}{2\varepsilon_0 A^2}[v(Q-q)^2 + Adq^2 + v(Q'+q)^2]$$

The condition that this is minimum is $dU/dq = 0$, so we differentiate U with respect to q to obtain

$$\frac{dU}{dq} = \frac{1}{\varepsilon_0 A^2}[v(2q - Q + Q') + Adq]$$

Setting this equal to zero and rearranging, gives

$$q = \frac{Q - Q'}{2 + \frac{Ad}{v}}$$

Now we can assume that $Ad/v << 1$, which gives $q = (Q - Q')/2$. [This implies that the charges on the two outermost surfaces of the capacitor are equal.] Thus the charge per unit area on each surface, reading from left to right on the diagram, is

$$(Q+Q')/2A,\ (Q-Q')/2A,$$
$$(Q'-Q)/2A,\ (Q'+Q)/2A,$$

and the fields are

$$E_1 = (Q+Q')/2\varepsilon_0 A,$$

$$E_2 = (Q - Q')/2\varepsilon_0 A,$$
$$E_3 = (Q + Q')/2\varepsilon_0 A,$$

If $Q' = -Q$, the external fields E_1 and E_3 are zero and the internal field $E_2 = Q/\varepsilon_0 A$. The potential difference across the gap d is thus

$$V = \frac{Qd}{\varepsilon_0 A},$$

and since the capacitance C is defined by $Q = CV$, this gives $C = \varepsilon_0 A/d$.

Example 118

Three large conducting plates are placed a distance d apart in air. The space between the first two plates is completely filled with a dielectric slab of dielectric constant $K = 2$ as shown in Fig. 1E.118(a). The plates are given charges $Q_1 = 7Q$, $Q_2 = 3Q$ and $Q_3 = 2Q$ respectively. The outer two plates are now connected with a conducting wire. Find the charges on all the six surfaces.

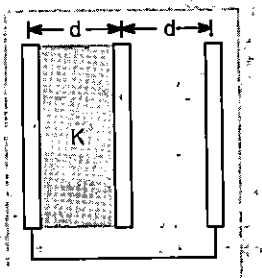


Fig. 1E.118 (a)

Solution: Charges on the facing surfaces will be equal and opposite from Gauss's theorem. Let the distribution be as shown in the following diagram:

Charges on the six surfaces are as shown in the Fig. 1E.118(b)

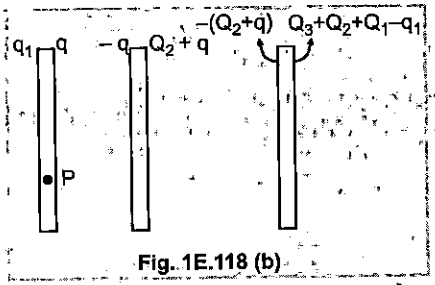


Fig. 1E.118 (b)

As electric field inside the conductor (let at point P) is zero.

$$q_1 = Q_3 + Q_2 + Q_1 - q_1 \quad \text{or} \quad q_1 = \frac{Q_3 + Q_2 + Q_1}{2}$$

As the outer plates are connected, their potential will be same

$$\frac{Q_2 + q}{\varepsilon_0 A} d + \frac{q}{K\varepsilon_0 A} d = 0 \Rightarrow Q_2 + q + \frac{q}{K} = 0$$

$$KQ_2 + (K+1)q = 0 \Rightarrow q = \frac{-KQ_2}{K+1} = -2Q$$

Thus the final charge configuration on the six surfaces will be as shown.

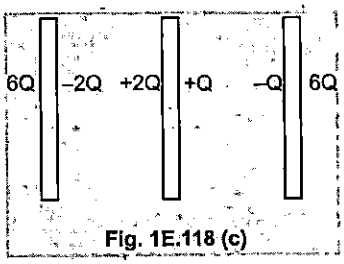


Fig. 1E.118 (c)

Example 119

Three identical metallic plates are kept parallel to one another at a separation of a and b. The outer plates are connected by a thin conducting wire and a charge Q is placed on the central plate. Find the final charges on all the six plates surfaces.

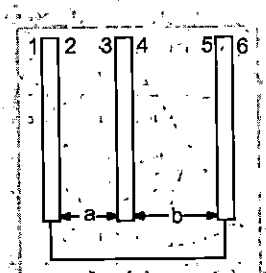


Fig. 1E.119 (a)

Solution: Figure shows the charge distribution in all the six faces. Facing surfaces of capacitor plates have equal and opposite charges distributed on them. Plates A and C are initially neutral, so net charge after redistribution is zero.

Here $q_2 - q_1 + q_3 + q_1 - Q = 0$

or $q_2 + q_3 = Q$ ...(1)

A and C are connected through a conducting wire, hence they have same potentials.

$$V_B - V_A = V_B - V_C$$

or $$\left(\frac{Q_1}{A\varepsilon_0}\right) a = \frac{Q - Q_1}{A\varepsilon_0} \cdot b$$

or $$q_1 = -\frac{Qb}{a+b} \quad ...(2)$$

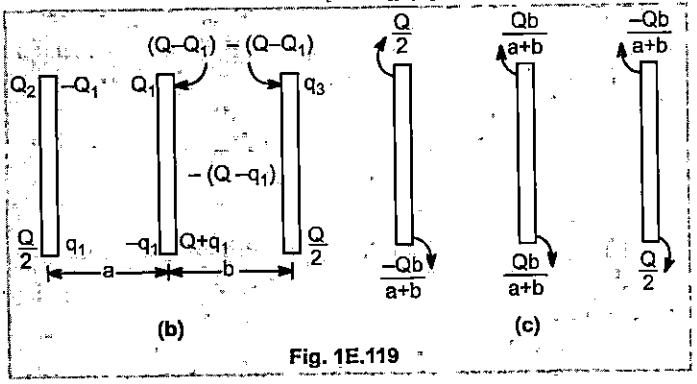


Fig. 1E.119

Now we will use the fact that the electric field inside a conductor is zero. Therefore, electric filed in C is

$$\frac{q_2}{2A\varepsilon_0}-\frac{q_1}{2A\varepsilon_0}+\frac{q_1}{2A\varepsilon_0}+\frac{Q-q_1}{2A\varepsilon_0}+\frac{q_1-Q}{2A\varepsilon_0}-\frac{q_3}{2A\varepsilon_0}=0$$

or $$q_2-q_3=0 \quad ...(3)$$

Solving eqns. (1), (2) and (3), we get

$$q_1=\frac{Qb}{a+b},\ q_2=q_3=\frac{Q}{2}$$

Hence, charges on different faces are shown in Fig. 1E.119(c).

Conceptual Example: A charge Q is placed at the center of a conducting spherical shell of inner radius R_1 and outer radius R_2 (see Fig. 1.190). The sphere has no net charge.

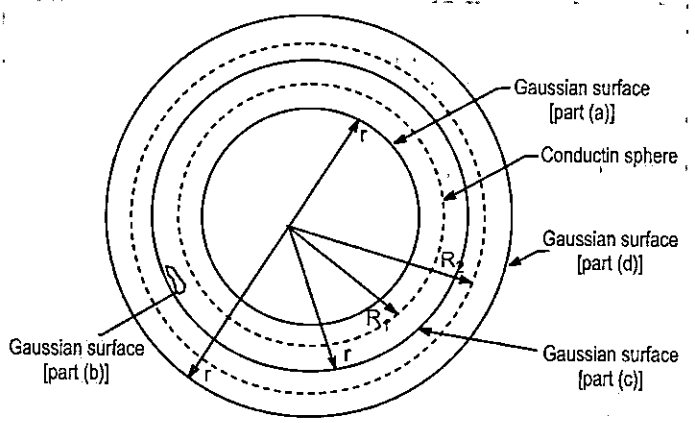


Fig. 1.190

(a) Use Gauss's law to find the electric field in the hollow spherical region of the sphere, $r < R_1$.

(b) Show, material that for a conductor in equilibrium (no moving charges) the electric field in the interior of the conductor is zero.

(c) Calculate the charge collected on the inner surface of the sphere (at R_1).

(d) Calculate the field outside the sphere, *i.e.*, for $r > R_2$.

(e) If the sphere had a net charge of Q', what change, if any, would there be to the answers in parts (a), (c) and (d) ?

Solution: (a) Due to symmetry E will point radially. If we choose a closed spherical surface of radius r as our Gaussian surface (see Fig. 1.190) the magnitude of the field will be the same at every point of this surface, and its direction is perpendicular to the surface at every point so that E is parallel to A. By Gauss' Law $E(4\pi r^2)=Q/\varepsilon_0$, and $E=Q/(4\pi\varepsilon_0 r^2)=kQ/r^2$.

(b) If an electrical field existed in the interior, charges would be not be in equilibrium and hence be moving. In equilibrium, the charges must arrange themselves on the surface of the conductor so that the net field (due to *all* the charges everywhere) is zero throughout the material of the conductor.

(c) Draw a Gaussian surface as a concentric spherical shell within the conductor, at r such that $R_1 < r < R_2$. At every point on this surface the field is zero, since it is in the conducting region. Thus the total flux through the surface is zero. From Gauss' law the total charge within the sphere is zero. Inside the Gaussian sphere there is a charge Q at its center and some other possible charge on the inner surface of the conducting sphere. For the total charge to be zero the charge on the inner surface of the conducting sphere must be equal to $-Q$. Note that this means that the outer surface of the sphere has a charge of $+Q$, since the net charge on the conducting sphere was given as zero.

(d) Draw the Gaussian surface as a sphere with a radius, r, greater than R_2. The total flux through the Gaussian surface will again equal $E(4\pi r^2)$. The total charge within the sphere is the charge at the center plus the charge on the conducting sphere. There is no net charge on the conducting sphere (although there is $-Q$ on its inner surface and $+Q$ on its outer surface). Thus the total charge within the Gaussian surface is Q. Using Gauss' law gives $E(4\pi r^2)=Q/\varepsilon_0$ or $E=Q/4\pi\varepsilon_0 r^2=kQ/r^2$.

(e) All of the charge Q' must appear on the surface of the conductor, and the arguments of part (b) still hold. Thus nothing is changed about parts (a) or (c), since they depend only on the central charge and the fact that the field inside the conductor vanishes. Part (c), however, does tell us that since $(-Q)$ appears on the inner surface of the conductor, $(Q'+Q)$ must appear on the outer surface. In part (d) the total charge within the Gaussian surface is changed to $(Q+(-Q)+Q+Q')=(Q+Q')$, since the conducting sphere now has a net charge Q'. Then the field outside the conducting sphere is $E=k(Q+Q')/r^2$.

Conceptual example: If Fig. 1.191 a small sphere carrying charge $+Q$ is located at the center of a spherical cavity in a large uncharged metal sphere. Find by Gauss' law the field E at points P_1 in the space between small sphere and cavity wall, at points P_2 in the metal of the large sphere, and at points P_3 outside the large sphere.

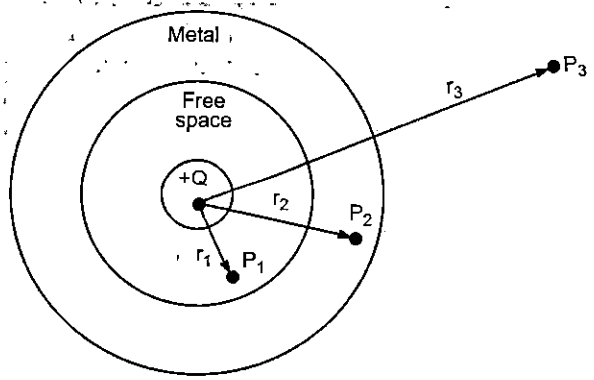


Fig. 1.191

For a spherical gaussian surface of radius r_1 enclosing $+Q$,

$$\text{Flux} = \phi = \frac{Q}{\varepsilon_0} = 4\pi r_1^2 E_1$$

or $$E_1 = \frac{Q}{4\pi\varepsilon_0 r_1^2}$$

In the same way, we draw spherical gaussian surfaces at radius r_2 and again at r_3. Since the field at r_2 is inside the conducting material we must have $E_2 = 0$. Then the total charge enclosed is $=0$, and a uniform negative charge $-Q$ must be induced on the inner surface of the hollow sphere. At radius r_3, again by symmetry and the fact that the large hollow sphere is uncharged, we again get $\phi = Q/\varepsilon_0 = 4\pi r_3^2 E_3$ or $E_3 = Q/(4\pi\varepsilon_0 r_3^2)$.

Note that a uniform positive charge $+Q$ is induced on the outer surface of the hollow sphere. This follows from conservation of charge.

Conceptual Example: A coaxial cable consists of a long, conducting wire, of radius R_1 with a linear charge density of λ, and a long conducting coaxial cylindrical shell, with an inner radius R_2 and an outer radius R_3, and with a symmetric linear charge density of $-\lambda$. We assume the length to be much greater than any of the radial distances of interest.

(a) What is the potential due to the cable at a point at a radial distance from the axis r, such that $r > R_3$?

(b) What is the potential at a point within the outer cylindrical shell, at $R_2 < r < R_3$?

(c) What is the potential at a point between the wire and the cylinder at $R_1 < r < R_2$?

(d) What is the potential at a point within the wire, at $r < R_1$?

Solution: (a) For each of the three surface charges since the point in question is outside both cylindrical distributions. Then $V = 0$, since the total enclosed linear charge density is $\lambda - \lambda = 0$.

(b) We note that the charge on the outer cylinder is all on the inner surface. This is because the field within the conductor is zero, and therefore, from **Gauss' law** the total charge within a Gaussian surface must be zero. Then the charge on the inner surface must cancel the charge on the wire, and equal $-\lambda$. Therefore the point within the cylinder is also outside all the charge distributions, and the result is the same as in (a), *i.e.*, $V = 0$.

(c) In this case the point in question is outside of the wire but within the surface distribution on the outer cylinder. For the wire and for the cylinder we have for the potential: $V = V_1 + V_2 = (-\lambda/2\pi\varepsilon_0)\ln(r/R') + (-\lambda/2\pi\varepsilon_0)\ln(R_2/R') = (-\lambda/2\pi\varepsilon_0)\ln(r/R_2)$ (where we recall $\ln(A/B) = \ln A - \ln B$).

(d) Since we are now within the inner conducting cylinder where the field is zero, the potential must equal its value at the surface. Thus, $V = (-\lambda/2\pi\varepsilon_0)\ln(R_1/R_2)$.

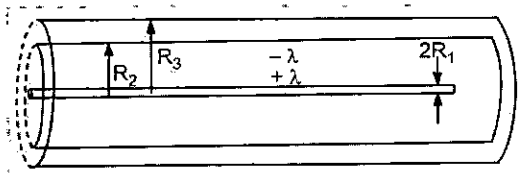


Fig. 1.192

Concepts: *One could also get this result by adding the contribution of the two surface charge distribution. Then* $V = V_1 + V_2 = (-\lambda/2\pi\varepsilon_0)\ln(R_1/R') - (-\pi/2\pi\varepsilon_0)\ln(R_2/R') = (-\lambda/2\pi\varepsilon_0)\ln(R_1/R_2)$.

1. The metal plate on the left in Fig. 1.193 (a) carries a surface charge of $+\sigma$ per unit area. The metal plate on the right has a surface charge of -2σ per unit area. Find the charge densities on the two surfaces of the metal plate in the center. The center plate is assumed to be connected to the earth, so it need not be neutral. Assume the plates to be very large.

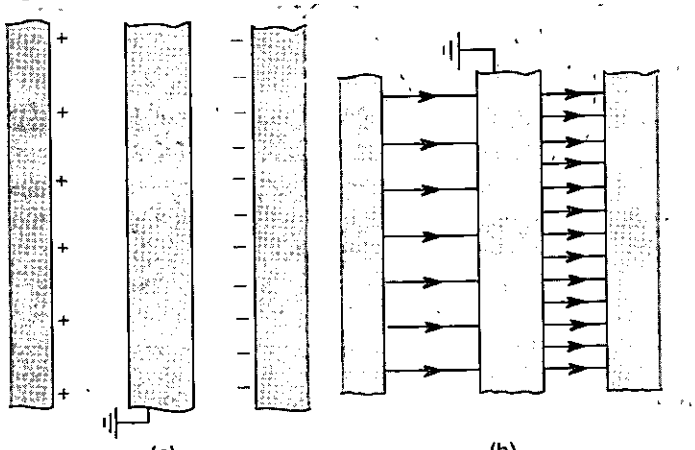


Fig. 1.193

As we know that charge density on the facing surface must be same also [Fig. 1.193 (a)]. The lines coming from the plate on the left must end on the left side of the center plate. Therefore the charge density there must be $-\sigma$ per unit area. Since the lines which end on the surface of the right-hand plate originate on the right-hand surface of the center plate, the charge densities on these two surfaces must be equal and opposite. As a result, we see that the center plate carries a charge $+2\sigma$ on its right-hand side.

2. A thin, long, straight wire carries a charge λ_1 per unit length. The wire lies along the axis of a long metal cylinder, which carries a net charge λ_2 per unit length. The inner radius of the cylinder is b, and its outer radius is c. Find the electric field in the following three regions: $r < b$, $b<r<c$, $r>c$. How much charge per unit length exists on the inner surface of the cylinder ? The outer surface ?

Use gaussian cylinders of length L in each of the given region as shown in Fig. 1.194. No flux passes through the ends. In each case $2\pi rLE$ = (charge enclosed)/ε_0. For $r < b$, the enclosed charge is $\lambda_1 L$ and $E = \lambda_1/2\pi\varepsilon_0 r$. For $b < r < c$, the point lies in the metal, so $E = 0$. At $r > c$, $(\lambda_1 + \lambda_2)L$ is the enclosed charge; $E = (\lambda_1 + \lambda_2)/(2\pi\varepsilon_0 r)$. The charge on the inner surface is $-\lambda_1$, since λ_1 must be cancelled by it (from Gauss' law case $b<r<c$). This leaves $\lambda_2 - (-\lambda_1) = \lambda_2 + \lambda_1$ on the outer surface.

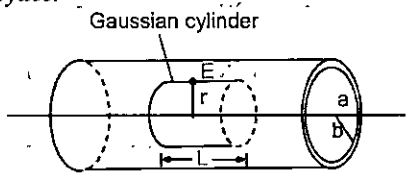


Fig. 1.194

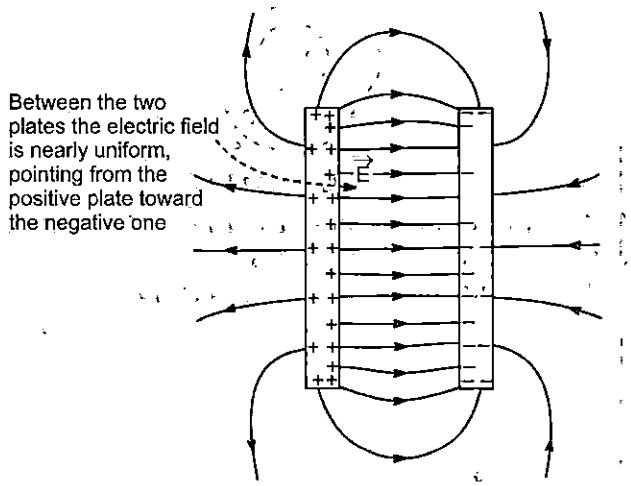
3. Two long straight parallel wires carry charges λ_1 and λ_2 per unit length. The separation between their axes is b. Find the magnitude of the force exerted on unit length of one due to the charge on the other.

The field at 2 due to 1 is $E = \lambda_1/(2\pi\varepsilon_0 b)$ exerting a force of $E\lambda_2$ on a unit length of 2; thus $F/L = (\lambda_1\lambda_2)/(2\pi\varepsilon_0 b)$.

FIELD BETWEEN OPPOSITELY CHARGED PARALLEL CONDUCTING PLATES

Two large parallel conducting plates are given charges of equal magnitude and opposite sign; the charge per unit area is $+\sigma$ for one $-\sigma$ for the other between the plates.

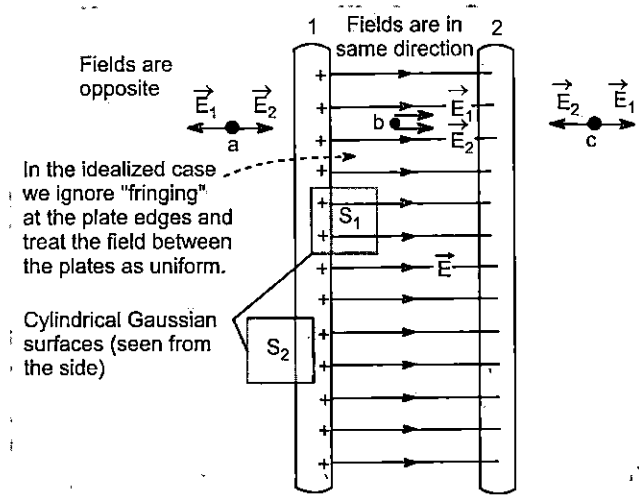
The field between and around the plates is approximately as shown in Fig. 1.195 (a). As we have shown in example 180 that facing surfaces have equal and opposite



(a) Realistic model with fringing field

Fig. 1.195

charges. A small amount of charge resides on the outer surfaces of the plates, and there is some "fringing" of the field at the edges. But if the plates are very large in comparison to the distance between them, the amount of charge on the outer surfaces is negligibly small, and the fringing can be neglected except near the edges. In this case we can assume that the field is uniform in the interior region



(b) Idealized model without fringing field

Fig. 1.195

between plates, as in Fig. 1.195 (b) and that the charge are distributed uniformly over the opposing surfaces.

We use the Gaussian surfaces S_1, S_2 as cylinders with ends of area A Fig. 1.195 (b). One end of each surface lies within one of the conducting plates.

For the surface labeled S_1, the left-hand end is within plate 1 (the positive plate). Since the field is zero within the volume of any solid conductor under electrostatic conditions, there is no electric flux through this end. The electric field between the plates is perpendicular to the right-hand end, the flux is EA; this is positive, since $\vec{E}$ is directed out of the Gaussian surface. There is no flux through the side curved walls of the cylinder, since these walls are parallel to $\vec{E}$. So the total flux integral in Gauss's law is EA. The net charge enclosed by the cylinder is σA, so Gauass's law yields

$$EA = \frac{\sigma A}{\varepsilon_0}$$

and

$$E = \frac{\sigma}{\varepsilon_0}$$

(Resultant field between oppositely charged conducting plates)

The field is uniform and perpendicular to the plates, and its magnitude is independent of the distance from either plate. The surface S_2 can be used to show that $E = 0$ to the left of plate 1 and to the right of plate 2.

Example 120

A conducting sphere of radius R has two spherical cavities of radius a and b. The cavities have charges q_a and q_b respectively at their centres. Find:

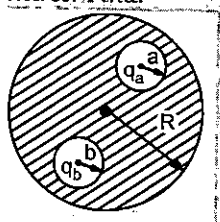


Fig. 1E.120 (a)

(a) The electric field and electric potential at a distance r
(i) r (distance from O, the centre of sphere > R)
(ii) r (distance from B, the centre of cavity b) < b
(iii) r inside sphere but out side cavities.
(b) Surface charge densities on the surface of radius R, radius a and radius b.
(c) What is the force on q_a and q_b?

Solution: In electrostatic equilibrium condition distribution of charges on all the surfaces is shown in Fig. 1E.120 (b)

Surface	Charge
S_1 :	$-q_a$
S_2 :	$-q_b$
S_3 :	$q_a + q_b$

(a) (i) At point $P(r > R)$, outside conductor

$$V = \frac{k(q_a + q_b)}{r} \Rightarrow E = k\frac{(q_a + q_b)}{r^2}$$

Entire sphere behaves as if its entire charge is concentrated at its centre

Electric field and potential due to induced charges on the inner surface of cavity and charge at centre is zero for all the points just outside it.

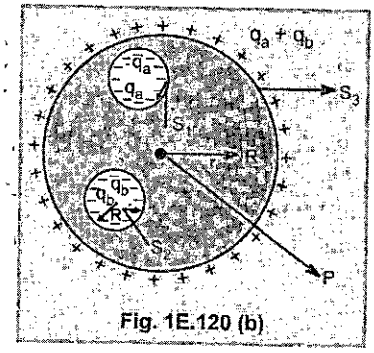


Fig. 1E.120 (b)

(ii) For a point R inside cavity B electric field and potential due to all the charges of cavity is zero.

Electric field due to charges on the outer surface of sphere is zero for any inside point.

Electric potential due to charges on outer surface of conducting sphere for all the inside points is constant and equal to

$$V = \frac{k(q_a + q_b)}{R}$$

Electric potential due to charges on surface S_2 at any inside point of cavity b is $\frac{kq_b}{b}$

$V_R = V$ (due to charges of cavity a) + V (due to charges of cavity b) + V (due to charges on outer surface)

$$= 0 + \frac{-kq_b}{b} + \frac{kq_b}{r} + \frac{k(q_a + q_b)}{R}$$

(iii) At position Q electric potential and electric field due to charges of cavity is zero.

Electric field due to charges on outer surface is zero. As point Q lies inside material, resultant electric field here must be zero and potential must be constant.

$$E_Q = 0$$
$$V_Q = \frac{k(q_a + q_b)}{R}$$

Electric potential inside material is constant and equal to that at surface.

(q) Charge density on surface S_3: $\sigma_3 = \frac{(q_a + q_b)}{4\pi R^2}$

Charge density on surface $S_2 = \frac{q_b}{4\pi b^2}$

Charge density on surface $S_1 = \frac{q_a}{4\pi a^2}$

(c) Electric due to charges on inner surface cavity at any inside point is zero. Electric field due to charges on outer surface is zero for all inside points. Thus neither q_a nor q_b experiences any electric field and resultant force on them is zero.

Example 121

S_1 and S_2 are two conducting surfaces. Between S_1 and S_2 and inside S_1 is air. S_1 is spherical with A its centre S_1 has total charge Q. S_2 is uncharged. Determine the value of following quantities if it is possible, using the given data ?

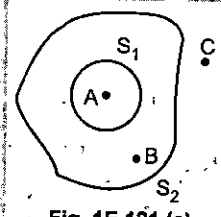


Fig. 1E.121 (a)

(i) Charges induced on inner and outer surface of S_2.
(ii) Total electric field at A and B.
(iii) Electric field at B due to induced charges on S_2.
(iv) Electric field at C due to induced charge on inner surface of S_2.
(v) Electric field produced by induced charge on outer surface of S_2 inside the material of S_2.
(vi) Electric field at C. (take the required distance from A).

Solution: Charge given on S_1 will distribute on outer surface of S_1, such that it is an equipotential surface in electrostatic equilibrium. Due to nonuniform nature of outer surface the contribution to potential at S_1 cannot be predicted. Charge distribution S_1 and inner surface of S_2 cannot be predicted.

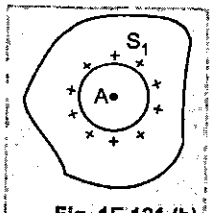


Fig. 1E.121 (b)

(i) $-Q, +Q$
(ii) $E_A = 0$
E_B can't be found
(iii) can't be found
(iv) can't be found
(v) zero
(vi) can't be found.

Example 122

A solid metal sphere of radius R has a charge +2Q. A hollow spherical shell of radius 3R placed concentric with the first sphere has net charge −Q.

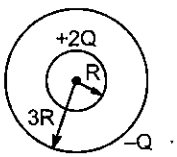


Fig. 1E.122 (a)

(a) Find the electric field between the spheres at a distance r from the centre of the inner sphere. [$R < r < 2R$]
(b) Calculate the potential difference between the spheres.
(c) What would be the final distribution of charges if the spheres are joined by a conducting wire.
(d) Instead of (c), if the inner sphere is earthed, what is the charge on it.

Solution: (a) Consider a spherical gaussian surface at $R < r < 2R$

$$E4\pi r^2 = \frac{+2Q}{\varepsilon_0}$$

$$E(R < r < 2R) = \frac{Q}{2\pi\varepsilon_0 r^2}$$

(b) $$V_{\text{inner sphere}} = V_1 = \frac{2Q}{4\pi\varepsilon_0 R} - \frac{Q}{4\pi E_0(3R)}$$

$$V_{\text{outer sphere}} = V_2 = \frac{2Q}{4\pi\varepsilon_0(3R)} - \frac{Q}{4\pi\varepsilon_0(3R)}$$

$$V_2 - V_1 = \frac{Q}{2\pi\varepsilon_0 R}$$

(c) When a wire connects inner sphere to outer sphere its entire charge gets transfered to outer surface. In final state inner sphere has zero charge and outer sphere Q.

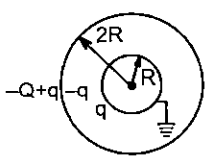


Fig. 1E.122 (b)

(d) If inner sphere is earthed, net charge appearing on it be q.

After earthing potential of inner sphere becomes zero

$$\frac{q}{4\pi\varepsilon_0 R} - \frac{Q}{4\pi\varepsilon_0(2R)} = 0$$

$$q = Q/2$$

(Note that potential due to induced charges on outer shell is zero only if shell is thin)

Example 123

The point charge q is within an electrically neutral conducting shell whose outer surface has spherical shape. Find the potential V at a point P lying outside the shell at a distance r from the centre O of the outer sphere.

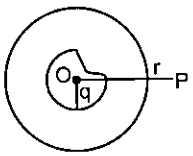


Fig. 1E.123 (a)

Solution: Charge on surface S_1 is $-q$, and on S_2 is $+q$.

Charge distribution S_1 is nonuniform but on S_2 it is uniform. Potential due to charges of cavity is zero.

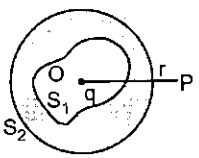


Fig. 1E.123 (b)

Thus potential at P is entirely due to charges an outer surface of S_2.

Example 124

In the figure shown, both the shells are conducting. The outer shell is thick and has a total charge of Q, inner shell is thin and is earthed.

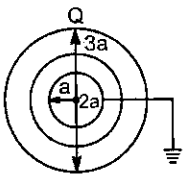


Fig. 1E.124 (a)

(a) Find charge on each surface.
(b) Draw graph of potential (V) v/s distance from centre (r).

Solution: (a) Let charge on S_2 after earthing be q. Potential of this surface must be zero.

$$V_{S_2} = \frac{kq}{a} - \frac{kq}{2a} + \frac{+k(Q+q)}{3a} = 0$$

$$q = -\frac{2Q}{5}$$

In final state charges on all the sphere is shown in Fig.

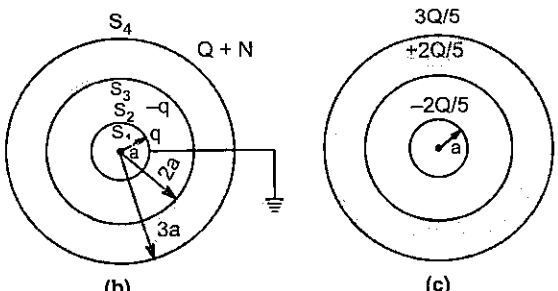


Fig. 1E.124

$$V(r < q) = 0$$

Potential at surface S_2 is zero and a conductor has equipotential volume.

$$V(q < r < 2a) = \frac{kq}{r} - \frac{-kq}{2a} + \frac{k(Q+q)}{3a}$$

$$= \frac{2kQ}{5}\left[\frac{1}{a} - \frac{1}{r}\right]$$

$$V(r = 3a) = \frac{k(3Q/5)}{3a} = \frac{kQ}{5a}$$

$$V(r > 3a) = \frac{kQ}{5r}$$

(b) Electric field in material of conductor is zero therefore potential is constant and equal to that at surface

$$V(2a < r < 3a) = \frac{kQ}{5a}$$

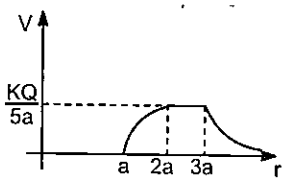


Fig. 1E.124 (d)

Example 125

Near an infinite line of charge carrying a linear charge density λ, a light rod of length $2l$ is placed in the plane of line charge such that its centre is at a distance d from the line charge ($d > l$), as shown in figure. Two point charges $+q$ and $-q$ are attached at the ends of the rod. The rod is hinged at its centre and initially perpendicular to the line charge. Find the work done in rotating the rod to the position when it becomes perpendicular to the line charge in the plane normal to the plane of paper.

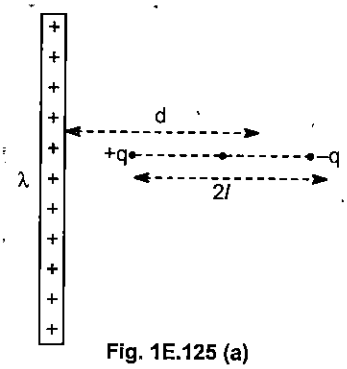


Fig. 1E.125 (a)

Solution: Potential difference between two points at r_1 and r_2 near a long wire is given by

$$V_2 - V_1 = \frac{\lambda}{2\pi\varepsilon_0}\ln\frac{r_2}{r_1}$$

Change in potential energy of charge $-q$

$$\Delta U_1 = \frac{-\lambda q}{2\pi\varepsilon_0}\ln\frac{\sqrt{d^2+l^2}}{(d+l)}$$

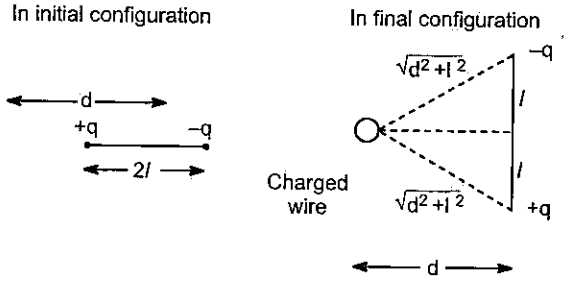


Fig. 1E.125 (b)

Change in potential energy of charge $+q$

$$\Delta U_2 = \frac{+\lambda q}{2\pi\varepsilon_0}\ln\frac{\sqrt{d^2+l^2}}{(d-l)}$$

Net change in potential energy of system

$$\Delta U = \Delta U_1 + \Delta U_2$$

$$= \frac{-\lambda q}{2\pi\varepsilon_0}\ln\frac{\sqrt{d^2+l^2}}{(d+l)} + \frac{\lambda q}{2\pi\varepsilon_0}\ln\frac{\sqrt{d^2+l^2}}{(d-l)}$$

$$= \frac{\lambda q}{2\pi\varepsilon_0}\ln\left(\frac{d+l}{d-l}\right)$$

Example 126

Two mutually perpendicular wire carry charge densities λ_1 and λ_2. The electric line of force makes angle α with second wire then find λ_1/λ_2. (in tern of angle α)

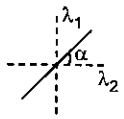


Fig. 1E.126 (a)

Solution: $E_1 = \frac{\lambda_1}{2\pi\varepsilon_0 x}$; $E_2 = \frac{\lambda_2}{2\pi\varepsilon_0 y}$

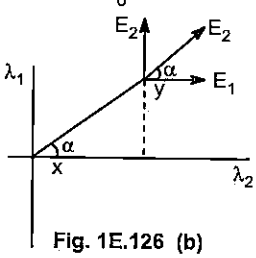


Fig. 1E.126 (b)

$$\tan\alpha = \frac{E_2}{E_1} = \frac{\lambda_2}{\lambda_1}\times\frac{x}{y}$$

also $\tan\alpha = \frac{y}{x}$

Thus $\cot^2\alpha = \lambda_1/\lambda_2$

Example 127

Two equal point charges are fixed at $x = -a$ and $x = +a$ on the x-axis. Another point charge Q is placed at the origin. Find the change in electrical energy of Q (approximately) when it is displaced by a small distance along the x-axis.

Solution: Initially, potential energy of Q is,

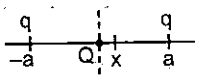


Fig. 1E.127

$$U_1 = \frac{1}{4\pi\varepsilon_0}\cdot\left[\frac{Qq}{a}+\frac{Qq}{a}\right]$$
$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{2Qq}{a}$$

When Q is displaced by small amount x to the right, the potential energy of Q becomes,

$$U_2 = \frac{1}{4\pi\varepsilon_0}\left[\frac{Qq}{(a+x)}+\frac{Qq}{(a-x)}\right]$$
$$= \frac{1}{4\pi\varepsilon_0}\frac{2Qqa}{(a^2-x^2)}$$
$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{2Qq}{a}\left(1-\frac{x^2}{a^2}\right)^{-1}$$

Expanding binomially and neglecting higher powers of $\frac{x^2}{a^2}$ as $x \ll a$.

$$U_2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2Qq}{a}\left(1+\frac{x^2}{a^2}\right)$$

∴ The change in electrostatic potential energy of system,

$$dU = U_2 - U_1$$
$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{2Qq}{a}\cdot\frac{x^2}{a^2}$$
$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{2Qqx^2}{a^3}$$

Example 128

An electron enters the region between the plates of a parallel plate capacitor at a point equidistant from either plate. The capacitor plates are 2×10^{-2} m apart and 10^{-1} m long. A potential difference of 300 V is maintained across the plates. Assuming that the initial velocity of the electron is parallel to the capacitor plates, calculate the largest value of the velocity of the electrons so that they do not fly out of the capacitor at the other end. [$m_e = 9\times10^{-3}$ kg and $e = 1.6\times10^{-19}$ C]

Solution: The electron will experience a force $F_e = eE$ opposite to the field as shown in Fig. 1E.128 and hence acceleration of electron along y-axis.

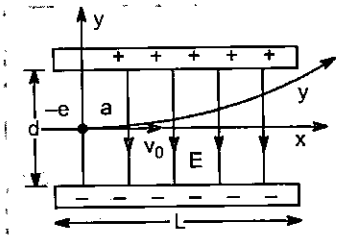


Fig. 1E.128

$$a = \frac{F}{m} = \frac{eE}{m} = \text{constt.} \qquad ...(1)$$

So from equation of motion,

$$s = ut + \frac{1}{2}at^2$$

along x-axis, $x = v_0 t$ [as $a = 0$] ...(2)

And along y-axis, $y = \frac{1}{2}at^2$ [as $u = 0$] ...(3)

Eliminating t between Eqs. (2) and (3)

$$y = \frac{1}{2}a\left[\frac{x}{v_0}\right]^2 = \frac{1}{2}\frac{qE}{m}\frac{x^2}{v_0^2}$$

$\left[\text{as from Eq. (1) } a = \frac{qE}{m}\right]$

As electron is entering the plates midway between them, it will come out of the plates if

$$(y)_{x=L} < \frac{d}{2}$$

i.e., $$\frac{1}{2}\frac{qE}{m}\frac{L^2}{v_0^2} < \frac{d}{2}$$

i.e., $$v_0 > L\sqrt{\frac{qE}{md}}$$

so $$(v_0)_{\min} = \frac{L}{d}\sqrt{\frac{qV}{m}} \qquad \left[\text{as } E = \frac{V}{d}\right]$$

Substituting the given data

$$(v_0)_{\min} = \frac{10^{-1}}{2\times10^{-2}}\sqrt{\frac{1.6\times10^{-19}\times300}{9\times10^{-31}}}$$
$$= 3.65\times10^7 \text{ m/s}$$

As this is the minimum velocity so that electron may come out of the plates, it is also the maximum velocity so that the electron may not come out of the plates.

Example 129

A block having mass m and charge q is resting on a frictionless plane at a distance L from the wall as shown in Fig. 1E.129. Discuss the motion of the block when a uniform electric field E is applied horizontally towards the wall assuming that collision of the block with the wall is perfectly elastic.

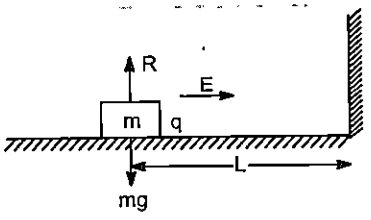


Fig. 1E.129

Solution: The situation is shown in Fig. 1E.129. Electric force $\vec{F} = q\vec{E}$ will accelerate the block towards the wall producing an acceleration,

$$a = \frac{F}{m} = \frac{qE}{m}$$

As initially the block is at rest and acceleration is constant, from second equation of motion, time taken by the block to reach the wall

$$L = \frac{1}{2}at^2$$

i.e., $$t = \sqrt{\frac{2L}{a}} = \sqrt{\frac{2mL}{qE}}$$

As collision with the wall is perfectly elastic, the block will rebound with same speed and as now its motion is opposite to the acceleration, it will come to rest after travelling same distance L in same time t. After stopping it will be again accelerated towards the wall and so *the block will execute oscillatory motion* with 'span' L and time period

$$T = 2t = 2\sqrt{\frac{2mL}{qE}}$$

However, as the restoring *force* $F(= qE)$ when the block in moving away from the wall is constant and not proportional to displacement x, *the motion is not simple harmonic.*

Example 130

A block having mass $m = 4$ kg and charge $q = 50$ μC is connected to a spring having a force constant $k = 100$ N/m. The block lies on a frictionless horizontal track and a uniform electric field $E = 5 \times 10^5$ V/m acts on the system as shown in Fig. 1E.130. The block is released from rest when the spring is unstretched (at $x = 0$). (i) By what maximum amount does the spring expand? (ii) What is the equilibrium position of the block? (iii) Show that the block's motion is simple harmonic and determine the amplitude and time period of the motion.

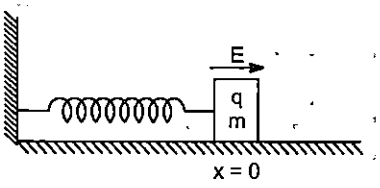


Fig. 1E.130

Solution: (i) As x increases, electric force qE will accelerate the block while elastic force in the spring kx will oppose the motion. The block will move away from its initial position $x = 0$ till it comes to rest, *i.e.*, work done by the electric force is equal to the energy stored in the spring. So if x_{max} is the maximum stretch of the spring.

$$\frac{1}{2}kx_{max}^2 = (qE)x_{max}$$

i.e., $$x_{max} = \frac{2qE}{k}$$

i.e., $$x_{max} = \frac{2 \times (50 \times 10^{-6}) \times (5 \times 10^5)}{100} = 0.5 \text{ m}$$

(ii) In equilibrium position $F_R = 0$, so if x_0 is the stretch of the spring in equilibrium position,

$$kx_0 = qE,$$

i.e., $$x_0 = (qE/k) = \frac{1}{2}x_{max} = 0.25 \text{ m}$$

(iii) If the displacement of the block from equilibrium position (x_0) is x, restoring force will be

$$F = k(x \pm x_0) \mp qE = kx \quad [\text{as } kx_0 = qE]$$

And as the restoring force is linear *the motion will be simple harmonic* with time period

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{4}{100}} = 0.4\pi \text{ sec}$$

and amplitude $= x_{max} - x_0 = 0.5 - 0.25 = 0.25$ m

Example 131

An infinite plane of positive charge has a surface charge density σ. A metal ball B of mass m and charge q is attached to a thread and tied to a point A on the sheet PQ. Find the angle θ which AB makes with the plane PQ.

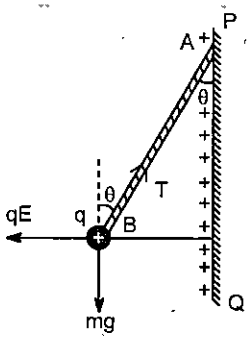


Fig. 1E.131

Solution: Due to positive charge the ball will experience electrical force $F_e = qE$ horizontally away from the sheet while the weight of the ball will act vertically downwards and hence if T is the tension in the string, for equilibrium of ball:

Along horizontal, $T \sin\theta = qE$

And along vertical, $T \cos\theta = mg$

So that $$\tan\theta = \frac{qE}{mg}$$

and $$T = [(mg)^2 + (qE)^2]^{1/2} \quad ...(1)$$

However, as here field E is produced by the sheet of charge PQ having charge density σ.

i.e., $$E = \frac{\sigma}{2\varepsilon_0}$$

So, $$\tan\theta = \frac{q\sigma}{2\varepsilon_0 mg}$$

i.e., $$\theta = \tan^{-1}\left[\frac{q\sigma}{2\varepsilon_0 mg}\right]$$

Example 132

An infinite number of charges each equal to q are placed along the x-axis at x = 1, x = 2, x= 4, x = 8 ... and so on . Find the potential and the electric field at the point x = 0 due to this set of charges. What will be the potential and field in the above set, if the consecutive charges have opposite signs ?

Solution: As potential V and intensity E due to a point charge at position x are respectively given by:

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{x}$$

and

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{x^2}$$

and electric interaction is a two body interaction, *i.e.*, principle of super-position holds good.

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{1}+\frac{q}{2}+\frac{q}{4}+\ldots\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}\ldots\right]$$

i.e.,

$$V = \frac{q}{4\pi\varepsilon_0}\times\frac{1}{[1-(1/2)]}$$

$$= \frac{1}{4\pi\varepsilon_0}(2q)$$

$$E = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{1^2}+\frac{q}{2^2}+\frac{q}{4^2}+\ldots\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[1+\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+\ldots\right]$$

i.e.,

$$E = \frac{q}{4\pi\varepsilon_0}\frac{1}{[1-(1/4)]} = \frac{1}{4\pi\varepsilon_0}\left[\frac{4}{3}q\right]$$

When consecutive charges have opposite signs:

$$V = \frac{q}{4\pi\varepsilon_0}\frac{1}{[1-(1/2)]}$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{2}{3}q\right]$$

And

$$E = \frac{q}{4\pi\varepsilon_0}\frac{1}{[1-(-1/4)]}$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{4}{5}q\right]$$

Example 133

Two small charged blocks of charges 5 μC and 3μC are kept on a rough surface (μ = 0.5) at a separation of 0.1 m. Find the separation between the two blocks when they come to rest.

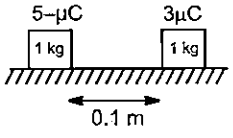


Fig. 1E.133

Solution: When the charges are released from rest, they tend to move due to electrostatics repulsion. In this process two forces perform work electrostatic and function. From work energy theorem we get

$$kQ_1Q_2\left(\frac{1}{l}-\frac{1}{l_0}\right)-\mu mg\,(l-l_0) = 0$$

on solving we get

$$l = 0.27\text{m}$$

Example 134

The bob of a pendulum has mass m =1 kg and charge q = 40 μC. Length of pendulum is l = 0.9 m. The point of suspension also has the same charge 40 μC. What the minimum speed u should be imparted to the bob so that it can complete vertical circle ?

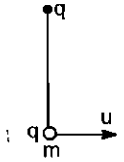


Fig. 1E.134

Solution: In the given problem which should first determine wether string can get slack or net.

As $\frac{kq^2}{l^2} > mg$ string will no slack for any value of a.

Thus now from conservation of energy, we get networkdone by electric force is zero

$$\frac{1}{2}mu^2 = mg(2l)$$

$$\Rightarrow \quad u = \sqrt{4gl} = 6\text{m/s}$$

Example 135

Charge q and $-\frac{cq}{b}$ are placed at (b, 0) and $\left(\frac{c^2}{b}, 0\right)$. Find the locus of points in $x-y$ plane, having zero potential. What is its shape ?

Solution: Potential at point (x, y) has zero potential thus

$$\frac{kq}{[(x-b)^2+y^2]^{1/2}}+\frac{k\left(\frac{-cq}{b}\right)}{\left[\left(x-\frac{c^2}{b}\right)^2+y^2\right]^{1/2}} = 0$$

Which simplifying gives

$$\left(x-\frac{c^2}{b}\right)^2+y^2 = \frac{c^2}{b^2}(x-b)^2+\frac{c^2y^2}{b^2}$$

$$\left(x-\frac{c^2}{b}-\frac{cx}{b}+c\right)\left(x-\frac{c^2}{b}-\frac{cx}{b}-c\right) = y^2\left(\frac{c^2}{b^2}-1\right)$$

$$\left(x\left(1-\frac{c}{b}\right)+c\left(1-\frac{c}{b}\right)\right)\left(x\left(1-\frac{c}{b}\right)-c\left(1+\frac{c}{b}\right)\right)$$
$$= y^2\left(\frac{c}{b}-1\right)\left(\frac{c}{b}+1\right)$$
$$(x+c)\left(1-\frac{c}{b}\right)(x-c)\left(1+\frac{c}{b}\right)=y^2\left(\frac{c}{b}-1\right)\left(\frac{c}{b}+1\right)$$
$$c^2-x^2=y^2$$
$$x^2+y^2=c^2$$

Which is equation of circle.

Example 136

A solid sphere having uniform volume charge density ρ and radius R is shown in fig. A spherical cavity of radius R/2 hollowed out. Find the potential difference between the centre of sphere O and centre of cavity O_1. (As shown in Fig. 1E.136)

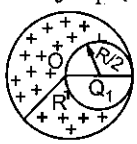


Fig. 1E.136

Solution: Potential at any position can be determined from principle of superposition. System can be imagined to be superposition of a complete solid sphere, charge density +ρ and a negatively charged sphere of charge density −ρ

$$V_0 = V_+ + V_-$$
$$V_+ = \frac{3}{2}\frac{Q}{4\pi\varepsilon_0 R}$$
$$= \frac{3}{2}\frac{\rho\times\left(\frac{4}{3}\pi R^3\right)}{4\pi\varepsilon_0 R}=\frac{\rho R^2}{2\varepsilon_0}$$
$$V_- = \frac{-Q}{4\pi\varepsilon_0 R/2}$$
$$= \frac{-\rho\times\frac{4}{3}\pi(R/2)^3}{4\pi\varepsilon_0(R/2)}=\frac{-\rho R^2}{4\varepsilon_0}$$
$$V_0 = \frac{\rho R^2}{2\varepsilon_0}-\frac{\rho R^2}{4\varepsilon_0}=\frac{\rho R^2}{4\varepsilon_0}$$

Similarly $V_{0_1} = V_+ + V_-$

$$V_+ = \frac{Q}{8\pi\varepsilon_0 R^3}(3R^2-r^2)$$
$$= \frac{\rho\times\left(\frac{4}{3}\pi R^3\right)}{8\pi\varepsilon_0 R^3}[3R^2-(R/2)^2]$$
$$= 11\rho R^2/24\varepsilon_0$$
$$V_- = -\frac{3}{2}\frac{Q'}{4\pi\varepsilon_0(R/2)}$$
$$= -\frac{3}{2}\frac{\rho\times\frac{4}{3}\pi(R/2)^3}{4\pi\varepsilon_0(R/2)}=\frac{-\rho R^2}{8\varepsilon_0}$$
$$V_{0_1} = \frac{11\rho R^2}{24\varepsilon_0}-\frac{\rho R^2}{8\varepsilon_0}=\frac{\rho R^2}{3\varepsilon_0};$$

Hence $V_0 - V_{0_1} = \frac{\rho R^2}{12\varepsilon_0}$

Alternately: Determine electric field inside cavity and apply $\int \vec{\mathbf{E}}\cdot d\vec{\mathbf{s}} = E\frac{R}{2}$.

Example 137

A metal sphere A of radius a is charged to potential V. What will be its potential if it is enclosed by a spherical conducting shell B of radius b and the two are connected by a wire ?

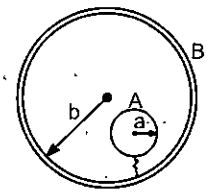


Fig. 1E.137

Solution: If the charge on sphere of radius *a* is *q*,

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{a}$$

i.e., $q = (4\pi\varepsilon_0 a)V$ ...(1)

Now, when sphere *A* is enclosed by spherical conductor *B* and the two are connected by a wire, charge will reside on outer surface of *B* and so the potential of *B* will be,

$$V_B = \frac{1}{4\pi\varepsilon_0}\frac{q}{b}$$
$$= \frac{1}{4\pi\varepsilon_0}\frac{4\pi\varepsilon_0 a}{b}V=\frac{a}{b}V$$

Now as sphere *A* is inside *B* so its potential,

$$V_A = V_B = \frac{a}{b}(V) \qquad [< V \text{ as } a < b]$$

Example 138

A solid conducting sphere having a charge Q is surrounded by an uncharged concentric conducting hollow spherical shell. Let the potential difference between the surface of the solid sphere and that of the outer surface of hollow shell be V. What will be the new potential difference between the same two surfaces if the shell given a charge −3Q?

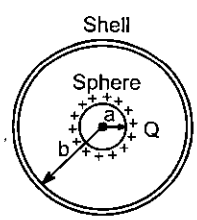


Fig. 1E.138

Solution: In case of a charged conducting sphere,

$$V_{in} = V_C = V_S = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$$

and $$V_{out} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

So if a and b are the radii of sphere and spherical shell respectively, potential at their surfaces will be,

$$V_{sphere} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{a}$$

and $$V_{shell} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{b}$$

Ans so according to give problem

$$V = V_{sphere} - V_{shell} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{a} - \frac{1}{b}\right] \quad ...(1)$$

Now when the shell is given a charge $(-3Q)$ the potential at its surface and also inside will change by

$$V_0 = \frac{1}{4\pi\varepsilon_0}\left[\frac{-3Q}{b}\right]$$

So that now

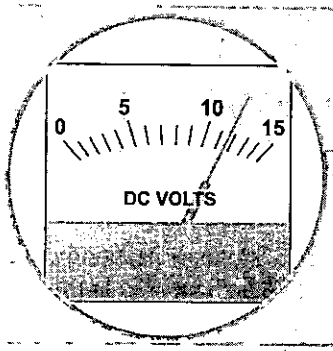
$$V'_{sphere} = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{a}\right] + V_0$$

and $$V'_{shell} = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{b}\right] + V_0$$

And hence

$$V'_{sphere} - V'_{shell} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{a} - \frac{1}{b}\right] = V \quad \text{[from Eq. (1)]}$$

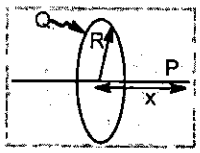
i. e., if any charge is given to external shell, the potential difference between sphere and shell will not change. This is because by presence of charge on outer shell, potential everywhere inside and on the surface of shell will change by same amount and hence potential difference between sphere and shell will remain unchanged.



PROBLEMS

Level 1 Only One Alternative is Correct

1. A small particle of mass m and charge $-q$ is placed at point P and released. If $R >> x$, the particle will undergo oscillations along the axis of symmetry with an angular frequency that is equal to:



(a) $\sqrt{\dfrac{qQ}{4\pi\varepsilon_0 mR^3}}$ (b) $\sqrt{\dfrac{qQx}{4\pi\varepsilon_0 mR^4}}$

(c) $\dfrac{qQ}{4\pi\varepsilon_0 mR^3}$ (d) $\dfrac{qQ}{4\pi\varepsilon_0 mR^4}$

2. A small electric dipole is placed at origin with its dipole moment directed along positive x-axis. The direction of electric field at point $(2, 2\sqrt{2}, 0)$ is
(a) along z-axis
(b) along y-axis
(c) along negative y-axis
(d) along negative z-axis

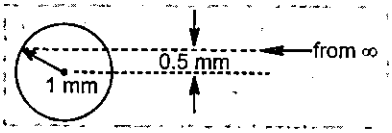
3. Two particles X and Y, of equal mass and with unequal positive charges, are free to move and are initially far away from each other. With Y at rest, X begins to move towards it with initial velocity u. After a long time finally:
(a) X will stop, Y will move with velocity u.
(b) X and Y will both move with velocities $u/2$ each
(c) X will stop, Y will move with velocity $< u$
(d) both will move with velocities $< u/2$

4. A uniform electric field of magnitude E and directed along positive x-axis exists in a certain region of space. If at $x = 0$ the electric potential V is zero, then the potential at $x = +x_0$ is:
(a) zero (b) $-E\,x_0$
(c) $-\dfrac{E}{x_0}$ (d) $\dfrac{E}{x_0}$

5. Three points A, B and C are at a distance of 1 m, 2 m and 1 m from an infinitely long charged wire of linear density λ coulomb/meter. A charge q is taken from A to B, B to C and finally C to A. Which of the following statements is correct about the work done in the above process?
(a) $W_{AB} = 2W_{BC}$ (b) $W_{AB} = -W_{BC}$
(c) $W_{BC} = 0$ (d) $W_{CA} \neq 0$

6. The electric intensity at a point at distance 2 m from charge q is E. The amount of work done in bringing a charge of 2 coulomb from infinity to this point will be:
(a) $2E$ joules (b) $4E$ joules
(c) $\dfrac{E}{2}$ joules (d) $\dfrac{E}{4}$ joules

7. A particle of mass 1 kg & charge $\dfrac{1}{3}\mu$C is projected towards a non conducting fixed spherical shell having the same charge uniformly distributed on its surface. Find the minimum initial velocity of projection required it the particle just grazes the shell:



(a) $\sqrt{\dfrac{2}{3}}$m/s (b) $2\sqrt{\dfrac{2}{3}}$ m/s
(c) $\dfrac{2}{3}$ m/s (d) none of these

8. The potential field of an electric field $\vec{\mathbf{E}} = (y\,\hat{\mathbf{i}} + x\,\hat{\mathbf{j}})$ is
(a) $V = -xy$ + constant
(b) $V = -(x + y)$ + constant
(c) $V = -(x^2 + y^2)$ + constant
(d) V = constant

9. The electric field in a region is given by the vector $\vec{\mathbf{E}} = (4\hat{\mathbf{i}} + 1\hat{\mathbf{j}})\left(\dfrac{\text{N}}{\text{C}}\right)$. The maximum drop in potential will be along:
(a) X-axis (b) Y-axis
(c) the line $4y = 3x$ (d) the line $3y = 4x$

10. Two point charges of $+Q$ each have been placed at the positions $(-a/2, 0, 0)$ and $(a/2,0,0)$. The locus of the points where $-Q$ charge can be placed such the that total electrostatic potential energy of the system can become equal to zero, is represented by which of the following equations?

(a) $Z^2 + (Y-a)^2 = 2a$

(b) $Z^2 + (Y-a)^2 = 27a^2/4$

(c) $Z^2 + Y^2 = 15a^2/4$

(d) None of these

11. A hemispherical body of radius R is placed in a uniform electric field E. The flux linked with it, field is parallel to the base, is:

(a) zero (b) $\pi R^2 E$

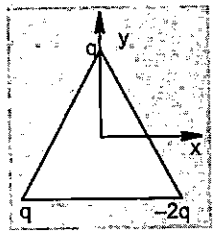
(c) $2\pi R^2 E$ (d) $2\pi RE$

12. Three identical metallic uncharged spheres A, B and C of radius a are kept at the corners of an equilateral triangle of side $d(d >> a)$. The fourth sphere (of radius a), which has a charge q, touches A and is then removed to a position far away. B is earthed and then the earth connection is removed. C is then earthed. The charge on C is:

(a) $\frac{qa}{2d}\left(\frac{2d-a}{2d}\right)$ (b) $\frac{qa}{2d}\left(\frac{2d-a}{d}\right)$

(c) $-\frac{qa}{2d}\left(\frac{d-a}{d}\right)$ (d) $\frac{2qa}{d}\left(\frac{d-a}{2d}\right)$

13. An equilateral triangle wire frame of side L having 3 point charges at its vertices is kept in x-y plane as shown. Component of electric field due to the configuration in z direction at $(0, 0, L)$ is [origin is centroid of triangle]:



(a) $\frac{9\sqrt{3}kq}{8L^2}$

(b) zero

(c) $\frac{9kq}{8L^2}$

(d) None

14. A uniform electric field exists in $x-y$ plane. The potential of points $A(2m, 2m), B(-2m, 2m)$ and $C(2m, 4m)$ are $4V, 16V$ and $12V$ respectively. The electric field is

(a) $(4\hat{\mathbf{i}} + 5\hat{\mathbf{j}})\frac{V}{m}$ (b) $(3\hat{\mathbf{i}} + 4\hat{\mathbf{j}})\frac{V}{m}$

(c) $-(3\hat{\mathbf{i}} + 4\hat{\mathbf{j}})\frac{V}{m}$ (d) $(3\hat{\mathbf{i}} - 4\hat{\mathbf{j}})\frac{V}{m}$

15. Two free positive charges $4q$ and q are a distance 1 apart. What charge Q is needed to achieve equilibrium for the entire system and where should it be placed form charge q?

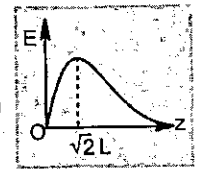
(a) $Q = \frac{4}{9}q$ (negative) at $\frac{l}{3}$

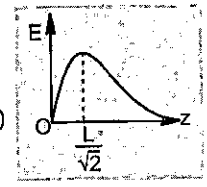
(b) $Q = \frac{4}{9}q$ (positive) at $\frac{l}{3}$

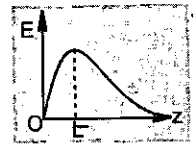
(c) $Q = q$ (positive) at $\frac{l}{3}$

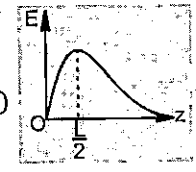
(d) $Q = q$ (negative) at $\frac{l}{3}$

16. Four equal positive charges are fixed at the vertices of a square of side L. z-axis is perpendicular to the plane of the square. The point z = 0 is the point where the diagonals of the square intersect each other. The plot of electric field due to the four charges, as one moves on the z-axis.

(a) 

(b) 

(c) 

(d) 

17. A nonconducting ring of radius R has uniformly distributed positive charge Q. A small part of the ring of length d, is removed $(d << R)$. The electric field at the centre of the ring will now be

(a) directed towards the gap, inversely proportional to R^3

(b) directed towards the gap, inversely proportional to R^2

(c) directed away from the gap, inversely proportional to R^3

(d) directed away from the gap, inversely proportional to R^2

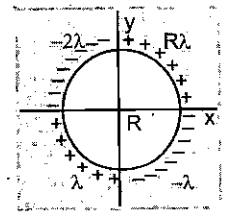
18. The charge per unit length of the four quadrant of the ring is 2λ, -2λ, λ and λ respectively. The electric field at the centre is:

(a) $\frac{\lambda}{2\pi\varepsilon_0 R}\hat{\mathbf{i}}$

(b) $\frac{\lambda}{2\pi\varepsilon_0 R}\hat{\mathbf{j}}$

(c) $\frac{\sqrt{2}\lambda}{4\pi\varepsilon_0 R}\hat{\mathbf{i}}$

(d) None of these



19. Electric field on the axis of a small electric dipole at a distance r is $\vec{\mathbf{E}}_1$ and $\vec{\mathbf{E}}_2$ at a distance of $2r$ on a line of perpendicular bisector. Then:

(a) $\vec{\mathbf{E}}_2 = \frac{-\vec{\mathbf{E}}_1}{8}$ (b) $\vec{\mathbf{E}}_2 = -\frac{\vec{\mathbf{E}}_1}{16}$

(c) $\vec{\mathbf{E}}_2 = -\frac{\vec{\mathbf{E}}_1}{4}$ (d) $\vec{\mathbf{E}}_2 = \frac{\vec{\mathbf{E}}_1}{8}$

20. a charged particle having some mass is resting in equilibrium at a height H above the centre of a uniformly charged non-conducting horizontal ring of radius R. The force of gravity acts downwards. The equilibrium of the particle will be stable:

(a) for all values of H

(b) only if $H > \frac{R}{\sqrt{2}}$

(c) only if $H < \frac{R}{\sqrt{2}}$

(d) only if $H = \frac{R}{\sqrt{2}}$

21. The positively charged particles X and Y are initially far away from each other and at rest. X begins to move towards Y with some initial velocity. The total momentum and energy of the system are p and E.

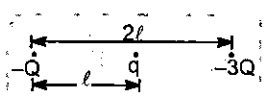
(a) If Y is fixed, both p and E are conserved.

(b) If Y is fixed, E is conserved, but not p.

(c) If both are free to move, p is conserved but not E

(d) If both are free, E is conserved, but not p.

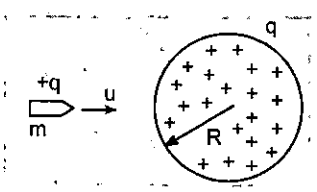
22. Three charges $-Q, q$ and $-3Q$ are arranged as shown in figure. The system of charges will have positive configuration energy if:



(a) $q > \frac{3Q}{8}$ (b) $q < \frac{8Q}{3}$

(c) $q > \frac{3Q}{2}$ (d) $q < \frac{3Q}{8}$

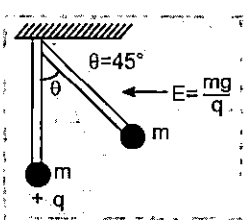
23. A bullet of mass m and charge q is fired towards a solid uniformly charged sphere of radius R and total charge $+q$. If it strikes the surface of sphere with speed u, find the minimum speed u so that it can penetrate through the sphere. (Neglect all resistance forces or friction acting on bullet except electrostatic forces.):



(a) $\frac{q}{\sqrt{2\pi\varepsilon_0 mR}}$ (b) $\frac{q}{\sqrt{4\pi\varepsilon_0 mR}}$

(c) $\frac{q}{\sqrt{2\pi\varepsilon_0 mR}}$ (d) $\frac{\sqrt{3}q}{\sqrt{4\pi\varepsilon_0 mR}}$

24. In space of horizontal EF ($E = mg/q$) exist as shown in figure and a mass m attached at the end of a light rod. If mass m is released from the position shown in figure find the angular velocity of the rod when it passes through the bottom most position:



(a) $\sqrt{\frac{g}{l}}$ (b) $\sqrt{\frac{2g}{l}}$

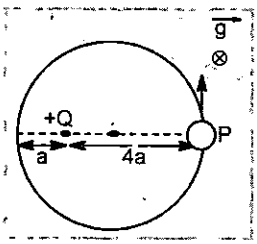
(c) $\sqrt{\frac{3g}{l}}$ (d) $\sqrt{\frac{5g}{l}}$

25. A point charge $q = 50\mu C$ is located in the $x-y$ plane at the point of position vector $\vec{r_0} = 2\hat{i} + 3\hat{j}$. What is the electric field at the point of position vector $\vec{r} = 8\hat{i} - 5\hat{j}$?.

(a) $1200\frac{V}{m}$ (b) $4\times10^{-2}\frac{V}{m}$

(c) $900\frac{V}{m}$ (d) $4500\frac{V}{m}$

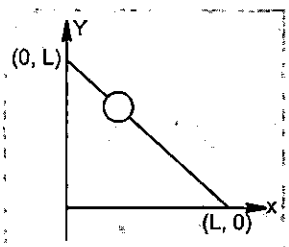
26. The diagram shows a small bead of mass m carrying charge q. The bead can freely move on the smooth fixed ring placed on a smooth horizontal plane. In the same plane a charge $+Q$ has also been fixed as shown. The potential at the point P due to $+Q$ is V. The velocity with which the bead should projected from the point P so that is can complete a circle should be greater than:



(a) $\sqrt{\frac{6qV}{m}}$ (b) $\sqrt{\frac{qV}{m}}$

(c) $\sqrt{\frac{3qV}{m}}$ (d) none of these

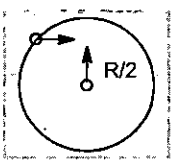
27. Electric field given by the vector $E = x\hat{i} + y\hat{j}$ is present in the XY plane. A small ring carrying charge $+Q$, which can freely slide on a smooth non conducting rod, is projected along the rod from the point $(0, L)$ such that it can reach the other end of the rod. What minimum velocity should be given to the ring? (Assume zero gravity)



(a) $(QL^2/m)^{1/2}$ (b) $2(QL^2/m)^{1/2}$

(c) $4(QL^2/m)^{1/2}$ (d) $(QL^2/2m)^{1/2}$

28. A unit positive point charge of mass m is projected with a velocity v inside the tunnel as shown. The tunnel has been made inside a uniformly charged non-conducting sphere. The minimum velocity with which the point charge should be projected such it can it reach the opposite end of the tunnel, is equal to:



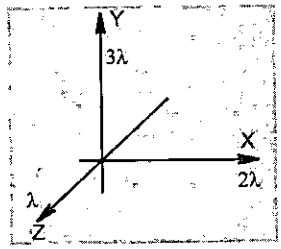
(a) $[\sigma R^2 / 4m\varepsilon_0]^{1/2}$

(b) $[\sigma R^2 / 24m\varepsilon_0]^{1/2}$

(c) $[\sigma R^2 / 6m\varepsilon_0]^{1/2}$

(d) zero because the initial and the final points are at same potential

29. The diagram shows three infinitely long uniform line charges placed on the X, Y and Z axis. The work done in moving a unit positive charge from (1, 1, 1) to (0, 1, 1) is equal to:



(a) $(\lambda \ln 2)/2\pi\varepsilon_0$
(b) $(\lambda \ln 2)/\pi\varepsilon_0$
(c) $(3\lambda \ln 2)/2\pi\varepsilon_0$
(d) None of these

30. A charged particle of charge Q is held fixed and another charged particle of mass m and charge q (of the same sign) is released from a distance r. The impulse of the force exerted by the external agent on the fixed charge by the time distance between Q and q becomes $2r$ is:

(a) $\sqrt{\frac{Qq}{4\pi\varepsilon_0 mr}}$ (b) $\sqrt{\frac{Qqm}{4\pi\varepsilon_0 r}}$
(c) $\sqrt{\frac{Qqm}{\pi\varepsilon_0 r}}$ (d) $\sqrt{\frac{Qqm}{2\pi\varepsilon_0 r}}$

31. Three point charges q, $-2q$ and $-2q$ are placed at the vertices of an equilateral triangle of side a. The work done by some external force to increase their separation to $2a$ will be:

(a) $\frac{1}{4\pi\varepsilon_0}\cdot\frac{2q^2}{a}$ (b) negative
(c) zero (d) $\frac{1}{4\pi\varepsilon_0}\cdot\frac{3q^2}{a}$

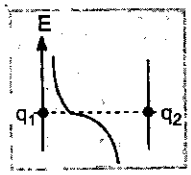
32. The equation of an equi-potential line in an electric field is $y = 2x$, then the electric field strength vector at (1, 2) may be:

(a) $4\hat{i}+3\hat{j}$ (b) $4\hat{i}+8\hat{j}$
(c) $8\hat{i}+4\hat{j}$ (d) $-8\hat{i}+4\hat{j}$

33. The electric field in a region is given by: $E=(4axy\sqrt{z})\hat{i}+(2ax^2\sqrt{z})\hat{j}+(ax^2/\sqrt{z})\hat{k}$, where a is a positive constant. The equation of an equipotential surface will be of the form:

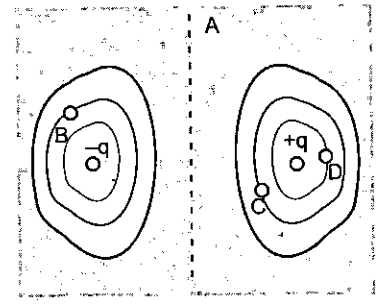
(a) $z = \text{constant}/[x^3y^2]$
(b) $z = \text{constant}/[xy^2]$
(c) $z = \text{constant}/[x^4y^2]$
(d) None of these

34. The variation of electric field between the two charges q_1 and q_2 along the line joining the charges is plotted against distance from q_1 (taking rightwards direction of field as positive) as shown, then the correct statement is:



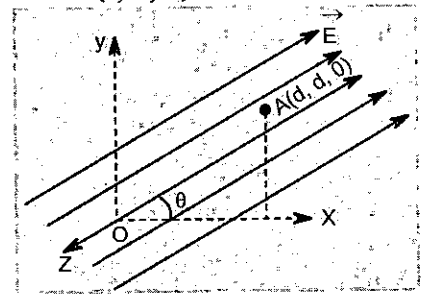
(a) q_1 and q_2 are positive and $q_1 < q_2$
(b) q_1 and q_2 are positive and $q_1 > q_2$
(c) q_1 is positive and q_2 is negative and $q_1 < q_2$
(d) q_1 and q_2 are negative and $q_1 < q_2$

35. Figure shows equi-potential surfaces for a two charges system. At which of the labeled points point will an electron have the highest potential energy?



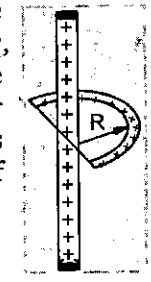
(a) Point A
(b) Point B
(c) Point C
(d) Point D

36. A uniform electric field having strength $\vec{E}$ is existing in x-y plane as shown in figure.Find the p.d. between origin O & A $(d, d, 0)$:



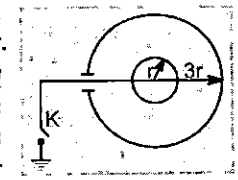
(a) $Ed(\cos\theta+\sin\theta)$ (b) $-Ed(\cos\theta+\sin\theta)$
(c) $\sqrt{2}Ed$ (d) none of these

37. Find the force experienced by the semicircular rod charged with a charge q, placed as shown in figure. Radius of the wire is R and the line of charge with linear charge density λ is passing through its centre and perpendicular to the plane of wire:



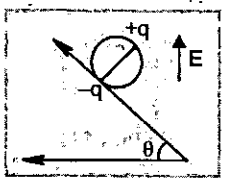
(a) $\frac{\lambda q}{2\pi^2\varepsilon_0 R}$ (b) $\frac{\lambda q}{\pi^2\varepsilon_0 R}$
(c) $\frac{\lambda q}{4\pi^2\varepsilon_0 R}$ (d) $\frac{\lambda q}{4\pi\varepsilon_0 R}$

38. Figure shows two conducting thin concentric shells of radii r and $3r$. The outer shell carries charge q and inner shell is neutral. The amount of charge which flows from inner shell to the earth after the key K is closed, is equal to:



(a) $-\frac{q}{3}$ (b) $\frac{q}{3}$
(c) $3q$ (d) $-3q$

39. A wheel having mass m has charges $+q$ and $-q$ on diametrically opposite points. It remains in equilibrium on a rough inclined plane in the presence of uniform vertical electric field E =:



(a) $\frac{mg}{q}$ (b) $\frac{mg}{2q}$

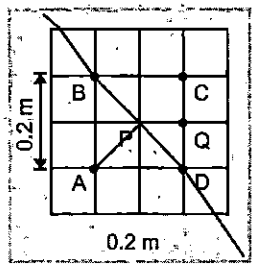
(c) $\frac{mg\tan\theta}{2q}$ (d) none

40. A non-conducting ring of radius 0.5 m carries a total charge of 1.11×10^{-10}C distributed non-uniformly on its circumference producing an electric field $\vec{E}$ everywhere in space. The value of the line integral $\int_{l=\infty}^{l=0} -\vec{E}.d\vec{l}$ ($l = 0$ at the centre of the ring) in volt is:

(a) + 2 (b) – 1

(c) – 2 (d) zero

41. *A, B, C, D, P* and *Q* are points in uniform electric field. The potentials a these points are V (A) = 2 volt. V (P) = V (B) = V (D) = 5 volt. V(C) = 8 volt. The electric field at *P* is:



(a) 10 Vm^{-1} along *PQ*

(b) $15\sqrt{2}$ Vm^{-1} along *PA*

(c) 5 Vm^{-1} along *PC*

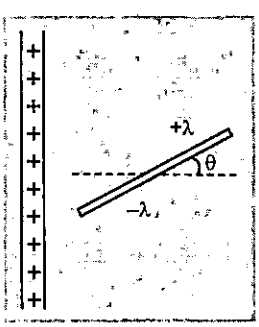
(d) 5 Vm^{-1} along *PA*

42. A solid conducting sphere having a charge *Q* is surrounded by an unchanged concentric conducting hollow spherical shell. The potential difference between the surface of the solid sphere and that of the outer surface of the hollow shell is *V*. If the shell is now given a charge of $-3Q$, the new potential difference between the same two surface is:

(a) *V* (b) 2*V*

(c) 4*V* (d) –2*V*

43. A large sheet carries uniform surface charge density σ. A rod of length 21 has a linear charge density λ on one half and –λ on the second half. The rod is hinged at mid point *O* and makes an angle θ with the normal to the sheet. The torque experienced by the rod is:



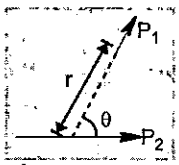
(a) zero

(b) $\frac{\sigma\lambda l^2}{2\varepsilon_0}\sin\theta$

(c) $\frac{\sigma\lambda l^2}{\varepsilon_0}\sin\theta$

(d) $\frac{\sigma\lambda l^2}{2\varepsilon_0}$

44. Two short electric dipoles are placed as shown. The energy of electric interaction between these dipoles will be:



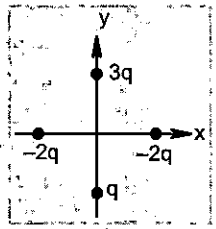
(a) $\frac{2kP_1P_2\cos\theta}{r^3}$

(b) $\frac{-2kP_1P_2\cos\theta}{r^3}$

(c) $\frac{2kP_1P_2\sin\theta}{r^3}$

(d) $\frac{-4kP_1P_2\cos\theta}{r^3}$

45. 4 charges are placed each at a distance 'a' from origin. The distance 'a' from origin. The dipole moment of configuration is:



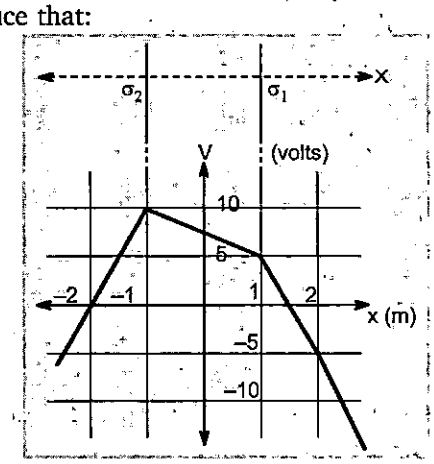
(a) $2qa\hat{j}$

(b) $3qa\hat{j}$

(c) $2aq[\hat{i}+\hat{j}]$

(d) none of these

46. The figure to the right shows the potential due to two similarly charged infinite sheets with charge per unit area σ_1 and σ_2. From examining this plot we can deduce that:



(a) $\sigma_2 > \sigma_1$

(b) $\sigma_2 < \sigma_1$

(c) $\sigma_2 = \sigma_1$

(d) None of these

47. In a region of space, the electric field is in the x direction and is given as $\vec{E} = \vec{E}_0\times\hat{i}$. Consider an imaginary cubical volume of edge *a*, with a its edges parallel to the axes of coordinates. The charge inside this volume is:

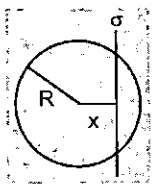
(a) zero (b) $\varepsilon_0E_0a^3$

(c) $\frac{1}{\varepsilon_0}E_0a^3$ (d) $\frac{1}{6}\hat{\varepsilon}_0E_0a^2$

48. Electric flux through a surface of area 100 m^2 lying in the xy plane is (in V-m) if $\vec{E} = \hat{i} + \sqrt{2}\hat{j} + \sqrt{3}\hat{k}$:

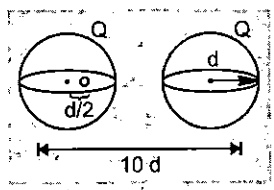
(a) 100 (b) 141.4
(c) 173.2 (d) 200

49. An infinite, uniformly charged sheet with surface charge density σ cuts through a spherical Gaussian surface of radius R at a distance x from its centre, as shown in the figure. The electric flux Φ through the Gaussian surface is:



(a) $\frac{\pi R^2 \sigma}{\varepsilon_0}$ (b) $\frac{2\pi(R^2 - x^2)\sigma}{\varepsilon_0}$
(c) $\frac{\pi(R-x)^2\sigma}{\varepsilon_0}$ (d) $\frac{\pi(R^2 - x^2)\sigma}{\varepsilon_0}$

50. Two spherical, nonconducting, and very thin shells of uniformly distributed positive charge Q and radius d are located a distance 10d from each other. A positive point charge q is placed inside one of the shells at a distance $d/2$ from the center, on the line connecting the centers of the two shells, as shown in the figure. What is the net force on the charge q?



(a) $\frac{qQ}{361\pi\varepsilon_0 d^2}$
(b) $\frac{qQ}{361\pi\varepsilon_0 d^2}$ to the right
(c) $\frac{362qQ}{361\pi\varepsilon_0 d^2}$ to the left
(d) $\frac{362qQ}{361\pi\varepsilon_0 d^2}$ to the right

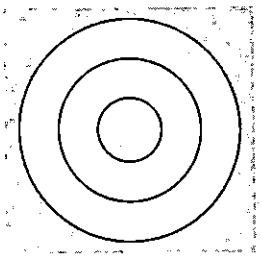
51. A positive charge q is placed in a spherical cavity made in a positively charged sphere. The centres of sphere and cavity are displaced by a small distance $\vec{l}$. Force on charge q is:

(a) in the direction parallel to vector $\vec{l}$
(b) in radial direction
(c) in a direction which depends on the magnitude of charge density in sphere
(d) direction can not be determined

52. There are four concentric shells A, B, C and D of radii a, $2a$ and $4a$ respectively. Shells B and D are given charges $+q$ and $-q$ respectively. Shell C is now earthed. The potential difference $V_A - V_C$ is:

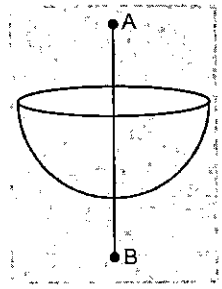
(a) $\frac{Kq}{2a}$ (b) $\frac{Kq}{3a}$
(c) $\frac{Kq}{4a}$ (d) $\frac{Kq}{6a}$

53. A metal ball of radius R is placed concentrically inside a hollow metal sphere of inner radius $2R$ and outer radius $3R$. The ball is given a charge $+2Q$ and the hollow sphere a total charge $-Q$. The electrostatic potential energy of this system is:



(a) $\frac{5Q^2}{24\pi\varepsilon_0 R}$ (b) $\frac{5Q^2}{16\pi\varepsilon_0 R}$
(c) $\frac{5Q^2}{8\pi\varepsilon_0 R}$ (d) None

54. The diagram shows a uniformly charged hemisphere of radius R. It has volume charge density ρ. If the electric field at a point $2R$ distance above its centre is E then what is the electric field at the point which is $2R$ below its centre ?

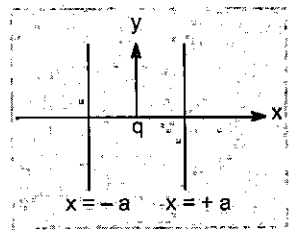


(a) $\rho R/6\varepsilon_0 + E$
(b) $\rho R/12\varepsilon_0 - E$
(c) $-\rho R/6\varepsilon_0 + E$
(d) $\rho R/24\varepsilon_0 + E$

55. A uniform electric field $\vec{E} = a\hat{i} + b\hat{j}$, intersects a surface of area A. What is the flux through this area if the surface lies in the yz plane ?

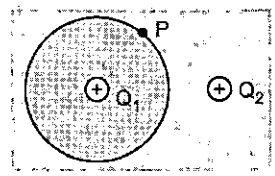
(a) aA (b) 0
(c) bA (d) $A\sqrt{a^2 + b^2}$

56. Two identical infinite positive line charges are placed along the lines $x = \pm a$, in the x-y plane. A positive point charge placed at origin is restricted to move along y-axis. Its equilibrium is:



(a) Stable (b) Neutral
(c) Unstable (d) None of these

57. In the figure below, a point charge $+Q_1$ is at the centre of an imaginary spherical Gaussian surface and another point charge $+Q_2$ is outside of the Gaussian surface. Point P is on the surface of the sphere. Which one of the following statements is true ?



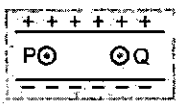
(a) Both charges $+Q_1$ and $+Q_2$ contribute to the net electric flux through the sphere but only charge $+Q_1$ contributes to the electric field at point P on the sphere.
(b) Both charges $+Q_1$ and $+Q_2$ contribute to the net electric flux through the sphere but only charge

$+Q_2$ contributes to the electric field at point P on the sphere.

(c) Only the charges $+Q_1$ contributes to the net electric flux through the sphere but both charge $+Q_1$ and $+Q_2$ contribute to the electric field at point P.

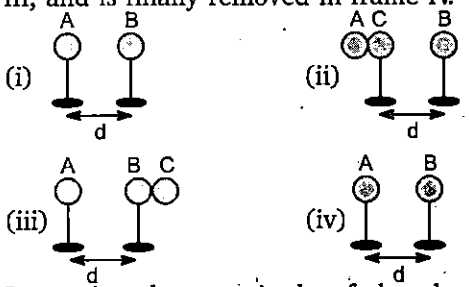
(d) Only the charges $+Q_2$ contributes to the net electric flux through the sphere but both charge $+Q_1$ and $+Q_2$ contribute to the electric field at point P.

58. In a mulikan-type experiment there are two oil droplets P and Q between the charged horizontal plates, as shown in the diagram. Droplet P is in rest while droplet Q is moving upwards. The polarity of the charges on P and Q is



	P	Q
(a)	+	+
(b)	Neutral	−
(c)	−	−
(d)	+	−

59. In frame I, two identical conducting spheres, A and B, carry equal amounts of excess charge that have the same sign. The spheres are separated by a distance d; and sphere A exerts an electrostatic force on sphere B which has a magnitude F. A third sphere, C, which is handled only by an insulated rod, is introduced in frame II. Sphere C is identical to A and B except that it is initially uncharged. Sphere C is touched first to sphere A, in frame II, and then to sphere B, in frame III, and is finally removed in frame IV.



Determine the magnitude of the electrostatic force that sphere A exerts on sphere B in frame IV:

(a) $F/2$ (b) $F/3$

(c) $3F/4$ (d) $3F/8$

60. A charge q is placed at the centroid of an equilateral triangle. Three charges equal Q are placed at the vertices of the triangle. The system of four charges will be in equilibrium if q is equal to:

(a) $-Q\sqrt{3}$ (b) $-Q/3$

(c) $-Q\sqrt{3}$ (d) $Q/\sqrt{3}$

61. Consider a point particle P of unknown charge and mass. When a point particle of mass m_1 and charge $q_1 > 0$ is placed at a distance r from P, the particle has an acceleration a, directed toward P. When a second point particle of mass m_2 and charge $q_2 < 0$ is placed at a distance r from P, it also has an acceleration of a, directed toward P. Let G and K be the universal gravitational constant and the electrostatic constant, respectively. Determine the mass M and charge Q of P:

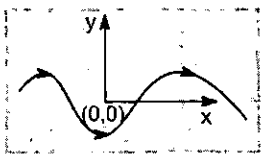
(a) $M = \dfrac{m_1 + m_2}{2}, Q = \dfrac{q_1 - q_2}{2}$

(b) $M = \dfrac{ar^2}{G} Q = 0$

(c) $M = m_1 = m_2, Q = 0$

(d) $M = m_1 = m_2, Q = q_1 = |q_2|$

62. The field line to the right is a field line of the electric field, then its representation can be:



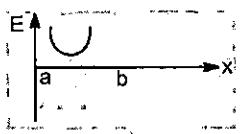
(a) $\vec{E}(x, y) = \hat{i} + \sin(x)\hat{j}$

(b) $\vec{E}(x, y) = \hat{i} - \cos(x)\hat{j}$

(c) $\vec{E}(x, y) = \hat{i} - \sin(x)\hat{j}$

(d) $\vec{E}(x, y) = \hat{i} + \cos(x)\hat{j}$

63. Two point like charges a and b whose magnitudes are same are positioned at a certain distance from each other, a is at origin. Graph is drawn between electric field strength and distance x from a. E is taken positive if it is along the line joining from a to b:



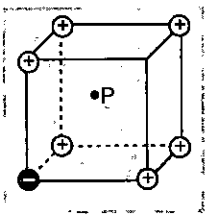
(a) a is positive, b is negative

(b) a and b both are positive

(c) a and b both are negative

(d) a is negative, b is positive

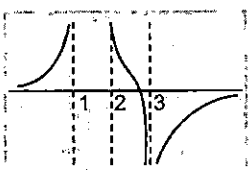
64. Consider a regular cube with positive point charge $+Q$ in all corners except for one which has a negative point charge $-Q$. Let the distance from any corner to the center of the cube be r. What is the magnitude of electric field at point P, the center of the cube ?



(a) $E = 7k_e Q/r^2$ (b) $E = 1k_e Q/r^2$

(c) $E = 2k_e Q/r^2$ (d) $E = 6k_e Q/r^2$

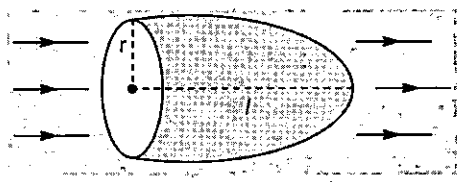
65. Three charges lie on the x-axis each at distance a apart from the nearest one. The charges are numbered from 1 to 3 moving from left to right. A representation of the electric potential V of the three charges at different points is shown above. Which one of the following statements is true ?



(a) The electric field is zero at some point between charges 1 and 2 and also at some point between charges 2 and 3.

(b) The electric field is zero at some point between charges 1 and 2 but it is never zero between charges 2 and 3.
(c) The electric field is never zero between charges 1 and 2 but it is zero at some point between charges 2 and 3
(d) The electric field is never zero between charges 1 and 2 and it is never zero between charges 2 and 3.

66. Fig. shows a circular surface and a paraboloidal surface. It is placed in a uniform electric field of magnitude E such that the circular surface is oriented at right-angles to the direction of field. Electric flux through the paraboloidal surface is:



(a) zero (b) $\pi r^2 lE$
(c) $\frac{1}{2}\pi r^2 E$ (d) $\pi r^2 E$

67. A uniformly charged and infinitely long line having a linear charge density 'λ' is placed at a normal distance y from a point O. Consider a sphere of radius R with O as centre and $R > y$. Electric flux through the surface of the sphere is :

(a) zero (b) $\frac{2\lambda R}{\varepsilon_0}$
(c) $\frac{2\lambda\sqrt{R^2-y^2}}{\varepsilon_0}$ (d) $\frac{\lambda\sqrt{R^2+y^2}}{\varepsilon_0}$

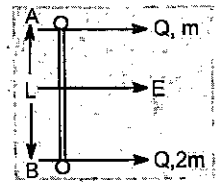
68. An insulating solid sphere of radius 'R' is charged in a non-uniform manner such that volume charge density $\rho = \frac{A}{r}$, where A is a positive constant and r is the distance from centre. Electric field strength at any inside point at distance r_1 is:

(a) $\frac{1}{4\pi\varepsilon_0}\frac{4\pi A}{r_1}$ (b) $\frac{1}{4\pi\varepsilon_0}\frac{A}{r_1}$
(c) $\frac{A}{\pi\varepsilon_0}$ (d) $\frac{A}{2\varepsilon_0}$

69. A thin, metallic spherical shell contains a charge Q on it. A point charge q is placed at the centre of the shell and another charge q_1 is placed outside it as shown in Fig.. All the three charges are positive. The force on the central charge due to the shell is:

(a) towards left (b) towards right
(c) upward (d) zero

70. Two small balls A & B of positive charge Q each and masses m and $2m$ respectively are connected by a non conducting light rod of length L. This system is released in a uniform electric field of strength E as shown. Just after the release (assume no other force acts on the system) :

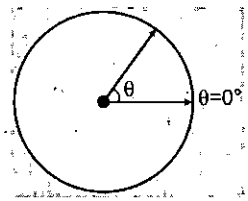


(a) rod has zero angular acceleration
(b) rod has angular acceleration $\frac{QE}{2mL}$ in anticlockwise direction.
(c) acceleration of point A is $\frac{2QE}{3m}$ towards right
(d) acceleration of point A is $\frac{QE}{m}$ towards right

71. Which of the following is sufficient condition for finding the electric flux ϕ_E through a closed surface ?

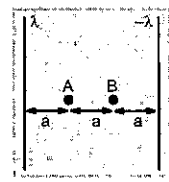
(a) If the magnitude of $\vec{E}$ is known everywhere on the surface
(b) If the total charge inside the surface is specified
(c) If the total charge outside the surface is specified
(d) Only if the location of each point charge inside the surface is specified

72. The linear charge density on a dielectric ring of radius R is varying with θ as $\lambda = \lambda_0 \cos(\theta/2)$. The potential at the centre of the ring is:



(a) Zero
(b) $\frac{\lambda_0}{2\pi\varepsilon_0}$
(c) $\frac{\lambda_0}{4\pi\varepsilon_0}$
(d) $\frac{\lambda_0}{\pi\varepsilon_0}$

73. Between two infinitely long wires having linear charge densities λ and $-\lambda$ there are two points A and B as shown in the figure. The amount of work done by the electric field in moving a point charge q_0 from A to B is equal to:



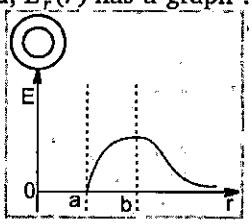
(a) $\frac{\lambda q_0}{2\pi\varepsilon_0}\ln 2$ (b) $-\frac{2\lambda q_0}{\pi\varepsilon_0}\ln 2$
(c) $\frac{2\lambda q_0}{\pi\varepsilon_0}\ln 2$ (d) $\frac{\lambda q_0}{\pi\varepsilon_0}\ln 2$

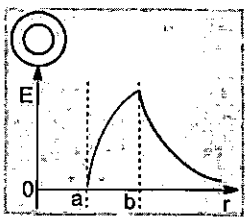
74. You are moving a negative charge $q < 0$ at a small constant speed away from a uniformly charged non-conducting spherical shell on which resides a negative charge $Q < 0$. The electrostatic field of Q is E. Let U be the total energy of the system, W_a the work done by the force F_a you exert on q and W_E the work done by the electrostatic force F_E on q. Then, as q is being moved:

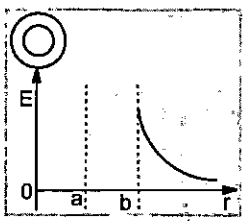
(a) $W_a = -W_E$, therefore U remains constant
(b) $F_a = -F_E$

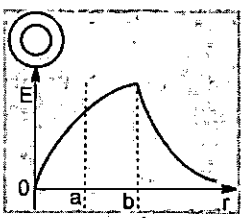
(c) U increases
(d) U decreases

75. An insulating spherical shell of inner radius a and outer radius b is uniformly charged with a positive charge density. The radial component of the electric field, $E_r(r)$ has a graph :

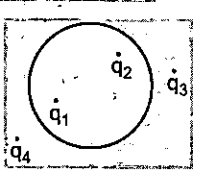
(a) 

(b) 

(c) 

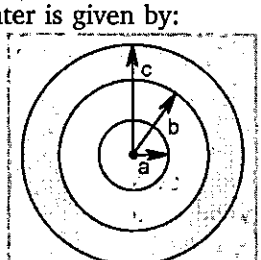
(d) 

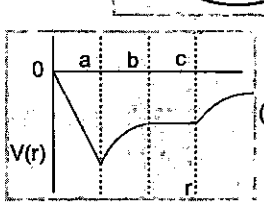
76. Consider a non-conducting shell as shown in Fig. Two point charges are inside the shell and two are outside the shell. If we apply Gauss's law over the non-conducting shell as Gaussian surface, the $\vec{E}$ on LHS of Gauss equation is due to:

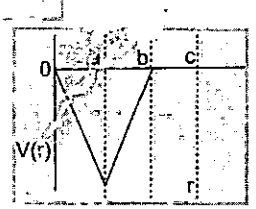


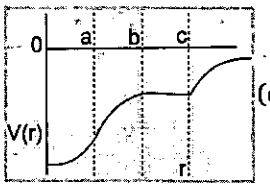
(a) q_1 and q_2 alone
(b) all charges q_1, q_2, q_3 and q_4
(c) q_1, q_2 and q_3 alone
(d) We cannot take the non-conducting shell as a Gaussian surface

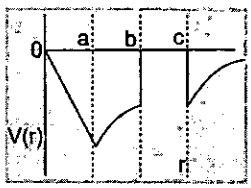
77. A non-conducting sphere with radius a is concentric with and surrounded by a conducting spherical shell with inner radius b and outer radius c. The inner sphere has a negative charge uniformly distributed throughout its volume, while the spherical shell has no net charge. The potential $V(r)$ as a function of distance from the center is given by:



(a) 

(b) 

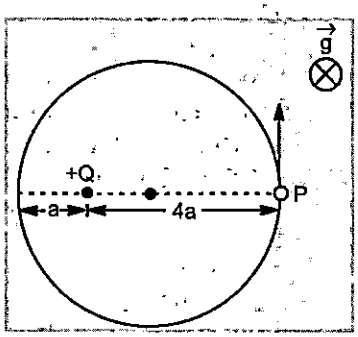
(c) 

(d) 

78. A spherical conducting shell of inner radius R_1 and outer radius R_2 has charge Q. Now a charge q is placed inside the shell but not at centre, then surface charge densities with their nature on inner and outer surfaces of the shell are respectively,

(a) $\frac{-q}{4\pi R_1^2}$ (non-uniform), $\frac{Q}{4\pi R_2^2}$ (uniform)
(b) $\frac{-q}{4\pi R_1^2}$ (non-uniform), $\frac{Q+q}{4\pi R_2^2}$ (uniform)
(c) $\frac{q}{4\pi R_1^2}$ (uniform), $\frac{Q+q}{4\pi R_2^2}$ (non-uniform)
(d) $\frac{-q}{4\pi R_1^2}$ (uniform), $\frac{Q+q}{4\pi R_2^2}$ (non-uniform)

79. Fig. shows a small bead of mass m carrying charge q. The bead can freely move on the smooth fixed ring placed on a smooth horizontal plane. In the same plane a charge + Q has also been fixed as shown. The potential at the point P due to + Q is V. The velocity with which the bead should projected from the point P so that it can complete a circle should be greater than :

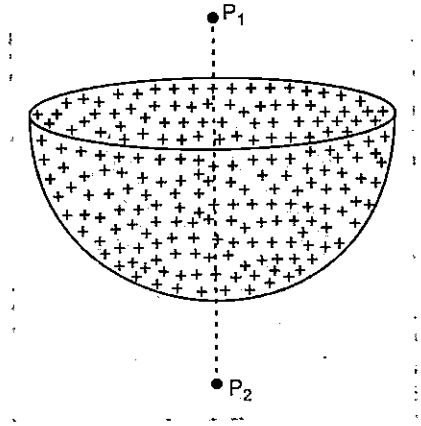


(a) $\sqrt{\frac{6qV}{m}}$ (b) $\sqrt{\frac{qV}{m}}$
(c) $\sqrt{\frac{3qV}{m}}$ (d) None

80. An infinite plane sheet of aluminium of area A has total charge Q uniformly distributed over its surface. The same charge is spread uniformly on upper surface of glass slab having same face area. The electric field intensities just above the centre of the plates, upper faces are $\vec{E}_1$ and $\vec{E}_2$ for aluminium and glass slabs respectively, then:

(a) $|\vec{E}_1| = |\vec{E}_2|$
(b) $|\vec{E}_1| = |\vec{E}_2| = 0$
(c) $|\vec{E}_1| \neq |\vec{E}_2|$
(d) Cannot say anything

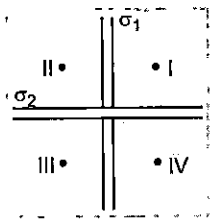
81. The diagram shows a uniformly charged sphere of radius R. It has volume charge density ρ. If the electric field at a point $2R$ distance above its centre is E then the electric field at the point which is $2R$ below its centre is :



(a) $\frac{\rho R}{6\varepsilon_0}+E$ (b) $\frac{\rho R}{12\varepsilon_0}-E$

(c) $-\frac{\rho R}{6\varepsilon_0}+E$ (d) $\frac{\rho R}{24\varepsilon_0}+E$.

82. Two infinite sheets having charge densities σ_1 and σ_2 are placed in two perpendicular planes whose two dimensional view is shown in the figure. The charges are distributed uniformly on sheets in electrostatic equilibrium condition. Four points are marked I, II, III and IV, the electric field intensities at these points are $\vec{E}_1, \vec{E}_2, \vec{E}_3$ and $\vec{E}_4$, respectively. The correct expression for electric field intensities is:



(a) $|\vec{E}_1|=|\vec{E}_2|=\frac{\sqrt{\sigma_1^2+\sigma_2^2}}{2\varepsilon_0}\neq|\vec{E}_4|$

(b) $|\vec{E}_2|=|\vec{E}_4|=\frac{\sqrt{\sigma_1^2-\sigma_2^2}}{2\varepsilon_0}$

(c) $|\vec{E}_1|=|\vec{E}_2|=|\vec{E}_3|=|\vec{E}_4|=\frac{\sqrt{\sigma_1^2+\sigma_2^2}}{2\varepsilon_0}$

(d) None of the above

83. Two point charges q and $-q$ are placed at x and y, respectively. If V_1 is the electric potential at some point P due to q alone and V_2 be due to both charges, then [*Position of P, X and Y are not same*]

(a) $V_1 > V_2$ for all locations of P

(b) $V_1 = V_2$ for some points.

(c) $V_1 > V_2$ for some points

(d) $V_1 < V_2$ for some points

84. Select the correct statement:

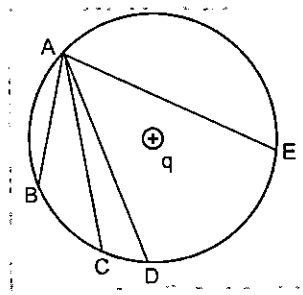
(a) Attraction is a true test of electrification.

(b) Mass can exist without charge and charge can also exist without mass.

(c) Charge is discrete in general and can be continuous two.

(d) Mass and charge, both are invariant physical quantities.

85. In the electric field of a point charge q, a certain charge is carried from point A to B, C, D, and E, the work done:



(a) is least along the path AB

(b) is least along the path AD

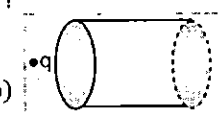
(c) is zero along any one of the path AB, AC and AE

(d) is least along AE.

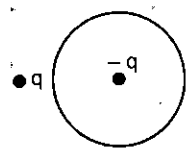
86. In which of the following cases, the flux crossing through the surface is zero ?

(a)

Hemispherical surface, without base

(b) 

Cylindrical surface with one end open

(c) 

Spherical surface

(d)

Hemispherical surface with base

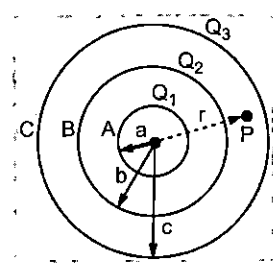
87. Three concentric spherical conductors are arranged as shown in the figure. The potential at point P will be:

(a) $\frac{1}{4\pi\varepsilon_0}\left[\frac{Q_1}{r}+\frac{Q_2}{r}+\frac{Q_3}{r}\right]$

(b) $\frac{1}{4\pi\varepsilon_0}\left[\frac{Q_1+Q_2}{r}+\frac{Q_3}{c}\right]$

(c) $\frac{1}{4\pi\varepsilon_0}\left[\frac{Q_1}{a}+\frac{Q_2}{b}+\frac{Q_3}{c}\right]$

(d) $\frac{1}{4\pi\varepsilon_0 c}\times[Q_1+Q_2+Q_3]$



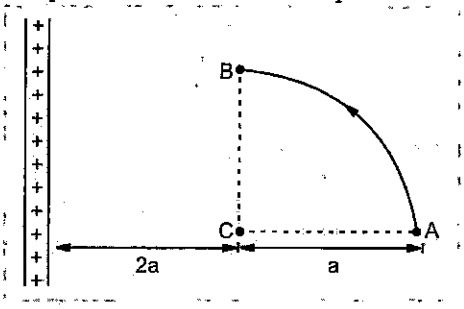
88. For the situation shown in the figure below, find the force experience by the dipole.

(a) $\frac{|\vec{P}|Q}{2\pi\varepsilon_0 r^3}$ (b) $\frac{PQ}{2\pi\varepsilon_0 r^2}$

(c) $\frac{3QP}{2\pi\varepsilon_0 r^3}$ (d) None of these

89. The arc AB with the center C and the infinitely long wire having linear charge density λ are lying in same

plane. The minimum amount of work to be done to move a point charge q_0 from point A to B through a circular path AB of radius a is equal to:



(a) $\frac{q_0^2}{2\pi\varepsilon_0}\ln\left(\frac{2}{3}\right)$ (b) $\frac{q_0\lambda}{2\pi\varepsilon_0}\ln\left(\frac{3}{2}\right)$

(c) $\frac{q_0\lambda}{2\pi\varepsilon_0}\ln\left(\frac{2}{3}\right)$ (d) $\frac{q_0\lambda}{\sqrt{2}\pi\varepsilon_0}$

90. A charge is distributed with a linear density λ over the length L along a radius vector drawn from the point where a point charge q is located. The distance between q and the nearest point on linear charge is R. The electrical force experienced by the linear charge due to q is:

(a) $\frac{q\lambda L}{4\pi\varepsilon_0 R^2}$ (b) $\frac{q\lambda L}{4\pi\varepsilon_0 R(R+L)}$

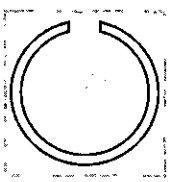
(c) $\frac{q\lambda L}{4\pi\varepsilon_0 RL}$ (d) $\frac{q\lambda L}{4\pi\varepsilon_0 L^2}$

91. A very long uniformly charged circular cylinder (radius R) has a surface charge density σ. A very long uniformly charged line charge (linear charge density λ) is placed along the cylinder axis. If electric field intensity vector outside the cylinder is zero, then:

(a) $\lambda = R\sigma$ (b) $\lambda = -R\sigma$

(c) $\lambda = 2\pi R\sigma$ (d) $\lambda = -2\pi R\sigma$

92. A small hole is cut into a charged hollow conductor of arbitrary shape as shown. If the local surface charge density near the hole is σ, then $\vec{E}$ inside the hole is:



(a) $\frac{\sigma}{\varepsilon_0}$ along outward normal

(b) $\frac{\sigma}{\varepsilon_0}$ along inward normal

(c) $\frac{\sigma}{2\varepsilon_0}$ along outward normal

(d) $\frac{\sigma}{2\varepsilon_0}$ along inward normal

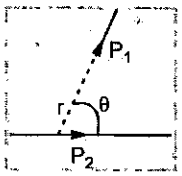
93. Two short electric dipoles are placed as shown in Fig. The energy of electric interaction between these dipoles will be:

(a) $\frac{2kP_1P_2\cos\theta}{r^3}$

(b) $-\frac{2kP_1P_2\cos\theta}{r^3}$

(c) $-\frac{2kP_1P_2\sin\theta}{r^3}$

(d) $-\frac{4kP_1P_2\cos\theta}{r^3}$



94. For a gaussian surface, through which the net flux is zero, which of the following statements must be true ?

(a) The net charge inside the surface is zero

(b) The number of electric field lines entering the surface is equal to the number of field lines leaving the surface

(c) The electric field is zero everywhere on the surface.

(d) Both (a) and (b) are correct

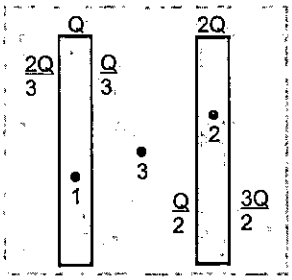
95. Two non-conducting infinite plane sheets having charges Q and $2Q$ are placed parallel to each other as shown in figure. The charge distribution on four faces of two plates are as also shown. The electric field intensities at three points 1, 2 and 3 are $\vec{E}_1$, $\vec{E}_2$ and $\vec{E}_3$ respectively, then the magnitudes of $\vec{E}_1$, and $\vec{E}_2$ and $\vec{E}_3$ are respectively, [S is face area of plates]:

(a) zero, $\frac{Q}{\varepsilon_0 S}$, zero

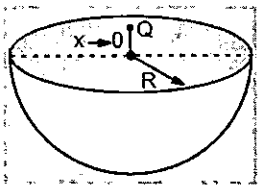
(b) $\frac{5Q}{6\varepsilon_0 S}, \frac{Q}{2\varepsilon_0 S}$, zero

(c) $\frac{5Q}{6\varepsilon_0 S}, \frac{Q}{\varepsilon_0 S}, \frac{Q}{3\varepsilon_0 S}$

(d) zero, $\frac{Q}{2\varepsilon_0 S}$, zero



96. A point charge Q is located just above the centre of the flat face of hemisphere as shown in figure. The electric flux through the flat face and curved face of hemisphere are respectively,



(a) $\frac{Q}{2\varepsilon_0}, -\frac{Q}{2\varepsilon_0}$ (b) $-\frac{Q}{2\varepsilon_0}, \frac{Q}{2\varepsilon_0}$

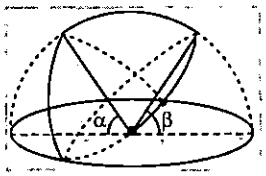
(c) $\frac{Q}{\varepsilon_0}, -\frac{Q}{\varepsilon_0}$ (d) $-\frac{Q}{\varepsilon_0}, \frac{Q}{\varepsilon_0}$

97. The electric field intensity at the centre of a uniformly charged hemispherical shell is E_0. Now two portions of the hemisphere are cut from either side and remaining portion is shown in figure. If $\alpha = \beta = \frac{\pi}{3}$, then field intensity due to remaining portion at centre is:

(a) $\frac{E_0}{3}$

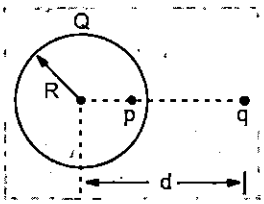
(b) $\frac{E_0}{6}$

(c) $\frac{E_0}{2}$



(d) Information insufficient

98. A point charge q is placed at a distance d from centre of a conducting charged spherical shell of radius R and having charge Q as shown. The electric field intensity at point P just inside the shell due to charge on the shell is :



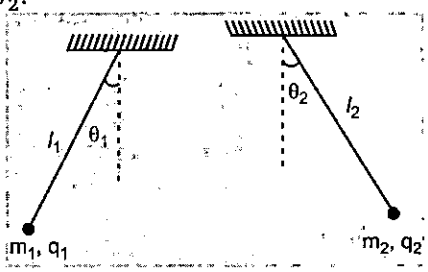
(a) zero

(b) $\frac{Q}{4\pi\varepsilon_0 R^2}$

(c) $\frac{q}{4\pi\varepsilon_0 d^2}$

(d) $\frac{q}{4\pi\varepsilon_0 (d-R)^2}$

99. Two small spheres with mass m_1 and m_2 hang from massless, insulating threads of length l_1 and l_2. The two spheres carry charges q_1 and q_2 respectively. The spheres hang such that they are on same horizontal level and the threads are inclined to the vertical at angles θ_1 and θ_2. Which of the condition is required if $\theta_1 = \theta_2$:



(a) $m_1 = m_2$

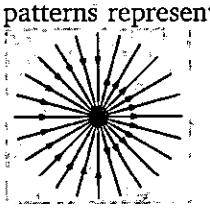
(b) $|q_1| = |q_2|$

(c) $l_1 = l_2$

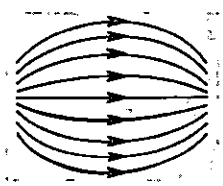
(d) $\frac{q_1}{m_1} = \frac{q_2}{m_2}$

100. Consider the four field patterns shown. Assuming there are no charges in the regions shown, which of the patterns represents a possible electrostatic field:

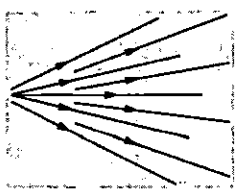
(a)



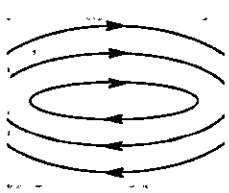
(b)



(c)



(d)



101. A positively charged insulator is brought near (but does not touch) two metallic sphere that are in contact. The metallic spheres are then separated. The sphere which was initially farthest from the insulator will have:

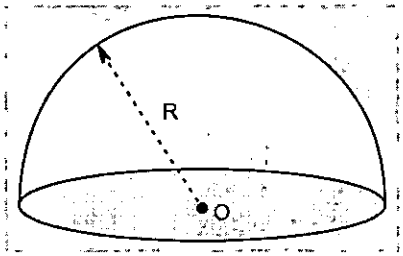
(a) no net charge

(b) a negative charge

(c) a positive charge

(d) either a negative or a positive charge.

102. Charge Q coulombs is uniformly distributed throughout the volume of a solid hemisphere of radius R metres. Then the potential at centre O of the hemisphere in volts is:



(a) $\frac{1}{4\pi\varepsilon_0}\frac{3Q}{2R}$

(b) $\frac{1}{4\pi\varepsilon_0}\frac{3Q}{4R}$

(c) $\frac{1}{4\pi\varepsilon_0}\frac{Q}{4R}$

(d) $\frac{1}{4\pi\varepsilon_0}\frac{Q}{8R}$

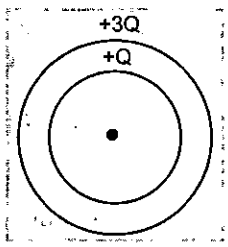
103. Two concentric conducting thin shells of radius R and $2R$ carry charges $+Q$, $+3Q$ respectively. The magnitude of electric field at a distance x outside and inside from the surface of outer sphere is same. Then the value of x is:

(a) $\frac{R}{3}$

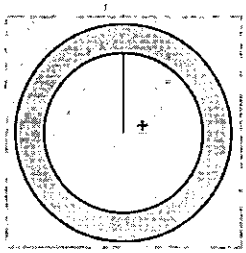
(b) $\frac{2R}{3}$

(c) $\frac{R}{4}$

(d) $\frac{R}{2}$



104. An electrically isolated hollow (initially uncharged), conducting sphere has a small positively charged ball suspended by an insulating rod from its inside surface, see diagram. This causes the inner surface of the sphere to become negatively charged. When the ball is centered in the sphere the electric field outside the conducting sphere is approximately.



(a) zero

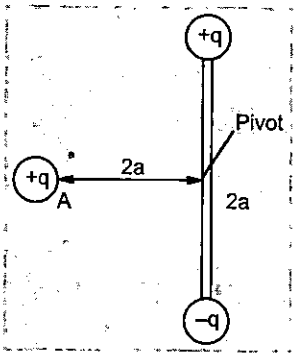
(b) the same as if the sphere wasn't there

(c) twice what it would be if the sphere wasn't there

(d) equal in magnitude but opposite in direction to what is would be if the sphere wasn't there

105. In a certain region of space, the potential is given by $V = k[2x^2 - y^2 + z^2]$. The electric field at the point (1, 1, 1,) has magnitude =
(a) $k\sqrt{6}$ (b) $2k\sqrt{6}$
(c) $2k\sqrt{3}$ (d) $4k\sqrt{3}$

106. Fig shows a ball having a charge q fixed at a point A. Two identical balls of mass m having charge $+q$ and $-q$ are attached to the end of a light rod of length $2a$. The system is released from the situation shown in fig. Find the angular velocity of the rod when the rod turns through 90°:



(a) $\dfrac{\sqrt{2}q}{3\pi\varepsilon_0 ma^3}$

(b) $\dfrac{q}{\sqrt{3\pi\varepsilon_0 ma^3}}$

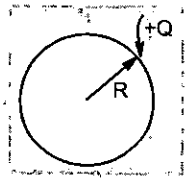
(c) $\dfrac{q}{\sqrt{6\pi\varepsilon_0 ma^3}}$ (d) $\dfrac{\sqrt{2}q}{4\pi\varepsilon_0 ma^3}$

107. The net force on an electric dipole oriented parallel to the x-axis in this field is:
(a) directed along the x-axis.
(b) directed along the y-axis.
(c) directed along the z-axis,.
(d) None of the above

108. The net torque on an electric dipole parallel to the x-axis in this field is:
(a) directed along the x-axis.
(b) directed along the y-axis.
(c) directed along the z-axis.
(d) None of the above

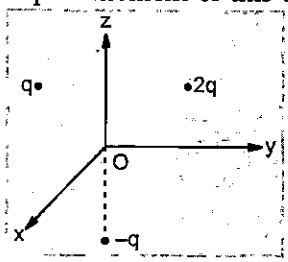
109. In a certain region of space, the electric field is zero. From this we can conclude that the electric potential in this region is:
(a) constant (b) zero
(c) positive (d) negative

110. Consider a uniformly charged spherical shell of radius R and total charge + Q as shown in the Fig. The electric field inside the sphere is:



(a) Constant and equal to $\dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{R_2}$
(b) Constant and equal to zero
(c) Not a constant and equal to $\dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r^2}$
(d) Not possible to determine from the information given

111. Three point charges $2q$, q and $-q$ are located respectively at $(0, a, a)$, $(0, -a, a)$ and $(0, 0, -a)$ as shown. The dipole moment of this distribution is:



(a) $2qa$ in the yoz plane at $\tan^{-1}\left(\dfrac{1}{4}\right)$ with z-axis
(b) $\sqrt{17}qa$ in the xoy plane at $\tan^{-1}\left(\dfrac{1}{4}\right)$ with z-axis
(c) $\sqrt{5}qa$ in the xoy plane at $\tan^{-1}(4)$ with y-axis
(d) $4qa$ in the xoy plane at $\tan^{-1}(4)$ with y-axis

112. A proton with mass m is propelled at an initial speed V_0 directly towards a Uranium nucleus from a distance x_0 away. The proton is repelled by the Uranium nucleus with a force of magnitude $\mathcal{E} = \alpha / x^2$, where x is the separation between two objects and α is a positive constant. As the proton approaches the Uranium nucleus, it comes momentarily to rest at a distance x_{min} after which the proton moves away from the Uranium nucleus. The value of x_{min} is:

(a) $\dfrac{\alpha}{\dfrac{\alpha}{x_0} - \dfrac{1}{2}mv_0^2}$ (b) $\dfrac{\alpha}{\dfrac{\alpha}{2x_0} - \dfrac{1}{2}mv_0^2}$

(c) $\dfrac{\alpha}{\dfrac{\alpha}{x_0} + \dfrac{1}{2}mv_0^2}$ (d) $\dfrac{\alpha}{\dfrac{1}{2}mv_0^2 - \dfrac{\alpha}{x_0}}$

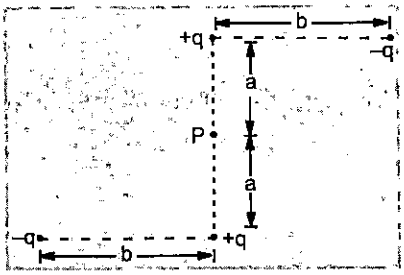
113. Four charges $+q$, $-q$, $+q$ and $-q$ are placed in order on the four consecutive corners of a square of side q. The work done in interchanging the positions of any two neighboring charges of opposite sign is:

(a) $\dfrac{q^2}{4\pi\varepsilon_0 a}(-4+\sqrt{2})$ (b) $\dfrac{q^2}{4\pi\varepsilon_0 a}(4+2\sqrt{2})$

(c) $\dfrac{q^2}{4\pi\varepsilon_0 a}(4-2\sqrt{2})$ (d) $\dfrac{q^2}{4\pi\varepsilon_0 a}(4+\sqrt{2})$

114. Three point charges q, $2q$ and $8q$ are to be placed on a straight line 9 cm long. The system possesses minimum potential energy when.
(a) $2q$ and q lie at ends with $8q$ at 3 cm from $2q$
(b) $2q$ and $8q$ lie at ends with q at 6 cm from $8q$
(c) q and $8q$ lie at ends with $2q$ at 6 cm from $8q$
(d) $2q$ and $8q$ lie at ends with q a. 3 cm from $8q$

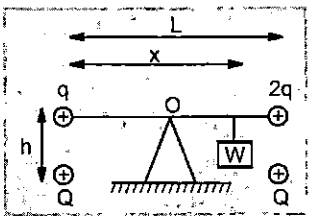
115. The electrostatic potential due to the charge configuration at point P as shown in fig for $b << a$.



(a) $\frac{2q}{4\pi\varepsilon_0 a}$ (b) $\frac{2qb^2}{4\pi\varepsilon_0 a^3}$

(c) $\frac{qb^2}{4\pi\varepsilon_0 a^3}$ (d) zero

116. An insulating long light rod of length L pivoted at is centre O and balanced with a weight W at a distance x from the left end as shown in Fig. Charges q and $2q$ are fixed to the ends of the rod. Exactly below each of these charges at a distance h a positive charge Q is fixed. Then x is:



(a) $\frac{QLq+\varepsilon_0^2 h^2 LW}{h^2 W}$ (b) $\frac{QLq+\varepsilon_0 h^2 LW}{\varepsilon_0 h^2 W}$

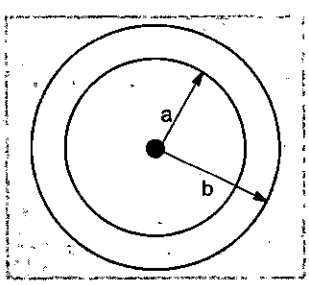
(c) $\frac{4QLq+\varepsilon_0 h^2 LW}{8\pi h^2 W}$ (d) $\frac{QLq+4\pi\varepsilon_0 h^2 LW}{8\pi\varepsilon_0 h^2 W}$

117. A thin dielectric rod of length l lies along the x-axis with one end at the origin and the other end at the point $(l, 0)$. It is charged uniformly along its length with a total charge Q. The potential at a point $(x,0)$ when $x > l$ is:

(a) $\frac{Q}{4\pi\varepsilon_0 l}$ (b) $\frac{Q}{4\pi\varepsilon_0 l}\log_e\left(\frac{x}{l}\right)$

(c) $\frac{Q}{4\pi\varepsilon_0 l}\log_e\left(\frac{x}{x-l}\right)$ (d) $\frac{Q}{4\pi\varepsilon_0 l}\log_e\left(\frac{x-l}{x}\right)$

118. In a spherical distribution the charge density varies as $\rho(r)=\frac{A}{r}$ for $a < r < b$(as shown) where A is a constant. A point charge Q lies at the centre of the sphere at $r = 0$. The electric field in the region $a < r < b$ has a constant magnitude for:



(a) $A = 0$ (b) $A = Q$

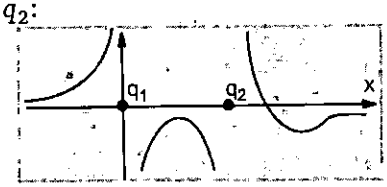
(c) $A=\frac{Q}{2\pi a^2}$ (d) $A=\frac{Q}{4\pi a^2}$

119. A square of side b centred at the origin with sides parallel to axes of x and y has surface charge density $\sigma(x,y)=\sigma_0 xy$ within its boundaries. Total charge on the square is :

(a) Zero (b) $\sigma_0 b^2$

(c) $2\sigma_0 b^2$ (d) $4\sigma_0 b^2$

120. Two charges q_1 & q_2 are kept on x-axis and electric field at different points an x-axis is plotted against x. Choose correct statement about nature and magnitude of q_1 & q_2:



(a) q_1 + ve, q_2 − ve; $|q_1| > |q_2|$

(b) q_1 + ve, q_2 − ve; $|q_1| < |q_2|$

(c) q_1 − ve, q_2 + ve; $|q_1| > |q_2|$

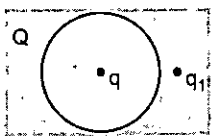
(d) q_1 − ve, q_2 + ve; $|q_1| < |q_2|$

121. A calculator runs on a solar cell with an area of $1.0\times10^{-3}\,m^2$. When it is illuminated by light of an intensity of $1.0\times10^2\,W/m^2$ it produces an emf of $0.60\,V$ and provides a current of 0.0030 A to power the calculator. What is the efficiency of the solar cell?

(a) 0.18% (b) 1.8%

(c) 0.018% (d) 18 %

122. A thin, metallic spherical shell contains a charge Q on it. A point charge q is placed at the centre of the shell and another charge q_1 is placed outside it as shown in Fig. All the three charges are positive. The force on the central charge due to the shell is:

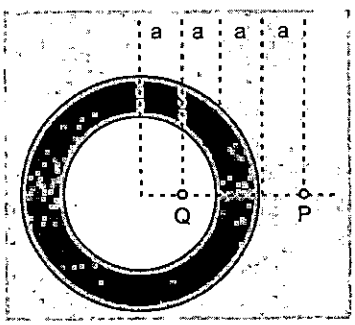


(a) towards left

(b) towards right

(c) upward (d) zero

123. A solid conducting sphere having net charge Q and radius 3 a contains a hollowed spherical region of radius 2 a. A point charge $+Q$ is placed at a position a distance a from the common center of the spheres. What is the magnitude of the electric field at the position $r = 4a$ from the center of the spheres as marked in the Fig. by P?



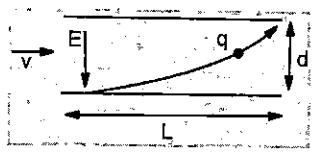
(a) $\frac{kQ}{16a^2}$

(b) $\frac{3kQ}{16a^2}$

(c) $\frac{kQ}{8a^2}$

(d) can't be determined due to non-uniform distribution

124. In an ink-jet printer, an ink droplet of mass m is given a negative charge q by a computer-controlled charging unit, and then enters at speed v the region between two deflecting parallel plates of length L separated by distance d (see Fig). All over this region exists a downward electric field which you can assume to be uniform. Neglecting the gravitational force on the droplet, the maximum charge that it can be given so that it will not hit a plate is most closely approximated by:



(a) $\frac{mv^2E}{dL^2}$ (b) $\frac{mv^2d}{EL^2}$

(c) $\frac{2dmv^2}{EL^2}$ (d) none

125. An uncharged aluminium block has a cavity within it. The block is placed in a region permeated by a uniform electric field which is directed upwards. Which of the following is a correct statement describing conditions in the interior of the block's cavity?

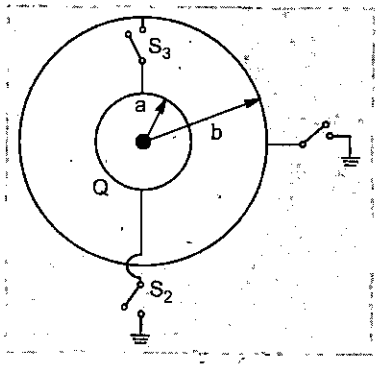
(a) The electric field in the cavity is directed upwards

(b) The electric field in the cavity is directed downwards

(c) There is no electric field in the cavity

(d) The electric field in the cavity is of varying magnitude and is zero at the exact center.

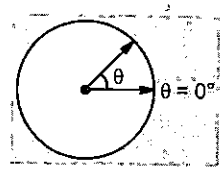
126. The Fig. shows a conducting sphere 'A' of radius 'a' which is surrounded by a neutral conducting spherical shell B of radius 'b' ($>a$). Initially switches S_1, S_2 and S_3 are open and sphere 'A' carries a charge Q. First the switch 'S_1' is closed to connect the shell B with the ground and then opened. Now the switch 'S_2' is closed so that the sphere 'A' is grounded and then S_2 is opened. Finally, the switch 'S_3' is closed to connect the spheres together. The heat (in joule) which is produced after closing the switch S_3 is [Consider $b=4$ cm, $a=2$ cm and $Q=8\mu C$]



(a) 1.8 (b) 3.4

(c) 6.8 (d) 2.8

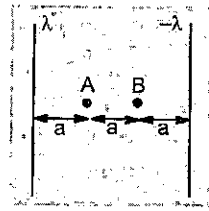
127. The linear charge density on a dielectric ring of radius R is varying with θ as $\lambda=\lambda_0\cos(\theta/2)$. The potential at the centre of the ring is:



(a) 0 (b) $\frac{\lambda_0}{2\pi\varepsilon_0}$

(c) $\frac{\lambda_0}{4\pi\varepsilon_0}$ (d) $\frac{\lambda_0}{\pi\varepsilon_0}$

128. Between two infinitely long wires having linear charge densities λ and $-\lambda$ there are two points A and B as shown in the Fig. The amount of work done by the electric field in moving a point charge q_0 from A to B is equal to:



(a) $\frac{\lambda q_0}{2\pi\varepsilon_0}\ln 2$ (b) $-\frac{2\lambda q_0}{\pi\varepsilon_0}\ln 2$

(c) $\frac{2\lambda q_0}{\pi\varepsilon_0}\ln 2$ (d) $\frac{\lambda q_0}{\pi\varepsilon_0}\ln 2$

129. Mullikan's oil drop experiment attempts to measure the charge on a single electron, e, by measuring the charge of tiny oil drops suspended in an electrostatic field. It is assumed that the charge on the oil drop is due to just a small number of excess electrons. The charges 3.90×10^{-19}C, 6.50×10^{-19}C and 9.10×10^{-19}C are measured on three drops of oil. The charge of an electron is deduced to be:

(a) 1.3×10^{-19}C (b) 1.6×10^{-19}C

(c) 2.6×10^{-19}C (d) 3.9×10^{-19}C

130. Two point-charges, each with a charge of $+1\mu$C, lie some finite distance apart. On which of the segments of an infinite line going through the charges is there a point, a finite distance away from the charges, where the electric potential is zero, assuming that it vanishes at infinity?

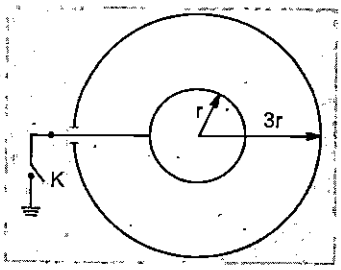
(a) Between the charges only

(b) On either side outside the system

(c) Impossible to tell without knowing the distance between the charges

(d) Nowhere

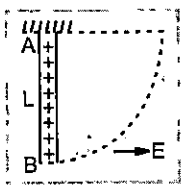
131. Figure shows two conducting thin concentric shells of radii r and $3r$. The outer shell carries charge q and inner shell is neutral. The amount of charge which flows from inner shell to the earth after the key K is closed, is equal to:



(a) $-q/3$ (b) $q/3$
(c) $3q$ (d) $-3q$

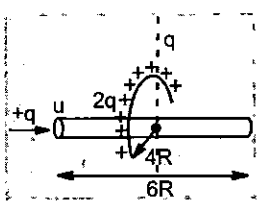
132. A point charge $q = 50\mu C$ is located in the x-y plane at the point of position vector $\vec{r}_0 = 2\hat{i} + 3\hat{j}$. What is the electric field at the point of position vector $\vec{r} = 8\hat{i} - 5\hat{j}$?
(a) 1200 V / m (b) 4×10^{-2} V / m
(c) 900 V / m (d) 4500 V / m

133. A rod AB of length L and mass m is uniformly charged with a charge Q and it is freely suspended from end A as shown in Fig. An electric field E is suddenly switched on in the horizontal direction due to rod which get turned by a maximum angle 90°. The magnitude of E is:



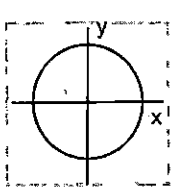
(a) $\frac{2Mg}{Q}$ (b) $\frac{4Mg}{Q}$
(c) $\frac{3Mg}{Q}$ (d) $\frac{Mg}{Q}$

134. On a semicircular ring of radius $=4R$, charge $+3q$ is distributed in such a way that on one quarter $+q$ is uniformly distributed and on another quarter $+2q$ is uniformly distributed. Along its axis a smooth non-conducting and uncharged pipe of length $6R$ is fixed axially as shown. A small ball of mass m and charge $+q$ is thrown from the other end of pipe. The ball can come out of the pipe if:



(a) $u > \sqrt{\frac{7q^2}{40\pi\varepsilon_0 Rm}}$ (b) $u > \sqrt{\frac{3q^2}{40\pi\varepsilon_0 Rm}}$
(c) $u \geq \sqrt{\frac{3q^2}{40\pi\varepsilon_0 Rm}}$ (d) $u > \sqrt{\frac{9q^2}{40\pi\varepsilon_0 Rm}}$

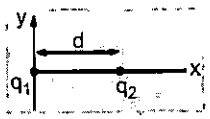
135. $1\mu C$ charge is uniformly distributed on a spherical shell given by equation $x^2 + y^2 + z^2 = 25$. What will be intensity of electric field at a point (1, 1, 2)?



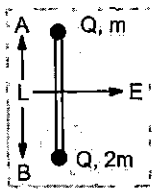
(a) 5 N/C (b) 45 N/C
(c) $\frac{5\sqrt{3}}{2}$ N/C (d) Zero

136. Two particles of charge q_1 and q_2 are separated by distance d as shown in Fig. Charge q_1 is situated at the origin. The net electric field due to the particles is zero at $x = d/4$. With $V = 0$ at infinity, the location of a point in terms of d on the x-axis (other than at infinity) at which the electrical potential due to the two particles is zero, is given by:

(a) $d/4$
(b) $16d$
(c) $-16d$
(d) no point on the axis



137. Two small balls A & B of positive charge Q each and masses m and $2m$ respectively are connected by a non-conducting light rod of length L. This system is released in a uniform electric field of strength E as shown. Just after the release (assume no other force acts on the system):

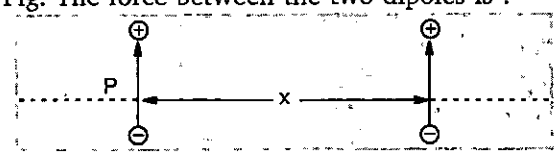


(a) rod has zero angular acceleration
(b) rod has angular acceleration $\frac{QE}{2mL}$ in anticlockwise direction
(c) acceleration of point A is $\frac{2QE}{3m}$ towards right
(d) acceleration of point A is $\frac{QE}{m}$ towards right

138. A metallic rod of length l rotates at angular velocity ω about an axis passing through one end and perpendicular to the rod. If mass of electron is m and its charge is $-e$ then the magnitude of potential difference between its two ends is :
(a) $m\omega^2 l^2 / (2e)$ (b) $m\omega^2 l^2 / e$
(c) $m\omega^2 l / e$ (d) none of these

139. An electric dipole is placed at the origin O such that its equator is y-axis. At a point P far away from dipole, the electric field direction is along y-direction. OP makes an angle α with the x-axis such that :
(a) $\tan\alpha = \sqrt{3}$ (b) $\tan\alpha = \sqrt{2}$
(c) $\tan\alpha = 1$ (d) $\tan\alpha = \frac{1}{\sqrt{2}}$

140. Two small dipoles of moment p are placed as shown in Fig. The force between the two dipoles is :



(a) $\frac{3kp^2}{2x^4}$ (b) $\frac{3kp^2}{x^4}$
(c) $\frac{3kp}{4x^4}$ (d) $\frac{3kp^2}{5x^4}$

141. Select the correct statement(s):
(a) The electrical interaction between two identical point charges A and B is $\vec{F}$, when another identical point charge C is placed near to B, then electrical interaction between A and B is $\vec{F}$.

(b) The electrical interaction between two charged bodies A and B is $\vec{F}$, if another charged body is brought nearby to them then electrical interaction between A and B is $\vec{F}$.

(c) Principle of superposition is not valid for charged extended objects.

(d) Both (a) and (b) are correct

142. Two point charges $2Q$ and $-18Q$ are placed at a separation r. Find the location and charge of a test charge, so that the entire system is in equilibrium.

(a) Outside the line joining the charges and on side of $2Q$ at $x = \frac{r}{2}, \frac{-9Q}{2}$

(b) Outside the line joining the charges and on side of $2Q$ at $x = r, -9Q$

(c) For any value of test charge at $x = \frac{r}{2}$ from $2Q$ outside the charges

(d) Not possible for all the three charges to remain in equilibrium

143. A particle of charge $-q$ and mass m moves in a circular orbit of radius r about a fixed charge $+Q$. The relation between the radius of the orbit r and the time period T is :

(a) $r = \frac{Qq}{16\pi^2\varepsilon_0 m}T^2$ (b) $r^3 = \frac{Qq}{16\pi^2\varepsilon_0 m}T^2$

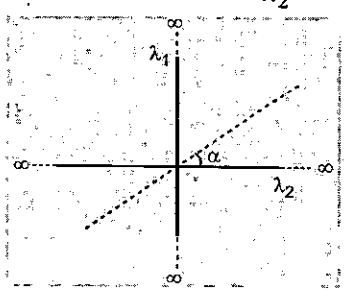
(c) $r^2 = \frac{Qq}{16\pi^3\varepsilon_0 m}T^3$ (d) $r^2 = \frac{Qq}{4\pi\varepsilon_0 m}T^3$

144. A long straight wire is surrounded by a long metallic cylinder such that axis of cylinder coincides with that of the wire. If wire is having uniform linear charge density λ and cylinder is having charge per unit length of 2λ, then in electrostatic equilibrium condition, the electric field outside the cylinder at a distance r from the axis of cylinder is:

(a) $\frac{\lambda}{2\pi\varepsilon_0 r}$ (b) $\frac{2\lambda}{3\pi\varepsilon_0 r}$

(c) $\frac{3\lambda}{2\pi\varepsilon_0 r}$ (d) $\frac{\lambda}{\pi\varepsilon_0 r}$

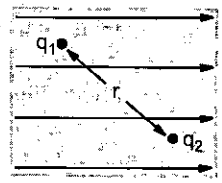
145. Two mutually perpendicular wire carry charge densities λ_1 and λ_2. The electric lines of force makes angle α with second wire then $\frac{\lambda_1}{\lambda_2}$ is :



(a) $\tan^2\alpha$ (b) $\cot^2\alpha$

(c) $\sin^2\alpha$ (d) $\cos^2\alpha$

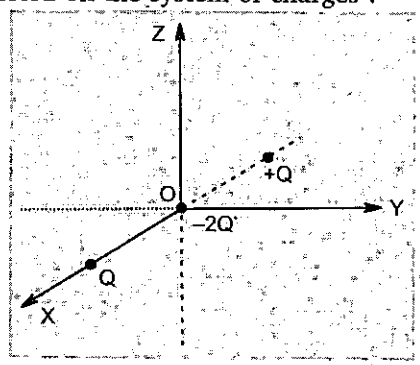
146. Two point charges q_1 and q_2 are placed in an external uniform electric field as shown in figure. The potential at the location of q_1 and q_2 are V_1 and V_2 respectively, the effect of q_1, q_2 are not included in V_1 and V_2, *i.e.*, V_1 and V_2 are potentials at location of q_1 and q_2 due to external unspecified charges only, then electric potential energy for this configuration of two charged particles is:



(a) $\frac{q_1V_1 + q_2V_2}{2}$ (b) $q_1V_1 + q_2V_2$

(c) $q_1V_1 + q_2V_2 + \frac{q_1q_2}{4\pi\varepsilon_0 r}$ (d) $\frac{q_1q_2}{4\pi\varepsilon_0 r}$

147. A distribution of charges is held fixed by rigid insulators as shown in Fig. A charge $+Q(-a, 0, 0)$ and a charge $+Q(a, 0, 0)$ and a charge $-2Q(0, 0, 0)$. Which of the following electric fields will cause a net torque to be exerted on the system of charges :



(a) $\vec{E} = (\text{constant})\hat{j}$ (b) $\vec{E} = (E_0x + \text{constant})\hat{i}$

(c) $\vec{E} = (E_0x + \text{constant})\hat{k}$ (d) $\vec{E} = E_0|x|\hat{i}$

148. A positively charged rod is brought near to a non-conducting object. The rod is attracted to the object, from this observation we can't predict whether the object is charged or uncharged. Some additional experiments have been performed to identify whether the object is charged or uncharged, these experiments with their observations and conclusions withdrawn are given. Select the option in which conclusion withdrawn is correct.

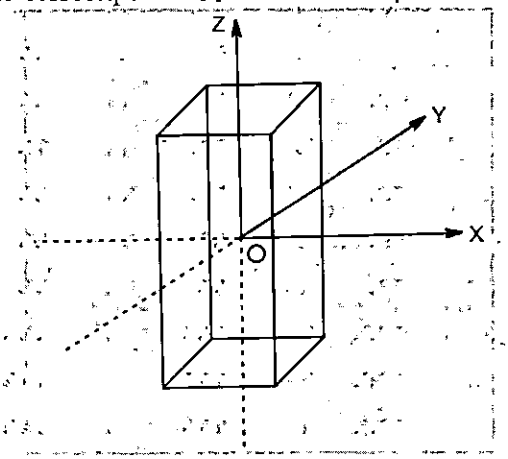
(a) A negatively charged rod is brought near to object and the two attract, this shows that the object is neutral.

(b) A negatively charged rod is brought near to object and two repel, this shows that the object is negatively-charged.

(c) A neutral rod is brought near to object and two attract, this shows that the object is negatively-charged.

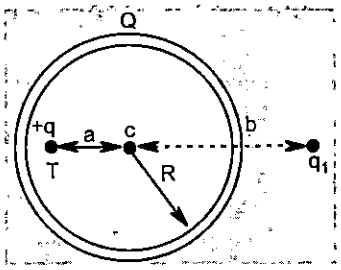
(d) All of the above

149. Two identical non-conducting cubes are charged uniformly with the same volume charge density. They joined face to face and placed with their common surface coinciding with the x-y plane and z-axis passes through the centre of the common structure as shown in the Fig. Electric field intensity and the electric potential at any point on the x-axis are measured as E and V. One of the cubes is removed and taken far away. The electric field intensity and the electric potential on the same point are now measured as E' and V'. Select the correct potion:



(a) $E > E'$ and $V > V'$
(b) $E < E'$ and $V < V'$
(c) $E < E'$ and $V > V'$
(d) $E > E'$ and $V < V'$

150. A thin metallic spherical shell contains a charge Q over it. A point charge $+q$ is placed inside the shell at point T separated from the centre by distance "a". Another point charge q_1 is placed outside the shell at a distance b from the centre as shown in the adjacent Fig. Now select the correct statement(s) from the following.



(a) Electric field at the centre due to charge over outer surface of the shell is zero.

(b) Electric field at the centre due to charge over outer surface of the shell is $\frac{q_1}{4\pi\varepsilon_0 b^2}$.

(c) Electric potential at the centre due to all charges in space is $\frac{1}{4\pi\varepsilon_0}\left[\frac{q}{a}-\frac{q}{R}+\frac{Q}{R}+\frac{q_1}{b}\right]$.

(d) Electric potential at the centre due to all charges in space is $\frac{1}{4\pi\varepsilon_0}\left[\frac{q}{a}+\frac{Q}{R}+\frac{q_1}{b}\right]$.

151. The electric field in a region is given by $\vec{E} = 200\hat{i}$ N/C for $x > 0$ and $-200\hat{i}$ N/C for $x < 0$. A closed cylinder of length 2 m and cross-section area $10^{-2}\,m^2$ is kept in such a way that the axis of cylinder is along X-axis and its centre coincides with origin. The total charge inside the cylinder is [Take $\varepsilon_0 = 8.55\times10^{-12}C^2m^2$ - N]

(a) Zero (b) 1.86×10^{-5}C
(c) 1.77×10^{-11}C (d) 1.86×10^{-11}C

152. A quantity of charge, Q, is distributed uniformly through a sphere of radius R. A smaller sphere, of radius d and concentric with the large sphere, is not removed from it; leaving a spherical cavity with no charge in it. The charge density of the remaining shell has not changed. The electrostatic potential at a distance $r > R$, outside the shell, is :

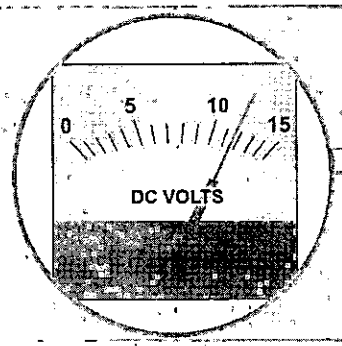
(a) $\frac{1}{4\pi\varepsilon_0}\frac{Q(R^3-d^3)}{rR^3}$ (b) $\frac{1}{4\pi\varepsilon_0}\frac{Qr^3}{rd^3}$
(c) $\frac{1}{4\pi\varepsilon_0}\frac{QR^3}{rd^3}$ (d) $\frac{1}{4\pi\varepsilon_0}\frac{Qd^3}{rR^3}$

153. The number of electric field lines crossing an area ΔS is n_1 when $\Delta\vec{S} \parallel \vec{E}$, while number of field lines crossing same area is n_2 when $\Delta\vec{E}$ makes an angle of 30° with $\vec{E}$, then:

(a) $n_1 = n_2$
(b) $n_1 > n_2$
(c) $n_1 < n_2$
(d) Cannot say anything

ANSWERS

1.	(a)	**2.**	(b)	**3.**	(a)	**4.**	(b)	**5.**	(b)	**6.**	(b)	**7.**	(b)	**8.**	(a)
9.	(c)	**10.**	(c)	**11.**	(a)	**12.**	(c)	**13.**	(b)	**14.**	(d)	**15.**	(a)	**16.**	(d)
17.	(a)	**18.**	(a)	**19.**	(b)	**20.**	(b)	**21.**	(b)	**22.**	(d)	**23.**	(b)	**24**	(b)
25.	(d)	**26.**	(a)	**27.**	(d)	**28.**	(a)	**29.**	(b)	**30.**	(b)	**31.**	(c)	**32.**	(d)
33.	(c)	**34.**	(a)	**35.**	(b)	**36.**	(a)	**37.**	(b)	**38.**	(b)	**39.**	(b)	**40.**	(a)
41.	(b)	**42.**	(a)	**43.**	(b)	**44.**	(a)	**45.**	(a)	**46.**	(a)	**47.**	(b)	**48.**	(c)
49.	(d)	**50.**	(a)	**51.**	(a)	**52.**	(a)	**53.**	(a)	**54.**	(b)	**55.**	(a)	**56.**	(b)
57.	(c)	**58.**	(c)	**59.**	(d)	**60.**	(c)	**61.**	(b)	**62.**	(a)	**63.**	(a)	**64.**	(c)
65.	(b)	**66.**	(d)	**67.**	(c)	**68.**	(d)	**69.**	(d)	**70**	(d)	**71.**	(b)	**72.**	(a)
73.	(d)	**74.**	(d)	**75.**	(b)	**76.**	(b)	**77.**	(c)	**78.**	(b)	**79.**	(a)	**80.**	(a)
81.	(b)	**82.**	(c)	**83.**	(a)	**84.**	(c)	**85.**	(c)	**86.**	(d)	**87.**	(b)	**88.**	(a)
89.	(b)	**90.**	(b)	**91.**	(d)	**92.**	(c)	**93.**	(b)	**94.**	(d)	**95.**	(b)	**96.**	(b)
97.	(c)	**98.**	(d)	**99.**	(a)	**100.**	(b)	**101.**	(c)	**102.**	(a)	**103.**	(b)	**104.**	(b)
105.	(b)	**106.**	(c)	**107.**	(a)	**108.**	(c)	**109.**	(a)	**110.**	(b)	**111.**	(b)	**112.**	(c)
113.	(c)	**114.**	(b)	**115.**	(c)	**116.**	(d)	**117.**	(c)	**118.**	(c)	**119.**	(a)	**120.**	(c)
121.	(b)	**122.**	(b)	**123.**	(c)	**124.**	(c)	**125.**	(c)	**126.**	(a)	**127.**	(a)	**128.**	(d)
129.	(a)	**130.**	(d)	**131.**	(a)	**132.**	(d)	**133.**	(d)	**134.**	(b)	**135.**	(c)	**136.**	(d)
137.	(d)	**138.**	(a)	**139.**	(b)	**140.**	(b)	**141.**	(a)	**142.**	(a)	**143.**	(b)	**144.**	(c)
145.	(b)	**146.**	(c)	**147.**	(c)	**148.**	(d)	**149.**	(a)	**150.**	(b)	**151.**	(c)	**152.**	(a)
153.	(b)														



SOLUTION

Level-1: Only One Alternative is Correct

2. (b) $\tan\theta = \frac{y}{x} = \sqrt{2}$

$\cot\theta = \frac{1}{\sqrt{2}}$

Also $\tan\alpha = \frac{\tan\theta}{2} = \frac{1}{\sqrt{2}} = \cot\theta$

$\Rightarrow \quad \theta + \alpha = 90^\circ$

i.e., $\vec{\mathbf{E}}$ is along positive y-axis.

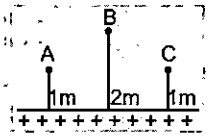
4. Errata (b)

$E = E\,\hat{\mathbf{i}}$

$\int_0^V dV = -\int_0^{x_0} E dx$

$V = -Ex_0$

5. (b)



Since $V_A = V_C$

So $W_{AC} = 0$

$\Rightarrow \quad W_{AB} + W_{BC} = 0$

$\Rightarrow \quad W_{AB} = -W_{AB}$

$W_{CA} = 0$ (as $V_A = V_C$)

6. (b) $E = \frac{Kq}{r^2}, V = \frac{Kq}{r}$

So $V = Er = 2E$,

Work $= qV = 2 \times 2E = 4E$ joules

8. (a) $dV = -\vec{\mathbf{E}}.d\vec{\mathbf{r}}$

$= -(y\,\hat{\mathbf{i}} + x\,\hat{\mathbf{j}}).(dx\,\hat{\mathbf{i}} + dy\,\hat{\mathbf{j}})$

$= -(ydx + xdy) = -d(xy)$

Integrating, we get

$V = -xy$ + constant.

9. (c) The potential drops maximum along the resultant electric field.

11. (a) Since E is perpendicular to area vector of base.

12. (c) Charge on A after contacted with fourth sphere

$q_1 = \frac{q}{2}$.

When B is earthed, let charge induced on it be q_2 then for $V = 0$

$K\left(\frac{q_1}{d} + \frac{q_2}{a}\right) = 0 \Rightarrow q_2 = -\frac{aq_1}{d}$

When C is earthed let charge q_3 induced on it then

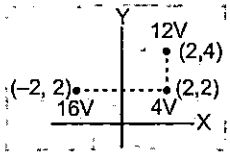
$\left(\frac{q_1}{d} + \frac{q_2}{d} + \frac{q_3}{a}\right) = 0$

$\Rightarrow \quad q_3 = -\frac{qa}{2d}\left(\frac{d-a}{d}\right)$

14. (d) $\vec{\mathbf{E}}_x = \frac{16-4}{4}\hat{\mathbf{i}} = 3\,\hat{\mathbf{i}}$

$\vec{\mathbf{E}}_y = -\frac{12-4}{2}\hat{\mathbf{j}} = -4\hat{\mathbf{j}}$

$\vec{\mathbf{E}} = 3\,\hat{\mathbf{i}} - 4\,\hat{\mathbf{j}}$

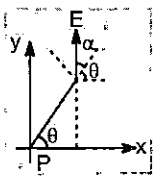


19. (b) $\vec{\mathbf{E}}_{axis} = \frac{1}{4\pi\varepsilon_0}\frac{2P}{r^3}$ (along $\vec{\mathbf{P}}$)

$E_{bisector} = \frac{1}{4\pi\varepsilon_0}\frac{P}{(2r)^3}$

(opposite to $\vec{\mathbf{P}}$)

So, $E_2 = -\frac{\vec{\mathbf{E}}_1}{16}$



22. (d) Total potential energy of system

$U = \frac{K(-Q)(-3Q)}{2l} + \frac{K(-3Q)q}{l} + \frac{K(-Q)q}{l}$

$K = \left(\frac{3Q^2}{2l} - \frac{4Qq}{l}\right)$

If will be positive for

$\frac{3Q^2}{2l} > \frac{4qQ}{l} \Rightarrow q < \frac{3Q}{8}$

25. (d) $r = \sqrt{(8-2)^2 + (-5-3)^2}$

$= \sqrt{6^2 + -8^2} = 10$

$E = \frac{9 \times 10^{+9} \times 50 \times 10^{-6}}{10^2}$

$= \frac{9}{2} \times 10^3 = 4500\frac{\text{V}}{\text{m}}$

31. (c) Word done $= V_f = V_i$

$V_i = \frac{K}{a}(q(-2q) + q(-2q) + (-2q)(-2q)) = 0$

Similarly, $V_f = 0$

Work done = zero.

34. (a) Since field between the charges is zero, so they must be of same sign. Also, the net field is zero near q_1 so its magnitude should be smaller than q_2.

38. (b) Suppose after earthing charge on inner shell becomes q' then for its potential to become zero.

$$\frac{Kq'}{r} + \frac{Kq}{3R} = 0 \Rightarrow q' = -\frac{q}{3}$$

So charge flow from shell to earth is $+\frac{q}{3}$.

40. (a) $$\int_{t=\infty}^{t=0} -\vec{\mathbf{E}}.d\vec{\mathbf{l}} = V(0) - V(\infty)$$

$V(0)$ = Potential at centre $= \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$

$= 2$ volts

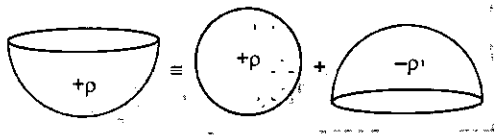
42. (a) By putting any charge on the outer sphere, change in potential through out the sphere will be same. So the difference will remain same.

46. (a) Negative of slope of potential *Vs* x curve gives electric field. In this case slope is negative therefore electric field is positive. Thus field due to σ_2 is greater *i.e.*, $\sigma_2 > \sigma_1$

54. (b) Apply principle of superposition

Electric field due to a uniformly charged sphere

$$= \frac{\rho R}{12\varepsilon_0}$$



$$E_{\text{Resultant}} = \frac{\rho R}{12\varepsilon_0} - E$$

56. (b) Resultant electric field due to both the wires is zero along y-axis

66. (d) Flux through the circular surface, since the field is uniform, will be

$$\phi = \vec{\mathbf{E}} \cdot \vec{\mathbf{S}}$$

$$= ES\cos 180°$$

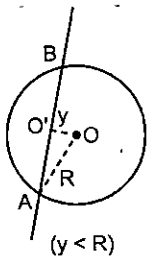
$= -E\pi r^2$ [$\vec{\mathbf{S}}$ being outward normal to surface is opposite to $\vec{\mathbf{E}}$]

All the lines of force that enter at the circular opening also leave through the paraboloidal surface, so that, flux through the paraboloidal surface has the same magnitude, but opposite sign.

i.e., $\phi = E\pi r^2$

67. (c) Electric flux $\oint_S \vec{\mathbf{E}} \cdot \vec{\mathbf{ds}} = \frac{q_{in}}{\varepsilon_0}$

q_{in} is the charge enclosed by the Gaussian-surface which, in the present case, is the surface of given sphere. As shown, **length *AB* of the line lies inside the sphere.**



In $\Delta OO'A$

$$R^2 = y^2 + (O'A)^2$$

$$\therefore \quad O'A = \sqrt{R^2 - y^2}$$

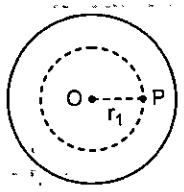
and $$AB = 2\sqrt{R^2 - y^2}$$

Charge on length $AB = 2\sqrt{R^2 - y^2} \times \lambda$

$$\therefore \text{ Electric flux } = \oint_S \vec{\mathbf{E}} \cdot \vec{\mathbf{ds}} = \frac{2\lambda\sqrt{R^2 - y^2}}{\varepsilon_0}$$

68. (d) P is any inside point at distance r_1 from O. We take a spherical surface of radius r_1 as Gaussian-surface.

$$\oint_S \vec{\mathbf{E}} \cdot \vec{\mathbf{ds}} = \frac{q_{in}}{\varepsilon_0}$$



By symmetry, E at all points on the surface is same and angle between $\vec{\mathbf{E}}$ and $\vec{\mathbf{ds}}$ is zero everywhere.

$$\therefore \quad \oint_S \vec{\mathbf{E}} \cdot \vec{\mathbf{ds}} = Es = \frac{q_{in}}{\varepsilon_0}$$

or $$E4\pi r_1^2 = \frac{q_{in}}{\varepsilon_0} \quad \ldots(1)$$

q_{in} : The sphere can be regarded as consisting of a large number of spherical shells. Consider a shell of inner and outer radii r and $r + dr$. Its volume will be $dV = 4\pi r^2 dr$. Charge in the shell,

$$dq = \rho dV = \frac{A}{r} 4\pi r^2 dr$$

Total charge enclosed by Gaussian-surface,

$$q_{in} = \int dq = \int_0^{r_1} \frac{A}{r} 4\pi r^2 dr$$

$$q_{in} = 4\pi A \int_0^{r_1} r dr = 4\pi A \frac{r_1^2}{2}$$

From Eq. (1), $E4\pi r_1^2 = 4\pi A \frac{r_1^2}{2} \Big/ \varepsilon_0$

$$\therefore \quad E = \frac{A}{2\varepsilon_0}$$

69. (d) Net force on q is zero in accordance with electrostatic shielding. Due to q_1, it is towards left to make net force zero due to the shell it is towards right.

70. (d) Resultant force on arrangement is $2QE$, thus acceleration of centre of mass is given by

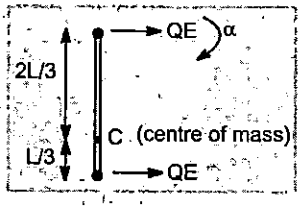
$$a_C = \frac{2QE}{3m}$$

Net torque on system is

$$QE\left(\frac{2L}{3}\right)-QE\left(\frac{L}{3}\right)=\left[m\times\frac{4L^2}{9}+2m\times\frac{L^2}{9}\right]\alpha$$

$$\alpha=\frac{QE}{2mL}\text{ (clockwise)}$$

From constraint equation, we get



$$a_A=a_C+\alpha\left(\frac{2L}{3}\right)$$

$$=\frac{2QE}{3m}+\frac{QE}{2mL}\left(\frac{2L}{3}\right)$$

$$=\frac{3QE}{3m}=\frac{QE}{m}$$

72. (a) Potential is a scalar quantity. The charge density function suggests that net charge is zero. Net charge on the ring

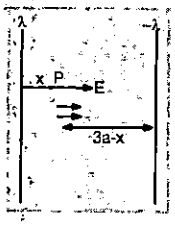
$$Q=\int_0^{2\pi}\lambda R d\theta$$

$$=\lambda_0 R\int_0^{2\pi}\cos(\theta/2)d\theta=0$$

$$V=\frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{R}\right)=0$$

73. (d) Electric field at P is given by

$$E=\frac{\lambda}{2\pi\varepsilon_0}\left(\frac{1}{x}+\frac{1}{3a-x}\right)$$

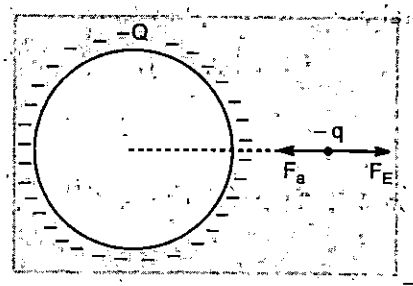


Work done is given by

$$W=\int_a^{2a}q_0E dx$$

$$=\frac{\lambda q_0}{2\pi\varepsilon_0}\left[\int_a^{2a}\frac{dx}{x}+\int_a^{2a}\frac{dx}{3a-x}\right]=\frac{\lambda q_0}{\pi\varepsilon_0}\ln 2$$

74. (d)



if charge is being moved slowly, $\vec{F}_a=-\vec{F}_E$

$\omega_a+\omega_E=\Delta KE=0$ [charge is moved slowly]

i.e. $\omega_a=-\omega_E$

also $\omega_E=-4U$

thus $\omega_a=+4U$

As force F_a and displacement are 180°

$\vec{F}.d\vec{r}<0$

$\omega_a<0$

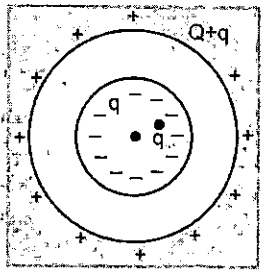
thus $\Delta U<0$

Therefore U decreases

76. (b) In gauss's theorem, $\oint\vec{\mathbf{E}}\cdot d\vec{\mathbf{S}}=\frac{q_{in}}{\varepsilon_0}$

The $\vec{\mathbf{E}}$ on LHS of equation, the $\vec{\mathbf{E}}$ at any point on surface is due to all the charges present in space. Here, only four point charges are given so $\vec{\mathbf{E}}$ is due to all four charges.

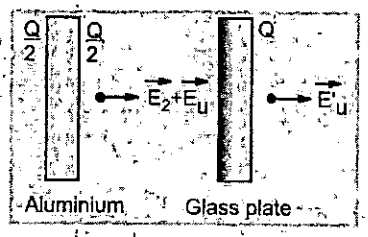
78. (b) Due to induction, $-q$ will induce on inner surface of shell. As the total charge of shell is Q, so charge on outer surface in electrostatic equilibrium condition in $Q+q$.



The charge distribution on inner surface of shell would be non-uniform, because electric field at any outside point ($r>R_1$) due to inside charges (q and $-q$) has to be zero which is possible only when $-q$ is distributed non-uniformly.

Charge on outer surface would be uniform.

80. (a)



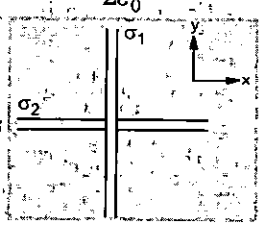
$$\vec{\mathbf{E}}_2=\frac{Q}{2\varepsilon_0 A},\ \vec{\mathbf{E}}_u=\frac{Q}{2\varepsilon_0 A}$$

So, $$|\vec{\mathbf{E}}|=\frac{Q}{\varepsilon_0 A}$$

$$|\vec{\mathbf{E}}_2|=|\vec{\mathbf{E}}_u|=\frac{Q}{\varepsilon_0 A}$$

Both are away from the plates.

82. (c) Using principle of superposition $\vec{\mathbf{E}}$ due to infinite plane sheet having surface charge density σ on its one face is, $\frac{\sigma}{2\varepsilon_0}$.



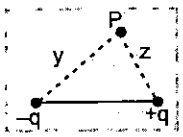
So, $$\vec{\mathbf{E}}_1=\frac{\sigma_1}{2\varepsilon_0}\hat{\mathbf{i}}+\frac{\sigma_2}{2\varepsilon_0}\hat{\mathbf{j}}$$

$$\vec{\mathbf{E}}_2=-\frac{\sigma_1}{2\varepsilon_0}\hat{\mathbf{i}}+\frac{\sigma_2}{2\varepsilon_0}\hat{\mathbf{j}}$$

$$\vec{E}_3 = -\frac{\sigma_1}{2\varepsilon_0}\hat{i} - \frac{\sigma_2}{2\varepsilon_0}\hat{j}$$

$$\vec{E}_4 = \frac{\sigma_1}{2\varepsilon_0}\hat{i} - \frac{\sigma_2}{2\varepsilon_0}\hat{j}$$

83. **(a)** $V_1 = \dfrac{q}{4\pi\varepsilon_0 x}$



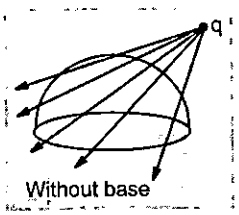
$$V_2 = \frac{q}{4\pi\varepsilon_0 x} - \frac{q}{4\pi\varepsilon_0 V}$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{x} - \frac{1}{y}\right] < V_1$$

For all values of x and y.

84. **(c)** As such actual charge is discrete in nature but if charge density is of the order of few nC/m^2, then we can take it as continuous. All other statements are wrong.

86. **(d)** Check that for the given surface, number of field lines entering the surface are same or different, than the number of field lines leaving the surface.



For example, if we observe the diagram, then for drawn field of lines, 5 lines of force are entering the surface while only, 2 are leaving, so flux is not zero.

87. **(b)** Point P is outside the spheres A and B and inside C. By applying principle of superposition, potential of point P,

$$V = \frac{Q_1}{4\pi\varepsilon_0 r} + \frac{Q_2}{4\pi\varepsilon_0 r} + \frac{Q_3}{4\pi\varepsilon_0 c}$$

88. **(a)** Force experienced by dipole in non-uniform electric field, is given by $F = P\dfrac{\partial E}{\partial l}$

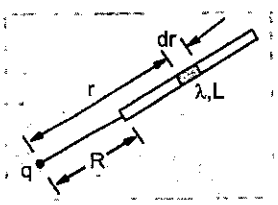
Here, $E = \dfrac{Q}{4\pi\varepsilon_0 r^2}$

We have to take derivative of E along dipole length, *i.e.*, along $\vec{\mathbf{r}}$.

So, $\dfrac{\partial E}{\partial r} = -\dfrac{2Q}{4\pi\varepsilon_0 r^3}$

So, required force (magnitude) $= \dfrac{|\vec{\mathbf{P}}|Q}{2\pi\varepsilon_0 r^3}$

90. **(b)** As coulomb's force is an action reaction pair, so force experienced by the line charge is equal and opposite to the force experienced by point charge q. Here, we are computing electric force experienced by q due to the charge.

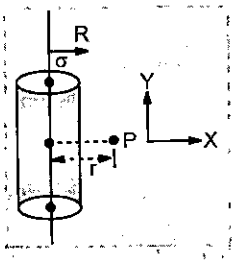


Consider a small element on line charge as shown, then force experienced by q due to this element is,

$$dF = \frac{q\lambda dr}{4\pi\varepsilon_0 r^2}$$

$$F = \int dF = \int_R^{R+L} \frac{q\lambda dr}{4\pi\varepsilon_0 r^2} = \frac{q\lambda L}{4\pi\varepsilon_0 R(R+L)}$$

91. **(d)** $\vec{E}$ at P would be due to two charge configurations, one due to linear charge and other due to charge on cylinder.



$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

$$= \frac{\lambda}{2\pi\varepsilon_0 r}\hat{i} + \frac{\sigma R}{\varepsilon_0 r}\hat{i}$$

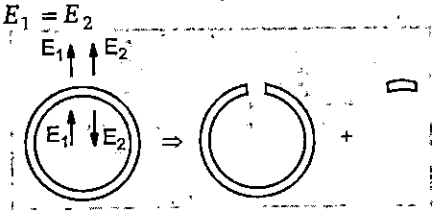
[using Gauss's law, we can find $\vec{E}_2$].

For $\vec{E}$ to be zero,

$$\frac{\lambda}{2\pi\varepsilon_0 r} = -\frac{\sigma R}{\varepsilon_0 R}$$

$$\lambda = -2\pi\sigma R$$

92. **(c)** Consider the full conductor (*i.e.*, without cutting the hole, which can be considered as superposition of hole and given structure) E_1 is the electric field intensity at the location of hole due to all remaining charges except charge at the hole's location and E_2 is due to the hole element. Electric field inside conductor is, zero, *i.e*, $E_1 = E_2$



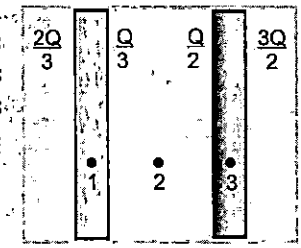
$$E_2 = \frac{\sigma}{2\varepsilon_0}$$

$$E_1 = \frac{\sigma}{2\varepsilon_0}$$

When hole has been cut, the $\vec{E}$ inside the hole is only due to remaining charges *i.e.*, $\vec{E}$ inside the hole is equal to $\vec{E}_1$.

94. **(d)** Electric field at some points or at all points on surface may be non-zero, in such a way that flux would be zero.

95. (b) Let us take rightward as +ve and leftward as –ve. These four faces can be considered as four thin infinite plane sheets.



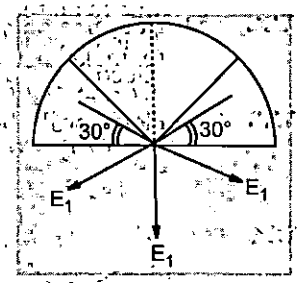
$$\vec{E}_1 = \frac{1}{2\varepsilon_0 S}\left[\frac{2Q}{3} - \frac{Q}{3} - \frac{Q}{2} - \frac{3Q}{2}\right] = \frac{-5Q}{6\varepsilon_0 S}$$

$$\vec{E}_2 = \frac{1}{2\varepsilon_0 S}\left[\frac{2Q}{3} + \frac{Q}{3} - \frac{Q}{2} - \frac{3Q}{2}\right] = \frac{-Q}{2\varepsilon_0 S}$$

$$\vec{E}_3 = \frac{1}{2\varepsilon_0 S}\left[\frac{2Q}{3} + \frac{Q}{3} + \frac{Q}{2} - \frac{3Q}{2}\right] = 0$$

96. (b) Place another identical hemisphere on given one so that flat faces of two coincide. Then total flux linked with cross-section of bottom half (lower hemisphere) is $\frac{Q}{2\varepsilon_0}$, as charge enclosed by given hemisphere is zero, so flux linked with flat face of hemisphere is $\frac{-Q}{2\varepsilon_0}$.

97. (c) Here, we are showing the 2D arrangement of the situation. We will use two concepts here:



1. $\vec{E}$ due to an uniformly charged circular arc or part of shell is acting along the angular bisector of arc or part of shell.
2. Principle of superposition. Here, as all the three portions, two cut-out portions and one remaining are identical, the $\vec{E}$ due to these portions would be have the same magnitude, let it be E_1. So, the electric field at centre due to full hemisphere is,

$$E = E_1 + 2E_1 \sin 30° = 2E_1 = E_0 \quad \text{(given)}$$

Hence, $E_1 = \frac{E_0}{2}$

98. (d) The force experience by an electric dipole placed in a non-uniform electric field is given by, $F = p\frac{\partial E}{\partial l}$ where $\frac{\partial E}{\partial l}$ is directional derivative of $\vec{E}$ along the dipole moment.

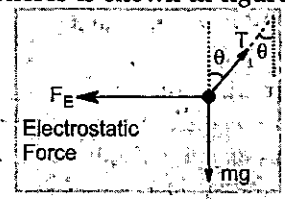
Here, dipole is placed along X-axis, so $\frac{\partial E}{\partial l}$ corresponding to component of $\frac{\partial E}{\partial x}$ is along X-axis.

$$\frac{\partial \vec{E}}{\partial x} = 6x\hat{i} + 6y\hat{j} + 0\hat{k}$$

$$\vec{F} = |\vec{P}|\left(\frac{\partial \vec{E}}{\partial x}\right)_{X\text{-component}} = p(6\times 4\hat{i})$$

$$\vec{F} = 24p\hat{i}$$

99. (a) Since both the small spheres are at same horizontal level, the electrostatic forces on both spheres are in horizontal direction. The FBD of left spheres is shown in figure.



$\because$ The sphere is in equilibrium

$\Rightarrow$ $T\cos\theta = mg$ and $T\sin\theta = F_E$

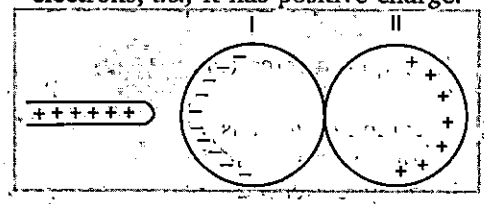
$$\tan\theta = \frac{F_E}{mg}$$

The magnitude of electrostatic force on each spheres is same irrespective of its charge

$\therefore$ for $\theta_1 = \theta_2$ the necessary conditions is $m_1 = m_2$

100. (b) Pattern (a) can be eliminated because field lines cannot simultaneously originate from and converge at a single point; (c) can be eliminated because there are no charges in the region, and so there are no sources of field lines; (d) can be eliminated because electrostaic field lines do not close on themselves.

101. (c) The positively charged insulator pulls the free electron in spheres towards left till electrostatic equilibrium is reached. Hence if both spheres are now separated, the sphere II has deficiency of electrons, *i.e.*, it has positive charge.



102. (a) The potential at centre of sphere in which q charge is uniformly distributed throughout the volume is:

$$V_C = \frac{1}{4\pi\varepsilon_0}\frac{3q}{2R}$$

By symmetry the potential at centre due to half sphere will be half of the complete sphere.

$$\therefore \quad V_C = \frac{1}{4\pi\varepsilon_0}\frac{3q/2}{2R}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{3Q}{R} \qquad \left[\because \frac{q}{2} = Q\right]$$

103. (b) Electric field at a distance x outside from surface of outer shell $= \frac{1}{4\pi\varepsilon_0}\frac{Q+3Q}{(2R+x)^2}$

Electric field at a distance x inside from surface of outer shell $= \frac{1}{4\pi\varepsilon_0}\frac{Q}{(2R-x)^2}$

$$\therefore \quad \frac{1}{4\pi\varepsilon_0}\frac{4Q}{(2R+x)^2} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{(2R-x)^2}$$

$$x = \frac{2R}{3}$$

104. (b) $E = kq/r^2$ outside a uniformly charged sphere and also for a point charge.

105. (b) $E = -\left[\frac{\partial V}{\partial x}\hat{\mathbf{i}} + \frac{\partial V}{\partial y}\hat{\mathbf{j}} + \frac{\partial V}{\partial z}\hat{\mathbf{k}}\right] = -2k[2x\hat{\mathbf{i}} - y\hat{\mathbf{i}} + z\hat{\mathbf{k}}]$

Thus, $|E|$ at $(1,1,1) = 2k\sqrt{6}$

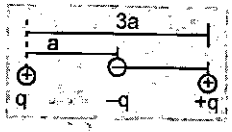
106. (c) $U = \frac{-kq^2}{a} + \frac{+kq^2}{3a} = \frac{-2kq^2}{3a}$

$\Delta U + \Delta KE = 0$

$$\frac{1}{2}(2ma^2)\omega^2 = \frac{2kq^2}{3a}$$

$$\omega = \sqrt{\frac{2kq^2}{3ma^3}}$$

$$\omega = \frac{q}{\sqrt{6\pi\varepsilon_0 ma^3}}$$

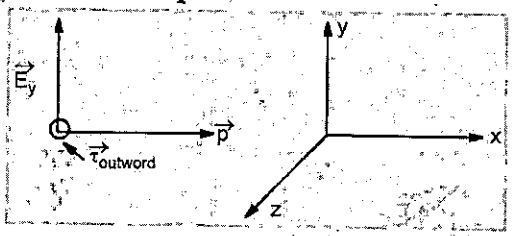


107. (a) $E_y \neq 0; \quad \frac{\partial \vec{\mathbf{E}}}{\partial x} \neq 0$

$$E_x = p\frac{\partial \vec{\mathbf{E}}}{\partial x}$$

Force is directed along x-axis.

108. (c) $\vec{\tau} = \vec{\mathbf{p}} \times \vec{\mathbf{E}}$



Torque is due E_y therefore it points along z-axis.

109. (a) $E = -\frac{dV}{dr}$

$$E = 0 \quad \Rightarrow \frac{dV}{dr} = 0 \quad \Rightarrow V = \text{Constant}$$

112. (c) W = work done by the force as it travels from x_0 to x_{min}

$$= \int_{x_0}^{x_{min}} F_x\,dx = \int_{x_0}^{x_{min}} \frac{\alpha}{\alpha^2}\,dx$$

$$= -\alpha\left[\frac{1}{x_{min}} - \frac{1}{x_0}\right]$$

Using work-energy theorem,

$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2 = -\alpha\left[\frac{1}{x_{min}} - \frac{1}{x_0}\right]$$

$$v_f = 0 \Rightarrow \quad x_{min} = \frac{\alpha}{\frac{\alpha}{x_0} + \frac{1}{2}mv_0^2}$$

120. (c) (1) Sign of q_1 and q_2 opposite because no point of zero field between them

(2) q_2 smaller in magnitude

(3) field between q_1 and q_2 is –ve so q_2 must be +ve and q_1 must be –ve.

121. (b) Power provided by light

$I \times A$ (I – intensity) $= 0.1$ W

Power delivered by cell $= VI$ (I – current)

$= 0.6 \times 00.003$

$= 1.8 \times 10^{-3}$ W

$$\eta = \frac{1.8\times10^{-3}}{0.1} \times 100 = 1.8\%$$

122. (b) Net force on q is zero. Due to q, it is towards left to make net force zero due to the shell it is towards right.

123. (c) Field will be uniform at the outside so, $E = \frac{k(2Q)}{(4a)^2} = \frac{kQ}{8a^2}$

124. (c) $d = \frac{1}{2}at^2, t = \frac{L}{v}$

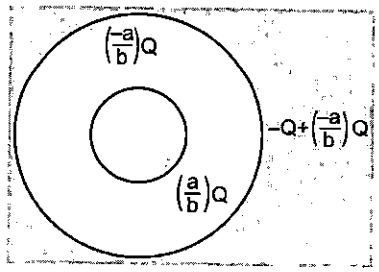
$$d = \frac{1}{2} \times \left(\frac{Eq}{m}\right) \times \left(\frac{L}{v}\right) \Rightarrow q = \frac{2dmv^2}{EL^2}$$

126. (a) When outer surface is grounded charge '$-Q$' resides on the inner surface of sphere 'B' Now sphere A is connected to earth potential on its surface becomes zero.

Let the charge on the surface A becomes q.

$$\frac{kq}{a} - \frac{kQ}{b} = 0 \quad \Rightarrow q = \frac{a}{b}Q$$

Consider the Fig. In this position energy stored



$$E_1=\frac{1}{8\pi\varepsilon_0 b}\left[\frac{a}{b}Q\right]^2+\frac{Q^2}{8\pi\varepsilon_0 b}+\frac{1}{4\pi\varepsilon_0 b}\left[\frac{a}{b}Q\right](-Q)$$

When 'S_3' is closed, total charge will appear on the outer surface of shell 'B'. In this position energy stored

$$E_2=\frac{1}{8\pi\varepsilon_0 b}\left(\frac{a}{b}-1\right)^2Q^2$$

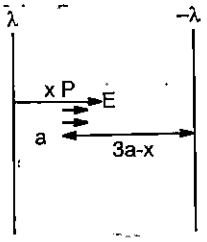
$$\text{Heat produced}=E_1-E_2=\frac{Q^2a(b-a)}{8\pi\varepsilon_0 b^3}=1.8$$

127. (a) Net charge on the ring

$$Q=\int_0^{2\pi}\lambda R\,d\theta=\lambda_0 R\int_0^{2\pi}\cos(\theta/2)\,d\theta=0$$

$$V=\frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{R}\right)=0$$

128. (d) Electric field at P



$$E=\frac{\lambda}{2\pi\varepsilon_0}\left(\frac{1}{x}+\frac{1}{3a-x}\right)$$

$$W=\int_a^{2a}q_0E\,dx$$

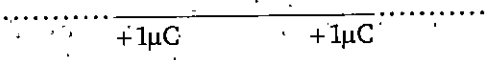
$$=\frac{\lambda q_0}{2\pi\varepsilon_0}\left[\int_a^{2a}\frac{dx}{x}+\int_a^{2a}\frac{dx}{3a-x}\right]$$

$$=\frac{\lambda q_0}{\pi\varepsilon_0}\ln 2$$

129. (a) $3.9\times10^{-19}=n_1e$, $6.5\times10^{-19}=n_2e$, $9.1\times10^{-19}=n_3e$

n_1, n_2 and n_3 are integers for $e=1.3\times10^{-19}$.

130. (d)



Since both charges are positive hence potential will not be zero any where (except at infinity).

131. (a) $V_{in}=0=\frac{kq_1}{r}+\frac{kq}{3r}$

$$q_1=-q/3$$

132. (d) $\vec{\mathbf{E}}=\frac{kq(\vec{\mathbf{r}}-\vec{\mathbf{r}}_0)}{\left|\vec{\mathbf{r}}-\vec{\mathbf{r}}_0\right|^3}$

$$=\frac{9\times10^9\times50\times10^{-6}(6\hat{\mathbf{i}}+8\hat{\mathbf{j}})}{10^3}$$

$$=4500\,\text{V/m}$$

133. (d) $\frac{qEL}{2}-\frac{mgL}{2}=0$

134. (b) $U_C+K_C=U_e+K_e$

$$q\times\frac{k\times3q}{\sqrt{(4R)^2+(3R)^2}}+\frac{1}{2}mu^2=\frac{k\times3q^2}{4R}$$

$$\frac{1}{2}mu^2=\frac{3kq^2}{R}\left[\frac{1}{4}-\frac{1}{5}\right]$$

$$u=\sqrt{\frac{3q^2}{40\pi\varepsilon_0 Rm}}$$

135. (c) $r=\sqrt{1+1+4}=\sqrt{6}<5$

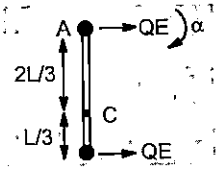
$\Rightarrow E_{in}=0$

136. (d) $\frac{kq_1}{\left(\frac{d}{4}\right)^2}-\frac{kq_2}{\left(\frac{3d}{4}\right)^2}=0$

$\Rightarrow 9q_1=q_2$

$\Rightarrow$ Both positive so no point except ∞.

137. (d) $a_c=\frac{2QE}{3m}$



$$QE\left(\frac{2L}{3}\right)-QE\left(\frac{L}{3}\right)=\left[m\times\frac{4L^2}{9}+2m\times\frac{L^2}{9}\right]\alpha$$

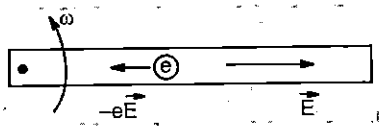
$$\alpha=\frac{QE}{2mL}\quad\text{(clockwise)}$$

$$a_A=a_C+\alpha\left(\frac{2L}{3}\right)=\frac{2QE}{3m}+\frac{QE}{2mL}\left(\frac{2L}{3}\right)$$

$$\frac{3QE}{3m}=\frac{QE}{m}$$

138. (a) When rod rotates the centripetal acceleration of electron comes from electric field

$$eE=mr\omega^2$$



$$E=\frac{mr\omega^2}{e}$$

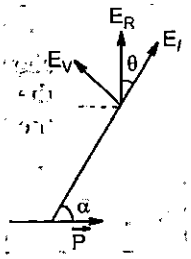
Thus, $\Delta V=-\int\vec{\mathbf{E}}\,dr=-\int_0^l\frac{mr\omega^2}{e}dr=\frac{m\omega^2l^2}{2e}$

139. (b) Given E_R along y-axis

thus $\theta=90^\circ-\alpha$

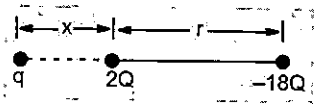
also $\tan\theta=\frac{1}{2}\tan\alpha$

or $\tan\alpha=\sqrt{2}$



141. (a) When a charged body is brought nearby to two charged bodies A and B, the charge distribution on A and B gets changed (*while this is not the case with point charges*) as a result interaction force between A and B changes due to presence of other charged bodies. But this is not a violation of principle of superposition.

142. (a) When two charges are of same sign, then point of zero $\vec{\mathbf{E}}$ in between the charges while if two charges are of opposite signs, then



the point of zero $\vec{\mathbf{E}}$ lies outside the line joining the charges and nearer to charge having smaller magnitude.

So, according to given situation we have to place a test charge q as shown in the figure.

Let us find the value of x by considering that test charge is in equilibrium, then

$$\frac{q \times 2Q}{4\pi\varepsilon_0 x^2} = \frac{18Q \times q}{(r+x)^2}$$

$$\Rightarrow \quad x = \frac{r}{2}$$

Now, let us consider the equilibrium of $2Q$, then

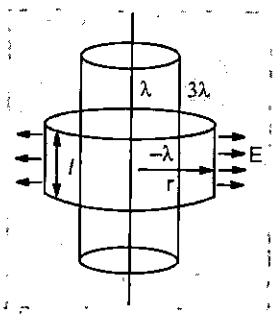
$$\frac{q \times 2Q}{4\pi\varepsilon_0 (r/2)^2} = \frac{2Q \times 18Q}{4\pi\varepsilon_0 (r^2)}$$

$$\Rightarrow \quad q = \frac{9}{2}Q$$

For system to be in equilibrium, $q = \frac{-9Q}{2}$

For $x = \frac{r}{2}$ and $q = -\frac{9Q}{2}$, we can check that $-18Q$ is also in equilibrium. So, the entire system is in equilibrium.

144. (c) The situation is shown clearly in the figure. The figure also shows the charges densities on various faces of the cylinder in electrostatic equilibrium condition. Consider the cylindrical Gaussian surface as shown. From symmetry, the direction of electric field intensity is along the normal. Applying Gauss's law,



$$E \times 2\pi r l = \frac{3\lambda \times l}{\varepsilon_0}$$

$$\Rightarrow \quad E = \frac{3\lambda}{2\pi\varepsilon_0 r}$$

146. (c) The potential energy of the system can be found by computing the work done by external agent in assembling the system without changing the kinetic energy of the system.

Let us first bring q_1 from infinity to the desired location, then in doing so we have to do work against external electric field which is equal to $W_1 = q_1 V_1$ [*Potential at infinity is considered as zero*].

Now, we will bring q_2 from infinity to the desired location, to do so, we have to do work against external electric field and against electric force of q_1, so work done is,

$$W_2 = q_2 V_2 + \frac{q_1 q_2}{4\pi\varepsilon_0 r}$$

So, total work done $= U$

$$= q_1 V_1 + q_2 V_2 + \frac{q_1 q_2}{4\pi\varepsilon_0 r}$$

148. (d) This question is based on the factual statement "*True test of electrification is repulsion*".

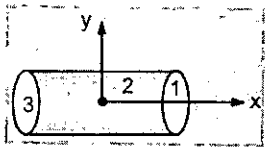
As mentioned in the question, a positively charged rod attracts a non-conducting object, so the two possibilities about the object are whether it is negatively charged or neutral in nature.

If it is neutral, then it would be attracted by some other negatively charged body and won't be attracted or repelled by neutral body [*For option a*]. If it is negatively charged, then it would be repelled by negatively charged object and will attract neutral objects. [*For option 'b' and 'c'*]

151. (c) Total flux crossing the closed cylinder is

$$\phi = \phi_1 + \phi_2 + \phi_3$$

where, ϕ_1, ϕ_2, ϕ_3 are the flux linked with surfaces, 1, 2 and 3, respectively.



$$\phi_1 = \vec{\mathbf{E}} \cdot \vec{\mathbf{A}}_1 = 200\hat{\mathbf{i}} \cdot 10^{-2}\hat{\mathbf{i}} = 2\text{N-m}^2/\text{C}$$

$$\phi_2 = \vec{\mathbf{E}} \cdot \vec{\mathbf{A}}_2 = 0$$

$$\phi_3 = \vec{\mathbf{E}} \cdot \vec{\mathbf{A}}_2 = -200\hat{\mathbf{i}} \cdot (10^{-2}\hat{\mathbf{i}}) = 2\text{N-m}^2/\text{C}$$

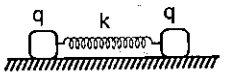
so, from Gauss's law, $\phi = \frac{q_{in}}{\varepsilon_0}$

$$\Rightarrow \quad q_{in} = \phi \times \varepsilon_0 = 2 \times 8.85 \times 10^{-12}\text{C} = 1.77 \times 10^{-11}\text{C}$$

153. (b) The no. of field lines crossing an area is directly related to electric flux passing through surface. In first case the flux crossing the surface is more than the flux crossing through surface in 2nd case, so $n_1 > n_2$.

Level ② More than One Alternative is/are Correct

1. Two identical particle of charge q each are connected by a massless spring of force constant k. They are placed over a smooth horizontal surface. They are released when the separation between them is r and spring is unstretched. If maximum extension of the spring is r, the value of k is (neglected gravitational effect):



(a) $k=\frac{q^2}{8\pi\varepsilon_0 r^3}$ (b) $k=\frac{q^2}{4\pi\varepsilon_0 r^3}$

(c) $k=\frac{q^2}{\pi\varepsilon_0 r^3}$ (d) $k=\frac{4q^2}{\pi\varepsilon_0 r^3}$

2. Two point charges $2q$ and $8q$ are placed at a distance r apart. Where should a third charge $-q$ be placed between them so that the electrical potential energy of the system is a minimum:

(a) at a distance of $\frac{r}{3}$ from $2q$

(b) at a distance of $\frac{2r}{3}$ from $2q$

(c) at a distance of $\frac{r}{16}$ from $2q$

(d) none of these

3. Two identical charges $+Q$ are kept fixed some distance apart. A small particle P with charge q is placed midway between them. If P is given a small displacement Δ, it will undergo simple harmonic motion if:

(a) q is positive and Δ is along the line joining the charges.

(b) q is positive and Δ is perpendicular to the line joining the charges.

(c) q is negative and Δ is perpendicular to the line joining the charges.

(d) q is negative and Δ is along the line joining the charges.

4. A point charge q is placed at origin. Let $\vec{E}_A, \vec{E}_B$ and $\vec{E}_C$ be the electric field at three points $A(1, 2, 3), B(1, 1, -1)$ and $C(2, 2, 2)$ due to charge q. Then

(a) $\vec{E}_A \perp \vec{E}_B$ (b) $\vec{E}_A \| \vec{E}_C$

(c) $|\vec{E}_B| = 4|\vec{E}_C|$ (d) $|\vec{E}_B| = 8|\vec{E}_C|$

5. The charge density of a spherical charge distribution is given by $\rho(r)=\begin{cases}\left(\frac{\rho_0}{a}\right)r & \text{for } r \le a \\ 0 & \text{for } r>a\end{cases}$

The total charge on the distribution is:

(a) $\frac{2}{3}\pi a^3\rho_0$ (b) $\frac{1}{3}\pi a^3\rho_0$

(c) $2\pi a^3\rho_0$ (d) $\pi a^3\rho_0$

6. Potential at a point A is 3 volt and at a point B is 7 volt, an electron is moving towards A from B.

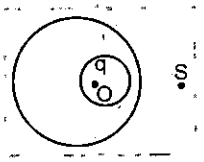
(a) It must have some K.E. at B to reach A

(b) It need not have any K.E. at B to reach A

(c) to reach A is must have more than or equal to 4 eV K.E. at B.

(d) when it will reach A, it will have K.E. more then or at least equal to 4 eV if it was released from rest at B.

7. A charge q is placed at O in the cavity in a spherical uncharged conductor. Point S is outside the conductor. If the charge is displaced from O towards S still remaining within the cavity:



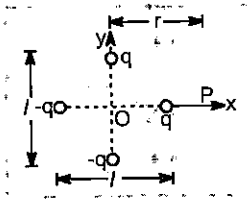
(a) electric field at S will increase

(b) electric field at S decrease

(c) electric field at S will first increase and then decrease

(d) electric field at S will not change

8. Four charges are arranged as shown in figure. A point P is located at distance r from the centre of the configuration. Assume $r >> l$ the field at point P



(a) is of magnitude $\frac{2\sqrt{5}ql}{4\pi\varepsilon_0 r^3}$

(b) is of magnitude $\frac{\sqrt{5}ql}{4\pi\varepsilon_0 r^3}$

(c) makes an angle $\tan^{-1}(2)$ with x-axis

(d) makes an angle $\tan^{-1}\left(\frac{1}{2}\right)$ with x-axis

9. A distance of 5 cm and 10 cm outwards from the surface of a uniformly charged solid sphere, the potentials are 100 V and 75 V respectively. Then:

(a) potential at its surface is 1560 V

(b) the charge on the sphere is $(5/3) \times 10^{10}$C

(c) the electric field on the surface is 1500 V/m

(d) the electric potential at its centre is 225 V

10. A point charge is brought in an electric field. The electric field at a nearby point:

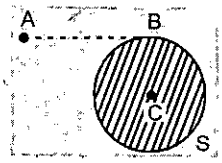
(a) will increase if the charge is positive

(b) will decrease if the charge is negative

(c) may increase if the charge is positive
(d) may decrease if the charge is negative

11. The electric potential decreases uniformly from 120 V to 80 V as one moves on the X-axis from $x = -1$ cm to $x = +1$ cm. The electric field at the origin:
(a) must be equal to $20\frac{V}{cm}$
(b) may be equal to $20\frac{V}{cm}$
(c) may be greater than $20\frac{V}{cm}$
(d) may be less than $20\frac{V}{cm}$

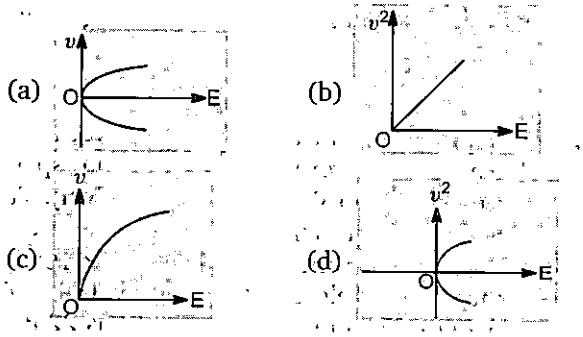
12. S is a solid neutral conducting sphere. A point charge q of 1×10^{-6} C is placed at point A.C is the centre of sphere and AB is a tangent. BC = 3m and AB = 4m:



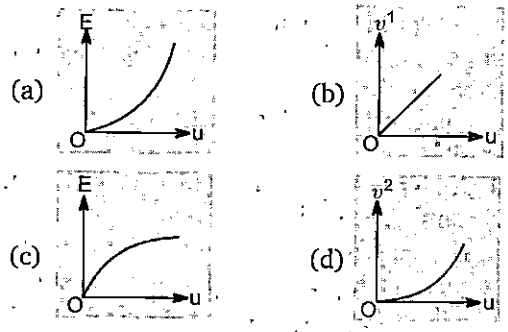
(a) The electric potential of the conductor is 1.8 kV
(b) The electric potential of the conductor is 2.25 kV
(c) The electric potential at B due to induced charges on the sphere is –0.45 kV.
(d) The electric potential at B due to induced charges on the sphere is 0.45 kV.

13. A particle of charge $-q$ and mass m moves in a circle around a long wire of linear charge density $+\lambda$. If r = radius of the circular parth and T = time period of the motion circular path. Then:
(a) $T = 2\pi r(m / 2K\lambda q)^{1/2}$
(b) $T^2 = 4\pi^2 mr^3 / 2qK\lambda$
(c) $T = 1/2\pi r(2K\lambda q / m)^{1/2}$
(d) $T = 1/2\pi r(m / K\pi\lambda q)^{1/2}$
where $K = 1/4\pi\varepsilon_0$

14. If at distance r from a positively charged particle, electric field strength and potential are E and V respectively, which of the following graphs is/are correct?

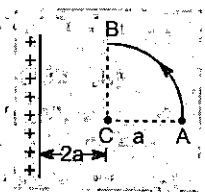


15. If at a distance r from a positively charged particle, electric field strength, potential and energy density are E; V and U respectively, which of the following graphs is/are correct?



16. Two points charges Q and $-\frac{Q}{4}$ are separated by a distance x. Then:
(a) potential is zero at a point on the axis which is $\frac{x}{3}$ on the right side of the charge $-\frac{Q}{4}$
(b) potential is zero at a point on the axis which is $\frac{x}{5}$ on the left side of the charge $-\frac{Q}{4}$
(c) Electric field is zero at a point on the axis which is at a distance x on the right side of the charge $-\frac{Q}{4}$
(d) There exist two points on the axis, where electric field is zero

17. The arc AB with the centre C and the infinitely long wire having linear charge density λ are lying in the same plane. The minimum amount of work to be done to move a point charge q_0 from point A to B through a circular path AB of radius a is equal to:



(a) $\frac{q_0^2}{2\pi\varepsilon_0}\log\left(\frac{2}{3}\right)$
(b) $\frac{q_0\lambda}{2\pi\varepsilon_0}\log\left(\frac{3}{2}\right)$
(c) $\frac{q_0\lambda}{2\pi\varepsilon_0}\log\left(\frac{2}{3}\right)$
(d) $\frac{q_0\lambda}{\sqrt{2}\pi\varepsilon_0}$

18. A charge $+Q$ is uniformly distributed in a spherical volume of radius R. A particle of charge $+q$ and mass m is projected with velocity v_0 from the surface of the spherical volume to its centre. The minimum value of v_0 such that it just reaches the centre (assume that there is no resistance on the particle except electrostatic force) of the spherical volume is:
(a) $\sqrt{\frac{Qq}{2\pi\varepsilon_0 mR}}$
(b) $\sqrt{\frac{Qq}{\pi\varepsilon_0 mR}}$
(c) $\sqrt{\frac{2Qq}{\pi\varepsilon_0 mR}}$
(d) $\sqrt{\frac{1}{4\pi\varepsilon_0 mR}}$

19. A hollow closed conductor of irregular shape is given some charge. Which of the following statements are correct?

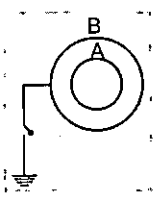
(a) The entire charge will appear on its outer surface.
(b) The points on the conductor will have the same potential.
(c) All points on its surface will have the same charge density.
(d) All points near its surface and outside it will have the same electric intensity.

20. Two points are at distance r_1 and $r_2 (r_1 < r_2)$ from a long string having charge per unit length σ. The potential difference between the points is proportional to:

(a) σ (b) $\log\left(\frac{r_2}{r_1}\right)$

(c) $\frac{1}{\sigma}$ (d) $\frac{r_2}{r_1}$

21. Two concentric shells have radii R and $2R$, charges q_A and q_B and potential $2V$ and $\left(\frac{3}{2}\right)V$ respectively. Now shell B is earthed and let charges on them become q'_A & q'_B. Then:



(a) $\frac{q_A}{q_B} = \frac{1}{2}$

(b) $\frac{q'_A}{q'_B} = 1$

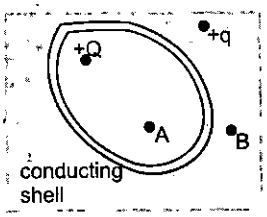
(c) potential of A after earthing becomes $\left(\frac{3}{2}\right)V$

(d) potential difference between A and B after earthing becomes $\frac{V}{2}$

22. An electric field converges at the origin whose magnitude is given by the expression $E = 100$ rNt/Coul, where r is the distance measured from the origin.
(a) total charge contained in any spherical volume with its centre at origin is negative.
(b) total charge contained at any spherical volume, irrespective of the location of its centre, is negative.
(c) total charge contained in a spherical volume of radius 3 cm with its centre at origin has magnitude 3×10^{-13}C.
(d) total charge contained in a spherical volume of radius 3 cm with its centre at origin has magnitude 3×10^{-9} Coul.

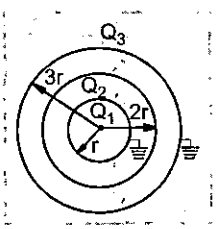
23. Two fixed charges $-2Q$ and $+Q$ are located at points $(-3a, 0)$ and $(+3, 0)$
(a) points where the electric potential due to the two charges is zero, lie on a circle of radius $4a$ and centre $(5a, 0)$
(b) potential is zero at $x = 0$ and $x = 9a$
(c) If a particle of charge $+q$ is released from the centre of the circle obtained in part (a) it will eventually cross the circle
(d) Electric field at origin is along x-axis

24. A point charge $+Q$ lies somewhere inside a closed conducting shell as shown in the figure. Another point charge $+q$ lies outside the shell as shown. There are two points A and B, one inside and other outside the shell. Choose correct alternatives(s):



(a) When charge $+Q$ is shifted from its position keeping it inside the shell then the charge distribution on inner surface of the shell will change and that on outer surface of shell will remain unchanged
(b) When charge $+q$ is shifted from its position keeping it outside the shell then the charge distribution on the inner surface of the shell will remain unchanged and that on outer surface will change
(c) When the charge $+Q$ is shifted from its position keeping it inside the shell then the electric field at B will remain same but that at A will change
(d) When charge $+q$ is shifted from its position keeping it outside the shell then the electric field at B will change but that at A will remain same

25. Three concentric conducting spherical shells have radius r, 3r and 3r and Q_1, Q_2 and Q_3 are final charges respectively. Innermost and outermost shells are already earthed as shown in figure. Select the wrong statement:



(a) $Q_1 + Q_3 = -Q_2$ (b) $Q_1 = \frac{-Q_2}{4}$

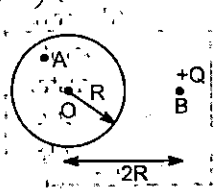
(c) $\frac{Q_3}{Q_1} = 3$ (d) $\frac{Q_3}{Q_2} = \frac{-1}{3}$

26. A point charge $+q$ is projected from point A towards an infinitely long line charge having linear charge density λ with kinetic energy K_0. The distance of the closest approach will be:

(a) $ae^{-\left(\frac{2\pi\varepsilon_0 K_0}{q\lambda}\right)}$ (b) $ae^{-\left(\frac{\pi\varepsilon_0 K_0}{q\lambda}\right)}$

(c) $a\left(1 - \frac{2\pi\varepsilon_0 K_0}{q\lambda}\right)$ (d) $\left(\frac{2\pi\varepsilon_0 K_0}{q\lambda}\right)$

27. A point charge $+Q$ is placed at point B at a distance $2R$ from the center O of an uncharged thin conducting shell of radius R as shown in the figure. If V_A be the potential at point A which is at a radial distance of $\frac{R}{2}$ from the center of the shell, then:

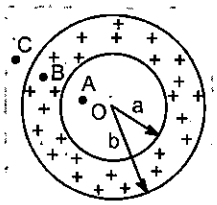


(a) $V_A > \frac{Q}{8\pi\varepsilon_0 R}$ (b) $V_A < \frac{Q}{8\pi\varepsilon_0 R}$

(c) $V_A = \frac{Q}{8\pi\varepsilon_0 R}$ (d) $V_A = 0$

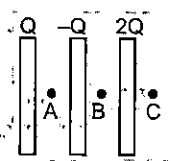
28. A particle of mass m and charge q is thrown in a region where uniform gravitational field and electric field are present. The path of particle:
(a) may be a straight line
(b) may be a circle
(c) may be a parabola
(d) may be a hyperbola

29. A thick spherical shell with its center at O and having inner radius 'a' and outer radius 'b', carries uniformly distributed positive charge as shown in the figure. If V refers to the potential then



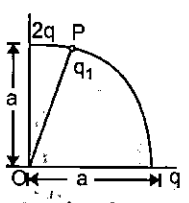
(a) $V_0 = V_A > V_B > V_C$ (b) $V_0 = V_A < V_B < V_C$
(c) $V_0 > V_A > V_B > V_C$ (d) $V_0 = V_A = V_B > V_C$

30. Three large identical conducting parallel plates carrying charge $+Q$, $-Q$ and $+2Q$ respectively are placed as shown in the figure. If E_A, E_B and E_C refer the magnitude of electric field at point A, B and C respectively then:



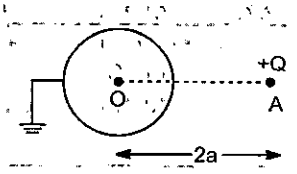
(a) $E_A > E_B > E_C$
(b) $E_A = E_B > E_C$
(c) $E_A = 0$ and $E_B < E_C$
(d) $E_A = 0$ and $E_B = E_C$

31. Two point charges q and 2q are placed at (a, 0) and (0, q). A point charge q_1 is placed at a point P on the quarter circle of radius a as shown in the diagram so that the electric field at the origin becomes zero:



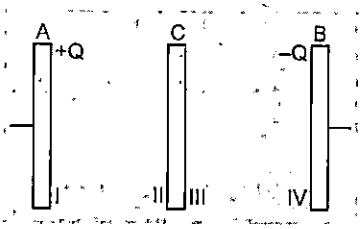
(a) the point P is $\left(\frac{a}{\sqrt{3}}, \frac{\sqrt{2}a}{\sqrt{3}}\right)$
(b) the point P is $\left(\frac{a}{\sqrt{5}}, \frac{2a}{\sqrt{5}}\right)$
(c) $q_1 = -5q$
(d) none of these

32. A point charge is placed at a distance $2a$ from the center of a thin conducting uncharged spherical shell A of radius 'a' as shown in the figure. Then the charge on the shell will be:



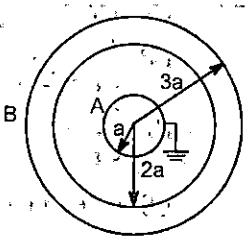
(a) $-\frac{Q}{2}$ (b) Q
(c) $-Q$ (d) Zero

33. Plates A and B constitute an isolated, charge parallel-plate capacitor. The inner surface (I and IV) of A and B have charges $+Q$ and $-Q$ respectively. A third plate C with charge $+Q$ is now introduced midway between A and B. Which of the following statements is not correct?



(a) The surface I and II will have equal and opposite charges.
(b) The surfaces, III and IV will have equal and opposite charges.
(c) The charges on surface III will be greater than Q.
(d) The potential difference between A and C will be equal to the potential difference between C and B.

34. An uncharged thin conducting spherical shell A of radius 'a' is placed concentrically with a thick conducting spherical shell B of inner radius $2a$ and outer radius $3a$ as shown in the figure. If the shell B is given a total charge $+Q$, then:



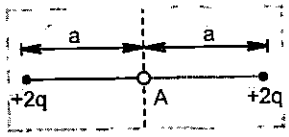
(a) Charge on shell A will be $-\frac{Q}{5}$
(b) Charge on shell A will be $-\frac{2Q}{5}$
(c) Charge on outer surface of shell B will be $\frac{3Q}{5}$
(d) Charge on outer surface of shell B will be $\frac{Q}{5}$

35. The electric potential in a region is given by the relation $V(x) + 4 + 5x^2$. If a dipole is placed at position $(-1, 0)$ with dipole moment $\vec{\mathbf{P}}$ pointing along positive Y-direction, then:
(a) Net force on the dipole is zero
(b) Net torque on the dipole is zero
(c) Net torque on the dipole is not zero and it is clockwise direction
(d) Net torque on the dipole is not zero and it is anticlockwise direction

36. Mid way between the two equal and similar charges, e placed the third equal and similar charge. Which of the following statements is correct, concerned to the equilibrium along the lien joining the charges?
(a) The third charge experienced a net force inclined to the line joining the charges

(b) The third charge is in stable equilibrium
(c) The third charge is in unstable equilibrium
(d) The third charge experiences a net force perpendicular to the line joining the charges.

37. A negative point charge placed at the point A is:



(a) in stable equilibrium along x-axis
(b) in unstable equilibrium along y-axis
(c) in stable equilibrium along y-axis
(d) in unstable equilibrium along x-axis

38. Two fixed charges $4Q$ (positive) and Q (negative) are located at A and B, the distance AB being 3 m:

(a) The point P where the resultant field due to both is zero is on AB outside AB.
(b) The point P where the resultant field due to both is zero is on AB inside AB.
(c) If a positive charge is placed at P and displaced slightly along AB it will execute oscillations.
(d) If a negative charge is placed at P and displaced slightly along AB is will execute oscillations.

39. An electric field is given by $\vec{\mathbf{E}} = (y\hat{\mathbf{i}} + x\hat{\mathbf{j}})\frac{\text{N}}{\text{C}}$. The work done in moving a 1 C charge from $\vec{\mathbf{r}}_A = (2\hat{\mathbf{i}} + 2\hat{\mathbf{j}})$m to $\vec{\mathbf{r}}_B = (4\hat{\mathbf{i}} + \hat{\mathbf{j}})$m is:

(a) + 4 J (b) − 4 J
(c) + 8 J (d) zero

40. Select the correct statement: (Only force on a particle is due to electric field)
(a) A charged particle always moves along the electric line of force.
(b) A charged particle may move along the line of force
(c) A charged particle never moves along he line of force
(d) A charged particle moves along the line of force only if released from rest.

41. The electric potential decreases uniformly from V to –V along x-axis is a coordinate system as we moves from a point $(-x_0, 0)$ to $(x_0, 0)$, then the electric field at the origin.
(a) must be equal to $\frac{V}{x_0}$;
(b) may be equal to $\frac{V}{x_0}$
(c) must be greater than $\frac{V}{x_0}$
(d) may be less than $\frac{V}{x_0}$

42. The electric potential decreases uniformly from 120 V to 80 V as one moves on the x-axis from $x = -1$ cm to $= +1$ cm. The electric field at the origin:
(a) must be equal to 20 V/cm
(b) may be equal to 20 V/cm
(c) may be greater than 20 V/cm
(d) may be less than 20 V/cm

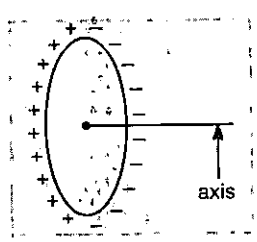
43. A thin wire ring of radius r has an electric charge q. A point charge q_0 is placed at the rings centre. The increment of the force stretching the wire is:
(a) $\frac{qq_0}{8\pi^2\varepsilon_0 r^2}$ (b) $\frac{qq_0}{4\pi^2\varepsilon_0 r^2}$
(c) $\frac{qq_0}{2\pi^2\varepsilon_0 r^2}$ (d) $\frac{qq_0}{\pi^2\varepsilon_0 r^2}$

44. A ring of radius R carries charge Q distributed uniformly over this ring. P is a point on its axis, at a distance r from its centre. the electric field at P due to ring is E. Which of the following is correct?
(a) $E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Qr}{(r^2+R^2)^{3/2}}$
(b) E is maximum for $r = R/\sqrt{2}$
(c) $E \neq 0$ at the centre of the ring.
(d) As r increases, E will first increase, then decrease.

45. A conducting sphere of radius r has a charge. Then
(a) The charge is uniformly distributed over its surface, if there is an external electric field.
(b) Distribution of charge over its surface will be non uniform if no external electric field exist in space.
(c) Electric field strength inside the sphere will be equal to zero only when no external electric field exists
(d) Potential at every point of the sphere must be same

46. For a spherical shell
(a) If potential inside it is zero then it necessarily electrically neutral
(b) electric field in a charge d conducting spherical shell can be zero only when the charge is uniformly distributed.
(c) electric potential potential due to induced charges at a point inside it will always be zero
(d) none of these

47. The figure shows a nonconducting ring which has positive and negative charge non uniformly distributed on it such that the total charge is zero.



(a) The potential at all the points on the axis will be zero.
(b) The electric field at all the points on the axis will be zero.

(c) The direction of electric field at all a points on the axis will be along the axis.
(d) If the ring is placed inside a uniform external electric field then net torque and force acting on the ring would be zero.

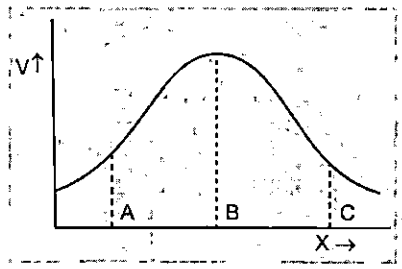
48. A ring of radius R is made out of a thin metallic wire of area of cross section A. The ring has a uniform charge Q distributed on it. A charge q_0 is placed at the centre of the ring. If Y is the young's modulus for the material of the ring and ΔR is the change in the radius of the ring then:

(a) $\Delta R = \dfrac{q_0 Q}{4\pi\varepsilon_0 RAY}$ (b) $\Delta R = \dfrac{q_0 Q}{4\pi^2\varepsilon_0 RAY}$

(c) $\Delta R = \dfrac{q_0 Q}{8\pi^2\varepsilon_0^2 RAY}$ (d) $\Delta R = \dfrac{q_0 Q}{8\pi\varepsilon_0 RAY}$

49. Four identical charges are placed at the point (1, 0, 0), (0, 1, 0), (–1, 0, 0) and (0, –1, 0):
(a) The potential at the origin is zero
(b) The field at the origin is zero
(c) The potential at all points on the z-axis, other than the origin, is zero
(d) The field at all points on the z-axis, other than the origin acts along the z-axis.

50. Variation of electrostatic potential along x-direction is shown in the graph. The correct statement about electric field is:



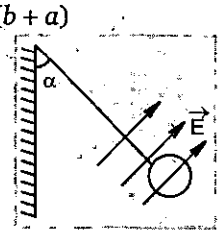
(a) x component at point B is maximum
(b) x component at point A is towards positive x-axis
(c) x component at point C is along negative x-axis
(d) x component at point C is along positive x-axis

51. A charge Q is uniformly distributed over a circular annulus of inner radius 'a' and outer radius 'b'. Taking potential at infinity to be zero, the potential at the center of the annulus will be given by:

(a) $\dfrac{Q}{4\pi\varepsilon_0 b}$ (b) $\dfrac{Q}{4\pi\varepsilon_0 a}$

(c) $\dfrac{Q}{2\pi\varepsilon_0(b-a)}$ (d) $\dfrac{Q}{2\pi\varepsilon_0(b+a)}$

52. A charged cork of mass ma suspended by a light string is placed in uniform electric field of strength $E = (\hat{\mathbf{i}} + \hat{\mathbf{j}}) \times 10^5 \text{NC}^{-1}$ as shown in the fig. If in equilibrium



position tension in the string is $\dfrac{2mg}{(1+\sqrt{3})}$ then angle 'α' with the vertical is:

(a) 60° (b) 30°
(c) 45° (d) 18°

53. Two particles of same mass and charge are thrown in the same direction along the horizontal with same velocity v from two different heights h_1 and $h_2 (h < h_2)$. Initially they were located on the same vertical line. Choose the correct statements:
(a) Both the particle will always lie on a vertical line
(b) Acceleration of the centre of mass of two particles will be g downwards
(c) Horizontal displacement of the particle lying at h_1 is less and the particle lying at h_2 is more than the value, which would had been in the absence of charges on them.
(d) all of these

54. If the flux of the electric field through a closed surface is zero:
(a) the electric field must be zero every where on the surface
(b) the electric field may be zero everywhere on the surface
(c) the charge inside the surface must be zero
(d) the charge in the vicinity of the surface must be zero

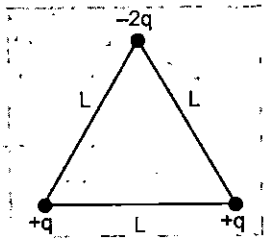
55. Change Q is distributed non-uniformly over a ring of radius R, P is a point on the axis of ring at a distance $\sqrt{3}R$ from its centre. Which of the following is a wrong statement.
(a) Potential at P is $\dfrac{KQ}{2R}$
(b) Magnitude of electric field at P may be greater than $\dfrac{\sqrt{3}KQ}{8R^2}$
(c) Magnitude of electric field at P must be equal to $\dfrac{\sqrt{3}KQ}{8R^2}$
(d) Magnitude of electric field at P cannot be less than $\dfrac{\sqrt{3}KQ}{8R^2}$

56. An electric dipole moment $\vec{\mathbf{p}} = (2.0\hat{\mathbf{i}} + 3.0\hat{\mathbf{j}})\mu\text{C}$. m is placed in a uniform electric field $\vec{\mathbf{E}} = (3.0\hat{\mathbf{i}} + 2.0\hat{\mathbf{k}}) \times 10^5 \text{NC}^{-1}$.
(a) The torque that $\vec{\mathbf{E}}$ exerts on $\vec{\mathbf{p}}$ is $(0.6\hat{\mathbf{i}} - 4.0\hat{\mathbf{j}} - 0.9\hat{\mathbf{k}})\text{Nm}$
(b) The potential energy of the dipole is – 0.6 J.
(c) The potential energy of the dipole is 0.6 J.
(d) If the dipole is rotated in the electric field, the maximum potential energy of the dipole is 1.3 J.

57. Select the correct options:
(a) Gauss's law is valid only for uniform charge distributions.
(b) Gauss's law is valid only for charges placed in vacuum.
(c) The electric field calculated by Gauss's law is the field due to all the charges.
(d) The flux on the electric field through a closed surface due to all the charges is equal to the flux due to the charges enclosed by the surface.

58. A charge of mass 50 mg and carrying 5×10^{-9} C is approaching a fixed charge of 10^{-8} C with a velocity of 0.5 m/s. The distance of closest approach of the charge is:
(a) 7.2 cm (b) 8.6 cm
(c) 4.2 cm (d) 1.2 cm

59. Three points charges are placed at the corners of an equilateral triangle of side L as shown in the figure:

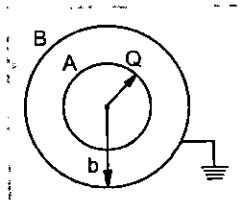


(a) The potential at the centroid of the triangle is zero.
(b) The electric field at the centroid of the triangle is zero.
(c) The dipole moment of the system is $\sqrt{2}qL$
(d) The dipole moment of the system is $\sqrt{2}qL$.

60. An electric dipole is placed at the centre of a sphere. Select the correct statement:
(a) the flux of the electric field through the sphere is zero
(b) the electric field is zero at every point of the sphere
(c) the electric potential is zero everywhere on the sphere
(d) the electric potential is zero on a circle on the surface

61. Two free point charges $+4Q$ and $+Q$ are placed at a distance r. A third charge q is so placed such that all the three are in equilibrium then:
(a) q is placed at a distance $\frac{1}{3}r$ from $4Q$
(b) q is placed at a distance $\frac{1}{3}r$ from Q
(c) $q=\frac{4Q}{9}$ (d) $q=-\frac{4Q}{9}$

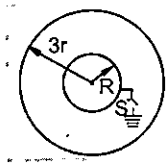
62. A conducting sphere A of radius a, with charge Q, is placed concentrically inside a conducting shell B of radius b. B is earthed C is the common centre of the A and B :



(a) The field is a distance r from C, where $a \le r \le b$ is $\frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$
(b) The potential at a distance r from C, where $a \le r \ge b$ is $\frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$
(c) The potential difference between A and B is $\frac{1}{4\pi\varepsilon_0}Q\left(\frac{1}{a}-\frac{1}{b}\right)$
(d) The potential at a distance r from C, where $a \le r \le b$, $\frac{1}{4\pi\varepsilon_0}Q\left(\frac{1}{r}-\frac{1}{b}\right)$

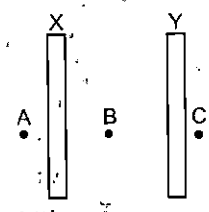
63. Uniform electric field exists in a region and is given by $\vec{E}=E_0\hat{i}+E_0\hat{j}$. There are four points $A(-a,0), B(0,-a,), C(a,0)$ and $D(0,a)$ lying $x-y$ plane. Which of the following is the correct relation for the electric potential:
(a) $V_A=V_C>V_B=V_D$ (b) $V_A=V_B>V_C=V_D$
(c) $V_A>V_C>V_B=V_D$ (d) $V_A<V_C<V_B<V_D$

64. Two thin conducting shells of radii R and 3R are shown in the figure. The outer shell carries a charge $+Q$ and the inner shell is neutral. The inner shell is earthed with the help of a switch S.:



(a) With the switch S open, the potential of the inner sphere is equal to that of the outer.
(b) When the switch S is closed, the potential of the inner sphere becomes zero.
(c) With the switch S closed, the charge attained by the inner sphere is $-q/3$
(d) By closing the switch the capacitance of the system increases.

65. X and Y are large, parallel conducting plates closed to each other. Each face has an area A. X is given a charge Q. Y is without any charge. Points A, B and C are as shown in figure:



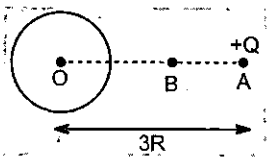
(a) The field at B is $\frac{Q}{2\varepsilon_0 A}$
(b) The field B is $\frac{Q}{\varepsilon_0 A}$
(c) The field at A, B and C are the same magnitude.
(d) The field at A and C are of the same magnitude, but in opposite directions.

66. A small positive charge located at origin experiences an electrostatic force directed along x-axis. One can conclude that at origin (V refers to potential):

(a) $\frac{\partial V}{\partial x} \neq 0$ (b) $\frac{\partial^2 V}{\partial^2 x} \neq 0$

(c) $\frac{\partial V}{\partial y} = 0$ (d) $V \neq 0$

67. A point charge $+Q$ is placed at point A at a distance $3R$ from the center O of an uncharged thin conducting shell of radius R as shown in figure, then the potential (V_B) at point B will be:



(a) $\frac{Q}{4\pi\varepsilon_0 R}$ (b) $> \frac{Q}{4\pi\varepsilon_0 R}$

(c) $< \frac{Q}{4\pi\varepsilon_0 R}$ (d) Zero

68. An oil drop has a charge -9.6×10^{-19} Coul and has a mass 1.6×10^{-15} gm. When allowed to fall, due to air resistance it attains a constant velocity. Then if a uniform electric field is to be applied vertically to make the oil drop ascend up with the same constant speed, which of the following are correct?

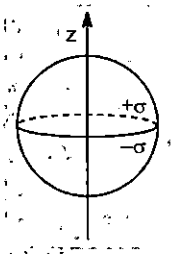
(a) the electric field is directed upward

(b) the electric field is directed downward

(c) the intensity of the electric field is $\frac{1}{3}\times10^2$ Nt/Coul

(d) the intensity of the electric field is $\frac{1}{6}\times10^5$ Nt/Coul

69. A hollow, insulating spherical shell has a surface charge distribution placed upon it, such that the upper hemisphere has a uniform surface charge density $+\sigma$, while the lower hemisphere has a uniform surface charge density $-\sigma$, as shown in the figure. Their interface lies in x-y plane. Which of the following statement(s) is/are correct ?



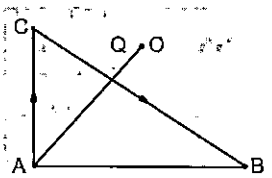
(a) The field at all points of x-y plane within the sphere points in the $-$ ve z-direction

(b) All points of the x-y plane within the sphere are equipotential.

(c) The field at all points on z-axis outside the sphere point along positive z-direction

(d) The field at points on z-axis which are on either side of origin outside the sphere is in opposite directions.

70. There is a fixed positive charge Q at O and A and B are points equidistant from O. A positive charge $+q$ is taken slowly by an external agent from A to B along the line AC and then along the line CB.



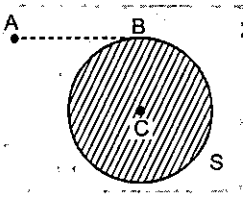
(a) The total work done on the charge is zero.

(b) The work done by the electrostatic force from A to C is negative

(c) The work done by the electrostatic force from C to B is positive

(d) The work done by electrostatic force in taking the charge from A to B is dependent on the actual path.

71. S is a solid neutral conducting sphere. A point charge q of 1×10^{-6}C is placed at point A. C is the centre of sphere and AB is a tangent. BC = 3m and AB = 4m.



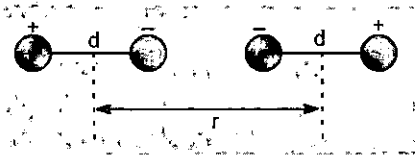
(a) The electric potential of the condctor is 1.8 kV

(b) The electric potential of the conductor is 2.25 kV

(c) The electric potential at B due to induced charges on the sphere is –0.45 kV

(d) The electric potential at B due to induced charges on the sphere is 0.45 kV.

72. We have two electric dipoles. Each dipole consists of two equal and opposite point charges at the ends of an insulating rod of length d. The dipoles sit along the x-axis at a distance r apart, oriented as shown:



Their separation $r >> d$. The dipole on the left:

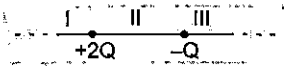
(a) will feel a force to the left.

(b) will feel a force to the right.

(c) will feel a torque trying to make it rotate counterclockwise.

(d) will feel no torque

73. The figure shows, two point charges $q_1 = 2Q\,(>0)$ and $q_2 = -Q$. The charges divide the line joining them in three parts I, II and III:



(a) Region III has a local maxima of electric field

(b) Region I has a local minima of electric field

(c) Equilibrium position for a test charge lies in region II

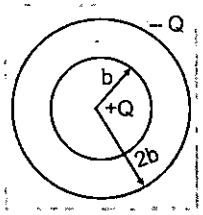
(d) The equilibrium for constrained motion along the line joining the charges is stable for a negative test charge

74. Three equal point changes (Q) are kept at the three corners of an equilateral triangle ABC of side a. P is a point having equal distance a from A, B and C. If E is the magnitude of electric field and V is the potential at point P, then

(a) $E = \frac{3Q}{4\pi\varepsilon_0 a^2}$ (b) $E = \frac{\sqrt{6}Q}{4\pi\varepsilon_0 a^2}$

(c) $V = \frac{3Q}{4\pi\varepsilon_0 a}$ (d) $E = \frac{3\sqrt{6Q}}{4\pi\varepsilon_0 a^2}$

75. The accompanying figure shows two concentric spherical shells isolated from each other. The smaller shell has radius b and net charge $+Q$. The larger shell has radius $2b$ and net charge $-Q$. If R is the distance from the common center:



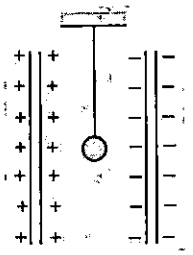
(a) the highest electric field magnitude E occurs immediately outside the smaller ($R = b$) shell
(b) the highest electric field magnitude E occurs immediately outside the larger ($R = 2b$) shell
(c) At $R = b$ potential is maximum
(d) At $R = 0$ potential is maximum answers

76. Select the correct statement(s) w.r.t. charge:
(a) The additive property of charge is not an obvious property, but is related to the fact that charge is a scalar physical quantity.
(b) Charge is invariant *i.e.,* its value is same in different frames of reference having relative motion.
(c) Charge is conserved for an electrically isolated system, this can be concluded from the scalar nature of charge.
(d) Charge is conserved for an electrically isolated system, this can't be concluded from scalar nature of charge.

77. Select the correct statements w.r.t. electric field intensity due to various charge configuration:
(a) $\vec{\mathbf{E}}$ due to discrete point charge is not defined at the location of discrete point charge.
(b) $\vec{\mathbf{E}}$ due to volume charge distribution can be defined at any point.
(c) $\vec{\mathbf{E}}$ due to surface charge distribution is discontinuous at surface.
(d) $\vec{\mathbf{E}}$ due to line charge distribution is not defined at any point on line charge itself.

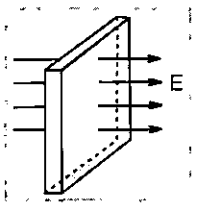
78. A Gaussian surface:
(a) must not pass through any discrete charge.
(b) must pass through any material medium whether conducting or non-conducting.
(c) may pass through a continuous charge distribution
(d) may pass through a region where no material medium is present.

79. A small metallic uncharged object is suspended between two vertical metal plates having equal and opposite charges, as shown. Due to the charges on plates a uniform electric field appears between the plates. Now, the ball is touched to one of the plates and then released. For this arrangement, Select the correct statements:



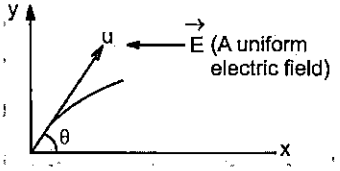
(a) Ball moves to and fro between the plates and in both to and fro motion, it is accelerated (speeding up).
(b) The charge of both the plates is continuously decreasing and becomes neutral at infinite time.
(c) The charge of the system remains conserved and it is only transferred between the plates with the help of ball.
(d) Finally, +ve plate acquires –ve charge and –ve plate acquires +ve charge, but magnitude of charge on both the plates still remains the same.

80. An uncharged conducting thin square plate of side 1 m is placed in an uniform electric field of magnitude 200 N/C as shown in the figure. The field direction is perpendicular to the plane of plate. For this situation, mark out the correct statement(s). [In the options only big faces has been considered]:



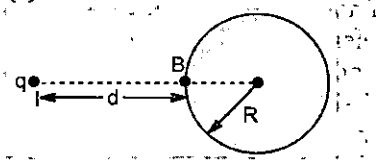
(a) The net charge on each face of the plate is zero.
(b) The net charge on each face of the plate is non-zero and having the magnitude equal to 1.77×10^{-9} C.
(c) The net electric field intensity inside the plate is zero.
(d) The electric field intensity at the surface of plate is discontinuous while potential is continuous.

81. A particle of mass m and charge $-q$ has been projected from ground as shown in the figure. Mark out the correct statement(s).



(a) The path of motion of the particle is parabolic
(b) The path of motion of the particle is a straight line.
(c) Time of flight of the particle is $\frac{2u\sin\theta}{g}$.
(d) Range of motion of the particle can be less than, greater than or equal to $\frac{u^2\sin 2\theta}{g}$.

82. For the situation shown in the figure, select the correct statement(s):



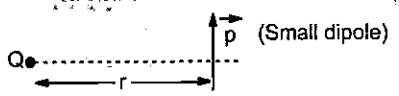
(a) Potential of the conductor is $\frac{q}{4\pi\varepsilon_0(d+R)}$.

(b) Potential of the conductor is $\frac{q}{4\pi\varepsilon_0 d}$.

(c) Potential of the conductor can't be determined as nature of distribution of induced charges is not known.

(d) Potential at point B due to induced charges is $\frac{-qR}{4\pi\varepsilon_0(d+R)d}$.

83. For the situation shown in the figure (assume $r >>$ length of dipole) select the correct statement(s):



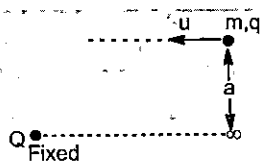
(a) Force acting on the dipole is zero.

(b) Force acting on the dipole is approximately $\frac{pQ}{4\pi\varepsilon_0 r^3}$ and is acting upwards.

(c) Torque acting on the dipole is $\frac{pQ}{4\pi\varepsilon_0 r^2}$ in clockwise direction.

(d) Torque acting on the dipole is $\frac{pQ}{4\pi\varepsilon_0 r^2}$ in anti-clockwise direction.

84. A charge particle q is projected in an electric field produced by a fixed point charge Q as shown in the figure.



Select the correct statements:

(a) The path taken by q is a straight line.

(b) The path taken by q is not a straight line.

(c) The minimum distance between the two particles is $\frac{\frac{qQ}{2\pi\varepsilon_0}+\sqrt{\left(\frac{qQ}{2\pi\varepsilon_0}\right)^2+4m^2u^4a^2}}{2mu^2}$

(d) Velocity of the particle of charge q is changing in magnitude and direction both.

85. A spherical shell is uniformly charged by a charge q. A point charge q_0 is placed at its centre. The expansion of the shell is taking place from R_1 to R_2 ($R_2 > R_1$). For this, situation select the correct statement(s):

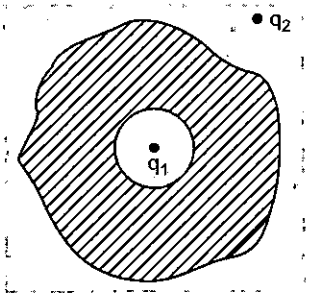
(a) If an external force is acting, then work done by the external agent is negative.

(b) If no external force is acting, then energy would be released in this expansion.

(c) If no external force is acting, then energy would be dissipated in this process.

(d) The work of electric forces in this process is $\frac{q(q_0+q/2)}{4\pi\varepsilon_0}\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$.

86. A point charge q_1 is placed in a cavity inside a conductor and another point charge q_2 is placed outside it as shown in Fig. Which of the following statements are correct ?



(a) If q_1 is slightly shifted, the induced charge density on the inner surface of the cavity only changes

(b) If q_1 is slightly shifted, the induced charge density on the outer surface of the cavity only changes

(c) If q_2 is slightly shifted, the induced charge density on the inner surface of the cavity only changes

(d) If q_2 is slightly shifted, the induced charge density on the outer surface of the cavity only changes

87. Select the correct statements :

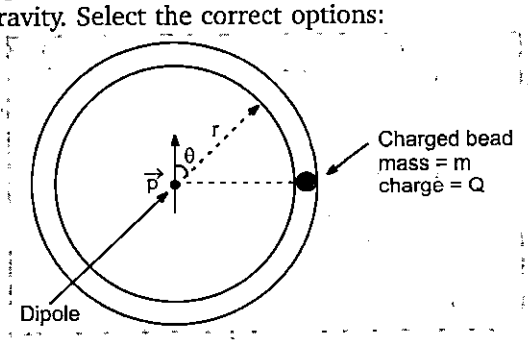
(a) If a point charge is placed off-centre inside an electrically neutral spherical metal shell then induced charge on its inner surface is uniformly distributed.

(b) If a point charge is placed off-centre inside an electrically neutral, isolated spherical metal shell, then induced charge on its outer surface is uniformly distributed.

(c) A non-metal shell of uniform charge attracts or repels a charged particle that is outside the shell as if all the shell's charge were concentrated at the centre of the shell.

(d) If a charged particle is located inside a non-metal shell of uniform charge, there is no electrostatic force on the particle due to the shell.

88. A small charged bead can slide on a circular frictionless, insulating wire frame. A point like dipole is fixed at the centre of circle, dipole moment is $\vec{p}$. Initially the bead is on the plane of symmetry of the dipole. Bead is released from rest. Ignore the effect of gravity. Select the correct options:



(a) Magnitude of velocity of bead as function of its angular position is $\sqrt{\frac{Qp\cos\theta}{2\pi\varepsilon_0 mr^2}}$

(b) Normal force exerted by the string on bead is zero

(c) If the wire frame were not present bead executes circular motion and returns to initial point after tracing a complete circle.

(d) Bead would move along a circular path until it reached the opposite its starting position and then executes periodic motion

89. Which statement about a system of point charges that are fixed in space is necessarily true? Assuming electrostatic potential energy at infinity to be zero

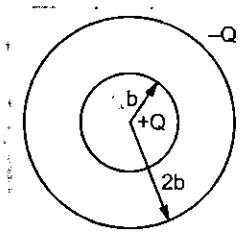
(a) if the electrostatic potential energy of the system is negative, net positive work by an external agent is required to take the charges in the system back to infinity.

(b) if the electrostatic potential energy of the system is zero, all charges in the configuration cannot have same sign.

(c) if the electrostatic potential energy of the system is negative, net positive work by an external agent was required to assemble the system of charges

(d) if the electrostatic potential energy of the system is negative, then there is no electric force anywhere in space on any other charged particle not part of the system

90. The accompanying figure shows two concentric spherical shells isolated from each other. The smaller shell has radius b and net charge $+Q$. The larger shell has radius $2b$ and net charge $-Q$. If R is the distance from the common center:



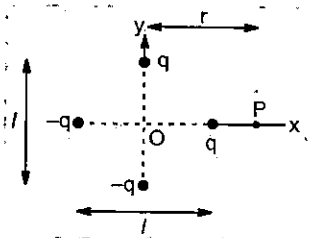
(a) the highest electric field magnitude E occurs immediately outside the smaller ($R = b$) shell

(b) the highest electric field magnitude E occurs immediately outside the larger ($R = 2b$) shell

(c) At $R = b$ potential is maximum

(d) At $R = 0$ potential is maximum

91. Four charges are arranged as shown in Fig. A point P is located at distance r from the center of the configuration. Assume $r >> l$ the field at point P:



(a) is of magnitude $\frac{2\sqrt{5}ql}{4\pi\varepsilon_0 r^3}$

(b) is of magnitude $\frac{\sqrt{5}ql}{4\pi\varepsilon_0 r^3}$

(c) makes an angle $\tan^{-1}(2)$ with x-axis

(d) makes an angle $\tan^{-1}\left(\frac{1}{2}\right)$ with x-axis

92. An electric dipole moment $\vec{p}=(2\hat{i}+3\hat{j})\mu$cm is placed in a uniform electric field $\vec{E}=(3\hat{i}+2\hat{k})\times 10^5$ N/C:

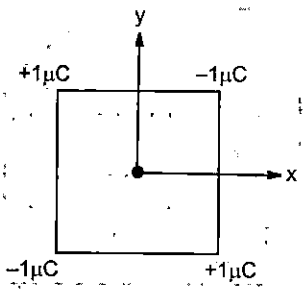
(a) the torque that $\vec{E}$ exerts on P, is $(0.6\hat{i}-0.4\hat{j}-0.9\hat{k})$ Nm

(b) the potential energy of the dipole is -0.6 J

(c) the potential energy of the dipole is 0.6 J

(d) if the dipole is rotated in the electric field, the maximum potential energy of the dipole is 1.3 J

93. Point charges are located on the corner of a square as shown. Find the components of electric field at any point on the z-axis which is axis of symmetry of the square:



(a) $E_z = 0$ (b) $E_x = 0$

(c) $E_y = 0$ (d) none of these

94. An electric dipole is placed in an electric field generated by an infinitely long uniformly charged wire:

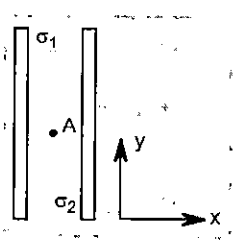
(a) The net electric force on the dipole must be zero

(b) The net electric force on the dipole may be zero

(c) The torque on the dipole due to the field must be zero

(d) The torque on the dipole due to the field may be zero

95. Two large conducting sheets are kept parallel to each other as shown in Fig. In equilibrium, the charge density on facing surfaces is σ_1 and σ_2. What is the value of electric field at A:

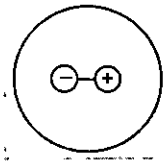


(a) $\frac{\sigma_1}{\varepsilon_0}\hat{i}$

(b) $-\frac{\sigma_2}{\varepsilon_0}\mathbf{i}$

(c) $\frac{\sigma_1+\sigma_2}{2\varepsilon_0}\mathbf{i}$ (d) $\frac{\sigma_1-\sigma_2}{2\varepsilon_0}\mathbf{i}$

96. Consider a Gaussian spherical surface, covering a dipole of charge q and $-q$, then:



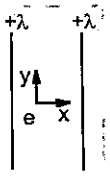
(a) $q_{in} = 0$ (Net charge enclosed by the spherical surface)

(b) $\phi_{net} = 0$(Net flux coming out the spherical surface)

(c) $E = 0$ at all points on the spherical surface

(d) $\int \vec{E} \cdot d\vec{s} = 0$ (Surface integral of $\vec{E}$ over the spherical surface)

97. An electron is placed just in the middle between two long fixed line charges of charge density $+\lambda$ each.



The wires are in the x-y plane (Do not consider gravity):

(a) The equilibrium of the electron will be unstable along x-direction

(b) The equilibrium of the electron will be neutral along y-direction

(c) The equilibrium of the electron will be stable along z-direction

(d) The equilibrium of the electron will be stable along y-direction

98. Select the correct statements:

(a) If a point charge is placed off-centre inside an electrically neutral spherical metal shell then induced charge on its inner surface is uniformly distributed.

(b) If a point charge is placed off-centre inside an electrically neutral, isolated spherical metal shell, then induced charge on its outer surface is uniformly distributed.

(c) A non-metal shell of uniform charge attracts or repels a charged particle that is outside the shell as if all the shell's charge were concentrated at the centre of the shell.

(d) If a charged particle is located inside a non-metal shell of uniform charge, there is no electrostatic force on the particle due to the shell.

ANSWER

More than One Alternatives are Correct

1.	(a)	**2.**	(d)	**3.**	(a, c)	**4.**	(a, c)	**5.**	(d)	**6.**	(a, c)
7.	(d)	**8.**	(b, d)	**9.**	(a, c, d)	**10.**	(c, d)	**11.**	(b, c)	**12.**	(a, c)
13.	(a)	**14.**	(b, c)	**15.**	(c)	**16.**	(a, b, c)	**17.**	(b)	**18.**	(d)
19.	(a, b)	**20.**	(a, b)	**21.**	(a, d)	**22.**	(a, b, c)	**23.**	(a, c, d)	**24.**	(a, b, c, d)
25.	(d)	**26.**	(a)	**27.**	(c)	**28.**	(a, c)	**29.**	(a)	**30.**	(d)
31.	(b)	**32.**	(a)	**33.**	(d)	**34.**	(a, c)	**35.**	(a, d)	**36.**	(b)
37.	(c, d)	**38.**	(a, d)	**39.**	(d)	**40.**	(b)	**41.**	(b)	**42.**	(b, c)
43.	(a, c)	**44.**	(a, b, c)	**45.**	(d)	**46.**	(d)	**47.**	(a)	**48.**	(d)
49.	(a, b)	**50.**	(d)	**51.**	(d)	**52.**	(a, b)	**53.**	(d)	**54.**	(b, c)
55.	(c)	**56.**	(a, b, d)	**57.**	(c, d)	**58.**	(a)	**59.**	(a, d)	**60.**	(a, d)
61.	(b, d)	**62.**	(a, c, d)	**63.**	(b)	**64.**	(a, b, c, d)	**65.**	(a, c, d)	**66.**	(a)
67.	(c)	**68.**	(b, c)	**69.**	(a, b, c)	**70.**	(a, b, c)	**71.**	(a, c)	**72.**	(a, d)
73.	(a, d)	**74.**	(b, c)	**75.**	(a, c, d)	**76.**	(a, b, d)	**77.**	(a, b, c, d)	**78.**	(a, c, d)
79.	(a, b, c)	**80.**	(b, c, d)	**81.**	(b, c)	**82.**	(a, d)	**83.**	(b, c)	**84.**	(b, c, d)
85.	(a, b, d)	**86.**	(a, d)	**87.**	(b, c, d)	**88.**	(a, b, d)	**89.**	(a, b)	**90.**	(a, c, d)
91.	(b, d)	**92.**	(a, b, d)	**93.**	(a, b, c)	**94.**	(b, d)	**95.**	(a, b)	**96.**	(a, b, d)
97.	(a, b, c)	**98.**	(b, c, d)								

More than One Alternatives are Correct

1. [b] By conservation of energy

$$\frac{1}{2}kr^2 = \frac{1}{4\pi\varepsilon_0}\left(\frac{q^2}{r} - \frac{q^2}{r+r}\right) = \frac{q^2}{8\pi\varepsilon_0 r}$$

$$\Rightarrow \quad k = \frac{q^2}{4\pi\varepsilon_0 r^3}$$

2. [a] Let the third charge $-q$ is placed at a distance x from charge $2q$, then

$$V = K\left[\frac{(2q)(8q)}{r} - \frac{(2q)(q)}{x} - \frac{(8q)q}{r-x}\right]$$

V will be minimum /maximum.

$$\frac{dV}{dx} = 0 \Rightarrow \frac{-2}{x^2} + \frac{8}{(r-x)^2} = 0$$

$$\frac{x}{r-x} = \sqrt{\frac{2}{8}} = \frac{1}{2} \Rightarrow x = \frac{r}{3}$$

for $x = \frac{r}{3}$ $\frac{\partial V^2}{\partial x^2}$ is negative so no minima exist.

4. [a, c]

$\vec{E}_A$ is along $\overrightarrow{OA}$ and $\vec{E}_B$ is along $\overrightarrow{OB}$

$$\overrightarrow{OA} = \hat{i} + 2\hat{j} + 3\hat{k}$$

and $$\overrightarrow{OB} = \hat{i} + \hat{j} - \hat{k}$$

Since $\overrightarrow{OA}.\overrightarrow{OB} = 0 \quad \therefore \vec{E}_A \perp \vec{E}_B$

Further $|\vec{E}| \propto \frac{1}{r^2} \Rightarrow |\overrightarrow{OC}| = 2|\overrightarrow{OB}|$

$$\vec{E}_B = 4\vec{E}_C$$

5. [d] $$Q = \int_0^a 4\pi r^2 \rho dr$$

$$\int_0^a 4\pi r^2 \frac{P_0}{a} r dr = \pi\rho_0 a^3$$

8. [b, d]

Point P lies on the axial line for one dipole and on the equilateral line for the other.

Hence,

$$E_{axial} = \frac{2ql}{4\pi^2\varepsilon_0 r^3} \quad \text{along positive } x\text{-axis}$$

$$E_{equilatorial} = \frac{ql}{4\pi\varepsilon_0 r^3} \quad \text{along positive } y\text{-axis}$$

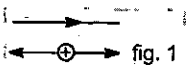
$$E_{net} = \sqrt{E_{axial}^2 + E_{equilatorial}^2}$$

$$= \sqrt{5}\frac{ql}{4\pi\varepsilon_0 r^3}$$

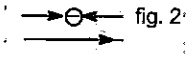
at an angle $\alpha = \tan^{-1}\left(\frac{1}{2}\right)$ below x-axis.

10. [c, d]

In figure 1, the field on left of charge decreases while on right increases.



In figure 2, field on left increases and on right decreases.



11. [b, c] $$\vec{E}_x = -\frac{\partial v}{\partial x}\hat{i}$$

$$\Rightarrow \quad |E_x| = \frac{40}{20} = 20\,\text{N/C}$$

E_y and E_z may also exist so.

If E_y & $E_z = 0$ the $E_x = 20$ N/C

If E_y & $E \neq 0$ then $E > E_x$

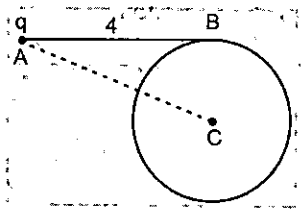
12. [a, c)

$AC = 5$ m

$$V = \frac{kq}{AC} = \frac{9\times10^9 \times 1\times10^{-6}}{5}$$

$$= 1.8\times10^3 = 1.8\,\text{kV}$$

$V_B = (V_B)_{\text{due to q}} + (V_B)_i$



$(V_B)_i$ = Potential at B due to induced charge

$$\therefore \quad 1.8\times10^3 = \frac{kq}{AB} + (V_B)_i$$

$$1.8\times10^3 = 2.25\times10^3 + (V_B)_i$$

$$(V_B)_i = -0.45\,\text{kV}$$

So, (a) and (c) are correct.

14. [b, c]

$$E = \frac{kq}{r^2}, V = \frac{kq}{r}$$

$$\Rightarrow \quad E \propto V^2$$

So E *vs.* V^2 plot is a parabola, symmetrical to E-axis.

15. [c] $$U = \frac{1}{2}\varepsilon_0 E^2$$

so the plot will be parabolic.

16. [a, b, c]

Potential at point at a distance $\frac{x}{3}$ on the right side of charge $-\frac{Q}{4} = \frac{KQ}{x+\frac{x}{3}} - \frac{KQ}{4\frac{x}{3}} = \text{zero}$

Potential at point which is $\frac{x}{5}$ on the left side of charge $-\frac{Q}{4} = \frac{KQ}{x-\frac{x}{5}} - \frac{KQ}{4.\frac{x}{5}} = \text{zero}$

Electric field at a point which is at a distance x on right of charge $\frac{-Q}{4} = \frac{KQ}{(x+x)^2} - \frac{KQ}{4x^2} = \text{zero}$

17. [b] $W = q_0(V_B - V_A) = q_0(V_C - V_A)$

$dV = -E.dr$

$$\int_C^A dV = -\int_{2a}^{3a} E.dr = -\int_{2a}^{3a} \frac{\lambda}{2\pi\varepsilon_0 r} dr$$

$$V_A - V_C = -\frac{\lambda}{4\pi\varepsilon_0} \ln\frac{3}{2}$$

$$V_C - V_A = \frac{\lambda}{2\pi\varepsilon_0} \ln\frac{3}{2}$$

$$W = \frac{q_0\lambda}{2\pi\varepsilon_0} \ln\frac{3}{2}$$

18. [d] By energy conservation :

$$\frac{1}{2}mv_0^2 = q(V_i - V_f) = q\left[+\frac{3KQ}{2R} - \frac{KQ}{R}\right]$$

$$\frac{mv_0^2}{2} = \frac{KqQ}{2R}$$

$$\Rightarrow \quad v_0^2 = \frac{KQq}{mR}$$

$$v_0 = \sqrt{\frac{Qq}{4\pi\varepsilon_0 mR}}$$

20. [a, b]

$$\int dV = \int_{r_1}^{r_2} E dr = \int_{r_1}^{r_2} \frac{\sigma}{2\pi\varepsilon_0 r} dr$$

$$= \frac{\sigma}{2\pi\varepsilon_0} \ln\left(\frac{r_2}{r_1}\right)$$

21. [a, d]

$$\frac{Kq_A}{R} + \frac{Kq_B}{2R} = 2V \quad \text{... (i)}$$

and $$\frac{Kq_A}{2R} + \frac{Kq_B}{2R} = \frac{3}{2}V \quad \text{... (ii)}$$

Solving eqn. (i) and (ii) :

$$\frac{q_A}{q_B} = \frac{1}{2}$$

After B is earthed.

$$V_B = 0.$$

$$\therefore \quad q'_B = -q'_A = -q_A$$

(charge on A remains same)

Also after earthing

$$V_A - V_B = Kq_A\left(\frac{1}{R} - \frac{1}{2R}\right) = \frac{Kq_A}{2R}.$$

Substituting $q_B = 2q_A$ in equation (i)

$$\frac{Kq_A}{2R} = \frac{V}{2}$$

$$\Rightarrow \quad V_A - V_B = \frac{V}{2}$$

$$\because \quad V_B = 0$$

So, $$V_A = \frac{V}{2}$$

23. [a, c, d]

For $V_P = 0$

$$K\frac{2Q}{\sqrt{(x+3a)^2+y^2}} = \frac{KQ}{\sqrt{(3a-x)^2+y^2}}$$

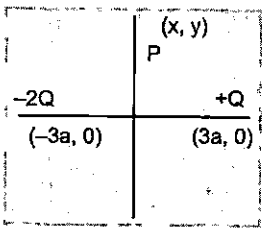
$$\Rightarrow \quad 4(x-3a)^2 + 4y^2 = (x+3a)^2 + y^2$$

$$\Rightarrow \quad x^2 + y^2 - 10ax + 9a^2 = 0$$

$$\Rightarrow \quad (x-5a)^2 + y^2 = 16a^2$$

$\Rightarrow$ Locus is a circle of radius $4a$ and centre $(5a, 0)$

→ Let potential is O for $X = x$ on x-axis.



$$\therefore \quad \frac{-2Q}{(x+3a)} \times \frac{1}{(4\pi\varepsilon_0)} + \frac{Q}{(x-3a)} \frac{1}{\pi\varepsilon_0} = 0$$

$$\Rightarrow \quad 2x - 6a = x + 3a$$

$$\Rightarrow \quad x = 9a \text{ only.}$$

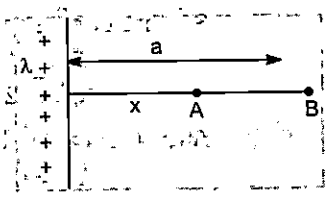
→ Centre is on x-axis at $(5a, 0)$ at this position net force on $+q$

$$F = \frac{-2Qq}{4\pi\varepsilon_0(8a)^2} + \frac{Qq}{4\pi\varepsilon_0(4a^2)} = +\text{ve}$$

$\therefore$ Particle will move in +ve X and eventually cross the circle.

→ Electric field at origin is along –ve X-axis.

26. **[a]** Work done by electric field during motion of charge $+q$ from B to A ('x' is distance of closest approach)



$$q.(V_A - V_B) = -q.\int_a^x \frac{\lambda}{2\pi\varepsilon_0 x}dx$$

$$W_1 = q\frac{\lambda}{2\pi\varepsilon_0}\ln\frac{a}{x}$$

By work energy theorem

$$\Rightarrow \quad K_0 = \frac{q\lambda}{-2\pi\varepsilon_0 x}\ln\frac{a}{x}$$

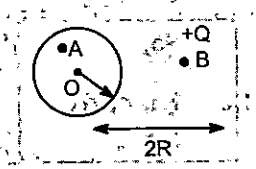
$$\Rightarrow \quad x = ae^{-2\pi\varepsilon_0 K_0/\lambda q}$$

27. **[c]** Since electric field inside the conducting shell is zero

$$\therefore \quad V_0 = V_A$$

and $$V_0 = \frac{Q}{8\pi\varepsilon_0 R}$$

$$V_A = \frac{Q}{8\pi\varepsilon_0 R}$$



29. **[a]** Electric field is radially outwards and in direction of field, potential decreases

$$V_B > V_C$$

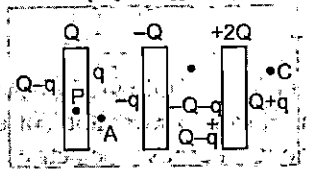
Inside the radius a $E = 0 \Rightarrow$ Potential will be same $\Rightarrow V_0 = V_A$

$V_0 = V_A = V_a$ (potential at distance 'a' from centre)

and $V_a > V_B$

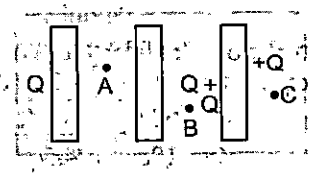
$$\therefore \quad V_0 = V_A > V_B > V_C$$

30. **[d]**



$E_P = 0$ (since it is pt. inside conductor)

$$\Rightarrow \quad K\left[\frac{Q-q}{2\varepsilon_0} - \frac{q}{2\varepsilon_0} + \frac{q}{2\varepsilon_0} + \frac{Q-q}{2\varepsilon_0} - \frac{(Q+q)}{2\varepsilon_0} - \frac{(Q-q)}{2\varepsilon_0}\right] = 0$$



$$\Rightarrow \quad q = 0$$

Clearly $E_A = 0$

$$E_B = -\frac{2KQ}{2\varepsilon_0} - \frac{KQ}{\varepsilon_0}$$

$$E_C = \frac{2KQ}{2\varepsilon_0} = \frac{KQ}{\varepsilon_0}$$

In magnitude $E_B = E_C$

32. **[a]** Since it is a conductor

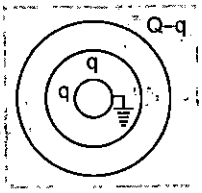
V_0 = potential at surface = 0

$$\frac{q}{4\pi\varepsilon_0 a} + \frac{Q}{4\pi\varepsilon_0 2a} = 0$$

$$\Rightarrow \quad q = -\frac{Q}{2}$$

34. **[a, c]**

$$V_A = \frac{K(Q-q)}{3a} + \frac{Kq}{2a} + \frac{Kq'}{a} = 0$$



also $q' = -q$

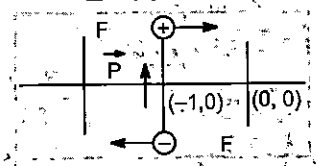
$$\Rightarrow \quad \frac{Q-q}{3a} + \frac{q}{2a} - \frac{q}{a} = 0$$

$$\Rightarrow \quad q = \frac{2Q}{5}$$

$$\therefore \quad q' = -\frac{2Q}{5}$$

35. **[a, c]** $V_{(x)} = 4 + 5x^2$

$$\vec{\mathbf{E}} = 10x\,\hat{\mathbf{i}}$$



Net force will be zero and torque not zero

And rotation will be along clockwise direction.

39. **[d]** $A = (2,2)$ and $B = (4,1)$

$$W_{A\to B} = q(V_B - V_A) = q\int_A^B dV$$

$$= -\int_A^B q\vec{\mathbf{E}}.d\vec{\mathbf{r}}$$

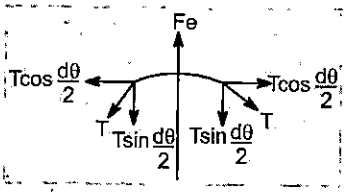
$$= -q\int_A^B (y\,\hat{\mathbf{i}} + x\,\hat{\mathbf{j}}).(dx\,\hat{\mathbf{i}} + dy\,\hat{\mathbf{j}})$$

$$= -q\int_A^B (ydx + xdy)$$

$$= -q\int_{(2,2)}^{(4,1)} d(xy) = -q[xy]_{(2,2)}^{(4,1)}$$

$$= -q[4-4] = \text{zero.}$$

43. **[a]** From diagram it is clear that



$$F_e = 2\ T\sin\frac{d\theta}{2}$$

$$\frac{1}{4\pi\varepsilon_0}\frac{q_0 dq}{r^2} = 2\ T\sin\frac{d\theta}{2}$$

$$\frac{1}{4\pi\varepsilon_0}\frac{q_0}{r^2}\cdot\left(\frac{q}{2\pi r}\right).rd\theta = 2\ T.\frac{d\theta}{2}$$

$$\Rightarrow \quad T = \frac{qq_0}{8\pi\varepsilon_0 r^2}$$

48. **[d]** $$\lambda = \frac{Q}{2\pi R}$$

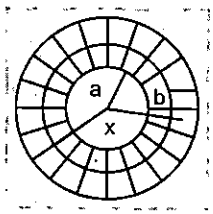
As obtained in above question.

$$T = \frac{q_0\lambda}{4\pi\varepsilon_0 R} = \frac{q_0 Q}{8\pi^2\varepsilon_0 R^2}$$

$$Y = \frac{\text{stress}}{\text{strain}} = \frac{\frac{T}{A}}{\frac{\Delta R}{R}}$$

$$\Rightarrow \quad \Delta R = \frac{q_0 Q}{8\pi^2\varepsilon_0 RAY}$$

51. **[d]** $$6 = \frac{Q}{\pi(b^2 - a^2)}$$



$$V = \int_a^b \frac{Q}{\pi(b^2-a^2)}\frac{2\pi x dx}{4\pi\varepsilon_0 x}$$

$$= \frac{Q}{2\pi\varepsilon_0(b+a)}$$

54. **[b, c]**

$$\oint E.ds = \frac{q}{\varepsilon_0} = 0$$

$$\Rightarrow \quad q = 0 \text{ or } E = 0$$

58. **[a]** By energy conservation

$$\frac{1}{2}mu^2 = \frac{kq_1q_2}{r}$$

$$\frac{1}{2}(0.5)(.5)^2 = \frac{9\times10^9\times5\times10^{-9}\times10^{-8}}{r}$$

Solving we get

r = 7.2 cm.

61. **[b, d]**

For equilibrium $\frac{4Qq}{(r-x)^2} = \frac{qQ}{x^2} \Rightarrow x = \frac{r}{3}$

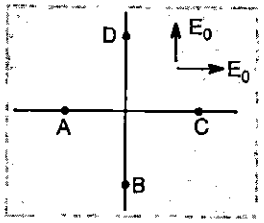
For all the charges to be in equilibrium

$$\frac{4Q^2}{r} + \frac{4Qq}{(r-x)} + \frac{Q_q}{x} = 0$$

For $x = \frac{r}{3}$

we get $q = -\frac{4Q}{9}$

63. **[b]** Since potential decreases along electric field as



$$E = \frac{-dV}{dx}$$

$\therefore \quad V_A > V_C$

and $V_B > V_D$ also due to symmetry

$$V_A = V_B, V_C = V_D$$

$$V_A = V_B > V_C = V_D$$

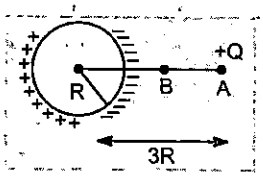
66. **[a]** Particle experiences a force along x-axis

$$\Rightarrow \quad E_x \neq 0$$

$$\Rightarrow \quad \frac{dv}{dx} \neq 0$$

There is no statement for y-direction so we can't say any thing for y-direction.

67. **[c]** Due to +Q some charges are induced on shell. Now at B potential due to $+Q = \frac{Q}{4\pi\varepsilon_0 R}$ and due to –ve and +ve charges induced on shell potential will be –ve (since –ve charges are closer to B)



$\therefore$ Net Potential at $B < \frac{Q}{4\pi\varepsilon_0 R}$.

70. **[a, b, c]**

Point A and B are equipotential because they are equil distant from O work done by electric is path independent because electric field is a conservative field and $V_C > V_A$.

71. **[a, c]**

Distance AC is given by,

AC = 5 m

Potential at due to charge q at A is given by

$$V = \frac{kq}{AC} = \frac{9\times10^9\times1\times10^{-6}}{5}$$

$$= 1.8\times10^3 = 1.8\text{kV}$$

Potential at B is given by

$$V_B = (V_B)_{\text{due to q}} + (V_B)_{\text{induced}}$$

$\{(V_B)_i$ = Potential at B due to induced charge$\}$

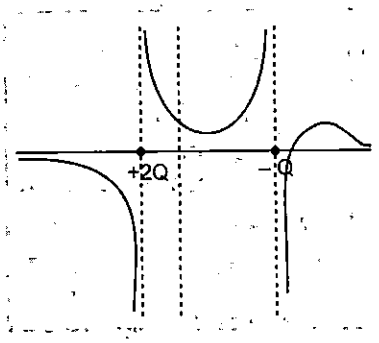
Thus $1.8\times10^3=\frac{kq}{AB}+(V_B)_i$

$1.8\times10^3=2.25\times10^3+(V_B)_i$

$(V_B)_i=-0.45\,\text{kV}$

So option (a) & (c) are correct.

73. [a, d]



74. [b, c]

This problem can be solved more easily by using vector method. Let the points charges have the coordinate

$$P(-\frac{a}{2},0,0), Q\equiv(\frac{a}{2},0,0)$$

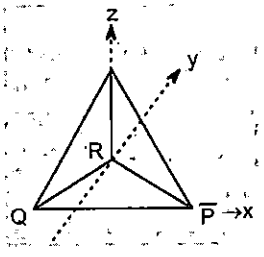
and $$R\equiv(0,\frac{\sqrt{3}}{2}a,0)$$

coordinate of point at which E has to be calculated is $(0,\frac{a}{2\sqrt{3}},\sqrt{\frac{2}{3}}a)$

Also we have

$$\vec{\mathbf{E}}=\frac{1}{4\pi\varepsilon_0}\frac{Q}{r^3}\vec{\mathbf{r}}$$

From the principle of superposition we get



$$\vec{\mathbf{E}}_{net}=\vec{\mathbf{E}}_1+\vec{\mathbf{E}}_2+\vec{\mathbf{E}}_3$$

$$\vec{\mathbf{E}}_{net}=\frac{1}{4\pi\varepsilon_0}\frac{Q}{a^3}(\overline{\mathbf{PO}}+\overline{\mathbf{QO}}+\overline{\mathbf{RO}})$$

$$=\frac{Q}{4\pi\varepsilon_0 a^3}\cdot\left(3\sqrt{\frac{3}{2}}a\right)\hat{\mathbf{k}}=\frac{\sqrt{6}Q}{4\pi\varepsilon_0 a^2}\hat{\mathbf{k}}$$

75. [a, c, d]

Electric field exists between the region between shell; and given by

$$E(R)=\frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}$$

$$E_{max}=\frac{1}{4\pi\varepsilon_0}\frac{Q}{b^2}\quad\text{at } (R=b)$$

E inside inner shell = 0

or V is constant in the shell and maximum

76. [a, b, d]

For 'a': As the charge is not having any direction associated with it, *i.e.*, being a scalar, it is additive in nature.

Option (b) is a standard factual statement for charge, but many other physical quantities like KE, mass etc are not invariant.

Option (c) and (d): Conservation of physical quantity refers to invariance with time in a given frame of reference, which is not at all can be concluded from scalar nature of the physical quantity.

77. [a, b, c, d]

$\vec{\mathbf{E}}$ due to discrete point charge at the location of discrete point charge is not defined as $\vec{\mathbf{E}}$ grows without any bound near to charge and becomes infinite at the location of charge.

Same as above is the reasoning for option (d).

In support of (b) and (c) you can take as example, $\vec{\mathbf{E}}$ due to solid and hollow uniformly charged spheres, respectively.

78. [a, c, d]

Gaussian surface must not pass through discrete charge as at the location of discrete charge $\vec{\mathbf{E}}$ is not defined, while it can pass through continuous charge distribution as $\vec{\mathbf{E}}$ is defined at all points due to continuous charge distribution (in general).

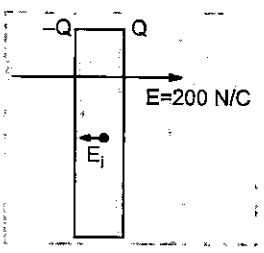
It is totally immaterial that some material must be present at the location of Gaussian surface. The Gaussian surface can be any closed surface whether material one or imaginary.

79. [a, b, c]

Let us consider that ball is first touched to +ve plate, as a result, it acquires +ve charge and due to electric field from + ve to –ve plate, it accelerates and comes into contact with –ve plate. There it releases +ve charge and acquires –ve charge, thus again accelerating from –ve to +ve plate and this process repeats.

80. [b, c, d]

As in electrostatics, the electric field inside the bulk of the material of the conductor is zero. So some charge gets induced on two faces of the plate to make electric field zero inside the conductor.



Let Q and $-Q$ appears on right and left face as shown, then induced electric field,

$$E_i = \frac{Q}{2\varepsilon_0 A} + \frac{Q}{2\varepsilon_0 A}$$

$$= \frac{Q}{\varepsilon_0 A} \quad (\textit{towards left})$$

For $\vec{\mathbf{E}}$ inside plate to be zero

$$E_i = E$$

$$\Rightarrow \quad Q = \varepsilon_0 A \times 200 = 1.77 \times 10^{-9}\text{C}$$

The electric field is discontinuous at surface but potential is continuous.

81. [b, c]

Along X-axis,

$$u_x = u\cos\theta$$

$$a_x = -\frac{qE}{m}$$

Along Y-axis

$$u_y = u\sin\theta$$

$$a_y = -g$$

Equations of motion along X and Y-axes would be

$$x = u\cos\theta t - \frac{qE}{2m}t^2$$

and $$y = u\sin\theta t - \frac{1}{2}gt^2$$

Solving above equations, we get an equation of the form $Ax + By = Ct$, which is a linear equation.

Time of flight remains unchanged as vertical motion is not affected by $\vec{\mathbf{E}}$. Range of the particle in the present case is always less than $\frac{u^2\sin 2\theta}{g}$ whatever be the value of E.

$$R = x_{t=T} = u\cos\theta \times \frac{2u\sin\theta}{g} - \frac{1}{2}\times\frac{qE}{m}\left[\frac{2u\sin\theta}{g}\right]^2$$

$$< \frac{u^2\sin 2\theta}{g}$$

82. [a, d]

As no charge is present inside the conductor, potential at any point inside the conductor is same as that of the potential or conductor.

So, potential of the conductor = Potential at the centre

$$= V_q + V_{\text{induced charges}}$$

$$V_{\text{conductor}} = \frac{q}{4\pi\varepsilon_0(d+R)} + 0$$

As the total induced charge at conductor's surface is equal to zero and hence to the potential at centre due to the induced charge.

For point B,

$$V_{\text{conductor}} = V_{\text{at point } B} = V_q + V_{\text{induced charges}}$$

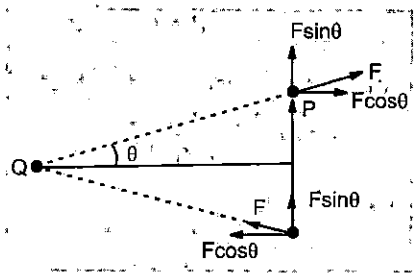
$$\Rightarrow \quad V_{\text{induced charges}} = \frac{q}{4\pi\varepsilon_0(d+R)}$$

$$-\frac{q}{4\pi\varepsilon_0 d} = \frac{-qR}{4\pi\varepsilon_0 d(d+R)}$$

83. [b, c]

The situation is shown in the figure below:

$$\vec{\mathbf{F}}_{\text{net}} = 2F\sin\theta\hat{\uparrow} = \frac{PQ}{4\pi\varepsilon_0 r^3}$$

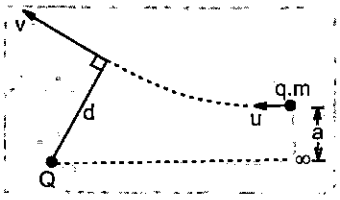


$$\vec{\tau} = F\cos\theta \times 2a \text{ in clockwise direction}$$

$$= \frac{PQ}{4\pi\varepsilon_0 r^2}$$

84. [b, c, d]

The path traced by q is shown in figure, the path is curvilinear and acceleration is due to the force exerted by Q on q.



The separation between them is minimum if relative velocity of the particles along the line joining them is zero. Let d be the minimum separation between them. As torque about Q is zero, so angular momentum remains conserved.

$$mua = mvd$$

$$\Rightarrow \quad v = \frac{ua}{d}$$

From energy conservation law,

$$\frac{mu^2}{2}=\frac{mv^2}{2}+\frac{1}{4\pi\varepsilon_0}\frac{qQ}{d}$$

85. [a, b, d]

$$v_i=\frac{q_0}{4\pi\varepsilon_0 R_1}\times q+\frac{q^2}{8\pi\varepsilon_0 R_1}$$

$$v_f=\frac{qq_0}{4\pi\varepsilon_0 R_2}+\frac{q^2}{8\pi\varepsilon_0 R_2}$$

$$\Rightarrow \quad dU=U_f-U_i=\frac{q(q_0+q/2)}{4\pi\varepsilon_0}\left[\frac{1}{R_2}-\frac{1}{R_1}\right]<0$$

It means electric potential energy is decreasing and work done by electric force would be positive as given by

$$W_{el}=-dU$$

From work-energy theorem,

$$dK=W_{el}+W_{el}+\Delta H_{\text{dissipated}}$$

88. [a, b, d]

$$\frac{1}{2}mv^2+Q\left(\frac{p\cos\theta}{4\pi\varepsilon_0 r^2}\right)=0$$

or $$v=\sqrt{\frac{-2Qp\cos\theta}{4\pi\varepsilon_0 mr^2}} \qquad \left\{\frac{\pi}{2}\le\theta\le\pi\right\}$$

circular motion of bead requires a centripetal force

$$E_r=-\frac{\partial V}{\partial r}=\frac{2p\cos\theta}{4\pi\varepsilon_0 r^3}$$

note that $QE_r=\dfrac{mv^2}{r}$

thus wire frame does not exert any force on the bead to sustain circular motion. Bead will reach the point opposite its starting position and then repeatedly retrace its path executing a periodic motion.

89. [a, b]

(a) $W_{ext}=\Delta U=U_f-U_i=+\text{ ve}$

(b) $U=k\dfrac{\sum_{i\neq j} q_iq_j}{r_{ij}}$ If it is 0, at least one charge should be –ve

(c) $W_{ext}=\Delta U=U_f-U_i=+\text{ ve}$

(d) Force is independent of potential energy

90. [a, c, d]

Electric field exists between the region between shell; and given by

$$E(R)=\frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}$$

$$E_{max}=\frac{1}{4\pi\varepsilon_0}\frac{Q}{b^2} \quad \text{at } (R=b)$$

E inside inner shell = 0

$\Rightarrow$ V is constant in the shell and maximum.

92. [a, b, d]

$$\vec{p}=2\hat{i}+3\hat{j}-0\hat{k} \qquad (\mu\text{ cm})$$

$$\vec{E}=3\hat{i}+0\hat{j}+2\hat{k} \qquad (10^5\text{ NC}^{-1})$$

$\vec{\tau}=$ Torque on the electric dipole

$$=\vec{p}\times\vec{E}=\left[(2\hat{i}+3\hat{j})\times(3\hat{i}+2\hat{k})\right](0.1\text{ Nm})$$

$$=\left[4(-\hat{j})+9(-\hat{k})+6\hat{i}\right](0.1\text{ Nm})$$

$$\left|\vec{\tau}\right|=0.1\sqrt{16+81}=\sqrt{0.97}\text{ Nm}$$

P.E. of the dipole $=-\vec{p}\cdot\vec{E}=-0.6\text{ J}$

Maximum P.E. of the dipole $=|\vec{p}||\vec{E}|$

$$=(\sqrt{2^2+3^2})(\sqrt{2^2+3^2})\times10^{-1}\text{ J}=1.3\text{ J}$$

93. [a, b, c]

Diagonally opposite charges will produce field in z-axis, but fields due to +ve and –ve charges will cancel.

94. [b, d]

Since electric field due a uniformly charged wire is non-uniform so force and torque on a dipole may or may not be zero.

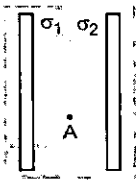
95. [a, b, d]

Two conducting surfaces facing each other have equal and opposite charges

$$E_A=\frac{\sigma_1-\sigma_2}{\varepsilon_0}$$

so $$E_A=\frac{\sigma_1}{\varepsilon_0}=-\frac{\sigma_2}{\varepsilon_0}$$

Since $\sigma_1=-\sigma_2$



Level ③ Comprehension Based Problems

Passage 1

In a certain region, uniform electric field exist as $E = E_0\hat{j}$. Proton and electron are projected from origin at $t = 0$ with certain velocities along the +ve x-axis direction. Due to the electric field, they experience force and so they move in the x-y plane along different trajectory.

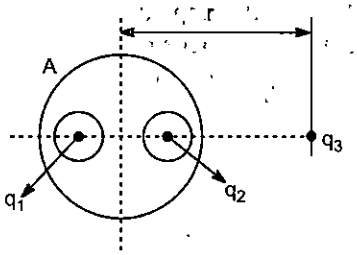
1. The path followed by the particles will be:
(a) Parabola (b) Circular
(c) Hyperbola (d) Spiral

2. If they have same initial kinetic energy then for the same displacement along x-direction, deflection is:
(a) more for proton
(b) more for electron
(c) equal for both
(d) independent of kinetic energy

3. If they have same initial velocity then for same x-axis displacement, deflection is:
(a) more for proton
(b) more for electron
(c) equal for both
(d) independent of kinetic energy

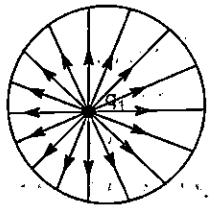
Passage 2

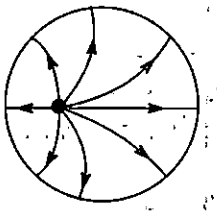
A spherical conductor A contains two spherical cavities as shown in figure. The total charge on conductor itself is zero. However, there is a point charge q_1 at centre of one cavity and q_2 at the centre of other cavity. Another charge q_3 is placed at large distance 'r' from the center of the spherical conductor.



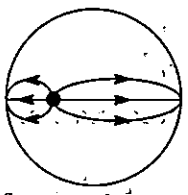
1. Which of the following statements are true?
(a) Charge q_3 applies larger force on charge q_2 than on charge q_1
(b) Charge q_3 applies smaller force on charge q_2 than on charge q_1
(c) Charge q_3 applies equal force on both the charges
(d) Charge q_3 applies no force on any of the charges

2. If q_1 is displaced from its centre slightly (being always inside the cavity) then the correct representation of field lines inside the same cavity is:

(a) 

(b) 

(c) Then will be no field lines inside cavity

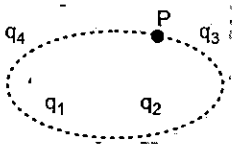
(d) 

3. The force acting on conductor A will be:
(a) Zero (b) $\frac{q_3(q_1+q_2)}{4\pi\varepsilon_0 r^2}$
(c) $\frac{-q_3(q_1+q_2)}{4\pi\varepsilon_0 r^2}$ (d) $\frac{q_3q_1+q_2q_3+q_1q_2}{4\pi\varepsilon_0 r^2}$

Passage 3

Gauss's law and Coulomb's law expressed in different forms, although are equivalent ways of describing the relation between charge and electric field in static conditions. Gauss's law is $\varepsilon_0\phi = q_{encl}$ in which q_{encl} is the net charge inside an imaginary closed surface called Gaussian surface and ϕ is the net flux of the electric field through the surface. $\phi = \oint \vec{E}\cdot\vec{d}A$ gives electric flux through Gaussian surface. The two equations hold only when the net charge is in vacuum or air.

1. A Gaussian surface encloses two of the 4 positively charged particles. The particles which contribute to the electric field at point P on the surface are:
(a) q_1 and q_2
(b) q_2 and q_3



(c) q_4 and q_3
(d) q_1, q_2, q_3 and q_4

2. The net flux of the electric field through the surface is:
(a) due to q_1 and q_2 only
(b) due to q_3 and q_4 only
(c) equal due to all the four charges
(d) cannot say

3. The net flux of the electric field through the surface due to q_3 and q_4 is:
(a) zero (b) positive
(c) negative (d) can't say

4. If the charges q_3 and q_4 are displaced (always remaining outside the Gaussian surface), then consider the following two statements
A: Electric field at each point on Gaussian surface will remain same
B: The value of $\oint \vec{E} \cdot \vec{d}A$ for the Gaussian surface will remains same.
(a) Both A and B are true
(b) Both A and B are false
(c) A is true but B is false
(d) B is true but A is false

Passage 4

A point charge $+Q$ having mass m is fixed on horizontal smooth surface. Another point charge having magnitude $+2Q$ & mass $2m$ is projected horizontal towards the charge $+Q$ from far distance with velocity V_0.

1. Force applied by floor on the fixed charge in horizontal direction, when distance between charges becomes 'd':
(a) $\frac{2KQ^2}{d^2}$ (b) $\frac{KQ^2}{d^2}$
(c) Zero (d) None

2. The impulse acting on the system of particles $(Q + 2Q)$ in the time interval when distance between them becomes 'd':
(a) $2m\left[\sqrt{V_0^2 = \frac{2KQ^2}{md}} - V_0\right]$
(b) $2mV_0$
(c) $2m\left[\sqrt{V_0^2 = \frac{2KQ^2}{md}}\right]$
(d) None

3. Minimum distance of approach:
(a) $\frac{2KQ^2}{mV_0^2}$ (b) $\frac{KQ^2}{mV_0^2}$
(c) $\frac{4KQ^2}{mV_0^2}$ (d) None

4. Acceleration of particle $2Q$ when it is closed to fixed particle Q:
(a) Zero (b) $\frac{mV_0^4}{2KQ^2}$
(c) $\frac{mV_0^4}{4KQ^2}$ (d) None

5. If particle $+Q$ is free to move, then what will be the closest distance between the particles:
(a) zero (b) $\frac{6KQ^2}{mV_0^2}$
(c) $\frac{3KQ^2}{mV_0^2}$ (d) None

Passage 5

We have two electric dipoles. Each dipole consists of two equal and opposite point charge at the end of an insulating rod of length d. The dipoles are placed along the x-axis at a large distance r apart oriented as shown below :

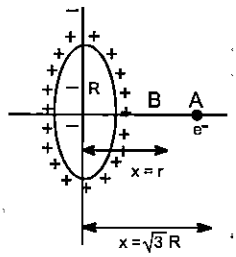
1. The dipole on the left
(a) will feel a force upwards and a torque trying to make it rotate clockwise.
(b) will feel a force upwards and a torque trying to make it rotate counter clockwise.
(c) will feel a force upwards and no torque about its centre.
(d) will feel a force downwards and a torque trying to make it rotate clockwise.

2. The dipole on the right
(a) will feel a force downwards and a torque trying to make it rotate clockwise
(b) will feel a force downwards and a torque trying to make it rotate counter clockwise
(c) will feel a force upwards and no torque about its centre.
(d) will feel no force and a torque trying to make it rotate counter clockwise.

3. Question given below consist of two statement each printed as **Assertion (A)** and **Reason (R)** ; while answering these questions you are required to choose any one of the following four responses:
Assertion (A): Angular momentum of the two dipole system is not conserved.
Reason (R): There is a net torque on the system.
(a) both (A) and (R) are true and (R) is the correct explanation of (A)
(b) (A) is correct and (R) is incorrect
(c) (A) is incorrect and (R) is correct
(d) (A) and (R) both incorrect

Passage 6

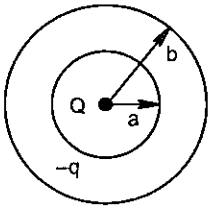
There is a uniformly charged ring having radius R. An infinite line charge (charge per unit length λ) is placed along a diameter of the ring (in gravity free space). Total charge on the ring $Q = 4\sqrt{2}\lambda R$. An electron of mass m is released from rest on the axis of the ring at a distance $x = \sqrt{3}R$ from the centre.



1. Magnitude of initial acceleration of the electron:
   (a) $\frac{e\lambda}{\pi\varepsilon_0 mR}\left(\frac{3-2\sqrt{2}}{4\sqrt{6}}\right)$ (b) $\frac{e\lambda}{\pi\varepsilon_0 mR}\left(\frac{3+2\sqrt{2}}{4\sqrt{6}}\right)$
   (c) $\frac{e\lambda}{\pi\varepsilon_0 mR}\left(\frac{3+2\sqrt{2}}{4\sqrt{3}}\right)$ (d) none
2. The distance from centre of ring on the axis where the net force on the electron is zero:
   (a) $2R$ (b) $\sqrt{2}R$
   (c) R (d) none of these
3. Potential difference between points $A(x=\sqrt{3}R)$ and $B(x=R)$ i.e. $(V_A - V_B)$ is:
   (a) $-\frac{\lambda}{\pi\varepsilon_0}\left[\left(1-\frac{1}{\sqrt{2}}\right)-\frac{\ln 3}{4}\right]$
   (b) $\frac{\lambda}{\pi\varepsilon_0}\left[\left(1-\frac{1}{\sqrt{2}}\right)-\frac{\ln 3}{4}\right]$
   (c) $-\frac{\lambda}{\pi\varepsilon_0}\left[\left(1+\frac{1}{\sqrt{2}}\right)-\frac{\ln 3}{4}\right]$
   (d) none

Passage 7

Both question (a) and (b) refer to the system of charges as shown in the figure. A spherical shell with an inner radius 'a' and an outer radius 'b' is made of conducting material. A point charge $+Q$ is placed at the centre of the spherical shell and a total charge $-q$ is placed on the shell.



1. Charge $-q$ is distributed on the surfaces as:
   (a) $-Q$ on the inner surface, $-q$ on outer surface
   (b) $-Q$ on the inner surface, $-q + Q$ on the outer surface
   (c) $+Q$ on the inner surface, $-q - Q$ on the outer surface
   (d) The charge $-q$ is spread uniformly between the inner and outer surface.
2. Assume that the electrostatic potential is zero at an infinite distance from the spherical shell. The electrostatic potential at a distance R $(a < R < b)$ from the centre of the shell is:
   (a) 0 (b) $\frac{KQ}{a}$
   (b) $k\frac{Q-q}{R}$ (d) $K\frac{Q-q}{b}$

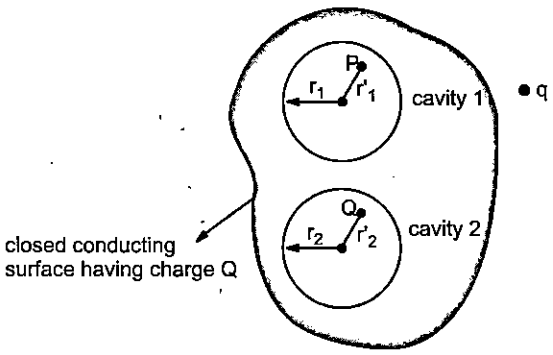
Passage 8

A point charge q is located at a distance r from the centre O of an uncharged conducting spherical layer whose inside and outside radii are equal to R_1 and R_2, respectively. It is given $r < R_1$. Assume zero potential at infinity.

1. The potential at point O is:
   (a) zero
   (b) $\frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r}-\frac{1}{R_1}+\frac{1}{R_2}\right]$
   (c) $\frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r}-\frac{1}{R_1}\right]$
   (d) can not be determined
2. The electric field intensity at point O is:
   (a) zero
   (b) $\frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r^2}-\frac{1}{R_1^2}+\frac{1}{R_2^2}\right]$
   (c) $\frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r^2}-\frac{1}{R_1^2}\right]$
   (d) can not be determined
3. The electric potential of the conductor is:
   (a) zero
   (b) $\frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r}-\frac{1}{R_1}+\frac{2}{R_2}\right]$
   (c) $\frac{q}{4\pi\varepsilon_0 R_2}$
   (d) can not be determined

Passage 9

A point charge q_1 is placed inside the cavity 1 and another point charge q_2 is inside cavity 2. A point charge q is placed outside the conductor.



1. The charge on outer surface of the conductor would be:
(a) $Q + q_1 + q_2$ and non-uniformly distributed
(b) $Q + q_1 + q_2$ and its uniform or non-uniform distribution depends upon location of q_1 and q_2
(c) $Q + q_1 + q_2$ would be distributed uniformly
(d) $Q + q_1 + q_2$ and the distribution depends upon the location of q_1, q_2 and q

2. If q_1 is at the centre of cavity 1, then $\vec{E}$ at a point S, at the distance r from centre of cavity 1($r > r_1$) due to induced charge on the surface of cavity 1, is:
(a) $\frac{q_1}{4\pi\varepsilon_0 r^2}$ away from centre of cavity 1
(b) $\frac{q_1}{4\pi\varepsilon_0 r_1^2}$ away from centre of cavity 1
(c) zero
(d) $\frac{q_1}{4\pi\varepsilon_0 r^2}$ towards centre of cavity 1

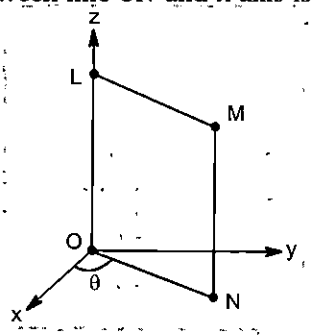
3. $\vec{E}$ inside the conductor at point S distant r from point charge q due to charge on outer surface of conductor would be:
(a) $\frac{Q + q_1 + q_2}{4\pi\varepsilon_0 r^2}$ away from charge q
(b) $\frac{q}{4\pi\varepsilon_0 r^2}$ towards charge q
(c) zero
(d) can not be determined

4. If charge q_2 is at point Q (inside cavity 2), then $\vec{E}$ at the centre of cavity 2 due to induced charge on the surface of cavity 2 would be:
(a) $\frac{q_2}{4\pi\varepsilon_0 r_2'^2}$ towards q_2
(b) $\frac{q_2}{4\pi\varepsilon_0 r_2'^2}$ away from q_2
(c) zero
(d) can not be determined

5. If the potential of the conductor is V_0 and charge q_2 is placed at centre of cavity 2, then potential at point Q, is:
(a) $\frac{q_2}{4\pi\varepsilon_0 r_2'} + V_0$
(b) $\frac{q_2}{4\pi\varepsilon_0}\left(\frac{1}{r_2'} + \frac{1}{r_2}\right) + V_0$
(c) $\frac{q_2}{4\pi\varepsilon_0}\left(\frac{1}{r_2'} - \frac{1}{r_2}\right) + V_0$
(d) V_0

Passage 10

The electric field intensity at all points in space is given by $\vec{E} = \sqrt{3}\,\hat{i} - \hat{j}$ volts/metre. A square frame *LMNO* of side 1 metre is shown in Fig. The point N lies in *x*-*y* plane. The initial angle between line *ON* and *x*-axis is $\theta = 60°$.



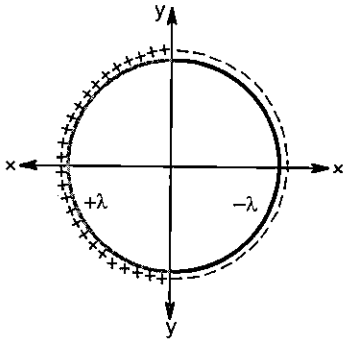
1. The magnitude of electric flux through area enclosed in square frame *LMNO* is:
(a) 0 volt metre (b) 1 volt metre
(c) 2 volt metre (d) 4 volt metre

2. The work done by electric field in taking a point charge 1μC from origin *O* to point *M* is:
(a) 0μJ (b) 1μJ
(c) 2μJ (d) 4μJ

3. The square frame *LMNO* is now rotated about *z*-axis by an angle 30°, such that θ either increases or decreases. Then pick up the correct statement.
(a) The magnitude of electric flux increases from initial value as θ is increased.
(b) The magnitude of electric flux increases from initial value as θ is decreased
(c) The magnitude of electric flux may increase or decrease from initial value as θ is changed.
(d) The magnitude of electric flux will decreases from initial value as θ is changed.

Passage 11

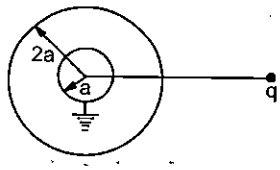
A thin ring of radius R metres is placed in x-y plane such that its centre lies on origin. The half ring in region $x < 0$ carries uniform linear charge density $+\lambda$C/m and the remaining half ring in region $x > 0$ carries uniform linear charge density $-\lambda$ C/m.



1. Then the electric potential (in volts) at point P whose coordinates are $(0m, +\frac{R}{2}m)$ is:
   (a) $\frac{1}{4\pi\varepsilon_0}\frac{\lambda}{2}$
   (b) 0
   (c) $\frac{1}{4\pi\varepsilon_0}\frac{\lambda}{4}$
   (d) cannot be determined
2. Then the direction of electric field at point P whose coordinates are $(0m, +\frac{R}{2}m)$ is:
   (a) Along positive x-direction
   (b) Along negative x-direction
   (c) Along negative y-direction
   (d) None of these
3. Then the dipole moment of the ring in C-m is:
   (a) $-(2\pi R^2\lambda)\hat{\mathbf{i}}$ (b) $(2\pi R^2\lambda)\hat{\mathbf{i}}$
   (c) $-(4R^2\lambda)\hat{\mathbf{i}}$ (d) $(4R^2\lambda)\hat{\mathbf{i}}$

Passage 12

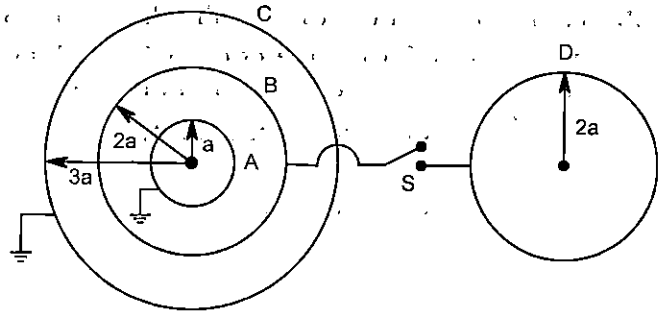
A solid conducting sphere of radius 'a' is surrounded by a thin uncharged concentric conducting shell of radius $2a$. A point charge q is placed at a distance $4a$ from common centre of conducting sphere and shell. The inner sphere is then grounded.



1. The charge on solid sphere:
   (a) $-\frac{q}{2}$ (b) $-\frac{q}{4}$
   (c) $-\frac{q}{8}$ (d) $-\frac{q}{16}$
2. Select the correct statement:
   (a) Charge on surface of inner sphere is non-uniformly distributed.
   (b) Charge on inner surface of outer shell is non-uniformly distributed.
   (c) Charge on outer surface of outer shell is non-uniformly distributed.
   (d) All the above statements are false.
3. The potential of outer shell is:
   (a) $\frac{q}{32\pi\varepsilon_0 a}$ (b) $\frac{q}{16\pi\varepsilon_0 a}$
   (c) $\frac{q}{8\pi\varepsilon_0 a}$ (d) $\frac{q}{4\pi\varepsilon_0 a}$

Passage 13

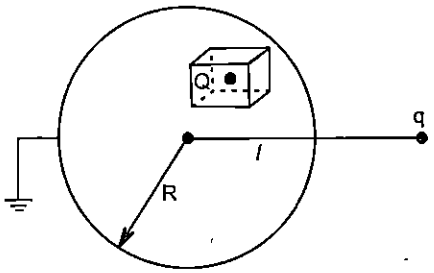
Three concentric conducting spherical shells A, B and C having radii a, $2a$ and $3a$ respectively are placed as shown in Fig. Shell B is having net charge $+Q$, while shells A and C are earthed. An uncharged conducting spherical shell D of radius $2a$ is placed at a large distance from C. There is a switch 'S' connecting shell B and D.



1. Charges on shell A and C are when switch 'S' is open:
   (a) $\frac{-5Q}{8}, \frac{-3Q}{8}$ (b) $\frac{-3Q}{8}, \frac{-5Q}{8}$
   (c) $\frac{-Q}{4}, \frac{-3Q}{4}$ (d) $\frac{3Q}{4}, \frac{5Q}{4}$
2. Charges on shell A and C are when switch 'S' is closed:
   (a) $\frac{-Q}{5}, \frac{-3Q}{5}$ (b) $\frac{-5Q}{8}, \frac{-3Q}{8}$
   (c) $\frac{-3Q}{8}, \frac{-5Q}{8}$ (d) $\frac{-3Q}{4}, \frac{-Q}{4}$
3. Charges flow through switch from B to D when it is closed, is:
   (a) $\frac{5Q}{3}$ (b) $\frac{Q}{5}$
   (c) $\frac{-2Q}{3}$ (d) None of these

Passage 14

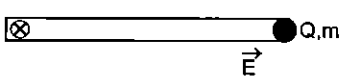
There is a cubical cavity inside a conducting sphere of radius R. A positive point charge Q is placed at the centre of the cube and another positive charge q is placed at a distance $l(>R)$ from the centre of the sphere. The sphere is earthed as shown in Fig.



1. Charge induced on the inner surface of cavity is :
   (a) $-Q$, uniformly distributed
   (b) $-Q$, non-uniformly distributed
   (c) $-(Q+q)$, non- uniformly distributed
   (d) None
2. Net charge on the outer surface of conducting sphere is :
   (a) $+Q$ (b) $Q-\frac{qR}{l}$
   (c) $-\frac{qR}{l}$ (d) None
3. Potential at a point inside the cavity is :
   (a) zero
   (b) positive
   (c) negative
   (d) cannot be determined

Passage 15

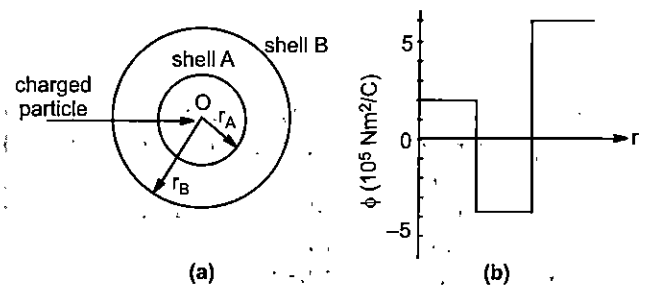
A thin, homogeneous stick of mass m and length L may rotate in the vertical plane around a horizontal axle pivoted at one end of the stick. A small ball of mass m and charge Q is attached to the opposite end of this stick. The whole system is positioned in a constant horizontal electric field of magnitude $E=\frac{mg}{2Q}$. The stick is held horizontally at the beginning.



1. What is the acceleration of the small ball at the instance of releasing the stick ?
   (a) $\frac{3g}{2}$ (b) $\frac{3g}{4}$
   (c) $\frac{9g}{8}$ (d) None of these
2. What is the speed of ball when rod becomes vertical :
   (a) $\sqrt{\frac{3gl}{2}}$ (b) $\sqrt{2gl}$
   (c) $\sqrt{\frac{3gl}{4}}$ (d) None of these
3. Magnitude of acceleration of the end of the stick when it swings through the vertical position ?
   (a) $\frac{3\sqrt{5}g}{2}$ (b) $\frac{3\sqrt{17}g}{8}$
   (c) $\frac{3\sqrt{2}g}{4}$ (d) None of these

Passage 16

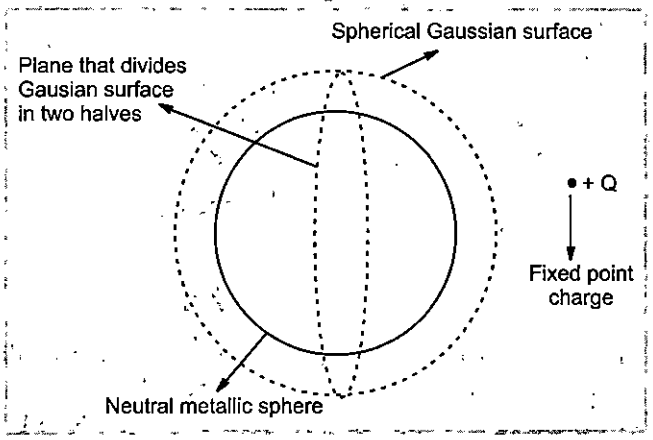
A charged particle is suspended at the centre of two thin concentric spherical charged shells, made of non-conducting material. Fig (a) shows cross-section of the arrangement. Fig (b) gives the net flux ϕ through a Gaussian sphere centered on the particle, as a function of the radius r of the sphere.



1. What is the charge on the central particle?
   (a) 0.2μC (b) 2μC
   (c) 1.77μC (d) 3.4μC
2. What is the charge on shell A?
   (a) 5.31×10^{-6}C (b) -5.31×10^{-6}C
   (c) -3.54×10^{-6}C (d) -1.77×10^{-6}C
3. In which range of the values of r is the electric field zero?
   (a) 0 to r_A
   (b) r_A to r_B
   (c) for $r > r_B$
   (d) for no range of r, electric field is zero

Passage 17

Figure shows a neutral metallic sphere with a point charge $+Q$ placed near its surface. Electrostatic equilibrium conditions exist on metallic sphere.



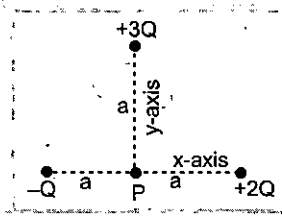
1. Net flux through right half of Gaussian surface is :
(a) negative (b) positive
(c) zero (d) cannot be determined

2. Mark the correct statements :
(a) Net flux through Gaussian surface due to external charge is zero
(b) Net flux through Gaussian surface due to charges appearing on the outer surface of metallic sphere must be zero
(c) Resulatnt electric field on the surface of Gaussian surface must be zero
(d) Gauss's law can not be applied

3. If external charge is displaced towards metallic sphere:
(a) Net flux through Gaussian surface will change
(b) Net flux through right half of Gaussian surface will increase in magnitude
(c) Charge distribution on outer surface of sphere will change
(d) Flux through Gaussian surface due to charges appearing on metallic shell only is always zero

Passage 18

Three charges $+3Q$, $+2Q$ and $-Q$ respectively are located at a distance a from the origin as shown in the Fig. The point P is located at a distance a from the origin. $\left(k=\frac{1}{4\pi\varepsilon_0}\right)$



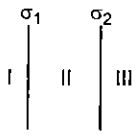
1. The magnitude of x and y-components of the electric field $\vec{E}$ at P are:
(a) $E_x = 2E_y = \frac{k3Q}{a^2}$ (b) $E_x = E_y = \frac{k3Q}{a^2}$
(c) $E_x = \frac{1}{2}E_y = \frac{k3Q}{a^2}$ (d) $E_x = 3E_y = \frac{k3Q}{a^2}$

2. The electric potential energy 'U' for the configuration three charges is
(a) $\frac{kQ^2}{a}\left(1-\frac{1}{\sqrt{2}}\right)$ (b) $\frac{kQ^2}{a}\left(\frac{1}{\sqrt{2}}+1\right)$
(c) $\frac{kQ^2}{a}\left(\frac{3}{\sqrt{2}}+1\right)$ (d) $\frac{kQ^2}{a}\left(\frac{3}{\sqrt{2}}-1\right)$

3. A fourth charge $+3Q$ is slowly moved in from infinitely to point P. How much work must be done by an external agent in this process?
(a) $\frac{kQ^2}{a}\left(\frac{3}{\sqrt{2}}-1\right)$ (b) $\frac{kQ^2}{a}\left(\frac{3}{\sqrt{2}}+1\right)$
(c) $\frac{kQ^2}{a}\frac{4}{3}$ (d) $\frac{kQ^2}{a}(12)$

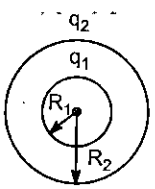
MATCHING TYPE PROBLEMS

1. Two parallel metallic plates have surface charge densities σ_1 and σ_2 as shown in figure. Match the following:



	Column I		Column II
(a)	If $\sigma_1 + \sigma_2 = 0$	(p)	Electric field in region III is towards right
(b)	If $\sigma_1 + \sigma_2 > 0$	(q)	Electric field in region I is zero
(c)	If $\sigma_1 + \sigma_2 < 0$	(r)	Electric field in region I is towards right
		(s)	None
		(t)	Nothing can be said

2. Two spherical shells are as shown in figure. Suppose r is the distance of a point from their common centre. Then,

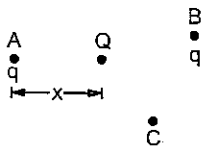


	Column I		Column II
(a)	Electric field for $r < R_1$	(p)	is constant for q_2 and vary for q_1
(b)	Electric potential for $r < R_1$	(q)	is zero for q_2 and vary for q_1
(c)	Electric potential for $R_1 < r < R_2$	(r)	Electric field in region I is towards right
(d)	Electric field for $R_1 < r < R_2$	(s)	is zero

3. A spherical metallic conductor has a spherical cavity. A positive charge is placed inside the cavity at its centre. Another positive charge is placed outside it. The conductor is initially electrically neutral.

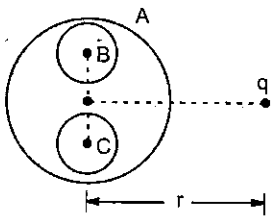
	Column I (Cause)		Column II (Effect)
(a)	If outside charge is shifted to other position	(p)	distribution of charge on inner surface of cavity changes
(b)	If inside charge is shifted to other position within cavity.	(q)	distribution of charge on outer surface of conductor changes
(c)	If magnitude of charge inside cavity is increased	(r)	electric potential at centre of conductor due to charges present on outer surface of conductor changes
(d)	If conductor is earthed	(s)	force on the charge placed inside cavity changes.

4. Two point charges A and B each having charge q are placed as shown in figure. Point C is considered in the surrounding region. Then suddenly A is moved from its initial position to Q (shown in figure), as a result electric field intensity may change in the surrounding region. In Column I, the point (location) is specified where electric field intensity has to be analysed while in Column II some statements of electric field intensity are given. Math the entries of Column I with the entries of Column II.



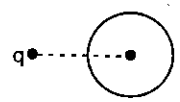
	Column I		Column II
(a)	$\vec{E}$ at new location of A (*i.e.*, at Q) due to B	(p)	Changes after some time of arrival of A and Q.
(b)	$\vec{E}$ at location of B due to A (now at Q)	(q)	Remains the same
(c)	$\vec{E}$ at location of C due to A (now at Q)	(r)	Changes instantaneously (as soon as A is moved)
(d)	$\vec{E}$ at location of C due to B	(s)	Increases

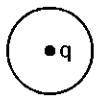
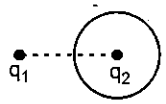
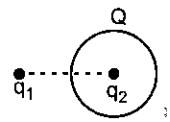
5. A neutral spherical conductor A has two spherical cavities B and C. Two point charges q_B and q_C are placed at centres of cavities B and C, respectively. Another point charge q is fixed at a large distance r from the centre of sphere. In Column I, information about force experienced by the particle is given while in Column II the details of the forces. Match the entries of Column I with the entries of Column II.



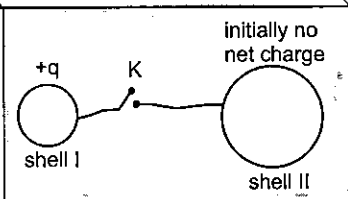
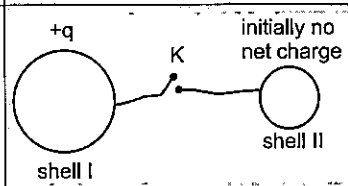
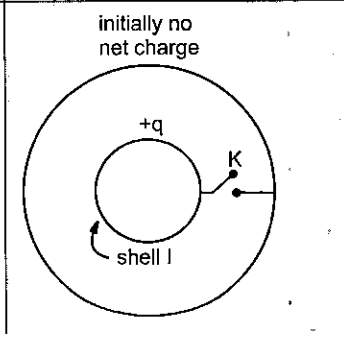
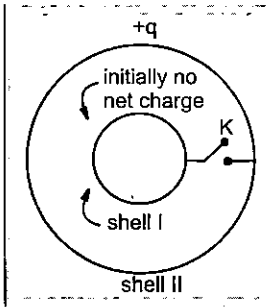
	Column I		Column II
(a)	Force on q_B	(p)	Zero
(b)	Force on q_C	(q)	Non-zero
(c)	Force on q	(r)	$\frac{(q_B + q_C)q}{4\pi\varepsilon_0 r^2}$
(d)	Force on conductor	(s)	Rightward

6. Match the entries of Column I with entries of Column II :

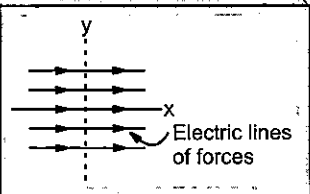
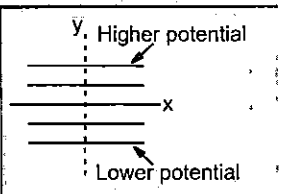
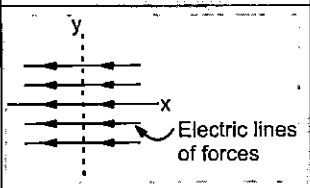
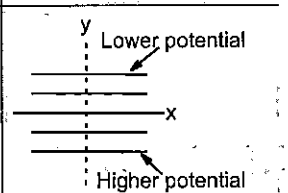
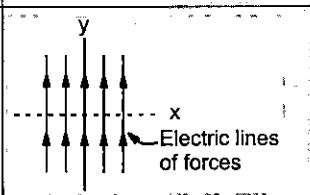
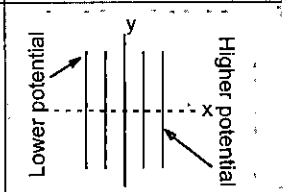
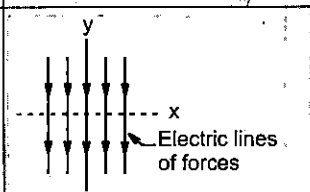
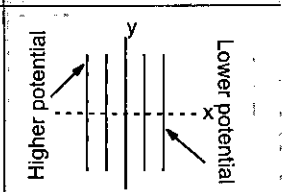
	Column I		Column II
(a)	Hollow neutral conductor 	(p)	$\vec{E}$ inside the conductor is zero

(b)	Hollow neutral conductor 	(q)	$\vec{E}$ inside the conductor is constant
(c)	Hollow neutral conductor 	(r)	$\vec{E}$ inside the conductor is varying
(d)	Hollow charged conductor 	(s)	Potential inside the conductor is same as that of conductor
		(t)	Potential inside the conductor is varying

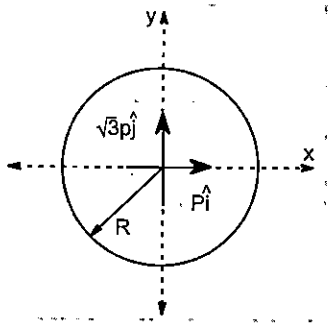
7. Column I gives certain situations involving two thin conducting shells connected by a conducting wire via a key K. In all situations one sphere has net charge $+q$ and other sphere has no net charge. After the key K is pressed, column II gives some resulting effect. Match the figures in Column I with the statements in Column II.

Column I		Column II	
(a)		(p)	Charge flows through connecting wire
(b)		(q)	Potential energy of system of sphere decreases.
(c)		(r)	No heat is produced.
(d)		(s)	The sphere I has no charge after equilibrium is reached.

8. Column I gives certain situation in which electric field is represented by electric lines of forces in x-y plane. Column II gives corresponding representation of equipotential lines in x-y plane. Match the figures in Column I with the figures in Column II.

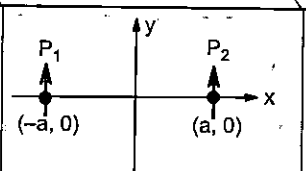
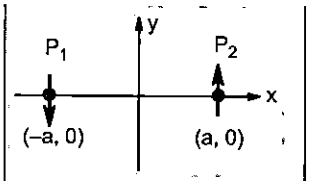
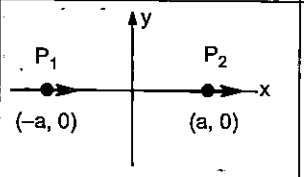
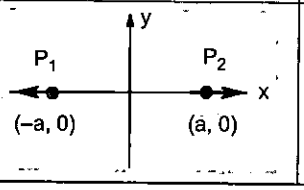
Column I		Column II	
(a)		(p)	
(b)		(q)	
(c)		(r)	
(d)		(s)	

9. Column I gives a situation in which two dipoles of dipole moment $p\hat{i}$ and $\sqrt{3}p\hat{j}$ are placed at origin. A circle of radius R with centre at origin is drawn as shown in Fig. Column II gives coordinates of certain positions on the circle. Match the statements in Column I with the statements in Column II.

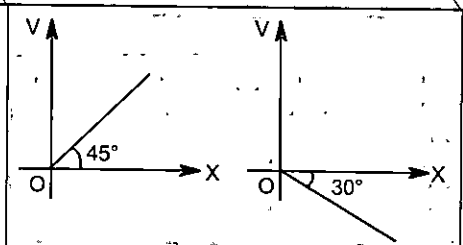
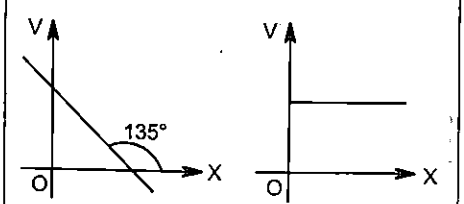
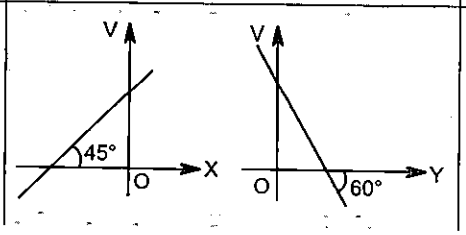


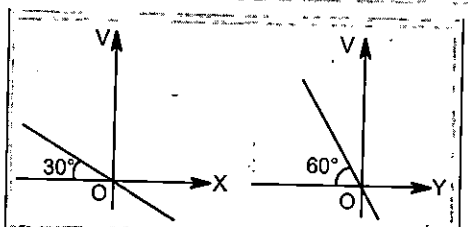
	Column I		Column II
(a)	The coordinate(s) of point on circle where potential is maximum	(p)	$\left(\frac{R}{2}, \frac{\sqrt{3}R}{2}\right)$
(b)	The coordinate(s) of point on circle where potential is zero	(q)	$\left(-\frac{R}{2}, -\frac{\sqrt{3}R}{2}\right)$
(c)	The coordinate(s) of point on circle where magnitude of electric field intensity is $\frac{1}{4\pi\varepsilon_0}\frac{4p}{R^3}$	(r)	$\left(-\frac{\sqrt{3}R}{2}, \frac{R}{2}\right)$
(d)	The coordinate(s) of point on circle where magnitude of electric field intensity is $\frac{1}{4\pi\varepsilon_0}\frac{2p}{R^3}$	(s)	$\left(\frac{\sqrt{3}R}{2}, -\frac{R}{2}\right)$

10. In each situation of column-I, two electric dipoles having dipole moments $\vec{\mathbf{p}}_1$ and $\vec{\mathbf{p}}_2$ of same magnitude (that is, $p_1 = p_2$) are placed on x-axis symmetrically about origin in different orientations as shown. In column-II certain inferences are drawn for these two dipoles. Then match the different orientations of dipoles in column-I with the corresponding results in column-II.

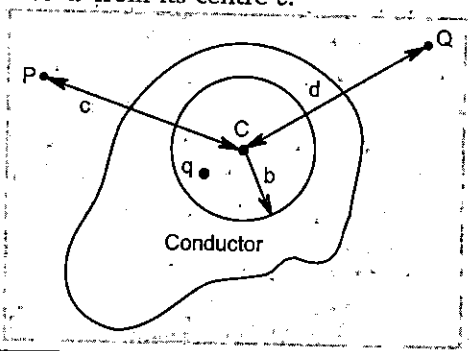
	Column I		Column II
(a)	 $\vec{\mathbf{P}}_1$ and $\vec{\mathbf{P}}_2$ are perpendicular to x-axis as shown	(p)	The torque on one dipole due to other is zero
(b)	 $\vec{\mathbf{P}}_1$ and $\vec{\mathbf{P}}_2$ are perpendicular to x-axis as shown	(q)	The potential energy of one dipole in electric field of other dipole is negative
(c)	 $\vec{\mathbf{P}}_1$ and $\vec{\mathbf{P}}_2$ are parallel to x-axis as shown	(r)	There is one straight line in x-y plane (not at infinity) which is equipotential
(d)		(s)	Electric field at origin is zero

11. Column I shows graphs of electric potential V *versus* x and y in a certain region for four situations. Column II gives angle which the electric field vector makes with positive X-direction.

	Column I		Column II
(a)		(p)	0°
(b)		(q)	$\tan^{-1}(3)$
(c)		(r)	120°

(d)		(s)	150°

12. In the Fig. shown, the conductor is uncharged and a charge q is placed inside a spherical cavity at a distance 'a' from its centre c.

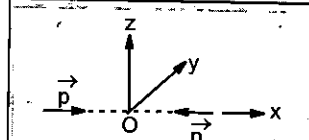
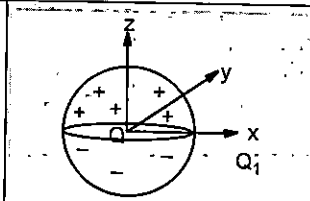
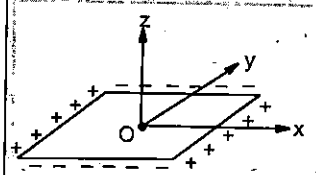
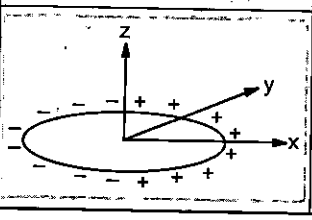


	Column I		Column II
(a)	Electric field due to induced charges on the inner surface of cavity at point P	(p)	zero
(b)	Electric potential due to charges on the inner surface of cavity and q at P	(q)	non-zero
(c)	Electric field due to induced charges on the outer surface of conductor and Q at c	(r)	value can be stated with the given data
(d)	Electric potential due to induced charges on the inner surface of cavity at c	(s)	value cannot be stated from the given data

13. A spherical metallic conductor has a spherical cavity. A positive point charge is placed inside cavity at its centre. Another positive point charge is placed outside (near) the conductor. The conductor is initially electrically neutral.

	Column I (Cause)		Column II (Effect)
(a)	When outside charge is shifted to other position	(p)	Distribution of charge on inner surface of cavity changes
(b)	When inside charge is shifted to other position within cavity	(q)	Distribution of charge on outer surface of conductor changes
(c)	When magnitude of charge inside cavity is increased	(r)	Electric potential at centre of conductor due to charges present on outer surface of conductor changes
(d)	When conductor is earthed	(s)	Force on charge inside cavity changes

14.

	Column I (Cause)		Column II (Effect)
(a)	 Two identical dipoles placed on x-axis at same distance from origin O	(p)	Electric potential at any pint on z-axis is zero
(b)	 Uniformly charged hemispherical shells; charge density σ	(q)	Electric field at any point on z-axis is either zero or $E_z > 0$
(c)	 Four uniformly charged rod of length L, forming a square charge density λ	(r)	Electric field at any point on z-axis with $z > 0$ is $E_z > 0$ and with $z < 0; E_z < 0$
(d)		(s)	Electric field at origin is either zero or points along $z > 0$

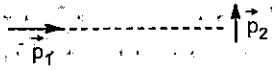
ASSERTION AND REASON

Direction: In the questions that follow two statements are given. Statement 2 is purported to be the explanation for statement 1. Study both the statements carefully and then Select your answers, according to the codes given below:

(a) If Statement 1 is true, Statement 2 is true; Statement 2 is the correct explanation for Statement 1.
(b) If Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation for Statement 1.
(c) If Statement 1 is true; Statement 2 is false.
(d) If Statement 1 is false; Statement 2 is true.

1. **Statement-1 :** Any net charge of a conductor resides on its outer surface.
   Statement-2 : The electrostatic energy of a conductor is minimum when charge is spread over the surface.

2. **Statement-1:** If a dipole ($\vec{p_1}$) is moved along the line normal to the axis (dotted line shown) of another dipole ($\vec{p_2}$), their interaction energy does not change.



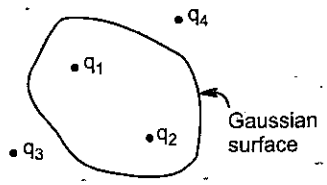
   Statement-2 : Electric field of $\vec{p_2}$ at the position of $\vec{p_1}$ is normal to $\vec{p_1}$.

3. **Statement 1:** In the frame of reference where all considered charges are rest, the force experienced by a moving test charge due to all considered charges are purely electrical.
   Statement 2: A stationary charge produces electric field only.

4. **Statement 1:** A device used to measure $\vec{E}$ is located a some distance from a fixed point charge. In this situation, the device measures E_0 as the magnitude of electric field intensity. Now an uncharged conducting sphere with a very small hole is lowered by an insulating thread so that it surrounds the point charge. Now, the reading of the device becomes zero.
   Statement 2: Electrostatic shielding is the phenomenon in which inside of hollow conductor is shielded for outside electric field.

5. **Statement 1:** $\vec{E}$ in outside velocity of a conductor depends only on the local charge density σ.
   Statement 2: $\vec{E}$ in outside vicinity of a conductor is given by $\frac{\sigma}{\varepsilon_0}$.

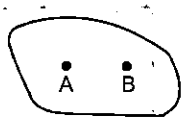
6. **Statement-1:** Four point charges q_1, q_2, q_3 and q_4 are as shown in Fig. The flux over the shown Gaussian surface depends only on charges q_1 and q_2.



   Statement-2: Electric field at all points on Gaussian surface depends only on charges q_1 and q_2.

7. **Statement-1:** Total work done by non-uniform electric field on a charged particle starting from rest till any time is non-negative. (assume no other forces act on the charged particle).
   Statement-2: The angle between electrostatic force an velocity of the charged particle released from rest in non-uniform electric field is always acute. (assume no other forces act on the charged particle).

8. **Statement-1:** A point charge q is placed near an arbitary shaped solid conductor as shown in figure. The potential difference between the points A and B within the conductor remain same irrespective of the magnitude of charge q.



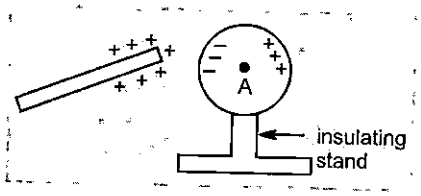
   Statement-2: The electric field inside a solid conductor is zero under electrostatic conditions.

9. **Statement-1:** For a non-uniformly charged thin circular ring with net charge zero, the electric field at any point on axis of the ring is zero.
   Statement-2: For a non-uniformly charged thin circular ring with net charge zero, the electric potential at each point on axis of the ring is zero.

10. **Statement-1:** Two point charges $+Q$ and $-Q$ are fixed at point $A(+a,0,0)$ and point $B(-a,0,0)$ respectively. Then the magnitude of electric flux due to electric field of either point charge through infinite y-z plane (that is $x = 0$ plane) is less than magnitude of net electric flux due to electric field of both charges through that plane ($x = 0$ plane).
    Statement-2: The magnitude of net electric flux through a surface due to a system of point charges is equal to sum of magnitude of electric flux through that surface due to each of the point charge of the system.

11. **Statement-1:** In a region where uniform electric field exists, the net charge within volume of any size is zero.
    Statement-2: The electric flux within any closed surface in region of uniform electric field is zero.

12. **Statement-1:** A positively charged rod is held near a neutral conducting solid sphere as illustrated below. The sphere lies on a insulated stand. The potential of ground (or earth) is zero. The potential at point A (point A need not be centre of the sphere) is higher compared to potential of ground (earth).



Statement-2: In the situation of statement-1, the potential at the centre of conducting sphere is positive. The solid sphere being conducting, potential at each point in the sphere is same.

13. **Statement-1:** A small uncharged solid conducting sphere is suspended from a fixed point O by a light insulating string (Fig.-1). The system is in equilibrium. Now an horizontal uniform electric field E is switched on. As a result the conducting sphere is deflected towards left as shown (Fig.-2)

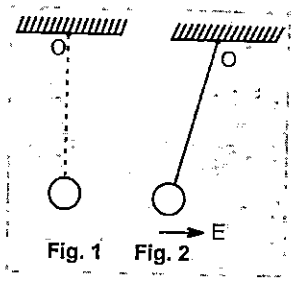
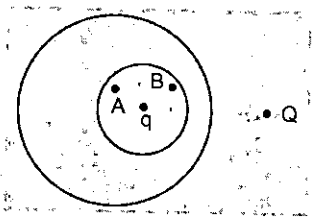


Fig. 1 Fig. 2

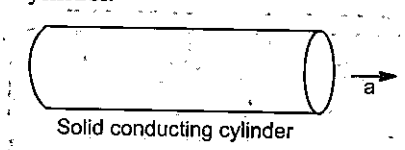
Statement-2: When a solid conducting sphere having zero net charge is placed in uniform electric field, charges are induced on the surface of sphere.

14. **Statement-1:** A point charge q is placed at centre of spherical cavity inside a spherical conductor as shown. Another point charge Q is placed outside the conductor as shown in Fig. Now as the point charge Q is pushed away from conductor, the potential difference $(V_A - V_B)$ between two points A and B within the cavity of sphere remains constant.



Statement-2: The electric field due to charges on outer surface of conductor and outside the conductor is zero at all points inside the conductor.

15. **Statement-1:** A solid uncharged conducting cylinder moves with acceleration a (w.r.t ground). As a result of acceleration of cylinder, an electric field is produced within cylinder.



Statement-2: When a solid conductor moves with acceleration a, then from frame of conductor a pseudoforce (of magnitude ma; where m is mass of electron) will act on free electrons in the conductor. As a result some portion of the surface of conductor acquires negative charge and remaining portion of surface of conductor acquires positive charge.

16. **Statement-1:** Two concentric conducting spherical shells are charged. The charge on the outer shell is varied keeping the charge on inner shell constant, as a result the electric potential difference between the two shells does not change.
Statement-2: If charge is changed on an isolated thin conducting spherical shell, the potential at all points inside the shell changes by same amount.

17. **Statement-1:** There is an isolated system of two charged conducting spheres A and B. The resultant electric field at point P is the sum of electric field at P due to charged sphere A only (that is, assuming sphere B and all its effects to be absent) and the electric field at P only due to sphere B (that is, assuming sphere A and all its effects to be absent).
Statement-2: Superposition theorem for electric field due to point charges states that resultant electric field at a point due to point charges is the sum of electric field at that point due to individual charges.

18. **Statement-1 :** Any net charge of a conductor resides on its outer surface.
Statement-2 : The electrostatic energy of a conductor is minimum when charge is spread over the surface.

19. **Statement-1 :** Assuming zero potential at infinity, gravitational potential at a point cannot be positive.
Statement-2 : Magnitude of gravitational force between two particles has inverse square dependence on distance between two particles

20. **Statement-1 :** A positive point charge initially at rest in a uniform electric field starts moving along electric lines of forces. (Neglect all other forces except electric forces)
Statement-2 : Electric lines of force represents path of charged particle which is released from rest in it.

21. **Statement-1 :** If electric potential while moving in a certain path is constant, then the electric field must be zero.
Statement-2 : Component of electric field $E_r = -\frac{\partial V}{\partial r}$.

22. **Statement-1 :** The electrostatic force on a charged particle located on an equipotential surface is zero.
Statement-2 : x-component of electric field is given by, $E_x = -\frac{\partial V}{\partial x}$ and on equipotential surface potential V is constant.

23. **Statement-1 :** We cannot produce electric field in a neutral conductor.
Statement-2 : Neutral conductor cannot produce electric field.

24. **Statement-1 :** An uncharged conducting slab is placed normally in a uniform electric field. The resultant electric field inside the slab is zero.
Statement-2 : The equal and opposite charges appearing on two surfaces of slab cancel the external field.

ANSWERS

Level-3: Comprehension Based Problems

Passage-1 1. (a) 2. (c) 3. (b)

Passage-2: 1. (d) 2. (b) 3. (b)

Passage-3 1. (d) 2. (a) 3. (a) 4. (d)

Passage-4: 1. (a) 2. (a) 3. (a) 4. (c) 5. (b)

Passage-5: 1. (b) 2. (b) 3. (d)

Passage-6: 1. (a) 2. (c) 3. (b)

Passage-7: 1. (b) 2. (d)

Passage-8 1. (b) 2. (d) 3. (c)

Passage-9 1. (a) 2. (d) 3. (b) 4. (d) 5. (c)

Passage-10 1. (c) 2. (a) 3. (d)

Passage-11 1. (b) 2. (a) 3. (c)

Passage-12 1. (b) 2. (c) 3. (a)

Passage-13 1. (c) 2. (a) 3. (b)

Passage-14 1. (b) 2. (c) 3. (d)

Passage-15 1. (c) 2. (a) 3. (b)

Passage-16 1. (c) 2. (b) 3. (d)

Passage-17 1. (a) 2. (a, b) 3. (b, c)

Passage-18 1. (b) 2. (d) 3. (d)

Matching Type Problem

1. (a) – q; (b) – p; (c) – r

2. (a) – s; (b) – r; (c) – p; (d) – q

3. (a) – q; (b) – p, s; (c) – p, q, r; (d) – q, r

4. (a)-q; (b)-p, s; (c)-p, s; (d)-q

5. (a)-p; (b)-p; (c)-q, r, s; (d)- q, r

6. (a)-p, q, s; (b)-r, t; (c)-r, t; (d)-r, t

7. (a)–p, q; (b)–p, q; (c)–p, q, s (d)–r, s

8. (a)–s; (b)– r; (c) – q; (d) – p

9. (a)–p; (b)–r; (c)–p, r; (d)–r, s

10. (a)–p, q, r; (b)–p, r, s; (c)–p, s; (d)–p, q

11. (a)–s; (b)–p; (c)–r; (d)–q

12. (a)–q, s; (b)–p, r; (c)–p, r; (d)–q, r

13. (a)–q; (b)–p, s; (c)–p, q, r; (d)–q, r

14. (a)–p, s; (b)–q, s; (c)–p, q, s; (d)–p, s

Assertion and Reason

1.	(b)	2.	(a)	3.	(a)	4.	(c)	5.	(d)	6.	(c)
7.	(c)	8.	(a)	9.	(d)	10.	(c)	11.	(a)	12.	(a)
13.	(d)	14.	(a)	15.	(a)	16.	(a)	17.	(d)	18.	(a)
19.	(b)	20.	(c)	21.	(d)	22.	(d)	23.	(c)	24.	(a)

Level-3: Comprehension Based Problems

Passage-1

1. (a) Since the acceleration provided by electric field is perpendicular to initial velocity, therefore path will be parabola.

2. (c) $a_x = 0, \quad a_y = \frac{qE}{m}$

$U_x = V_0$

$\Rightarrow \quad x = V_0 t$

and $\quad y = \frac{1}{2}\frac{qEt^2}{m}$

$\Rightarrow \quad y = \frac{qEx^2}{2mV_0^2}$

For same value of x and initial kinetic energy 'y' is same.

3. (b) Since mass of proton > mass of electron
Therefore deflection is greater for electron as

$$y = \frac{qEx^2}{2mV_0^2}$$

i.e., $\quad y \propto \frac{1}{m}$

Passage-2

1. (d) Charges outside cavity of a conductor have no influence on the field inside cavity.

2. (b) There is charge inside cavity so field lines will be present but the potential at surface of conductor is same everywhere. So field lines should be perpendicular.

3. (b) We see that the surface of two spherical cavities carry charge $-q_1$, $-q_2$ respectively and, since the sphere A was not charged originally its spherical surface must carry induced charges $(q_1 + q_2)$

$\therefore$ force acting on conductor $A = \frac{q_3(q_1 + q_2)}{4\pi\varepsilon_0 r^2}$

Passage-3

1. (d) Electric field is always net electric field in the formulae

$$\int \vec{E} \cdot d\vec{s} = \frac{q}{\varepsilon_0}$$

While $q \rightarrow$ charge inside the closed surface

$\therefore$ E_p is due to q_1, q_2, q_3 and q_4

2. (a) Net flux depends on charge enclosed *i.e.*, due to q_1 & q_2 only

4. (d) Electric field will change since two of the charges are displaced from their earlier position while flux will not change since q_1 and q_2 are still inside the surface.

Passage-6

Electric field $E = -\frac{\lambda}{2\pi\varepsilon_0 x} + \frac{Q}{4\pi\varepsilon_0}\frac{x}{(R^2 + x^2)^{3/2}}$

(considering right direction as positive)

$$= \frac{\lambda}{2\pi\varepsilon_0}\left[-\frac{1}{x} + \frac{x4\sqrt{2}R}{2(R^2 + x^2)^{3/2}}\right]$$

$$= \frac{\lambda}{2\pi\varepsilon_0}\left[-\frac{1}{x} + \frac{2\sqrt{2}xR}{(R^2 + x^2)^{3/2}}\right]$$

1. (a) Initially $x = \sqrt{3}R$

$$E = \frac{\lambda}{2\pi\varepsilon_0 R}\left[-\frac{1}{\sqrt{3}} + \frac{2\sqrt{2}\sqrt{3}}{8}\right]$$

$$\frac{\lambda}{2\pi\varepsilon_0 R}\left[\frac{-2\sqrt{2}+3}{\sqrt{3}(2\sqrt{2})}\right] = \frac{\lambda}{2\pi\varepsilon_0 R}\left[\frac{3-2\sqrt{2}}{2\sqrt{6}}\right]$$

(acceleration) $a = \frac{(-e)(E)}{m} = \frac{e\lambda}{\pi\varepsilon_0 mR}\left(\frac{3-2\sqrt{2}}{4\sqrt{6}}\right)$

2. (c) Force on electron is zero at point where

$$E = 0 \Rightarrow x = R$$

3. (b) Potential difference between two points

$$\Delta V = -E\,dx$$

Pd. due to line charge between $x = R$ & $x = \sqrt{3}R$

$$V_A - V_B = -\int_R^{\sqrt{3}R} \frac{\lambda dx}{2\pi\varepsilon_0 x} = \frac{\lambda}{\pi\varepsilon_0}\left(\frac{\ln 3}{4}\right)$$

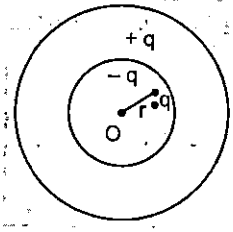
Potential difference due to ring between $x = \sqrt{3}R$ and $x = R$

$$V_A - V_B = \frac{1}{4\pi\varepsilon_0}\left(\frac{\sqrt{2}\lambda R}{2R} - \frac{4\sqrt{2}\lambda R}{\sqrt{2}R}\right)$$

Net $V_A - V_B = \frac{\lambda}{\pi\varepsilon_0}\left[\left(1 - \frac{1}{\sqrt{2}}\right)\frac{\ln 3}{4}\right]$

Passage-8

The situation is as shown in the Fig.



The charge $-q$ induces on inner surface and due to neutral nature of shell $+q$ gets induced on outer surface as shown in the figure.
The distribution of charge on inner surface is non-uniform while on outer surface is uniform.

1. (b)

Potential at point O, V = potential at O due to point charge placed + potential due to induced charges on inner surface + potential due to induced charge on outer surface.

$$V = \frac{q}{4\pi\varepsilon_0 r} - \frac{q}{4\pi\varepsilon_0 R_1} + \frac{q}{4\pi\varepsilon_0 R_2}$$

[For 2nd term, even though the charge is distributed non-uniformly, the entire charge is at a distance R_1 from O].

2. (d)

Electric field intensity at point O can't be determined as the nature of distribution of charge induced on inner surface is not known.

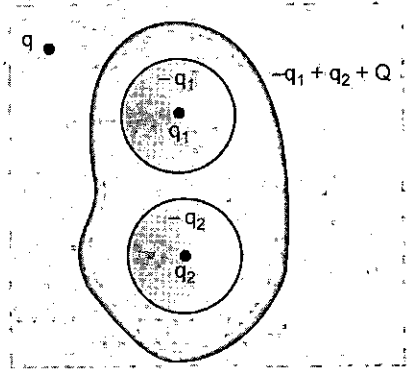
3. (c)

Electric potential of the conductor is 3rd term of the point at point O. From basic definition also we can find this.

Passage-9

1. (a)

The charge distribution on various surfaces is as shown in the figure, $-q_1$ on the surface of cavity



1 will spread uniformly if q_1 is at the centre, otherwise, the distribution would be non-uniform. Same is the case with $-q_2$.

The charge appearing on outer surface of the conductor is q_1+q_2+Q which would be non-uniformly distributed as radius of curvature at various points of conductor's surface is different. The presence of q_1 and q_2 (and their locations) have no effect on the distribution of charge on the outer surface. The presence of q will change the distribution of charge on the outer surface, but still it remains non-uniform.

2. (d)

Electric field intensity outside the cavity due to q_1 and $-q_1$ would be zero.

$$\vec{\mathbf{E}}_{q_1} + \vec{\mathbf{E}}_{-q_1} = 0$$

$$\Rightarrow \quad \vec{\mathbf{E}}_{-q_1} = -\vec{\mathbf{E}}_{q_1} = \frac{q_1}{4\pi\varepsilon_0 r^2}$$

towards centre of cavity 1.

3. (b)

$\vec{\mathbf{E}}$ inside to conductor due to outside charges = 0

$$\Rightarrow \quad \vec{\mathbf{E}}_q + \vec{\mathbf{E}}_{q_1+q_2+Q} = 0$$

or, $$\vec{\mathbf{E}}_{q_1+q_2+Q} = -\vec{\mathbf{E}}_q$$

$$= \frac{q}{4\pi\varepsilon_0 r^2} \text{ towards } q.$$

4. (d)

If q_2 is at point Q, then induced charge $-q_2$ would be non-uniformly distributed, so we cannot determine $\vec{\mathbf{E}}$ due to $-q_2$ at any inside point.

5. (c)

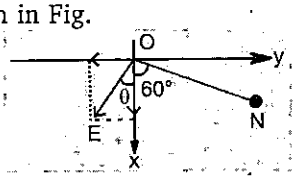
Potential at point Q = potential of conductor + Potential due to q_2 + Potential due to $-q_2$

$$= V_0 + \frac{q_2}{4\pi\varepsilon_0 r_2'} + \frac{-q_2}{4\pi\varepsilon_0 r_2}$$

$$= \frac{q_2}{4\pi\varepsilon_0}\left(\frac{1}{r_2'} - \frac{1}{r_2}\right) + V_0$$

Passage-10

The direction of electric field is in x-y plane as shown in Fig.



The magnitude of electric field is

$$E = \sqrt{E_x^2 + E_y^2} = \sqrt{3+1} = 2\text{V/m}$$

The direction of electric field is given by

$$\theta = \tan^{-1}\frac{E_y}{E_x} = \tan^{-1}\frac{1}{\sqrt{3}} = 30°$$

Hence electric field is normal to square frame $LMNO$ as shown in figure.

$\therefore$ electric flux $= \vec{\mathbf{E}}\cdot\vec{\mathbf{A}} = E\,A\cos 0 = 2\times 1 = 2$ V/m

$\therefore$ (c) is correct option of question-1

Since flux is maximum at $\theta = 60°$, rotation by 30° either way would lead to decrease in flux.

$\therefore$ (d) is correct option of question-3

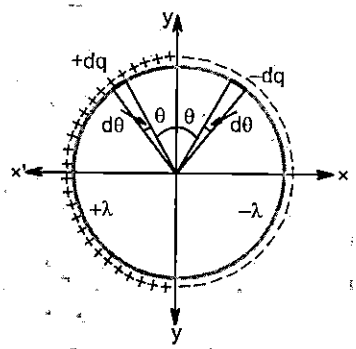
Lines ON and NM are both normal to uniform electric field $\vec{\mathbf{E}}$. Hence work done by electric field as a point charge 1 μC is taken from O to M is zero.

$\therefore$ (a) is correct option of question-2.

Passage-11

1. (b)

Consider two small elements of ring having charges $+dq$ and $-dq$ symmetrically located about y-axis.



The potential due to this pair at any point on y-axis is zero. The sum of potential due to all such possible pairs is zero at all points on y-axis.

Hence potential at $P\left(0, \frac{R}{2}\right)$ is zero.

2. (a)

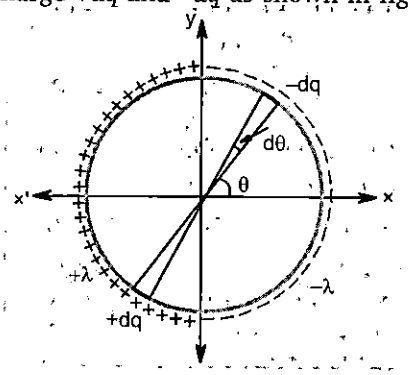
Since all charge lies in x-y plane, hence direction of electric field at point P should be in x-y plane

Also y-axis is an equipotential (zero potential) line. Hence direction of electric field at all point of y-axis is should be normal to y-axis.

$\therefore$ The direction of electric field at P should be in x-y plane and normal to y-axis. Hence direction of electric field is along positive x direction.

3. (c)

Consider two small elements of ring having charge $+dq$ and $-dq$ as shown in figure.



The pair constitutes a dipole of dipole moment.

$$dp = dq.2R$$
$$= (\lambda R d\theta)2R$$

The net dipole moment of system is vector sum of dipole moments of all such pairs of elementary charges.

By symmetry the resultant dipole moment is along negative x-direction.

$\therefore$ Net dipole moment

$$= -\int_{-\pi/2}^{+\pi/2} (dp\cos\theta)\,\hat{\mathbf{i}}$$
$$= -\int_{-\pi/2}^{+\pi/2} (2\lambda R^2\cos\theta\, d\theta)\hat{\mathbf{i}}$$
$$= -4R^2\lambda\,\hat{\mathbf{i}}$$

Passage-12

The inner sphere is grounded, hence its potential is zero. The net charge on isolated outer sphere is zero. Let the charge on inner sphere be q'.

$\therefore$ Potential at centre of inner sphere is

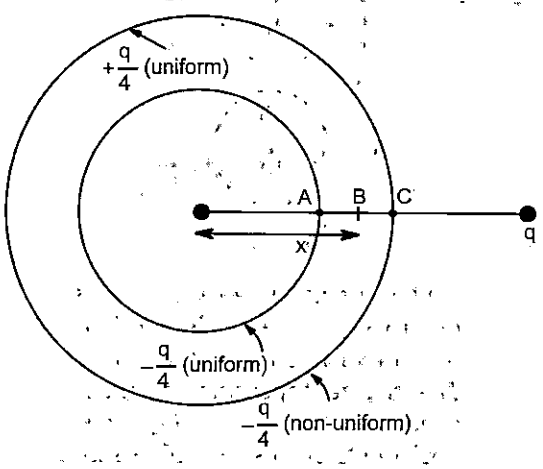
$$= \frac{1}{4\pi\varepsilon_0}\frac{q'}{a} + 0 + \frac{1}{4\pi\varepsilon_0}\frac{q}{4a} = 0$$

$\therefore$ $$q' = -\frac{q}{4}$$

The region in between conducting sphere and shell is shielded from charges on and outside the outer surface of shell. Hence charge distribution on surface of sphere and inner surface of shell is uniform.

The distribution of induced charge on outer surface of shell depends only on point charge q, hence is non-uniform.

The charge distribution on all surfaces, is as shown in Fig.



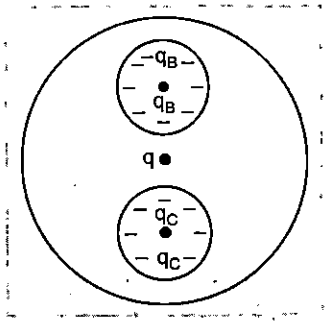
The electric field at B is $= \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{4x^2}$ towards left.

$\therefore$ $$V_C = V_C - V_A = -\int_{2a}^{a}\frac{1}{4\pi\varepsilon_0}\frac{q}{4x^2}dx$$
$$= \frac{1}{32\pi\varepsilon_0}\cdot\frac{q}{a}$$

Matching Type Problem

4. Electric field propagates with finite speed equal to speed of light. On the basis of the concept you can solve the things.

5. Here , we have to use the concept "Electric field at any outside point of conducting shell due to all inside charges is zero and *vice-versa*". The charge distribution on various faces are as shown in figure.



Distribution of charge on surfaces of cavities are uniform and hence electric field due to induced charges on surface of cavities at their respective centres is zero.

So, $F_{qB} = F_{qC} = 0$

The charge on conductor A is $q_B + q_C$ and is distributed uniformly, so force experienced by q (which is only due to $q_B + q_C$) is $\frac{(q_B + q_C)q}{4\pi\varepsilon_0 r^2}$.

6. For A, as no charge is there inside the conductor, so $\vec{\mathbf{E}}$ inside the conductor is zero and hence potential is constant and equal to that of potential of the conductor.

For other cases, $\vec{\mathbf{E}}$ inside conductor is non-zero and varying as we are going from centre to periphery and so potential inside the conductor is varying.

7. In situation a, b and c, shells I and II are not at same potential. Hence charge shell flow from sphere I to sphere II till both acquire same potential.

If charge flows, the potential energy of system decreases and heat is produced.

In situations a and b charges shall divide in some fixed ratio, but in situation (c) complete charge shall be transferred to shell II for potential of shell I and II to be same.

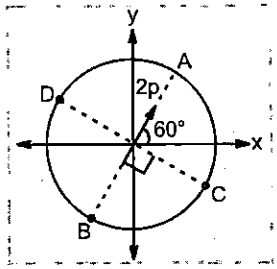
In situation (d) both the shells are at same potential, hence no charge flows through connecting wire.

8. Electric field is uniform in all four cases.

Equipotential lines in x-y plane shall be normal to corresponding electric lines of forces. Also direction of electric field is from region of higher potential to lower potential.

9. The resultant dipole moment has magnitude $\sqrt{(\sqrt{3}P)^2 + P^2} = 2P$ at an angle.

$\theta = \tan^{-1}\frac{\sqrt{3}P}{P} = 60°$ with positive x-direction.



Diameter AB is along net dipole moment and diameter CD is normal to net dipole moment.

Potential at $A\left(\frac{R}{2}, \frac{\sqrt{3}R}{2}\right)$ is maximum

Potential is zero at $C\left(\frac{\sqrt{3}R}{2}, -\frac{R}{2}\right)$ and $D\left(-\frac{\sqrt{3}R}{2}, \frac{R}{2}\right)$

Magnitude of electric field is $\frac{1}{4\pi\varepsilon_0}\frac{4p}{R^3}$

at $A\left(\frac{R}{2}, \frac{\sqrt{3}R}{2}\right)$ and $B\left(-\frac{R}{2}, -\frac{\sqrt{3}R}{2}\right)$

Magnitude of electric field is $\frac{1}{4\pi\varepsilon_0}\frac{2p}{R^3}$

at $C\left(\frac{\sqrt{3}R}{2}, -\frac{R}{2}\right)$ and $D\left(-\frac{\sqrt{3}R}{2}, \frac{R}{2}\right)$

10. The electric field due to one dipole at centre of other dipole is parallel to that dipole. Hence torque on all given dipoles is zero.

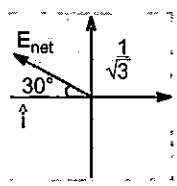
In case B and C the electric field at second dipole due to first is along the second dipole. Hence electrostatic potential energy of second dipole is positive.

In case A and B x-axis is line of zero potential. In case B and C electric field at origin is zero.

11. (a) $E_x = \left(-\frac{dv}{dx}\right)\hat{\mathbf{i}} = -1\hat{\mathbf{i}}$

$E_y = \left(-\frac{dv}{dx}\right)\hat{\mathbf{j}} = \frac{1}{\sqrt{3}}\hat{\mathbf{j}}$

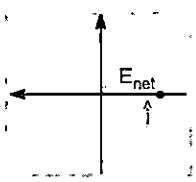
$\tan\theta = \frac{1}{\frac{\sqrt{3}}{1}} = 30° \quad \Rightarrow \theta = 150°$



(b) $E_x = \left(-\frac{dv}{dx}\right)\hat{\mathbf{i}} = +1\hat{\mathbf{i}}$

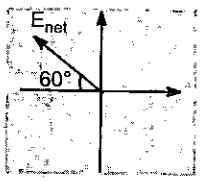
$E_y = \left(-\frac{dv}{dx}\right)\hat{\mathbf{j}} = 0$

$\theta = 0°$



(c) $E_x = \left(-\frac{dv}{dx}\right)\hat{i} = -1\hat{i}$

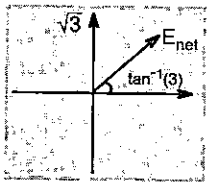
$E_y = \left(-\frac{dv}{dx}\right)\hat{j} = +\sqrt{3}\,\hat{j}$



$\Rightarrow \quad \theta = 120°$

(d) $E_x = \left(-\frac{dv}{dx}\right)\hat{i}$

$= -\left(-\frac{1}{\sqrt{3}}\right)\hat{i} = \frac{1}{\sqrt{3}}\hat{i}$



$E_y = \left(-\frac{dv}{dy}\right)\hat{j}$

$= -(-\sqrt{3})\hat{j} = \sqrt{3}\hat{j}$

Alternative

$\vec{E} = -\frac{\delta V}{\delta x}\hat{i} - \frac{\delta V}{\delta y}\hat{j}$

(a) $\vec{E} = -\hat{i} + \frac{1}{\sqrt{3}}\hat{j}$ $\quad \theta = 150°$

(b) $\vec{E} = -\hat{i} + 0\hat{j}$ $\quad \theta = 0°$

(c) $\vec{E} = -\hat{i} + \sqrt{3}\hat{j}$ $\quad \theta = 120°$

(d) $\vec{E} = \frac{1}{\sqrt{3}}\hat{i} + \sqrt{3}\hat{j}$ $\quad \theta = \tan^{-1}(3)$

Assertion and Reason

1. (b) Since charge on a conductor is movable, it moves out to repel each other. Their potential energy will be minimum when they are farthest apart *i.e.* at the surface because minimum potential energy ensures electrostatic equilibrium.

2. (a) Potential energy of a dipole is given by

$$U = -\vec{p}.\vec{E} = -pE\cos\theta$$

if $\theta = 90°, U = 0$

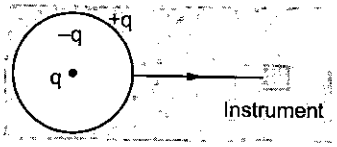
3. (a) A moving charge produces both electric and magnetic fields while a stationary charge produces electric field only in its surrounding region. In the frame of reference where all charges are at rest, only electric field would be present and hence a moving test charge will experience only electrostatic force (electrical force).

4. (c) Statement 2 is correct but Statement 1 is wrong. When point charge is surrounded by uncharged conductor, some charge gets induced on inner and outer surface of sphere as shown.
For any position of charge inside the sphere the charge on outer surface of sphere is uniform so the electric field measured by device would be same as before.

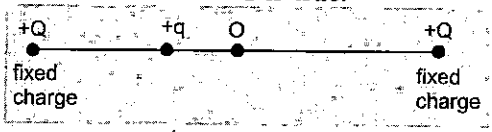
5. (d) $\vec{E}$ in outside vicinity of conductors surface is depending on all the charges present in the space, but expression $\vec{E} = \frac{\sigma}{\varepsilon_0}$ leads to a misconception.

6. (c) Statement-1 is true directly from Gauss Theorem. Statement-2 is false. Electric field at any point on Gaussian surface depends on all four charges.

7. (c) From work energy theorem
Final K.E. = Initial K.E. + work done by non-uniform electric field.



$\because$ initial K.E. = 0 and final K.E. cannot be negative.
$\therefore$ Work done by non-uniform electric field on a charged particle starting from rest is non-negative.
Hence Statement-1 is true.

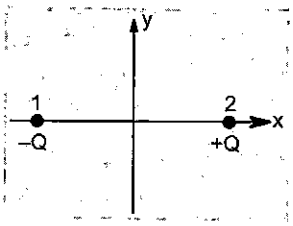


Consider a situation in which two point charges +Q are fixed some distance apart. At some distance left of equilibrium point *O*, a charge +*q* is released from rest. After the charge +*q* moving towards right crosses *O*, it experiences a force towards left.
Hence statement-2 is false.

8. (a) Since electric field inside the conductor is zero, *A* and *B* are always at same potential.
Hence $V_A - V_B = 0$. Hence statement-1 and 2 are true and statement-2 is correct explanation of statement-1.

9. (d) For a non-uniformly charged thin circular ring with net zero charge, electric potential at each point on its axis is zero. Hence electric field at each point on its axis must be perpendicular to the axis. Therefore statement-1 is false and statement-2 is true.

10. (c) Take direction of area of *y-z* plane in negative *x*-direction.
The electric flux through *y-z* plane due to charge 1 and charge 2 is



$$\phi_1 = \int \vec{E}_1 \cdot \vec{dS}$$

and $\phi_2 = \int \vec{E}_2 \cdot \vec{dS}$

where $\vec{E}_1$ and $\vec{E}_2$ is electric field due to charge 1 and charge 2 respectively.

Net flux due to both charges is

$$\phi_1 = \int \vec{E} \cdot \overrightarrow{dS} = \int (\vec{E}_1 + \vec{E}_2) \cdot \overrightarrow{dS}$$

$$= \int \vec{E}_1 \cdot \overrightarrow{dS} + \int \vec{E}_2 \cdot \overrightarrow{dS} = \phi_1 + \phi_2$$

$\therefore$ $|\phi_1| = |\phi_1 + \phi_2|$ Hence statement-1 is true.

But $|\phi| \neq |\phi_1| + |\phi_2|$ hence statement-2 is false.

11. (a) Electric flux within any closed surface in region of uniform electric field is zero because the total number of electric lines of forces entering the closed surface equals that exiting the surface. Hence from Gauss theorem the net charge enclosed within such a closed surface is zero.

12. (a) The potential at centre of sphere is only due to the charged rod and hence positive. The potential at centre of sphere due to induced charges on its surface is zero. Hence the net potential at centre is positive. The solid sphere being conducting, potential at each point in the sphere is same. Hence statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for statement-1

13. (d) Charge is induced on surface of conducting sphere when electric field is switched on, but net charge on conducting sphere remains zero. Electric force on a charged body placed in uniform electric field is the product of charge and electric field. Hence force exerted by electric field on conducting sphere is zero. Therefore statement-1 is false.

14. (a) The electric field inside the cavity depends only on point charge q. Hence $V_A - V_B$ remains constant even if point charge Q is shifted. Here statement-2 is correct explanation of statement-1

15. (a) From the frame of cylinder, the pseudo force acting on free electrons shifts them towards left. As a result, left end of cylinder becomes negatively charged and right end becomes positively charged. The electric field thus produced balances the pseudo force when a stage of equilibrium is reached.

Here statement-2 is correct explanation of statement-1.

16. (a) The potential at each point within an isolated thin conducting charged shell is same. Hence if charge is changed on this shell, potential at all points within the shell change by same amount. Hence statement-2 is true.

The potential 'V' at any point on or inside the outer shell in statement-1 is sum of potential due to outer shell (V_1) and potential due to inner shell (V_2).

$\therefore$ $V = V_1 + V_2$

From statement-2 'V_1' (potential due to outer shell) at all points on or inside the outer shell is same. Further as the charge on outer shell is changed, potential at all points on and inside outer shell change by same amount. Hence potential difference between both the shells remains same. Hence statement-1 is true and statement-2 is correct explanation.

17. (d) In absence of the charged conducting sphere B, charge on the sphere A will no longer be non-uniform, that is, charge on the sphere will be uniformly distributed. Hence electric field due to sphere A to P in presence of sphere B and in absence of sphere B will be different. So we cannot add electric field in this manner. For the superposition to be valid the nature of charge distribution should not change or should not be change while taking individual contributions. Statement-1 is false.

18. (a) Since charge on a conductor is movable, it moves out to repel each other. Their potential energy will be minimum when they are farthest apart *i.e.*, at the surface.

19. (b) Gravitational potential is always –ve because

$V = -\int_{\infty}^{x} \vec{E} \cdot d\vec{s}$, here E is always –ve in case of gravitation because force is attractive

$F = -\frac{GmM}{r^2}$, but this is not reason for potential to be –ve. Real reason is that force is attractive, so the potential energy decreases as a mass moves from ∞ to r {K.E. increases}. So U is always –ve.

2 ELECTRIC CURRENT

ELECTRIC CURRENT

The electric current characterizes the flow of charge through a material Fig. 2.1 (a) shows a section of conducting wire with positive charge carriers moving to the right. Let dQ be the magnitude of the charge that passes through the plane cross-sectional surface labeled S in time dt.

The electric current I in the wire is the rate at which charge passes through this surface:

$$I = \frac{dQ}{dt}$$

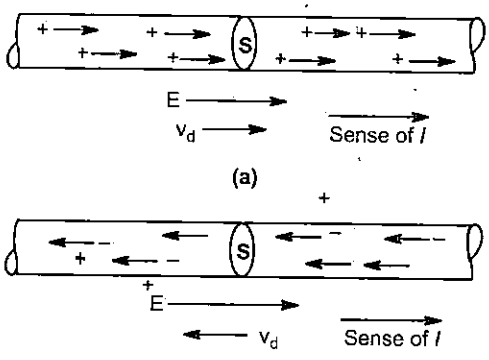


Fig. 2.1

The SI unit of electric current is the **ampere** (A), equal to one coulomb per second: 1A = 1C/s.

Electric current I is a scalar quantity. Even though the electric current is not a vector quantity, it is common practice to speak of the "direction" of the current. This direction corresponds to the direction of flow of positive charge carriers. To emphasize that current is a scalar, we shall refer to the **sense** of the current. The sense of the current in a conductor is given by the direction of motion of positive charge carriers. For example, the current in Fig. 2.1 (a) is to the right.

Consider the effect of the sign of the charge carriers on the sense of the current. Positive carriers moving to the right tend to cause the region to the right to become more positive and the region to the left to become more negative. In Fig. 2.1 (b), negative carriers moving to the left also tend to cause the region to the right to become more positive and the region to the left to become more negative. That is, the carrier motion shown in both Fig. 2.1 (a) and (b) gives the same result. Thus the sense of the current is the same in Fig. 2.1 (a) as it is in Fig. 2.1 (b) in each case it is to the right. **This means that we need not be concerned with the sign of the carriers when dealing with the external effects of a current; these effects are the same for carriers having either sing.**

Concepts: ***1.*** *Conductors are materials in which electric charges readily flow. In many metals, these electrons are free to move when an electric field is applied to the material. Under static conditions the electric field in the interior of a conductor is zero, even if the conductor carries a net charge. Otherwise the free electrons would be accelerated, which would violate the assumption of a static charge distribution.*

2. *In an insulator, on the other hand, the electrons are bound rather tightly to the atoms and are not free to move under the electric fields. An insulator can carry any distribution of electric charges on its surface or in its interior, and the electric field in the interior of an insulator can have non-zero values.*

3. *In an insulating material, molecules are not easily ionized, the electrical properties may depend on the electric dipole moment of the molecules. Materials in which the molecules have permanent dipole moments are called* ***polar****, and* ***electric fields*** *can align the dipole moments of molecules. In some materials, the alignment of the dipoles remains even when we remove the applied field; these materials are called* ***ferroelectric.***

Even in non-polar materials the applied electric field can induce a dipole moment in the molecules. In an insulator, a sufficiently large electric field can ionize the atoms, and as a result there are electrons available to move through the material. Under these circumstances an insulator can behave more like a conductor. This situation is called ***breakdown*** *and requires fields typically in the range of* 10^6 *V/m in air .*

4.

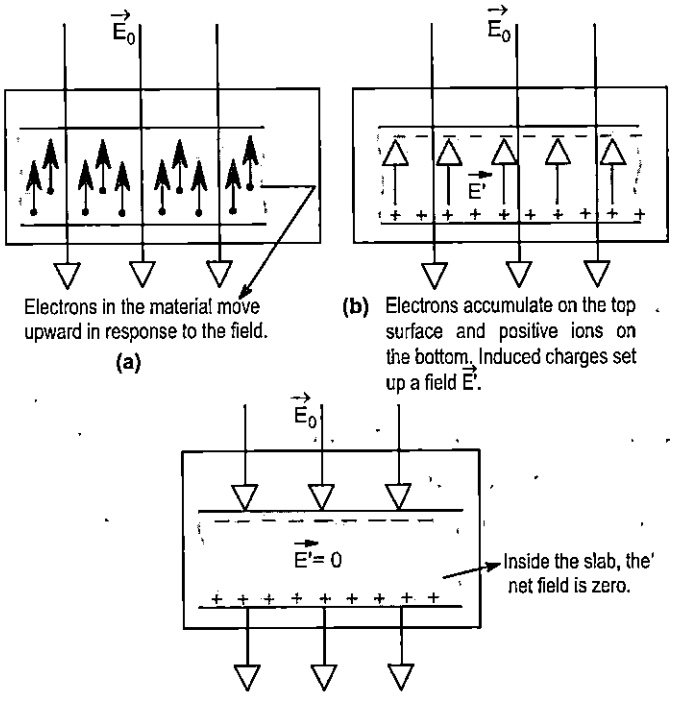


Fig. 2.2

In Fig. 2.2 (a) electrons move from the bottom of the slab of copper to the top under the action of the applied electric field, until the concentration of electrons at the top of the slab (and of positive ions at the bottom) creates a field that cancels the applied field in the interior of the copper and prevents the flow of additional electrons. Suppose there were a mechanism to remove electrons from the top of the slab, carry them around an external path, and re-inject them at the bottom of the slab (shown schematically in Fig. 2.3). In this case, there would be no build-up of charge on the top and bottom of the slab.

The continuous loop of flowing electrons is a simple representation of an electric circuit, and the flow of electrons (or other charged particles) is called an ***electric current.***

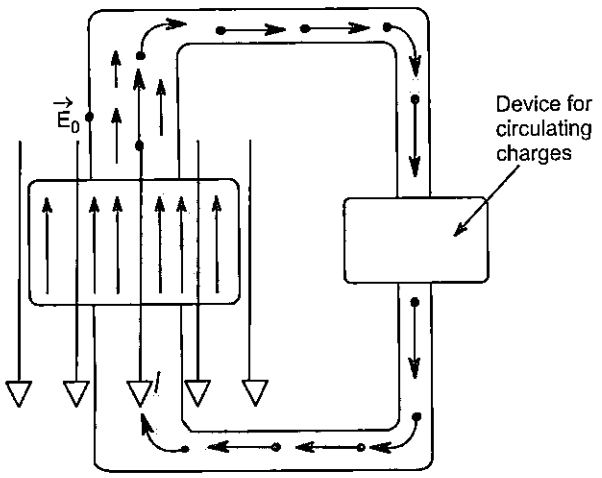


Fig. 2.3. The electric field $\vec{E}_0$ moves electrons through the slab of copper. The electrons collected at the top of the slab are transported through an external path to the bottom of the copper slab by a device.

ELECTRIC CURRENT DENSITY

Let the density of charge carriers be n (number of charge carriers per unit volume of the material). Let q be the charge on each charge carrier. We imagine the current carrying wire to be cylindrically shaped. We consider a reference location, as shown in Fig. 2.4. All the charges within $v_d\,dt$ to the left of the reference location will pass the location during the time interval dt. The charge conservation implies that net charge entering and leaving any cross-section must be the same. The number of charge carriers within this volume is :

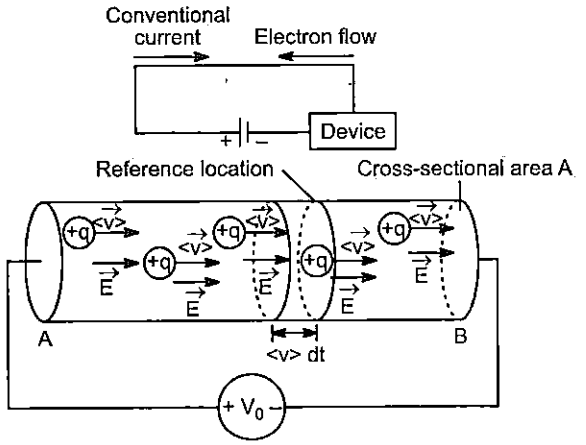


Fig. 2.4

No. of charge carriers per unit volume × volume $= nAv_d dt$

If each charge carrier has charge q, the charge dQ within this volume is

$$dQ = qnA\,v_d dt$$

As current is defined as $I = \dfrac{dQ}{dt}$, so we have

$$I = qnAv_d$$

If the charge flowing through an area varies with position across that area as shown in Fig. 2.5, the upper part has more charges crossing per unit area than the lower part. Analogous to the idea of electric flux, the electric field crossing a unit area, we define the current I as the flux of current density through an area.

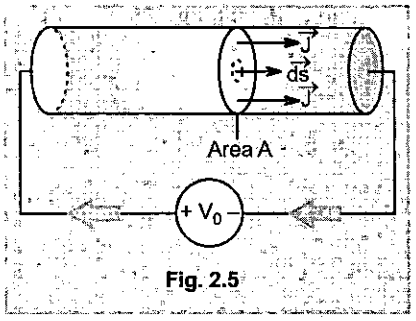


Fig. 2.5

$$I = \int_{\text{area}} \vec{\mathbf{J}} \cdot d\vec{\mathbf{s}}$$

Regardless of the orientation of the cross-sectional area through the flux is taken, we obtain the same current or flux of the current density $\vec{\mathbf{J}}$. (See Fig. 2.6.)

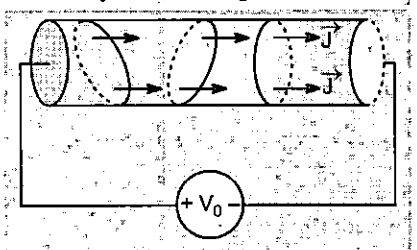


Fig. 2.6

To describe the flow of charge at points within a conductor, we use the current density J, which is a vector quantity. If the current density is uniform the magnitude J of the current density is the current I divided by the cross-sectional area A of the wire:

$$J = \frac{I}{A} \quad \text{(uniform } \vec{\mathbf{J}}\text{)}$$

J in terms of the drift speed v_d:

$$J = \frac{nAv_d|q|}{A} = nv_d|q|$$

This result can be expressed as a vector equation using the drift velocity $\vec{\mathbf{v}}_d$:

$$\vec{\mathbf{J}} = nq\vec{\mathbf{v}}_d$$

Note that the absolute value sign has been taken from $|q|$. Thus the current density points in the direction of $\vec{\mathbf{v}}_d$ for positive carriers, and it points opposite $\vec{\mathbf{v}}_d$ for negative carriers. Consequently, the direction of $\vec{\mathbf{J}}$ coincides with the sense of the current in a wire.

Concept: *If the conductor contains more than one type of charge carrier, then there is a contribution to $\vec{\mathbf{J}}$ from each type of carrier. Suppose there are two types of charge carriers, a and b. Then,*

$$\vec{\mathbf{J}} = n_a q_a \vec{\mathbf{v}}_{da} + n_b q_b \vec{\mathbf{v}}_{db}$$

where the subscripts a and b designate the quantities for each type of charge carrier.

If the drift velocity of the carriers varies from point to point within a material, as shown in Fig. 2.7, then the current density varies correspondingly. In this case the current I through a surface can be found from the surface integral of the current density $\vec{\mathbf{J}}$:

$$I = \int \vec{\mathbf{J}} \cdot d\vec{\mathbf{S}}$$

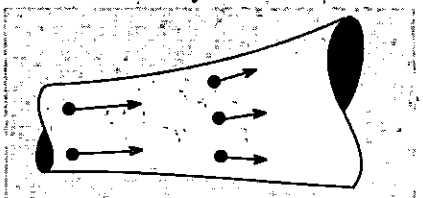


Fig. 2.7 The drift velocity is shown as it varies in a conductor with varying cross section. The arrows indicate values of $\vec{\mathbf{v}}_d$ at a few representative points. If the carriers are positive, these arrows can be used to represent $\vec{\mathbf{J}}$ also. What if the carriers are negative.

The current through a surface is the flux of the current density for that surface.

Model of a Metal

From combining eqn. $\vec{\mathbf{J}} = nq\vec{\mathbf{v}}_d$, and eqn. $\vec{\mathbf{J}} = \sigma\vec{\mathbf{E}}$, we see that $\vec{\mathbf{v}}_d$ and $\vec{\mathbf{E}}$ are proportional. That is, an applied field causes the carriers to move with a constant average velocity $\vec{\mathbf{v}}_d$. But if the field were to furnish the only force on a carrier, then the carrier's acceleration would be constant, not its velocity. This means that when a carrier moves through the metal, there must be other forces on it. Indeed, since the average velocity is constant, the sum of all forces on a carrier must be zero on the average.

The situation is analogous to a marble rolling down a pegboard (Fig. 2.8). When first released, a marble will accelerate down the board because of the unbalanced component of the gravitational force down the board. As it collides with the pegs, we notice that its motion, averaged over many collisions, is characterized by a constant average velocity down the board. Averaged over many collisions, the force on a marble due to the pegs is equal and opposite the component of the gravitational force down the board. A free electron is similar to a marble, the lattice ions are similar to the pegs, and the applied field is similar to the component of the gravitational field down the board.

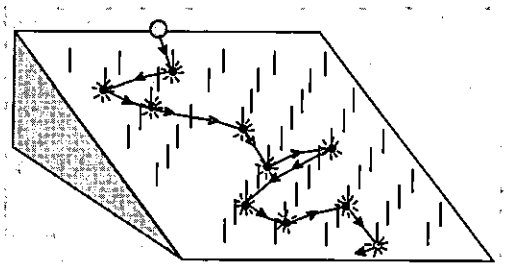


Fig. 2.8

The free electrons in metals are the valence electrons that are weakly bound to the atoms when the atoms are isolated (not part of a metal). When the atoms are side by side in a solid, these electrons are free to move through the material. Thus the number density n of carriers is the product of a small integer and the number density of atoms in the material.

The average velocity $\langle \vec{\mathbf{v}} \rangle$ of the free electrons is the carrier drift velocity $\vec{\mathbf{v}}_d$: $\langle \vec{\mathbf{v}} \rangle = \vec{\mathbf{v}}_d$. Since $\vec{\mathbf{E}} \propto \vec{\mathbf{v}}_d$, the average velocity is zero, when there is no applied field. That is, if $\vec{\mathbf{E}} = 0$, then $\langle v_x \rangle = \langle v_y \rangle = \langle v_z \rangle = 0$. The behavior of the free electrons is similar to the behavior of the molecules of a gas in that their velocities are randomly directed.

The force by the applied field $\vec{\mathbf{E}}$ on an electrons is $\vec{\mathbf{F}} = -e\vec{\mathbf{E}}$. Since this is the only force on a free electron between collisions, Newton's second law, $\Sigma \vec{\mathbf{F}} = m\vec{\mathbf{a}}$, gives the acceleration between collisions as $\vec{\mathbf{a}} = -e\vec{\mathbf{E}}/m$. If we align our x-axis along $\vec{\mathbf{E}}$, then the x-component of a free electron's velocity at a time t after a collision is

$$v_x = v_{x0} + a_x t = v_{x0} - \left(\frac{eE}{m}\right)t$$

where v_{x0} is the x-component of the electron's velocity immediately after the collision. On the average, we have

$$\langle v_x \rangle = \langle v_{x0} \rangle - \left(\frac{eE}{m}\right)\tau$$

where τ is the time interval between collisions. This time interval is often called the mean free time, or the relaxation time.

Concept: *For a rather sizable current, the drift speed v_d is only about 10^{-4} m/s, whereas the average speed $\langle v \rangle$ of the free electrons is about 10^6 m/s. Since $\langle v \rangle$ is a factor of about 10^{10} larger than v_d, the contribution to the motion of the free electrons due to the applied field is negligible at the microscopic level. Therefore it is valid to assume that the velocity of an electron immediately after each collision is randomly directed relative to $\vec{\mathbf{E}}$, or $\langle v_{x0} \rangle = 0$.*

Thus $$\langle v_x \rangle = \frac{-eE\tau}{m}$$

The drift velocity is $\vec{\mathbf{v}}_d = \langle \vec{\mathbf{v}} \rangle = \langle v_x \rangle \vec{\mathbf{i}}$, so that

$$\vec{\mathbf{v}}_d = \frac{-eE\tau}{m}\vec{\mathbf{i}}$$

Using eqn. $\vec{\mathbf{J}} = nq\vec{\mathbf{v}}_d$, we obtain

$$\vec{\mathbf{J}} = n(-e)\left(\frac{-eE\tau}{m}\vec{\mathbf{i}}\right)$$

$$= \frac{ne^2\tau}{m}E\vec{\mathbf{i}} = \frac{ne^2\tau}{m}\vec{\mathbf{E}}$$

Comparing this with eqn. $\vec{\mathbf{J}} = \sigma\vec{\mathbf{E}}$, gives

$$\sigma = \frac{ne^2\tau}{m} \qquad ...(1)$$

Since $\rho = 1/\sigma$, we also have

$$\rho = \frac{m}{ne^2\tau} \qquad ...(2)$$

Concepts: *If $\sigma = ne^2\tau/m$ is independent of $\vec{\mathbf{E}}$, then the model yields Ohm's law. The factors n, e, and m are plainly independent of $\vec{\mathbf{E}}$, but what about τ? Expect τ to depend on $\langle v_0 \rangle$, and $\vec{\mathbf{E}}$ may change $\langle v \rangle$ by no more than v_d. But noted earlier that $\langle v \rangle \approx 10^6$ m/s and $v_d \approx 10^{-4}$ m/s. Because of this vast difference, we expect τ to be essentially independent of $\vec{\mathbf{E}}$.*

We can express σ and ρ in terms of the average speed $\langle v \rangle$ by introducing the average distance an electron travels between collisions, the mean free path λ:

$$\lambda = \langle v \rangle \tau$$

Substitution for τ into eqn. (1) and (2) gives

$$\sigma = \frac{ne^2\lambda}{m\langle v \rangle} \quad \text{and} \quad \rho = \frac{m\langle v \rangle}{ne^2\lambda}$$

Concept: *It is inappropriate to apply Newton's second law to the motion of an electron in a metal. An electron in a metal must be described according to quantum mechanics.*

CURRENT DENSITY AND DRIFT SPEED

As electrons are accelerated by an electric field, which exerts a force $-e\vec{\mathbf{E}}$ on the electrons. Here static conditions do not apply and $\vec{\mathbf{E}}$ can be non-zero inside a conductor.

The electrons collide with the ions of the lattice and transfer energy to them. The motion of individual electrons is therefore very irregular, consisting of a short interval of acceleration in a direction opposite to the electric field, followed by a collision with an ion that might send the electron into motion in any direction, followed by another acceleration, and so on. The net effect is a drift of electrons in a direction opposite to the field. There is no net acceleration of electrons, because they continually lose energy in collisions with the lattice of copper ions. In effect, energy is transferred from the applied field to the lattice (in the form of internal energy of the conductor, often observed

as a temperature increase). On the average, electrons can be described as moving with a constant drift velocity $\vec{v}_d$ in a direction opposite to the field.

Concepts: 1. *If the electrons drift at such a low speed, why do electrical effects seem to occur immediately after a switch is thrown, such as when you turn on the room lights ? Distinguish between the drift speed of the electrons and the speed at which changes in the electric field configuration travel along wires. This latter speed approaches that of light. Similarly, when you turn the valve on your garden hose, with the hose full of water, a pressure wave travels along the hose at the speed of sound in water. The speed at which the water moves through the hose is much lower.*

2. *Between collisions with the lattice ions, the electrons in a conducting material are accelerated by the electric field* $\vec{E}$, *and so their drift velocity is proportional to* $\vec{E}$. *The current density* $\vec{J}$ *is also proportional to* $\vec{v}_d$, *so* $\vec{J}$ *should be proportional to* $\vec{E}$. *The proportionality constant between the current density and electric field is the electrical conductivity* σ *of the material:*

$$\vec{J} = \sigma \vec{E} \qquad ...(1)$$

A large value of σ *indicates that the material is a good conductor of electric current. The conductivity is a property of the material, not of any particular sample of the material.*

Also resistivity, which is the inverse of the conductivity:

$\rho = 1/\sigma$

Hence

$$\vec{E} = \rho \vec{J} \qquad ...(2)$$

Note that 1 ohm = (1 siemens)$^{-1}$

Equation (1) and (2) are valid only for isotropic materials, whose electrical properties are the same in all directions. In these materials, $\vec{J}$ *will always be in the same direction as* $\vec{E}$.

For certain materials, the resistivity does not depend on the strength of the applied field for a wide range of applied fields. For such materials, a plot of E against j gives a straight line, whose slope is the resistivity ρ. *These materials are known as ohmic materials. Such materials are said to satisfy Ohm's law.*

The resistivity (or conductivity) of a material is independent of the magnitude and direction of the applied electric field.

We obtain an expression for the resistance *R*,

$$R = \rho \frac{L}{A}$$

The resistance *R* is characteristic of a particular object and depends on the material of which it is made as well as on its length and cross-sectional area; the resistivity ρ is characteristic of the material in general.

The resistance of an object is independent of the magnitude or sign of the applied potential difference.

Concepts:1. *Potential difference* ΔV, *I and R are macroscopic quantities, applying to a particular body or extended region. The corresponding microscopic quantities are* $\vec{E}$, $\vec{j}$ *and* ρ *(or* σ*); they have values at every point in a body. The macroscopic quantities are related by* $\Delta V = IR$ *and the microscopic quantities by* $\vec{E} = \rho \vec{j}$

The macroscopic quantities ΔV, *I and R are the quantities whose values are indicated on meters. The microscopic quantities* $\vec{E}$, $\vec{j}$, *and* ρ *are of primary importance when we are concerned with the fundamental behavior of matter (rather than of specimens of matter).*

2. *Ohm's law is not a fundamental law of electromagnetism because it depends on the properties of the conducting medium.*

In a metal, the valence electrons are not attached to individual atoms but are free to move about within the lattice and are called conduction electrons.

In the free-electron model, the conduction electrons are assumed to move throughout the conducting material, somewhat like molecules of gas in a container. In fact, the assembly of conduction electrons is sometimes called an ***electron gas.***

In the absence of an electric field, the electrons move randomly, an electron collides with an ionic core of the lattice, suffering a sudden change in direction in the process. We can associate a mean free path λ *and a mean free time* τ *to the average distance and time between collisions. (Collisions between the electrons themselves are rare and do not affect the electrical properties of the conductor.)*

In an ideal metallic crystal at 0 K, electron-lattice collisions would not occur, according to the predictions of quantum physics; that is, $\lambda \to \infty$ *as* $T \to 0$ *K for ideal crystals. Cause of collisions in actual crystals .*

(1) the ionic cores at any temperature T are vibrating about their equilibrium positions in a random way;

(2) impurities may be present; and

(3) the crystal may contain lattice imperfections, such as missing atoms and displaced atoms. Consequently, the resistivity of a metal can be increased by (i) raising its temperature, (ii) adding small amounts of impurities, and (iii) straining it severely, to increase the number of lattice imperfections.

When we apply an electric field to a metal, the electrons modify their random motion in such a way that they drift slowly, in the opposite direction to that of the field, with an average drift speed v_d.

Fig. 2.9 suggests the relationship between these two speeds. The solid lines suggest a possible random path followed by an electron in the absence of an applied field. The dashed lines show how this same event might have occurred if an electric field $\vec{E}$ had been applied. When a field is applied to an electron in the metal, it experiences a force eE, by Newton's second law,

$$a = \frac{eE}{m}$$

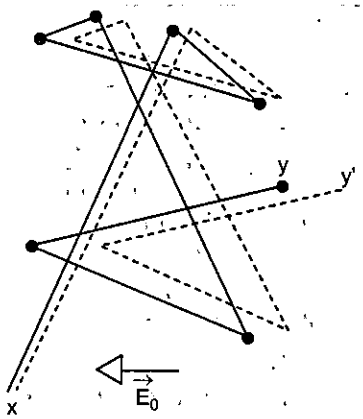


Fig. 2.9

Consider an electron that has just collided with an ion core. The electron has a truly random direction after the collision. During the time interval to the next collision, the electron's speed changes, on the average, by an amount $a(\lambda/v_{av})$ or $a\tau$, where τ is the mean time between collisions. We identify this with the drift speed v_d,

$$v_d = a\tau = \frac{eE\tau}{m}$$

We may also express v_d in terms of the current density which gives

$$v_d = \frac{J}{ne} = \frac{eE\tau}{m}$$

Combining this with $(\rho = E/J)$, we finally obtain

$$\rho = \frac{m}{ne^2\tau}$$

The quantity τ depends on the speed distribution of the conduction electrons that, is affected only very slightly by the application of even a relatively large electric field, since v_{av} is of the order of 10^6 m/s, and v_d is only of the order of 10^{-4} m/s. Whatever the value of τ is in the absence of a field, it remains unchanged when the field is applied. Thus the ρ is independent of E and the material obeys Ohm's law.

EXPRESSION FOR RESISTANCE

We have already derived that,

$$v_d = \left(\frac{eE}{m}\right)\tau$$

but $$E = \frac{\Delta V}{l}$$

therefore $$v_d = \left(\frac{e\Delta V}{ml}\right)\tau$$

also $$I = neAv_d$$

therefore $$I = ne$$

$$A = \frac{e\Delta V}{ml}\tau = \frac{ne^2(\Delta V)A\tau}{ml}$$

$$\frac{I}{\Delta V} = \frac{ne^2A\tau}{ml}$$

$$\frac{I}{R} = \frac{ne^2A\tau}{ml} \quad \Rightarrow \quad R = \frac{\boldsymbol{ml}}{\boldsymbol{ne^2A\tau}}$$

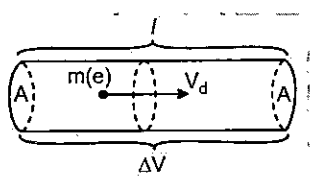


Fig. 2.10

But we have already derived that,

$$\rho = \left(\frac{m}{ne^2\tau}\right)$$

therefore $$R = \frac{\rho l}{A}$$

For $l = 1$ and $A = 1$, $R = \rho$

i.e., the specific resistance is the resistance offered by a conductor of unit length and unit cross-section.

Note: The formula $R = \frac{\rho l}{A}$ is applicable only when cross-sectional area A and resistivity ρ are same throughout the conductor.

If conducting rod of uniform cross-section is stretched or compressed (volume remains constant) then,

$$\text{volume} = Al \Rightarrow \text{constant}$$

$$R = \rho\frac{l}{A} \times \frac{l}{l}$$

$$\Rightarrow \quad R = \rho\frac{l^2}{Al} \quad \Rightarrow \quad R \propto l^2$$

$$R = \rho\frac{l}{A} \times \frac{A}{A}$$

$$\Rightarrow \quad R = \rho\frac{lA}{A^2} \quad \Rightarrow \quad R \propto \frac{l}{A^2}$$

A graph of the current through a circuit element *versus* the potential difference is called the **characteristic curve** of the circuit element.

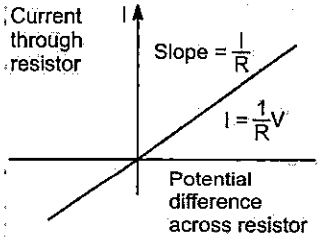


Fig. 2.11. The characteristic curve of a resistor

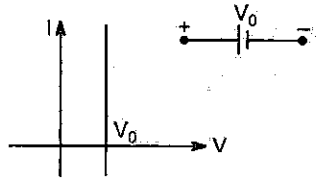


Fig. 2.12. The characteristic curve of an independent voltage source

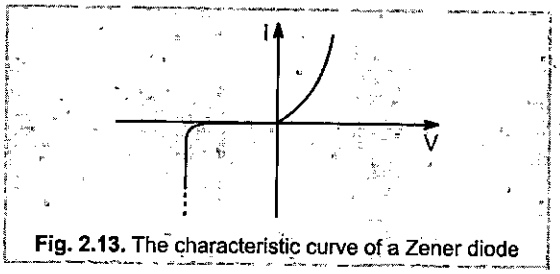


Fig. 2.13. The characteristic curve of a Zener diode

Fig. 2.13 shows the characteristics of a non-linear device. We define the resistance of the device as the inverse of the slope of the characteristics curve at any point.

$$R = \frac{1}{dI/dV} = \frac{dV}{dI}$$

Concept: *The vibrational energy of the lattice increases with temperature; as a result the atoms are moving more rapidly and are arranged in a less orderly manner. So interference in the flow of electron is increased. Therefore, the resistivity of metals increases with temperature. The resistivity of metals usually increases approximately linearly with temperature.*

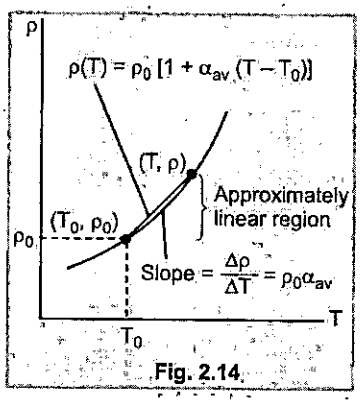


Fig. 2.14

$$\rho = \rho_0[1 + \alpha_{av}(T - T_0)]$$

where α_{av} is the mean temperature coefficient of resistivity.

$$\rho_{av} = \frac{1}{\rho_0}\frac{\Delta\rho}{\Delta T}$$

Since resistance is proportional to resistivity, we have

$$R = R_0[1 + \alpha_{av}(T - T_0)]$$

Electric Power

The energy transformed when an infinitesimal charge dq moves through a potential difference V is $dU = V\,dq$. The power P is the rate at which energy is transformed.

Thus we have

$$P = \frac{dU}{dt} = \frac{dq}{dt}V = IV \quad \ldots(i)$$

$$= I(IR) = I^2R \quad \ldots(ii)$$

$$\left(\frac{V}{R}\right)V = \frac{V^2}{R} \quad \ldots(iii)$$

Eqns. (ii) and (iii) apply to resistors only, whereas (i) is applicable to any device. The SI unit of electric power is same as for any kind of power, the **watt** (1 W = 1 J/s).

The total energy consumed by any device is simply its power consumption multiplied by the time it works. The household electricity connections use the unit **kilowatt-hour** (kWh).

$$\text{One kWh} = (1000)(3600) = 3.60 \times 10^6 \text{ J}$$

The average electric current is defined as the net amount of charge that passes through the wire's full cross-section at any point per unit time.

$$I_{av} = \frac{\Delta Q}{\Delta t} \qquad \text{(average current)}$$

where ΔQ is the charge passing through cross-section during time interval Δt. If the amount of charge ΔQ charges in time, we define the instantaneous current as

$$I = \lim_{\Delta t \to 0}\frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}$$

The SI unit of current is ampere.

$$1 \text{ A} = 1 \text{C/s}$$

Conventionally the direction of current is taken to be the direction of flow of positive charge.

Resistors in Series

In the series connection current through each resistor is same. The potential difference across each resistor is found from Ohm's law.

$$V_1 = IR_1, \quad V_2 = IR_2, \quad V_3 = IR_3$$

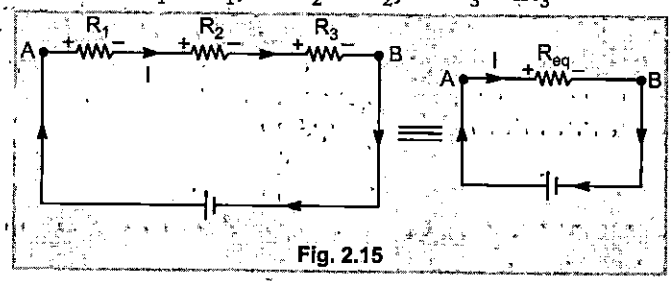


Fig. 2.15

We wish to replace a series of resistors with a single equivalent resistor $R_{eq.}$ connected between the same two terminals A and B. The total potential difference between A and B is

$$V = V_1 + V_2 + V_3$$

Since the potential difference between A and B across $R_{eq.}$ also must be V, we have

$$IR_{eq.} = IR_1 + IR_2 + IR_3$$

$$R_{eq.} = R_1 + R_2 + R_3 \quad \ldots(i)$$

As current is same through each resistor in series combination,

therefore $V = IR$

gives $V \propto R$

i.e., $V_1 : V_2 : V_3 = R_1 : R_2 : R_3$

and if $V_1 + V_2 + V_3 = V$

then $$V_1 = \frac{R_1}{R_1 + R_2 + R_3}V$$

Similarly expression for V_2 and V_3 can be obtained.

Power dissipated in the resistor in time 't',

$$P = I^2R$$

$$P \propto R$$

i.e., $$P_1 : P_2 : P_3 = R_1 : R_2 : R_3$$

Dividing equation (i) by V^2,

$$\frac{R}{V^2} = \frac{R_1}{V^2} + \frac{R_2}{V^2} + \frac{R_3}{V^2}$$

$$\frac{1}{W} = \frac{1}{W_1} + \frac{1}{W_2} + \frac{1}{W_3}$$

Resistors in Parallel

If three resistors are connected to same nodes A and B, the potential difference V across each resistor is the same. We wish to replace the set with an equivalent resistor with resistance $R_{eq.}$ so that the potential difference across $R_{eq.}$ is same as across any of the resistors in parallel.

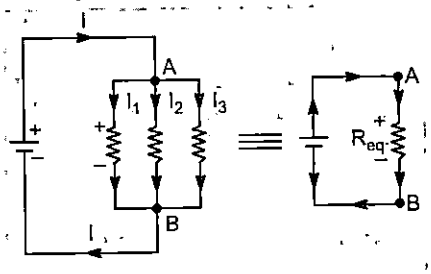


Fig. 2.16

The current I reaching node A is divided into I_1, I_2 and I_3 in the branches containing resistors R_1, R_2 and R_3 respectively.

$$I = I_1 + I_2 + I_3 \quad \ldots(1)$$

and $$V = I_1 R_1, \; V = I_2 R_2, \; V = I_3 R_3 \quad \ldots(2)$$

$$\frac{V}{R_{eq.}} = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}$$

$$\frac{1}{R_{eq.}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

In parallel combination, the equivalent resistance is smaller than the smallest one in the combination.

Since V is same for all resistors in parallel.

Therefore $$I \propto \frac{1}{R}$$

or $$I_1 : I_2 : I_3 = \frac{1}{R_1} : \frac{1}{R_2} : \frac{1}{R_3}$$

if $$I_1 + I_2 + I_3 = I$$

then $$I_1 = \frac{\frac{1}{R_1}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} \times I$$

Power $$(P) = V^2/R$$

$$P \propto (1/R)$$

$$P = P_1 + P_2 + P_3$$

ELECTROMOTIVE FORCE AND ITS SOURCES

Fig. 2.17 schematically shows the water analogy that will guide us in our study of electric currents. The pump produces a pressure difference $\Delta p = p_{hi} - p_{lo}$ between its outlet O and inlet I. Driven by this pressure difference, water flows through the pipes in the sense of decreasing pressure, as shown by the arrows. If its pumping capacity is sufficient, the pump will maintain the pressure difference Δp even if the valve is opened wider.

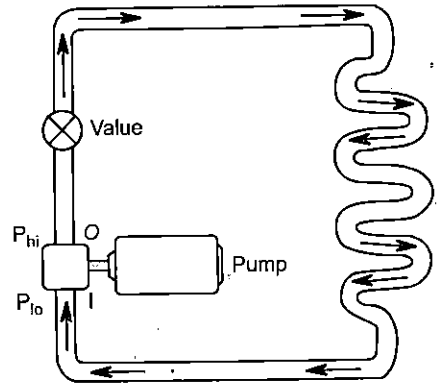


Fig. 2.17

1. External mechanical energy is converted by the pump into potential energy of water.

2. The potential energy is converted into kinetic energy of flow as the water moves through the pipes outside the pump.

3. The kinetic energy is converted into heat energy by frictional processes.

4. The process continues as long as external mechanical energy is supplied to the pump.

The device that produces the emf called a **source of emf** is usually part of an **electric circuit,** which is a closed path for circulation of charge, just as the system of Figure is a closed path for circulation of water. Indeed, we can describe the cycle of charge circulation in terms closely analogous to those used for the water system:

1. External energy is converted by the source of emf into potential energy of electric charge.

2. The potential energy is converted into kinetic energy of flow as the charge moves through the part of the circuit external to the source of emf.

3. The kinetic energy is converted into heat by the "frictional" process that is equivalent to the phenomenon of electrical resistance.

4. The process continues as charge continuously arrives at the source of emf.

A battery is connected to a "device." The device may be a single circuit element, such as a resistor or a capacitor, or it may be a combination of circuit elements. The battery maintains the upper terminal at a potential V_+ and the lower terminal at a potential V_-. For an ideal battery, the potential difference $V_+ - V_-$ between its terminals is independent of the amount of current that it is providing to the circuit.

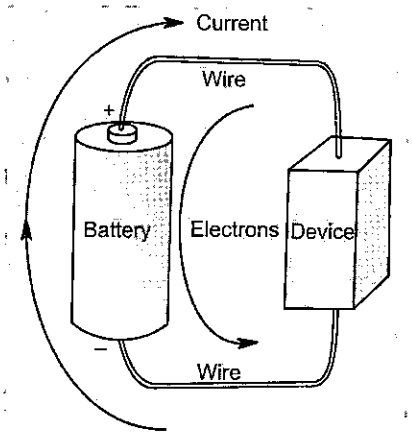


Fig. 2.18

The battery can be considered a "pump" for charge, as if it were bringing positive charge through the battery from the negative terminal to the positive one. In actuality, it is usually the motion of the negatively charged electrons that is responsible for the current flow.

Concept: *The function of the battery in the circuit is to maintain the potential difference that enables the flow of charge. The battery is not a source of electrons. Electrons pass through the battery and have their energy raised as the move inside the battery from the positive to the negative terminal.*

When we say that a battery is "drained," we do not mean that it has "used up" its supply of electrons; instead, we mean that we have exhausted the source of energy (often a chemical reaction) that was responsible for raising the energies of the electrons. Note in Fig. 2.18 that the electrons move throughout the entire circuit; they do not "come from" the battery.

The direction of current is the direction that positive charges would move, even if the actual charge carriers are negative.

We assume that, under steady conditions, charge does not collect at or drain away from any point in our idealized wire.

Concept: *The electric current I is the same for all cross-sections of a conductor, even though the cross-sectional area may be different at different points. The current density* $\vec{\mathbf{J}}$ *(current per unit area) will change as the cross-sectional area changes, but the current I remains the same.*

At any junction in an electric circuit, the total current entering the junction must be equal to the total current leaving the junction.

A source of emf must be able to do work on charge carriers that enter it. The source acts to move positive charges from a point of low potential (the negative terminal) through its interior to a point of high potential (the positive terminal). The charges then move through the external circuit, in the process dissipating the energy provided to them by the source of emf. Eventually, they return to the negative terminal, from which the emf raises them to the positive terminal again, and the cycle continues.

When a steady current has been established in the circuit a charge dq passes through any cross-section of the circuit in time dt. The emf $\mathcal{E}$ of the source is defined as the work per unit charge, or

$$\mathcal{E} = dW/dq$$

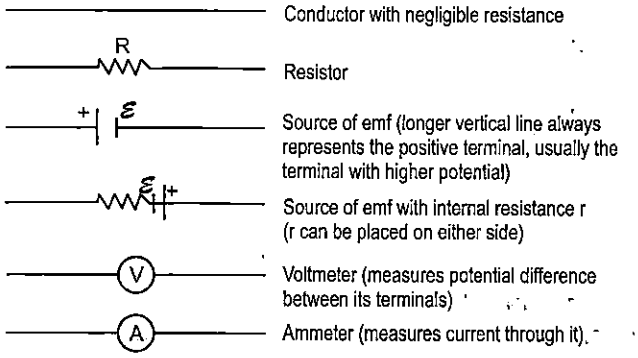
The unit of emf is the joule/coulomb, which is the same as the volt.

The source of emf provides energy to the circuit. Its energy might be obtained from a variety of processes: chemical (as in a battery or a fuel cell), mechanical (a generator), thermal (a thermopile), or radiant (a solar cell).

The current in the circuit transfers energy from the source of emf to the device. If the device is a resistor, the transferred energy appears as internal energy (observed perhaps as an increase in temperature). If the device is a capacitor, the energy transferred is stored as potential energy in its electric field.

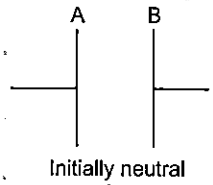
A battery can either be charged (meaning an external source adds to the battery's supply of energy, not that we are forcing more charge into the battery) or discharged (meaning we take energy from the battery).

Symbols for Circuit Diagrams



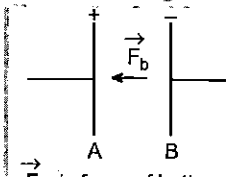
Concept: 1.

Battery and EMF: *When a device is connected to a circuit element it maintains a potential difference between terminals of element. The battery has an internal mechanism that exerts force on charges, we refer is as* $\vec{\mathbf{F}}_b$.



Initially neutral conductors

Fig. 2.19



$\vec{F}_b$ is force of battery on positive charges

Fig. 2.20

Force of battery $\vec{F}_b$ is a non-electrostatic force. The battery that can maintain $\vec{F}_b$ for q longer duration is a good battery. Due to $\vec{F}_b$ positive charge accumulate on terminal A and negative charge on B. Due to this charge separation on electric field $\vec{E}$ is developed in the material from A to B.

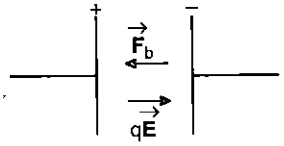


Fig. 2.21

The charge separation occurs till the $\vec{F}_b = q\vec{E}$ when a charge q is transferred from A to B workdone by $\vec{F}_b$ is $F_b d$ and emf is defined as

$$\mathcal{E} = \frac{F_b d}{q}$$

In the absence of any entered circuit current

$$\mathcal{E} = \frac{F_b d}{q} = qEd = qV$$

Where Ed is potential difference between terminals A and B.

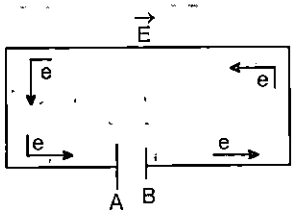


Fig.2.22

When an external circuit is connected across terminals A and B. Free electrons more toward A, at the same instant electron are driven at each cross-section of circuit. When an electron reaches A and another moves away from B, the electrostatic field $\vec{E}$ decreases. Now electrostatic force $q\vec{E}$ decrease but force $\vec{F}_b$ due to internal mechanics of battery remains uncharged. Thus there is a net force on the positive charges of material from B to A. Thus the potential difference between A and B is maintained.

2. *When a current is driven into battery, positive charges enter battery from positive terminal and leave at B such a process is called charging of battery. When we draw current it leaves out of positive terminal, this process is called discharging of battery.*

3. *Electric Fields in Circuits:* *The relationship between current and electric field in a conductor: $\vec{E} = \rho \vec{J}$ where ρ is the resistivity of the material and $\vec{J}$ is the current density (current per unit cross-sectional area).*

The wires in our circuits are conductors, and so there must be an electric field present to establish and sustain the current. Where does this electric field come from ?

The battery provides the emf to the circuit; its role is to "pump" charges from low potential to high potential. The emf is defined as the work per unit charge done by the battery. With the conventional definition of work done by a force $\vec{F}$ as $W = \int \vec{F} \cdot d\vec{S}$, the work per unit charge W/q (the emf) must then be related to the force per unit charge $\vec{F}/q$:

$$\mathcal{E} = \oint (\vec{F}/q) \cdot d\vec{S}$$

The force $\vec{F}$ in this case is the one that acts inside the source of emf; it might be a force of mechanical, chemical, thermodynamic, or magnetic origin, but it is not necessarily associated with an electric field. The emf depends only on the net effect of the source on a charge that makes a complete loop around the circuit. Conservative external fields cannot give an emf, because integral $\int \frac{\vec{F}}{q} \cdot d\vec{S}$ vanishes for such fields.

Inside the wires, there is an electric field. This field must be present for charge to flow in the wires.

When the battery is first connected to the circuit, initial transient currents are established. These currents distribute charge along the surfaces of the wires in just the precise way necessary to establish the electric field that maintains the steady current in the wires– and the entire process takes place in a time that is typically of the order of nanoseconds !

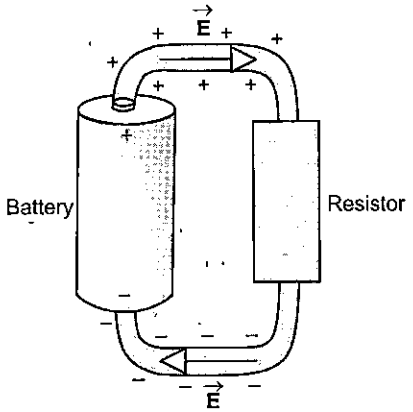


Fig. 2.23

4. *In practice, the current is the same all the way around the loop, at any given moment.*

If the current is not the same all the way around then charge may collect somewhere, and the electric field of this accumulating charge is in such a direction as to even out the flow. Suppose for the instance, that the current into the bend in Fig. 2.24 is greater than the current out.

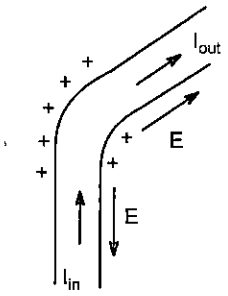


Fig. 2.24

Then charge collects at the "knee," and this produces a field directed away from the kink. This field opposes the current flowing in and promotes the current flowing out until these currents are equal, at which point there is no further accumulation of charge, and equilibrium is established.

5. *In a similar fashion, we can ask how the current "knows" to change direction when it encounters a bend in the wire. Fig. 2.25 shows a schematic view of a right-angle bend, for which the surface charges must be distributed roughly as shown. The negative charge sets up a field near the bend that opposes the motion of the oncoming current, and the positive charge provides an initial "push" in the new direction.*

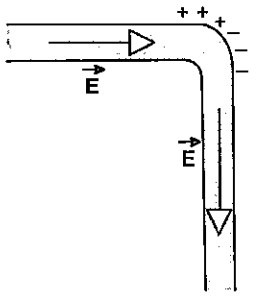


Fig. 2.25

The battery provides the initial "burst" of current to the circuit, and almost instantly the charge finds its way to the locations where it guides the steady current and prevents further build-up of charge on the surface of the wires. This equilibrium is maintained as long as the battery continues to pump charge around the circuit.

EMF AND INTERNAL RESISTANCE OF A BATTERY

For an electric circuit to have a continuous current, the circuit must contain an element that is a source of electric energy. Such an element is called a source of emf.

The battery produces steady current by maintaining a nearly constant potential difference across its terminals. The terminal that is at the higher potential is called the positive terminal and the terminal that is at the lower potential is called the negative terminal. Thus the sense of the current outside the battery (through the resistor) is from the positive terminal toward the negative terminal, and the sense of the current inside the battery is from the negative terminal toward the positive terminal.

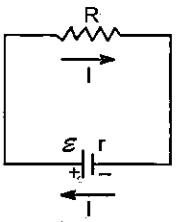


Fig. 2.26

Two important characteristics of a battery are its emf $\mathcal{E}$ and its internal resistance r. The emf characterizes the energy that the battery provides the charge carriers, and the internal resistance is the battery's own resistance. Fig. 2.26 shows how the emf and internal resistance can be determined. A voltmeter placed across the battery measures the battery's terminal potential difference V and an ammeter measures the current I. The current can be changed by changing the resistance of the variable resistor (shown as ⁄\/\/\⁄) Fig. 2.28 shows a graph of V versus I that is typical for such measurements. The equation that gives this graph is

$$V = \mathcal{E} - Ir \qquad ...(1)$$

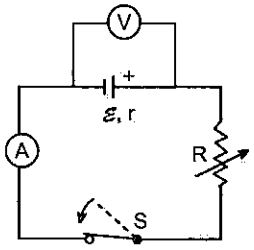


Fig. 2.27

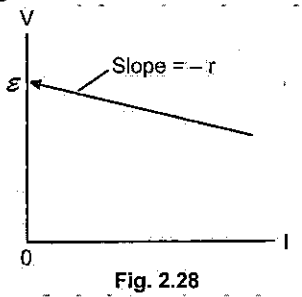


Fig. 2.28

The graph's intercept equals the value of the emf E, and its slope gives the internal resistance r. Thus the battery's emf E is its terminal potential difference when the current in the battery is zero:

$$\mathcal{E} = V \qquad \text{(when } I = 0\text{)}$$

The emf of a battery can be measured by placing a high-resistance voltmeter across its terminals while the terminals are not connected to anything else. In this way the current is so small that the Ir term in eqn. (1) is negligible compared with $\mathcal{E}$.

emf has the same dimension as electric potential, namely, energy per unit charge. The dimension energy per unit charge indicates the physical nature of emf.

Concept: *If we consider a small current so that we can neglect the Ir term then we can describe the emf of a battery as the electric potential energy per unit charge given to the charge carriers by non-electrostatic forces in the battery as the carriers pass from one terminal to the other. These force are a result of the chemical action of the battery.*

In Fig. 2.29, we show the internal resistance separate from the emf, even though they cannot be physically separated. Traversing the battery alor the sense of the current, we find that the potential increases by the amount E because of the chemical action of the battery and decrease by Ir because of the resistance of the battery, which illustrates the relation $V = \mathcal{E} - Ir$

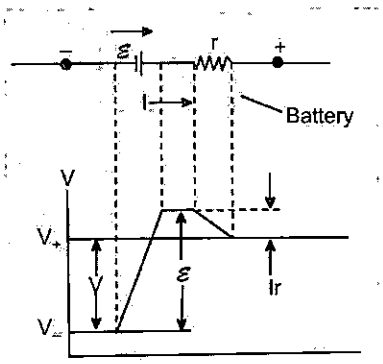


Fig. 2.29 A battery with current I in it. The sense of I is the same as the sense of $\mathcal{E}$. The terminal potential difference is $V = V_+ - V_-$

If the sense of the current in a battery is the same as the sense of its emf, as seen in Fig. 2.29, then the battery is said to be "discharging," and eq. $V = \mathcal{E} - Ir$ is valid for this case. On the other hand, if the sense of the current is opposite the emf, as shown in Fig. 2.30, then the battery is said to be "charging" or is "being charged." From the figure we see that if we traverse a charging battery along the sense of the current, then the potential decreases by the amount $\mathcal{E}$ because of chemical reactions in the battery. As with a discharging battery, the potential decreases by Ir because of the battery's resistance. Thus the terminal potential difference V across a battery that is being charged is

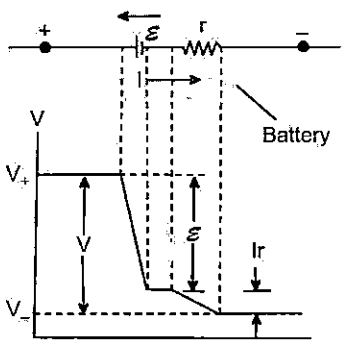


Fig. 2.30 A battery being charged. The sense of I is opposite the sense of $\mathcal{E}$. The terminal potential difference is $V = V_+ - V_-$

$$V = \mathcal{E} + Ir$$

ELECTRIC ENERGY AND POWER

When a current exists in a circuit element, energy is transformed. We now investigate energy transformations due to currents in circuit elements.

Energy Dissipated in a Resistor

Consider the energy transformed when a resistor of resistance R carries a current I, as shown in Fig. 2.31. The potential difference across the resistor is $V = V_a - V_b$ and the sense of the current is from a to b. In a time interval Δt, a number of carriers with total charge ΔQ enter the resistor at point a where the potential is V_a and a number of carriers with an equal total charge ΔQ leave the resistor at point b where the potential is V_b ($V_a > V_b$). The change in the electric point energy ΔU of the carriers is

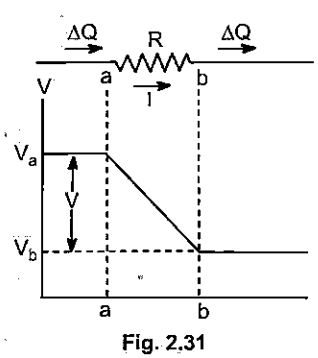


Fig. 2.31

$$\Delta U = V_b \Delta Q - V_a \Delta Q$$
$$= -(V_a - V_b)\Delta Q = -V\Delta Q$$

The quantity ΔU is negative because the potential decreases along the sense of the current. The rate at which the carriers lose electric potential energy is $-\Delta U/\Delta t$, and we call this rate the power P_R dissipated in the resistor. Thus

$$P_R = -\frac{\Delta U}{\Delta t} = -\frac{-V\Delta Q}{\Delta t} = V\frac{\Delta Q}{\Delta t}$$

Since $I = \Delta Q/\Delta t$,

$$P_R = VI \quad ...(1)$$

We can write P_R in terms of the resistance R of the resistor by using $V = IR$. We have $P_R = IV = I(IR)$, or

$$P_R = I^2R \quad ...(2)$$

Alternatively, $P_R = VI = V(V/R)$,

or $$P_R = \frac{V^2}{R} \quad ...(3)$$

Equation (1), (2) and (3) is called Joule's law.

Concept: *The carriers lose energy in the collisions that are responsible for the resistance of the resistor. When a resistor carries a current, the temperature of the resistor tends to increase as a result of these collisions. If the temperature of the resistor rises above that of its surroundings, then the resistor transfers heat to its surroundings. (Recall that heat is the transfer of energy due to a temperature difference.) Under steady conditions the energy is continuously transferred as heat to the surroundings. These effects are sometimes called I^2R heating, sometimes called Joule heating or sometimes called ohmic heating.*

Energy to or From a Battery

If charge carriers pass through a battery in the direction such that the sense of the current is the same as the sense of the emf (the battery is discharging as in Fig. 2.26 and 2.29) then their electric potential energy increases. Let ΔQ be the amount of charge that passes through the battery in time Δt. The change in the electric potential energy of the carriers is $\Delta U = V\Delta Q$, where V is the terminal potential difference across the battery. In this case, ΔU is positive because the electric potential increases through the battery along the sense of the current (see Fig. 2.29) The rate at which the carriers gain electric potential energy is $\Delta U/\Delta t$, and we call this rate the power output P_o from the battery:

$$P_o = \frac{\Delta U}{\Delta t} = V\frac{\Delta Q}{\Delta t} = VI$$

Using the expression for the terminal potential difference across a discharging battery, $V = \mathcal{E} - Ir$,

we have $$P_o = IV = I(\mathcal{E} - Ir)$$

Concept: $P_o = \mathcal{E}I - I^2r$ ...(4)

The term $\mathcal{E}I$ in eqn. (4) represents the rate at which the electric potential energy of the carriers is increased by chemical reactions in the battery. We shall call this the power $P_{\mathcal{E}}$ expended by the emf of the battery: $P_{\mathcal{E}} = \mathcal{E}I$. We recognize the I^2r terms as the power P_r dissipated in the battery due to its resistance r. (The temperature of the battery tends to increase.) This term represents a rate of loss of electric potential energy for the carriers and properly enters the expression with a minus sign. Thus eqn. (4) states that the power output P_o of a battery is equal to the power $P_{\mathcal{E}}$ expended by the emf minus the power P_r dissipated as heat: $P_o = P_{\mathcal{E}} - P_r$.

Equation (4) is valid for a discharging battery. If a battery is being charged, then the sense of the current is opposite the sense of the battery's emf. In this case the potential decreases along the sense of the current and the electric potential energy of the carriers decreases as they pass through the battery. The power input P_i to the battery is equal to the rate at which the carriers lose electric potential energy in passing through the battery: $P_i = IV$. Since the terminal potential difference across a charging battery is

$$V = \mathcal{E} + Ir,$$

we have $P_i = IV = I(\mathcal{E} + Ir)$,

or $P_i = \mathcal{E}I + I^2r$

In this case the product $\mathcal{E}I$ represents the power delivered to the emf of the battery by the charge carriers.

Energy to or From Any Circuit Element

Next we consider the rate of energy transformation P in any type of circuit element. If V is the potential difference across the element and I is the current in the element, then

$$P = IV, \quad \text{...(5)}$$

because V is the change in the potential energy per unit charge for carriers that pass through the element and I is rate at which charge passes through the element. The product IV gives the rate at which the electric potential energy of the carriers changes as they pass through the element. If the sense of I is along the direction the potential decreases (as in a resistor or a charging battery), then P gives the rate at which the carriers lose electric potential energy. If the sense of I is along the direction the potential increases (as in a discharging battery), then P gives the rate at which the carriers gain electric potential energy.

Discharging of a Cell

In the discharging of a cell the current flows from cathode to anode inside the cell (in the direction of emf).

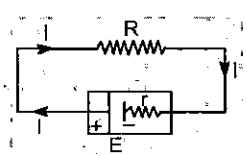


Fig. 2.32

Applying the Kirchoff's law,

$$\mathcal{E} - IR - Ir = 0$$

$$\mathcal{E} = I(R + r)$$

Where $IR (= V)$ is the terminal potential difference and Ir is the internal drop.

Hence $V = \mathcal{E} - Ir$

Thus during the discharging of a cell the terminal potential difference is less than the emf of the cell and if the circuit is open *i.e.*, no current is drawn from the cell,

then $V = \mathcal{E}$

The internal resistance of the cell (r) can be expressed in terms of E and V as,

$$r = R\left(\frac{\mathcal{E}}{V} - 1\right)$$

During the discharging of a cell, it continuously losses the energy.

Energy lost by cell $= q\mathcal{E}$ (where 'q' is the charge flown)

$$= \mathcal{E}\int I dt$$

Power Transferred to the Load Resistance

$$P = I^2R$$

$$= \left(\frac{\mathcal{E}}{R+r}\right)^2 R = \frac{\mathcal{E}^2}{(R+r)^2}R$$

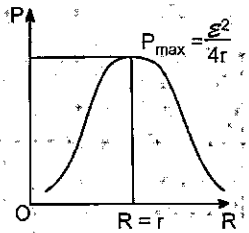


Fig. 2.33

For P to be maximum,

$$\frac{dP}{dR} = 0; \quad R = r$$

Therefore the power transferred to the load by a cell is maximum when the load resistance of the circuit equals the internal resistance of the cells the statements is named as maximum power transfer theorem.

Total Power Consumed in the Circuit

Total power,

$$P_{total} = \frac{\mathcal{E}^2}{R+r} = \mathcal{E}I$$

For P_{total} to be maximum, $R = 0$,

$$(P_{total})_{max} = \frac{\mathcal{E}^2}{r}$$

And simultaneously the current is also maximum as,

$$I = \frac{\mathcal{E}}{R+r}$$

$$I_{max} = \frac{\mathcal{E}}{r}$$

or $R = 0$

Efficiency of a Cell (η) or Power Transfer Efficiency

$$\eta = \frac{\text{Power developed in } R \text{ (load resistance)}}{\text{Total power supplied}}$$

$$= \frac{I^2R}{I^2(R+r)} = \frac{R}{R+r} = \frac{1}{1+(r/R)}$$

When the power is maximum *i.e.*, $R = r$, then,

$$\eta = \frac{r}{2r} = \frac{1}{2}$$

$$\eta = 50\%$$

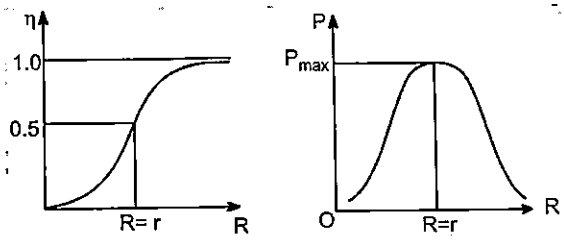


Fig. 2.34

Efficiency will be maximum when the resistance (load) is infinitely large or (r/R) is very small.

Charging of a Cell

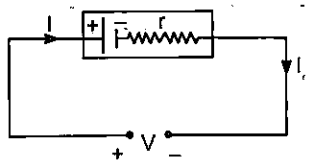


Fig. 2.35

During charging current inside the cell is from anode to cathode and from the figure.

$$V = \mathcal{E} + Ir$$

i.e., during the charging of a cell the terminal potential difference (V) is greater than emf of the cell.

When cell is short circuited- the resistance outside the cell becomes zero,

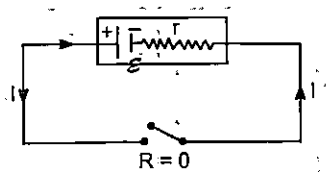


Fig. 2.36

therefore $$I = \frac{\mathcal{E}}{R + r} = \frac{\mathcal{E}}{r}$$

and $$V - IR = 0$$

$$V = \mathcal{E} - Ir = 0$$

Hence current is maximum and terminal potential difference (V) is zero.

Short and Open Circuits

When two points of a circuit are connected together by a conducting wire, they are said to be **short-circuited.** The connecting wire is assumed to have zero resistance. No voltage can exist across a 'short', secondly current through it is very large (theoretically infinity).

☛Two points are said to be **open-circuited** when there is no direct connection between them, a break in the continuity of circuit exists. Due to this break the resistance between the two points is infinite and there is no flow of current between the two points.

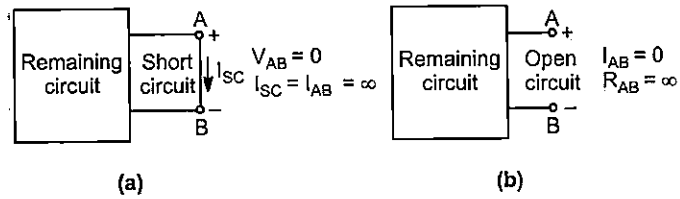


Fig. 2.37

☛Whole of the applied voltage is felt across the 'open', *i.e.*, across terminals A and B.

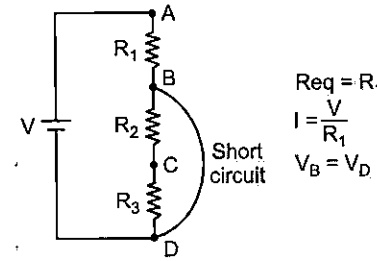


Fig. 2.38

☛Bulb B_1 acts as open circuit, bulb B_1 will not glow. However, other bulb B_2 remains connected across the voltage supply, it will operate normally.

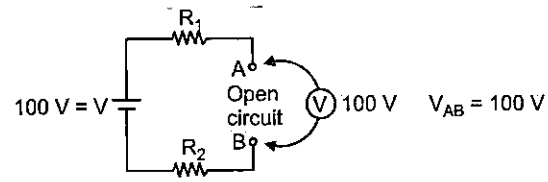


Fig. 2.39

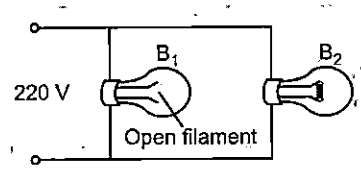


Fig. 2.40

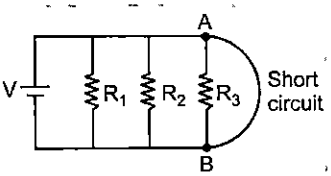


Fig. 2.41

☛Short circuit across R_3 shorts R_1 and R_2 as well, short across one branch in parallel means across all branches. There is no current in shorted resistors. The shorted components are not damaged, they will function normally when short circuit is removed.

☛In Fig. 2.42 shown, short circuit across R_3 may short out R_2 but not R_1, because it is protected by R_4.

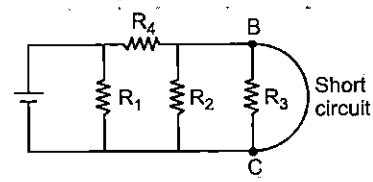


Fig. 2.42

Voltage Divider

In a series circuit, current through each resistor is same.

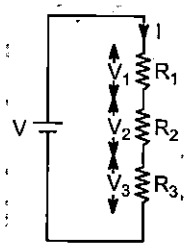


Fig. 2.43

$$V = V_1 + V_2 + V_3$$
$$= IR_1 + IR_2 + IR_3$$
$$I = \frac{V}{R_1 + R_2 + R_3}$$
$$V_1 = \frac{VR_1}{R_1 + R_2 + R_3}$$
$$V_2 = \frac{VR_2}{R_1 + R_2 + R_3}$$
$$V_3 = \frac{VR_3}{R_1 + R_2 + R_3}$$

Each resistor has its own potential drops. Voltage across each resistor is additive.

Current Divider

From Fig. 2.44 we have

$$I = I_1 + R_2 \quad ...(1)$$
$$I_1R_1 = I_2R_2 \quad ...(2)$$

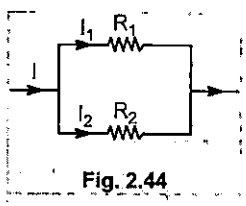


Fig. 2.44

On solving eqns. (1) and (2), we get

$$I_1 = \left(\frac{R_2}{R_1 + R_2}\right)I$$
$$I_2 = \left(\frac{R_1}{R_1 + R_2}\right)I$$

The division of current in the branches of a parallel circuit is inversely proportional to their resistances.

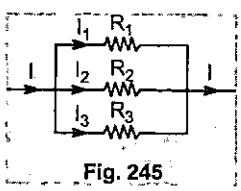


Fig. 245

$$I = I_1 + I_2 + I_3$$
$$I_1R_1 = I_2R_2 = I_3R_3$$

and $$R_{eq.} = \frac{R_1R_2R_3}{R_1R_2 + R_2R_3 + R_3R_1}$$

$$I_1 = I\left[\frac{R_2R_3}{R_1R_2 + R_2R_3 + R_3R_1}\right]$$
$$I_2 = I\left[\frac{R_1R_3}{R_1R_2 + R_2R_3 + R_3R_1}\right]$$
$$I_3 = I\left[\frac{R_1R_2}{R_1R_2 + R_2R_3 + R_3R_1}\right]$$

It is easy to remember the expression for I_1, I_2 and I_3, notice which resistance is missing in the numerator.

Heat Energy Produce in a Resistance

The heat produced in a resistance in time 't', if a current 'I' flows through it and the potential difference across its two ends is V is given by,

$$H = \int_0^t I^2R\,dt$$

If current is constant then,

$$H = I^2R\int_0^t dt$$

$$\left.\begin{aligned} H &= I^2Rt \\ &= VIt \\ &= (V^2/R)t \end{aligned}\right] \text{(using } V = IR\text{)}$$

As resistance of a given electrical appliance (bulb, heater, press etc.) is constant, it is given by, $R = \frac{V_s^2}{W}$

Where V_s and W are the voltage and wattage specified on the appliance.

An electric appliance consumes the specified power W only if it runs at the specified voltage V_s. If the applied voltage (V_A) is greater than specified voltage (V_s) then the appliance will get damaged as in this situation the current $I = (V_A/R)$ will exceed its current capacity (V_s/R). If an appliance is made to run at a lower voltage than specified, then the power consumption will be,

$$P = \frac{V_A^2}{R} = \frac{V_A^2}{(V_s^2/W)} = \left(\frac{V_A}{V_s}\right)^2 W$$

SERIES AND PARALLEL COMBINATION OF BULBS

Parallel Combination

If the different bulbs are in parallel and a voltage $V_A (\le V_s)$ is applied across them as in case of house wiring then the resistance of a bulb.

$$R \propto \frac{1}{W} \quad ...(1)$$

Since $$R = \frac{V_s^2}{W}$$

V_s: is specified voltage.

and W: is specified wattage.

$$I \propto \frac{1}{R} \quad ...(2)$$

$$P \propto \frac{1}{R} \quad ...(3)$$

Since $$P = \frac{V_A^2}{R}$$

From eqn. (1), (2) and (3), we get

$$P \propto I \propto W \propto \frac{1}{R}$$

In parallel grouping of bulbs the bulb of greater wattage will give more bright light and will pass greater current through it and will have smaller resistance and if one bulb gets fused other will still work.

Series Combination

In series combination same current passes through each bulb,

$$R \propto \frac{1}{W} \quad ...(1)$$

Since $$R = \frac{V_s^2}{W}$$

$$V \propto R \quad (\text{'}I\text{' is constant}) \quad ...(2)$$

$$P \propto R \quad (I \text{ is constant}) \qquad ...(3)$$

Since $P = I^2R$

From equation (1), (2) and (3), we get

$$P \propto V \propto R \propto \frac{1}{W}$$

In series combination the bulb of greater wattage will give less bright light and will have smaller resistance and potential difference across it. If one bulb gets fused the other will not work.

In an extended medium of conductivity σ, the power dissipated in an element of cross-section ΔS and length Δl is,

$$\Delta P = \Delta V J \Delta S = E \Delta l J \Delta S$$

or Power dissipated per unit volume,

$$= JE = \sigma EE = \sigma E^2$$

Example 1

A circuit contains a 48V battery and a single bulb whose resistance is 240Ω. A second identical bulb can be connected either in series or parallel with the first one. Determine the power in a single bulb when the circuit contains (a) only one bulb, (b) two bulbs in series and (c) two bulbs in parallel. Assume that the battery is ideal without any internal resistance.

Solution: Power consumed by a light bulb is related to its resistance R and the voltage across it by

$$P = \frac{V^2}{R}$$

(a) When only the bulb is connected in the circuit, the power it consumes is

$$P = \frac{V^2}{R} = \frac{(48)^2}{240} = 9.6\text{W}$$

(b) The more the power dissipated in a light bulb, the brighter it is.

When identical bulbs are wired in series, each receives one half the battery voltage V. The power consumed by each bulb is

$$P = \frac{(\frac{1}{2}V)^2}{R} = \frac{1}{4}\frac{V^2}{R}$$

The power dissipated in each bulb is reduced to only one-fourth the power dissipated in a single bulb circuit. Thus the brightness of each bulb decreases.

The equivalent resistance of two bulbs is $R_{eq.} = R + R$. The current in the circuit is given by $I = V/R_{eq.}$.

The power consumed by one of the light bulbs can be expressed as

$$P = I^2R = \left(\frac{V}{R+R}\right)^2 R = \frac{V^2}{4R}$$

$$= \frac{(48)^2}{4(240)} = 2.4\text{W}$$

The power dissipated in one-fourth as compared to that by a single bulb.

(c) When wires are connected in parallel, each receives full battery voltage V. Thus the power consumed by each bulb remains the same as when only one bulb is in the circuit, so the brightness does not change.

$$P = \frac{V^2}{R} = \frac{(48)^2}{240} = 9.6\text{ W}$$

Example 2

A potential difference V is applied to a copper wire of diameter d and length L. What is the effect on the electron drift speed of doubling (i) voltage V, (ii) length L and (iii) diameter d ?

Solution: According to *'electron theory of metals'* the drift velocity of an electron inside a metal in presence of an electric field E is given by:

$$v_d = \left(\frac{e\tau}{m}\right)E = \left(\frac{e\tau}{m}\right)\left(\frac{V}{L}\right) \qquad \left[\text{as} E = \frac{V}{L}\right]$$

So (i) As $v_d \propto V$, on doubling V, drift velocity will be doubled.

(ii) As $v_d \propto (1/L)$, on doubling L, drift velocity will be halved.

(iii) As drift velocity is independent of diameter d, it will not change on doubling the diameter.

Example 3

The area of cross-section, length and density of a piece of a metal of atomic weight 60 are $10^{-6}m^2$, 1 m and 5×10^3 kg/m^3 respectively. Find the number of free electrons per unit volume if every atom contributes one free electron.

Also find the drift velocity of electrons in the metal when a current of 16 A passes through it. Given that Avogadro's number $N_A = 6\times10^{23}$/mol and charge on an electron $e = 1.6\times10^{-19}$ C.

Solution: As according to Avogadro's hypothesis,

$$\frac{N}{N_A} = \frac{m}{M}$$

so, $$n = \frac{N}{V} = N_A\frac{m}{VM} = \frac{N_A d}{M} \qquad \left[\text{as} d = \frac{m}{V}\right]$$

$$\therefore \quad n = \frac{6\times10^{23}\times(5\times10^3)}{(60\times10^{-3})} = 5\times10^{28}/\text{m}^3$$

Now as each atom contributes one electron, the number of electrons per unit volume

$$n_e = 1\times n = 1\times5\times10^{28} = 5\times10^{28}/\text{m}^3$$

Further as here:

$$J = \frac{I}{A} = \frac{16}{10^{-6}} = 16\times10^6\ \frac{\text{A}}{\text{m}^2}$$

and as $J = nev_d$

$$v_d = \frac{J}{ne} = \frac{16\times10^6}{(5\times10^{28})\times(1.6\times10^{-19})}$$

$$= 2\times10^{-3}\text{m/s}$$

Example 4

A copper wire of cross-sectional area $3.00 \times 10^{-6} m^2$ carries a current of 10 A. Find:

(a) The drift speed of the electrons in the wire. Assume that each copper atom contributes one free electron to the body of material.

(b) The average time between collisions for electrons in the copper at 20 °C. The density of copper is 8.95 g/cm^3, motor mass of copper 63.5 g/mol, Avogadro number 6.02×10^{23} electron/mol and resistivity of copper.

Solution (a) The volume occupied by 63.5 g of copper

$$V = \frac{M}{\rho} = \frac{63.5}{8.95} = 7.09 \text{ cm}^3/\text{mol}$$

As each copper atom contributes one free electron to the body of the material, the density of free electrons is

$$n = \frac{6.02 \times 10^{23}}{7.09 \times 10^{-6}}$$
$$= 8.48 \times 10^{28} \text{ electron/m}^3$$

The drift speed

$$v_d = \frac{I}{neA}$$
$$v_d = \frac{10.0}{8.48 \times 10^{28} \times 1.60 \times 10^{-19} \times 300 \times 10^{-6}}$$
$$= 2.46 \times 10^{-4} \text{ m/s}$$

(b) Average time between collision for electrons

$$\tau = \frac{m_e}{ne^2\rho}$$
$$= \frac{9.10 \times 10^{-31}}{8.48 \times 10^{28} \times (1.6 \times 10^{-19})^2 \times 1.7 \times 10^{-8}}$$
$$= 2.5 \times 10^{-14}$$

Example 5

Find the total momentum of electron in a straight wire of length $l = 1000$ m carrying current $I = 704$ A.

Solution: Let n be no. of electron per unit volume.

No. of electron in l length

$$N = nSl \quad (S \text{ is cross-sectional area})$$

Momentum of one electron $= mv_d$

Total momentum $P = (nSl)mv_d$

as $$v_d = \frac{I}{neS}$$

$$P = (nSl)m\frac{I}{(neS)} = \frac{mlI}{e}$$

On substituting numerical values, we get

$$P = 0.40\mu \text{ Ns}$$

Example 6

If a copper wire is stretched to make it 0.1% longer. Find the percentage change in its resistance :

Solution: For a given wire, $R = \frac{\rho L}{s}$

with $L \times s = \text{volume} = V = \text{constant}$

So that, $R = \frac{\rho L}{s}$

$$\frac{\Delta R}{R} = 2\frac{\Delta L}{L} = 2(0.1\%) = 0.2\% \text{ increase}$$

Example 7

Fig. 2E.7 shows a thick copper rod X and a thin copper wire Y joined in series. They carry a current which is sufficient to make Y much hotter than X.

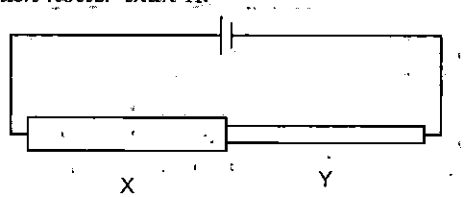


Fig.2E.7

Which one of the following is correct?

Number of density conduction electrons	Mean time between collisions of the electrons
(a) Same in X and Y	Less in X than in Y
(b) Same in X and Y	Same in X and Y
(c) More in X than Y	More in X than in Y
(d) More in X than Y	Less in X and Y
	Same in X and Y

Solution: (c) The number density n of conduction electrons in the copper is a characteristic of the copper and is about 10^{29} at room temperature for both the copper rod X and the thin copper wire Y.

Both X and Y carry the same current I since they are joined in series.

From $$I = nAv.q$$

Where q is the electron charge of 1.6×10^{-19}C, v is the drift velocity in the conductor and A is the cross-sectional area of the conductor.

We may conclude that rod X has a lower drift velocity of electrons compared to wire Y since rod X has larger cross-sectional area. This is so because the electrons in X collide more often with one another and with the copper ions when drifting towards the positive end. Thus, the mean time between collisions of the electrons is more in X than in Y.

Example 8

What is the equivalent resistance between points A and B in the networks shown in Fig. 2E.8 (a) and (b) if each resistance is R ?

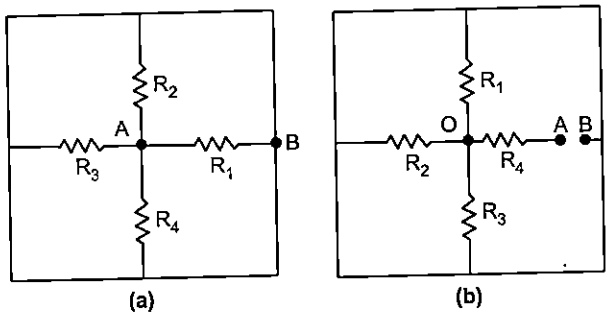


Fig. 2E.8

Solution: A careful study of the given networks reveals that:

(A) In this network one end of all the four resistances is connected to *A* while the other to *B*, *i. e.*, potential difference across each resistance is same and equal to applied between *A* and *B* while current divides, *i. e*,

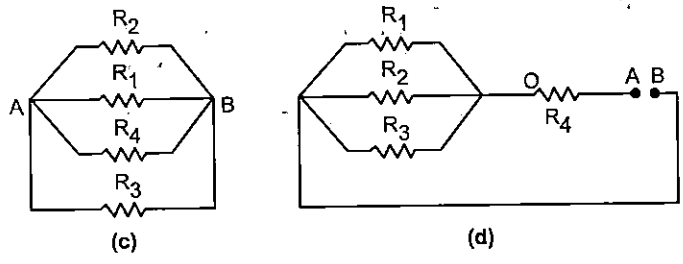


Fig. 2E.8

$V_1 = V_2 = V_3 = V_4 = V$ and $I = I_1 + I_2 + I_3 + I_4$

So the given four resistances are in parallel [as shown in Fig. 2E.8 (c)] and hence

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}$$

But as here,

$$R_1 = R_2 = R_3 = R_4 = R, R_{eq} = (R/4)$$

(B) In this network one end of each R_1, R_2 and R_3 are connected to point *O* while the other to *B*, so that R_1, R_2 and R_3 are in parallel resulting in a single resistance $(R/3)$ which is in series with resistance R_4 between points *O* and *A* as shown in Fig.2E.8 (d) so that:

$$R_{eq} = \frac{1}{3}R + R = \frac{4}{3}R$$

Example 9

When a metal rod is heated, not only its resistance but also its length and area of cross-section changes. Find the per cent change in R, l and A of a copper wire for a temperature rise of 1°C. Coefficient of linear expansion for copper is 1.7×10^{-5}/°C. and its thermal coefficient of resistance is 3.9×10^{-3}/°C.

Solution: If coefficient of linear expansion is α, percentage change in length is

$$\frac{\Delta l}{l} \times 100 = \alpha \Delta T \times 100$$

Coefficient of area expansion is 2α, so percentage change in area is

$$\frac{\Delta A}{A} \times 100 = 2\alpha \Delta T \times 100$$

percentage change in resistivity is

$$\frac{\Delta \rho}{\rho} \times 100 = \alpha_R \Delta T \times 100$$

where α_R denotes thermal coefficient of resistance.

All the variables l, A and ρ are function of T. Consequently, R is also a function of T. We find, therefore,

$$\frac{dR}{dT} = \frac{d}{dT}\left(\frac{\rho l}{A}\right)$$

$$= \frac{l}{A}\frac{d\rho}{dT} + \frac{\rho}{A}\frac{dl}{dT} + \rho l \frac{d}{dT}\left(\frac{l}{A}\right)$$

$$= \alpha_R \rho \frac{l}{A} + \alpha l \frac{\rho}{A} - \frac{1}{A^2} 2\alpha A \rho l$$

$$= (\alpha_R + \alpha - 2\alpha)\frac{\rho l}{A} = (\alpha_R - \alpha)\frac{\rho l}{A}$$

where we have written $\frac{d\rho}{dT} = \alpha_R \rho$ from equation $\Delta\rho = \alpha_R \rho \Delta T$; similarly $\frac{dl}{dT} = \alpha l$ and $\frac{dA}{dT} = 2\alpha A$.

Thus change in resistance due to temperature change is given by

$$\Delta R = (\alpha_R - \alpha) R \Delta T$$

We find that since α is quite small as compared to α_R, we have approximately

$$\Delta R = \alpha_R R \Delta T$$

and therefore, percentage change in R is the same as that in resistivity; changes in dimensions are not important.

Example 10

A long round conductor of cross-sectional area S is made of material whose resistivity depends only on a distance r from the axis of the conductor $\rho = \frac{\alpha}{r^2}$, where α is a constant. Find the resistance per unit length of conductor and the electric field strength due to which a current I flows in it.

Solution:

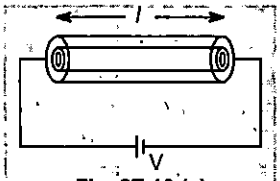


Fig. 2E.10 (a)

We consider a cylindrical shell of radius r and thickness dr, the entire conductor consists of such shell in parallel arrangement because p.d. across each is same.

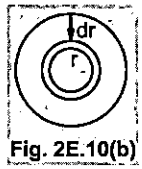


Fig. 2E.10(b)

$$dR = \frac{\rho l}{(2\pi r\, dr)} = \frac{\alpha l}{2\pi r^3 dr}$$

$$\frac{1}{R} = \int_{r=0}^{r=a} \frac{2\pi r^3 dr}{\alpha l} = \frac{2\pi a^4}{4\alpha l}$$

or $$R = \frac{4\alpha l}{2\pi a^4} \quad \text{as} \quad S = \pi a^2$$

$$\frac{R}{l} = \frac{2\pi\alpha}{S^2}$$

In accordance with Ohm's law.

$$E = \frac{V}{l} = \frac{IR}{l} = \left(\frac{R}{l}\right) I = \frac{2\pi\alpha I}{S^2}$$

Example 11

A portion of length L is cut out of a conical solid wire. The two ends of this portion have circular cross-sections of radii r_1 and $r_2 (r_2 > r_1)$. It is connected lengthwise to a circuit and a current I is flowing in it. The resistivity of the material of the wire is ρ. Calculate the resistance of the considered portion and the voltage developed across it.

Solution: Consider a disc of thickness dx, at a distance x from end AB.

Radius of the element

$$= \left(r_1 + \frac{(r_2 - r_1)}{L} x\right)$$

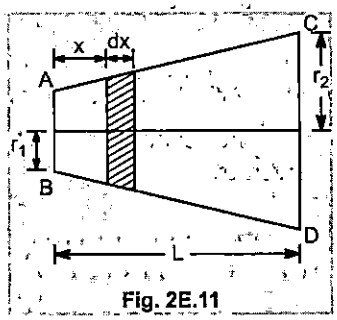


Fig. 2E.11

Resistance of this element $= \rho \dfrac{dx}{\pi\left[r_1 + \dfrac{(r_2 - r_1)}{L} x\right]^2}$

Resistance of the conical wire $= \displaystyle\int_0^L \frac{\rho dx}{\pi\left[r_1 + \dfrac{(r_2 - r_1)x}{L}\right]^2}$

$$= \frac{\rho L}{\pi r_1 r_2}$$

Voltage drop across it $= IR = I\dfrac{\rho L}{\pi r_1 r_2}$

Example 12

What is percentage change in its resistance if radius is increased by 1% and area is increased by 1%?

Solution: $\because \quad R = \rho \dfrac{V}{A^2} \quad \Rightarrow \quad \dfrac{dR}{R} = -2\dfrac{dA}{A}$

As cross-section area $A = \pi r^2$

$$\frac{dA}{A} = 2\frac{dr}{r}$$

so $$\frac{dR}{R} = -2\frac{dA}{A} = -4\frac{dr}{r}$$

If radius is increased by 1%, the resistance is decreased by 4%.

If area is increased by 1%, the resistance is decreased by 2%.

The significance of negative sign is that change in resistance is opposite to that of radius and area. Note that calculus can be employed for small percentage change only.

Example 13

An electric kettle has two coils. When one coil is switched on, it takes 15 minutes to boil water and when the second coil is switched on it takes 30 minutes. How long will it take to boil water when both the coils are used in (a) series, (b) parallel?

Solution: Heat produced in a resistance R in time t is

$$H = Pt = \frac{V^2}{R} t$$

For coil 1, $$H_1 = \frac{V^2}{R_1}(15 \times 60) \qquad \ldots(1)$$

and for coil 2, $$H_2 = \frac{V^2}{R_2}(30 \times 60) \qquad \ldots(2)$$

But according to given problem $H_1 = H_2$, *i.e.*,

$$\frac{15}{R_1} = \frac{30}{R_2}, \text{ i.e., } \quad R_2 = 2R_1 \qquad \ldots(3)$$

(a) Both the coils are used in series :

$$H_S = \frac{V^2}{(R_1 + R_2)} t_S$$

$$= \frac{V^2}{3R_1} \times t_S \qquad [\text{as } R_2 = 2R_1]$$

But as here, $H_S = H_1 (= H_2)$

So, $$\frac{V^2}{R_1}(15 \times 60) = \frac{V^2}{3R_1} \times t_S$$

$$t_S = (45 \times 60)\,\text{s} = 45 \text{ min}$$

(b) Both the coils are used in parallel :

$$H_P = \left[\frac{V^2}{R_1} + \frac{V^2}{R_2}\right] \times t_P$$

$$= \frac{3V^2}{2R_1} \times t_P \qquad [\text{as } R_2 = 2R_1]$$

According to given problem,

$$H_P = H_1$$

or $$\frac{3V^2}{2R_1} \times t_p = \frac{V^2}{R_1} \times (15 \times 30)$$

or $$t_P = (10 \times 60)\,\text{s} = 10\,\text{min}$$

Example 14

A wire of length 1.0 m and radius 10^{-3} m is carrying a heavy current and is assumed to radiate as a black body. At equilibrium its temperature is 900 K while that of the surroundings is 300 K. The resistivity of the material of the wire at 300 K is $\pi^2 \times 10^{-8}\ \Omega m$ and its temperature coefficient of resistance is 7.8×10^{-3} per °C. Find the current in the wire. Stefan's constant $= 5.68 \times 10^{-8} W\,m^{-2}K^{-4}$.

Solution: Resistance of the wire at 300 K is

$$R_0 = \frac{\rho L}{A} = \frac{\rho L}{\pi r^2}$$

$$= \frac{\pi^2 \times 10^{-8} \times 1.0}{\pi \times (10^{-3})^2} = \pi \times 10^{-2}\Omega$$

Resistance at 900 K is

$$R_t = R_0(1 + \alpha t)$$

$$= \pi \times 10^{-2}[1 + 7.8 \times 10^{-3} \times (900 - 300)]$$

$$= 5.68\pi \times 10^{-2}\Omega$$

The rate at which the wire radiates energy is

$$W = P = I^2 R_t$$

$$= 5.68\pi \times 10^{-2} I^2 \text{ watt} \qquad \ldots(1)$$

The rate at which a black body radiates energy is given by Stefan's law

$$W = \sigma A_S (T^4 - T_0^4)$$

where A_S = surface area of the wire $= (2\pi r)L$,

$$T = 900\,\text{K},\ T_0 = 300\,\text{K}$$

and $$\sigma = 5.68 \times 10^{-8}\,\text{W m}^{-2}\text{K}^{-4}$$

Thus $$W = 5.68 \times 10^{-8} \times (2\pi \times 10^{-3}) \times 1.0[(900)^4 - (300)^4]$$

$$= 5.68 \times 12.96\pi \text{ watt} \qquad \ldots(2)$$

Equating eqns. (1) and (2), we get

$$5.68\pi \times 10^{-2} \times I^2 = 5.68 \times 12.96\pi$$

$$I^2 = 1296$$

or $$I = 36\,\text{A}.$$

Example 15

(a) Two wires made of same tinned copper alloy having equal cross-sectional areas but of different lengths are to be used as fuses. Show that the fuses will melt at the same value of current which is independent of the lengths of the wires.

(b) A fuse wire of radius 0.10 mm blows when a current of 10 A passes through it. What should be the radius of a fuse wire made of the same material which will blow at a current of 20 A?

Solution: When a current is passed through a wire, its temperature rises with time due to heat produced. Since the fuse wire is exposed to the surroundings it also loses heat mainly due to radiation, heat loss depends upon temperature and it increases as temperature rises. The temperature attains a steady value when the rate at which heat is produced in the wire is equal to the rate at which heat is lost due to radiation. Let the steady temperature be equal to the melting point of the wire and I be the current at the melting point. The rate at which heat is produced in the wire is given by

$$P = I^2 R$$

where R is the resistance of the wire. The resistance of wire is

$$R = \frac{\rho l}{\pi r^2}$$

where ρ = resistivity, l = length and r = radius of the wire. Thus

$$P = \frac{I^2 \rho l}{\pi r^2} \qquad \ldots(1)$$

Now let h be the rate of loss of heat per unit surface area of the wire. The rate of loss of heat from the wire is

$$P' = 2\pi r l \times h \qquad \ldots(2)$$

In the steady state, $P = P'$. Therefore equating eqns. (1) and (2), we have

$$\frac{I^2 \rho l}{\pi r^2} = 2\pi r l h$$

or $$I = \left(\frac{2\pi^2 r^3 h}{\rho}\right)^{1/2} \qquad \ldots(3)$$

Thus the current at which the melting point is reached is independent of the length of the wire, it depends upon the values of r, ρ and h. If the two wires have the same value of r, they are made of the same material (ρ is the same for both wires), the rate of loss of heat per unit surface area (h) depends upon the material of the wire and the melting point which is same for both the wires, both fuses will melt at the same value of current, inspite of different lengths.

From eqn. (3), we have

$$I = \left(\frac{2\pi^2 r^3 h}{\rho}\right)^{1/2}$$

or $$\frac{I^2}{r^3} = \frac{2\pi^2 h}{\rho} = \text{constant}$$

Thus $$\frac{I_1^2}{r_1^3} = \frac{I_2^2}{r_2^3}$$

or $$r_2 = r_1 \times \left(\frac{I_2}{I_1}\right)^{2/3}$$

Substituting numerical values

$$r_1 = 0.10\text{mm},\ I_1 = 10\,\text{A}$$

and $$I_2 = 20\,\text{A}$$, we have

$$r_2 = 0.10\times\left(\frac{20}{10}\right)^{2/3} = 0.10(2)^{2/3}$$
$$= 0.159\,\text{mm} = 0.16\,\text{mm}$$

Example 16

What amount of heat will be generated in a coil of resistance R due to a charge q passing through it if the current in the coil:
(a) decreases down to zero uniformly during a time interval t_0 ?
(b) decreases down to zero halving its value every t_0 seconds ?

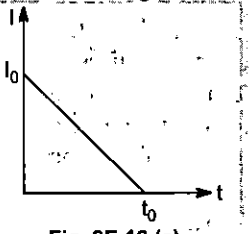


Fig. 2E.16 (a)

Solution: (a) The current decreases uniformly with time, therefore I *vs* t curve is a straight line as shown in Fig. 2E.16(a) with slope $m = -I_0/t_0$. Current as function of time can be written as

$$I = I_0 - \left(\frac{I_0}{t_0}\right)t \quad (y = -mx + c) \quad \ldots(1)$$

Area under I *vs* t graph gives the flow of charge q, therefore

$$q = \frac{1}{2}(t_0)(I_0)$$

$$\therefore \quad I_0 = \frac{2q}{t_0}$$

Substituting in eqn. (1), we get

$$I = \frac{2q}{t_0}\left(1 - \frac{t}{t_0}\right)$$

or $$I = \left(\frac{2q}{t_0} - \frac{2qt}{t_0^2}\right)$$

Heat produced in a time interval dt is

$$dH = I^2R\,dt$$
$$= \left(\frac{2q}{t_0} - \frac{2qt}{t_0^2}\right)R\,dt$$

or $$H = \int_0^{t_0}\left(\frac{2q}{t_0} - \frac{2qt}{t_0^2}\right)R\,dt$$
$$= \frac{4}{3}\frac{q^2R}{t_0}$$

(b) Here, current decreases from I_0 to zero exponentially with half-life of t_0. The I-t equation in this case is an exponential function like the radioactive decay law

$$I = I_0e^{-\lambda t}$$

where $$\lambda = \frac{\ln(2)}{t_0}$$

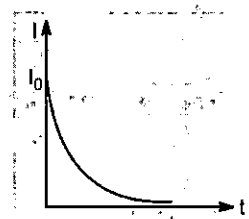


Fig. 2E.16 (b)

total charge,

$$q = \int_0^\infty I\,dt$$
$$= \int_0^\infty I_0e^{-\lambda t}dt = \left(\frac{I_0}{\lambda}\right)$$

or $$I_0 = \lambda q$$

or $$I = (\lambda q)e^{-\lambda t}$$

Heat produced in time interval dt,

$$dH = I^2R\,dt = \lambda^2q^2e^{-2\lambda t}R\,dt$$

ro $$H = \lambda^2q^2R\int_0^\infty e^{-2\lambda t}dt$$

$$= \frac{q^2\lambda R}{2} = \frac{q^2R\ln(2)}{2t_0}$$

ONE DIMENSIONAL CONDUCTION

In this type of conduction, current flows parallel to the length of conductor. Plane perpendicular to the direction of flow of current is always an equipotential plane and the decrease in potential per unit distance is maximum in the direction of flow of current.

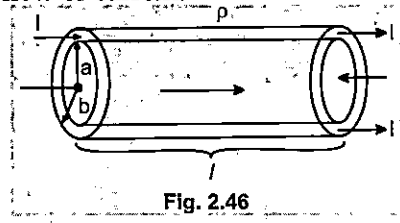


Fig. 2.46

In one dimensional conduction the equipotential surfaces are simple planes and parallel to each other.

Resistance of the tube shown in the figure between the two circular faces is given by (using $R = \frac{\rho l}{A}$)

$$R = \frac{\rho l}{\pi(b^2 - a^2)}$$

where ρ is the resistivity which is uniform throughout.

Resistance of a Conical Conductor Between its Two Circular Faces

Since the area of the equipotential surface (area of cross-section) varies as we move from one face to another, so to calculate the resistance, let us consider a co-axial disc of radius 'r' and thickness dx at a distance 'x' from one end as shown in the figure.

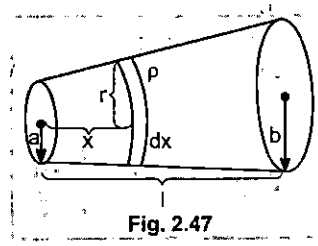


Fig. 2.47

Radius of the disc,

$$r = a + nx$$

where 'n' is the increament in radius per unit length.

$$i.e., \qquad n = \frac{(b-a)}{l}$$

The resistance of the element disc,

$$dR = \frac{\rho\, dx}{\pi r^2}$$

Since all such discs are in series therefore the net resistance of the conductor,

$$R = \int dr = \int \frac{\rho dx}{\pi r^2} = \frac{\rho}{\pi}\int_0^l \frac{dx}{(a+nx)^2}$$

$$= -\frac{1}{n}\frac{\rho}{\pi}\left(\frac{1}{a+nl} - \frac{1}{a}\right)$$

$$= \frac{\rho}{\pi}\frac{nl}{a(a+nl)n} = \frac{\rho l}{\pi a(a+b-a)}$$

$$R = \frac{\rho l}{\pi ab}$$

Tow dimensional conduction (Cylindrical Conduction), Resistance Between Inner and Outer Surface

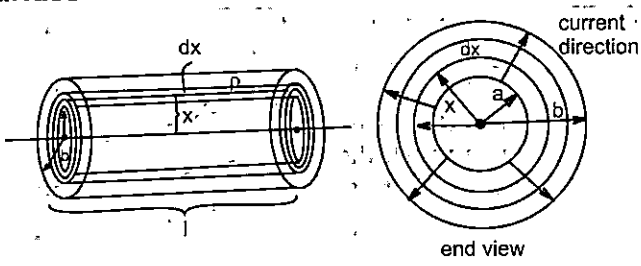


Fig. 2.48

To calculate the resistance let us consider an elemental concentric cylindrical shell of radius 'x' and thickness 'dx'. The resistance of this elemental shell is given by,

$$dR = \frac{\rho dx}{2\pi x l}$$

Since all such elemental shells are in series, therefore the net resistance of the conductor is,

$$R = \int dR = \int_a^b \frac{\rho\, dx}{2\pi x l} \quad \Rightarrow \quad R = \frac{\rho}{2\pi l}\ln\left(\frac{b}{a}\right)$$

Equipotential Surfaces are Spherical

To calculate the resistance between the inner and outer surface, let us consider a spherical shell of radius 'x' and thickness 'dx'.

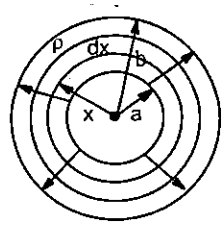


Fig. 2.49

$$dR = \frac{\rho\, dx}{4\pi x^2}$$

$$R = \int dR$$

$$= \frac{\rho}{4\pi}\int_a^b \frac{dx}{x^2}$$

$$R = \frac{\rho}{4\pi}\frac{(b-a)}{ab}$$

Example 17

The cross-section area and length of a cylindrical conductor are A and l, respectively. The specific conductivity varies as $\sigma(x) = \sigma_0 \frac{l}{\sqrt{x}}$, *where x is the distance along the axis of the cylinder from one of its ends [see Fig. 2E.17 (a)].*

(a) Compute the resistance of the system along the cylindrical axis.

(b) Compute the current density if the potential drop along the cylinder is V_0. *What is the electric field at each point in the cylinder in the case described?*

Solution: (a) Consider the cylinder as composed of thin discs of width dx connected in series [see Fig. 2E.17 (a)]. The resistance of a disc at a distance x away from the cylinder end is :

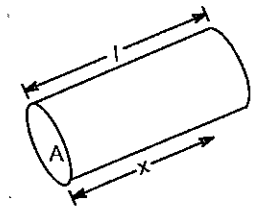


Fig. 2E.17 (a)

$$dR = \frac{1}{\sigma(x)}\frac{dx}{A} = \frac{\sqrt{x}dx}{Al\sigma_0} \qquad ...(1)$$

where A is the cross-section area of the disc and dx is its width. Since the discs are connected in series, the total resistance is

$$R = \int_0^l dR = \frac{1}{Al\sigma_0}\int_0^l \sqrt{x}\, dx = \frac{2\sqrt{l}}{3A\sigma_0} \qquad ...(2)$$

(b) From Ohm's law, we deduce that the current flowing across the cylinder is given by

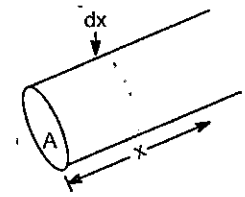


Fig. 2E.17 (b)

$$I = \frac{V_0}{R} = \frac{3A\sigma_0 V_0}{2\sqrt{l}} \qquad ...(3)$$

The current density is, therefore;

$$J = \frac{I}{A} = \frac{3\sigma_0 V_0}{2\sqrt{l}} \qquad ...(4)$$

The electric field in the cylinder may be found by using Ohm's law :

$$E(x) = \frac{J}{\sigma(x)} = \frac{J}{\sigma_0 \frac{l}{\sqrt{x}}} = \frac{3V_0\sqrt{x}}{2l^{3/2}} \qquad ...(5)$$

Example 18

The space between two coaxial cylinders, whose radii are a and b (where $a < b$ *as in Fig. 2E.18), is filled with a conducting medium. The specific conductivity of the medium is* σ.

(a) Compute the resistance between the cylinders in the radial direction, using two distinct methods.Assume that the cylinders are very long compared to their radii, i.e., $L >> b$, *where L is the length of the cylinders.*

(b) Calculate the resistance, assuming σ *varies as* $\sigma(r) = \frac{\sigma_0}{r}$.

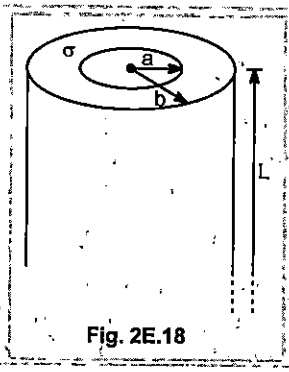


Fig. 2E.18

Solution: (a) From Ohm's law, we have

$$\vec{\mathbf{J}} = \sigma \vec{\mathbf{E}} \quad \ldots(1)$$

Assuming radial current density, $\vec{\mathbf{J}}$ becomes

$$\vec{\mathbf{J}} = \frac{I}{2\pi r L}\hat{\mathbf{r}} \quad \text{for} \quad a < r < b \quad \ldots(2)$$

and, therefore,

$$\vec{\mathbf{E}} = \frac{I}{2\pi\sigma r L}\hat{\mathbf{r}} \quad \ldots(3)$$

Here we have used the assumption that $L >> b$ so that $\vec{\mathbf{E}}$ and $\vec{\mathbf{J}}$ are in cylindrically symmetric form. The potential drop across the medium is thus :

$$V_{ab} = -\int_b^a \vec{\mathbf{E}}(r)\cdot d\vec{\mathbf{r}} = -\frac{I}{2\pi\sigma L}\int_b^a \frac{dr}{r} = \frac{I}{2\pi\sigma L}\ln\left(\frac{b}{a}\right) \quad \ldots(4)$$

The resistance is clearly:

$$R_{ab} = \frac{V_{ab}}{I} = \frac{\ln\left(\frac{b}{a}\right)}{2\pi\sigma L} \quad \ldots(5)$$

Method 2 : We split the medium into differential cylindrical shell elements, of width dr, in series. The current flow is cylindrically symmetric ($L >> b$). The area through which the current flows across a shell of radius r is $A(r) = 2\pi rL$. The length the current flows, passing through a shell of radius r is dr. Therefore, the resistance of the shell of radius r is :

$$dR = \frac{1}{\sigma}\frac{dr}{2\pi rL} \quad \ldots(6)$$

Since the shells are connected in a series, we have :

$$R_{ab} = \int_a^b dR = \frac{\ln\left(\frac{b}{a}\right)}{2\pi\sigma L} \quad \ldots(7)$$

(b) Using $\sigma(r) = \frac{\sigma_0}{r}$, we have from eqn. (6),

$$dR = \frac{dr}{2\pi rL\sigma(r)} = \frac{dr}{2\pi L\cdot r\cdot\frac{\sigma_0}{r}} = \frac{dr}{2\pi L\sigma_0} \quad \ldots(8)$$

Thus, $$R_{ab} = \int_a^b dR = \frac{b-a}{2\pi L\sigma_0} \quad \ldots(9)$$

Example 19

A conducting medium is shaped in the form of a quarter of an annulus of radii b and a ($b > a$) and thickness h. The medium's specific conductivity is σ. Compute the resistance between the three pairs of opposite faces in the $\hat{r}, \hat{z}$ and $\hat{\theta}$ directions.

Solution: (a) Calculate for the $\hat{z}$ direction : In this case, the electric current density is z-independent; $\vec{\mathbf{J}} = J\hat{\mathbf{z}}$, where J = constant. The cross-section area is $A = \frac{1}{4}\pi(b^2 - a^2)$ and the width of the medium through which the current passes is $l = t$. Therefore,

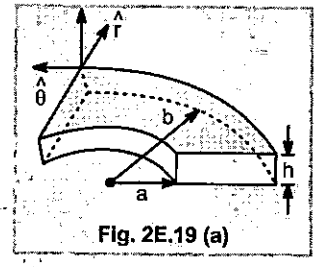


Fig. 2E.19 (a)

$$R_{\hat{z}} = \frac{1}{\sigma}\frac{h}{A} = \frac{1}{\sigma}\frac{h}{\frac{1}{4}\pi(b^2-a^2)} = \frac{4h}{\pi\sigma(b^2-a^2)} \quad \ldots(1)$$

(b) $\hat{r}$ direction : In this case, the radial current density is $\vec{\mathbf{J}} = \frac{I}{A(r)}\hat{\mathbf{r}}$, where $A(r)$ is the cross-section area at radius r through which the current flows. Therefore, $A(r) = \frac{1}{4}2\pi rh$; we divide the conductor of Fig. 2E.19 (b) into radial slices of width dr, as can be seen in figure. The area of such a slice is $A = \frac{1}{4}\cdot 2\pi rh$, and the width through which the current passes is $l = dr$. Thus, the resistance of a slice of radius r is

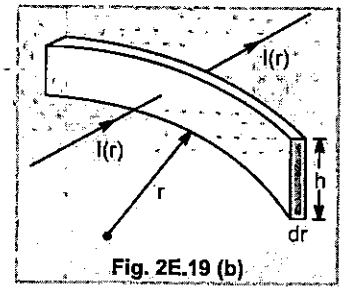


Fig. 2E.19 (b)

$$dR = \frac{1}{\sigma}\frac{l}{A} = \frac{1}{\sigma}\frac{dr}{\pi h\frac{r}{2}} \quad \ldots(2)$$

These slices are connected in a series, since all the current flowing out of the slice at r enters the adjacent slice of $r + dr$. Thus,

$$R_{\hat{r}} = \int_a^b dR = \frac{2}{\pi\sigma h}\int_a^b \frac{dr}{r} = \frac{2}{\pi\sigma h}\ln\left(\frac{b}{a}\right) \quad \ldots(3)$$

(c) $\hat{\theta}$ direction : In this case $\vec{\mathbf{J}} = J\hat{\theta}$, *i.e.*, the current flow lines are azimuthal. We split the conductor into radial slices. The current flows as shown in the figure. The slices are connected in parallel. The cross-section area of each slice is $h\,dr$, and the length along which the current flows in the slice of radius r is $\frac{1}{4}\cdot 2\pi r = \frac{\pi r}{2}$.

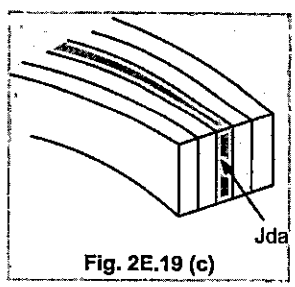


Fig. 2E.19 (c)

Therefore, $$dR = \frac{1}{\sigma}\frac{\pi\frac{r}{2}}{h\,dr} \quad \ldots(4)$$

Since the slices are connected in parallel, the equivalent resistance is given by

$$\frac{1}{R_{eq.}} = \int_a^b \frac{1}{dR} = \int_a^b \frac{2\sigma h\,dr}{\pi r} = \frac{2\sigma h}{\pi}\ln\left(\frac{b}{a}\right) \quad \ldots(5)$$

or, $$R_{\hat{\theta}} = \frac{\pi}{2\sigma h \ln\left(\frac{b}{a}\right)} \quad \ldots(6)$$

Concepts:1. *The resistivity or conductivity is independent of the magnitude and direction of electric field.*

2. *Resistance R is characteristic property of object. R is independent of potential difference.*

3. *ΔV, I and R are macroscopic quantities applying to a body or region. The corresponding macroscopic properties are $\vec{E}$, $\vec{J}$ and ρ or σ they have values at every point in a body.*

4. *Resistivity of a metal can be increased by (1) raising its temperature (2) adding small amounts of impurities and (3) increase lattice imperfections by doping.*

5. *ΔV, I, R are macroscopic quantities applying to a particular body or extended region.*

6. *$\vec{E}, \vec{J}$ and $\vec{a}$ are corresponding microscopic quantities they have values at every point of a body.*

7. *Temperature variation of resistivity.*

$$\alpha = \frac{1}{\rho}\frac{d\rho}{dt}$$

8. *Ohm's law is not a fundamental law of electromagnetism because it depends on properties of the conducting medium.*

(ρ = const. for same material not dependent on electric field.)

9. *Electrons move with a uniform average speed is result of quantum mechanics.*

The effective speed, obtained from the quantum distribution is nearly independent of temperature.

10. *Collisions between electrons themselves are rare do not effect the electrical properties of material.*

11. *Energy is transferred from the applied field to the lattice [in from of internal energy of the conductor, often observed as a temperature increase].*

12. *Drift speed is different from speed at which changes in the electric field configuration travels along wire.*

13. *$\vec{J} = \sigma\vec{E}$ is valid only for isotopic materials whose electric properties are same in all directions.*

14. *In case of ohmic materials the resistivity or conductivity of a material is independent of the magnitude and direction of the applied field.*

$\rho \rightarrow$ characteristic of material.

τ is independent of $\vec{E}$

ρ does not dependent on $\vec{E}$

for materials that obey ohm's law τ is unchanged when field is applied particular temperature.

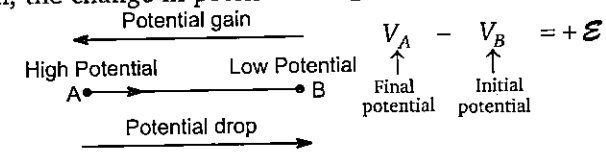
KIRCHHOFF'S LAWS FOR CIRCUIT ANALYSIS

Before moving on to the statement of Kirchhoff's law, we state some conventions to be followed in circuit analysis:

(1) Direction of conventional current is from high potential to low potential terminal.

(2) Current flows from high potential node *A* to low potential node *B*. If we traverse from point *A* to *B*, there is drop of potential; similarly from *B* to *A*, there is gain of potential.

If a source of emf is traversed from negative to positive terminal, the change in potential is $+\mathcal{E}$.



Note that while discharging, current is drawn from the battery, the current comes out from positive terminal and enters negative terminal, while charging of battery current is forced from positive terminal of the battery to negative terminal. Irrespective of direction of current through a battery the sign convention mentioned above holds.

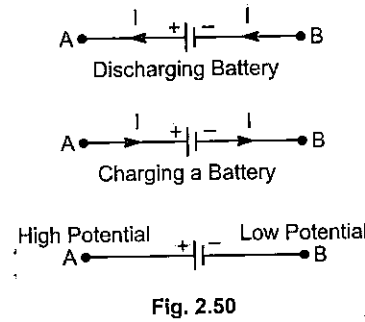


Fig. 2.50

The positive plate of a capacitor is at high potential and negative plate at low potential. If we traverse a capacitor from positive plate to negative plate, the change in potential is $-Q/C$.

$$V_B - V_A = -\frac{Q}{C}$$

(V_B: High potential; V_A: Low potential)

If we traverse a resistor in the direction of current, the change in potential is $-IR$

$$V_B - V_A = -IR$$

(V_B: Final potential; V_A: Initial potential)

If we transverse a resistor in the direction opposite to the direction of current, the change in potential is $+IR$.

$$V_A - V_B = +IR$$

(V_A: Final potential; V_B: Initial potential)

If a capacitor is traversed from negative plate to positive plate, the change in potential is $+Q/C$.

$$V_A - V_B = +\frac{Q}{C}$$

(V_A: High potential; V_B: Low potential)

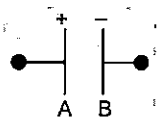


Fig. 2.51

KIRCHHOFF'S RULES

Two rules, called Kirchhoff's rules and named for G.R. Kirchhoff (1824-1887), guide us in finding the currents. These rules are referred to as the loop rule and the junction rule.

The Loop Rule

The loop rule states: The sum of the potential differences encountered in a round-trip around any closed loop in a circuit is zero.

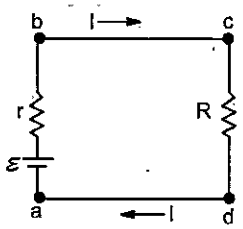
Concept: *Since the potential is directly related to the potential energy of the carriers, the loop rule is a statement of conservation of energy. We can write the loop rule as*

$$\Sigma V = 0$$

As we consider the potential in going around a loop in a circuit, the potential increases through some elements and decreases through others; the sum of the potential differences for a complete round-trip is zero.

By convention, I has a positive value when the sense of the current corresponds to the direction of motion of positive carriers.

Consider using the loop rule to find the current in the circuit of Fig. 2.52 (a). The sense of the current I is shown in the figure. We begin at point a and traverse the loop in the clockwise sense. The loop rule gives



Using the loop rule

Fig. 2.52 (a)

$$(V_b - V_a) + (V_c - V_b) + (V_a - V_c) + (V_a - V_d) = 0$$

The potential difference across the section from a to b is the terminal potential difference across the battery: $V_b - V_a = \mathcal{E} - Ir$. The connecting wires have negligible resistance, so the potential difference $V_c - V_b$ and $V_a - V_d$ are each zero.

The current through R is from c and d so that $V_c > V_d$. Therefore $V_d - V_c = -IR$.

Substitution into the loop rule gives

$$(\mathcal{E} - Ir) + (0) + (-IR) + (0) = 0$$

Solving for I, we have

$$I = \frac{\mathcal{E}}{r + R}$$

In the above analysis we chose to traverse the loop clockwise, but that choice is arbitrary. Suppose we traverse the loop counterclockwise, beginning at point a. The loop rule gives

$$(V_d - V_a) + (V_c - V_d) + (V_b - V_c) + (V_a - V_b) = 0$$

The potential difference $(V_c - V_a)$ is $+IR$ because $V_c > V_d$. Also, $V_a - V_b = -(V_b - V_a) = -(\mathcal{E} - Ir) = -\mathcal{E} + Ir$. This gives

$$IR - \mathcal{E} + Ir = 0$$

Solving for I we have

$$I = \frac{\mathcal{E}}{r + R}$$

This is the same result as before. The answer is independent of which way we go around the loop.

There are two rules we can use to give the algebraic sign of terms we enter into the loop-rule equation:

1. In traversing a resistance R along the sense of the current I, the potential difference across the resistance is entered as $-IR$. In traversing a resistance R opposite the sense of the current I, the potential difference is entered as $+IR$.

2. In traversing a source of emf along the sense of the emf, the potential difference across the source is entered as $+\mathcal{E}$. In traversing a source of emf opposite the sense of the emf, the potential difference across the source is entered as $-\mathcal{E}$.

In using these rules, we treat the internal resistance of the source as a separate resistance.

Now consider applying the loop rule to the single-loop circuit shown in Fig. 2.52 (b). Since the senses of the emf's of the two batteries are opposite one another, we are not certain about the sense of I. Let us assume that the sense of I is counterclockwise, as shown in the figure. Starting at point a and going counterclockwise around the loop, we write the sum of the potential difference as

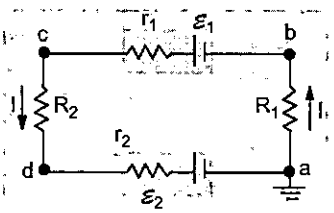


Fig. 2.52 (b)

$$(IR_1) + (+\mathcal{E}_1) + (-Ir_1)) + (-IR_2) + (-Ir_2) + (-\mathcal{E}_2) = 0$$

Solving for the current I, we have

$$I = \frac{\mathcal{E}_1 - \mathcal{E}_2}{R_1 + R_2 + r_1 + r_2}$$

Notice that eq. (1) yields a positive value for I if $\mathcal{E}_1 > \mathcal{E}_2$ and a negative value for I if $\mathcal{E}_2 > \mathcal{E}_1$. If $\mathcal{E}_2 > \mathcal{E}_1$, then the sense of the current is clockwise, opposite our assumed sense for I in Fig. 2.52 (b). Therefore, if we assume a particular sense for the current at the outset of a problem and the value of the current turns out to be negative, then the actual sense of the current is opposite the assumed sense. That is, the equation automatically gives the sense of the current.

We can assume the current has a particular sense, and if this assumption turns out to be wrong, then I has a negative value. In the case of Fig. 2.52 (b) the actual sense of the current is counterclockwise, the same as our assumed sense.

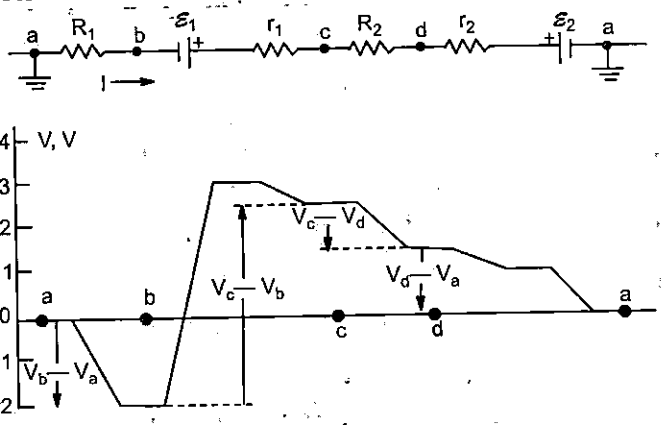


Fig. 2.53

Fig. 2.53 shows a graph of the loop rule for the circuit in Fig. 2.52(b). In our mind's we break the circuit at point a and string it along a straight line. Then we show the variation of the potential along the sense of the current. The potential of point a has been arbitrarily set to zero (that is, point a is "grounded").

The Junction Rule

The sum of the currents toward a junction point is equal to the sum of the currents away from the same point.

Points a and b in Fig. 2.54 are examples of points. Since charge does not accumulate at any point along the connecting wires, the point rule is simply a statement of the conservation of charge. The point rule can be written as

$$\Sigma I_{\text{toward}} = \Sigma I_{\text{away}} \qquad ...(1)$$

For example, the point rule applied to point a in the circuit in Fig. 2.54 gives

$$I_1 = I_2 + I_3 \qquad ...(2)$$

because current I_1 is toward a and current I_2 and I_3 are away from a.

We now use the loop rule and the point rule to find the currents I_1, I_2 and I_3 in the circuit of Fig. 2.54. Let loop *abcda* be loop 1 and loop *aefba* be loop 2. From the figure, our assumed sense for I_1 is chosen to be from b to c to d to a

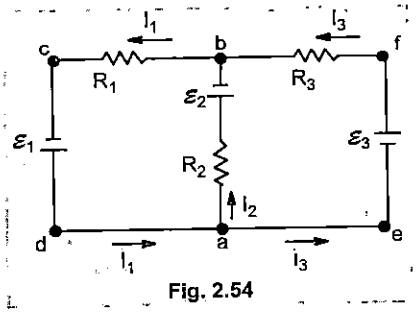


Fig. 2.54

(counterclockwise); our assumed sense for I_2 is chosen to be from a to b; and our assumed sense for I_3 is chosen to be from a to e to f to b. Also, for simplicity we include the internal resistance of each battery in the resistance that is in series with that battery. For example, the internal resistance of battery 1 is included in R_1.

The loop rule applied to loop 1 beginning at point a and going around counterclockwise gives

$$(-I_2R_2) + (-\mathcal{E}_2) + (-I_1R_1) + (\mathcal{E}_1) = 0$$

Rearranging, we have

$$\mathcal{E}_1 - \mathcal{E}_2 = I_1R_1 + I_2R_2 \qquad ...(3)$$

The loop rule applied to loop 2 beginning at point a and going around counterclockwise gives

$$(\mathcal{E}_3) + (-I_3R_3) + (\mathcal{E}_2) + (I_2R_2) = 0$$

Rearranging, we have

$$\mathcal{E}_3 + \mathcal{E}_2 = I_3R_3 - I_2R_2 \qquad ...(4)$$

We have three equations in three unknowns. The three equations are the point-rule equation (2) and the loop-rule equations (3 and 4). The three unknowns are the currents I_1, I_2 and I_3. The point-rule equation is simpler than the other two because each of the coefficient is 1. Using the point rule equation to eliminate I_1 in Eq. (3) gives

$$\mathcal{E}_1 - \mathcal{E}_2 = I_2(R_1 + R_2) + I_3R_1 \qquad ...(5)$$

Equations 4 and 5 represent two equations in two unknowns. Substitution of the numerical values from Fig. 2.54 into these equations gives.

COMBINATION OF CELLS

Series Combination

If the negative terminal of a cell is connected to the positive terminal of the second and the negative terminal of the second to positive terminal of the third and so on then the cells are said to be in series.

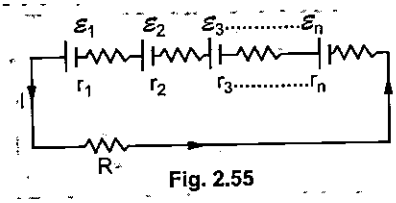


Fig. 2.55

Applying Kirchhoff's law, we get

$$I = \frac{\mathcal{E}_1 + \mathcal{E}_2 + \ldots \mathcal{E}_n}{r_1 + r_2 + r_3 \ldots r_n + R}$$

and equivalent emf.

$$\mathcal{E}_{eq} = \mathcal{E}_1 + \mathcal{E}_2 + \mathcal{E}_3 + \ldots \mathcal{E}_n$$

and $r_{eq} = r_1 + r_2 + r_3 + \ldots r_n$

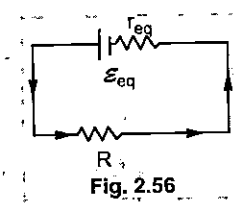


Fig. 2.56

If $\mathcal{E}_1 = \mathcal{E}_2 = \mathcal{E}_3 = \mathcal{E}_4 \ldots = \mathcal{E}_n = \mathcal{E}$ (say)

and $r_1 = r_2 = r_3 = r_4 \ldots = r_n = r$ (say)

then $I = \dfrac{n\mathcal{E}}{nr + R}$

$\mathcal{E}_{eq} = n\mathcal{E}$

$r_{eq} = nr$

case-1:

If $R >>> nr$

then $I = \dfrac{n\mathcal{E}}{R}$

i.e., current the circuit is 'n' times the current due to a single cell.

case-2:

If $R <<< nr$

then $I = \frac{n\mathcal{E}}{nr} = \frac{\mathcal{E}}{r}$

i.e., current in the circuit is same as due to a single cell.

Thus when internal resistance is negligible in comparison at the external resistance then cells are connected in series to get maximum current.

Parallel Combination

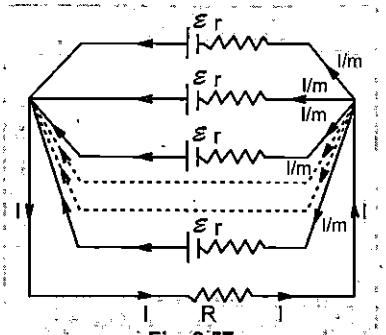


Fig. 2.57

In parallel connection of cells all the positive terminals are connected together at one point and all the negative terminals at another point.

Let there be 'm' branches with one cell each then applying Kirchhoff's law in any one loop.

$$-IR - \frac{I}{m}r + \mathcal{E} = 0$$

$$I = \frac{\mathcal{E}}{R + \left(\frac{r}{m}\right)}$$

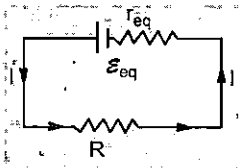


Fig. 2.58

$$r_{eq} = \frac{r}{m}$$

$$I' = \frac{\mathcal{E}_{eq}}{R + r_{eq}}$$

$I = I'$ so $\mathcal{E}_{eq} = \mathcal{E}$

Case-1:

If $R >>> r/m$,

then $I = \frac{\mathcal{E}}{R}$

i.e., the current in the circuit is equal to the current due to a single cell.

Case-2:

If $R <<< r/m$

then $I = \frac{m\mathcal{E}}{r}$

i.e., the current in the circuit is 'm' times the current produced by a single cell.

Hence when external resistance is negligible in comprising to the internal resistance (r/m) then the cells are connected in parallel to get maximum current.

If the cells have different emf, then the current in the circuit is solved by using kirchhoff's two laws.

Equivalent emf

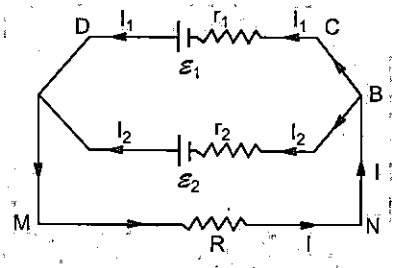


Fig. 2.59 (a)

From Kirchhoff's first law at any junction,

$$I = I_1 + I_2 \quad ...(1)$$

From second law for the loop DAMNBCD,

$$\mathcal{E}_1 - IR - I_1 r_1 = 0$$

$$I_1 = \frac{\mathcal{E}_1 - IR}{r_1} \quad ...(2)$$

Similarly from the loop AMNBA.

$$I_2 = \frac{\mathcal{E}_2 - IR}{r_2} \quad ...(3)$$

Putting the values of I_1 and I_2 from (ii) and (iii) in equation (i), we get

$$I = \left(\frac{\mathcal{E}_1}{r_1} + \frac{\mathcal{E}_2}{r_2}\right) - IR\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

$$I + IR\left(\frac{1}{r_1} + \frac{1}{r_2}\right) = \left(\frac{\mathcal{E}_1}{r_1} + \frac{\mathcal{E}_1}{r_2}\right)$$

$$I\left[1 + R\left(\frac{1}{r_1} + \frac{1}{r_2}\right)\right] = \left(\frac{\mathcal{E}_1}{r_1} + \frac{\mathcal{E}_2}{r_2}\right)$$

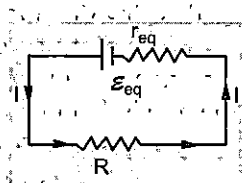


Fig. 2.59 (b)

$$I\left[R + \left(\frac{1}{r_1} + \frac{1}{r_2}\right)^{-1}\right] = \left(\frac{\mathcal{E}_1}{r_1} + \frac{\mathcal{E}_2}{r_2}\right)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)^{-1} \quad ...(4)$$

Now from the equivalent circuit,

$$I(R + r_{eq}) = \mathcal{E}_{eq} \quad ...(5)$$

Comparing equations (4) and (5), we get

$$r_{eq} = \left(\frac{1}{r_1} + \frac{1}{r_2}\right)^{-1} = \frac{r_2 r_1}{r_1 + r_2}$$

and $$\mathcal{E}_{eq} = \left(\frac{\mathcal{E}_1}{r_1} + \frac{\mathcal{E}_2}{r_2}\right)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)^{-1}$$

$$= \frac{\mathcal{E}_1 r_2 + \mathcal{E}_2 r_1}{r_1 + r_2}$$

In general for 'n' cells,

$$\mathcal{E}_{eq} = \left(\frac{\mathcal{E}_1}{r_1} + \frac{\mathcal{E}_2}{r_2} + \frac{\mathcal{E}_3}{r_3} \ldots + \frac{\mathcal{E}_n}{r_n}\right)\left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} \ldots + \frac{1}{r_n}\right)^{-1}$$

$$= \left(\Sigma \frac{\mathcal{E}}{r}\right)\left(\Sigma \frac{1}{r}\right)^{-1}$$

Mixed Combination of Cells

In the circuit N similar cells each of emf E and internal

resistance 'r' are connected as shown in figure. The arrangement consists of 'm' rows of cells, the rows being

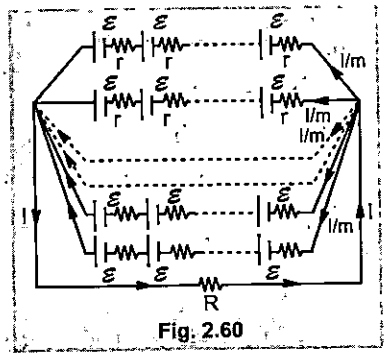


Fig. 2.60

connected in parallel each row has 'n' cells connected in series. Applying Kirchhoff's law in any of the loops, we get,

$$I = \frac{n\mathcal{E}}{R + \frac{nr}{m}}$$

$$I' = \frac{\mathcal{E}_{eq}}{R + \frac{nr}{m}} \quad I = I'$$

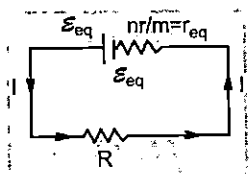


Fig. 2.61

Therefore $\mathcal{E}_{eq} = n\mathcal{E}$;

$$r_{eq} = \frac{n}{m} r$$

For current to be maximum,

$$\frac{dI}{dn} = 0$$

$$\frac{d}{dn}\left(\frac{mn\mathcal{E}}{mR + nr}\right) = 0$$

$$\frac{d}{dn}\left(\frac{N\mathcal{E}}{\frac{Nr}{n} + nr}\right) = 0$$

$$R = \frac{nr}{m}$$

$$I_{max} = \frac{n\mathcal{E}}{2R} = \frac{m\mathcal{E}}{2r}$$

Power Transferred to the Load

$$P = \left(\frac{n\mathcal{E}}{R + \frac{nR}{m}}\right)^2 R$$

For P to be maximum,

$$R = \frac{nr}{m} \text{ (by maximum power transfer theorem)}$$

$$P_{max} = \frac{n^2\mathcal{E}^2}{2R} = \frac{m^2\mathcal{E}^2 R}{4r^2}$$

Hence in mixed combination of cells, the current in the circuit and power dissipated in the load resistance are maximum for the same condition. Therefore it is preferred to series or parallel combination.

Net Work Analysis

The Kirchhoff's current law (KCL) states that the algebraic sum of the currents entering the junction must equal the sum of the currents leaving the junction. From the standpoint of physics, *KCL is a statement of charge conservation.*

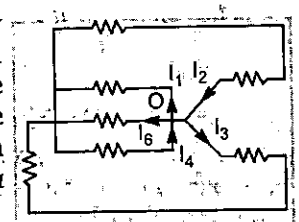


Fig. 2.62

The KCL applied to junction O yields

$$\underbrace{I_1 + I_6 + I_3}_{\text{Outgoing current}} = \underbrace{I_2 + I_4}_{\text{Incoming current}}$$

The Kirchhoff's law (KVL) sates that the algebraic sum of the potential difference around any closed loop of an electric circuit is zero. *The KVL is a statement of conservation of energy.* The KVL reflects that electric force is conservative, the work done by a conservative force on a charge taken around a closed path is zero.

- We can move clockwise or anticlockwise, it will make no difference because the overall sum of the potential difference is zero.
- We can start from any point on the loop, we just have to finish at the same point.
- An ideal battery is modelled by an independent voltage source of emf $\mathcal{E}$ and an internal resistance r as shown in Fig. 2.64. A real battery always absorbs power when there is a current through it, thereby offering resistance to flow of current.

Applying KVL around the single loop in anticlockwise direction, starting from point A, we have

$$\underbrace{+IR}_{\substack{\text{in the opposite} \\ \text{direction to current}}} + \underbrace{Ir}_{\substack{\text{In the opposite} \\ \text{direction to current}}} - \underbrace{\mathcal{E}}_{\substack{\text{From positive} \\ \text{to negative} \\ \text{terminal}}} = 0$$

Hence,

$$I = \frac{\mathcal{E}}{R + r}$$

Fig. 2.63 shows variation of potential in a closed loop.

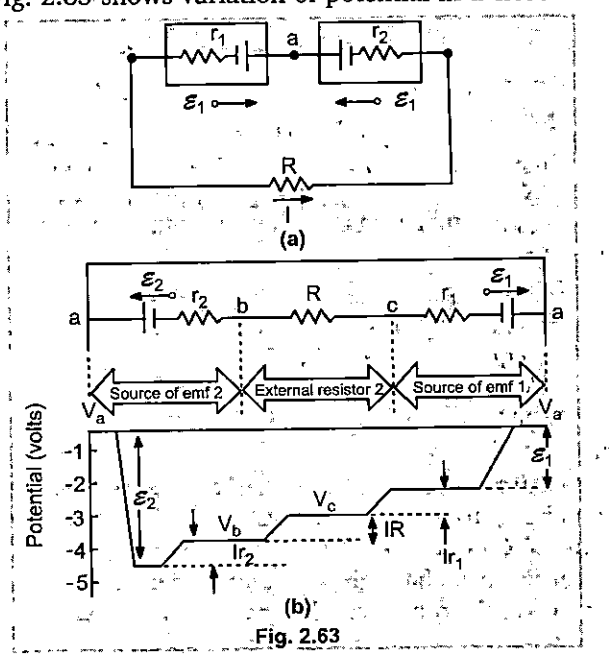


Fig. 2.63

Maximum power transfer theorem

The power absorbed by load resistor R is

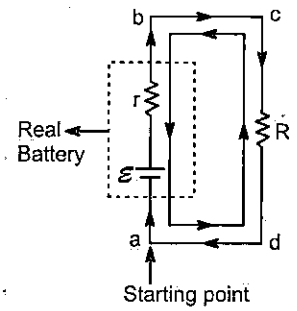


Fig. 2.64

$$P = I^2R = \left(\frac{\mathcal{E}}{R+r}\right)^2 R$$

For maximum power transfer we take the derivative of P w.r.t. R, set it equal to zero and solve the equation for R.

$$\frac{dP}{dR} = 0$$

$$\frac{dP}{dR} = \mathcal{E}^2 \frac{(R+r)^2 - R[2(R+r)]}{(R+r)^2} = 0$$

Solving for R, we have

$$(R+r)^2 - R(2)(R+r) = 0$$

$$(R+r) - 2R = 0$$

$$R = r$$

For a given real battery the load resistance maximizes the power if it is equal to the internal resistance of the battery.

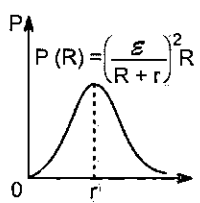


Fig. 2.65

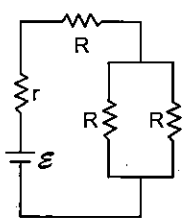


Fig. 2.66

The maximum power transfer theorem is general, it holds for any real voltage source. The resistance R may be a single resistor or R may be the equivalent resistance of a collection of resistors.

☛What value of R maximizes the power absorbed by all the resistors?

The total equivalent resistance connected to battery must be equal to the internal resistance of the real battery.

☛We assign a potential V_a to the point a and traverse to point b.

$$V_a + \mathcal{E} - Ir = V_b$$

When we reach the junction b, we must be at potential V_b.

Thus we have $V_b - V_a = \mathcal{E} - Ir$

where $V_b - V_a$ is the terminal potential difference across the battery.

$$\underbrace{V_b - V_a}_{\text{path 1}} = \mathcal{E} - \left(\frac{\mathcal{E}}{R+r}\right)r = \frac{\mathcal{E}R}{R+r}$$

We can traverse the circuit along path 2 to obtain $V_b - V_a$, we must get the same potential difference because potential difference is independent of path followed.

$$\underbrace{V_b - V_a}_{\text{path 2}} = +IR = \left(\frac{\mathcal{E}}{R+r}\right)R$$

☛The first step in analyzing a circuit is to simplify it to simplest equivalent configuration. In Fig. 2.67, each of the circuits has the same equivalent resistance between A and B, in each version, R_1, R_2 and R_3 are in parallel.

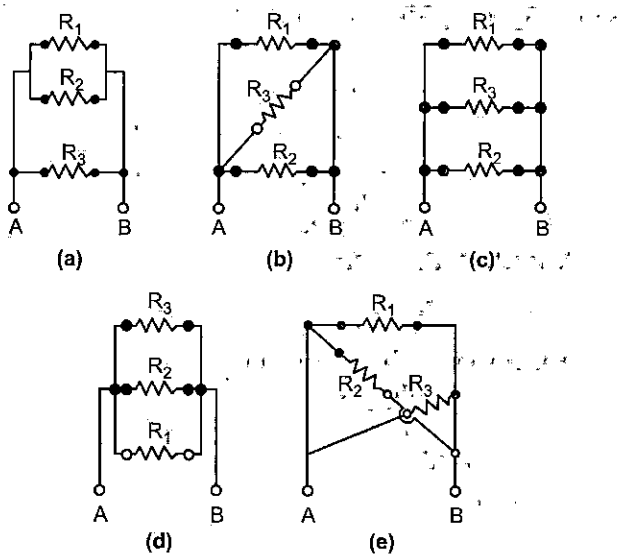


Fig. 2.67

☛All the resistors in Fig. 2.68 (a) are in parallel arrangement.

All the resistors in Fig. 2.68 (b) are in series arrangement.

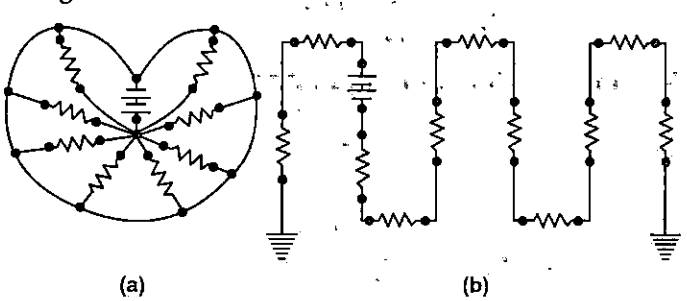


Fig. 2.68

☛Fig. 2.69 shows several circuits in which the circuit elements are neither in series nor in parallel.

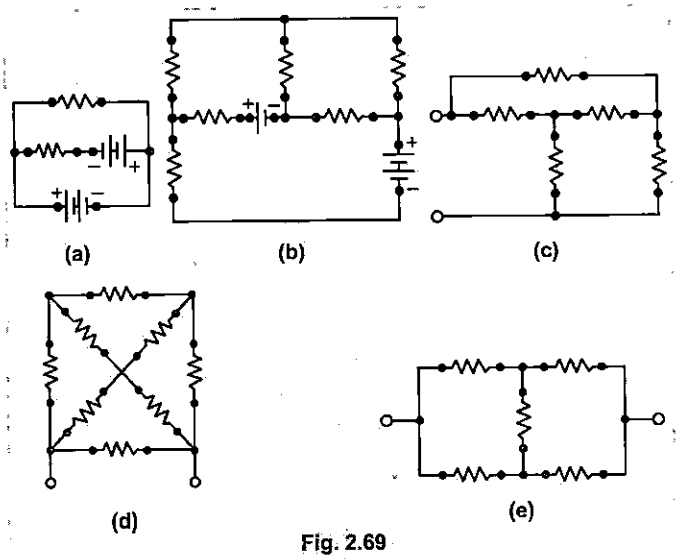


Fig. 2.69

☛If the same current passes through every resistor in a given branch [Fig. 2.70], irrespective of the presence of sources in that branch, the resistors are in series even though they are not directly connected to each other. Same is true about capacitors.

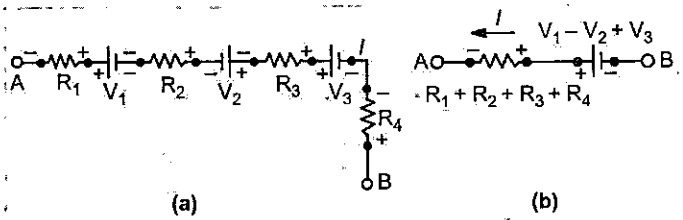


Fig. 2.70

☛In the given circuit [Fig. 2.71] we will determine (a) the equivalent resistance between C and G, (b) the current provided by the source, (c) voltage across points G and E.

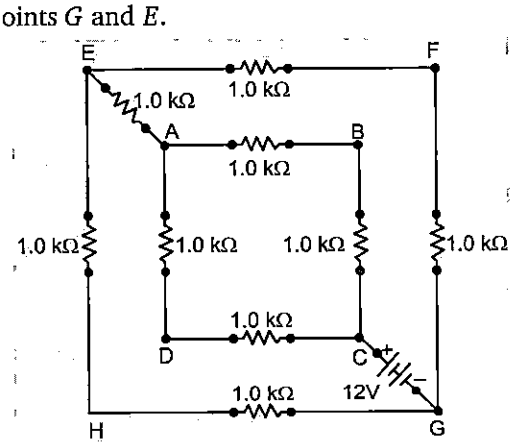


Fig. 2.71

(a) Imagine the wires to be flexible and lift up the inside square with the resistors and source attached.

Follows Fig. 2.72 (a), (b), (c) and (d) to arrive at equivalent circuit. The equivalent resistance is 3 kΩ.

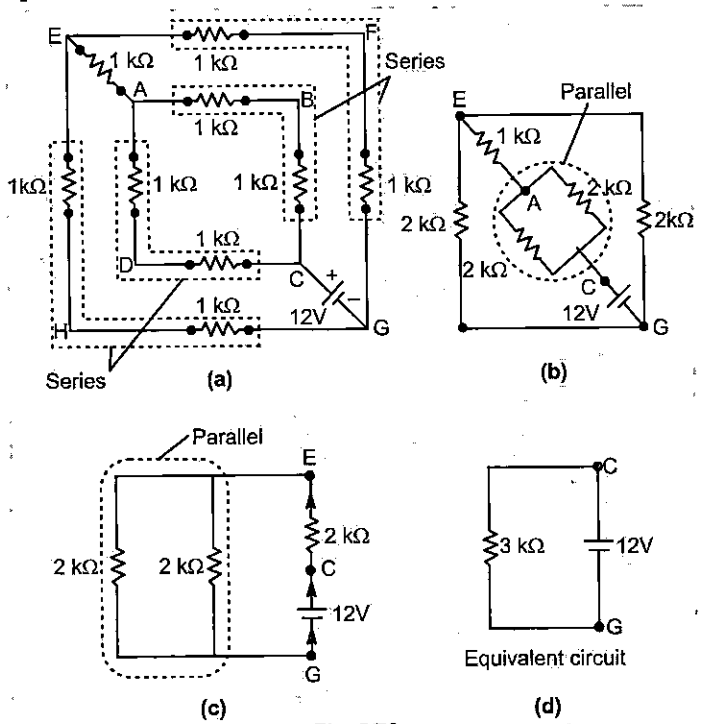


Fig. 2.72

(b) Since $V = IR_{eq.}$

$$I = \frac{V}{R_{eq.}} = 4\,\text{mA}$$

(c) Start at point G, assign it a potential V_G, proceed toward E along any path. When you reach point E after adding potential drops and gains you get potential of E.

$$V_G + 12 - IR = V_E$$

$$V_E - V_G = 12 - IR$$

$$= 12 - (4 \times 10^{-3} \times 2 \times 10^3)$$

$$= 4\,\text{V}$$

☛In the given figure we wish to determine (a) the current I in the circuit, (b) the potential at each of the labelled points a, b, c, d, e, f assuming that the potential at f is zero, (c) power input and output in the circuit.

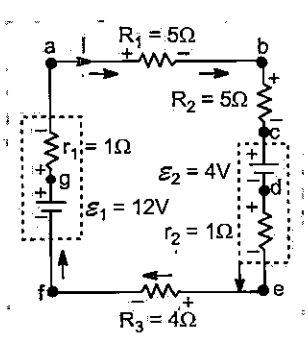


Fig. 2.73

(a) First we assume that current is clockwise. Now we apply KVL in the assumed direction of current.

Circuit element	Charges in potential	Sign
$R_1 (a \to b)$	Drop IR_1	minus
$R_2 (b \to c)$	Drop IR_2	minus
$E_2 (c \to d)$	Drop E_2	minus
$r_2 (d \to e)$	Drop Ir_2	minus
$R_3 (e \to f)$	Drop IR_3	minus
$E_1 (f \to g)$	Drop E_1	plus
$r_1 (g \to a)$	Drop Ir_1	minus

Always remember *the signs of emf are independent of the current.*

Hence we have

$$-IR_1 - IR_2 - \mathcal{E}_2 - Ir_2 - IR_3 + \mathcal{E}_1 - Ir_1 = 0$$

$$I = \frac{\mathcal{E}_1 - \mathcal{E}_2}{R_1 + R_2 + R_3 + R_4 + R_5}$$

Note that if $\mathcal{E}_2$ is greater than $\mathcal{E}_1$, we get a negative value of I, which shows that assumed direction of current is wrong.

$$I = \frac{12-4}{5+5+4+1+1} = 0.5 \text{ A}$$

(b) Now we determine the potential at each labelled point in the circuit.

$$V_g = V_f + \mathcal{E}_1 = 0 + 12 = 12\text{V}$$
$$V_a = V_g - Ir_1 = 12 - (0.5)(1) = 11.5\text{V}$$
$$V_b = V_a - IR_1 = 11.5 - (0.5)(5) = 9\text{V}$$
$$V_c = V_b - IR_2 = 9 - (0.5)(5) = 6.5\text{V}$$
$$V_d = V_c + \mathcal{E}_2 = 6.5 - 4 = 2.5\text{V}$$
$$V_e = V_d - Ir_2 = 2.5 - (0.5)(1) = 2.0\text{V}$$
$$V_f = V_e - IR_3 = 2.0 - (0.5)(4) = 0$$

Note that we have assigned f to be at zero potential, we can choose any point of the circuit to be at zero potential and then determine the potentials of the other points relative to it. The zero potential point is indicated by the ground symbol ⏚ at point f. The Earth can be considered to be a very large conductor with infinite capacity and unlimited supply of charge. Therefore the potential of the Earth remains essentially constant. In practice electric circuits are often grounded, *e.g.*, outside metal case of a washing machine is grounded by connecting it by a wire to a water pipe that is in contact with the Earth.

Since all the grounded points are a constant potential, it is customary to assign it a zero potential.

(c) Power delivered by source of emf $\mathcal{E}_1$,

$P_{\mathcal{E}_1} = \mathcal{E}_1 I = (12)(0.5) = 6$ W

Power dissipated in resistors,

$P_R = I^2(R_1 + R_2 + R_3 + r_1 + r_2)$

$= (0.5)^2(5+5+4+1+1) = 4$W

The power consumed by battery 2 in getting charged,

$P_{\mathcal{E}_2} = \mathcal{E}_2 I = 4(0.5) = 2$W

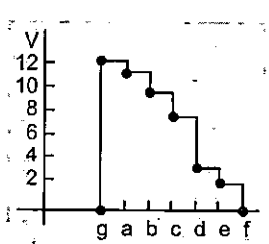


Fig. 2.74

Note that battery E_1 is discharging, the current comes out of its positive terminal. The terminal voltage across its terminals

$$= \mathcal{E}_1 - Ir_1$$
$$= 12 - (0.5 \times 1) = 11.5\text{V}$$

The battery $\mathcal{E}_2$ is charging, the current goes in from its positive terminal. The terminal voltage across it $= \mathcal{E}_2 + Ir_2 = 4 + (0.5)(1) = 4.5$V. The terminal voltage is greater than the emf of the battery.

☛When analyzing a circuit, study it carefully. Try to guess is there anything extraordinary in the circuit? Is there any shortcut? Is the circuit symmetrical in such a way that some sort of bridge is present so that the elements might be redundant. Take a look at the following illustrations.

☛In the circuit shown in Fig. 2.75 we wish to determine current and potential drop on resistor R_1. Note that points a, h, g and f have same potential, they are connected by conducting wires without any circuit elements between them. Similarly points b, c, d and e have same potential. Hence the potential drop across branch e and f, and a and b is same. The two resistors (6Ω and 4Ω in series) are directly connected across the terminals of 12 V battery.

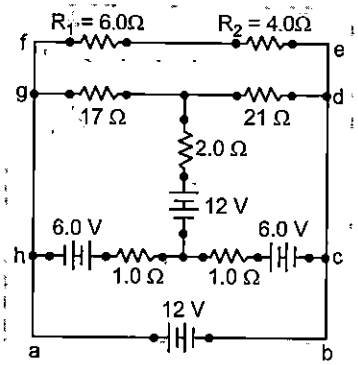


Fig. 2.75

The complex circuitry in the middle has no effect on the potential drop across the upper 10 Ω branch. If the current through it is I.

Potential drop across $R_1, V_1 = IR_1$

Potential drop across $R_2, V_2 = IR_2$

Potential drop across branch,

$V = V_1 + V_2 = I(R_1 + R_2)$

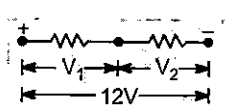


Fig. 2.76

The current $I = \frac{V}{R_1 + R_2} = \frac{12}{10} = 1.2$

Hence $V_1 = (1.2)(6) = 7.2\text{V}$

$V_2 = (1.2)(4) = 4.8\text{V}$

☛We wish to determine the current through the 8 Ω resistor in Fig. 2.77. Notice the polarity of batteries in the lowest horizontal branch and in the right vertical branch. The net emf in these branches is zero. So there is no current anywhere in the circuit.

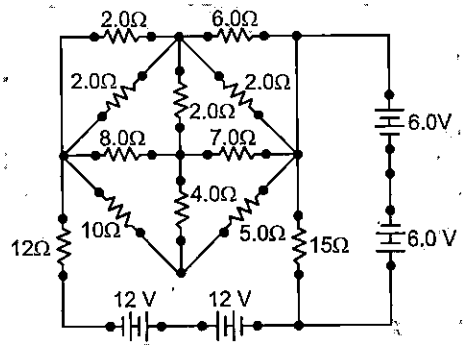


Fig. 2.77

What is the voltage drop across and the current through the 6 Ω resistor (Fig. 2.78)? Note that when current reaches node *A*, it "sees" two identical paths and therefore splits into

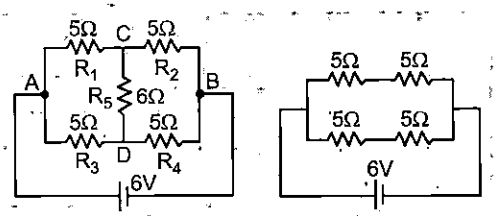


Fig. 2.78

equal parts. The voltage drop across R_1 is equal to the voltage drop across R_3. Hence points *C* and *D* are at same potential. There is no potential difference across R_5 and no current through it. So R_5 has no effect. By symmetry the voltage at *C* equals the voltage at *D*, so $V_{CD} = 0$. The resistors R_1 and R_2 are in series as are R_3 and R_4. Thus the equivalent resistance of circuit is 5 Ω. The given circuit is called **Wheatstone bridge.** We assume that R_4 is unknown. R_2 can be varied and it is adjusted till the current in the mid-branch is zero. In this condition:

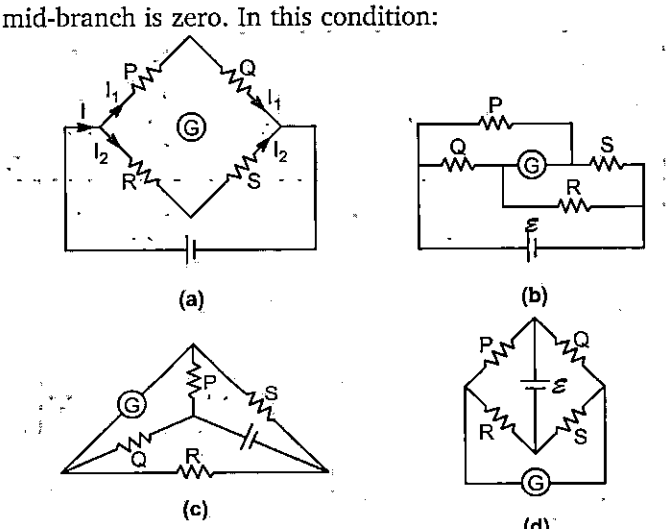


Fig. 2.79

P.D. across AC = P.D. across AD

Also, P.D. across CB = P.D. across DB

Therefore $I_1R_1 = I_2R_2$ ...(1)

and $I_1R_2 = I_2R_4$ ...(2)

From eqns.(1) and (2), we have

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

All the four circuit, (a), (b), (c) and (d) represent a **Wheatstone bridge** network.

☛In Fig. 2.80(a) shown, if a battery is connected between points *A* and *B*, emf, $\mathcal{E} = 18$ V, what is current through it? The resistors at *C* and *D* are redundant, they are open branches, therefore no current through them. The point *E* is grounded, but no current will leak to ground, no return path for the current. Points *a* and *d* are at the same potential, similarly points *b* and *c* are at the same potential, because these points are connected by conducting wires without any circuit element. Notice the 6Ω and 3Ω resistors, similarly 18Ω, 9/4Ω resistors, these pairs are a parallel arrangement.

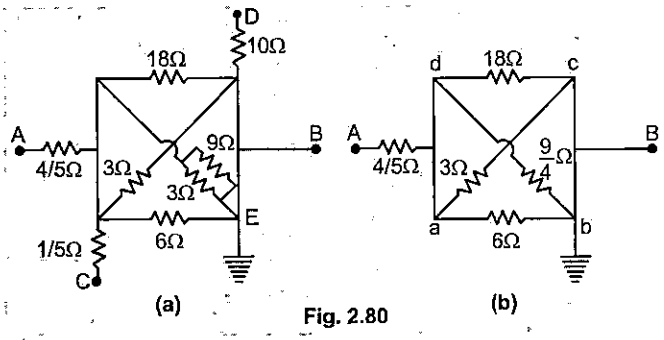
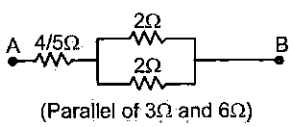


Fig. 2.80

Thus the equivalent resistance of the circuit between *A* and *B* is 1.8Ω. The current supplied by battery,



$$I = (18/1.8)\,\text{A} = 10\,\text{A}$$

☛In the Fig. 2.81 shown we wish to determine current provided by the battery. Just flip the circuit about the dotted line shown in the figure. Now the circuit is simple with equivalent resistance 2Ω and current $I = (10/2)\,\text{A} = 5\,\text{A}$.

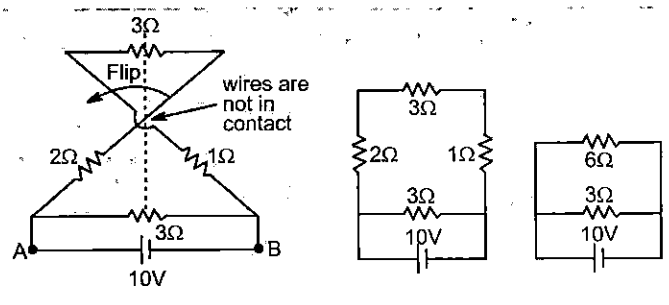


Fig. 2.81

☛In the Fig. 2.82 shown we wish to determine the current through each 5 Ω resistor. Note that both the 5Ω resistors are connected to same points *A* and *B*, across which battery is connected. Therefore each of

5Ω resistors is in parallel arrangement with battery, therefore the current through it, $I = (20/5)\text{A} = 4\text{A}$.

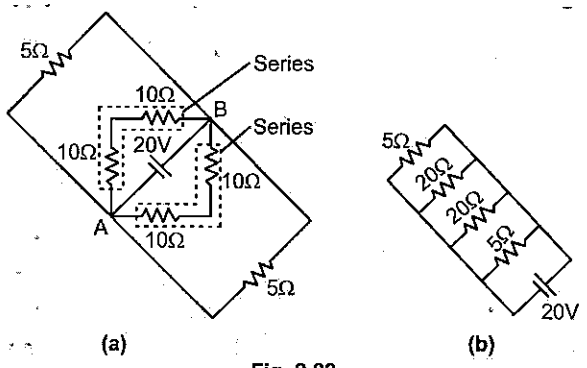


Fig. 2.82

☛The equivalent resistance between A and B is

$$\frac{1}{R_{eq.}} = \frac{1}{20} + \frac{1}{20} + \frac{1}{5} + \frac{1}{5} = \frac{10}{20}, \qquad R_{eq.} = 2\Omega$$

Thus the current supplied by battery $= (20/2)\text{A} = 10\text{A}$
Power supplied by battery $= VI = (20)(10) = 200$ W.

☛In Fig. 2.83 we wish to determine the current through 6Ω resistor. Both the ends of the bottom branch are grounded; so the net potential difference between E and A is zero. If we traverse from E to A there is a potential drop of 12 V across battery, so there must be a potential gain of 12 V in the resistor. Therefore the current in 6 Ω resistor is $I = (12/6)\text{A} = 2\text{A}$ and to the left. What is the current in the 9 Ω resistor? It is zero because there is no potential difference across E and A. The entire current of the battery goes from E into the zero resistance path back to A *via* ground. When ground connections are shown, it is assumed that all such points are wired to a common line even if not shown.

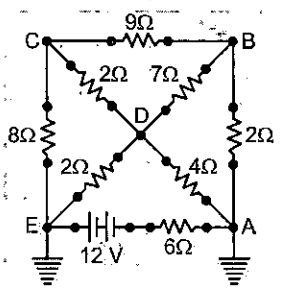


Fig. 2.83

☛In the Fig. 2.84 shown we wish to determine current in one of the 4 Ω resistors in the circuit. It is zero because the current will follow the path of least resistance, in fact a path of zero resistance is available, see figure.

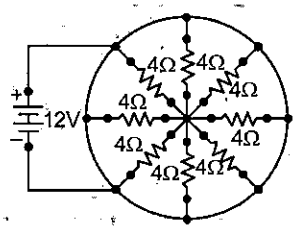


Fig. 2.84

☛When a circuit is symmetrical about a line (By symmetry we mean that two parts are mirror images of each others), then the potential and current must also be symmetrical. Therefore, currents in ab and ad are same (Fig. 2.85). Currents in dc and bc are same. Potentials of the points b, e and d are same. The equivalent circuit is redrawn, the equivalent resistance is $2/3R$. Note that there is no current in branches be and ef.

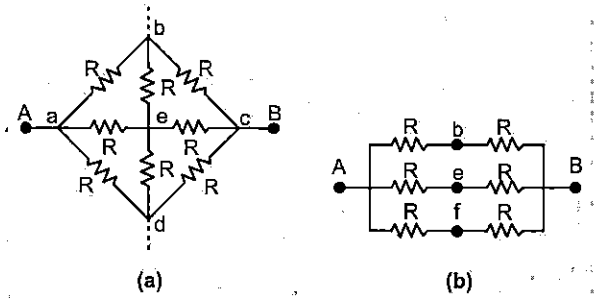


Fig. 2.85

Another symmetry is visible along line bd. The current flow is not a mirror image in branches ab and bc because the flow is in same direction. This is called asymmetric condition. The special thing about this asymmetry is that current incoming at b is equal to outgoing current, similar situation exists at b and d also. Thus resistor in branches be and de are ineffective.

In Fig. 2.86 there is asymmetry along line xy. The current reaching $O(I_2)$ is equal to outgoing current that means there is no mingling of current from upper branch and lower branch into middle branch.

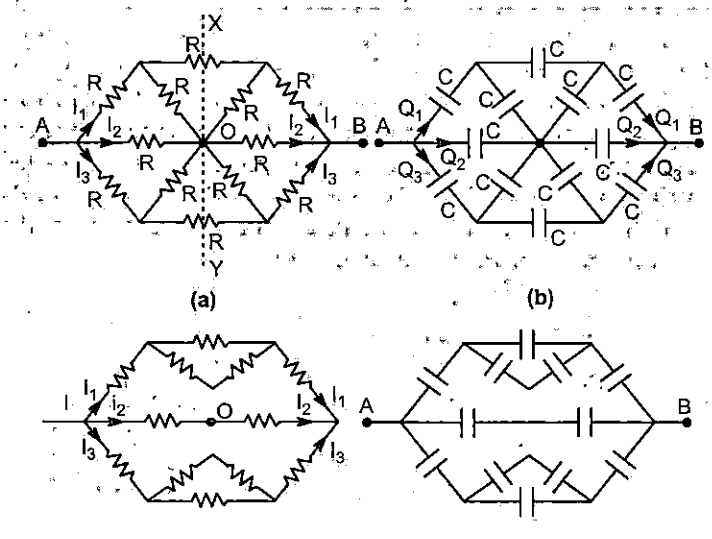


Fig. 2.86

The resulting circuit is simple enough, the equivalent resistance and capacitance are $4R/5$ and $5C/4$ respectively.

☛We wish to determine equivalent resistance between A and B. In Fig. 2.87 points (1, 2), (3, 4, 5) and (6, 7) are at same potential. Equivalent circuit can be redrawn as in Fig. 2.88.

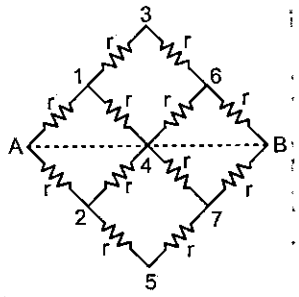


Fig. 2.87

The equivalent resistance of this series combination is

$$R_{eq.} = \frac{r}{2} + \frac{r}{4} + \frac{r}{4} + \frac{r}{2} = \frac{3r}{2}$$

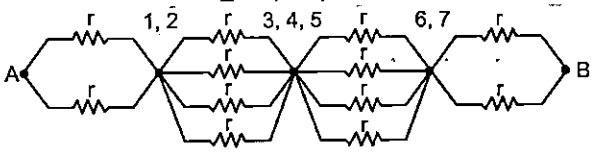


Fig. 2.88

☛In the Fig. 2.89 shown, the resistances specified are in ohms. We wish to determine the equivalent resistance between points *A* and *D*. Points *B* and *C*, *E* and *F* are at the same potential, so the circuit can be redrawn as in Fig. 2.90. Thus the equivalent resistance is 1 Ω.

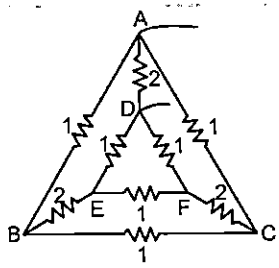


Fig. 2.89

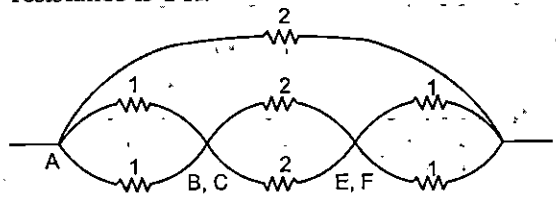


Fig. 2.90

☛In the network shown in Fig. 2.91 all the resistances are equal, we wish to determine equivalent resistance between *A* and *E*. Points *B* and *D* have same potential, similarly *F* and *H* have same potential. The equivalent circuit is shown in Fig. 2.92. The equivalent resistance of network is $7R/2$.

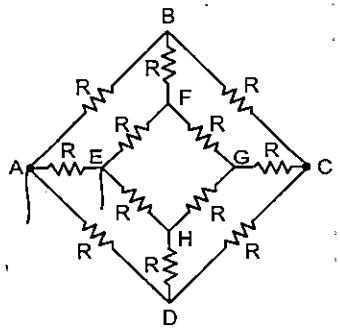


Fig. 2.91

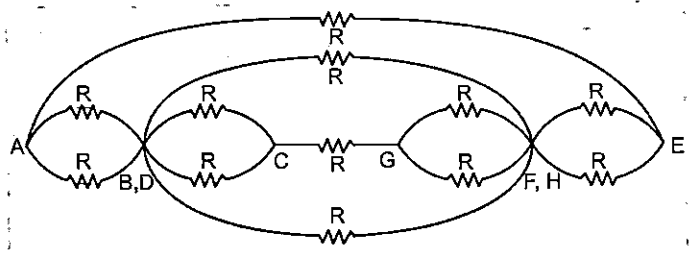


Fig. 2.92

☛Referring to Fig. 2.93 we see that symmetry demands that current only circulates in outer branch. Points *A* and *B* are at the same potential because the circuit is symmetrical. Therefore no current can go across the resistors in the branch. The current through both batteries is 2 A.

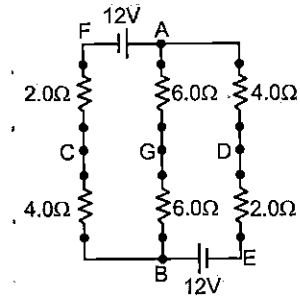


Fig. 2.93

NODAL ANALYSIS

1. It is based on Kirchhoff's current law eqn. At any node in electrical circuit $\Sigma I = 0$ (it will be reffered as node eqn).

2. In this technique to solve any numerical problem on electrical circuit assign potential of every junction of circuit taking potential of any one of the junction of the circuit zero (this will be called as reference node or reference junction).

3. Apply node eqn. to solve for unknown potential introduced in the circuits. Current in node eqn. will be written using resistor eqn. *i.e.*, $\left(I = \frac{V}{R}\right)$

Example 20

Find (i) Current through 4Ω resistor,
(ii) Find potential difference between A and B.

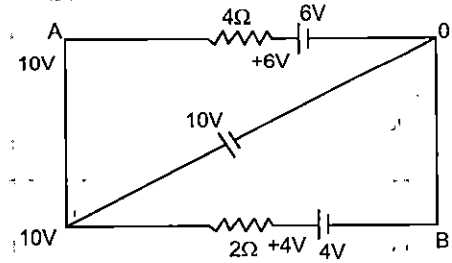


Fig. 2E.20 (a)

Solution: We assign zero potential at point B

$$V_B = 0$$

$$\therefore \quad V_A - V_B = 10$$

$$I_{4\Omega} = \frac{10-6}{4}$$

by $V = IR$

or $1\ A$

Remark

If any of the junction of circuit is earthed then potential of that point in circuit is zero.

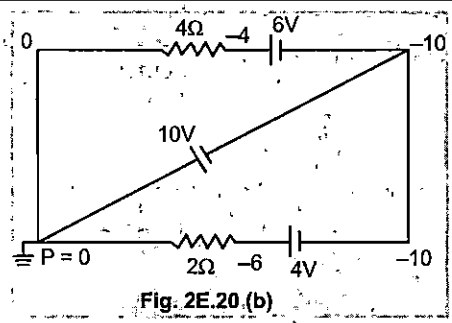


Fig. 2E.20 (b)

If we assign zero potential at P then potential at different point as shown in Fig. 2E.20.

Example 21

(i)Find the current through 5Ω resistor ($I_{5\Omega}$), and
(ii) Find potential difference between A and B ($V_A - V_B$).

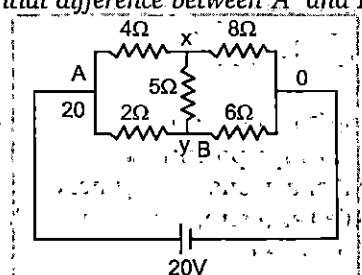


Fig. 2E.21 (a)

Solution: Node A:

$$\Sigma I = 0$$

$$\frac{x-0}{8} + \frac{x-y}{5} + \frac{x-20}{4} = 0 \qquad ...(1)$$

Node B:

$$\frac{y-x}{5} + \frac{y-0}{6} + \frac{y-20}{2} = 0 \qquad ...(2)$$

Remark:

While writing node eqn. at any junction, assume its potential to be highest or lowest and write node eqn. for that junction accordingly.

While writing node eqn. at any only junction in the same circuit the fredom of assuming the node potential to be highest or lowest is not affected by previous choice.

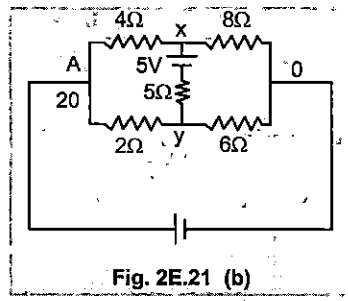


Fig. 2E.21 (b)

$$\frac{x-0}{8} + \frac{x-20}{4} + \frac{x-5-y}{5} = 0$$

$$\frac{y-0}{6} + \frac{y-20}{2} + \frac{y-5-x}{5} = 0$$

Example 22

Find: (i) $I_{4\Omega}$ (ii) $V_A - V_B$

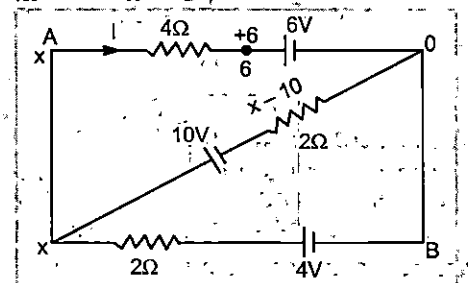


Fig. 2E.22

Solution: Node x

$$\frac{x-6}{4} + \frac{x-10}{2} + \frac{x-4}{2} = 0$$

$$x = \frac{34}{5} V$$

$$\therefore \quad I = \frac{34-30}{5\times 4} = \frac{1}{5} A$$

Remark:

While writing node eqn. at any junction consider all resistances connected to the junction taking potential of battery if connected appropriately.

EQUIVALENT RESISTANCE

In order to determine the equivalent resistance, we first reduce the network of resistance to a single resistance R_{eq}. Equivalent resistance means a single resistance that will have same effect as the network itself.

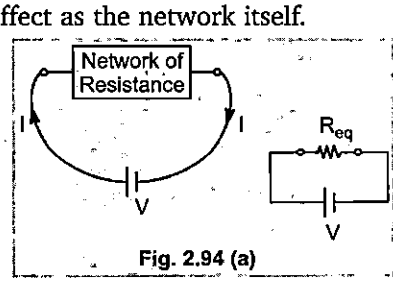


Fig. 2.94 (a)

$$R_{eq} = \frac{V}{I}$$

$$R_{eq} = \frac{\text{Battery voltage}}{\text{Current drawn by network from the battery}}$$

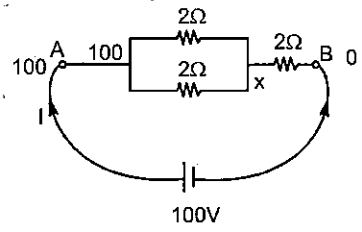


Fig. 2.94 (b)

$$\frac{x-0}{4}+\frac{x-100}{2}2=0$$

$$x = 80V$$

$$R_{AB} = \frac{100}{\frac{100-x}{2}+\frac{100-x}{2}} = 5\Omega$$

Example 23

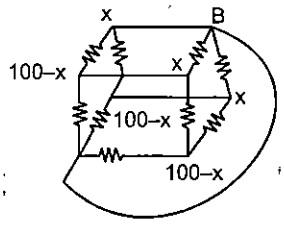


Fig. 2E.23

Solution: Applying KCL we get

$$\frac{x-0}{R}+\frac{2(2x-100)}{R}=0$$

$$x = 40V$$

$$R_{AB} = \frac{100}{\frac{x-(100x)}{R}\times 3} = \frac{5R}{6}$$

Example 24

Find the equivalent resistance of the network shown in the figure, across the points O and A. The resistance of each branch of the octagon is r_0.

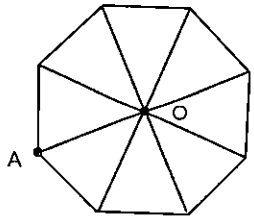


Fig. 2E.24 (a)

Solution: After connecting *A* and *O* to the terminals of a battery we find, there is symmetry about *OA*. Therefore the current distribution will the same as shown in the figure.

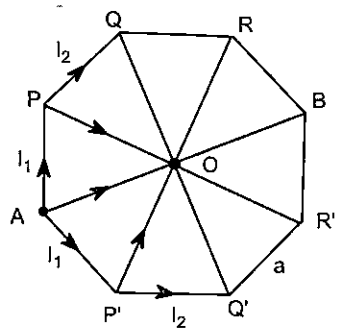


Fig. 2E.24 (b)

It is obvious that potentials

$$V_P = V'_P$$
$$V_R = V'_R$$
$$V_Q = V'_Q$$

∴ On superimposing *P* and *P'* and *Q* and *Q'* etc., we get the simplified arrangement of resistors as shown in figure.

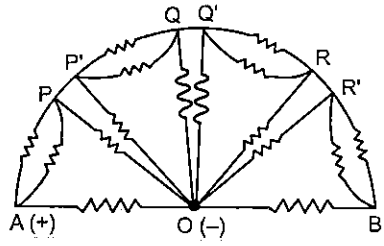


Fig. 2E.24 (c)

Hence $R_{eq} = \frac{69}{149}r_0$

Example 25

Each branch in the following circuit has a resistance R. The equivalent resistance of the circuit between the points A and B is:

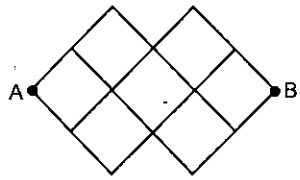


Fig. 2E.25 (a)

(a) R *(b) 2R* *(c) 4R* *(d) 8R*

Solution: Due to symmetry about line *AB* network can be represented as

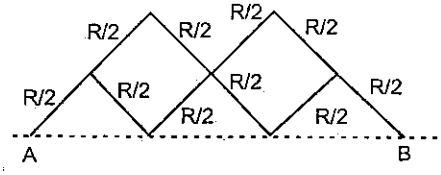


Fig. 2E.25 (b)

Equivalent resistance = 2*R*

Example 26

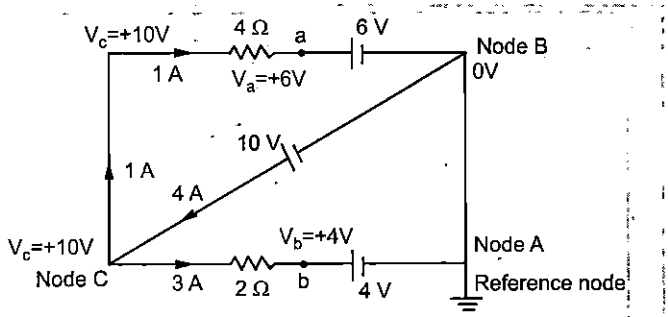


Fig. 2E.26

Solution:

Step-1: Assign node A as reference node

Step-2: Now we can assign potential to points

$$V_a = +6V$$
$$V_b = +4V$$
$$V_c = +10V$$

Step-3: Current through 2Ω resistor is

$$\frac{V_c - V_b}{2} = \frac{10-4}{2} = 3A$$

Current through 4Ω resistor is

$$\frac{V_c - V_a}{4} = \frac{10-6}{4} = 1A$$

Step-4: Apply KCL at node C to get current through 10V battery is 4A.

Example 27

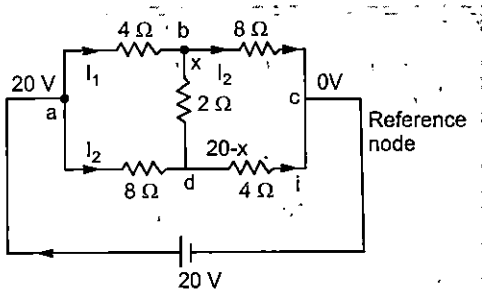


Fig. 2E.27

Solution: **Step-1:** Assign c as reference node.

Node a is at 0V

Step-2: Assign node b at x volt now node d is at $20-x$ volt.

Step-3: Note that I_1 entry and exist similarly current I_2 entry and exit potential drop an section ab is equal to that on section dc.

Step-4: Apply KCL at node b to get

$$\frac{x-20}{4} + \frac{x-0}{8} + \frac{x-(20-x)}{2} = 0$$

Thus $2x - 40 + x + 4x - 4[20-x] = 0$

or $x = \frac{-120}{11}$

Now we can get current in all the branches.

Example 28

In the given circuit we wish to determine the current through resistor given in box.

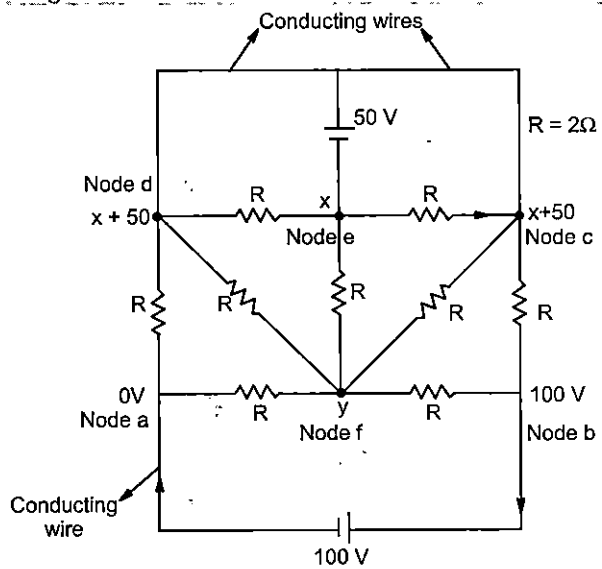


Fig. 2E.28

Solution:

Step-1: Assign voltage at all the nodes as shown in figure. Here x and y are unknown voltage at node e and f.

Step-2: Apply KCL at node f

$$\frac{y-100}{2} + \frac{y-(x+50)}{2} + \frac{y-x}{2} + \frac{y-(x+50)}{2} + \frac{y}{2} = 0 \quad ...(1)$$

Step-3: Note that outgoing current at b is equal to incoming current at node a, thus we get

$$\frac{100-(x+50)}{2} + \left(\frac{100-y}{2}\right) = -\left[\left(\frac{0-y}{2}\right) + \frac{0-(x+50)}{2}\right] \quad ...(2)$$

Note that we require first two equations to get x and y.

Solve x and y to get current in R as $\frac{y-x}{2}$.

$$x = \frac{-25}{2}, \quad y = \frac{125}{2} \text{ and } i = \frac{75}{2}A$$

Example 29

In the given circuit we wish to determine current through branch having indicated resistor 2Ω.

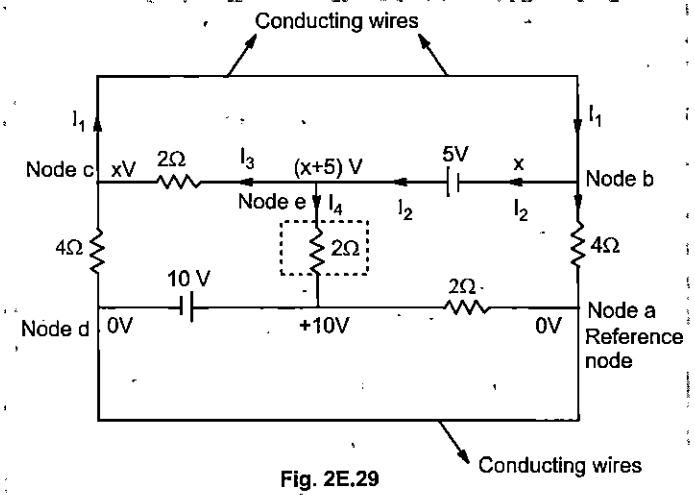


Fig. 2E.29

Solution:

Step-1: Assign potential at each node as shown in figure.

Step-2: Note that outgoing current at node C is equal to incoming current at node b. Secondly incoming current at e.

$$I_2 = I_3 + I_4$$

Thus our equation is at node c and node b

$$\frac{x-(x+5)}{2}+\left(\frac{x-0}{4}\right)=-\left[\underbrace{\frac{(x-0)}{4}}_{I_2}+\underbrace{\frac{x+5-x}{2}}_{I_3}+\underbrace{\frac{x+5-10}{2}}_{I_4}\right]$$

Now solve for x to get potential all the nodes.

On solving , we get

$$x = -2.5\,\text{V},$$

Current through 2Ω resistor is $\frac{7.5}{2}$A.

Equivalent Resistance by Nodal Analysis

Example 30

In the given figure we wish to determine equivalent resistance between point A and B.

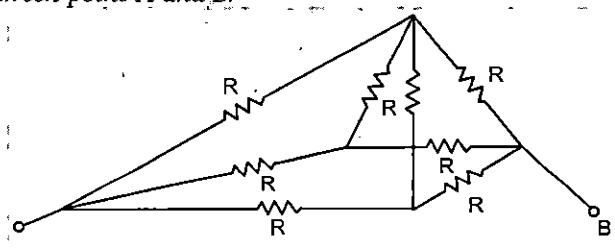


Fig. 2E.30 (a)

Solution:

Step-1: Attach a 100 V battery and assign potential at each node.

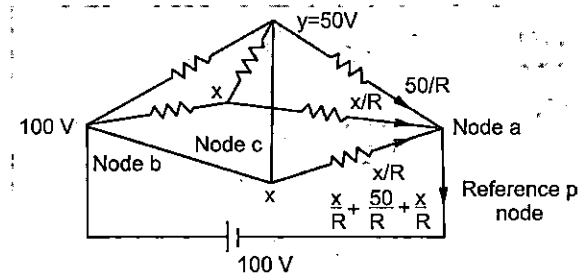


Fig. 2E.30 (b)

Step-2: By symmetry $100 - y = y - 0$ or $y = 50\text{V}$

Step-3: Now apply KCL at node c

$$\frac{x-100}{R}+\frac{x-50}{R}+\frac{x-0}{R}=0$$

or $$x = 50\text{V}$$

Step-4: If R_{eq} is equivalent resistance of this circuit. We have

$$\frac{100}{R_{eq}}=\frac{x}{R}+\frac{50}{R}+\frac{50}{12}$$

or $$R_{eq}=\frac{2}{3}R$$

Example 31

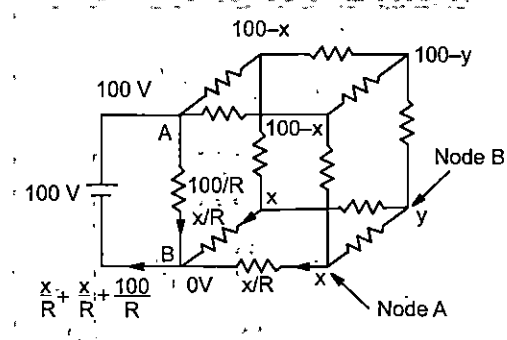


Fig. 2E.31

Solution: Here we wish to determine equivalent resistance between points A and B.

Step-1: By using concepts of symmetry assign potential at all nodes in terms of x and y.

Step-2: Apply KCL at node A and B

Node A: $$\frac{x-(100-x)}{R}+\frac{x}{R}+\frac{x-y}{R}=0$$

Node B: $$\frac{y-x}{R}+\frac{y-x}{R}+\frac{y-(100-y)}{R}=0$$

Step-3: Solve for x and y to get,

$$x=\frac{250}{7} \quad \text{and} \quad y=\frac{300}{7}$$

Step-4: Now apply KCL,

$$\frac{100}{R_{eq}}=\frac{x}{R}+\frac{x}{R}+\frac{100}{R}$$ to get,

$$R_{eq}=\frac{7R}{12}$$

Example 32

Determine equivalent resistance between A and B.

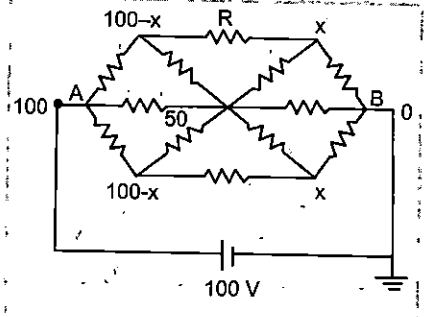


Fig. 2E.32

Solution: Equivalent resistance can be obtained by.

$$R_{AB} = \frac{100}{\frac{2x}{R} + \frac{50}{R}}$$

$$\frac{x}{R} + \frac{x-50}{R} + \frac{x(100-x)}{R} = 0$$

$$x = \frac{75}{R}$$

Example 33

Determine current through indicated 5Ω resistor.

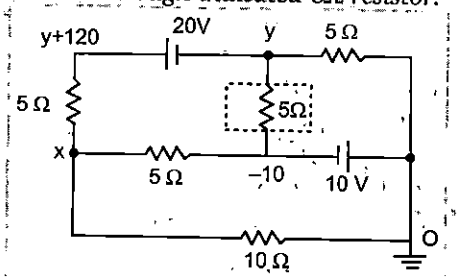


Fig. 2E.33

Solution:

$$\frac{x}{10} + \frac{x+10}{5} + \frac{x-y-20}{5} = 0$$

$$\frac{y}{5} + \frac{y+10}{5} + \frac{y+20-x}{5} = 0$$

On solving for x and y we get,

$$x = \frac{60}{13}, \quad y = \frac{-10}{13}$$

$$I_{5\Omega} = \frac{y-(-10)}{5}$$

$$I_{5\Omega} = \frac{\frac{-10}{13}+10}{5} = \frac{120}{65} = \frac{24}{13}\text{A}$$

Example 34

Each resistance is R determine equivalent resistance.

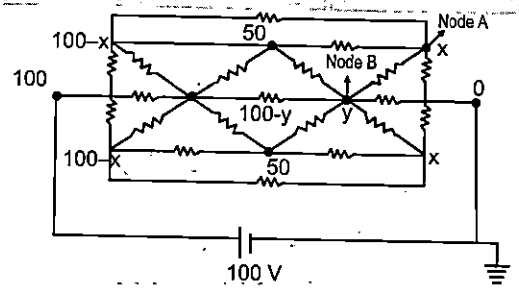


Fig. 2E.34

$$R_{AB} = \frac{100}{\frac{2x}{R} + \frac{y}{R}}$$

For node A

$$\frac{x-(100-x)}{R} + \frac{x-50}{R} + \frac{x-y}{R} + \frac{x-0}{R} = 0$$

For node B

$$\frac{y-x}{R} + \frac{y-50}{R} + \frac{y-(100-y)}{R} + \frac{y-50}{R} + \frac{y-x}{R} + \frac{y}{R} = 0$$

On solving for x and y, we get

$$y = 50 \quad \text{and} \quad x = \frac{625}{13}$$

which on substituting in R_{AB} gives desired result $\frac{13R}{19}$.

Example 35

ABCD is square (see Fig. 2E.35) where each side is a uniform wire of resistance 1Ω. A point E lies on CD such that if a uniform wire of resistance 1Ω is connected across AE and constant potential difference is applied across A and C then B and E are equipotential.

(a) $\frac{CE}{ED} = 1$ (b) $\frac{CE}{ED} = 2$

(c) $\frac{CE}{ED} = \frac{1}{\sqrt{2}}$ (d) $\frac{CE}{ED} = \sqrt{2}$

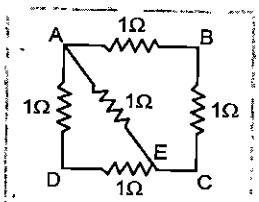


Fig. 2E.35 (a)

Solution: (d) Equivalent resistance between A and E:

$$y = \frac{x+1}{x+2}$$

For B and E to be equipotential

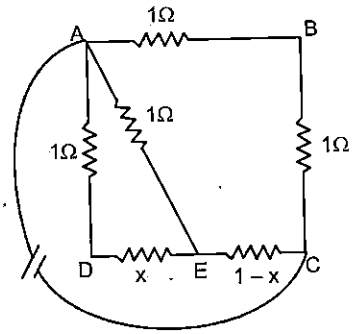


Fig. 2E.35 (b)

$$\frac{R_{AE}}{R_{AB}} = \frac{R_{EC}}{R_{BC}}$$

$$\Rightarrow \quad \frac{x+1}{(x+2)\times 1} = \frac{1-x}{1}$$

Solve to get: $x = \sqrt{2} - 1\Omega$

Now $\frac{CE}{ED} = \frac{1-x}{x} = \sqrt{2}\Omega$

Example 36

In the Fig. 2E.36 (a) the resistances are connected as shown. Determine the equivalent resistance between points A and D.

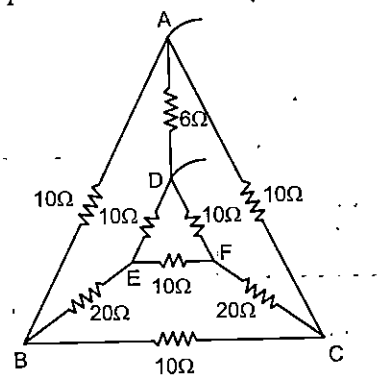


Fig. 2E.36 (a)

Solution:

Points B and C, and E and F are at the same potential, so the circuit can be redrawn as shown in figure. Thus, the equivalent resistance is 1Ω. There exists parallel axis of symmetry. The points across the parallel axis of symmetry can be treated as equipotential points.

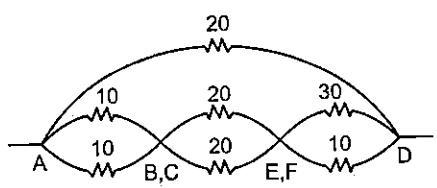


Fig. 2E.36 (b)

Example 37

The circuit diagram shown in the Fig. 2E.37-(a) consists of a large number of elements (each element has two resistors R_1 and R_2). The resistance of the resistors in each subsequent element differs by a factor of $k = \frac{1}{2}$ from the resistances of the resistors in the previous elements. Find the the equivalent resistance between A and B shown in Fig. 2E.37.

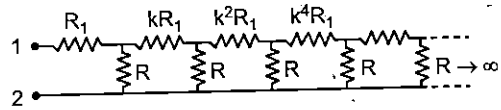
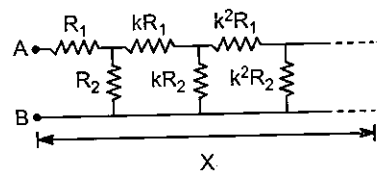


Fig. 2E.37 (a)

Solution: When each element of circuit is multiplied by a factor k then equivalent resistance also becomes k times.

Let the equivalent resistance between A and B be x.



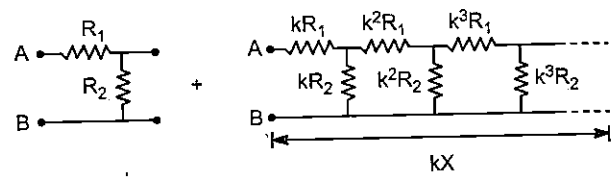


Fig. 2E.37 (b)

So the equivalent circuit becomes

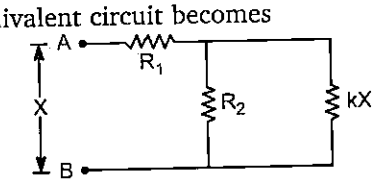


Fig. 2E.37 (c)

For $k = \frac{1}{2} \Rightarrow x = \frac{(R_1 - R_2) + \sqrt{R_1^2 + R_2^2 + 6R_1R_2}}{2}$

Example 38

Find the resistance R_{AB} of the frame made of a thin wire. Assume that the number of successively embedded equilateral triangles (with sides decreasing to half) tends to infinity [see Fig. 2E.38 (a)]

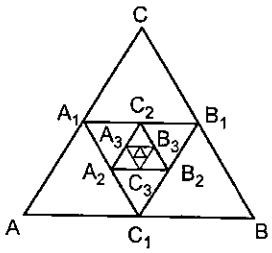


Fig. 2E.38 (a)

Side AB is equal to a and the resistance per unit length of wire is r.

Let $R_{AB} = x$ is equivalent resistance of system between A and B. As the resistance of a conductor is directly proportional to length, the equivalent resistance between A_1 and B_1 will be $\frac{x}{2}$.

Therefore, the equivalent circuit becomes as given below in Fig. 2E.38 (a)

Solution: Let $AB = 2r$,

then $A_1C = CB_1 = AA_1 = BB_1 = r$.

In the circuit $2r$ and $\frac{x}{2}$ are in parallel between A_1 and B_1, then their effective resistance is

$$R_1 = \frac{2r\,\frac{x}{2}}{2r + \frac{x}{2}} = \frac{2rx}{4r + x}$$

Now R_1 is in series with AA_1 and BB_1, therefore, their effective resistance is

$$R_2 = R_1 + 2r = \frac{2rx}{4r + x} + 2r$$

R_2 is in parallel with $2r$ (of AB), so the net effective resistance across AB is

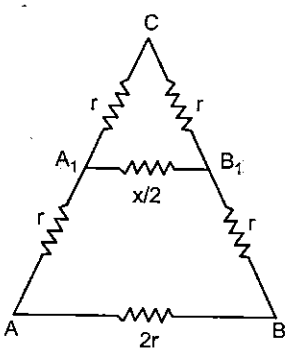


Fig. 2E.38 (b)

$$x = \frac{r_2 \times 2r}{R_2 + 2r} = \frac{\left(\frac{2rx}{4r + x} + 2r\right)}{\left(\frac{2rx}{4r + x} + 2r\right) + 2r}$$

$$\Rightarrow \quad 3x^2 + 4rx - 8r^2 = 0$$

or $$x = \frac{-4r + \sqrt{6r^2 + 4 \times 3 \times 8r^2}}{2 \times 3}$$

As x cannot be negative

$$\therefore \quad x = \frac{-4r \pm \sqrt{16r^2 + 96r^2}}{6} = \frac{(2\sqrt{7} - 2)}{3} r$$

But $$r = \frac{a}{2}\rho$$

$$\therefore \quad x = \frac{2(\sqrt{7} - 1)}{3} \times \frac{a}{2}\rho = 0.55a\rho$$

Example 39

Two circuits [as shown in Fig.2E.39 (a)] are called circuit A and circuit B. The equivalent resistance of circuit A is x and that of circuit B is y between 1 and 2.

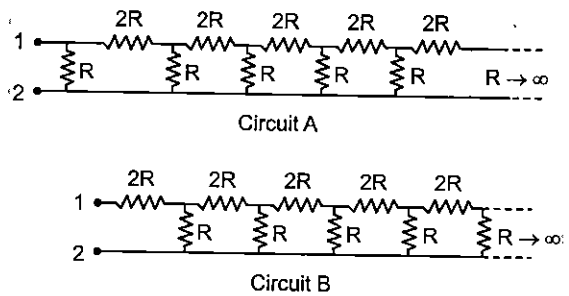


Fig. 2E.39 (a)

(a) $y > x$ (b) $y = (\sqrt{3} + 1)R$

(c) $xy = 2R^2$ (d) $x - y = 2R$

Solution: (a, b, c, d)

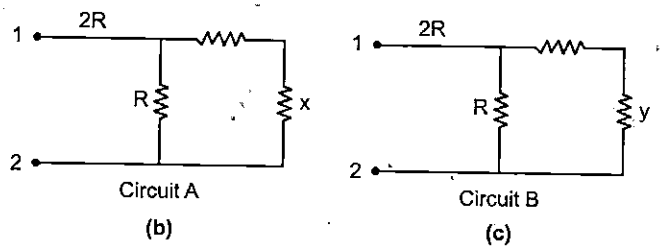


Fig. 2E.39

$$x = \frac{R(2R + x)}{3R + x}$$

$$\Rightarrow \quad 3Rx + x^2 = 2R^2 + Rx$$

$$\Rightarrow \quad x^2 + 2Rx - 2R^2 = 0$$

$$\Rightarrow \quad x = \frac{-2R \pm \sqrt{4R^2 + 8R^2}}{2}$$

$$\Rightarrow \quad x = \frac{-2R + 2\sqrt{3}R}{2}$$

$$\Rightarrow \quad x = (\sqrt{3} - 1)R$$

Example 40

For a cell, a graph is plotted between the potential difference V across the terminals of the cell and the current I drawn from the cell (see Fig.2E.40) Find the e.m.f. and internal resistance of the cell.

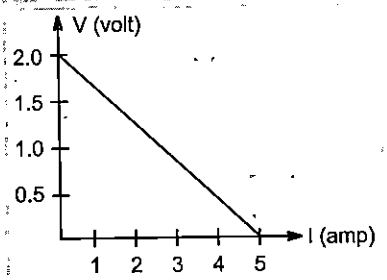


Fig. 2E.40

Solution: $r = \frac{V}{I} = \frac{2}{5} = 0.4\ \Omega$

when $I = 0$, the potential reading is 2V. Hence e.m.f.= 2V

Example 41

Find the current in each part of the circuit. Apply loop law in Fig.2E.41 (a).

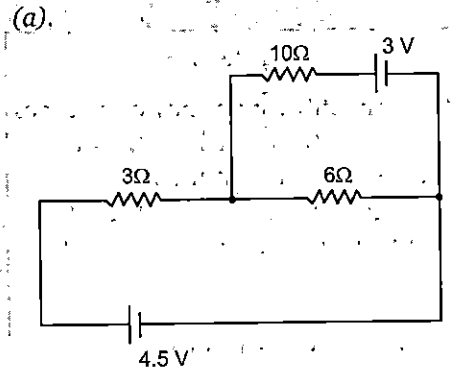


Fig. 2E.41 (a)

Solution:

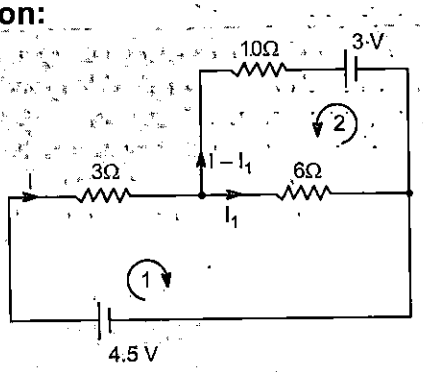


Fig. 2E.41 (b)

$$(1) - 3I - 6I_1 + 4.5 = 0$$

or $$I + 2I_1 = 1.5 \quad \ldots(i)$$

$$(2)10(I - I_1) + 3 - 6I_1 = 0$$

or $$10I - 10I_1 = -3 \quad \ldots(ii)$$

Solving equations (i) and (ii), we get

$$I = \frac{1}{2}\text{A}$$

and $$I - I_1 = \frac{1}{2} - \frac{1}{2} = 0$$

Example 42

What is the potential difference between the points M and N for the circuits shown in Fig. 2E.42 (a) and (b) for case I and case II.

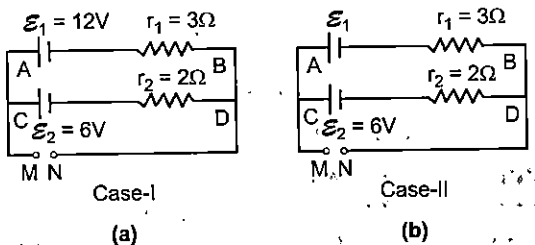


Fig. 2E.42

Solution: Case I: Current in the circuit

$$= \frac{12-6}{3+2} = \frac{6}{5} = 1.2\text{A}$$

$$\therefore \quad V_A - V_B = 12 - 3 \times 1.2$$

$$= 12 - 3.6 = 8.4\text{V}$$

$$V_C - V_D = 6 + 2 \times 1.2 = 6 + 2.4 = 8.4\text{V}$$

Hence $$V_M - V_N = 8.4\text{V}$$

Case II: $$\frac{12+6}{3+2} = \frac{18}{5} = 3.6\text{A}$$

$$V_A - V_B = 12 - 3.6 \times 3$$

$$= 12 - 10.8 = 1.2\text{V}$$

$$V_B - V_C = 6 - 3.6 \times 2 = 6 - 7.2 = -1.2\text{V}$$

$$V_C - V_D = +1.2\text{V}$$

$$\Rightarrow \quad V_M - V_N = 1.2\text{V}$$

Example 43

In the circuit diagram shown in Fig. 2E.43 (a) find the current through the 1Ω resistor.

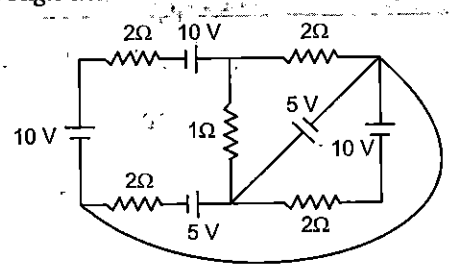


Fig. 2E.43 (a)

Solution: $$\frac{(V-10)-10}{2} + \frac{V-0}{2} + \frac{V-5}{1} = 0$$

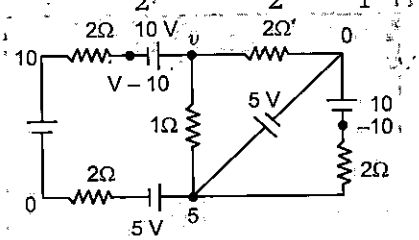


Fig. 2E.43 (b)

$$\frac{V-20}{2}+\frac{V}{2}+V-5=0$$

$$V-20+V+2(V-5)=0$$

$$\Rightarrow \quad 4V-20-10=0$$

$$V=\frac{30}{4}=\frac{15}{2}$$

$$V-5=\frac{15}{2}-5=\frac{15-10}{2}=\frac{5}{2}$$

$$i=\frac{5/2}{1}=\frac{5}{2}\text{A}$$

Example 44

In the given circuit of Fig. 2E.44 (a) all batteries have e.m.f. 10 V and internal resistance negligible. All resistors are in ohms. Calculate the current in the right most 2 Ω resistor.

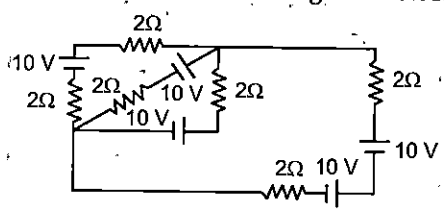


Fig. 2E.44 (a)

Solution: Let the potential of point P be O. Potential at other points are shown in Fig. 2E.44 (b) Apply Kirchhoff's current law at X_x.

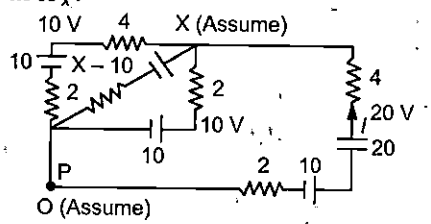


Fig. 2E.44 (b)

$$\frac{x-10}{4}+\frac{x-10}{2}+\frac{x-20}{4}+\frac{(x-10)-0}{2}=0$$

$$x-10+2x-20+x-20+2x-20=0$$

$$\Rightarrow \quad 6x=70$$

$$\Rightarrow \quad x=\frac{35}{3}\text{V},\ I=\frac{20-\frac{35}{3}}{4}=\frac{25}{12}\text{A}$$

Example 45

In the circuit shown in Fig. 2E.45 (a), $R_1=2\Omega$, $R_2=3\Omega$, $R_3=3\Omega$, $R_4=2\Omega$, $R_5=2\Omega$, $\mathcal{E}_1=16V$ and $\mathcal{E}_2=11V$. Calculate current through resistance R_5.

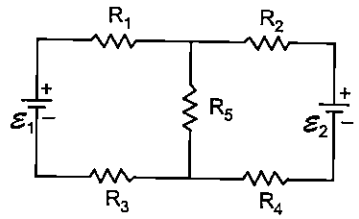


Fig. 2E.45 (a)

Solution: Let currents drawn from batteries $\mathcal{E}_1$ and $\mathcal{E}_2$ be I_1 and I_2 respectively. Then current through various resistors will be as shown in Fig. 2E.45(b)

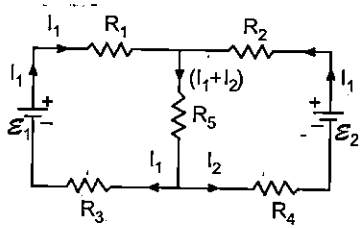


Fig. 2E.45 (b)

Applying Kirchhoff's voltage law on left mesh,

$$I_1R_1+(I_1+I_2)R_5+I_1R_3-\mathcal{E}_1=0 \qquad ...(1)$$

Now applying *KVL* on right mesh,

$$-I_2R_2+\mathcal{E}_2-I_2R_4-(I_1+I_2)R_5=0 \qquad ...(2)$$

From equations (1) and (2),

$$I_1=2\text{A}$$

and $$I_2=1\text{A}$$

$\therefore$ Current through resistance,

$$R_5=I_1+I_2=3\text{A}$$

Example 46

A resistance coil, wired to an external battery, is placed inside a thermally insulated cylinder fitted with a frictionless piston and containing an ideal gas. A current $I=2.40\,\text{m A}$ flows through the coil, which has a resistance $R=490\Omega$. At what speed v must the piston of mass $m=12\,\text{kg}$ move upward in order that the temperature of the gas remains unchanged? $[g=9.8\,\text{m/s}^2]$

Solution: As the gas is ideal and its temperature remains constant, the heat supplied by the resistance must be equal to the change in potential energy of the piston *i. e.*,

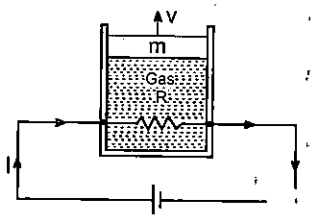


Fig. 2E.46

$$E=P\times t=I^2Rt=mgh$$

or, $$v = \frac{h}{t} = \frac{I^2R}{mg}$$

So, $$v = \frac{(0.24)^2 \times 490}{12 \times 9.8} = 0.24 \text{ m/s}$$

Example 47

12 cell each having the same emf are connected in series and are kept in a closed box. Some of the cells are wrongly connected. This battery is connected in series with an ammeter and two cells identical with series with an ammeter and two cells identical with the others. The current is 3A when the cells and the battery aid each other and 2 A when the cells and battery oppose each other. How many cells in the battery are wrongly connected?

Solution: Let n be the cells in the battery that are wrongly connected, then

$$\mathcal{E}_B = (12-n)\mathcal{E} - n\mathcal{E} = (12-2n)\mathcal{E} \text{ and } r_B = 12r$$

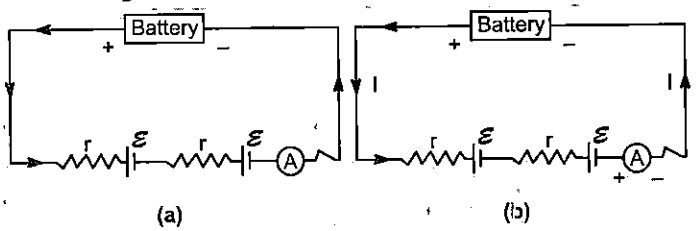


Fig. 2E.47

So according to the given problem as shown in Fig. 2E.47 (a), (b)

$$\frac{(12-2n)\mathcal{E} + 2\mathcal{E}}{12r + 2r} = 3 \quad \text{...(i)}$$

and $$\frac{(12-2n)\mathcal{E} - 2\mathcal{E}}{12r + 2r} = 2 \quad \text{...(ii)}$$

Dividing equation (i) by equation (ii),

$$\frac{14-2n}{10-2n} = \frac{3}{2}$$

i.e., $$n = 1$$

This means that in the battery only one cell is wrongly connected.

Example 48

Twelve equal resistors each R Ω are connected to form the edges of a cube. Find the equivalent resistance of the network.
(a) when current enters at A and leaves at A'.
(b) when current enters at A and leaves at B',
(c) when current enters at A and leaves at B.

Solution : (a) The potential difference between any two points is same no matter what path we take to arrive at the second beginning from first.

The circuit is symmetrical as shown in Fig. 2E.48, the entry point and exit point are identical. Therefore at A a circuit $I/3$ flows in each branch, similarly at exit point A' each branch have current $I/3$. At point B the current divides equally to $I/6$ and $I/6$ in branches BC and BD'.

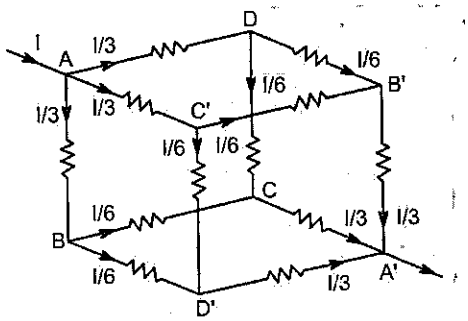


Fig. 2E.48 (a)

Similarly current divides equally to $I/6$ and $I/6$ in branches $C'D'$ respectively.

Let equivalent resistance between A and A' be $R_{eq.}$ and the potential drop across it $IR_{eq.}$

From given circuit the potential difference between A and A' can be determined as follows:

$$V_A - \frac{I}{3}R - \frac{1}{6}R - \frac{I}{3}R = V_B$$

$$V_A - V_B = \frac{I}{3}R + \frac{I}{3}R + \frac{I}{3}R$$

From equivalent circuit $V_A - V_B = IR_{eq.}$

Thus we have, $$IR_{eq.} = \frac{5}{6}IR \Rightarrow R_{eq.} = \frac{5}{6}R$$

Method 2 : In between A and A', symmetry of the circuit indicates that B, C', D are at equal potential and similarly B', C, D'. So the cube may be redrawn as

$$R_{eq.} = \frac{R}{3} + \frac{R}{6} + \frac{R}{3}$$

where '+' stands for the series

$$= \frac{R}{3} + \frac{R}{6} + \frac{R}{3} = \frac{5}{6}R$$

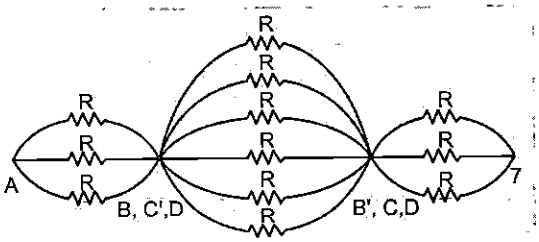


Fig. 2E.48 (b)

(b) Once again the circuit is symmetrical. Figure shows the current distribution at junctions A and B. The incoming current at D must be equal to outgoing current, similar situation exists at C'. Therefore the current in branches CD and $C'D'$ are zero or we can say that the points C and D are equipotential, similarly C' and D' have same potential.

The equivalent circuit is reduced to Fig. 2E.48 (a).

Resistors irrelevout from point of view of current.

Method 2 : In between points 1 and 3, 2 and 4 are at the same potential. Current in the wires 2 – 6 and 4 – 8 is zero. Potential difference between 2 – 6 and 4 – 8 is zero.

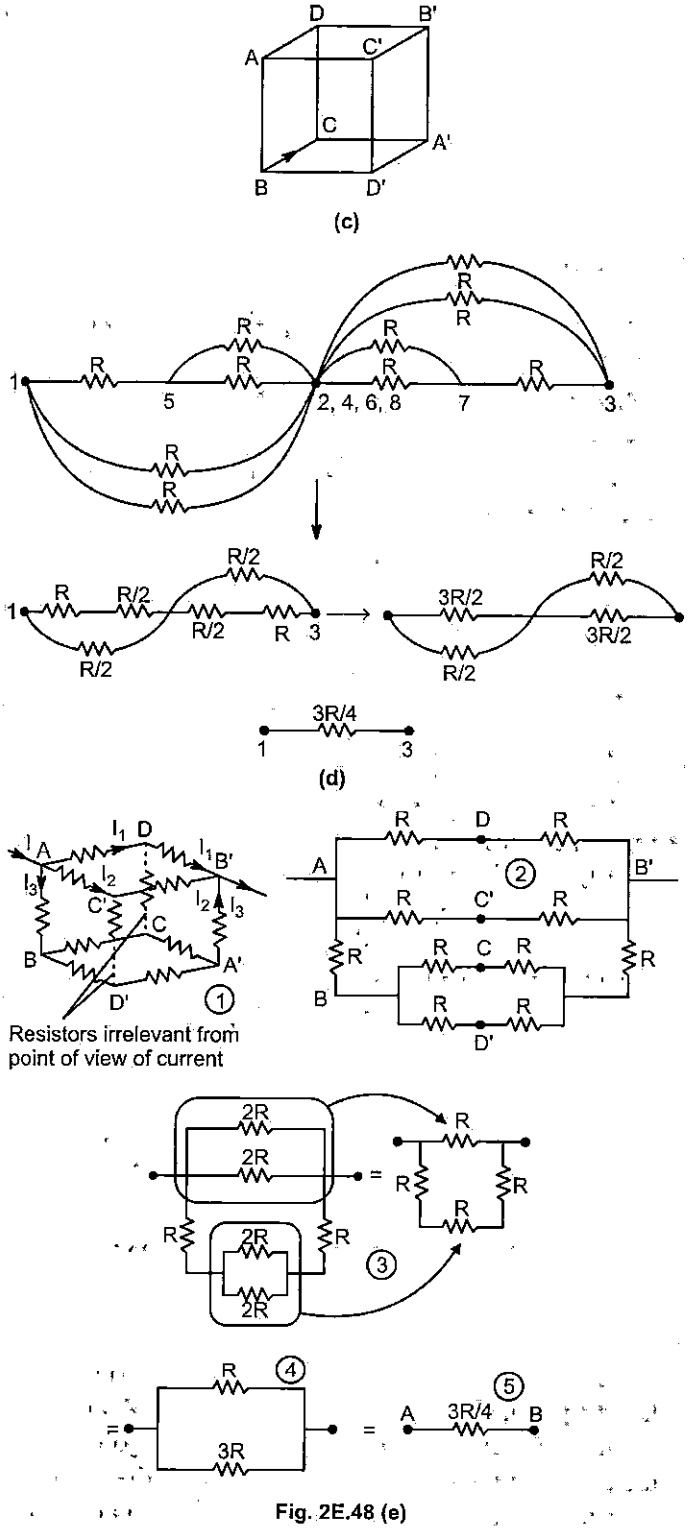


Fig. 2E.48 (e)

(c) Let a current I enter the point A and leave the point B. Figure shows distribution of current in the circuit. The wires AC' and AD have equal resistance and located symmetrically, so they have same current; similar situation exists with DC and $C'D'$.

From KCL we have

$$I = I_1 + 2I_2 \quad \ldots(1)$$

For the loop $AC'D'BA$, applying KVL, we have

$$-I_2R - I_3R - I_2R + I_1R = 0$$

or $$I_1 = 2I_2 + I_3 \quad \ldots(2)$$

For the loop $C'D'A'B'$, applying KVL, we have

$$-I_3R + (I_2 - I_3)R + 2(I_2 - I_3)R + (I_2 - I_3)R = 0$$

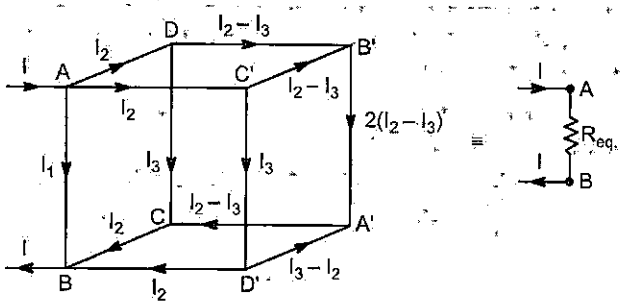


Fig. 2E.48 (f)

or $$4I_2 - 5I_3 = 0 \quad \ldots(3)$$

On solving eqns. (1), (2) and (3), we get

$$I_1 = \frac{7}{12}I$$

$$I_2 = \frac{5}{24}I$$

$$I_3 = \frac{1}{6}I$$

The potential difference across $AB = I_1R = \frac{7}{12}IR$

The potential difference across equivalent resistance $= IR_{eq.}$

Thus, $$IR_{eq.} = \frac{7}{12}IR$$

or $$R_{eq.} = \frac{7}{12}R$$

Method 2: From considerations of symmetry alone, points 3 and 6 must be at the same potential, and so must points 4 and 5.

If two points in a circuit have the same potential, the currents in the circuit do not change if they are connected at these points by a wire.There is no current in the wire because there is no potential difference between its ends. Points 3 and 6 may therefore be connected by a wire, and similarly points 4 and 5 may be connected.

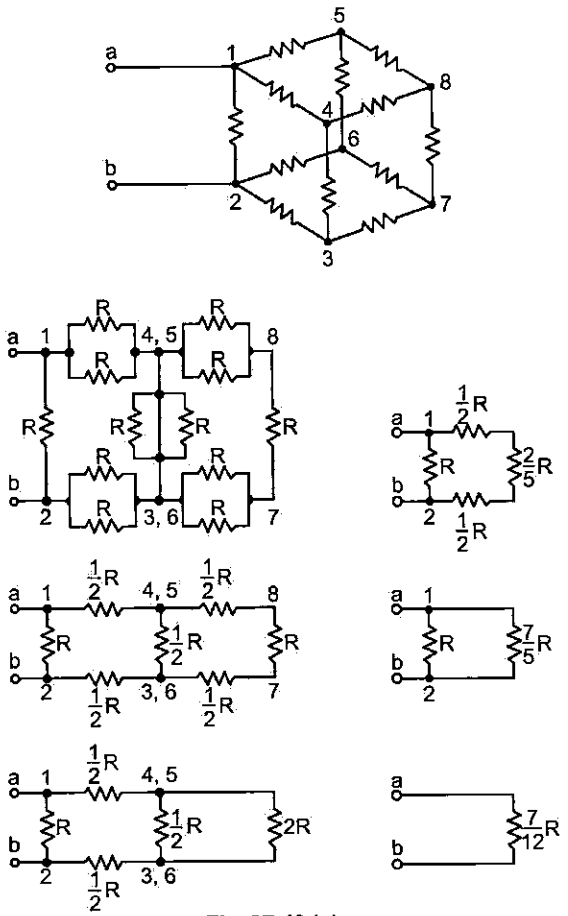


Fig. 2E.48 (g)

Thus the equivalent resistance between a and b is $\frac{7}{12}R$.

Example 49

In the circuit shown in Fig. 2E.49 (a), abcd is a square. All the wires forming the square and its diagonals are homogeneous and have same cross-section. Find the ratio of power dissipated in resistors ab and cd.

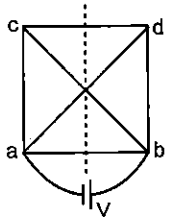
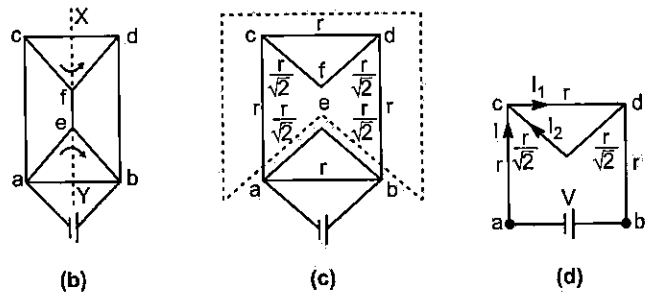


Fig. 2E.49 (a)

Solution: The circuit has asymmetry about line XY, *i.e.*, the current in left and right are not mirror images. Just imagine the central junction of wires in the form of two junctions connected by wire ef as shown in Fig. 2E.49 (b). Then it follows from symmetry consideration that there is no current in the wire ef, thus we can remove it form the circuit. The resistors of the wires will be proportional to their lengths.



(b) (c) (d)

Fig. 2E.49

$$R_{ab} = R_{ac} = R_{cd} = R_{bd} = r$$

$$R_{ae} = R_{be} = R_{cf} = R_{df} = r/\sqrt{2}$$

The power dissipated in resistor ab is

$$P_{ab} = \frac{V^2}{r}$$

By applying Ohm's law to the upper part, we can determine current through cd,

$$I_{cd} = \frac{V}{(\sqrt{2}+3)r}$$

The power dissipated in the condutor cd,

$$P_2 = I_{cd}^2 r = \frac{V^2}{(\sqrt{2}+3)^2 r}$$

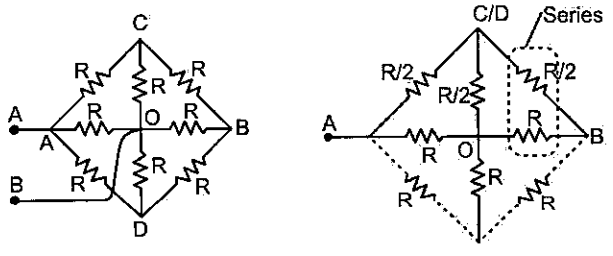
Thus the required ratio is

$$\frac{P_1}{P_2} = (\sqrt{2}+3)^2 = 11 + 6\sqrt{2}$$

Example 50

In the circuit shown in the Fig. 2E.50 (a) all the resistances are identical and equal to R. A constant voltage source of emf V volt is connected across AB. Determine the current supplied by the source.

Solution: From symmetry considerations, the potentials at points C and D are same. Thus the equivalent circuit can be redrawn as in Fig. 2E.50(b).



(a) (b)

Fig. 2E.50

Resistors in branches AC, AD; BC, BD are in parallel arrangement, their equivalent is $R/2$. Resistors in branches $OB, BC/D$ are in series, this combination is in parallel with resistor in branch OC/D.

Hence, $R_{eq.} = \frac{R/2(R/2+R)}{R/2+R/2+R} = \frac{3}{8}R$

Next we combine branches OC/D and AC/D that are in series [see Fig. 2E.50(c)].

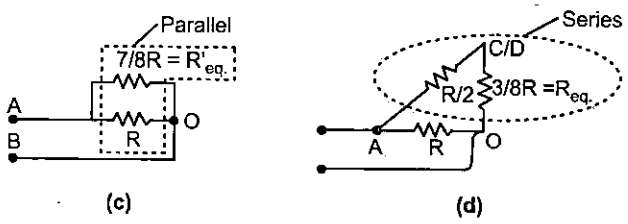


Fig. 2E.50

Hence $R'_{eq.} = \frac{R}{2} + \frac{3}{8}R = \frac{7}{8}R$

Finally see Fig. 2E.50(d).

$$R''_{eq.} = \frac{R(7/8)R}{R+(7/8)R} = \frac{7}{15}R$$

Thus, the current I from source

$$= \frac{V}{(7/15)R} = \frac{15}{7}\frac{V}{R}$$

Example 51

In the circuit shown in Fig. 2E.51(a) find the current flowing through the 100Ω resistor connecting points V and S.

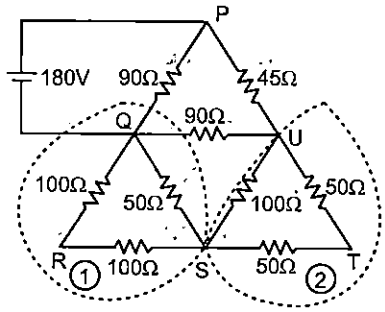


Fig. 2E.51 (a)

Solution: Fig. 2E.51(b) shows simplified circuit. The battery is directly attached to resistor 90Ω, hence current in it is 2 A, see Fig. 2E.51(c). The total resistance of second branch is also 90Ω, hence current divides equally. Now current through 45Ω resistor is 2 A and it is a combination of two equal 90Ω resistors. Once again current divides equally. 90Ω resistor is a series combination of 40Ω and 50Ω, hence current through them is equal, 1A. As 50Ω resistor is a parallel combination of two equal 100 Ω resistors, they must have the same current, *i.e.,* 0.5 A.

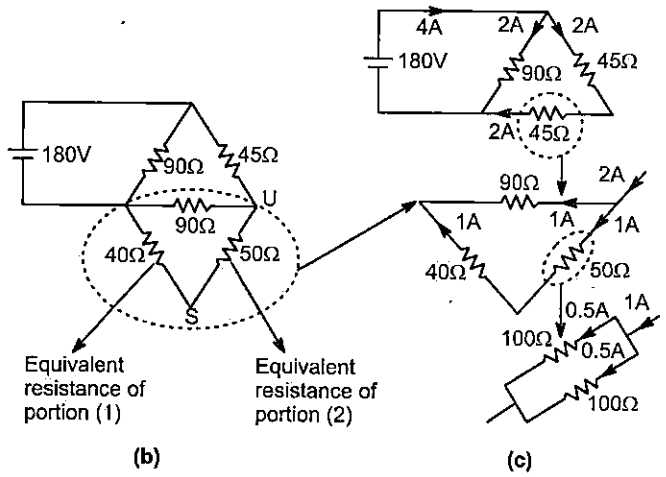


Fig. 2E.51

Example 52

Under what circumstances can the terminal potential difference of a battery exceeds its emf?

Solution: Two batteries AB and CD emf's $\mathcal{E}$ and $\mathcal{E}'$ ($\mathcal{E}' > \mathcal{E}$) and internal resistances r and r' respectively are connected in series as shown in the Fig. 2E.52.

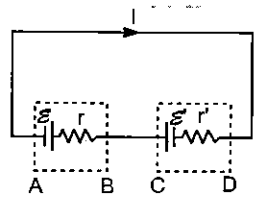


Fig. 2E.52

If I is the current in the circuit, the total potential drop in the circuit must be equal to net emf :

$$I(r+r') = \mathcal{E}' - \mathcal{E}$$

or $$I = \frac{\mathcal{E}' - \mathcal{E}}{r+r'}$$

The potential difference across terminals of battery AB is

$$V_B - V_A = Ir + \mathcal{E}$$

Obviously $(V_B - V_A) > \mathcal{E}$ by Ir, *i.e.*,

$$(V_B - V_A) - \mathcal{E} = \frac{(\mathcal{E}' - \mathcal{E})r}{r+r'}$$

when a current is flowing in any battery opposite to its emf, then terminal voltage is given by

$$V = \mathcal{E} + Ir$$

Example 53

Two batteries having the same emf $\mathcal{E}$ but different internal resistances r_1 and r_2 are connected in series with an external resistor R. For what value of R does the potential difference between the terminals of the first battery become zero?

Solution: Two batteries are connected in series. The effective emf in the circuit is therefore $2\mathcal{E}$ because both push the charge in the same direction. Hence emf's are added.

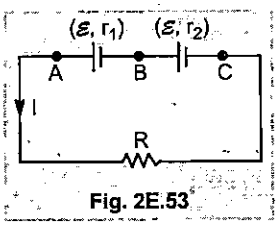


Fig. 2E.53

Net resistance in the circuit is $(r_1 + r_2 + R)$.

Therefore, current in the circuit

$$I = \frac{2\mathcal{E}}{(r_1 + r_2 + R)}$$

The potential difference between the terminals of first battery is $(V_A - V_B)$, terminal potential difference is given by

$$(V_A - V_B) = \mathcal{E} - Ir_1$$

where E is the emf of the battery and r_1 is its internal resistance. Substituting the value of I, we get

$$V_A - V_B = \mathcal{E} - \frac{2\mathcal{E}r_1}{r_1 + r_2 + R}$$

$$= \mathcal{E}\frac{(R + r_2 - r_1)}{(R + r_2 + r_1)}$$

For $(V_A - V_B)$ to be zero, we must have

$$R = (r_1 - r_2)$$

This gives meaningful result only if $r_1 > r_2$. Otherwise, if $r_2 > r_1$, then $R = r_2 - r_1$ will produce terminal voltage across second cell to be zero ($V_{BC} = 0$).

Example 54

An electrical circuit is shown in Fig. 2E.54 (a). Calculate the potential difference across the resistor of 400 ohm, as will be measured by the voltmeter V of resistance 400 ohm, either by applying Kirchhoff's rules or otherwise.

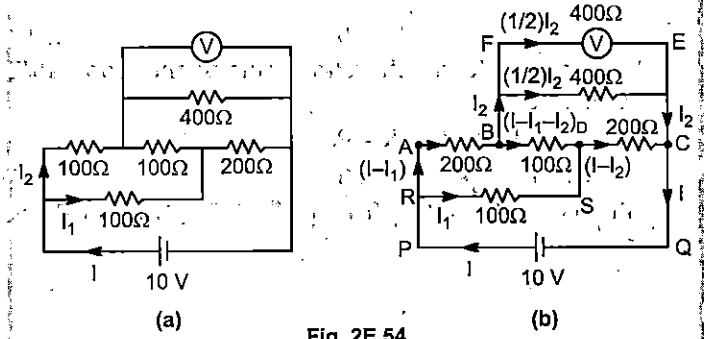


Fig. 2E.54

Solution: Method-1: The distribution of current in the given circuit is shown in Fig. 2E.54 (a). Applying KVL to loops *FECBF*, *ABDCQPRA* and *ABDSRA*, we have respectively:

$$-(1/2)I_2 \times 400 + 200(I - I_2) + 100(I - I_1 - I_2) = 0$$

i.e., $$3I - I_1 - 5I_2 = 0 \quad \ldots(1)$$

$$-(I - I_1)100 - (I - I_1 - I_2)100 - (I - I_2)200 + 10 = 0$$

i.e., $$4I - 2I_1 - 3I_2 = (1/10) \quad \ldots(2)$$

and $$-(I - I_1)100 - (I - I_1 - I_2)100 + 100I_1 = 0$$

i.e., $$2I - 3I_1 - I_2 = 0 \quad \ldots(3)$$

Substituting the value of I_1 from eqn. (1) in (2) and (3), we get

$$-2I + 7I_2 = (1/10) \quad \ldots(4)$$

and $$I = 2I_2 \quad \ldots(5)$$

Again substituting the value of I from eqn. (5) in (4), we get $I_2 = (1/30)$ A, so that the *PD* across 400 ohm resistance as read by the voltmeter.

$$V = \frac{1}{2}I_2 \times 400$$

$$= \frac{1}{2} \times \frac{1}{30} \times 400$$

$$= \frac{20}{3} \text{ volt}$$

Method 2. After combining the resistance of voltmeter with 400 Ω, the equivalent circuit is shown in Fig. 2E.54 (c), which is a balanced Wheatstone bridge, because

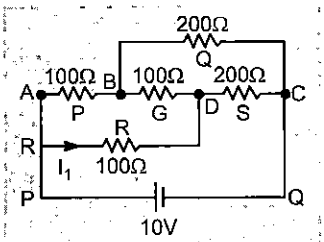


Fig. 2E.54 (c)

$$\frac{P}{Q}\left(=\frac{100}{200}\right) = \frac{R}{S}\left(=\frac{100}{200}\right)$$

There is no current through the resistance *G*. Now 10 V is applied along series combination of 100 and 200 V, so the voltmeter reading, *i.e.*, potential difference across *Q* will be

$$V = \frac{200}{(100 + 200)} \times (10) = \frac{20}{3} \text{V}$$

Example 55

A part of a circuit in steady state along with the current flowing in the branches, with value of each resistance is shown in Fig. 2E.55. Calculate the energy stored in the capacitor C.

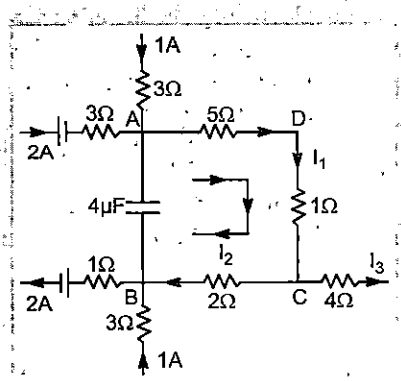


Fig. 2E.55

Solution: From KCL, at junctions A and B respectively, we have

$$2+1-I_1=0 \quad i.e., \quad I_1=3\,A$$

and $$I_2+1-2-0=0 \quad i.e., \quad I_2=1\,A$$

Let potential at points A and B be V_A and V_B respectively. Now we begin at A and after taking into account the potential drops and gains reach point B. We have

$$V_A-3\times5-3\times1-1\times2=V_B$$

$$i.e., \quad V_A-V_B=20\,V$$

So energy stored in the capacitor,

$$U=\frac{1}{2}CV^2=\frac{1}{2}(4\times10^{-6})\times(20)^2=8\times10^{-4}\ J$$

Example 56

In the given circuit,

$$\mathcal{E}_1=3\mathcal{E}_2=2\mathcal{E}_3=6V$$
$$R_1=2R_4=6\Omega$$
$$R_3=2R_2=4\Omega$$

and $$C=6\mu F$$

Find the current in R_3 and energy stored in the capacitor.

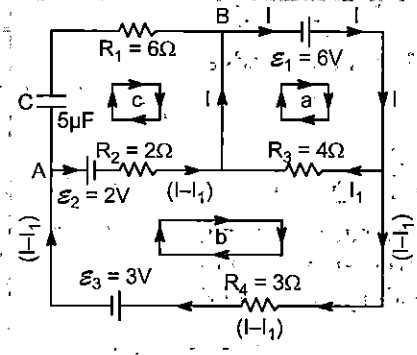


Fig. 2E.56

Solution: Let current I originates from the battery of emf $\mathcal{E}_1$, distribution of current in the circuit is shown in Fig. 2E.56, in accordance with KCL.

Applying Kirchhoff's voltage law to loop a in clockwise sense, we get

$$-4I_1+6=0,$$

$$i.e., \quad I_1=\frac{3}{2}=1.5\,A$$

Now applying KVL, to loop b, we get

$$-(I-I_1)\times3-3-2-(I-I_1)\times2+I_1\times4=0$$

or, $$-5I+9I_1=5,$$

$$i.e., \quad I=\frac{9}{5}I_1-1$$

or, $$I=\frac{9}{5}\times\left(\frac{3}{2}\right)-1=1.7\,A \qquad \left[\text{as } I_1=\frac{3}{2}\right]$$

and applying Kirchhoff's voltage law to mesh c, treating the capacitor as a seat of emf V in open circuit,

$$V_A-2-(1.7-1.5)\times2=V_B$$

$$i.e., \quad V_A-V_B=2.9V$$

So energy stored in the capacitor,

$$U=\frac{1}{2}CV^2=\frac{1}{2}\times(5\times10^{-6})\times(2.4)^2$$

$$=14.4\times10^{-6}\,J$$

Example 57

A piece of uniform wire is made up into two squares with a common side of length 4 inch. A current enters the rectangular system at one of the corners and leaves at the diagonally opposite corner. Show that the current in the common side is one-fifth of the entering current. What length of wire connected between input and output terminals would have an equivalent resistive effect?

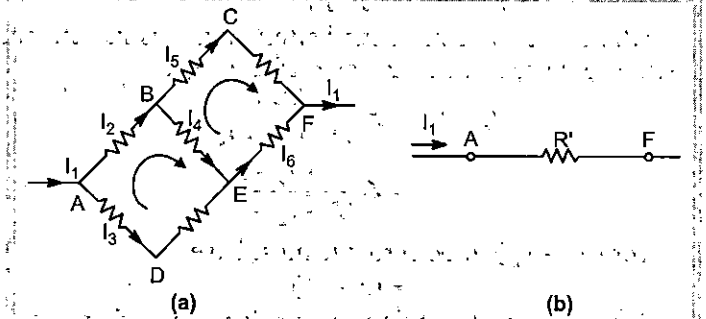


Fig. 2E.57

Solution: Let each side of the double square have resistance R, and let the lettering of the diagram and the currents flowing be as shown in Fig. 2E.57 (a). Applying the Kirchhoff current rule, $\Sigma I=0$, to the junctions A, B and E in turn (with the convention that current entering a junction is positive and current leaving a junction is negative), gives

$$I_1-I_2-I_3=0, \qquad ...(1)$$

$$I_2-I_4-I_5=0, \qquad ...(2)$$

$$I_3+I_4-I_6=0. \qquad ...(3)$$

Applying the Kirchhoff voltage rule to the loops $ABED$ and $BCFE$ gives

$$I_2R+I_4R\times2R=0, \qquad ...(4)$$

$$I_5\times2R-I_6R-I_4R=0 \qquad ...(5)$$

Eliminating I_5 and I_6 from eqns. (2), (3) and (5), we obtain

$$I_1-I_2-I_3=0 \qquad ...(6)$$

$$I_2-2I_3+I_4=0 \qquad ...(7)$$

$$2I_2-I_3-4I_4=0 \qquad ...(8)$$

Eliminating I_2 from these three equations gives

$$I_1-3I_3+I_4=0 \qquad ...(9)$$

and $$2I_1-3I_3-4I_4=0, \qquad ...(10)$$

or $$I_4=\frac{1}{5}I_1. \qquad ...(11)$$

Further, the potential drop from A to F by route $ADEF$ is

$$V_{AF}=I_3\times2R+I_0\times R=R(2I_3+I_3+I_4)$$

using eqn. (3). By (9) this becomes

$$V_{AF}=R(I_1+2I_4)$$

and using (11),

$$V_{AF} = I_1 R\left(1+\frac{2}{5}\right)=\frac{7}{5}RI_1$$

The equivalent effect is therefore obtained if a wire 7/5 times the length of any side of the square is connected between A and F, because if produces the same potential drop as the double square between these points [see Fig. 2E.57 (b)].

Example 58

(a) A network of 12 resistors each of value $R = 6\Omega$ are interconnected as shown in Fig. 2E.58 (a), being placed along the sides of a regular dodecagon. Each of the terminals 1, 2, 3,...,12 has been connected to each of the 9 terminals, (other than nearest) directly by insulated wires each of resistance R, there being 9 wires from each terminal making 108 wire connections totally. [Only one set of 9 wires, from terminal 1 have been shown]. Find the equivalent resistance of the network when the current enters from P at the terminal 1 and leaves Q at terminal 2.

(b) If the above network were generalised so that there are n (n = even) resistors each of resistance R placed along the sides of a regular n-gon and if each terminal point of a resistor were connected by $(n-2)$ insulated wires each of resistance R, directly to the $(n-2)$ terminals, other than its, nearest terminals, find the equivalent resistance across any two terminals of the network (i.e., current entering at one of the two terminals and leaving by the other.

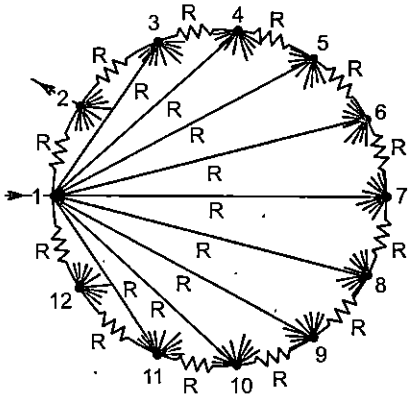
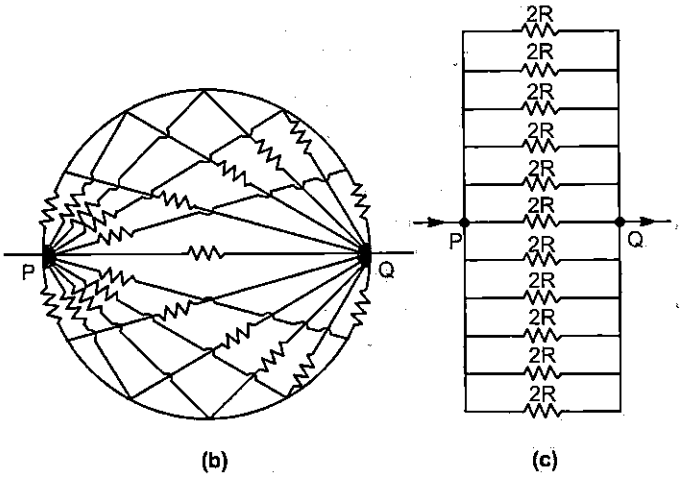


Fig. 2E.58 (a)

Solution: (a) Since each of the terminals is connected by an insulated wire to each of the remaining 11 terminals by a resistor R, the symmetry shows all the twelve terminals to be symmetrically equivalent before any voltage is applied.

However, asymmetry is introduced just at the point where current enters and also at the point where the current leaves the circuit. All other ten points are symmetry points, all at the same potential. Hence the given network reduces to the following network:

Thus if X be the equivalent resistance between P and Q,



(b) (c)

Fig. 2E.58

$$\frac{1}{X}=\frac{1}{2R}+\frac{1}{2R}+\ldots \text{ to 10 terms} +\frac{1}{R}$$

$$=\frac{10}{2R}+\frac{1}{R}=\frac{6}{R}$$

Hence $X=\frac{R}{6}$

The equivalent resistance $=\frac{R}{6}\Omega = 1\Omega$

(b) Proceeding in the same manner as in case (a) we find that the equivalent resistor is given by X where

$$\frac{1}{X}=\frac{1}{2R}+\frac{1}{2R}+\ldots \text{ to } (n-2) \text{ terms} +\frac{1}{R}$$

$$=\frac{n-2}{2R}+\frac{1}{R}=\frac{n}{2R}$$

Hence equivalent resistance $=\frac{2R}{n}$

Remark :

The above symmetry simplification can be done only if n is even.

FOLDING SYMMETRY

Resistance of each branch is R. Find R_{AB}

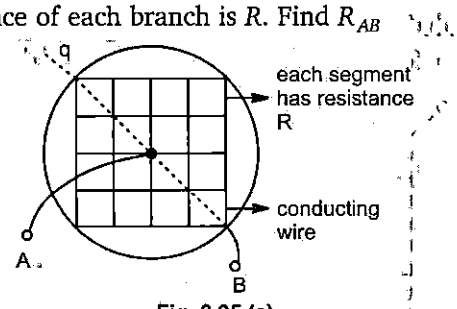
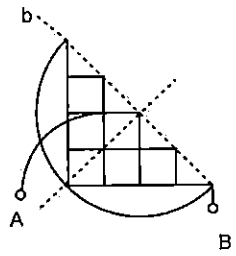


Fig. 2.95 (a)

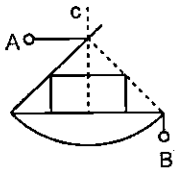
Figure is symmetrical about diagonal shown. We can fold the circuit about this diagonal.



each resistance is now $\frac{R}{2}$.

Fig. 2.95 (b)

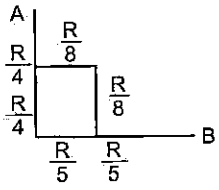
Now fold the circuit about diagonal *b*.



each resistance is now $\frac{R}{4}$

Fig. 2.95 (c)

Next fold the circuit about *c*.



$R_{AB} = \frac{21R}{40}$

Fig. 2.95 (d)

Example 59

Find R_{AB} given resistance of each branch is R.

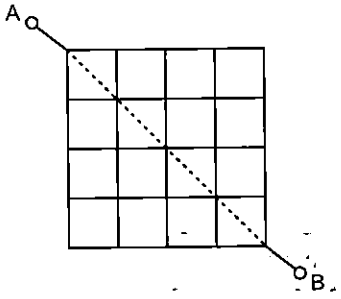
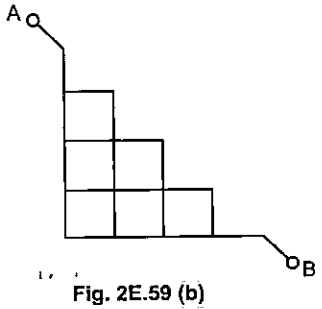


Fig. 2E.59 (a)

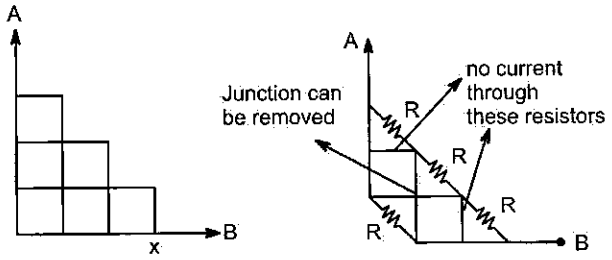
Solution:

First fold the circuit about diagonal *AB*.



Resistance is now $\frac{R}{2}$.

Fig. 2E.59 (b)



(c) (d)

Fig. 2E.59

**For folding the potentials of points must be same.

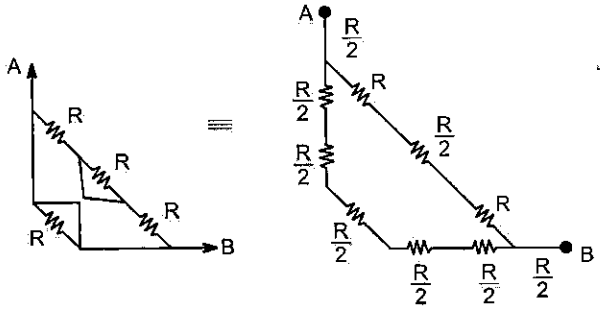


Fig. 2E.59 (e)

Now equivalent is $\frac{9R}{4}$

Example 60

Calculate the equivalent resistance of the networks shown in Fig.2E.60 (a) between the points A and C.

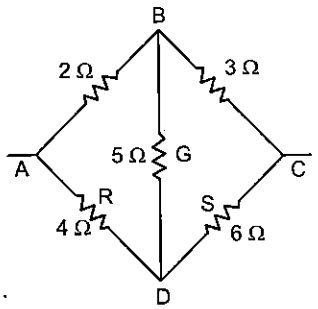


Fig. 2E.60 (a)

Solution: Given networks are *'balanced Wheatstone bridge'* as in these $(P/Q) = (R/S)$. Now as in a balanced Wheatstone bridge no current flows through resistance *G*, excluding *G*, effective resistance of the bridge between points *A* and *C* will be given by

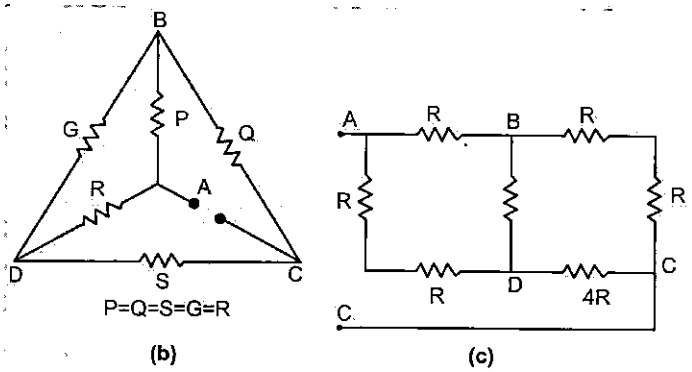


Fig. 2E.60

So in case (a)

$$i.e., \quad R_{eq} = \frac{5\times 10}{(5+10)} = \frac{10}{3}\,\Omega$$

while in case (b)

$$i.e., \quad R_{eq} = \frac{2R\times 2R}{(2R+2R)} = R$$

And in case (c)

$$R_{eq} = \frac{3R\times 6R}{(3R+6R)} = 2R$$

Example 61

1 m long metallic wire is broken into two unequal parts P and Q. P part of the wire is uniformly extended into another wire R. Length of R is twice the length of P and the resistance of R is equal to that of Q. Find the ratio of the resistances of P and R and also the ratio of lengths of P and Q.

Solution: Let the length of piece P be L then of Q will be $(1-L)$

So that, $$R_P = \rho\frac{L}{S}$$

and $$R_Q = \rho\frac{(1-L)}{S}$$

Now when part P is extended into another wire R of length twice of P, *i.e.*, $2L$ its resistance will be

$$R_R = \rho\frac{(2L)}{(S/2)} = 4\rho\frac{L}{S} \qquad [\text{as } SL = 2L\times S']$$

According to given problem

$$R_R = R_Q,$$

$$i.e., \quad 4\rho\frac{L}{S} = \rho\frac{(1-L)}{S}$$

$$i.e. \quad L = 0.2\text{ m}$$

So, $$\frac{R_P}{R_R} = \frac{\rho(L/S)}{4\rho(L/S)} = \frac{1}{4}$$

and $$\frac{L_P}{L_Q} = \frac{L}{(1-L)} = \frac{0.2}{(1-0.2)} = \frac{1}{4}$$

Example 62

Positive and negative ions are produced in the atmosphere due to cosmic rays from space and also due to radioactive elements in the soil. In some region in the atmosphere, the electric field strength is 100 V/m in the vertically downward direction. This field exerts force on the positive and negative ions in the given region in atmosphere.

As a result, positive ions, having a density 500/cm^3 drift downward while negative ions, having a density 300/cm^3 drift upward. All these ions are singly charged.It is observed that the conductivity in the given region is $4\times 10^{-13}(\Omega\text{ - m})^{-1}$. Find the average speed of ions, assuming it to be the same for positive and negative ions. Also find the current density.

Solution: Since the ion are singly charged, magnitudes charge on each ion is 1.6×10^{-19}C.

Using the expression for current density, $J = nev$

Magnitude of current density due to the drift of positive ions

$$J_1 = (500\times 10^6)(1.6\times 10^{-19})v = 800\times 10^{-13}v$$

Magnitude of current density due to drift of negative ions

$$J_2 = (300\times 10^6)(1.6\times 10^{-19})v = 480\times 10^{-13}v$$

(number density of ions has been converted to SI system; speed of positive and negative ions has been taken as v)

In a downward electric field, current density due to positive ions will be downward and that due to negative ions is also downward. So total current density

$$J = J_1 + J_2$$

$$\therefore \quad J = 1280\times 10^{-13}v \qquad ...(1)$$

but $$J = \sigma E$$

$$\therefore \quad J = 4\times 10^{-13}\times 100 = 4\times 10^{-11}\text{ A/m}^2 \qquad ...(2)$$

From eqs. (1) and (2),

$$1280\times 10^{-13}v = 4\times 10^{-11}$$

$$\therefore \quad v = \mathbf{0.31\,m/s}$$

Note: We will come across the following integration very frequently. So, remember the result as such.

Example 63

Suppose that you have a large number of identical batteries with emf $\mathcal{E}$ and internal resistance r, and a load resistor R. What is the current produced through a resistor R when (a) m batteries are connected in series? (b) n batteries are connected in parallel across R? (c) m batteries are in series and n such branches are connected in parallel across R? (d) What is the maximum power transfer to the load resistance in case (c) ?

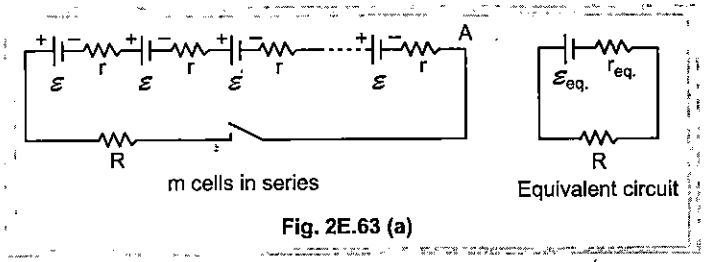


Fig. 2E.63 (a)

Solution: (a) If we start from A and traverse the circuit to B the equivalent emf is $\mathcal{E}_{eq.} = m\mathcal{E}$.

Equivalent internal resistance, $r_{eq.} = mr$

Total resistance of circuit $= mr + R$

Hence, current in the load resistance, $I = \dfrac{m\mathcal{E}}{mr + R}$

(b) Since all the branches are connected to same points, they are in parallel. The potential difference across each branch is equal to terminal potential difference across battery.

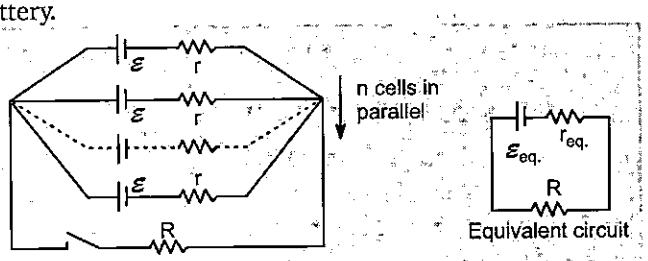


Fig. 2E.63 (b)

Thus, $\mathcal{E}_{eq.} = \mathcal{E} \Rightarrow$

$$r_{eq.} = \frac{r}{n}$$

Total resistance $= \dfrac{r}{n} + R$

Current through load resistance,

$$I = \frac{\mathcal{E}_{eq.}}{r_{eq.} + R} = \frac{\mathcal{E}}{\left(\dfrac{r}{n} + R\right)}$$

(c) Equivalent emf of a single branch $\neq m\mathcal{E}$

All the branches are in parallel arrangement, so

$$\mathcal{E}_{eq.} = m\mathcal{E}$$

$$\frac{1}{r_{eq.}} = \frac{1}{mr} + \frac{1}{mr} + \ldots \; n \text{ times}$$

$$r_{eq.} = \frac{mr}{n}$$

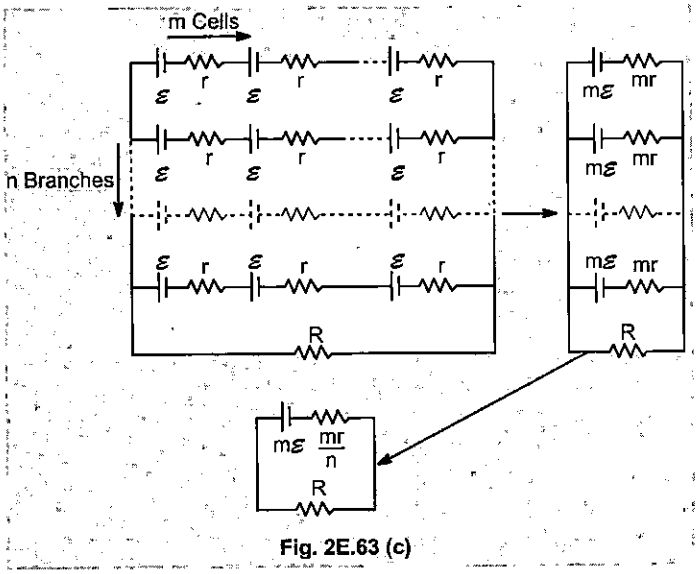


Fig. 2E.63 (c)

Total resistance of circuit $= \dfrac{mr}{n} + R$

Current through load resistance,

$$I = \frac{n\mathcal{E}}{R + \left(\dfrac{mr}{n}\right)} = \frac{\mathcal{E}}{\left(\dfrac{R}{m} + \dfrac{r}{n}\right)}$$

(d) The current in the circuit will be maximum when denominator is minimum when $\left(\dfrac{R}{m} + \dfrac{r}{n}\right)$ is minimum.

There are two variables in the expression to be minimised. Let p be the total number of cells, $m \times n = p$.

So $m = p/n$

Hence, $$\frac{d}{dn}\left[\frac{nR}{p} + \frac{r}{n}\right] = 0$$

$$\left[\frac{R}{p} + \frac{r}{n^2}\right] = 0$$

$$R = \frac{pr}{n^2} = \frac{mr}{n}$$

External resistance = Internal resistance

This is also **the condition for maximum power transfer.**

$$I_{max.} = \frac{m\mathcal{E}}{2R} = \frac{n\mathcal{E}}{2r}$$

$$P = \frac{\mathcal{E}^2 R}{\left(\dfrac{R}{m} + \dfrac{r}{n}\right)^2} = \frac{m^2\mathcal{E}^2}{4R} = \frac{n^2\mathcal{E}^2}{4r^2}R$$

AMMETER AND VOLTMETER

An **ammeter** measures current through circuit elements and a **voltmeter** measures voltage across circuit elements. A basic component of both of these types of meters is a **galvanometer**. The galvanometer works on the principles of magnetic field, to be discussed later. Here we treat it as a circuit element that has an internal resistance (G) and whose needle deflection is directly proportional to the current through it.

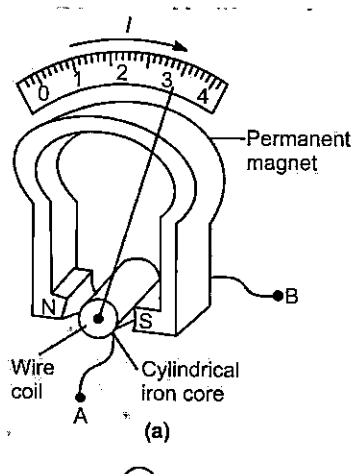


Fig. 2.96

Ammeter

The galvanometer coil has a small resistance, G, because the coil consists of metallic wire. A galvanometer measures current but due to small coil resistance only currents in the microampere range can be measured without burning out the coil. An ammeter that can be used to measure larger currents therefore must have a small **shunt resistor**, with resistance S, in parallel with a galvanometer to take most of the current. The shunt provides a bypass through which a large current I can bypass the galvanometer. The shunt and the galvanometer resistance are in parallel, thus to bypass most of the current the shunt must have the smaller resistance of the two. Because the voltage across the galvanometer and the shunt resistor are equal, we can write

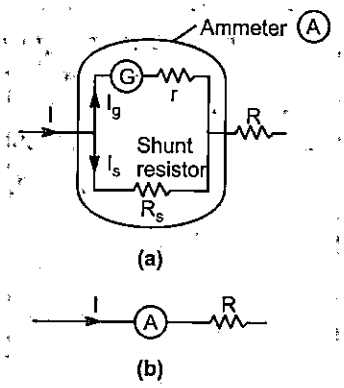


Fig. 2.97

$$V_g = V_s$$

$$I_g G = I_s S$$

From KCL, $$I = I_g + I_s$$

Hence, $$I_g G = (I - I_g) S$$

$$I_g = \frac{IS}{S+G}$$

This equation allows us to select the proper shunt resistance for a given current range and galvanometer.

Voltmeter

A voltmeter is capable of reading voltages higher than microvolt range. A large *multiplier resistor* is connected in series with a galvanometer. Due to multiplier resistor the galvanometer now has a large internal resistance, it draws little current from the main circuit. When connected across a circuit element, the voltage drop across the voltmeter [series combination of multiplier resistor and galvanometer] experiences a voltage drop of

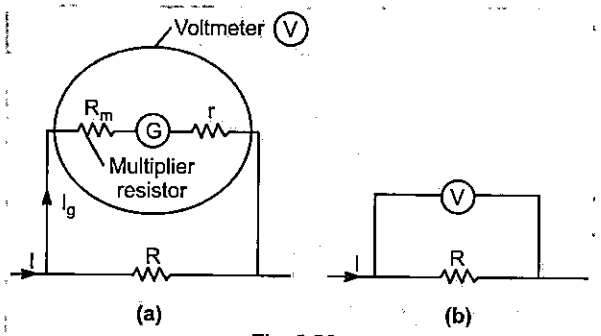


Fig. 2.98

$$V = V_g + V_m$$

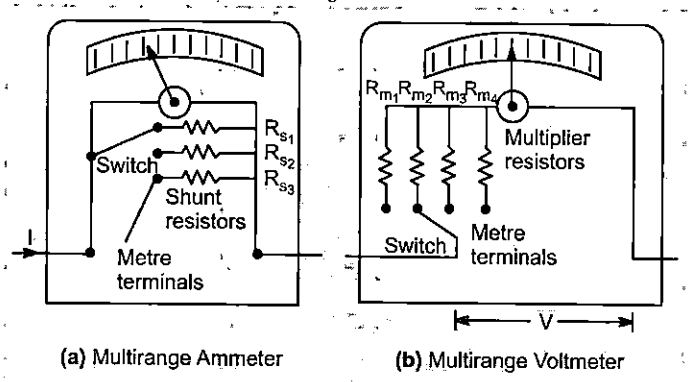


Fig. 2.99

Larger potential drop is across the multiplier resistor rather than the galvanometer coil.

$$V = V_g + V_m = I_g G + I_g R = I_g (R+G)$$

and $$I_g = \frac{V}{R+G}$$

The voltage V is also the potential difference across the circuit element having a resistance R because of parallel connection.

Example 64

A voltmeter of resistance R_V and an ammeter of resistance R_A are connected in series across a battery of e.m.f. $\mathcal{E}$ and of negligible internal resistance. When a resistance R is connected in parallel to voltmeter, reading of ammeter increases to three times while that of voltmeter reduces to one-third. Calculate R_A and R_V in terms of R.

Solution: Let initially a current I flow through the circuit as shown in Fig. 2E.64 (a).

Applying Kirchhoff's voltage law on the circuit,

$$IR_A + IR_V - \mathcal{E} = 0$$

or $$\mathcal{E} = IR_A + IR_V \quad ...(1)$$

Initial reading of ammeter is I and that of voltmeter is IR_V.

When resistance R is connected in parallel with voltmeter, reading of ammeter increases to three times, it means current $3I$ flows through ammeter but reading of voltmeter decreases to one third, it means current $I/3$ flows through the voltmeter. Hence, remaining $\left(3I - \frac{I}{3} = \frac{8I}{3}\right)$ current passes through R as shown in Fig. 2E.64 (b)

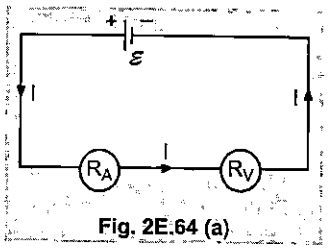


Fig. 2E.64 (a)

First, applying KVL on mesh 2 of Fig. ?

$$\frac{I}{3}R_V - \frac{8I}{3}R = 0 \quad \text{or} \quad R_V = 8R$$

Now applying KVL on mesh 1 of Fig. 2E.64 (b)

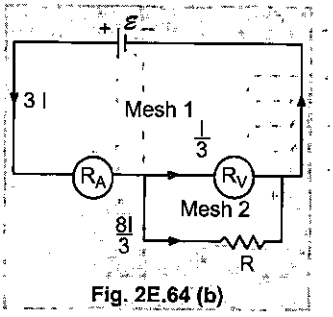


Fig. 2E.64 (b)

$$+\mathcal{E} - \frac{I}{3}R_V - 3IR_A = 0$$

or $$\mathcal{E} = \frac{1}{3}IR_V + 3IR_A \quad ...(2)$$

From equations (1) and (2),

$$\frac{1}{3}IR_V + 3IR_A = IR_A + IR_V$$

or $$2IR_A = \frac{2}{3}IR_V$$

Example 65

The full scale sensitivity of a galvanometer is 200 μA [maximum coil current], and coil resistance 50Ω. It is to be used in an ammeter designed to read currents up to 3.0 A (at full scale). What is the required shunt resistance?

Solution: The galvanometer can carry only a small current, most of the current has to be shunted through the shunt resistor. The given parameter are

$$I_g = 200\,\mu A = 2.00 \times 10^{-4}\,A$$

$$G = 50\Omega$$

$$I_{max.} = 3.0\,A$$

A full scale reading of 3.0 A implies that when a current of 3.0 A enters the ammeter, I_g should be 200 μA. As $I_g = \frac{IS}{S+G}$, we have

$$S = \frac{I_g G}{I_{max.} - I_g}$$

$$= \frac{(2.0\times10^{-4})(50)}{3.0 - 2.00\times10^{-4}}$$

$$= 3.3\times10^{-3}\,\Omega$$

Note that the shunt resistance is very small as compared to coil resistance $G = 80\Omega$. The current that passes through the shunt resistor branch is $(3.0 - 2\times10^{-4})$ A = 2.9998 A. The shunt resistor is made of a material that does not burn out as readily as the thin wire of galvanometer. The ammeter will read currents linearly up to 3 A. *i.e.,* for a current of 1.5 A flowing into the ammeter there will be a current of 200 μA in the coil of the galvanometer, which would give a half scale reading.

Example 66

What is the required multiplier resistor for a voltmeter with a full scale reading of 3.0 V in the galvanometer of the previous example?

Solution: Due to high multiplier resistor the major potential drop is on it and only a small fraction on the galvanometer.

$$I_g = 200\,\mu A = 2.00\times10^{-4}\,A$$

$$G = 50\Omega$$

$$V = 3.0\,V$$

As $$R_m = \frac{V - I_g G}{I_g}$$

$$= \frac{3.0 - (2.00\times10^{-4})(50)}{2.00\times10^{-4}}$$

$$= 1.5\times10^{4}\,\Omega = 15\,k\Omega$$

Example 67

To measure the resistance R of a resistor, a voltmeter of resistance R_v is placed across a resistor and an ammeter is placed in series with the combination as shown in Fig. 2E.67. (a) Find the resistance R in terms of the measured readings on the ammeter I_{meas} and voltmeter V_{meas}. (b) Discuss the result for $R_V >> V_{meas}/I_{meas}$.

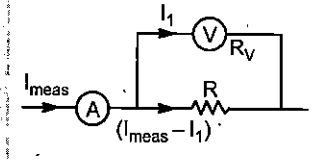


Fig. 2E.67

Solution: (a) Potential difference across $R = (I_{meas} - I_1)R$; potential difference across voltmeter $= I_1 R_V = V_{meas}$ and this is measured potential difference.

The two branches are in parallel.

$$(I_{meas} - I_1)R = V_{meas}$$

$$\left(I_{meas} - \frac{V_{meas}}{R_V}\right)R = V_{meas}$$

$$R = \frac{V_{meas}}{(I_{meas} - V_{meas}/R_V)}$$

(b) From the expression for R, we get

$$\frac{I}{R} = \frac{I_{meas}}{V_{meas}} - \frac{1}{R_V}$$

If $$R_V >> \frac{V_{meas}}{I_{meas}}$$

$$\frac{1}{R_V} << \frac{I_{meas}}{V_{meas}}$$

thus $$\frac{1}{R} \cong \frac{I_{meas}}{V_{meas}}$$

or $$R = V_{meas}/I_{meas}$$

Example 68

To measure the resistance R of a resistor, an ammeter of resistance R_A is placed in series with the resistor and the voltmeter is placed across the series combination as shown in Fig. 2E.68. (a) Find the resistance R in terms of the measured reading on the ammeter I_{meas} and voltmeter V_{meas}. (b) Discuss the result for $V_{meas}/I_{meas} >> R_A$.

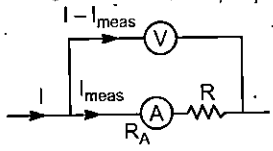


Fig. 2E.68

Solution: (a) The voltmeter and ammeter branches are in parallel arrangement. Hence

$$V_{meas} = (I - I_{meas})R_V = I_{meas}(R_A + R)$$

Hence, $$R_A + R = \frac{V_{meas}}{I_{meas}}$$

$$R = \left(\frac{V_{meas}}{I_{meas}} - R_A\right)$$

(b) For $\frac{V_{meas}}{I_{meas}} << R_A$, $R \simeq \frac{V_{meas}}{I_{meas}}$

Example 69

What shunt resistance is required to make the $1.00\,mA, 20.0\Omega$ galvanometer into an ammeter with a range of $0\,A$ to $50.0\,mA$?

Solution: The given parameters are

$$I_s = 1.00\,\text{mA} = 1.00 \times 10^{-3}\,\text{A}$$

$$G = 20\Omega$$

$$I_{max.} = 50.0 \times 10^{-3}\,\text{A}$$

as $$S = \frac{I_g G}{(I - I_g)}$$

$$= \frac{(1.00 \times 10^{-3})(20)}{50.0 \times 10^{-3} - 1.00 \times 10^{-3}}$$

$$= 0.408\Omega$$

The equivalent resistance of the instrument is

$$\frac{1}{R_{eq.}} = \frac{1}{G} + \frac{1}{S} = \frac{1}{20} + \frac{1}{0.408}$$

$$R_{eq.} = 0.400\Omega$$

Note that shunt resistance is so small in comparison to the galvanometer resistance that the equivalent is very nearly equal to the shunt resistance.

Example 70

A voltmeter has a resistance G ohm and range V volt. Calculate the resistance to be used in series with it to extend its range to nV volt.

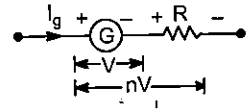


Fig. 2E.70

Solution: The maximum current through the galvanometer is

$$I_g = \frac{V}{G}$$

With a multiplier resistor in series with galvanometer the potential difference across the entire branch is

$$nV = I_g G + I_g R$$

$$= \left(\frac{V}{G}\right)G + \left(\frac{V}{G}\right)R$$

$$= V + \left(\frac{V}{G}\right)R$$

or $$R = (n-1)G$$

Usually n is referred to as multiplication factor, e.g., a galvanometer with a range 0 to 1 mV is to be converted to a voltmeter of range 0 to 1 V.

We have $$n = \frac{1\,\text{V}}{1\,\text{mV}} = 10^3$$

The multiplier resistor $R = (10^3 - 1)G = 999G$.

So you can guess how large the multiplier resistor is in comparison to the galvanometer resistance G.

Example 71

A shunt of resistance $1/n$ the value of galvanometer resistance is connected in parallel with a galvanometer. How will the current sensitivity of the galvanometer change?

Solution: The deflection per unit current is called **current sensitivity** of the galvanometer.

$$CS = \frac{\theta}{I}$$

Reciprocal of current sensitivity, *i.e.*, current per unit deflection, is called **figure of merit.**

$$FM = \frac{I}{\theta} = \frac{1}{CS}$$

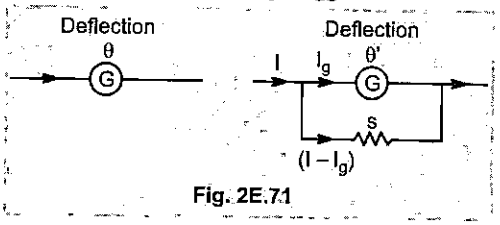


Fig. 2E.71

The deflection (θ) of the galvanometer is proportional to the current passing through the galvanometer.

$$\frac{\theta'}{\theta} = \frac{I_g}{I}$$

where θ' and θ are the deflection of a shunted galvanometer and simple galvanometer.

$$CS' = \frac{\theta'}{I} = \frac{1}{I}\left[\frac{I_g}{I}\theta\right] = \frac{I_g}{I}(CS)$$

For a shunted galvanometer with $S = \frac{G}{n}$.

$$(I - I_g)\frac{G}{n} = I_g G$$

$$I_g = \frac{I}{(1+n)}$$

Hence $$\frac{CS'}{CS} = \frac{I_g}{I} = \frac{1}{(1+n)}$$

So $$CS' = \frac{CS}{1+n}$$

This shows that current sensitivity of a galvanometer decreases by shunting.

Example 72

Consider two different ammeters in which the deflections of the needle are proportional to current. The first ammeter is connected to a resistor of resistance R_1 and the second to a resistor of unknown resistance R_x. At first the ammeters are connected in series between points A and B [as shown in Fig. 2E.72(a)]. In this case the readings of the ammeters are n_1 and n_2. Then the ammeters are connected in parallel between A and B as shown in Fig. 2E.72(b) and indicate n'_1 and n'_2. Determine the unknown resistance R_x of the second resistor.

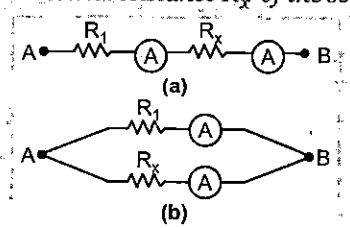


Fig. 2E.72

Solution: The currents in the ammeters are proportional to deflections produced, let α_1 and α_2 be the proportionally constants.

$$I_1 = \alpha_1 n_1, \; I_2 = \alpha_2 n_2$$

The ammeters are in series, hence

$$I_1 = I_2$$

$$\alpha_1 n_1 = \alpha_2 n_2 \qquad \ldots(1)$$

In the second arrangement the potential drops across resistors are equal as they are in parallel arrangement. Resistances of ammeters have been ignored assuming them to be ideal.

Thus we have

$$I'_1 R_1 = I'_2 R_x$$

Also $$I'_1 = \alpha_1 n'_1$$

and $$I'_2 = \alpha_2 n'_2$$

So $$R_1 \alpha_1 n'_1 = R_x \alpha_2 n'_2 \qquad \ldots(2)$$

From eqns. (1) and (2), eliminating α_1 and α_2, we get

$$\frac{R_1 n'_1}{n_1} = \frac{R_x n'_2}{n_2}$$

Therefore $$R_x = \frac{R_1 n_2 n'_1}{n_1 n'_2}$$

WHEAT STONE BRIDGE

It is an electrical arrangement with forms the basis of following instruments:

1. Slide wire bridge/meter bridge
2. Post office box
3. Cary- foster's bridge
4. Potentiometer

Wheat stone bridge consist of 4 resistance battery and a galvanometer as shown in Fig. 2.100.

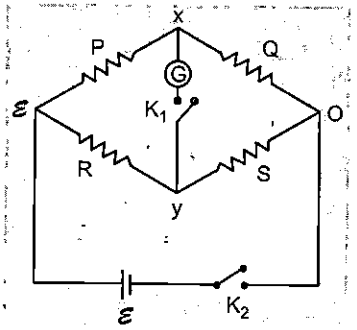


Fig. 2.100

Node x:

Closing K_1 and K_2

$$\frac{x-y}{G} + \frac{x-0}{Q} + \frac{x-\varepsilon}{P} = 0$$

Node y:

$$\frac{y-0}{S} + \frac{y-x}{G} + \frac{y-\varepsilon}{R} = 0$$

In balanced eqn. $I_G = 0 \Rightarrow x = y$

$$\frac{P}{Q} = \frac{R}{S} \quad \text{or} \quad \frac{P}{R} = \frac{Q}{S}$$

Note-1: Wheat stone bridge is used to determine unknown resistance provided the other there resistances of network are known and bridge is in balanced condition

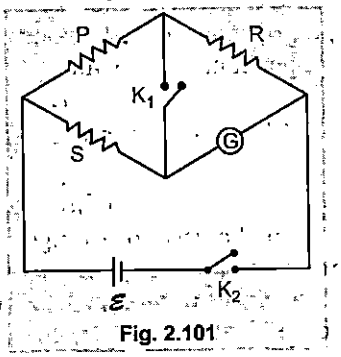


Fig. 2.101

Note-2: Different form of wheatstone bridge.

(a) In this situation, when key K_1 is closed deflection of galvanometer remains same when only K_2 was closed then bridge is in balanced condition.

$$\frac{P}{R}=\frac{S}{G} \Rightarrow G=\frac{RS}{P}$$

(b)

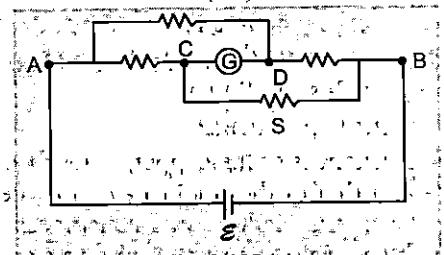


Fig. 2.102 (a)

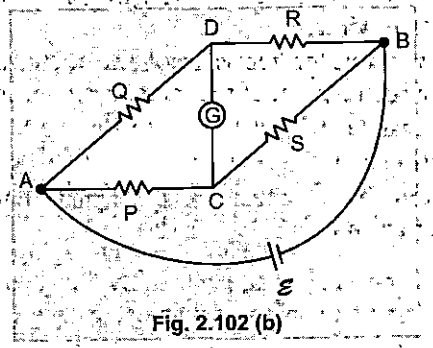


Fig. 2.102 (b)

Balanced conditions:

(i) $I_G = 0$

(ii) $\frac{Q}{P}=\frac{R}{S}$

(c)

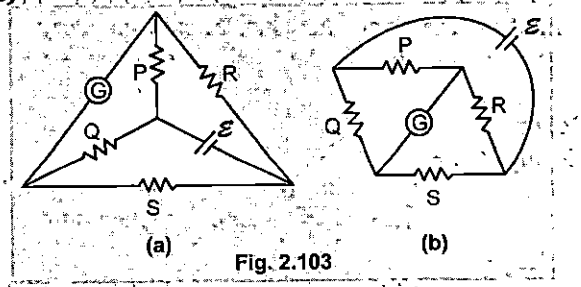


Fig. 2.103

(i) $I_G = 0$

(ii) $\frac{P}{Q}=\frac{R}{S}$

Note-3:

Application

(a) Slide wire bridge and meter bridge.

This is one of the practical form of Wheatstone's bridge used to determine unknown resistance.

(i) $l_{AB} = 1$ m

(ii) Wire is of uniform cross-section

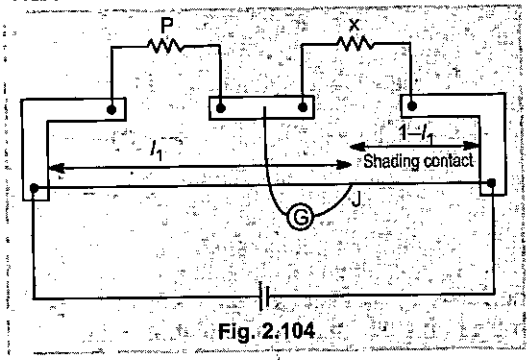


Fig. 2.104

Balanced condition:

(i) $I_G = 0$

(ii) $\frac{P}{Rl_1}=\frac{x}{R(l-l_1)}$

$$x=\frac{P\cdot R(l-l_1)}{Rl_1}$$

$$x \Rightarrow \frac{P_1(1-l_1)}{l_1}$$

The Potential Divider

A potential divider is used to supply a specified voltage from a fixed voltage supply, *e.g.*, a 5 V device can be powered from a 9 V battery using a potential divider. Fig. 2.105 shows a potential divider consisting of two resistors and a source of voltage. The current in each of the resistors is same. Since the current,

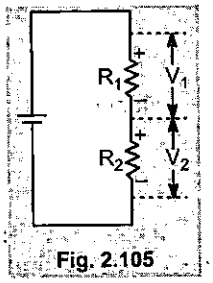


Fig. 2.105

$$I=\frac{V}{R_1+R_2}$$

The P.D. across resistor R_1,

$$V_1 = IR_1 = \frac{R_1}{R_1+R_2}V$$

and the P.D. across resistor R_2,

$$V_2 = IR_2 = \frac{R_2}{R_1+R_2}V$$

Note the ratio,

$$\frac{V_1}{V_2}=\frac{R_1}{R_2}$$

The means the potential difference across resistors in series is proportional to resistance.

Meter-Bridge

Meter-bridge is a sensitive device based on the principle of Wheatstone's bridge, for the determination of the resistance of a conductor (wire). Its sensitivity is much more than that of the post-office box.

Meter-bridge is shown in Fig. 2.106 *AC* is a 1-meter wire

of manganin or constantan which is fixed along a scale on wooden base. The area of cross-section of the wire is same at

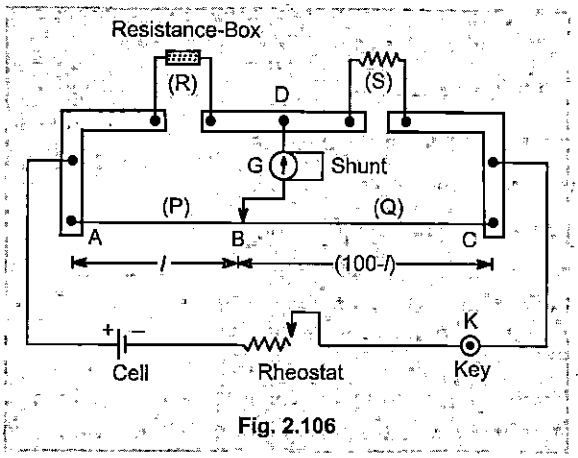


Fig. 2.106

all places. The ends A and C of the wire are joined to two L-shaped copper strips carrying binding-screws as shown. In between these strips, leaving a gap on either side, there is a third copper strip having three binding screws. The middle screw D is connected to a sliding jockey B through a shunted-galvanometer G. The knob of the jockey can be made to touch at any point on the wire.

Determination of resistance

The wire whose resistance (S) is to be determined is connected across the gap between the points C and D, and a resistance box across the gap between the points A and D. Between A and C are connected a cell, a rheostat and a key. In the experiment, when the sliding jockey thouches the wire AC at any point B then the wire is divided into two parts. These two parts AB and BC act as resistances P and Q of the Wheatstone's bridge.

First of all, a resistance R is taken in the resistance box and the key K is closed. Now the jockey is slided along the wire and a point is determined such that, on pressing the jockey on the wire at that points, there is no deflection in the galvanometer G. In this position the points B and D are at the same potential. The points B is called 'null-point. The lengths of both the parts AB and BC of the wire are read on the scale.

Suppose the resistance of the length AB of the wire is P, and that of the length BC is Q. Then, by the principle of Wheatstone bridge, we have,

$$\frac{P}{Q}=\frac{R}{S}$$

Let the length AB be l cm. Then the length BC will be $(100-l)$ cm.

$$\therefore \quad \text{resistance of } AB,\ P=\rho\frac{l}{a}$$

$$\text{and} \quad \text{resistance of } BC,\ Q=\rho\frac{(100-l)}{a}$$

where ρ (in ohm-cm) is the specific resistance of the material of the wire and a (in cm^2) is the area of cross-section of the wire. Thus

$$\frac{P}{Q}=\frac{l}{(100-l)}$$

Substituting this value of $\frac{P}{Q}$ in eq. (i), we get

$$\frac{l}{(100-l)}=\frac{R}{S}$$

or $$S=R\left(\frac{100-l}{l}\right).$$

R is the resistance taken in the resistance box and l is the length measured. Hence the value of resistance S can be determined from the above formula.

A number of observations are taken for different resistances in the resistance box and for each observation the value S is calculated.

Finally, the experiment is repeated by interchanging the unknown resistance S and the resistance box. The mean of the values of S is then obtained.

Errors and their Removal

(i) The resistances of the copper strips fitted at the ends of the wire and that of the solder have not been taken into account. These are called 'end-resistances'. These are determined by another experiment in terms of the length of the wire,and are added in the lengths of the two parts AB and BC of the wire.

(ii) If the end of the wire of the meter-bridge is not exactly at the zero-point of the meter scale, or the knife edge of the sliding jockey (which touches the wire) and its index point (by means of which of the scale is read) are not in line, then the length of the two parts AB and BC read on the scale will be different than the real lengths. To remove this error, the experiment is repeated after interchanging the unknown resistance S and the resistance-box. By doing so, error due to any non-uniformity in the thickness of the bridge-wire is also minimised.

(iii) The resistance R taken in the resistance -box is so chosen that the null-point is nearly in the middle of the wire. Then the percentage error will be least. In addition to it, the effect of end-resistances will also be minimum. There is one more advantage. When the null-point is in the middle, the sensitivity of the bridge becomes maximum because then all the four resistnances (P, Q, R and S) become nearly equal.

(iv) The current should not be allowed to flow in the wire for a long time otherwise the wire will become hot and its resistance will be changed.

(v) In the initial adjustments the shunt should be used with the galvanometer. Near the zero-deflection position the shunt should be removed.

(vi) The sliding-jockey should not be rubbed on the wire otherwise the thickness of the wire wi ot remain same at all places.

Rheostat

The rheostat is a device used for varying the strength of the current in an electrical circuit. It consists of an oxidised nichrome wire (whose specific resistance is high and

temperature-coefficient of resistance is low) wound on a hollow china-clay cylinder. As the wire is oxidised, its turns are insulated from each other. The ends of the wire are connected to the binding-screws A and B fixed at the base. A metal rod CD is fixed above the cylinder parallel to it. It carries a sliding metallic strip S. The strip can be slided to and for, pressing the coil of the wire. It is called the 'sliding contact'. A binding screw C is fixed at one end of the rod.

Using the binding screws A, B and C the rheostat can be used in two ways:

(i) As a current-controller: For this, one wire of the circuit is connected to one of the binding screws A and B fixed to the base and the other wire is connected to the binding screw C fixed to the rod [Fig. 2.107 (a), (b)]

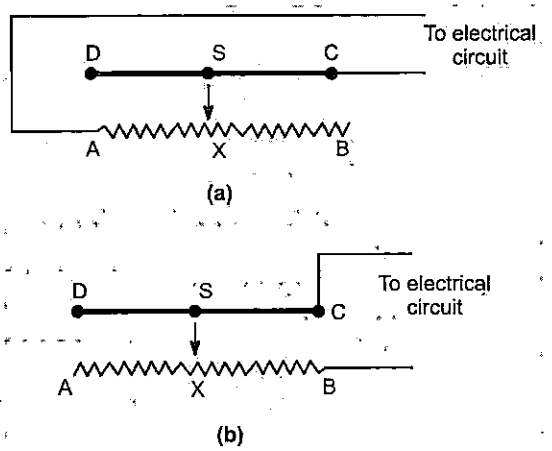


Fig. 2.107

Thus, in the first position [Fig. 2.107 (a)] the resistance of the length AX of the rheostat wire is included in the circuit, and in the second position [Fig. 2.107, (b)] the resistance of the length XB of the wire is included. By sliding the sliding-contact on the rod CD, the value of resistance included in the circuit, and hence the current in the circuit, can be changed. In the circuit, the rheostat is indicated by the symbol.

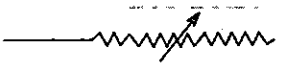


Fig. 2.108

(ii) As a potential-divider: The cell, or the battery, whose potential difference V is to be divided, is connected across the binding screws A and B; and the electric circuit is connected across A and C (or across B and C) as before (Fig. 2.109)

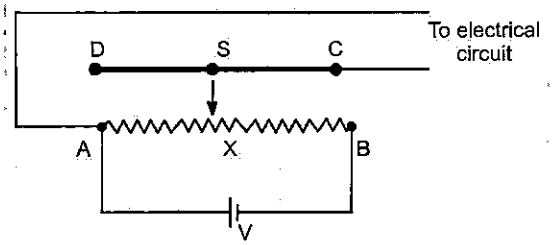


Fig. 2.109

If the contact-point X of the sliding-constant S be at a distance of three-fourth length of the wire AB from the screw A, then the potential difference between A and X will be three-fourth the total potential difference *i.e.*, $\frac{3V}{4}$. Hence a potential difference of $\frac{3V}{4}$ will be establised across the circuit. Thus, by sliding the contact-point X from A to B any desired potential difference from 0 to V can be establised across the circuit. In the circuit, the symbol of potential divider is:

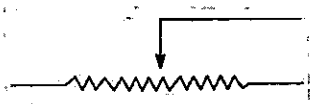


Fig. 2.110

The total resistance of the rheostat-wire and the maximum permissible current (which can flow without overheating) is written on the handle of the sliding-contact.

Necessity of High-resistance Measuring Instrument for the Measurement of E.M.F.

If we measure the emf of a cell by a voltmeter, then an accurate value of emf will not be obtained. The reason is that when the voltmeter is connected across the ends of the cell, it takes some current from the cell, that is, the cell does not remain on open-circuit. As a result of potential-drop across the internal resistance of the cell, the potential difference between the ends of the cell becomes less than the emf of the cell. Thus, the voltmeter measures the potential difference between the plates of the cell which is less than the emf.

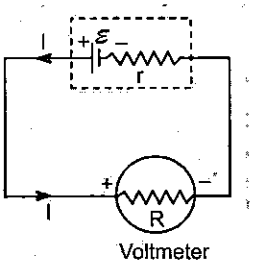


Fig. 2.111

If Fig. 2.111, the ends of a cell emf $\mathcal{E}$ and internal resistance r are connected to a voltmeter. Suppose the resistance of the voltmeter is r. Then the current in the circuit is given by

$$I = \frac{\mathcal{E}}{R + r} \quad \text{...(i)}$$

Let the potential difference between the plates of the cell be V. Since the plates of the cell are directly connected to the voltmeter, the potential difference across the voltmeter, that is, across the resistance R is also V. Therefore.

$$V = I R$$

Substituting the value of I from eqn. (i), we get

$$V = \frac{R}{R + r}\mathcal{E} \quad \text{...(ii)}$$

The reading of the voltmeter will give V which is less than $\mathcal{E}$.

Suppose the emf of the cell is $\mathcal{E} = 2$ volt and the internal resistance is $r = 5$ ohm. If the resistance of the voltmeter be $R = 100$ ohm, then the reading of the voltmeter, from eqn. (ii), will be

$$V = \frac{100}{100 + 5} \times 2 = 1.90 \text{ volt}$$

If the resistance of the voltmeter be 200 ohm, then

$$V = \frac{200}{200+5} \times 2 = 1.95 \text{ volt}$$

If the resistance of the voltmeter be 2000 ohm, then

$$V = \frac{2000}{2000+5} \times 2 = 1.99 \text{ volt.}$$

It is thus clear that higher the resistance of the voltmeter, nearer will be its reading to the emf of the cell (2 volt). Hence, for accurate measurement of the emf of a cell, the measuring-instrument should be of very high resistance.

Potentiometer

The potentiometer is simply a piece of resistance wire, usually a metre long, fixed between two points A and B with a cell of output V connected between the two ends. It can be used to measure the emf of a source without withdrawing any current from the source, measurement of internal resistance and comparison of resistances.

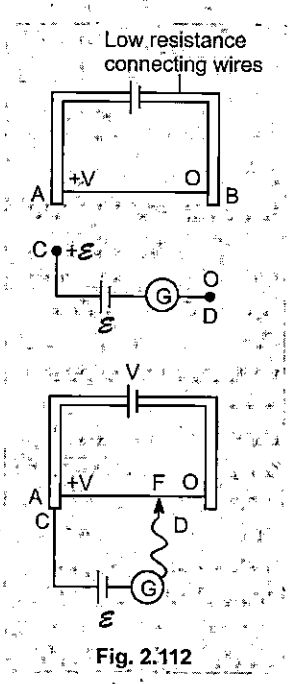


Fig. 2.112

Consider a source of emf $\mathcal{E}$ and a galvanometer connected in series as shown in Fig. 2.112. The potential at C is $+\mathcal{E}$ and at D will be zero. A potential difference of $\mathcal{E}$ volt exists between C and D.

When C is connected to A, the potential at junction AC becomes $+V$ and that at D becomes $V - \mathcal{E}$. When D is connected to the wire AB at F, the galvanometer will show no deflection if the potential drop down the wire is equal to that across the source and meter.

P.D. across AF = P.D. across CD

We assume that P.D. across AB decrease uniformly from A to B.

Let length $AB = 1$ m and $AF = l$ m.

Then $$\mathcal{E} = \frac{V}{l}$$

If length $AB = L$ m, then $\mathcal{E} = \frac{V}{L} l$

In practice the source is connected to the potentiometer wire by a sliding contact (or jockey). The jockey is moved along the wire AB until **null point** [point of zero deflection in galvanometer] is obtained.

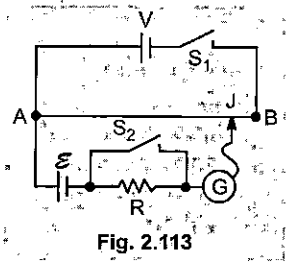


Fig. 2.113

The actual circuit is shown in Fig. 2.113. Key S_1 is used to cut out the supply emf to prevent overheating of potentiometer wire. A protective resistor is used with the galvanometer to prevent damage when far from the balance point. When the balance point is reached this resistor is shorted out. In the balance condition *no current flows in the galvanometer but the driver cell* (V) *supplies current throughout.*

Applications of Potentiometer

(i) Comparison of emfs

In the figure shown $\mathcal{E}_1$ is the unknown emf and $\mathcal{E}_0$ is a standard cell of known emf. The null point is first obtained with the standard cell; let it be l_0, and then with the unknown emf, let it be l_1. Then

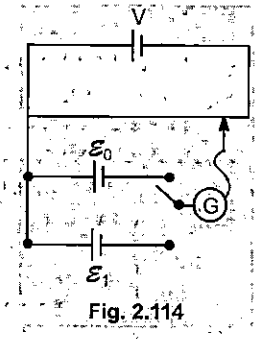


Fig. 2.114

$$\mathcal{E}_0 = \frac{V}{L} l_0$$

and $$\mathcal{E}_1 = \frac{V}{L} l_1$$

So we have $$\frac{\mathcal{E}_0}{\mathcal{E}_1} = \frac{l_0}{l_1}$$

(ii) Measurement of Internal Resistance

In the figure shown a resistor is connected in parallel with the cell of emf $\mathcal{E}$ and internal resistance r. First the null point is obtained with the switch open; this measures $\mathcal{E}$, since no current flows from cell, there is no potential drop across r. Therefore the balancing length is proportional to emf $\mathcal{E}$.

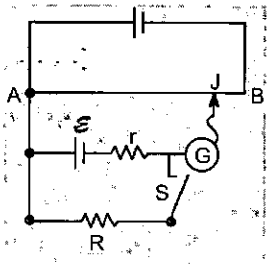


Fig. 2.115

$$\mathcal{E} \propto l_0$$

Next the switch is closed and a second balance point l_1 is found. This will measure output voltage of the cell,

i.e., $$V = E - Ir, \quad V \propto l_1$$

Since $$\frac{E}{V} = \frac{l_0}{l_1}$$

and $$E = V + Ir$$

therefore $$r = \left(\frac{l_0}{l_1} - 1\right) R$$

(iii) Comparison of Resistance

The figure shows circuit in this case. First the null point is obtained with one resistance (l_1) and then with the other (l_2).

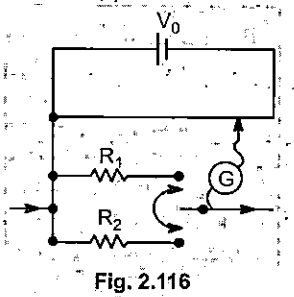


Fig. 2.116

Then $$\frac{l_1}{l_2}=\frac{IR_1}{IR_2}=\frac{R_1}{R_2}$$

(iv) Measurement of Small emfs (Thermocouple)

Since the balance length is proportional to the emf, therefore it will be small.

For a thermocouple the emf is of the order of a few mV, the null point will be very close to one end of the wire and therefore very difficult to measure. This situation can be tackled by putting a very large resistance in series with the potentiometer wire. Therefore there is a large P.D. across R and only a small P.D. across the wire.

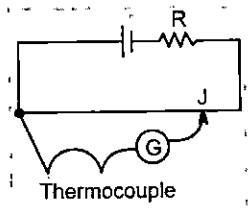


Fig. 2.117

Let the dry cell have emf of 1 V and the potentiometer wire have a resistance of 10 Ω. If $R = 500\Omega$ then the voltage drop down the potentiometer wire is

$$\frac{5\times 1}{510}=0.0098\text{V}$$

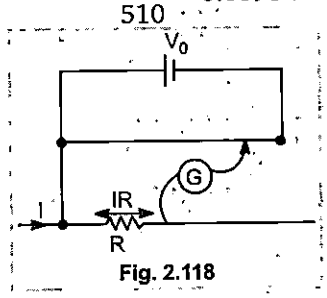


Fig. 2.118

Suppose we want to calculate the balance point for a thermocouple giving an emf of 5 mV.

Drop down 100 cm = 0.0098 V

Therefore for a drop of 5 mV,

$$\text{length}=\frac{0.005\times 100}{0.0098}=51.02\,\text{cm}$$

Superiority of Potentiometer over Voltmeter

(i) When we measure the emf of a cell by a potentiometer, then in the position of null-point no current flows in the cell circuit, that is, the cell is on open-circuit. Hence we obtain the actual value of the emf of the cell. Thus, a potentiometer is equivalent to an ideal voltmeter of infinite resistance.

(ii) For measuring the emf by a voltmeter, the deflection in the voltmeter is to be read. There may be some error in reading the deflection. On the other hand, the potentiometer method is a null method. One has to read null-point position on the wire. In reading the null-point position there can be a maximum error of 1 mm. If the wire is 4 meter long and its ends are directly connected to a cell of emf 2 volt, then the potential-gradient along the wire per mm will be 2/40000 volt = 0.0005 volt. Thus there can be a maximum error of 0.0005 volt in the emf. This can be further reduced by taking a still longer wire.

(v) Measurement of Current

The required circuit is shown in figure. The potential drop across R can be found by finding the null point. If R is known, the value of current can be found. This principle can be used to calibrate an ammeter or voltmeter.

Illustration:

In a potentiometer set up, the balance point for the unknown emf $\mathcal{E}$ is found at 70.4 cm from the left and of the meter wire. If the driver cell has an emf of 1.5 V, then determine the emf of the unknown cell.

Sol. We know that

$$\frac{\mathcal{E}}{l}=\frac{\mathcal{E}_0}{100}\ \text{(because length of the wires is 100 cm)}$$

$$\frac{\mathcal{E}}{70.4}=\frac{1.5}{100}$$

$$E=\frac{(1.5)(70.4)}{100}=1.056\text{V}$$

Checking of Connections:

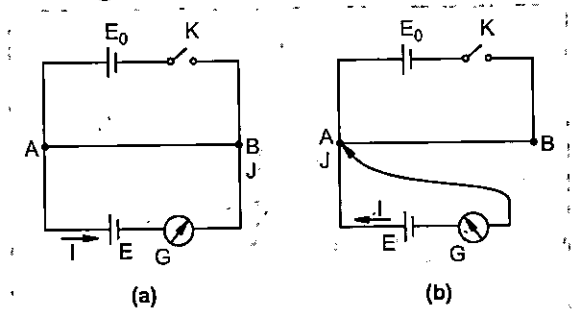


Fig. 2.119: The galvanometer should show opposite deflections when the jockey is first connected at and later on connected at A.

Consider a battery with emf $\mathcal{E}$, which is less than the potential difference V_{AB} across the full length of the wire. If the jockey J is connected with the point B, then a current I flows into the cell $\mathcal{E}$ (against its emf) and the galvanometer shows deflection towards right as shown in figure. If the jockey J is now connected to the point A, as shown in figure then a current I comes out of the cell $\mathcal{E}$ (in the direction of its emf) and the galvanometer shows deflection towards left. If the deflections at A and B are not opposite, then either the emf of $\mathcal{E}$ is greater than the potential difference between A and B, or the circuit been connected wrongly.

Note that: The most common mistake in connecting up is not joining both positive terminals at A.

Reading of a Potentiometer:

The reading of a potentiometer means determining a point on the wire AB at which the galvanometer shows no deflection. Suppose the jockey J is connected to the point C as shown in figure. If potential difference between A and C is less than the emf $\mathcal{E}$, then the current flows out of the cell and the galvanometer deflects toward left.

If figure, the jockey is connected to a point D which is nearer to the terminal B. If the potential difference between

A and B is more than the emf $\mathcal{E}$, the current flows into the cell and the galvanometer shows deflection toward right.

It implies that the point of zero deflection lies in between C and D. By trial and error (but no scraping of the jockey), we can find a point F such that, when the jockey is connected to this point the galvanometer shows no deflection. Thus, the potential difference V_{AF} is equal to the emf $\mathcal{E}$ of the cell.

THE POST OFFICE BOX

It is a compact form of the Wheatstone bridge. It consists of compact resistance so arranged that different desired values of resistances may be selected in the three arms of Wheatstone bridge, as shown in Fig. 2.120.

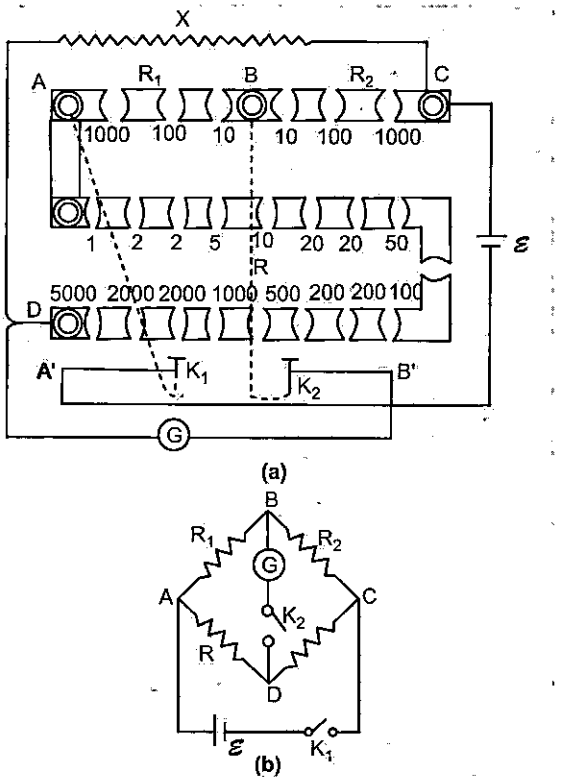


Fig. 2.120

Each of the arms AB and BC contains three resistances of 10, 10^2 and $10^3\Omega$, respectively. These are called the ratio arms. Using these resistances the ratio $\frac{R_2}{R_1}$ can be made to have any of the following values: 100 : 1, 10 : 1, 1 : 1, 1 : 10 or 1 : 100.

The arm AD is a complete resistance box containing resistance from 1 to 5000 Ω. The tap keys K_2 and K_2 are also provided in the post office box. The key K_1 is internally connected to the point A and the key K_2 to the point B (as shown by dotted line in the figure). The unknown resistance X is connected between C and D, the battery between C and the key K_1 and the galvanometer between D and the key K_2. The circuit shown in Fig. 2.120 (a) is exactly the same as that of the Wheatstone bridge shown in figure. Hence, the value of the unknown resistance is given by

$$X = R\left(\frac{R_2}{R_1}\right)$$

Note that:

1. The accuracy of the post office box depends on the choice at ratio arm $\frac{R_2}{R_1}$.

2. If $R_2:R_1$ is 1 : 1, then the value of the unknown resistance is obtained within ±1Ω.

3. If the ratio $R_2:R_1$ is selected as 1 : 10, then the unknown resistance $X = R\left(\frac{1}{10}\right)$ is accurately measured upto ±0.1Ω.

4. If the ratio $R_2:R_1$ is adjusted to 1 : 100, then the value of unknown resistance $X = R\left(\frac{1}{100}\right)$ is obtained to an accuracy of ±0.01Ω.

Illustration:

The value of an unknown resistance is obtained by using a post office box. Two consecutive readings of R are observed at which the galvanometer deflects in the opposite directions for three different value of R_1. These two values are recorded under the column-I and II in the following observation table.

S.N.	$R_1(\Omega)$	$R_2(\Omega)$	R lies in-between		$X = R(R_2/R_1)$	
			I (Ω)	II (Ω)	I (Ω)	II (Ω)
1	10	10	16	17		
2	100	10	163	164		
3	1000	10	1638	1639		

Determine the value of the unknown resistance.

Sol. The observation table may be completes as follows:

S.N.	$R_1(\Omega)$	$R_2(\Omega)$	R lies in-between		$X = R(R_2/R_1)$	
			I (Ω)	II (Ω)	I (Ω)	II (Ω)
1	10	10	16	17	16.0	17.0
2	100	10	163	164	16.3	16.4
3	1000	10	1638	1639	16.38	16.39

The value of the unknown resistance lies in-between 16.38 Ω and 16.39 Ω.

The unknown value may be the average of the two

i.e., $$X = \frac{16.38 + 16.39}{2}$$

or $$X = 16.385\Omega$$

Example 73

An ammeter is connected to measure the current intensity in a circuit with a resistance R. What relative error will be made if connection of the ammeter does not change the current intensity in the circuit? The voltage across the ends of the circuit is kept constant.

Solution: Before the ammeter is connected, $I_0 = \frac{V}{R}$, and after it is connected, $I = \frac{V}{R + R_0}$, where R_0 is the resistance of the ammeter. The error is

$$\varepsilon = \frac{I_0 - I}{I_0} = \frac{1}{1 + \frac{R}{R_0}}$$

When $R_0 << R$, the error may be neglected.

Example 74

Determine the voltage across a resistance R using a voltmeter connected to its ends. What relative error will be made if the readings of the voltmeter are taken as the voltage applied before it was switched on? The current intensity in the circuit is constant.

Solution: The error sought is $\varepsilon = \frac{V_0 - V}{V_0}$, where V_0 is the voltage across the resistance R before the voltmeter is switched on and V the voltage after it is switched on.

According to Ohm's law,

$$V_0 = IR \quad \text{and} \quad V = I\frac{RR_0}{R + R_0}$$

where R_0 is the resistance of the voltmeter. Hence,

$$\varepsilon = \frac{\frac{R}{R_0}}{1 + \frac{R}{R_0}}$$

is determined only by the ratio between the resistances of the section of the circuit and the voltmeter. When $R_0 >> R$, the error may be neglected.

Example 75

What resistance r should be used to shunt a galvanometer with an internal resistance $R = 10{,}000$ ohm to reduce its sensitivity $n = 50$ times?

Solution: A reduction in the sensitivity n times means that the galvanometer carries a current I_1 which is n times smaller than the current in the rest of the circuit before the branching off. Therefore, the current I_2 through the shunt is $\frac{n-1}{n}$ of the current I in the rest of the circuit. Hence,

$$\frac{r}{R} = \frac{I_1}{I_2} = \frac{1}{n-1}$$

Therefore, $$r = \frac{R}{n-1} \cong 204\Omega.$$

Example 76

A galvanometer (coil resistance 99Ω) is converted into an ammeter using a shunt of 1Ω and connected as shown in Fig. 2E.76 (a). The ammeter reads 3 A. The same galvanometer is converted into a voltmeter by connecting a resistance of 101Ω in series. This voltmeter is connected as shown in Fig. 2E.76 (b). Its reading is found to be 4/5 of the full scale reading. Find:

(i) internal resistance of the cell (r).

(ii) range of the ammeter and voltmeter.

(iii) full scale deflection current of the galvanometer.

Solution: For ammeter

$$99I_g = (I - I_g)1$$

or $$I = 100I_g \quad \ldots(1)$$

I_g is the full scale deflection current of the galvanometer and I is the range of ammeter.

For the circuit in Fig. 2E.76 (a),

$$\frac{12V}{2 + r + \frac{99 \times 1}{99 + 1}} = 3\text{A}$$

$$\Rightarrow \quad r = 1.01\Omega.$$

For voltmeter range,

$$V = I_g(99 + 101)$$

$$V = 200I_g \quad \ldots(2)$$

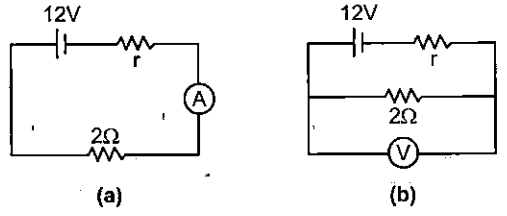


Fig. 2E.76

Also resistance of the voltmeter = 99 + 101 = 200Ω.

In Fig. 2E.76 (b), resistance across the terminals of the battery,

$$R_1 = r + \frac{200 \times 2}{202} = 2.99\Omega$$

∴ Current drawn from the battery.

$$I_1 = \frac{12}{2.99} = 4.01\text{ A}$$

∴ Voltmeter reading,

$$\frac{4}{5}\text{V} = 12 - I_1 r = 12 - 4.01 \times 1.01$$

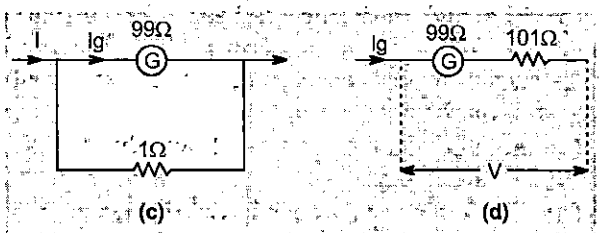


Fig. 2E.76

$$V = 7.96 \times \frac{5}{4} = 9.95\,\text{V}$$

Using eqn. (2), $I_g = \frac{9.95}{200} = 0.05\,\text{A}$

Using eqn. (1), range of the ammeter

$$I = 100 I_g = 5\,\text{A}.$$

Example 77

A galvanometer has an internal resistance of 50Ω and current required for full scale deflection is 1 mA. Find the series resistances required (as shown in Fig. 2E.77) to use it as a voltmeter with different ranges, as indicated in Fig. 2E.77.

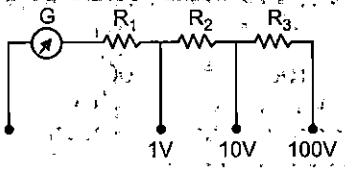


Fig. 2E.77

Solution: For range 1 volt, galvanometer and R_1 are in series

$$I_g = \frac{V}{(G + R_1)}$$

or $$10^{-3} = \frac{1}{(50 + R_1)}$$

or $$50 + R_1 = 1000$$

$$R_1 = 1000 - 50 = 950\Omega$$

For range 10 volt, galvanometer and R_2, R_3 are in series,

$$10^{-3} = \frac{10}{(G + R_1 + R_2)}$$

$$\therefore \quad G + R_1 + R_2 = \frac{10}{10^{-3}} = 10 \times 10^3$$

or $$R_2 = 10000 - (50 + 950) = 9000 = 9\text{k}\Omega$$

and for range 100 V, galvanometer, R_1, R_2 and R_3 are in series.

$$10^{-3} = \frac{100}{(G + R_1 + R_2 + R_3)}$$

$$\therefore \quad G + R_1 + R_2 + R_3 = \frac{100}{10^{-3}} = 100 \times 10^3$$

or $$R_3 = 100 \times 10^3 - (G + R_1 + R_2)$$

$$= 100 \times 10^3 - 10 \times 10^3$$

$$= 90 \times 10^3 = 90\text{k}\Omega$$

Example 78

The galvanometer shown in Fig. 2E.78 (a) has resistance 50Ω and current required for full scale deflection is 1 mA. Find the resistance R_1, R_2 and R_3 required to convert it into ammeter having ranges as indicated.

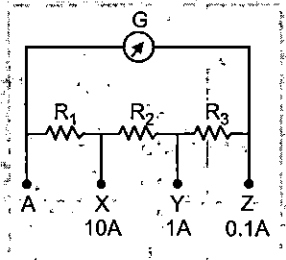


Fig. 2E.78 (a)

Solution: For the range 0.1 A, R_1, R_2 and R_3 are in series combination, equating potential difference across galvanometer and series combination of R_1, R_2 and R_3, we get

$$I_g G = (I - I_g)(R_1 + R_2 + R_3)$$

Hence

$$R_1 + R_2 + R_3 = \frac{I_g G}{(I - I_g)} = \frac{10^{-3} \times 50}{(0.1 - 10^{-3})} = \frac{50}{99}\,\text{Ohm} \quad \ldots(1)$$

For range 1 A, $(R_1 + R_2)$ is in parallel to $(G + R_3)$.

So, $$I_g (G + R_3) = (I - I_g)(R_1 + R_2)$$

or $$I_g (G + R_1 + R_2 + R_3) = I(R_1 + R_2)$$

Substituting for I_g, G, I and $(R_1 + R_2 + R_3)$, we have

$$R_1 + R_2 = 10^{-3}\left(\frac{50 + \frac{50}{99}}{1}\right) = \frac{5}{99}\,\text{Ohm} \quad \ldots(2)$$

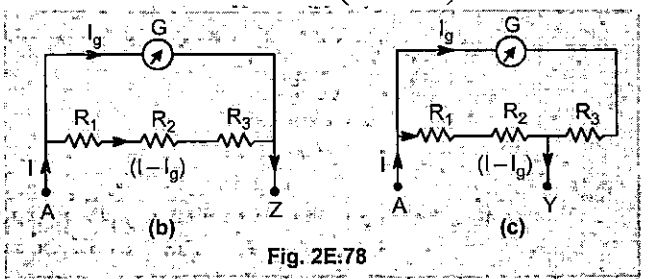


Fig. 2E.78

For range 10 A, R_1 is in parallel to $(G + R_2 + R_3)$.

So, $$I_g (G + R_2 + R_3) = (I - I_g) R_1$$

$$\therefore \quad R_1 = \frac{I_g (G + R_1 + R_2 + R_3)}{I}$$

$$= \frac{10^{-3}\left(50 + \frac{50}{99}\right)}{10} = \frac{5}{990}\,\text{Ohm} = \frac{1}{198}\,\text{Ohm} \quad \ldots(3)$$

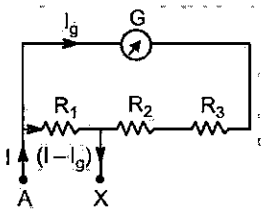


Fig. 2E.78 (d)

So, $R_2 = \frac{5}{99} - \frac{5}{990} = \frac{1}{22}$ Ohm

and $R_3 = \frac{50}{99} - (R_1 + R_2)$

$= \frac{50}{99} - \frac{5}{99} = \frac{45}{99} = \frac{15}{33}$ Ohm

Hence, $R_1 = \frac{1}{198}$ Ohm, $R_2 = \frac{1}{22}$ Ohm

and $R_3 = \frac{15}{33}$ Ohm

Example 79

Fig. 2E.79(a) shows a potentiometer using a cell C of emf 2.0 V and internal resistance 0.40Ω connected to a resistor wire AB. A standard cell of constant emf of 1.02 V gives a balance point at 67.3 cm length of the wire. A very high resistance $R = 600\,k\Omega$ is put in series with the standard cell. This resistance is shorted by inserting switch S when close to the balance point. The standard cell is then replaced by a cell of unknown emf $\mathcal{E}$ and the null point turns out to be 82.3 cm length of the wire.

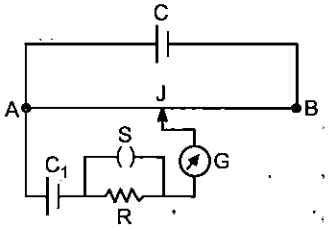


Fig. 2E.79 (a)

(a) What is the value of $\mathcal{E}$? (b) What is the purpose of using the high resistance R? (c) Is the null point affected by this high resistance? (d) Is the null point affected by the internal resistance of the cell C? (e) Would this method work if :

(i) the internal resistance of cell C were higher than the resistance of wire AB and

(ii) the emf of cell C were 1.0 V instead of 2.0 V? (f) Would the circuit work well for determining extremely small emf of the order of a few millivolts? What modification do you suggest in the circuit.

Solution: (a) The value of $\mathcal{E}$ is given by ($\mathcal{E}_1 = 1.02$ V is the emf of the standard cell C_1)

$$\mathcal{E} = \frac{1.02 \times 82.3}{67.3} = 1.25\,\text{V}$$

(b) The high resistance R keeps the current drawn from the standard cell within permissible limit and prevents a large current to flow through the galvanometer when far away from the balance point.

(c) At null point no current flows through R, hence no effect on null point.

(d) The null point depends on the terminal voltage of C and the emf of C_1 only, hence no effect.

(e) The method would not work if the potential difference across AB due to the driver cell C became less than the emf of cell C_1, because there would be no null point on the wire. (i) In this case the potential difference across AB due to cell C of emf 2.0 V will be less than 1.0 V. Since the emf of the standard cell is greater than this value, there will be no null point on the wire. (ii) Similarly, then method would not work if the emf of C were 1.0 V instead of 2.0 V.

(f) Suppose the emf $\mathcal{E}$ to be measured is say, 1 mV $= 1 \times 10^{-3}$ V, the potential difference across AB due to C is 2.0 V, the length AB of the wire is 100 cm and its resistance per centimeter is k. Then the balance length will be

1×10^{-3} V = P.D. across AJ due to C

= current in the main circuit × resistance of AJ

$$= \frac{2.0}{100k} \cdot kAJ = \frac{2.0AJ}{100}\,\text{V}$$

or $AJ = 0.05$ cm

Thus the null point will be very close to A and there is an extremely large percentage error in its measurement. To have a large balance length, the circuit shown in Fig. 2E.79 (b) is modified by putting a suitable resistor R'. The balance length will then be measurable and the percentage error will be much smaller.

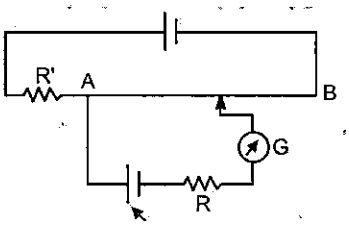


Fig. 2E.79 (b)

Example 80

A thin uniform wire AB of length 1 m, an unknown resistance X and a resistance of 12Ω are connected by thick conducting strips, as shown in the figure. A battery and a galvanometer (with a sliding jockey connected to it) are also available. Connections are to be made to measure the unknown resistance X using the principle of Wheatstone bridge. Answer the following questions:

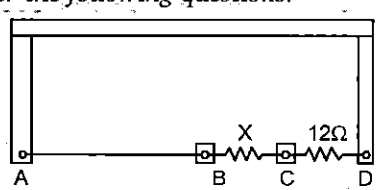


Fig. 2E.80 (a)

(a) Are there positive and negative terminals on the galvanometer?
(b) Copy the figure in your answer book and show the battery and the galvanometer (with jockey) connected at appropriate points.
(c) After appropriate connections are made, it is found that no deflection takes place in the galvanometer when the sliding jockey touches the wire at a distance of 60 cm from A. Obtain the value of the resistance X.

Solution: (a) There are no positive and negative terminals on the galvanometer because it detects current, and only zero deflection is required.

(b)

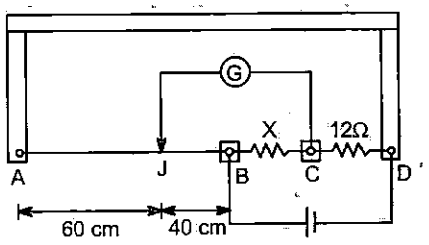


Fig. 2E.80 (b)

(c) From the figure, $AJ = 60$ cm, $BJ = 40$ cm.

The given circuit is a Wheatstone bridge, in no deflection mode it is balanced.

$$\frac{X}{12} = \frac{R_{BJ}}{R_{AJ}}$$

or $$\frac{X}{12} = \frac{40}{60} = \frac{2}{3}$$

or $$X = 8\Omega$$

Example 81

Fig. 2E.81 shows a potentiometer circuit for determining the internal resistance of a cell. When switch S is open, the balance point is found to be at 76.3 cm of the wire. When switch S is closed and the value of R is 4.0Ω, the balance point shifts to 60.0 cm. Find the internal resistance of cell C'.

Solution: Let $\mathcal{E}$ be the emf of the cell C' and r its internal resistance. Let $l = AJ$ be the balance length when switch S is open. When a resistance R is introduced by closing the switch a current begins to flow through the cell C' and resistance R. The potential difference between the terminals of the cell falls and the balance length decreases to $l' = AJ'$. The terminal resistance of the cell is given by

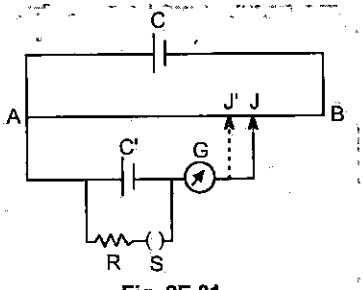


Fig. 2E.81

$$r = \frac{E - V}{I}$$

where V is the terminal voltage of C' and I is the current in the circuit involving C' and R. Also $I = V/R$. Hence

$$r = \left(\frac{E}{V} - 1\right)R$$

But $E/V = l/l'$. Hence

$$r = R\left(\frac{l - l'}{l'}\right)$$

$$= 4.0 \times \left(\frac{76.3 \times 60.0}{60.0}\right) = 1.1\Omega$$

Example 82

Fig. 2E.82 (a) shows a metre bridge consisting of two resistances X and Y together in parallel with a metre-long constantan wire AC of uniform cross-section. D is a movable contact that can slide along the wire AC.
The resistors X, Y and resistances of segments AD and DC of the wire constitute the four arms of the bridge. The length of wire AC is 100 cm. X is a standard 4.00Ω resistor and Y is a coil of wire. With Y immersed in melting ice the null point is found to be at a distance of 40.0 m from point A. When the coil Y is heated to 100°C, a 100Ω resistor has to be connected in parallel with Y in order to keep the bridge balanced at the same point. Calculate the temperature coefficient of resistance of the coil.

Solution: The resistances of the two segments of the wire AD and DC are in the ratio of their lengths. If R_0 is the resistance of Y in melting ice (0°C), the balance condition of Wheatstone bridge gives

$$\frac{X}{R_0} = \frac{kl}{k(100 - l)} = \frac{l}{100 - l}$$

where k is the resistance per centimetre of wire AC. Now, $l = 40.0$ cm and $X = 4.00\Omega$. Substituting these values, we get $R_0 = 6.00\Omega$. Let R_t be the resistance of Y when heated to a temperature $t = 100°C$. When it is connected in parallel with 100 Ω resistor as shown in Fig. 2E.82(b), the net resistance becomes

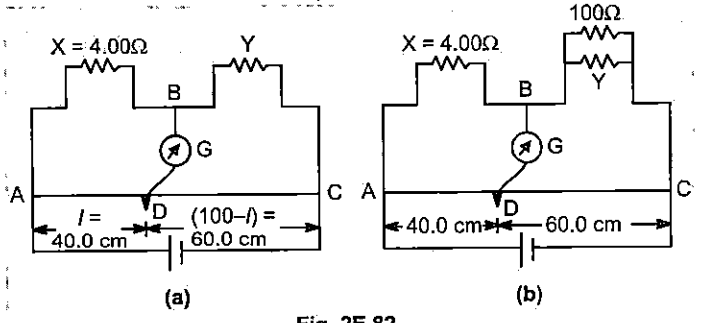


Fig. 2E.82

$$R' = \frac{100R_t}{R_t + 100}$$

Since the null point remains uncharged, we have

$$\frac{X}{R'} = \frac{40.0}{60.0}$$

$$\Rightarrow \quad R' = 6.00\Omega$$

Thus $$6.00 = \frac{100R_t}{R_t + 100}$$

$$\Rightarrow \quad R_t = 6.38\Omega$$

Temperature coefficient of resistance of the coil Y is

$$\alpha = \frac{R_t - R_0}{R_0 t}$$

$$= \frac{6.38 - 6.00}{6.00 \times 100}$$

$$= 6.3 \times 10^{-4}\,K^{-1}$$

Example 83

The wire AB of a meter bridge changes linearly from radius r to 2r from left end to right end. Where should the free end of the galvanometer be connected on AB so that the deflection in the galvanometer is zero?

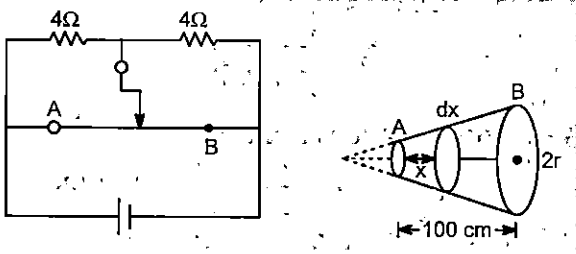


Fig. 2E.83

Solution: Let the galvanometer be connected at a point $x = x_1$ from end A where $x = 0$.

Let R_1 = resistance of left part, *i.e.*, AX_1 and

R_2 = resistance of right part, *i.e.*, X_1B

Length = 100 cm = 1 m.

Consider an element of thickness dx at a distance x from end A and of radius r_x.

Thus, $$r_x = \left(r + \frac{r}{1}x\right) = r(1 + x)$$

Resistance of this element will be;

$$dR_x = \frac{\rho\, dx}{\pi r_x^2}$$

$$R_1 = \int_0^{1} \frac{\rho\, dx}{\pi(1+x)^2 r^2} = \frac{\rho}{\pi r^2}\left[1 - \frac{1}{1+x_1}\right]$$

$$R_2 = \int_1 \frac{\rho\, dx}{(1+x)^2 r^2}$$

$$= \frac{\rho}{\pi r^2}\left[\frac{1}{1+x_1} - \frac{1}{1+1}\right]$$

For null point of zero deflection,

$$\frac{R_1}{R_2} = \frac{4}{4}$$

$$\Rightarrow \quad 1 - \frac{1}{1+x_1} = \frac{1}{1+x_1} - \frac{1}{1+1}$$

$$x_1 = \frac{1}{3}\text{m} = 33.33\,\text{cm}.$$

Example 84

In the simple potentiometer circuit, where the length AB of the potentiometer wire is 1m, the resistors X and Y have values 5Ω and 2Ω respectively. When X is shunted by a wire, the balance point is found to be 0.625 m from A. What is the resistance of the shunt? If the shunt wire is 0.75 m long and 0.25 mm in diameter, what is the resistivity of the material of the wire?

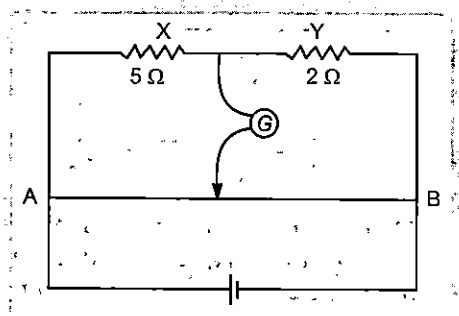


Fig. 2E.84

Solution: Let R be the resistance of the shunted wire, the effective resistance of R and 5Ω in parallel = $5 \times R/(5 + R)$

At balance point,

$$\frac{5R/(5+R)}{2} = \frac{0.625}{1 - 0.625}$$

$$= \frac{0.625}{0.0375} = \frac{5}{3}$$

On solving we get,

$$R = 10\Omega$$

Now, $$\rho = \frac{Ra}{l} = \frac{R\pi r^2}{l}$$

$$= \frac{10 \times (22/7) \times (0.125 \times 10^{-3})^2}{0.75}$$

$$= 6.54 \times 10^{-7}\,\Omega\text{m}$$

Example 85

You are given two resistors X and Y whose resistances are to be determined using an ammeter of resistance 0.5 Ω and a voltmeter of resistance 20 k Ω. It is known that X is in the range of a few ohms, while Y is in the range of several thousand ohms. In each case, which of the following two connections (Fig. 2E.85) would you choose for resistance measurement? Justify your answer quantitatively.

Solution: For each connection, determine the error in resistance measurement. The connection that corresponds to a smaller error (for a given range of resistance) is to be preferred.

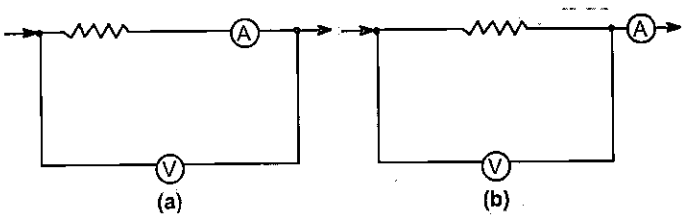


Fig. 2E.85

For X, use (b); for Y use (b).

Voltage drop across resistance and ammeter will be in the ratio of their resistances.

Arrangement (a) is preferred for Y, whose resistance is large as compared to ammeter resistance.

Arrangement (b) is preferred for X, whose resistance is not too large as compared to ammeter resistance. Voltage drop across the ammeter will be an appreciable fraction of that across the resistance and must be excluded.

Example 86

A cell of emf. 3.4 V and internal resistance 3 Ω is connected to an ammeter having resistance 2 Ω and to an external resistance of 100 Ω resistance, the ammeter reading is 0.04 A. Find the voltage read by the voltmeter and its resistance. Had the voltmeter been an ideal one, what would have been its reading.

Solution: Let R_V be resistance of the voltmeter. The equivalent resistance of voltmeter and 100 V resistance

$$R' = \frac{100R_V}{100 + R_V}$$

Net resistance of circuit, $R_{net} = R' = R_A + r$

$$= R' + 3 + 2 = \frac{100R_V}{10 + R_V} + 5$$

Current in circuit, $I = \dfrac{E}{R_{net}} = \dfrac{3.4}{\dfrac{100R_V}{100 + R_V} + 5}$

But $I = 0.04$ A,

$$\therefore \quad 0.04 = \frac{3.4}{\dfrac{100R_V}{100 + R_V} + 5}$$

Solving we get, $R_V = 400\Omega$

Reading of voltmeter, $V = IR' = 0.04 \times \dfrac{100R_V}{100 + R_V}$

$$= 0.04 \times \frac{100 \times 400}{100 + 400} = 0.04 \times 80 = 3.2 \text{ V}$$

Ideal voltmeter has infinite resistance. In that case net resistance of circuit,

$$R'_{net} = 100 + 3 + 2 = 105\Omega$$

$$\therefore \quad \text{Current } I' = \frac{E}{R_{net}} = \frac{3.4}{105} \text{A}$$

New voltmeter reading $V' = I' \times R = \dfrac{3.4}{105} \times 100 = 3.24$ V

Example 87

Potentiometer wire PQ of 1 m length is connected to a standard cell $\mathcal{E}_1$. Another cell, $\mathcal{E}_2$ of e.m.f. 1.02 V is connected with a resistance r and a switch S as shown in the circuit diagram. With switch S open null position is obtained at a distance of 51 cm from P. Calculate:

(i) potential gradient of the potentiometer wire.

(ii) e.m.f. of cell $\mathcal{E}_1$.

(iii) when switch S is closed, will null point move towards P or towards Q? Give reason for your answer?

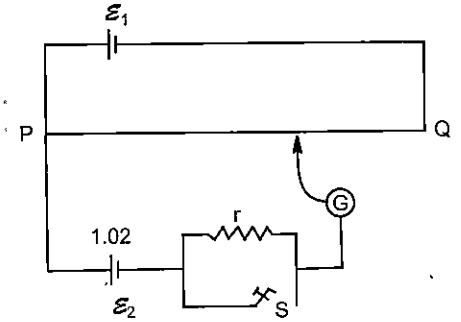


Fig. 2E.87

Solution: (i) Potential gradient,

$$k = \frac{V}{l} = \frac{1.02}{51} = 0.02 \text{ V/m}$$

(ii) The e.m.f. of cell $\mathcal{E}_1 = k \times 100 = 0.02 \times 100 = 2$ V

(iii) When switch S is closed, there is no shift in the position of null point as the position of null point depends upon the potential gradient along the potentiometer wire (which depends upon the e.m.f. of battery $\mathcal{E}_1$ and resistance of potentiometer wire circuit and length of potentiometer) and e.m.f. of the cell $\mathcal{E}_2$ which does not change when switch S is closed.

Example 88

In Fig. 2E.88, AB is a 1 m long uniform wire of 10 Ω resistance. Other data are shown in the diagram. Calculate: (a) potential gradient along AB, (b) length AO when galvanometer shows no deflection.

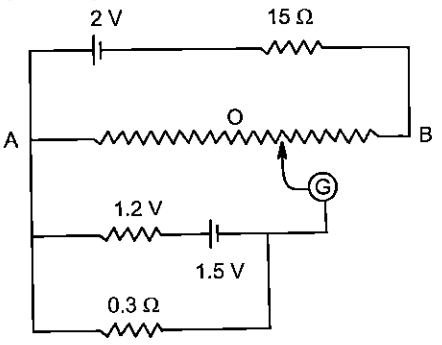


Fig. 2E.88

Solution: (a) Potential gradient along

$$AB = \left(\frac{2}{15+10}\right)\frac{10}{100} = 0.008 \text{ V cm}^{-1}$$

(b) Current through $0.3\ \Omega = \dfrac{1.5}{1.2+0.3} = 1$ A

Potential difference across $0.3\ \Omega = 1 \times 0.3 = 0.3$ V

Let l be the length AO then $0.3 = 0.008 \times l$

or $$l = \frac{0.3}{0.008} = 37.5 \text{ cm}$$

Example 89

Cells A and B and a galvanometer G are connected to a slide wire OS by two sliding contacts C and D as shown in Fig. 2E.89. The slide wire is 100 cm long and has a resistance of 12 Ω. With OD = 75 cm, the galvanometer gives no deflection when OC is 50 cm. If D is moved to touch the end of wire S, the value of OC for which the galvanometer shows no deflection is 62.5 cm. The e.m.f. of cell B is 1.0 V.

Calculate:

(i) The potential difference across O and D when D is at 75 cm mark from O.

(ii) The potential difference across OS when D touches S.

(iii) Internal resistance of cell A.

(iv) The e.m.f. of cell A.

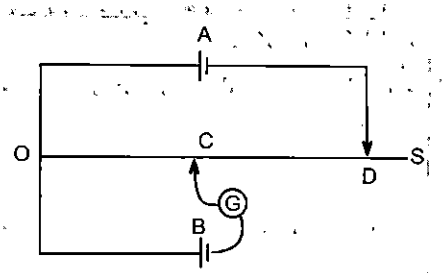


Fig. 2E.89

Solution: Resistance of wire $OD = \dfrac{12}{100} \times 75 = 9\Omega$. Let $\mathcal{E}$ and r be the e.m.f. and internal resistance of cell E.

(i) Potential gradient of wire = 1/50 V/cm. Therefore voltage drop across the wire OD of length 75 cm

$$= (1/50) \times 75 = 1.5 \text{ V}$$

(ii) Potential gradient of wire = 1/62.5 V/cm. Therefore voltage drop across the wire OS of length 100 cm = (1/62.5) × 100 = 1.6 V

(iii) $$\left(\frac{\mathcal{E}}{9+r}\right) \times 9 = 1.5 \quad \ldots(i)$$

$$\left(\frac{\mathcal{E}}{12+r}\right) \times 12 = 1.6 \quad \ldots(ii)$$

(iv) On solving eqn. (i) and (ii), we get $r = 3\Omega$ and $\mathcal{E} = 2$V

Example 90

In the given circuit, a meter bridge is shown in a balanced state. The bridge wire has a resistance of 1 Ω/cm. Find the value of the unknown resistance X and the current drawn from the battery of negligible internal resistance.

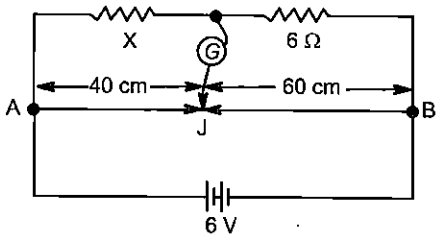


Fig. 2E.90

Solution: For the balanced bridge, the ratio of the two resistances is equal to the ratio of the lengths of the two parts AJ and JB of the wire *i.e.*,

$$\frac{X}{6\Omega} = \frac{40 \text{ cm}}{60 \text{ cm}} \text{ or } X = 4\Omega$$

No current flows through the galvanometer G, the resistance of the parts AJ and JB are 40 Ω and 60 Ω, respectively. If R be the equivalent resistance between the points A and B, then we have

$$\frac{1}{R} = \frac{1}{(X+6)\Omega} + \frac{1}{(40+60)\Omega}$$

or $$R = \frac{100}{11}\Omega,$$

$$I = \frac{V}{R} = \frac{6V}{(100/11)\Omega} = 0.66\text{A}$$

Example 91

A voltmeter reads 5.0 V at full scale deflection and is graded according to its resistance per volt at full scale deflection as 5000 Ω/V. How will you convert it into a voltmeter that reads 20 V at full scale deflection? Will it still be graded as 5000 Ω/V? Will you prefer this voltmeter to one that is graded as 2000 Ω/V?

Solution: Resistance per volt at full scale deflection = 5000 Ω V^{-1}

Reading of voltmeter at full scale deflection = 5 V

∴ Resistance of voltmeter $G = 5000 \times 5 = 25000\Omega$

Also current for maximum deflection,

$$I_g = \frac{1\text{V}}{5000\Omega} = 0.0002\text{A}$$

Range of voltmeter to be changed to $V = 20$ V

Now,

$$R = \frac{V}{I_g} - G = \frac{20}{0.0002} - 25000$$

$$= 100000 - 25000 = 75000\Omega$$

Thus, 75000Ω resistor is to be connected in series.

Resistance of 20 V voltmeter

$$= 75000 + 25000 = 1{,}00{,}000\Omega$$

Its grading becomes $= \frac{100000}{20} = 5000\,\Omega\,V^{-1}$ which is same as in the earlier case. A voltmeter with grading $2000\,\Omega\,V^{-1}$ will have less resistance and is therefore not preferred.

Example 92

A battery of e.m.f. 1.4 V and internal resistance 2 Ω is connected to a resistor of 100 Ω resistance through an ammeter. The resistance of the ammeter is 4/3 Ω. A voltmeter has also been connected to find the potential difference across the resistor.

(a) Draw the circuit diagram.

(b) The ammeter reads 0.02 A. What is the resistance of the voltmeter?

(c) The voltmeter reads 1.1 V. What is the error in the reading?

Solution: (a) The circuit diagram is shown in Fig. 2E.92

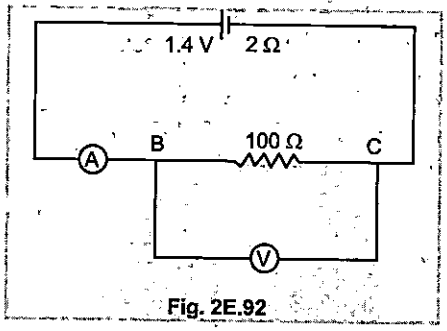


Fig. 2E.92

(b) Let resistance of the voltmeter be R ohm. The equivalent resistance of voltmeter (R ohm) and 100 Ω in parallel is

$$\frac{100\times R}{100+R} = \frac{100R}{100+R}$$

Resistance of the ammeter $= \frac{4}{3}\Omega$.

Total resistance of the circuit $= \frac{100R}{100+R} + \frac{4}{3} + 2\Omega$

Current in the circuit as read by the ammeter = 0.02 A

Now, $$0.002 = \frac{1.4}{\frac{100R}{100+R} + \frac{4}{3} + 2} \quad \left(\because I = \frac{V}{R}\right)$$

or $$R = 200\Omega$$

∴ Resistance of the voltmeter $= 200\,\Omega$

(c) Effective resistance between B and C

$$= \frac{100\times 200}{100+200} = \frac{200}{3}\Omega$$

The potential drop across this resistance

$$= \text{circuit current} \times \frac{200}{3} = 0.02\times\frac{200}{3} = \frac{4}{3}V = 1.333V$$

Reading of the voltmeter = 1.1 V

Error in the reading of the voltmeter

$$= 1.1 - 1.333 = -0.233\ V$$

Example 93

An experiment with a post office box, the ratio of arms are 1000 : 10. If the value of the third resistance is 999 Ω, find the unknown resistance.

Solution: In the given case, the ratio arms are 1000 : 10

$$\frac{P}{Q} = \frac{1000}{10} = 100$$

Third resistance, $R = 999V$

Let X be the unknown resistance. Then,

$$\frac{P}{Q} = \frac{R}{X} \quad \text{or} \quad \frac{Q}{P}\times R$$

$$= \frac{1}{100}\times 999 = 9.99\Omega$$

Example 94

A potentiometer wire has a length of 10 cm and resistance 4 Ω. An accumulator of e.m.f. 2 V and a resistance box are connected in series with it. Calculate the resistance to be introduced in the box so as to get a potential gradient of (a) 0.1 V/m and (b) 0.1 mV/m.

Solution: Current in the potentiometer wire is given by

$$I = \frac{\mathcal{E}}{R + l\times\rho}$$

where ρ is the resistance per unit length of the wire and l is the length of the wire.

Now, potential gradient

$$V = I\rho = \frac{\mathcal{E}\rho}{R + l\times\rho}$$

Here, $l = 10\,m, \rho = 4\Omega/m$

(a) For $V = 0.1\,V/m$, we have

$$0.1 = \frac{2\times 4}{R + 10\times 4} = \frac{8}{R+40}$$

or $$R = \frac{4}{0.1} = 40\Omega$$

(b) For $V = 0.1$ mV/m $= 0.1\times 10^{-3}$ V/m,

We have

$$0.1\times 10^{-3} = \frac{2\times 4}{R + 10\times 4}$$

or $$10^{-4} = \frac{8}{R+40}$$

or $$R = 79960\Omega.$$

Note: There is no current through the cell and galvanometer, battery $\mathcal{E}$, internal resistance r and potentiometer wire AB are in series.

Example 95

Fig. 2E.95 shows a 2.0 V potentiometer used for the determination of internal resistance of a 1.5 V cell. The balance point of the cell in open circuit is 76.3 cm. When a resistor of 9.5 Ω is used in the external circuit of the cell, the balance point shifts to 64.8 cm length of the potentiometer wire. Determine the internal resistance of the cell.

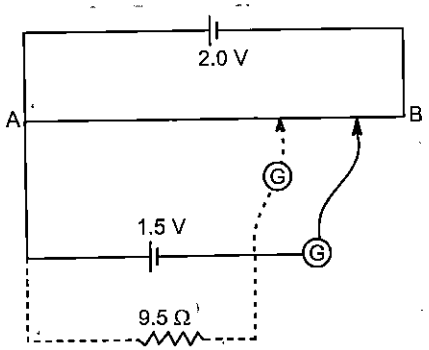


Fig. 2E.95

Solution: Internal resistance of a cell using potentiometer is given by,

$$r = R \times \frac{l_1 - l_2}{l_2}$$

Here $R = 9.5\Omega$, $l_1 = 76.3$ cm, $l_2 = 64.8$ cm

Hence $r = 9.5 \times \dfrac{76.3 - 64.8}{64.8} = 9.5 \times \dfrac{11.5}{64.8} = 1.7\Omega$

Example 96

A galvanometer has a resistance of 50 Ω and its full scale deflection current is 50 μ A. What resistance should be added to it so that it can have a range of 0 – 5 V?

Solution: Here, the maximum value of $I_G = 10\mu$ A. The upper limit gives the maximum voltage to be measured which is $V = 5$V. The galvanometer resistance, $G = 50\Omega$.

From the above relation, $R_h = 100\,\text{k}\Omega$

If we work out, we would understand that higher the range of voltmeter, higher is the value of shunt resistance.

Example 97

What is the value of shunt which passes 10% of the main current through a galvanometer of 99 Ω?

Solution. A shunt is a small resistance, S, in parallel with a galvanometer (of resistance G) as shown in Fig. 2E.97

$$(I - I_G)S = I_G \times G,$$

$$i.e., \quad S = \frac{I_G G}{(I - I_G)}$$

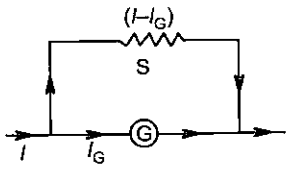


Fig. 2E.97

And as here, $G = 99\Omega$

and $I_G = (10/100)I = 0.1I$

$$S = \frac{0.1I \times 99}{(I - 0.1I)} = \frac{0.1}{0.9} \times 99 = 11\Omega$$

Example 98

A galvanometer has a resistance of 30 Ω and a current of 2 mA is needed for a given full scale deflection. What is the resistance and how is it to be connected to convert the galvanometer: (i) into an ammeter of 0.3 A range, (ii) into a voltmeter of 0.2 V range?

Solution: As here galvanometer resistance $G = 30\Omega$ and full scale deflection current $I_g = 2\,\text{m}$A, so,

(i) To convert the galvanometer into an ammeter of range 0.3 A, a resistance of value S is connected in parallel with it such that

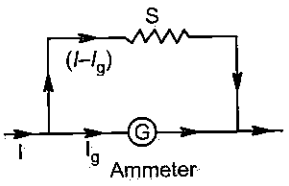


Fig. 2E.98 (a)

$$(I - I_g)S = I_g G$$

$$(0.3 - 0.002)S = 0.002 \times 3,$$

or, $$S = \frac{0.002 \times 30}{0.298}$$

$$= 0.2013\ \Omega$$

(ii) To convert the galvanometer into a voltmeter of range 0.2 V, a resistance R is connected in series with it such that

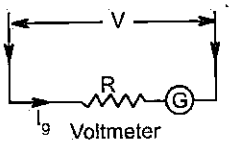


Fig. 2E.98 (b)

$$V = I_g (R + G),$$

$$i.e., \quad 0.2 = 2 \times 10^{-3}(30 + R)$$

$$i.e., \quad R = 100 - 30 = 70\Omega$$

Example 99

The scale of a galvanometer is divided into 150 equal divisions. The galvanometer has current sensitivity of 10 divisions per mA and a voltage sensitivity of 2 divisions per mV. How can the galvanometer be designed to read (i) 6 A, per division and (ii) 1 V, per division?

Solution: As per the resistance of galvanometer,

$$G = \frac{\text{Full scale voltage}}{\text{Full scale current}}$$

$$= \frac{75 \times 10^{-3}}{15 \times 10^{-3}} = 5\Omega$$

For conversion into ammeter of range I A.

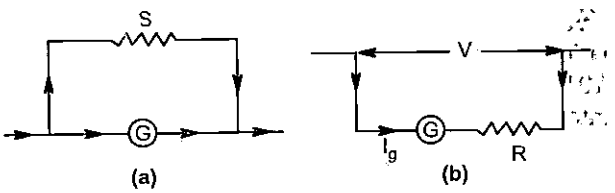


Fig. 2E.99

$$(I-I_g)S = I_gG$$

$$S = \frac{I_gG}{I-I_g}$$

$$= \frac{15\times10^{-3}\times5}{(150\times6-15\times10^{-3})}$$

$$= \frac{15\times10^{-3}\times5}{150\times6}$$

$$= 8.3\times10^{-5}\Omega$$

For conversion into voltmeter of range V volt,

$$I_g(G+R) = V$$

Example 100

The deflection is a moving coil galvanometer falls from 50 divisions to 10 divisions when a shunt of 12 ohm is applied. What is the resistance of the galvanometer?

Solution: In case of a galvanometer, $I \propto \theta$

So, $\frac{I_G}{I} = \frac{10}{50} = \frac{1}{5}$,

i.e., $I_G = \frac{1}{5}I$

Now in case of a shunted galvanometer as shown in Fig. 2E.100,

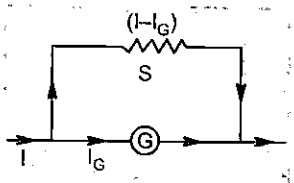


Fig. 2E.100

$$(I-I_G)S = I_GG$$

i.e., $(I-\frac{1}{5}I)\times12 = \frac{1}{5}IG$

So, $G = 4\times12 = 48\ \Omega$

Example 101

A galvanometer of resistance 95 Ω, shunted by a resistance of 5 Ω gives a deflection of 50 divisions when joined in series with a resistance of 20 Ω and a 2 V accumulator. What is the current sensitivity of the galvanometer (in div/μA)?

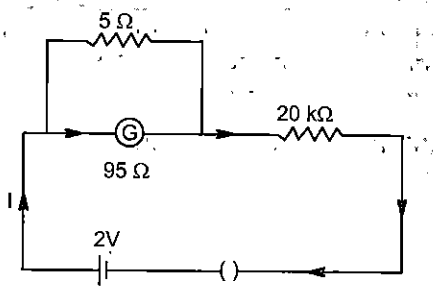


Fig. 2E.101

Solution: In accordance with the given problem, the situation is depicted by the circuit diagram in the Fig. 2E.101. As 20 k Ω is much greater than the resistance of shunted galvanometer (< 5Ω), the current in the circuit will be

$$I \simeq \frac{2}{20\times10^3} = 10^{-4}\ \text{A} = 100\mu\text{A}$$

And as this current produces a deflection of 50 divisions in the galvanometer,

$$\text{Current sensitivity} = \frac{\theta}{I} = \frac{50\text{div}}{100\mu\text{A}} = \frac{1}{2}\frac{\text{div}}{\mu\text{A}}.$$

Example 102

Consider the potentiometer circuit arranged as in the Fig. 2E.102 (a). The potentiometer wire is 600 cm long.

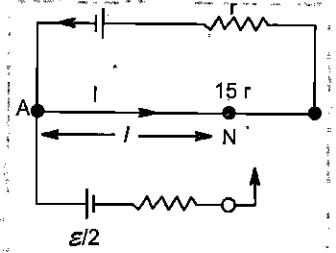


Fig. 2E.102 (a)

(i) At what distance from point A should the jockey touch the wire to get zero deflection in the galvanometer?
(ii) If the jockey touches the wire at a distance of 560 cm from A, what will be the current in the galvanometer?

Solution: (i) When the jockey is not connected.

$$I = \frac{\mathcal{E}}{16r}$$

Resistance per unit length;

$$\lambda = \frac{15r}{600}\Omega/\text{cm}$$

Let l be the length when we get zero deflection.

$$\left(\frac{\mathcal{E}}{2}\right) = (\lambda l)\frac{\mathcal{E}}{2} = \frac{\mathcal{E}}{16r}\times\frac{15r}{600}\times l$$

$$\Rightarrow \quad l = 320\ \text{cm}$$

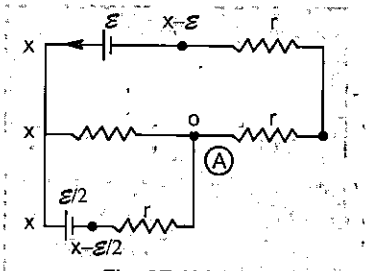


Fig. 2E.102 (b)

(ii) Let potential at A is zero
Then apply Kirchhoff's first law

$$\frac{x-0}{14r} + \frac{x-\frac{\mathcal{E}}{2}-0}{r} + \frac{(x-\mathcal{E}-0)}{2r} = 0$$

$$\Rightarrow \quad x = \frac{14\mathcal{E}}{22}$$

$$I_g = \frac{x-\frac{\mathcal{E}}{2}}{r} = \frac{\left(\frac{14\mathcal{E}}{22}\right)-\frac{\mathcal{E}}{2}}{r} = \frac{3\mathcal{E}}{22r}$$

Example 103

One of the circuits for the measurement of resistance by potentiometer is shown. The galvanometer is connected at point A and zero deflection is observed at length PJ = 30 cm. In second case the secondary cell is changed.

Take $\mathcal{E}_s = 10\,V$ *and* $r = 1\Omega$ *in* 1^{st} reading

and $\mathcal{E}_s = 5\,V$ *and* $r = 2\Omega$ *in* 2^{nd} reading.

In second case, the zero deflection is observed at length PJ = 10 cm. What is the resistance R (in ohm) is?

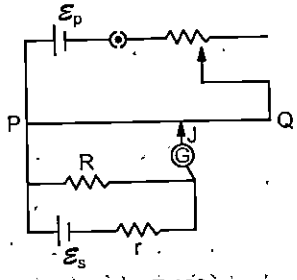


Fig. 2E.103

Solution: $\frac{V}{l} = \frac{I_1 R}{30} = \frac{I_2 R}{10}$

(1) $\mathcal{E}_{S_1} = I_1(r_1 + R)$

$\Rightarrow \quad 10 = I_1(1+R) \quad \Rightarrow I_1 = \frac{10}{1+R}$

(2) $\mathcal{E}_{S_2} = I_2(r_2 + R)$

$\Rightarrow \quad 5 = I_2(2+R)$

$\Rightarrow \quad I_2 = \frac{5}{2+R}$

$$\frac{10R}{(1+R)\times 30} = \frac{5R}{10\times(2+R)}$$

$\Rightarrow \quad 4 + 2R = 3 + 3R$

$\Rightarrow \quad R = 1\Omega$

Example 104

For the arrangement of the potentiometer shown in the Fig. 2E.104, the balance point is obtained at a distance 75 cm from A when the key k is open.

The second balance point is obtained at 60 cm from A when the key k is closed. Find the internal resistance (in Ω) of the battery $\mathcal{E}_1$.

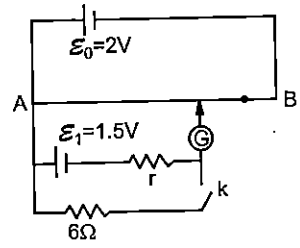


Fig. 2E.104 (a)

Solution: Let λ is resistance per unit length of wire *AB*. When *k* is opened

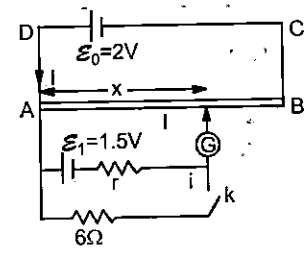


Fig. 2E.104 (b)

$$I(\lambda x_1) = \mathcal{E}_1 \qquad \ldots(1)$$

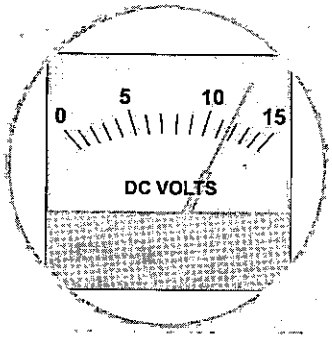
k is closed

$$I\lambda x_2 = \mathcal{E}_1 - Ir \qquad \ldots(2)$$

$$I = \frac{\mathcal{E}_1}{R+r} \qquad \ldots(3)$$

$\Rightarrow \quad r = \left(\frac{x_1}{x_2} - 1\right) R = \left(\frac{0.75}{0.60} - 1\right) 6$

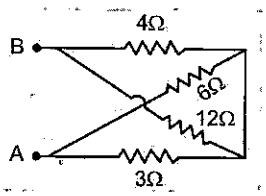
$r = 1.5\Omega$



PROBLEMS

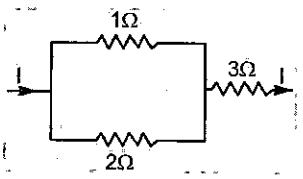
Level 1 Only One Alternative is Correct

1. In the given network, the equivalent resistance between A and B is:



(a) 6 Ω
(b) 16 Ω
(c) 7 Ω
(d) 5 Ω

2. In the circuit shown in the figure, power developed across 1 Ω, 2 Ω and 3 Ω resistance are in the ratio:



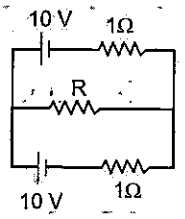
(a) 1 : 2 : 3
(b) 4 : 2 : 27
(c) 6 : 4 : 9
(d) 2 : 1 : 27

3. A galvanometer of resistance 20 Ω gives full scale deflection when a current of 0.04 A is passed through it. It is desired to convert it into an ammeter reading upto 20A. The only shunt available is 0.05 Ω resistance. The resistance that must be connected in series with the coil of the galvanometer is:
(a) 4.95 Ω (b) 5.94 Ω
(c) 9.45 Ω (d) 12.62 Ω

4. The length of the potentiometer wire is L. A cell of emf $\mathcal{E}$ is balanced at a length $\frac{L}{3}$ from the positive end of the wire. If the length of the wire is increased $\frac{L}{2}$. At what distance will the same cell give a balance point:
(a) $\frac{2L}{3}$ (b) $\frac{L}{2}$
(c) $\frac{L}{6}$ (d) $\frac{4L}{3}$

5. The terminals of a battery of emf 12 V and negligible internal resistance are connected to two coils, each of resistance 50Ω connected in series. A voltmeter of resistance 1000Ω is connected first across the coils and then across the terminals of the battery. The reading in three cases are:
(a) 5.85 V across each coil and 9.2 V across the battery
(b) 5.85 V across each coil and 12 V across the battery
(c) 3.5 V across each coil and 8 V across the battery
(d) 3.5 V across each coil and 12 V across the battery

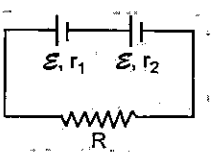
6. Maximum power developed across variable resistance R in the circuit shown in figure is:



(a) 50 watt
(b) 75 watt
(c) 25 watt
(d) 100 watt

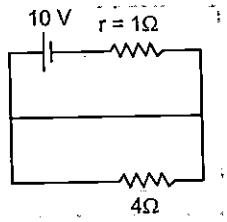
7. Two wires of same dimension but resistivities ρ_1 and ρ_2 are connected in series. The equivalent resistivity of the combination is:
(a) $\rho_1 + \rho_2$ (b) $\frac{1}{2}(\rho_1 + \rho_2)$
(c) $\sqrt{\rho_1 \rho_2}$ (d) $2(\rho_1 + \rho_2)$

8. Two batteries having same emf $\mathcal{E}$ but having different internal resistances r_1 and $r_2 (< r_1)$ are connected in series to an external resistance R as shown in figure. For this situation mark out the correct statement(s).



(a) Only one value of R exist for which potential difference across battery having internal resistance r_1 is zero.
(b) Only one value of R exist for which potential difference across battery having internal resistance r_2 is zero.
(c) No value of R exist for which potential difference across any of the battery is zero.
(d) For all value of R potential difference across both the batteries would be zero.

9. Potential difference across the terminals of the battery shown in the figure is:

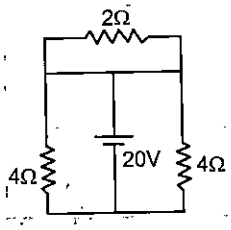


(a) 8 V
(b) 10 V
(c) 6 V
(d) zero

10. As the switch S is closed in the circuit as shown in the figure, current passed through it is:

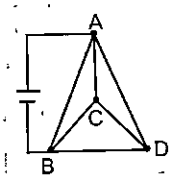
(a) 4.5 A
(b) 6 A
(c) 3 A
(d) zero

11. In the circuit shown in the figure:



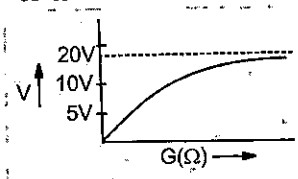
(a) Current passing through 2 Ω resistance is zero
(b) Current passing through 4 Ω resistance is 5 A
(c) Current passing through 5 Ω resistance is 4 A
(d) All of these

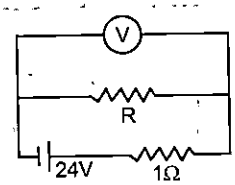
12. A tetrahedral is consisting of 6 identical wires as shown in figure. Each wire is having a resistance of 2Ω. When an ideal cell of emf 2 V is connected across AB, as shown then the current through CD is:



(a) 1 A
(b) $\frac{6}{19}$ A
(c) 4 A
(d) zero

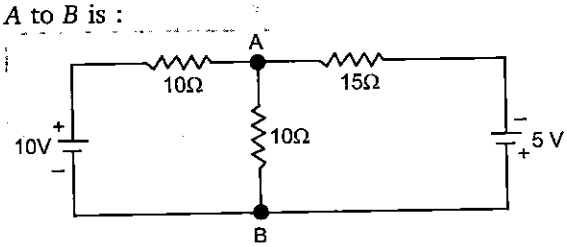
13. A cell of internal resistance 1 Ω is connected across a resistor. A voltmeter having variable resistance G is used to measure p.d. across resistor. The plot of voltmeter reading V against G is shown. What is value of external resistor R?





(a) 5 Ω
(b) 4 Ω
(c) 3 Ω
(d) 1 Ω

14. A circuit is arranged as shown. Then, the current from A to B is :

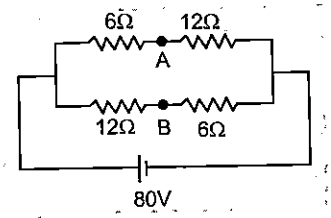


(a) +500 mA.
(b) +250 mA.
(c) –250 mA.
(d) –500 mA.

15. The potential difference across the terminal of the battery is 10 V when there is a current of 3 A in the battery from negative to positive terminal. When the current is 2 A in the reverse direction, the potential difference becomes 15 V. The internal resistance of the battery is:

(a) 2.5 Ω
(b) 5 Ω
(c) 2.83 Ω
(d) 1 Ω

16. In the circuit shown in the figure, if a wire is connected between A and B. How much current will flow through the wire:

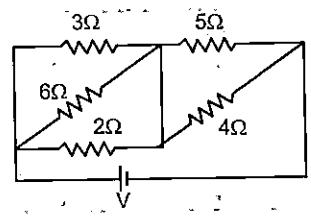


(a) 5 A
(b) $\frac{10}{3}$ A
(c) $\frac{20}{3}$ A
(d) $\frac{5}{3}$ A

17. n identical cells are joined in series with two cells A and B with reversed polarities. EMF of each cell is $\mathcal{E}$ and internal resistance is r. Potential difference across cell A or B is $(n > 4)$:

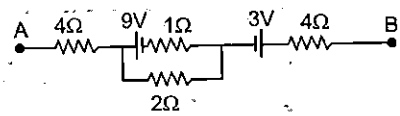
(a) $\frac{2\mathcal{E}}{n}$
(b) $2\mathcal{E}\left(1-\frac{1}{n}\right)$
(c) $\frac{4\mathcal{E}}{n}$
(d) $2\mathcal{E}\left(1-\frac{2}{n}\right)$

18. The resistor in which maximum heat will be produced is:



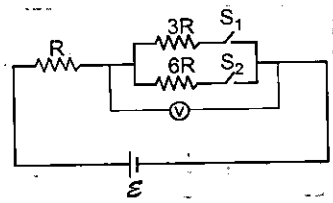
(a) 6 Ω
(b) 2 Ω
(c) 5 Ω
(d) 4 Ω

19. In the shown circuit, potential difference between points A and B is 16 V. The current passing through 2 Ω resistor will be:



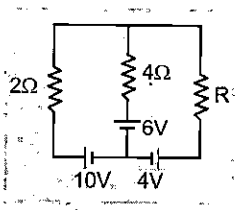
(a) 2.5 A
(b) 3.5 A
(c) 4 A
(d) zero

20. In the shown circuit, the reading of the voltmeter is V_1 when only S_1 is closed, reading of voltmeter is V_2 when only S_2 is closed and its reading is V_3 when both S_1 and S_2 are closed. Then:



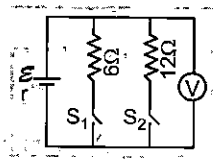
(a) $V_3 > V_2 > V_1$
(b) $V_2 > V_1 > V_3$
(c) $V_3 > V_1 > V_2$
(d) $V_1 > V_2 > V_3$

21. For what value of R in the circuit as shown, current through 4 Ω will be zero:



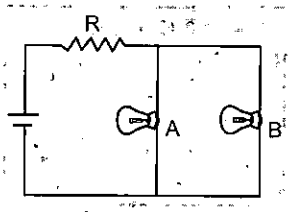
(a) 1 Ω
(b) 2 Ω
(c) 3 Ω
(d) 4 Ω

22. In the circuit shown in the figure, when switch S_1 is closed and S_2 is open, the ideal voltmeter shows a reading of 18 V. When switch S_2 is closed and S_1 is open, the reading of the voltmeter is 24 V. When both S_1 and S_2 are closed, the reading of the voltmeter will be :



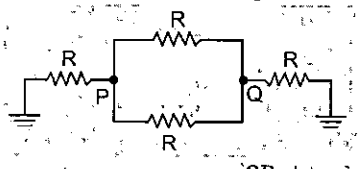
(a) 14.4 V (b) 20.6 V
(c) 24.2 V (d) 10.8 V

23. A circuit consists of a battery, a resistor R and two light bulbs A and B as shown:



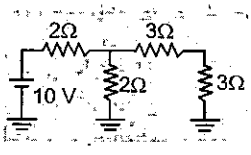
If the filament in lightbulb A burns out, then the following is true for light bulb B:
(a) it is turned off
(b) its brightness does not change
(c) it gets dimmer
(d) it gets brighter

24. The resistance between P and Q in the shown circuit is:



(a) $\frac{R}{2}$ (b) $\frac{2R}{5}$
(c) $\frac{3R}{5}$ (d) $\frac{R}{3}$

25. Current in 3 Ω resistance is :

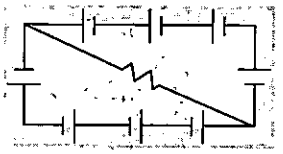


(a) 1 A
(b) $\frac{1}{7}$ A
(c) $\frac{5}{7}$ A
(d) $\frac{15}{7}$ A

26. A galvanometer of resistance R_G is to be converted into a ammeter, with the help of a shunt of resistance R. If the ratio of heat dissipated through galvanometer and shunt is 3 : 4, then:

(a) $R = \frac{3}{4}R_G$ (b) $R = \frac{4}{3}R_G$
(c) $R_G = \frac{9R}{16}$ (d) $R_G = \frac{16R}{9}$

27. A circuit is comprised of eight identical batteries and a resistor $R = 0.8\Omega$. Each battery has an emf of 1.0 V and internal resistance of 0.2Ω. The voltage difference across any of the battery is:



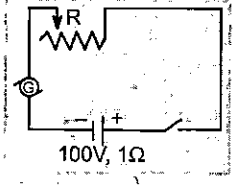
(a) 0.5 V (b) 1.0 V
(c) 0 V (d) 2 V

28. In order to determine the e.m.f. of a storage battery it was connected in series with a standard cell in certain circuit and a current I_1 was obtained. When the battery is connected to the same circuit opposite to the standard cell a current I_2 flow in the external circuit from the positive pole of the storage battery was obtained. What is the e.m.f. of the storage battery? The e.m.f., of the standard cell is $\mathcal{E}_2$.

(a) $\mathcal{E}_1 = \frac{I_1 + I_2}{I_1 - I_2}\mathcal{E}_2$ (b) $\mathcal{E}_1 = \frac{I_1 + I_2}{I_2 - I_1}\mathcal{E}_2$
(c) $\mathcal{E}_1 = \frac{I_1 - I_2}{I_1 + I_2}\mathcal{E}_2$ (d) $\mathcal{E}_1 = \frac{I_2 - I_1}{I_1 + I_2}\mathcal{E}_2$

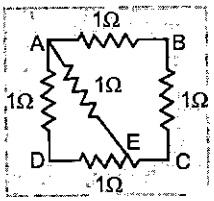
29. A wire of cross-section area A, length L_1, resistivity σ_1 and temperature coefficient of resistivity α_1 is connected to a second wire of length L_2, resistivity σ_2, temperature coefficient of resistivity α_2 and the same area A, so that wire carries same current. Total resistance r is independent of temperature for small temperature charge if (Thermal expansion effect is negligible)
(a) $\alpha_1 = -\alpha_2$
(b) $\sigma_1 L_1 \alpha_1 + \sigma_2 L_2 \alpha_2 = 0$
(c) $L_1\alpha_1 + L_2\alpha_2 = 0$
(d) None

30. The battery in the diagram is to be charged by the generator G. The generator has a terminal voltage of 120 volts when the charging current is 10 amperes. The battery has an emf of 100 volts and a internal resistance of 1 ohm. In order to charge the battery at 10 amperes charging current, the resistance R should be set at:



(a) 0.1Ω (b) 0.5Ω
(c) 1.0Ω (d) 5.0Ω

31. ABCD is a square where each side is a uniform wire of resistance 1Ω. A point E lies on CD such that if a uniform wire of resistance 1Ω is connected across AE and constant potential difference is applied across A and C then B and E are equipotential:



(a) $\frac{CE}{ED}=1$ (b) $\frac{CE}{ED}=2$

(c) $\frac{CE}{ED}=\frac{1}{\sqrt{2}}$ (d) $\frac{CE}{ED}=\sqrt{2}$

32. Power generated across a uniform wire connected across a supply is H. If the wire is cut into n equal parts and all the parts are connected in parallel across the same supply, the total power generated in the wire is:

(a) $\frac{H}{n^2}$ (b) n^2H

(c) nH (d) $\frac{H}{n}$

33. When electric bulbs of same power, but different marked voltage are connected in series across the power line, their brightness will be:

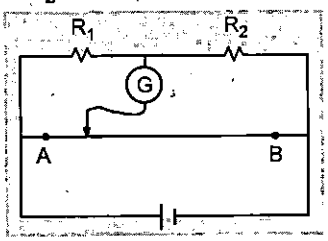
(a) proportional to their marked voltage.
(b) inversely proportional to their marked voltage.
(c) proportional to the square of their marked voltage.
(d) the same for all of them.

34. A galvanometer coil has a resistance 90Ω and full scale deflection current 10 mA. A 910Ω resistance is connected in series with the galvanometer to make a voltmeter. If the least count of the voltmeter is 0.1 V, the number of divisions on the scale is:

(a) 90 (b) 91

(c) 100 (d) none

35. In the figure shown for gives values of R_1 and R_2 the balance point for Jockey is at 40 cm from A. When R_2 is shunted by a resistance of 10Ω, balance shifts to 50 cm. R_1 And R_2 are: (AB =1 m)



(a) $\frac{10}{3}\Omega, 5\Omega$ (b) 20Ω, 30Ω

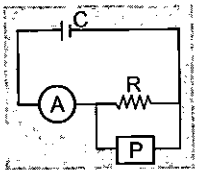
(c) 10Ω, 15Ω (d) 5Ω, $\frac{15}{2}\Omega$

36. A potentiometer wire has length 10 m and resistance 10Ω. It is connected to a battery of EMF 11 volt and internal resistance 1Ω, then the potential gradient in the wire is:

(a) 10 V/m (b) 1 V/m

(c) 0.1 V/m (d) none

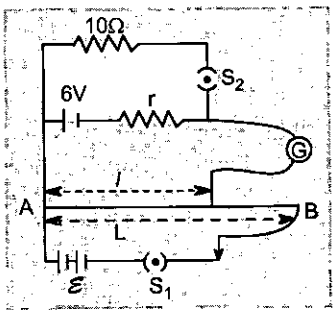
37. An ammeter A of finite resistance, and a resistor R are joined in series to an ideal cell C. A potentiometer P is joined in parallel to R. The ammeter reading is I_0 and the potentiometer reading is V_0. P is now replaced by a voltmeter of finite resistance.The ammeter reading now is I and the voltmeter reading is V :



(a) $I>I_0, V<V_0$ (b) $I>I_0, V=V_0$

(c) $I=I_0, V<V_0$ (d) $I<I_0, V=V_0$

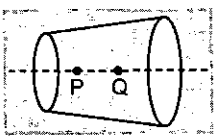
38. In the arrangement shown in figure when the switch S_2 is open, the galvanometer shows no deflection for $l=L/2$. When the switch S_2 is closed,the galvanometer shows no deflection for $l=5L/12$. The internal resistance (r) of 6 V cell, and the emf $\mathcal{E}$ of the other battery are respectively:



(a) 3Ω, 8 V (b) 2Ω, 12 V

(c) 2Ω, 24 V (d) 3Ω, 12 V

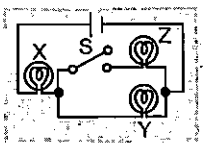
39. A wire has a non-uniform cross-section as shown in figure. A steady current flows through it. The drift speed of electrons at points P and Q is v_P and v_Q.



(a) $v_P=v_Q$ (b) $v_P<v_Q$

(c) $v_1>v_Q$ (d) Data insufficient

40. If X, Y and Z in figure are identical lamps, which of the following changes to the brightnesses of the lamps occur when switch S is closed?



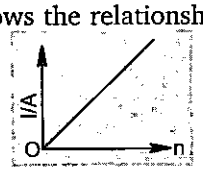
(a) X stays the same, Y decreases

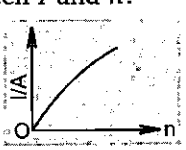
(b) X increases, Y decreases

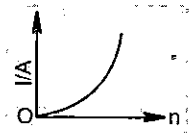
(c) X increases, Y stays the same

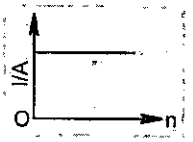
(d) X decreases, Y increases

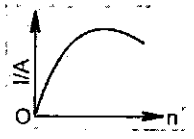
41. A battery consists of a variable number n of identical cells having internal resistance connected in series. The terminal of the battery are short circuited and the current I measured. Which one of the graph below, shows the relationship between I and n?

(a) 

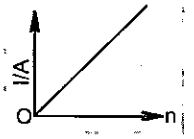
(b) 

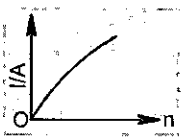
(c) 

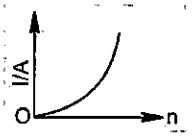
(d) 

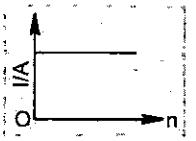
(e) 

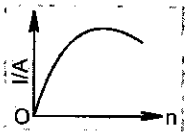
42. In previous problem, if the cell hand been connected in parallel (instead of in series) which of the above graphs would have shown the relationship between total current I and n:

(a) 

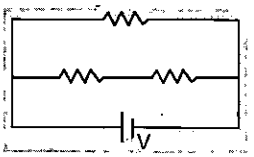
(b) 

(c) 

(d) 

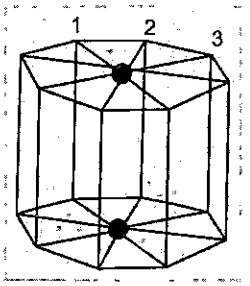
(e) 

43. Three identical resistors are connected across a voltage source V so that one of them is in parallel with two others which are connected in series as shown. The power dissipated through the first one, compared to the power dissipated by each of the other two, is approximately:



(A) the same
(b) half as much
(c) twice as much
(d) four times as much

44. In the diagram shown, all the wires have resistance R. The equivalent resistance between the upper and lower dots shown in the diagram is:

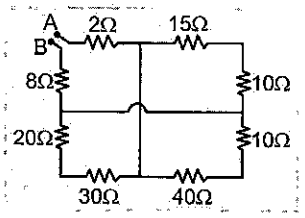


(a) $R/8$
(b) R
(c) $2R/5$
(d) $3R/8$

45. A wire of length L and 3 identical cells of negligible internal resistance are connected in series. Due to the current, the temperature of the wire is raised by ΔT in time t. N number of similar cells is now connected in series with a wire of the same material and cross section but of length $2L$. The temperature of the wire is raised by the same amount ΔT in the same time t. The value of N is:

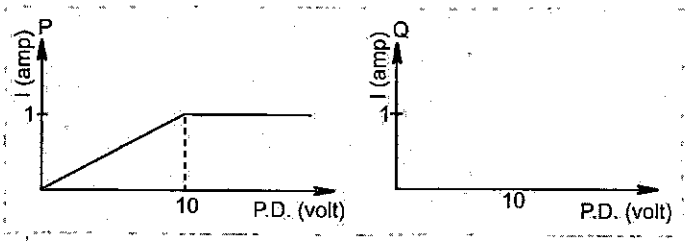
(a) 4 (b) 6
(c) 8 (d) 9

46. The equivalent resistance between A and B is :

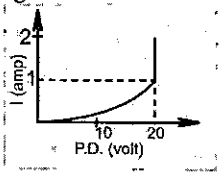


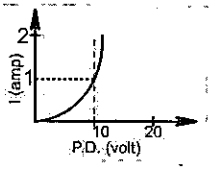
(a) 32.5 Ω
(b) 22.5 Ω
(c) 2.5 Ω
(d) 42.5 Ω

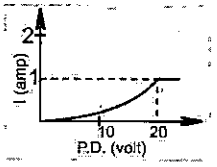
47. Two current elements P and Q have current voltage characteristics as shown below:

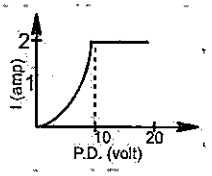


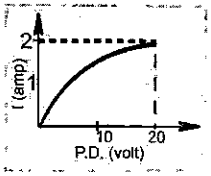
Which of the graph given below represents current voltage characteristics when P and Q are in series.

(a) 

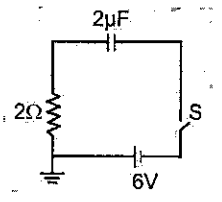
(b) 

(c) 

(d) 

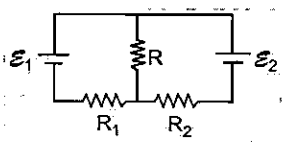
(e) 

48. In the given circuit, the quantity of charge that flows to ground long time after the switch is closed is:



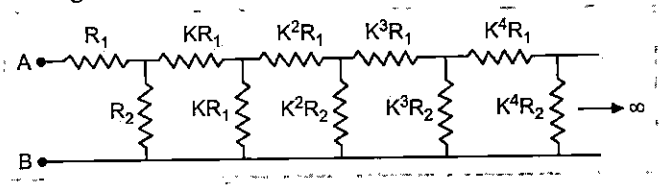
(a) 12 μC
(b) 9 μC
(c) 13 μC
(d) zero

49. For what value of R the thermal power developed in it is maximum :



(a) $\frac{R_1R_2}{R_1+R_2}$
(b) $\sqrt{R_1R_2}$
(c) R_1+R_2
(d) R_1-R_2

50. The circuit diagram shown consists of a large number of element (each element has two resistors R_1 and R_2). The resistance of the resistors ion each subsequent element differs by a factor of K = ½ from the resistance of the resistors in the previous elements. The equivalent resistance between A and B shown in Fig. is:



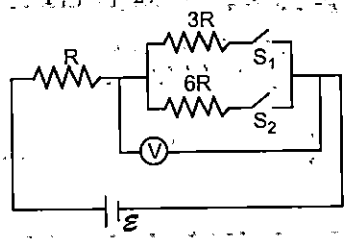
(a) $\frac{R_1 - R_2}{2}$

(b) $\frac{(R_1 - R_2) + \sqrt{6R_1R_2}}{2}$

(c) $\frac{(R_1 - R_2) + \sqrt{R_1^2 + R_2^2 + 6R_1R_2}}{2}$

(d) None of these

51. In the circuit shown in figure reading of voltmeter is V_1 when only S_1 is closed, reading of voltmeter is V_2 when only S_2 is closed. The reading of voltmeter is V_3 when both S_1 and S_2 are closed then:



(a) $V_2 > V_1 > V_3$ (b) $V_3 > V_2 > V_1$

(c) $V_3 > V_1 > V_2$ (d) $V_1 > V_2 > V_3$

52. A metallic resistor is connected across a battery. If the number of collisions of free electrons with the lattice is somehow decreased in the resistor (for example cooling it), the current will :

(a) Increase (b) Decrease

(c) remains same (d) become zero

53. A constant voltage is applied between the two ends of a uniform metallic wire. Some heat is developed in it. The heat developed is doubled if:

(a) both the length and the radius of the wire are halved.

(b) both the length and the radius of the wire are halved.

(c) the radius of the wire is doubled

(d) the length of the wire is doubled

54. In an electrical circuit containing a battery, the charge (assume positive) inside the battery :

(a) always goes from positive terminal to the negative terminal

(b) may go from positive terminal to negative terminal

(c) always goes from the negative terminal to positive terminal

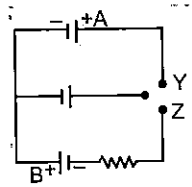
(d) does not move

55. Two bulbs rated (25 W – 220 V) and (100 W – 220 V) are connected is series to a 440 V line. Which one is likely to fuse?

(a) 25 W bulb (b) 100 W bulb

(c) both bulbs (d) none

56. In the circuit shown the cells are ideal and of equal emfs, the capacitance of the capacitor is C and the resistance of the resistor is R. X is first joined to Y and then to Z. After a long time, the total heat produced in the resistor will be:



(a) equal to the energy finally stored in the capacitor

(b) half of the energy finally stored in the capacitor

(c) twice the energy finally stored in the capacitor

(d) 4 times the energy finally stored in the capacitor

57. Two non-ideal batteries are connected in parallel. Consider the following statements :

(A) The equivalent emf is smaller than either of the two emfs

(B) The equivalent internal resistance is smaller than either of the two internal resistances

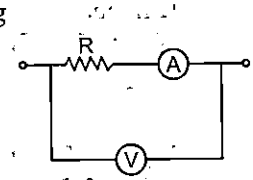
(a) Both (A) and (B) are correct

(b) (A) is correct and (B) is wrong

(c) (B) is correct but (A) is wrong

(d) Both (A) and (B) are wrong

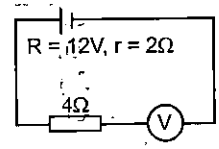
58. In the circuit shown the resistance of voltmeter is 10,000 ohm and that of ammeter is 20 ohm. The ammeter reading is 0.10 amp and voltmeter reading is 12 volt. Then R is equal to:



(a) 122Ω (b) 140Ω

(c) 116Ω (d) 100Ω

59. By error, a student places moving-coil voltmeter (V) nearly ideal in series with the resistance in a circuit in order to read the current, as shown. The voltmeter reading will be:



(a) 0 (b) 4 V

(c) 6 V (d) 12 V

60. In a balanced wheat stone bridge, current in the galvanometer is zero. It remains zero when:

[1] battery emf is increased

[2] all resistance are increased by 10 ohms

[3] all resistance are made five time

[4] the battery and the galvanometer are interchanged

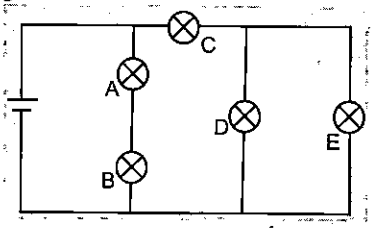
(a) only [1] is correct

(b) [1], [2] and [3] are correct

(c) [1], [3] and [4] are correct

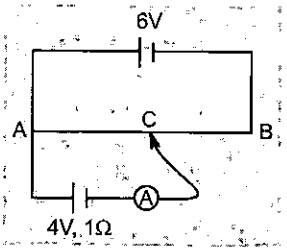
(d) [1] and [3] are correct

61. The circuit below is made up using identical light bulbs. The light bulbs of maximum brightness of the following will be :



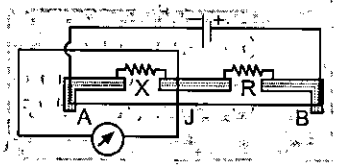
(a) A (b) C
(c) D (d) E

62. A 6 V battery of negligible internal resistance is connected across a uniform wire of length 1 m. The positive terminal of another battery of emf 4 V and internal resistance 1Ω is joined to the point A as shown in figure. The ammeter shows zero deflection when the jockey touches the wire at the point C. The AC is equal to :



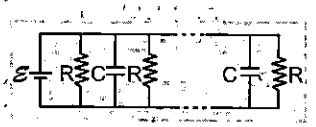
(a) 2/3 m (b) 1/3 m
(c) 3/5 m (d) 1/2 m

63. The figure shows a metre-bridge circuit, with AB = 100 cm, X = 12Ω and R = 18Ω, and the jockey J in the position of balance. If R is now made 8Ω, through what distance will J have to be moved to obtain balance?



(a) 10 cm (b) 20 cm
(c) 30 cm (d) 40 cm

64. n resistances each of resistance R are joined with capacitors of capacity C (each) and a battery of emf $\mathcal{E}$ as shown in the figure. In steady state condition ratio of charge stored in the first and the last capacitor is :

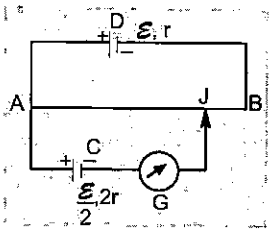


(a) $n : 1$ (b) $(n-1) : R$
(c) $\frac{n^2+1}{n^2-1}$ (d) 1 : 1

65. The length of a potentiometer wire is l. A cell of emf $\mathcal{E}$ is balanced at a length $l/3$ from the positive end of the wire. If the length of the wire is increased by $l/2$. At what distance will the same cell give a balance point:

(a) $\frac{2l}{3}$ (b) $\frac{l}{2}$
(c) $\frac{l}{6}$ (d) $\frac{4l}{3}$

66. In the figure, the potentiometer wire AB of length L and resistance $9r$ is joined to the cell D of emf $\mathcal{E}$ and internal resistance r. The cell C's emf is $\mathcal{E}/2$ and its internal resistance is $2r$. The galvanometer G will show no deflection when the length AJ is:

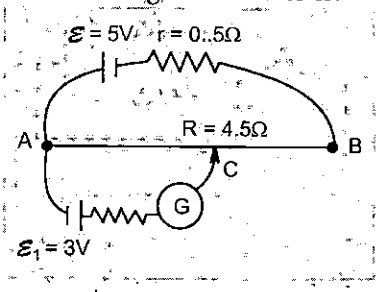


(a) $\frac{4L}{9}$ (b) $\frac{5L}{9}$
(c) $\frac{7L}{18}$ (d) $\frac{11L}{18}$

67. 24 identical cells, each of internal resistance 0.5 Ω, are arranged in a parallel combination of n rows, each row containing m cells in series. The combination is connected across a resistor of 3 Ω. In order to send maximum current through the resistor, we should have :

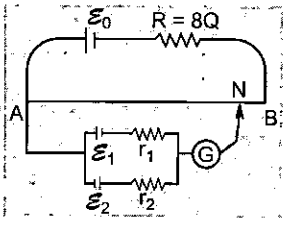
(a) $m = 12, n = 2$ (b) $m = 8, n = 3$
(c) $m = 2, n = 12$ (d) $m = 3, n = 8$

68. In the given potentiometer circuit length of the wire AB is 3 m and resistance is R = 4.5Ω. The length AC for no deflection in galvanometer is:



(a) 2 m (b) 1.8 m
(c) dependent on r_1 (d) none of these

69. A battery of emf $\mathcal{E}_0$ = 12V is connected across a 4 m long uniform wire having resistance 4Ω / m. The cells of small emfs $\mathcal{E}_1$ = 2V and $\mathcal{E}_2$ = 4V, having internal resistance 2Ω and 6Ω respectively, are connected as shown in the figure. If galvanometer shows no reflection at the point N, the distance of point N from the point A is equal to :

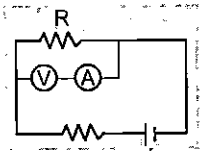


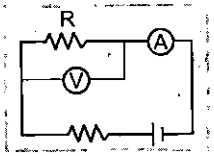
(a) $\frac{1}{6}$ m (b) $\frac{1}{3}$ m
(c) 25 cm (d) 50 cm

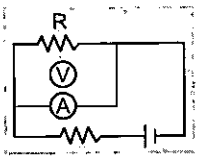
70. When a battery is getting charged :
(a) the voltage drop across its internal resistance is zero
(b) the terminal potential difference is less than its emf

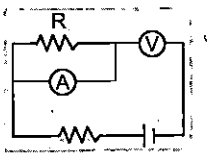
(c) the terminal potential difference is more than its emf
(d) its terminal potential difference is zero

71. Which of the following wiring diagrams could be used to experimentally determine R using ohm's law? Assume an ideal voltmeter and an ideal ammeter:

(a) 

(b) 

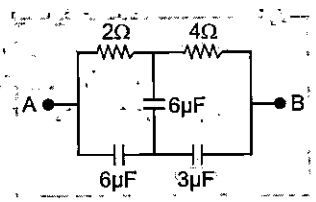
(c) 

(d) 

72. Current density in a cylindrical wire of radius R is given as $J = \begin{cases} J_0\left(\frac{x}{R}-1\right) \text{ for } 0 \le x \le \frac{R}{2} \\ J_0\frac{x}{R} \text{ for } \frac{R}{2} \le x \le \frac{R}{2} \end{cases}$. The current flowing in the wire is:

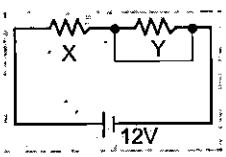
(a) $\frac{7}{24}\pi J_0 R^2$ (b) $\frac{1}{6}\pi J_0 R^2$
(c) $\frac{7}{12}\pi J_0 R^2$ (d) $\frac{5}{12}\pi J_0 R^2$

73. What is the equivalent capacitance between A and B in the circuit shown:



(a) 6 μ F
(b) 1.5 μ F
(c) zero
(d) 2 μ F

74. When an ammeter of negligible internal resistance is inserted in series with circuit it reads 1A. When the voltmeter of very large resistance is connected across X it reads 1V. When the point A and B are shorted by a conducting wire, the voltmeter measures 10 V across the battery. The internal resistance of the battery is equal to:



(a) zero (b) 0.5Ω
(c) 0.2Ω (d) 0.1Ω

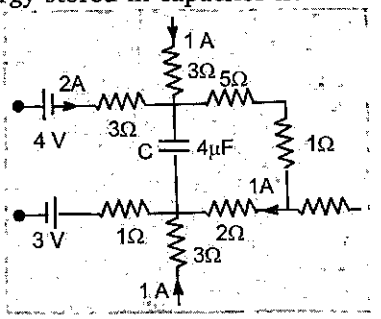
75. The ratios of lengths, masses, densities, and resistivities of two wires are in ratio 1 : 2, 1 : 1, 1 : 2 and 4 : 1 respectively. The ratio of their resistances is :

(a) 1 : 1 (b) 2 : 1
(c) 4 : 1 (d) 1 : 2

76. A curved rectangular bar forms a resistor. The curved sides are concentric circular arcs. If ρ is the resistivity of the material of bar, l_0 is the length of inner arc of radius r_0, $(r_0 + b)$ is the radius of the outer arc, and a is the width of the bar. The electric resistance of the bar across its rectangular ends is:

(a) $\frac{\rho l_0}{a r_0}$ (b) $\frac{\rho l_0}{a r_0 \ln\left[1+\frac{b}{r_0}\right]}$
(c) $\frac{2\rho l_0}{a r_0 \ln\left[1+\frac{b}{r_0}\right]}$ (d) None of these

77. Fig. shows a part of complete circuit. The current in various branches in steady state are shown in figure. The energy stored in capacitor is:

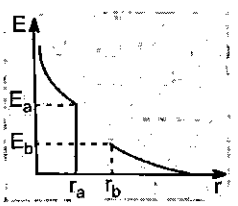


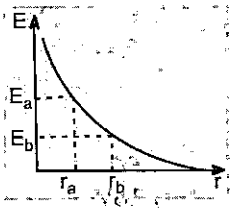
(a) 200μJ (b) 400μJ
(c) 600μJ (d) 800μJ

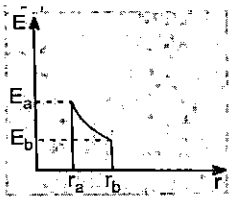
78. A cell develops the same power across two resistance R_1 and R_2 separately. The internal resistance of the cell is:

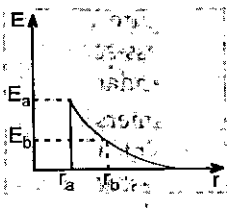
(a) $R_1 + R_2$ (b) $\frac{R_1 + R_2}{2}$
(c) $\sqrt{R_1 R_2}$ (d) $\frac{\sqrt{R_1 R_2}}{2}$

79. The region between two concentric spheres of radii r_a and r_b is filled with a conducting material. The inner sphere is maintained at a potential V_a and outer sphere is at lower potential V_b, as a result of which, current is there in radially outward direction. The variation of electric field intensity as a function of distance r from centre of sphere is given by:

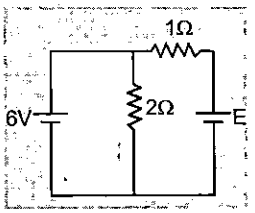
(a) 

(b) 

(c) 

(d) 

80. Current passing through 1 Ω resistance is zero. Then the emf $\mathcal{E}$ is:



(a) 8 V
(b) 6 V
(c) 4 V
(d) 12 V

81. The two batteries of emf $\mathcal{E}_1$ and $\mathcal{E}_2$ having internal resistances r_1 and r_2 respectively are connected in series to an external resistor R. Both the batteries are getting discharged. The above described combination of these two batteries has to produce a weaker current then when any one of the battery is connected to same resistor. For this requirement to be fulfilled:

(a) $\frac{\mathcal{E}_2}{\mathcal{E}_1}$ must not lie between $\frac{r_2}{r_1+R}$ and $\frac{r_2+R}{r_1}$

(b) $\frac{\mathcal{E}_2}{\mathcal{E}_1}$ must lie between $\frac{r_2}{r_1+R}$ and $\frac{r_2+R}{r_1}$

(c) $\frac{\mathcal{E}_2}{\mathcal{E}_1}$ must lie between $\frac{r_2}{r_1+R}$ and $\frac{r_1}{r_2+R}$

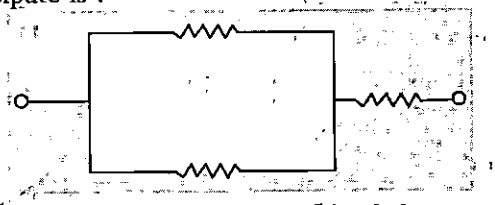
(d) $\frac{\mathcal{E}_2}{\mathcal{E}_1}$ must not lie between $\frac{r_2}{r_1+R}$ and $\frac{r_1}{r_2+R}$

82. An ammeter has resistance R_0 and range I. What resistance should be connected in parallel with it to increase its range to nI?

(a) $\frac{R_0}{n}$ (b) $R_0(n+1)$

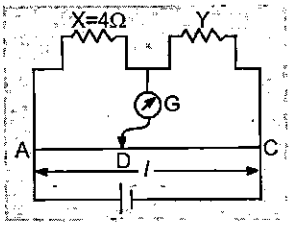
(c) $\frac{R_0}{n+1}$ (d) $\frac{R_0}{n-1}$

83. Each of the three resistors connected in a circuit as shown below has a resistance of 2Ω and can dissipate a maximum of 18W without becoming excessively heated. The maximum power that the circuit can dissipate is :



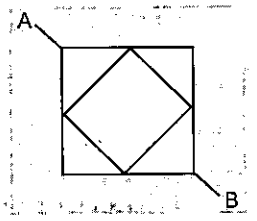
(a) 54W (b) 36W
(c) 18W (d) 27W

84. Figure shows a meter bridge, wire AC has uniform cross-section. The length of wire AC is 100 cm. X is a standard resistor of 4Ω and Y is a coil. When Y is immersed in melting ice the null point is at 40 cm from point A. When the coil Y is heated to 100°C, a 100 Ω resistor has to be connected in parallel with Y in order to keep the bridge balanced at the same point:



Temperature coefficient of resistance of the coil is :
(a) 6.3×10^{-4} K^{-1} (b) 4.3×10^{-4} K^{-1}
(c) 8.3×10^{-4} K^{-1} (d) 2.3×10^{-4} K^{-1}

85. A wire has linear resistance ρ (in Ohm/m). Find the resistance R between points A and B if the side of the "big" square is d:



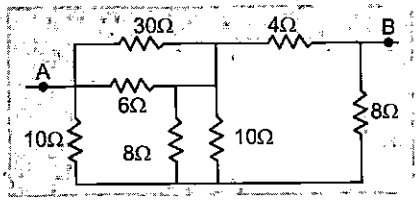
(a) $\frac{\rho d}{\sqrt{2}}$
(b) $\sqrt{2}\rho d$
(c) $2\rho d$
(d) None of these

86. The wattage rating of a light bulb indicates the power dissipated by the bulb if it is connected across 110 V DC potential difference. If a 50W and 100W bulb are connected in series to a 110 V DC source, how much power will be dissipated in the 50W bulb :
(a) 50W (b) 100W
(c) 22W (d) 11W

87. The same mass of copper is drawn into two wires A and B of radii r and $3r$ respectively. They are connected in series, and electric current is passed. The ratio of the heat produced in A and B is :
(a) 1 : 9 (b) 1 : 81
(c) 81 : 1 (d) 9 : 1

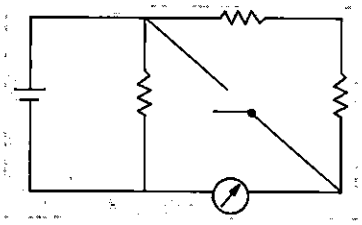
88. If in the **above** question A and B are connected in parallel between the terminals of a source of emf, the ratio of heat produced in A and B is :
(a) 1 : 9 (b) 1 : 81
(c) 81 : 1 (d) 9 : 1

89. Seven resistors are connected as shown in the diagram.



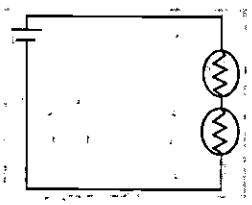
The equivalent resistance in ohms of this network between A and B is :
(a) 6 (b) 8
(c) 12 (d) 20

90. In the circuit shown, the reading of the ammeter is doubled after the switch is closed. Each resistor has a resistance =1Ω and the ideal cell has an emf.=10 V. Then, the ammeter has a coil resistance equal to:



(a) 2Ω (b) 1Ω
(c) 2.5Ω (d) None

91. Two lamps, each with a resistance of 50Ω, are connected in series. The lamps will fuse if a power of more than 200 W is dissipated in it. What is the maximum voltage that can be applied to the circuit?

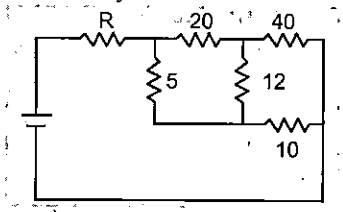


(a) 100 V (b) 140 V
(c) 200 V (d) None

92. A beam of fast moving electrons having cross-sectional area A falls normally on flat surface. The electrons are absorbed by the surface and the average pressure exerted by the electrons on this surface is found to be P. If the electrons are moving with a speed v, then the effective current through any cross-section of the electron beam is:

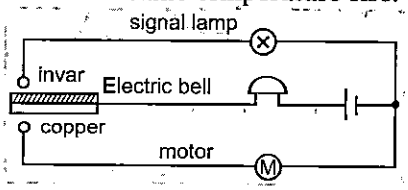
(a) $APe/(mv)$ (b) $APe/(mv^2)$
(c) $APe/(me)$ (d) $APm/(eV)$

93. What should be the value of R so that the electric power consumed by it is maximum:



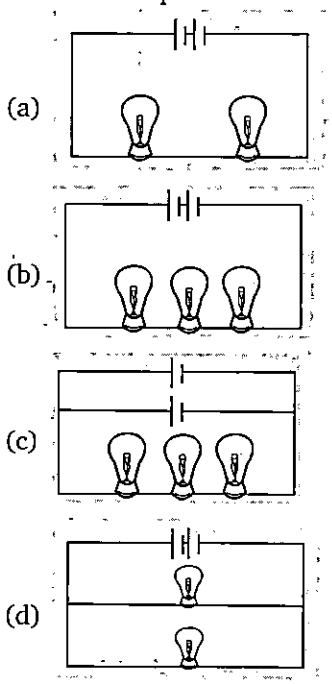
(a) 12Ω (b) 24Ω
(c) 6Ω (d) none of these

94. The diagram shows a bimetallic strip used as a thermostat in a circuit. The copper expands more than the invar for the same temperature rise.

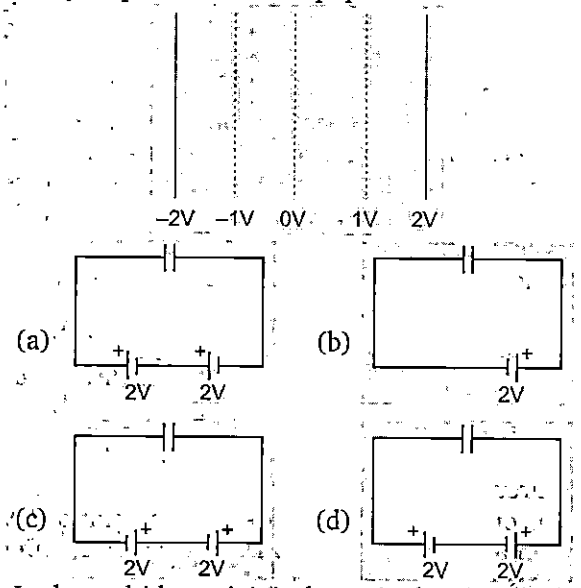


What will be switched on when the bimetallic strip becomes hot question
(a) bell only
(b) lamp and bell only
(c) motor and bell only
(d) lamp, bell and motor

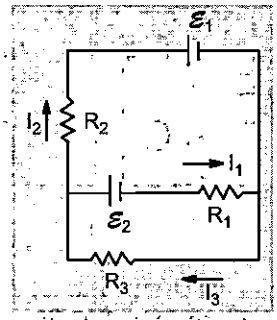
95. In the diagrams, all light bulbs are identical, all cells are ideal and identical. In which circuit will the bulbs be dimmest question :



96. A battery, or batteries, connected to two parallel plates produce the equipotential lines between the plates as shown. Which of the following configurations is most likely to produce these equipotential lines?



97. In the multi-loop circuit shown in the Fig. which of the following set of equations is correct?



$I_2 + I_3 = I_1$

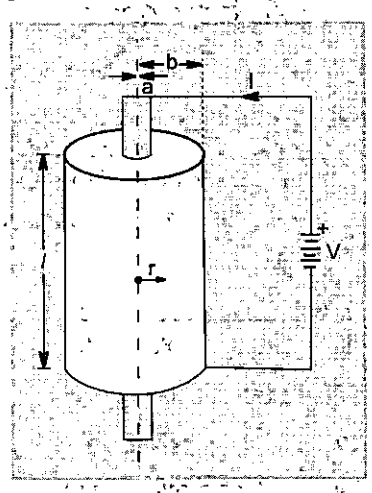
(a) $+I_2R_2 - \mathcal{E}_1 + I_1R_1 - \mathcal{E}_2 = 0$
$\mathcal{E}_2 - I_1R_1 - I_3R_3 = 0$
$I_2 + I_3 = I_1$

(b) $-I_2R_2 - \mathcal{E}_1 - I_3R_3 = 0$
$\mathcal{E}_2 - I_1R_1 - I_3R_3 = 0$
$I_1 + I_2 = I_3$

(c) $-I_2R_2 - \mathcal{E}_1 + I_1R_1 - \mathcal{E}_2 = 0$
$\mathcal{E}_2 - I_1R_1 - I_3R_3 = 0$
$I_1 + I_2 = I_3$

(d) $I_2R_2 + \mathcal{E}_1 + I_3R_3 = 0$
$\mathcal{E}_2 + I_1R_1 + I_3R_3 = 0$

98. A metal rod of radius a is concentric with a metal cylindrical shell of radius b and length l. The space between rod and cylinder is tightly packed with a high resistance material of resistively ρ. A battery having a terminal voltage V is connected across the combination as shown. Neglect resistance of rod and cylinder. If I is the total current in the circuit then:



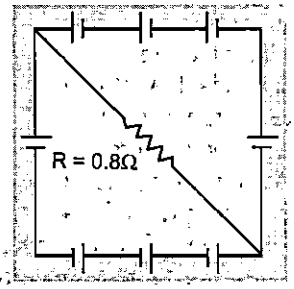
(a) $I = \dfrac{lV}{\rho}$

(b) $I = \dfrac{2\pi lV}{\rho(\ln b - \ln a)}$

(c) $I = \dfrac{4\pi lV}{\rho(\ln b - \ln a)}$

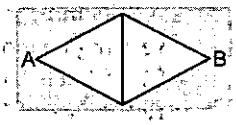
(d) $I = \dfrac{lV}{4\pi\rho(\ln b - \ln a)}$

99. A circuit is comprised of eight identical batteries and a resistor $R = 0.8\Omega$. Each battery has an emf of 1 V and internal resistance of 0.2Ω. The voltage difference across any of the battery is:



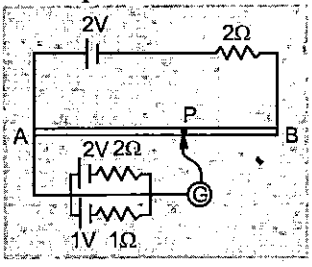
(a) 0.5 V (b) 1 V
(c) 0 V (d) 2 V

100. A uniform wire of resistance R stretched uniformly to n times and then cut to form five wires of equal length. These wire are arranged as shown in the Fig. The effective resistance between points A and B is:



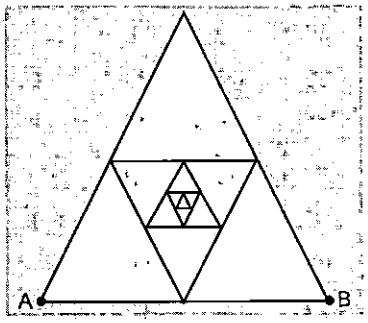
(a) $nR/5$ (b) $R/(5n)$
(c) $n^2R/5$ (d) $R/(5n^2)$

101. A battery of emf 2V is connected across a long uniform wire AB of length 1m and resistance per unit length $2\Omega\,m^{-1}$. Two cells of emf $\mathcal{E}_1 = 1$ V and $\mathcal{E}_2 = 2$ V are connected as shown in the fig. If the galvanometer shows no deflection at point P, the distance of point P from point A is equal to :



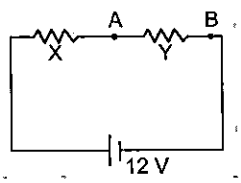
(a) 0 (b) 50 cm
(c) 100 cm (d) 25 cm

102. A frame made of thin homogeneous wire is shown in Fig. Assume that the number of successively embedded equilateral triangle with sides decreasing by half tends to infinity. The side AB has a resistance R_0. The equivalent resistance between A and B is χ.



(a) χ is infinite (b) χ is zero
(c) $\chi = 2R_0$ (d) $\chi = \left(\dfrac{\sqrt{7}-1}{3}\right)R_0$

103. When an ammeter of negligible resistance is inserted in series with circuit it reads 1 A. When the voltmeter of very large resistance is connected across X it reads 1 V. When the points A and B are shorted by a conducting wire, the voltmeter measures 10 V across the battery. The internal resistance of the battery is:

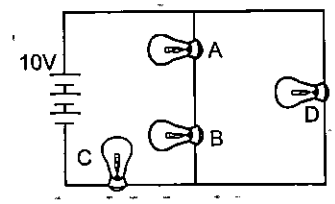


(a) zero (b) 0.2Ω
(c) 0.5Ω (d) 0.1Ω

104. A galvanometer is a sensitive instrument that gives a reading proportional to the current that flows through it. If such an instrument has a built-in (internal) resistance of 200 Ω and requires a current of 5.0 mA for full-scale reading, what resistance should be connected in parallel with this galvanometer to make it function as an ammeter that reads 10 A when the reading is full-scale?

(a) 400 Ω (b) 0.2 Ω
(c) 0.1 Ω (d) $4\times10^5\Omega$

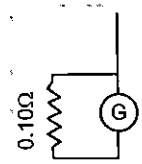
105. In the circuit diagram, all the bulbs are identical. Which bulb will be the brightest?

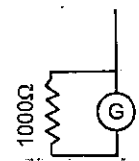


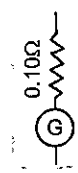
(a) A (b) B
(c) C (d) D

106. A galvanometer with an internal resistance of 100Ω will show a full scale deflection with a current of 10mA. Which of the following circuits would tum this galvanometer into an ammeter which will read 10A at full scale?

(a)

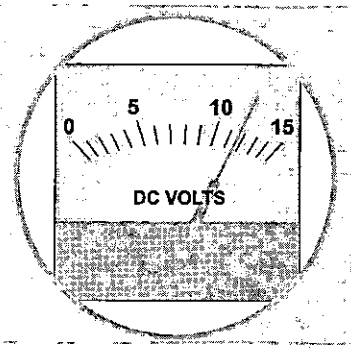
(b) 

(c) 

(d) 

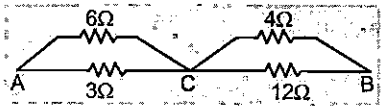
ANSWERS

1.	(d)	**2.**	(b)	**3.**	(a)	**4.**	(b)	**5.**	(b)	**6.**	(a)	**7.**	(b)	**8.**	(a)
9.	(d)	**10.**	(a)	**11.**	(d)	**12.**	(d)	**13.**	(a)	**14.**	(b)	**15.**	(d)	**16.**	(b)
17.	(d)	**18.**	(d)	**19.**	(b)	**20.**	(b)	**21.**	(a)	**22.**	(a)	**23.**	(d)	**24**	(b)
25.	(c)	**26.**	(a)	**27.**	(c)	**28.**	(a)	**29.**	(b)	**30.**	(c)	**31.**	(d)	**32.**	(b)
33.	(c)	**34.**	(c)	**35.**	(a)	**36.**	(b)	**37.**	(a)	**38.**	(b)	**39.**	(c)	**40.**	(b)
41.	(d)	**42.**	(a)	**43.**	(d)	**44.**	(d)	**45.**	(b)	**46.**	(b)	**47.**	(c)	**48.**	(d)
49.	(a)	**50.**	(c)	**51.**	(a)	**52.**	(a)	**53.**	(b)	**54.**	(b)	**55.**	(a)	**56.**	(d)
57.	(c)	**58.**	(d)	**59.**	(d)	**60.**	(c)	**61.**	(b)	**62.**	(a)	**63.**	(b)	**64.**	(d)
65.	(b)	**66.**	(b)	**67.**	(a)	**68.**	(d)	**69.**	(c)	**70.**	(c)	**71.**	(b)	**72.**	(d)
73.	(d)	**74.**	(b)	**75.**	(d)	**76.**	(b)	**77.**	(d)	**78.**	(c)	**79.**	(c)	**80.**	(b)
81.	(a)	**82.**	(d)	**83.**	(d)	**84.**	(a)	**85.**	(a)	**86.**	(c)	**87.**	(c)	**88.**	(b)
89.	(a)	**90.**	(a)	**91.**	(c)	**92.**	(a)	**93.**	(a)	**94.**	(b)	**95.**	(c)	**96.**	(b)
97.	(d)	**98.**	(b)	**99.**	(c)	**100.**	(c)	**101.**	(a)	**102.**	(d)	**103.**	(b)	**104.**	(c)
105.	(c)	**106.**	(b)												



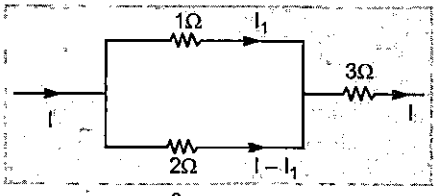
SOLUTION

1. [d] Redrawing the circuit



$\therefore$ net resistance between A and B is $5\ \Omega$.

2. [b] $I_1 \times 1 = (I - I_1) \times 2 \Rightarrow \frac{I_1}{I} = \frac{2}{3}$



$P = I^2R$

$\therefore \quad P_1 = 4\frac{I^2}{9} \times 1, \quad P_2 = \frac{I^2}{9} \times 2,$

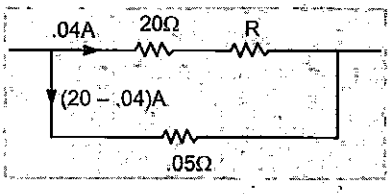
$P_3 = I^2 \times 3$

$P_1 : P_2 : P_3 = 4 : 2 : 27$

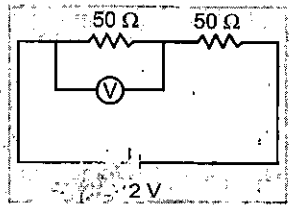
3. [a] Since the shunt is in parallel to $(20 + R)\Omega$

$\Rightarrow \quad 0.04 \times (20 + R) = 0.05 \times (20 - 0.04)$

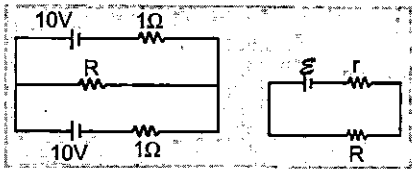
$\Rightarrow \quad R = 4.95\ \Omega$



5. [b] When the voltmeter is connected across any of the coil as shown in figure. Then by solving the circuit we will get the potential difference across voltmeter as 5.85 V which is nothing but equal to voltmeter reading. When voltmeter is connected across terminals of the battery then the reading of voltmeter is 12 V.



6. [a]



$$\varepsilon = \frac{10 \times 1 + 10 \times 1}{1 + 1} = 10\ \text{V}$$

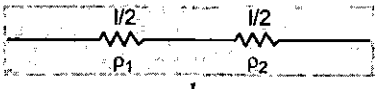
$$r = \frac{1 \times 1}{1 + 1} = \frac{1}{2}$$

For max. power

$$R = r = \frac{1}{2}$$

$\Rightarrow$ maximum power $= 100 \times \frac{1}{2} = 50$ watt.

7. [b] Let length of each wire be $\frac{l}{2}$



$$R_1 = \frac{\rho_1 \frac{l}{2}}{A}$$

$$R_2 = \frac{\rho_2 \frac{l}{2}}{A}$$

$$R = R_1 + R_2$$

$$= \frac{\rho \times l}{A} = \frac{(\rho_1 + \rho_2)\frac{l}{2}}{A}$$

$$\Rightarrow \quad \rho = \frac{\rho_1 + \rho_2}{2}$$

8. [a] Current in the circuit is; $I = \frac{2\varepsilon}{r_1 + r_2 + R}$

Potential difference across the battery having internal resistance r_1 is,

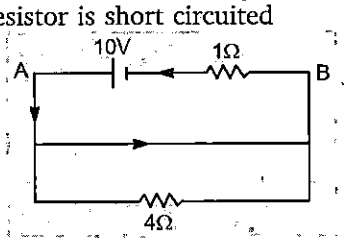
$$V_1 = \varepsilon - Ir_1 = \frac{\varepsilon(r_2 + R - r_1)}{r_1 + r_2 + R}$$

Potential difference across the battery having internal resistance r_2 is,

$$V_2 = \varepsilon - Ir_2 = \frac{\varepsilon(r_1 + R - r_2)}{r_1 + r_2 + R}$$

If $V_1 = 0$, then $R = r_1 - r_2 > 0$ as $r_1 > r_2$
If $V_2 = 0$, then $R = r_2 - r_1 < 0$ which is not possible.

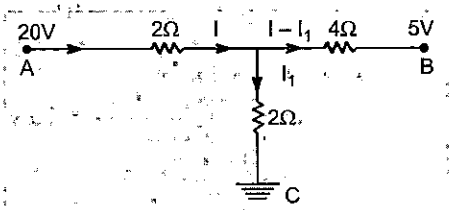
9. [d] 4 Ω resistor is short circuited



$$\Rightarrow \quad I = \frac{10}{1} = 10\ \text{A}$$

$$V_A - 10 + 10 \times 1 = V_B$$
$$V_A - V_B = 0$$

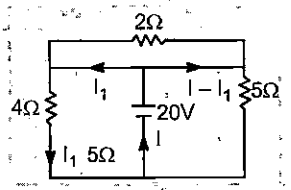
10. [a] Using Kirchhoff's law



$$20 - 2I - 2I_1 = 0 \quad \text{... (i)}$$
$$20 - 2I - 4(I - I_1) = 5$$
$$15 - 6 + 4I_1 = 0 \quad \text{... (ii)}$$

From eqns. (i) and (ii),
$$I_1 = 4.5\ \text{A}$$

11. [d] 2 Ω resistance will be short circuited as potential drop across it is 0 also 4 Ω and 5 Ω are in parallel



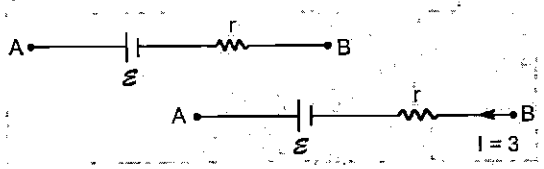
$$\frac{1}{R} = \frac{4+5}{4\times 5} = \frac{9}{20}$$
$$I = \frac{20}{20} \times 9 = 9\text{A}$$
$$\Rightarrow \quad I_1 \times 4 = (9 - I_1) \times 5$$
$$I_1 = 5\text{A}, I - I_1 = 4\ \text{A}$$

12. (d) From the symmetry of figure it is clear that C and D are at same potential so no current flows from CD branch.

Extra (1) Find R_{eq} across AB and across BC.
(2) Find the current through CD if battery is connected across BD.

13. [a] When galvanometer resistance tends to infinity
$G \to \infty$, P.d. across R is 20 V
$$20 = 24 - Ir = 24 - I \times 1$$
or $I = 4$ A
also $20 = 4 \times R$
or $R = 5\Omega$

15. [d] In first case



$$V_A - \mathcal{E} + 3r = V_B$$
$$V_A - V_B = 10 = \mathcal{E} - 3r \quad \text{...(i)}$$

In second case similarly
$$V_A - \mathcal{E} - 2r = V_B$$
$$\Rightarrow \quad 15 = \mathcal{E} + 2r \quad \text{... (ii)}$$

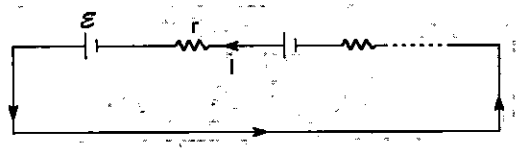
From eqns. (i) and (ii),
$$r = 1\ \Omega.$$

16. [b] Due to symmetry of circuit. Current distribution in circuit will be as shown

$$I = \frac{80}{4+4} = 10\ \text{A}$$

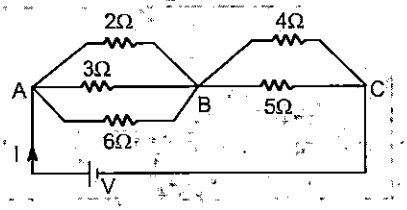
Now, $V_A = V_B$
$$\Rightarrow \quad 6(I - I_1) = 12 I_1$$
$$\Rightarrow \quad 6(10 - I_1) = 12 \times I_1$$
$$60 = 18 I_1$$
$$I_1 = \frac{60}{18} = \frac{10}{3}$$
$$\Rightarrow \quad I - 2I_1 = \frac{10 - 2\times 10}{3} = \frac{10}{3}\ \text{A}$$

17. [d]



$$I = \frac{(n-2)\mathcal{E} - 2\mathcal{E}}{nr} = \frac{(n-4)\mathcal{E}}{nr}$$
$$V = \mathcal{E} + Ir$$
$$= \mathcal{E} + \frac{(n-4)\mathcal{E} \times r}{nr} = 2\mathcal{E}\left(1 - \frac{2}{n}\right)$$

18. [d] 2, 3, 6 Ω resistors are in parallel
Among these resistors, maximum heat will be produced in 2 Ω; $\left[H = \frac{V^2}{R}t\right]$



Similarly in 4, 5 Ω max heat will be produced in 4 Ω.

$$R_{\text{equivalent}} = \frac{20}{9} + 1 = \frac{29}{9} \Rightarrow I = \frac{9V}{29}$$

$V_{AB} = 1 \times \frac{9V}{29}$ (1 is equivalent resistance of 6 Ω, 3 Ω, 2 Ω)

$V_{BC} = \frac{20}{9} \times \frac{9V}{29} = \frac{20V}{29}$ ($\frac{20}{9}$ is equivalent resistance of 4 Ω, 5 Ω)

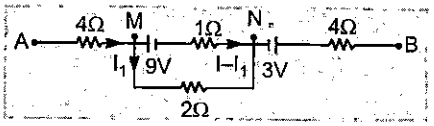
Heat generated in $2\Omega = \left(\frac{9V}{29}\right)^2 \times \frac{1}{2}t$

Heat generation in $4\Omega = \left(\frac{20V}{29}\right)^2 \times \frac{1}{4}t$

∴ Heat generated in 4Ω is max.

19. [b] $V_A - 4I - 9 - (I - I_1) + 3 - 4I = V_B$

$16 - 9I + I_1 - 6 = 0$



$10 = 9I - I_1$ ... (i)

$V_A - 4I - 2I_1 + 3 - 4I = V_B$

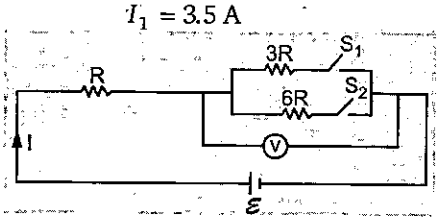
$16 - 8I - 2I_1 + 3 = 0$

$19 = 8I + 2I_1$ ... (ii)

From eqn. (i) and (ii),

$I_1 = 3.5$ A

20. [b]



When S_1 is closed

$$I = \frac{\varepsilon}{(3+1)R} = \frac{\varepsilon}{4R}$$

$$V_1 = \frac{\varepsilon}{4R} \times 3R = \frac{3\varepsilon}{4}$$

When S_2 is closed

$$I = \frac{\varepsilon}{7R} \Rightarrow V_2 = \frac{6\varepsilon}{7}$$

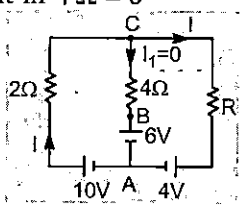
When both are closed equivalent of $6R$ and $3R$ is $2R$.

$$I = \frac{E}{3R}$$

$$V_3 = \frac{E}{3R} \times 2R = \frac{2E}{3}$$

$\Rightarrow$ $V_2 > V_1 > V_3$

21. [a] For current in 4 Ω = 0



$V_C = V_B$

$V_A + 6 = V_B$ ...(i)

$V_A + 10 - 2I = V_C = V_B$ ... (ii)

From eqn. (i) and (ii),

$10 - 2I = 6 \Rightarrow I = 2$A

Also $V_A + 4 + 2 \times R = V_C$ ...(iii)

From eqn. (iii) and (i),

$V_A + 4 + 2R = V_A + 6 \; (V_B = V_C)$

$2R = 2$

$R = 1\,\Omega$

22. [a] In first case

$$18 = \frac{\varepsilon}{6+r} \times 6 \quad \text{... (i)}$$

In second case

$$24 = \frac{\varepsilon}{12+r} \times 12 \quad \text{... (ii)}$$

From eqn. (i) and (ii) $r = 6\,\Omega$

Now let in the case S_1 and S_2 both are closed (Resultant of 12 and 6 Ω will be 4)

$$V = \frac{\varepsilon}{4+r} \times 4 = \frac{\varepsilon \times 4}{10} \quad \text{...(iii)}$$

(as $r = 6\,\Omega$)

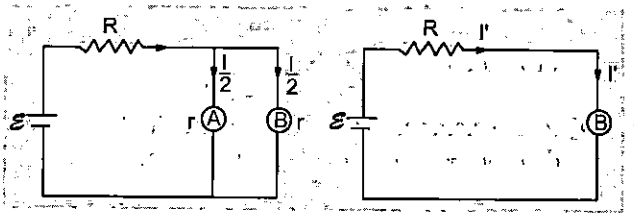
Eqn. (i) divided by eqn. (iii)

$$\frac{18}{V} = \frac{10}{4} \times \frac{6}{12}$$

$$V = \frac{72}{5} = 14.4\,\text{V}$$

23. [d] $R \uparrow I \downarrow \Rightarrow V_B \uparrow$

Alternative :-



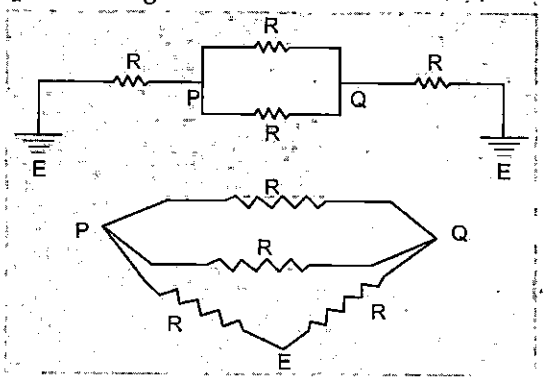
$$P_1 = \left(\frac{I}{2}\right)^2 r = \frac{I^2 r}{4} \quad P_2 = I'^2 r$$

$$P_1 = \left(\frac{\varepsilon}{R + \frac{r}{2}}\right)^2 \frac{r}{4} \; P_2 = \left(\frac{\varepsilon}{R+r}\right)^2 r$$

$$P_1 = \frac{\varepsilon^2 r}{(2R+r)^2} \quad \frac{P_2}{P_1} = \left(\frac{2R+r}{R+r}\right) > 1 \; ; \quad P_2 > P_1$$

So ball B will become brighter.

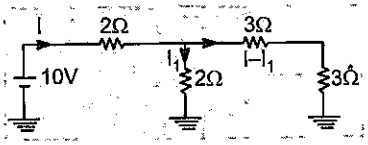
24. [b] Redrawing the circuit



$$R_{\text{equivalent}} = \frac{2R \times \frac{R}{2}}{2R + \frac{R}{2}} = \frac{2}{5}R$$

25. [c] $0 + 10 - 2I - 3(I - I_1) - 3(I - I_1) = 0$

$$10 = 8I - 6I_1 \quad \text{...(i)}$$



Also

$$0 + 10 - 2I - 2I_1 = 0$$

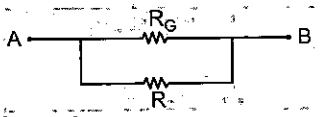
$$2I + 2I_1 = 10 \quad \text{...(ii)}$$

From eqn. (i) and (ii),

$$I_1 = \frac{15}{7}\text{A}, \; I = \frac{20}{7}\text{A}$$

$$\Rightarrow \quad i - i_1 = \frac{5}{7}\text{A}$$

26. [a] Let $V_A - V_B = V$

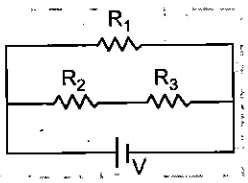


Given, $\dfrac{\frac{V^2}{R_G}}{\frac{V^2}{R}} = \dfrac{3}{4}, \; R = \dfrac{3}{4}R_G$

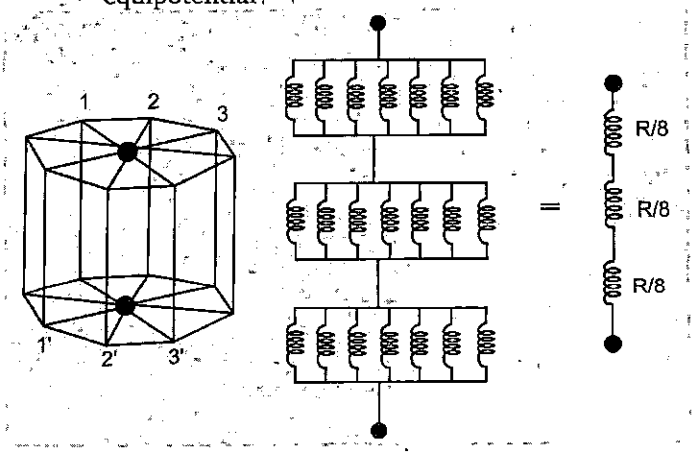
43. [d] Hint: $P_1 = V^2/R;$

$$P_3 = P_2 = \frac{(V/2)^2}{R}$$

$$= \frac{V^2}{4R}; \; \frac{P_1}{P_2} = 4$$

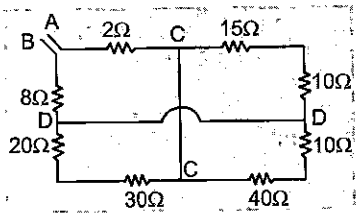


44. [d] Points 1,2,3,.....are equipotential

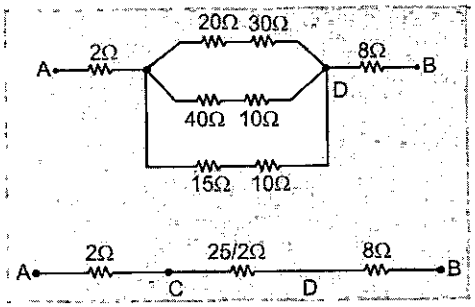


and 1', 2', 3;,.....are also equipotential.

46. [b]



Redrawing the circuit

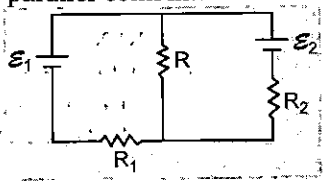


$R_{AB} = 22.5\ \Omega$

48. [d] Sum of charges on plates of capacitor was zero initially and will be zero finally.

∴ No charge is flown to ground.

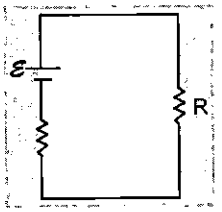
49. [a] This is parallel combination of batteries



$$\therefore \quad \text{equivalent e.m.f.} = \frac{\mathcal{E}_1 R_2 + \mathcal{E}_2 R_1}{R_1 + R_2}$$

$$\text{Resistance} = \frac{R_1 R_2}{R_1 + R_2}$$

The given circuit is



$$\mathcal{E} = \frac{\mathcal{E}_1 R_2 + \mathcal{E}_2 R_1}{R_1 + R_2}$$

and we know that power developed in 'R' in such circuits is max. when R = internal resistance

$$R = \frac{R_1 R_2}{R_1 + R_2}$$

52. [a] Due to decrease in number of collisions resistance will decrease

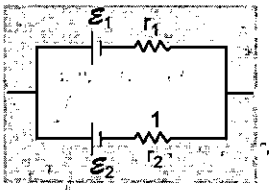
⇒ current will increase

54. [b] During charging positive charge flow from positive terminal to negative terminal.

57. [c] $\text{Equivalent e.m.f.} = \dfrac{\mathcal{E}_1 r_1 + \mathcal{E}_2 r_2}{r_1 + r_2}$,

$$\text{Equivalent resistance} = \frac{r_1 r_2}{r_1 + r_2}$$

Equivalent resistance will be less than either of the two resistance



Now let $\mathcal{E}_1 < \mathcal{E}_2$

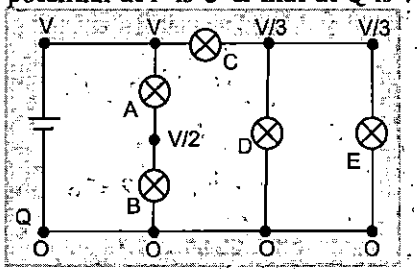
$\Rightarrow$ equivalent e.m.f. $> \dfrac{\mathcal{E}_1 r_1 + \mathcal{E}_1 r_2}{r_1 + r_2} = \mathcal{E}_1$

Also equivalent e.m.f. $< \dfrac{\mathcal{E}_2 r_1 + \mathcal{E}_2 r_2}{r_1 + r_2}$

Equivalent e.m.f. $< \mathcal{E}_2$

i.e. equivalent e.m.f. will be in between $\mathcal{E}_1$ and $\mathcal{E}_2$.

61. [b] Let potential at P is O & that at Q is V

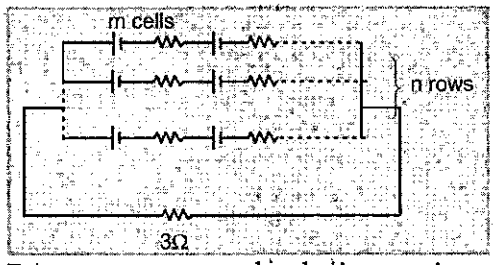


Potential difference across bulb C is maximum.

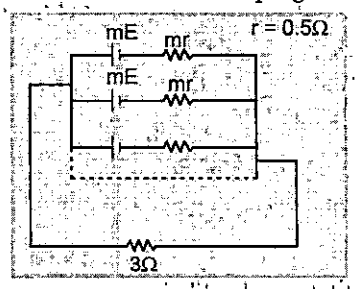
64. [d] Each resistor and capacitor are in parallel *i.e.*, there is no difference in first and last capacitors.

$\therefore$ equal charge will be stored on both capacitors.

67. [a]



For max. current or developing maximum power



$3 = r_{\text{equivalent}}$ (due to all cells) $3\ \Omega$

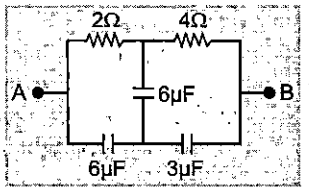
$$3 = \frac{mr}{n} = \frac{m}{2n}$$

On cross checking options $m = 12$, $n = 2$ satisfies.

70. [c] $V_A - \mathcal{E} - Ir = V_B$

$V_A - V_B = \mathcal{E} + Ir > \mathcal{E}$

73. [d] Consider the shown circuit.



Current through resistors

$$I = \frac{\mathcal{E}}{6}$$

$$\therefore \quad V_A - V_C = 2 \times \frac{\mathcal{E}}{6} = \frac{\mathcal{E}}{3} \qquad \text{...(i)}$$

$$V_A - V_B = \frac{\mathcal{E}}{3} \qquad \text{...(ii)}$$

From eqn. (i) and (ii)

$\Rightarrow$ $V_C = V_B$

$\therefore$ if 6 μF is connected through B and C it will be short circuited.

So net capacitance $= \dfrac{6 \times 3}{6+3} = 2\ \mu F$

75. [d]

$$\frac{l_1}{l_2} = \frac{1}{2}, \frac{m_1}{m_2} = 1, \frac{\rho_1}{\rho_2} = \frac{4}{1}$$

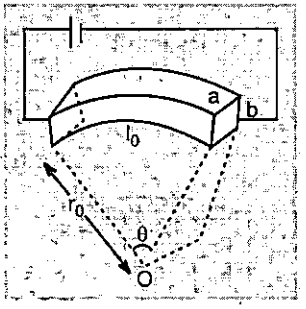
$$\frac{d_1}{d_2} = \frac{1}{2}$$

$$R = \frac{\rho l}{A} = \frac{\rho \times l^2}{A \times l} = \frac{\rho l^2}{V} = \frac{\rho \times l^2}{\left(\frac{m}{d}\right)}$$

$$\frac{R_1}{R_2} = \frac{\rho_1}{\rho_2} \times \left(\frac{l_1}{l_2}\right)^2 \times \frac{d_1}{d_2} \times \frac{m_2}{m_1}$$

$$= \frac{4}{1} \times \left(\frac{1}{2}\right)^2 \times \frac{1}{2} \times \frac{1}{1} = \frac{1}{2}$$

76. (b) Let us consider an elemental portion of the resistor. The element consider is a circular arc of radius r and thickness dr. The resistance of this element would be



$$dR = \frac{\rho \times r\theta}{a\,dr}$$

where $l_0 = r_0\theta$

So, $dR = \dfrac{\rho r \times l_0}{a r_0 \times dr}$

If we divide the entire resistor in these elemental portions, then these elemental resistors are joined in parallel, equivalent resista of which is given by

$$\frac{1}{R} = \int \frac{1}{dR} = \int_{r_0}^{r_0+b} \frac{a r_0 dr}{\rho l_0 r}$$

$$= \frac{a r_0}{\rho l_0} \times \ln \frac{r_0 + b}{r_0}$$

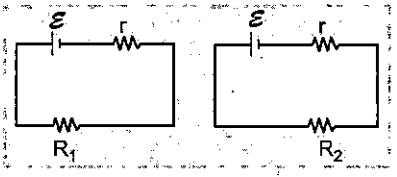
$$\Rightarrow \quad R = \frac{\rho l_0}{a r_0 \ln\left[1 + \frac{b}{r_0}\right]}$$

77. (d) Using KCL and KVL we can find the potential difference across capacitor which comes out to be 20 V.

So energy stored in capacitor is,

$$U = \frac{4 \times 10^{-6} \times 20^2}{2} = 800 \mu J$$

78. [c]



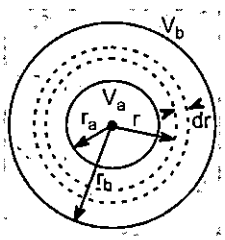
$$P_1 = \frac{\varepsilon^2}{(R_1 + r)^2} \times R_1$$

$$= P_2 \frac{\varepsilon_2}{(R_2 + r)^2} \times R_2$$

$$\Rightarrow \left(\frac{R_2 + r}{R_1 + r}\right)^2 = \frac{R_2}{R_1}$$

On solving $r = \sqrt{R_1 R_2}$

79. (c) Consider a shell of radius r and thickness dr as shown in figure. Resistance of this shell is



$dR = \frac{\rho\, dr}{4\pi r^2}$ where ρ is resistivity of conducting material.

$$R = \int dR = \int_{r_a}^{r_b} \frac{\rho\, dr}{4\pi r^2}$$

$$= \frac{\rho}{4\pi}\left[\frac{1}{r_a} - \frac{1}{r_b}\right]$$

$$I = \frac{V_a - V_b}{R}$$

The electric field intensity at the location of shell is, $E = \frac{\rho I}{4\pi r^2}$

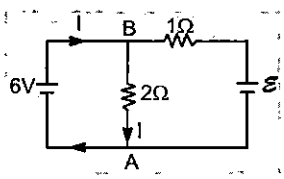
$$E = \frac{V_a - V_b}{\left(\frac{1}{r_a} - \frac{1}{r_b}\right)} \times \frac{1}{r^2}$$

As current is zero for $r < r_0$ and $r > r_b$ and hence E.

80. [b] Current through 1 Ω = 0

Current through

$$2\,\Omega = \frac{6}{2} = 3\,A$$



$$V_A + \varepsilon = V_B$$

$$V_A + 2 \times 3 = V_B$$

$$\Rightarrow \quad \varepsilon = 6\,V$$

81. (a) When two cells are connected in series, $I = \frac{\varepsilon_1 + \varepsilon_2}{r_1 + r_2 + R}$. When resistor is connected across battery of emf ε_1, $I_1 = \frac{\varepsilon_1}{r_1 + R}$

When resistor is connected across battery of emf ε_2, $I_2 = \frac{\varepsilon_2}{r_2 + R}$

For required condition to be fulfilled.

$$I < I_1$$

and $$I < I_2$$

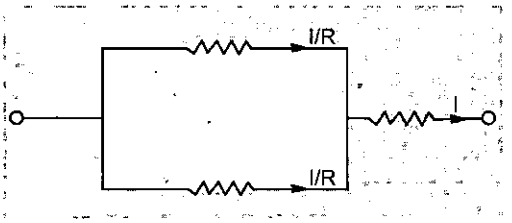
$$\Rightarrow \quad \frac{\varepsilon_2}{\varepsilon_1} < \frac{r_2}{r_1 + R} \quad \text{and} \quad \frac{\varepsilon_2}{\varepsilon_1} > \frac{r_2 + R}{r_1}$$

82. [d] To increase the range upto n times, the resistance S (shunt) should be such that

$$I_g = I / n$$

$$\therefore \quad S = \frac{(I/n)R_0}{I - \frac{I}{n}} = \frac{R_0}{n-1}$$

83. [d] $P = I^2 R = I^2 2 = 18$



$$I = 3A$$

$$P_{net} = \left[\left(\frac{I}{2}\right)^2 R\right] 2 + [I^2 R] = \frac{I^2 R}{4q} + I^2 R$$

$$= \frac{18}{2} + 18 = 27$$

84. [a] $$\frac{x}{R_0} = \frac{l}{100 - l}$$

Since null point remains unchanged

$$\frac{x}{R'} = \frac{40}{60}$$

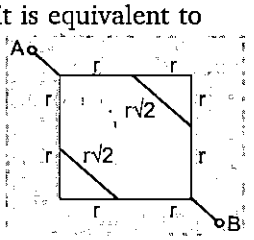
$$\Rightarrow \quad R' = 6\Omega$$

And $$6 = \frac{100 R_t}{R_t + 100}$$

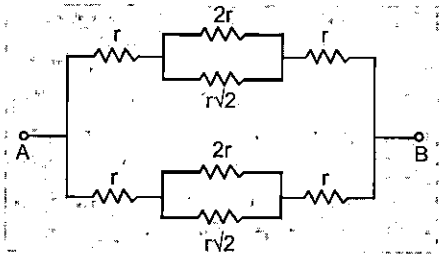
$$\Rightarrow \quad R_t = 6.38\Omega$$

$$\alpha = \frac{R_t - R_0}{R_0 t} = 6.3 \times 10^{-4}\ K^{-1}$$

85. [a] The circuit is equivalent to



Let each half side has resistance $r\,(=\rho d / 2)$

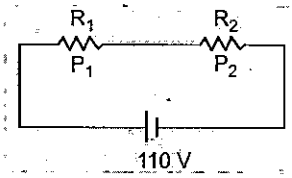


$$\therefore \quad R=\frac{1}{2}\left[2r+\frac{(2r)(r\sqrt{2})}{(2+\sqrt{2})r}\right]=r\sqrt{2}\ \text{(on solving)}$$

$$\therefore \quad R=\rho d/\sqrt{2}$$

86. [c] $R=\dfrac{V^2}{P}$

$$\therefore \quad R_1=\frac{110\times110}{50} \quad R_2=\frac{110\times110}{100}$$



$$I=\frac{110}{3R}=\frac{110\times110}{3\times110\times110}=\frac{10}{33}\text{ amp.}$$

$$P_1=I^2\times R_1=\frac{10}{33}\times\frac{10}{33}\times\frac{110\times110}{50}$$

$$=\frac{200}{9}\approx22\text{ W}$$

87. [c] Let R_A and R_B are resistances

$$R_A=\rho\frac{l_A}{A_A}$$

and $$R_B=\rho\frac{l_B}{A_B}$$

$$\frac{R_A}{R_B}=\frac{l_A}{l_B}\times\frac{A_B}{A_A}=\left(\frac{A_B}{A_A}\right)^2$$

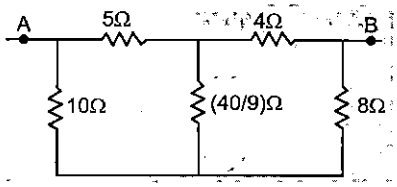
$$\left[\because l_AA_A=l_BA_B\Rightarrow\frac{A_B}{A_A}=\frac{l_A}{l_B}\right]$$

$$=\frac{(\pi\times9r^2)^2}{(\pi r^2)^2}=\frac{81}{1}$$

In series $H_A:H_B=R_A:R_B=81:1$

88. [b] In parallel $H_A:H_B=R_B:R_A=\dfrac{1}{81}:\dfrac{1}{1}=1:81$

89. [a] The given circuit is equivalent to



As $10\times4=5\times8$ this is balanced Wheatstone network

Therefore $$R=\frac{(5+4)\times(10+8)}{9+18}\ 6\Omega$$

90. [a] Ohm's law $V=IR\ \Rightarrow V\propto I$ (for ammeter)

$$\therefore \quad \frac{V_2}{V_1}=\frac{I_2}{I_1}=2$$

$$\Rightarrow \quad 10=2\times10\times\frac{R}{2+R}$$

$$\Rightarrow \quad R=2\Omega$$

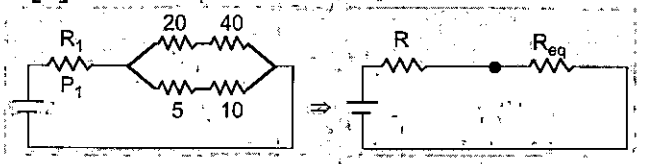
92. [a] Let n be the average no of electrons per unit volume, present in the beam of electrons, then impulse-momentum theorem gives

$$(nAv)(mv)=F, \qquad P=F/A$$

$$\Rightarrow \quad n=P/(mv^2).$$

$$\therefore \quad I=\nu Av\varepsilon=\frac{A\Pi\varepsilon}{\mu v}$$

93. [a]



Here $R_{eq}=12$

For maximum $P; R=R_{eq}=12$

97. [d] Junction rule

$$I_3=I_2+I_1$$

loop $DCEFD$

$$-\mathcal{E}_2-I_1R_1-I_3R_3=0$$

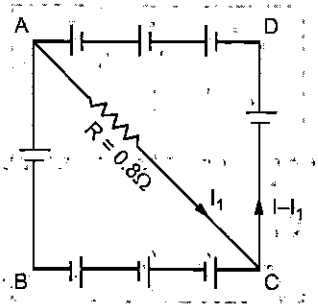
loop $ABEFA\ -\mathcal{E}_1-I_3R_3-I_2R_2=0$

99. [c] For loop $CDAC$, $4(1.0)-4(0.2)I-0.8I_1=0$

$$\Rightarrow \quad 5=I-I_1 \qquad \ldots(i)$$

For loop $ABCA$, $4(1)-4(0.2)(I-I_1)+0.8I_1=0$

$$\Rightarrow \quad 5=1-2I_1 \qquad \ldots(ii)$$



Using eqn. (i) and (ii), $I_1=0$ and $I=5$

$\therefore$ V = Voltage across any battery-

$$=\mathcal{E}-Ir=1.0-5(0.2)=0\text{ V}$$

100. [c] $$R=\frac{\rho l_1}{A_1} \qquad [\rho=\text{resistivity of the wire}]$$

$$l_2=nl_1$$

$$A_1l_1=A_2l_2$$

$$\Rightarrow \quad A_2=\frac{A_1}{l_2/l_1}=A_1/n$$

New resistance of the wire $= R' = \frac{\rho l_2}{A_2} = n^2 R$

Resistance of each wire $= n^2 R / 5$

Since the wires are arranged to form balanced Wheatstone's bridge

$$R_{AB} = \frac{(n^2R/5)+(n^2R/5)}{2} = \frac{n^2R}{5}$$

101. [a] $I = 2/(2+2) = (1/2)\text{A}$

$\therefore \quad \varsigma_A - \varsigma_{II} = \Xi I = \Xi/2 \quad \ldots(i)$

Also, $2 - 2I_1 + I - I_1 = 0$

$\Rightarrow \quad I_1 = 1\text{A}$

$V_A - V_P = I_1 - 1 = 1 - 1 = 0 \quad \ldots(ii)$

using eqn. (i) and (ii), $x = 0$

Point A and P are at same potential.

103. [b] $I = 1\text{A}$

R = internal resistance of the body

$\therefore \quad 12 = (X + Y + r)\,(1) \quad \ldots(1)$

Also $1 = (X)(1) \Rightarrow X = 1\Omega$

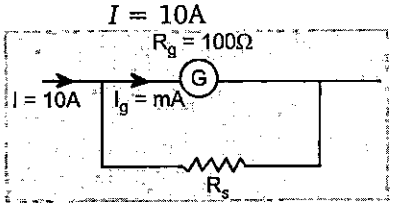
And 10 = voltage across battery = voltage across X (when A and B are shorted)

$\therefore \quad 10 = \left(\frac{12}{X+r}\right)X \Rightarrow 10 = \frac{12}{1+r} \Rightarrow 1 + r = \frac{6}{5}$

$\therefore \quad r = 0.2\Omega$

105. [c] As all bulbs are identical, the power (related to brightness) is most useful as $P = I^2R$, so the resistor with the most current will be brightest. As bulb C has the same current as that through the battery, while the branches with bulbs A, B and D are in parallel and have the total current split between the branches, the current through bulb C is greatest.

106. [b] $I = 10\text{A}$



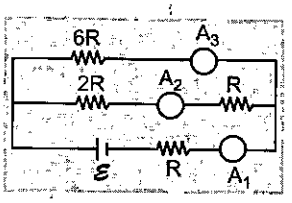
$$I_g R_g \approx I R_s$$

$$R_s = \frac{I_g R_g}{I}$$

$$= \frac{10 \times 10^{-3} \times 100}{10} = 0.1\Omega$$

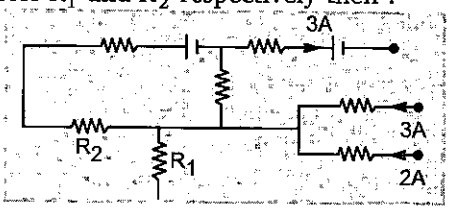
Level 2 More than One Alternative is/are Correct

1. In the given circuit ammeters are ideal then, which of the following statements are true?



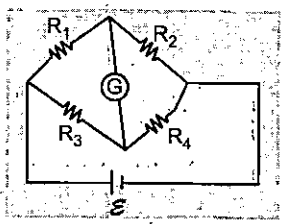
(a) Reading of A_3 will be half as shown by A_2
(b) Reading of A_1 will be thrice as shown by A_2
(c) Reading of A_3 will be lowest
(d) Reading of A_1 will be thrice as shown by A_3

2. In the given circuit if I_1 and I_2 be the current in resistors R_1 and R_2 respectively then :



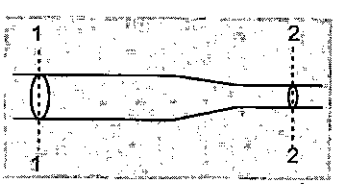
(a) $I_1 = 3A$, $I_2 = 2A$
(b) $I_1 = 0$, $I_2 = 2A$
(c) $I_1 = 2A$, $I_2 = 2A$
(d) $I_1 = 2A$ and I_2 can't be determined with given data

3. In the given circuit $R_1R_4 = R_2R_3$. Then choose the correct statement :



(a) If positions of battery and galvanometer are interchanged then galvanometer will still show zero deflection.
(b) Rate of heat dissipation through R_1 will change if position of battery and galvanometer are inter changed.
(c) If e.m.f. $\mathcal{E}$ is doubled still no deflection is shown by galvanometer.
(d) If galvanometer is not ideal it will show deflection if e.m.f. is doubled.
(d) Initial rate of heat dissipation in 1 will be less than in 2.

4. The area of cross section of a current carrying conductor is A_0 and $\frac{A_0}{4}$ at section (1) and (2) respectively. If $v_{\alpha_1}, v_{\alpha_2}$ and E_1, E_2 be the drift velocity and electric field at sections 1 and 2 respectively then :



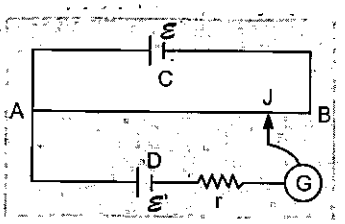
(a) $v_{\alpha_1} : v_{\alpha_2} = 1 : 4$ (b) $v_{\alpha_1} : v_{\alpha_2} = 4 : 1$
(c) $E_1 : E_2 = 4 : 1$ (d) $E_1 : E_2 = 1 : 4$

5. A cuboids has longest dimension double of the shortest dimension. Then the ratio of maximum to minimum resistance across its parallel faces is :
(a) 2 : 1 (b) 4 : 1
(c) 8 : 1 (d) 3 : 2

6. If temperature for a metallic conductor increases then which of the following quantity decreases :
(a) μ_e (b) ρ
(c) σ (d) V_d

7. A voltmeter and an ammeter are joined in series to an ideal cell, giving readings V and A respectively. If a resistance equal to resistance of ammeter is now joined parallel to ammeter then :
(a) V will decrease slightly
(b) V will increase slightly
(c) A will become half of its initial value
(d) A will become slightly more than half of its initial value

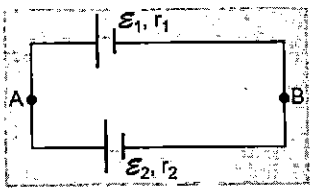
8. The figure shows a potentiometer arrangement $\mathcal{E}$ is e.m.f. of driving cell. $\mathcal{E}'$ is to be determined. Then which of the following are essential condition?



(a) $\mathcal{E}$ must be greater than $\mathcal{E}'$
(b) The positive terminals of C and D must be joined to A only
(c) Either the +ve terminal of C and D or –ve terminal of both C and D must be joined to A
(d) The resistance r must be smaller than total resistance of wire AB

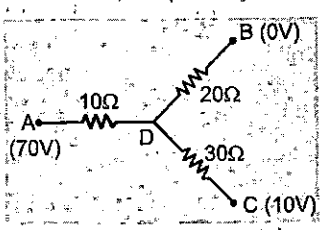
9. A uniform wire of resistance R is shaped into a regular n-sided polygon (n is even). The equivalent resistance between any two corners can have :
(a) the maximum value $\frac{R}{4}$
(b) the maximum value $\frac{R}{n}$
(c) the minimum value $R\left(\frac{n-1}{n^2}\right)$
(d) the minimum value $\frac{R}{n}$

10. Two cells of unequal emfs, $\mathcal{E}_1$ and $\mathcal{E}_2$ and internal resistances r_1 and r_2 are joined as shown. V_A and V_B are the potentials at A and B respectively :



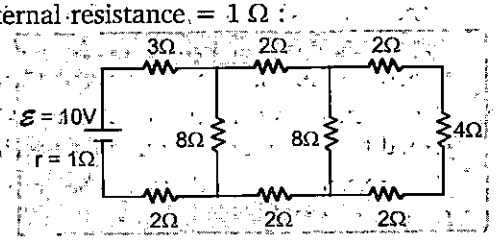
(a) One cell will continuously supply energy to the other.
(b) The potential difference across both the cells will be equal.
(c) The potential difference across one cell will be greater than its emf.
(d) $V_A - V_B = \dfrac{(\mathcal{E}_1 r_2 + \mathcal{E}_2 r_1)}{(r_1 + r_2)}$

11. In the network shown, points, A, B and C are at potentials of 70 V, zero and 10 V respectively.



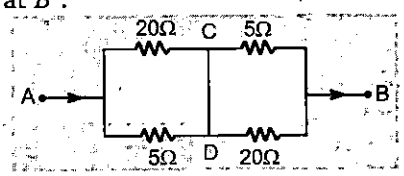
(a) Point D is at a potential of 40 V :
(b) The currents in the sections AD, DB, DC are in the ratio 3 : 2 : 1.
(c) The currents in the sections AD, DB, DC are in the ratio 1 : 2 : 3.
(d) The network draws a total power of 200 W.

12. In the circuit shown, the cell has emf = 10 V and internal resistance = 1 Ω :



(a) The current through the 3 Ω resistor is 1 A
(b) The current through the 3 Ω resistor is 0.5 A
(c) The current through the 4 Ω resistor is 0.5 A
(d) The current through the 4 Ω resistor is 0.25 A

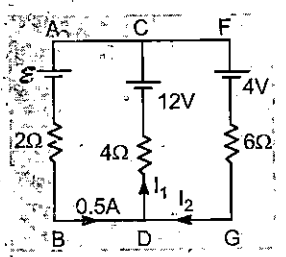
13. When some potential difference " is maintained between A and B, current I enters the network at A and leaves at B :



(a) The equivalent resistance between A and B is 8 Ω
(b) C and D are at the same potential
(c) No current flows between C and D
(d) Current $\dfrac{3I}{5}$ flows from D to C

14. A galvanometer has a resistance of 100 Ω and a full scale range of 50 μ A. It can be used as a voltmeter or as a higher range ammeter provided a resistance is added to it. Select the correct range of resistance combination(s) :
(a) 50 V range with 10 k Ω resistance in series
(b) 10 V range with 200 k Ω resistance in series
(c) 5 mA range with 1 Ω resistance in parallel
(d) 10 mA range with 1 Ω resistance in parallel

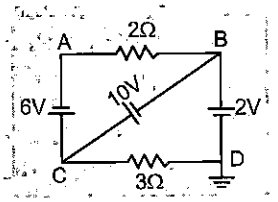
15. In the circuit shown in the figure :



(a) $\mathcal{E} = 6.6$ V
(b) $I_1 = 1.1$ A
(c) $I_2 = 0.5$ A
(d) all of these

16. Voltmeter reads the potential difference between the terminals of an old battery as 1.4 V while a potentiometer reads its voltage to be 1.55 V. The voltmeter resistance is 280 Ω. Then
(a) the battery of the cell is 1.4 V
(b) the battery of the cell is 1.55 V
(c) the internal resistance r of the battery is 30 Ω
(d) the internal resistance r of the battery is 5 Ω

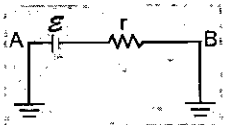
17. In the shown circuit :



(a) current passing through 2 Ω resistance is 2A
(b) current passing through 3 Ω resistance is 4A
(c) current in wire D to earth is zero
(d) potential of point A is 10 V

18. Two concentric metallic shells of radius R and $2R$, out of which the inner shell is having charge Q and outer shell is uncharged. If they are connected with a conducting wire, then:
(a) Q amount of charge flow from inner to outer shell
(b) $\dfrac{Q}{e}$ number of electrons flow from outer to inner shell where $e = 1.6 \times 10^{-19}$C
(c) $\dfrac{KQ^2}{4R}$ amount of heat is produced in the wire
(d) $\dfrac{KQ^2}{2R}$ amount of heat is produced in the wire

19. Both terminals of a battery of emf $\mathcal{E}$ and internal resistance r are grounded as shown. Select the correct alternative(s) :

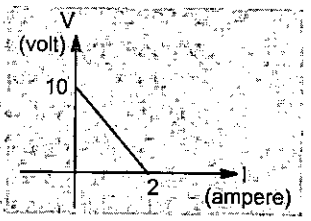


(a) Potential difference across A and B is zero
(b) Potential difference across A and B is $\mathcal{E}$
(c) Current across AB is zero
(d) Current across AB is $\dfrac{\mathcal{E}}{r}$

20. A current passes through a wire of non-uniform cross-section. Which of the following quantities are independent of the cross-section?
(a) the charge crossing in a given time interval
(b) drift speed
(c) current density
(d) free-electron density

21. A battery is of emf $\mathcal{E}$ is being charged from a charger such that positive terminal of the battery is connected to terminal A of charger and negative terminal of the battery is connected to terminal B of charger. The internal resistance of the battery is r.
(a) Potential difference across points A and B must be more than $\mathcal{E}$.
(b) A must be at higher potential than B
(c) In battery, current flows from positive terminal to the negative terminal
(d) No current flows through battery

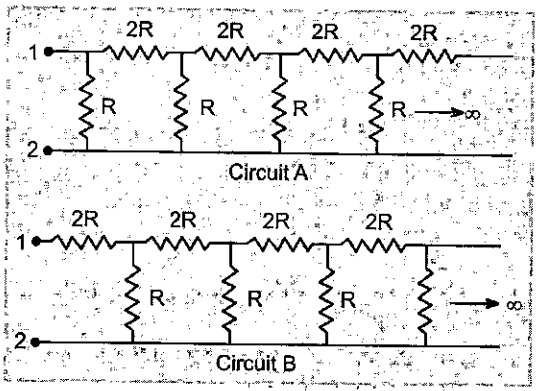
22. A battery of emf $\mathcal{E}$ and internal resistance r is connected across a resistance R. Resistance R can be adjusted to any value greater than or equal to zero. A graph is plotted between the current (I) passing through the resistance and potential difference (V) across it. Select the correct alternative (s):



(a) internal resistance of battery is 5Ω
(b) emf of the battery is 20 V
(c) maximum current which can be taken from the battery is 4 A
(d) V - I graph can never be a straight line as shown in figure.

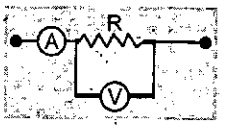
23. Two identical fuses are rated at 10A. If they are joined:
(a) in parallel, the combination acts as a fuse of rating 20A
(b) in parallel, the combination acts as a fuse of rating 5A
(c) in series, the combination acts as a fuse of rating 10A
(d) in series, the combination acts as a fuse of rating 20A

24. Two circuits (shown below) are called 'Circuit A' and 'Circuit B'. The equivalent resistance of 'Circuit a' is x and that of 'Circuit B' is y between 1 and 2.



(a) $y > x$ (b) $y = (\sqrt{3}+1)R$
(c) $xy = 2R^2$ (d) $x - y = 2R$

25. In the circuit shows the readings of ammeter and voltmeter are 4A and 20V respectively. The meters are non ideal, the R is:

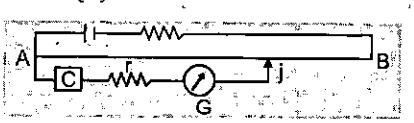


(a) 5Ω (b) less than 5Ω
(c) greater than 5Ω (d) between 4Ω & 5Ω

26. In a potentiometer arrangement. $\mathcal{E}_1$ is the cell establishing current in primary circuit. $\mathcal{E}_2$ Is the cell to be measured. A B is the potentiometer wire and G is a galvanometer. Which of the following are the essential condition for balance to be obtained.
(a) The emf of $\mathcal{E}_1$ must be grater than the emf of $\mathcal{E}_2$.
(b) Either the positive terminals of both $\mathcal{E}_1$ and $\mathcal{E}_2$ or the negative terminals of both $\mathcal{E}_1$ and $\mathcal{E}_2$ must be joined to one end of potentiometer wire.
(c) The positive terminals of $\mathcal{E}_1$ and $\mathcal{E}_2$ must be joined to one end of potentiometer wire.
(d) The resistance of G must be less than the resistance of AB

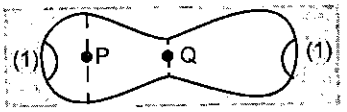
27. In a potentiometer wire experiment the emf of a battery in the primary circuit is 20 V and its internal resistance is 5Ω. There is a resistance box in series with the battery and the potentiometer wire, whose resistance can be varied from 120Ω to 170Ω. Resistance of the potentiometer wire is 75Ω. The following potential differences can be measured using this potentiometer.
(a) 5 V (b) 6 V
(c) 7 V (d) 8 V

28. In the given potentiometer circuit, the resistance of the potentiometer wire AB is R_0. C is a cell of internal resistance r. The galvanometer G does not give zero deflection for any position of the jockey. J which of the following cannot be a reason for this?



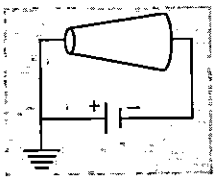
(a) $r > R_0$
(b) $R >> R_0$
(c) emf of C > emf of D
(d) The negative terminal of C is connected to A

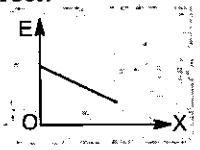
29. A metallic conductor of irregular cross-section is as shown in the figure. A constant potential difference is applied across the ends (1) and (2). Then:

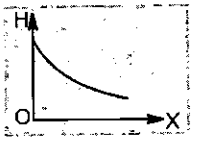


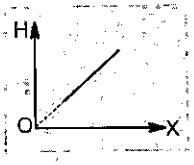
(a) the current at the cross-section P equals the current at the cross-section Q
(b) the electric field intensity at P is less than that at Q
(c) the rate of heat generated per unit time at Q is greater than that at P
(d) the number of electrons crossing per unit area of cross-section at P is less than that at Q.

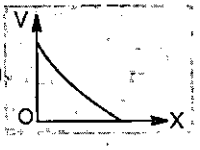
30. A conductor is made of an isotropic material and has shape of a truncated cone. A battery of constant emf is connected across it and its left end is earthed as shown in figure. If at a section distant x from left and, electric field intensity, potential and the rate of generation of heat per unit length are E, V and H respectively, which of the following graphs is/are correct?



(a) 

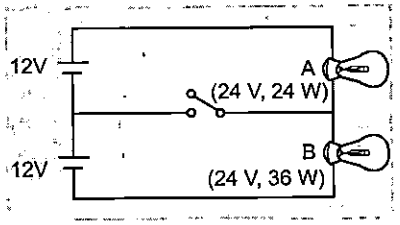
(b) 

(c) 

(d) 

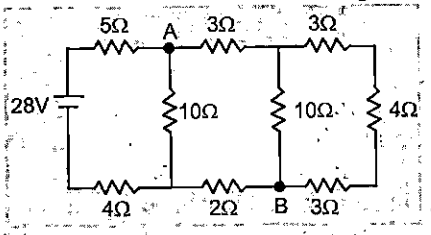
31. Two bulbs consume same power when operated at 200 V and 300 V respectively. When the two bulbs are connected in series across a DC source of 500 V then :
(a) ratio of potential difference across them is 3 : 2
(b) ratio of potential difference across them is 4 : 9
(c) ratio of power consume across them is 4 : 9
(d) ratio of power consume across them is 2 : 3

32. Two light bulbs shown in the circuit have ratings A(24 V, 24 W) and B(24 V and 36 W) as shown. When the switch is closed:



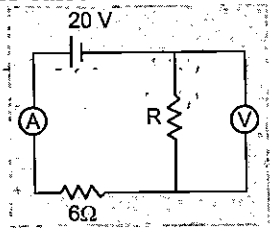
(a) the intensity of light bulb A increases
(b) the intensity of light bulb A decreases
(c) the intensity of light bulb B increases
(d) the intensity of light bulb B decreases

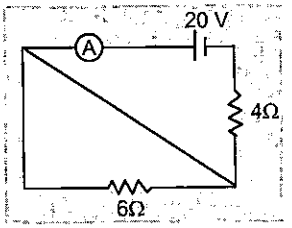
33. Consider the circuit shown in the Fig.

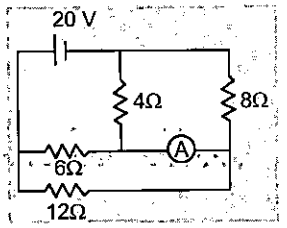


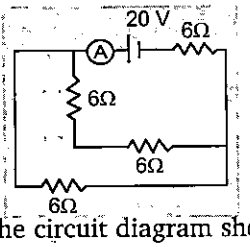
(a) the current in the 5Ω resistor is 2A
(b) the current is the 5Ω resistor is 1A
(c) the potential difference $V_A - V_B$ is 8V
(d) the potential difference $V_A - V_B$ is − 5V

34. The ammeter connected in following circuits has zero resistance. The voltmeter in (B) has infinite resistance and a reading 8V. The value of resistance R has not been specified. Which of the following circuit(s) has same current in the ammeter?

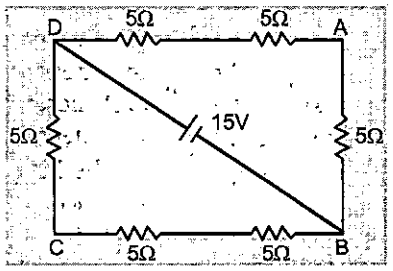
(a) 

(b) 

(c) 

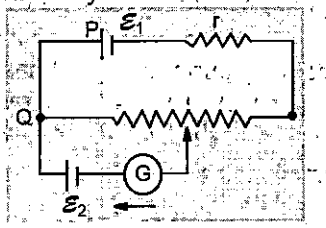
(d) 

35. In the circuit diagram shown in the Fig. Which of the following is true:



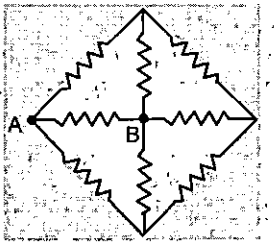
(a) The points A and C are at the same potential
(b) A is at a higher potential than C
(c) Magnitude of p.d. between A and C is 5 volt
(d) C is at higher potential than A

36. In the potentiometer circuit of given Fig. the galvanometer reveals a current in the direction shown wherever the sliding contact touches the wire. This could be caused by:



(a) $\mathcal{E}_1$ being too low (b) r being too high
(c) a break in PQ (d) $\mathcal{E}_2$ being too low

37. In the circuit diagram each resistor of resistance 5Ω. The points A and B are connected to the terminals of a cell of emf 9 volt and internal resistance 2/3 Ω.



(a) The rate at which heat is produced in the cell is 6W.
(b) The current in the resistor connected directly between A and B is 1.2A.
(c) The current in the resistor connected directly between A and B is 1.8A.
(d) None of the above

38. Which of the following does not have the same dimensions as the henry?

(a) $\frac{\text{joule}}{(\text{ampere})^2}$ (b) $\frac{\text{tesla}\cdot \text{m}^2}{(\text{ampere})^2}$
(c) ohm-second (d) $\frac{1}{\text{farad}\cdot\text{second}}$

ANSWERS

Level-2: More than One Alternatives are Correct

1.	(a, c, d)	2.	(d)	3.	(a, b, c)	4.	(a, d)	5.	(b)	6.	(a, c, d)
7.	(b, d)	8.	(a, c)	9.	(a, c)	10.	(a, b, c, d)	11.	(a, b, d)	12.	(a, d)
13.	(a, b, d)	14.	(b, c)	15.	(a, b)	16.	(b, c)	17.	(a, b, c)	18.	(a, b, c)
19.	(a, d)	20.	(a, d)	21.	(a, b, c)	22.	(a)	23.	(a, c)	24.	(a, b, c)
25.	(c)	26.	(a, b)	27.	(a, b, c)	28.	(a)	29.	(a, b, c, d)	30.	(b, c)
31.	(b, c)	32.	(b, c)	33.	(a, b, c)	34.	(b, d)	35.	(c, d)	36.	(a, b, c)
37.	(a, b)	38.	(b, d)								

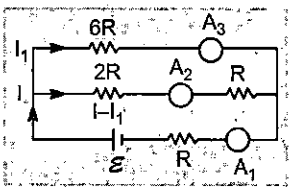
Level-2: More than One Alternatives are Correct

1. [a, c, d]

From Kirchhoff's law

$$-6RI_1 - IR + \mathcal{E} = 0 \quad ...(i)$$

$$-2R(I - I_1) - (I - I_1)R - IR + \mathcal{E} = 0$$



On solving $I_1 = \frac{I}{3}$

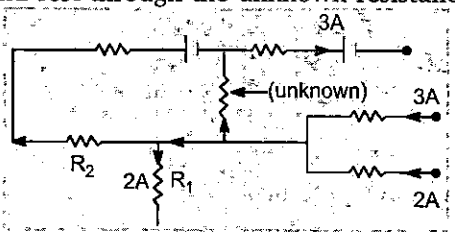
$$I - I_1 = \frac{2I}{3}$$

$\Rightarrow$ Reading of $A_1 = I$

$\therefore$ Reading of $A_2 = \frac{2I}{3}$

Reading of $A_3 = \frac{1}{3}$

2. [d] Since 3A is in upper part of circuit. Therefore out of 5A coming in lower part, 3A has to go to the upper part. Out of which some part will flow through R_2 and rest through the unknown resistance.

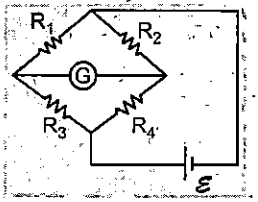


$\therefore$ 2A will go through R_1.

3. [a, b, c]

On interchanging Galvanometer and battery

Since still $R_1R_4 = R_2R_3$ (wheatstone bridge condition is satisfied)



$\therefore$ No deflection

Initially current through R_1 was

$$I_1 = \frac{\mathcal{E}}{R_1 + R_2}$$

Now current through R_1 after interchanging is

$$I_2 = \frac{\mathcal{E}}{R_1 + R_3}$$

Since current has changed.

$\therefore$ Rate of heat dissipation changes.

Since current through galvanometer is zero.

$\therefore$ It will always show zero deflection.

4. [a, d]

Current does not depend on area

$\Rightarrow$ Current will be same through the two sections

$$\Rightarrow \quad neA_0 \times v_{\alpha_1} = ne\frac{A_0}{4} v_{\alpha_2}$$

$$\Rightarrow \quad \frac{v_{\alpha_1}}{v_{\alpha_2}} = \frac{1}{4}$$

Also $$v_\alpha = \frac{eE\tau}{m_e} \Rightarrow \frac{v_{\alpha_1}}{v_{\alpha_2}} = \frac{E_1}{E_2}$$

5. [b] $$R_{max} = \frac{R \cdot 2a}{6 \cdot a}$$

$$R_{min} = \frac{R \cdot a}{2a \cdot b}$$

$$\Rightarrow \quad \frac{R_{max}}{R_{min}} = \frac{4}{1}$$

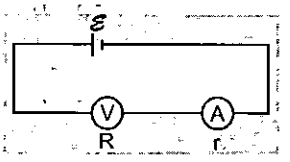
7. [b, d]

In first case

$$V = \frac{\mathcal{E} \times R}{R + r} \quad \text{and} \quad A = \frac{\mathcal{E}}{R + r}$$

In second case

$$V = \frac{\mathcal{E} \times R}{R + \frac{r}{2}}$$



$$A = \left(\frac{\mathcal{E}}{R + \frac{r}{2}}\right) \times \frac{r}{2}$$

$$= \frac{\mathcal{E}}{2R + r} > \frac{\mathcal{E}}{2(R + r)}$$

Clearly V increases and A becomes slightly more then half of its initial value.

8. [a, c]

If $\mathcal{E}$ is less than $\mathcal{E}'$ then no neutral point will be found. Since potential drop across whole wire AB is $\mathcal{E}$ which is less than $\mathcal{E}'$.

If different terminals of C and D are joined to either of A and B. Starting from A if potential increase along AB then it will decrease in wire containing 'r' by $\mathcal{E}$.

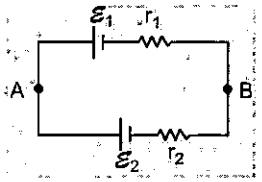
No neutral point will be obtained.

9. [a, c]

The resistance will be maximum if it is divided into two equals parts *i.e.*, $\frac{R}{2}$ each, and it will be minimum if it is obtained across one of the sides; because if two resistances are connected in parallel, the resultant is smaller than the smallest resistor.

10. [a, b, c, d]

Let $\mathcal{E}_1 < \mathcal{E}_2$



$$\Rightarrow \quad I = \frac{\mathcal{E}_2 - \mathcal{E}_1}{r_1 + r_2}$$

$$V_A - \mathcal{E}_1 - Ir_1 = V_B$$

$$V_A - V_B = \mathcal{E}_1 + Ir_1$$

$$= \mathcal{E}_1 + \frac{(\mathcal{E}_2 - \mathcal{E}_1)r_1}{r_1 + r_2}$$

$$V_A - V_B = \frac{\mathcal{E}_1 r_2 + \mathcal{E}_2 r_2}{r_1 + r_2}$$

Potential difference across each cell will be

$$V_A - V_B.$$

i.e., equal also $V_A - V_B = \mathcal{E}_1 + Ir_1 > \mathcal{E}_1$

Cell of e.m.f. $\mathcal{E}_1$ is absorbing energy provided by cell of e.m.f. $\mathcal{E}_2$.

11. [a, b, d]

Let V = potential at D

$$70 - D = 10I_1$$

$$V - 0 = 20I_2$$

$$V - 10 = 30(I_1 - I_2)$$

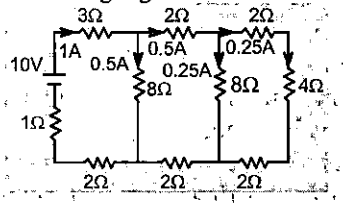
Solve for I_1, I_2 and V

12. [a, d]

It is obvious that net resistance = 9 Ω + internal resistance (1Ω) = 10 Ω

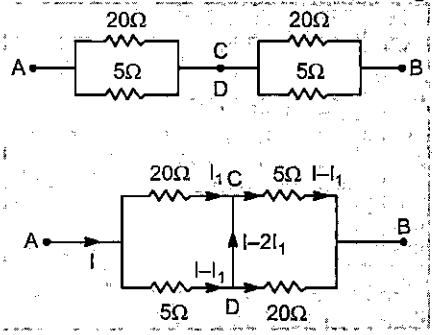
So, current through cell or 3Ω resistor $= \frac{10}{10} = 1$A.

Current distribution in different resistors is shown in the following figure.



13. [a, b, d]

As C and D are joined, they must be at the same potential, and may be treated as the same point. This gives the equivalent resistance as 8 Ω. If we distribute current in the network, using symmetry.



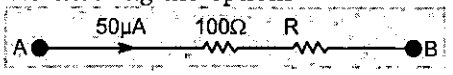
$$V_A - V_D = V_A - V_C$$

or $\quad 20I_1 - 5(I - I_1) = 0$ or $I_1 = \frac{I}{5}$

$$\therefore \; I - 2I_1 = I - \frac{2I}{5} = \frac{3I}{5} = \text{current flowing from } D \text{ to } C.$$

14. [b, c]

For converting it to voltmeter 'R' is joined in series cross checking the options



$$V_{AB} = 50(100 + R)$$

For $\quad R = 10 \times 10^3 \Omega$

$$V_{AB} = 50(100 + 100 \times 100) \times 10^{-6} \text{ V}$$

$$= 50 \times 100 \times 100 \times 10^{-6}$$

$$= 50 \times 10^{-2}$$

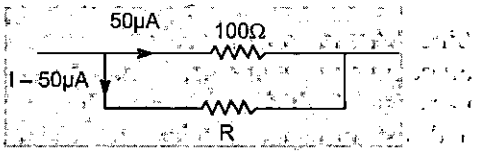
For $\quad R = 200 \times 10^3 \Omega$

$$V_{AB} = 50 \times 10^{-6}(100 + 200 \times 10^3)$$

$$= 50 \times 10^{-6} \times 2 \times 10^5$$

$$= 10 \text{ V}$$

For converting to ammeter low resistance in parallel



For $\quad R = 1, \; I = 5 \times 10^{-3}$

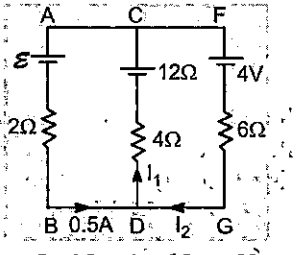
$5 \times 10^{-3} \times 1 = 50 \times 10^{-6} \times 100$

$5 \times 10^{-3} = 5 \times 10^{-3}$ *i.e.*, it satisfied.

15. [a, b]

$$I_1 = I_2 + 0.5 \qquad \ldots(i)$$

Using Kirchhoff's law



$$V_D - 4I_1 + I \times 12 - 4 - 6I_2 = V_D$$

$$4I_1 + 6I_2 = 8 \qquad \ldots(ii)$$

From eqn. (i) and (ii),

$$I_1 = 1.1 \text{ A}, \; I_2 = 0.6 \text{ A}$$

Now

$$V_B - 4(1.1) + 12 - \mathcal{E} - 2 \times 0.5 = V_B$$

$$\mathcal{E} = 12 - 1 - 4.4$$

$$\mathcal{E} = 6.6 \text{ V}$$

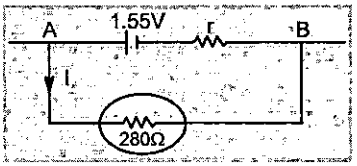
16. [b, c]

Potentiometer reads the voltage of battery and voltmeter reads the potential across the terminals of battery

$\therefore$ battery of cell is of 1.55 V

$$V_{AB} = 1.4 \text{ V}$$

$$I = \frac{1.55}{280 + r}$$



$$1.4V = 280 \times \frac{1.55}{280 + r}$$

$$r \times 1.4 = 280 + (1.55 - 1.4)$$

$$r \times 1.4 = 280 \times 0.15$$

$$r = \frac{2.8}{1.4} \times 15 = 30\ \Omega$$

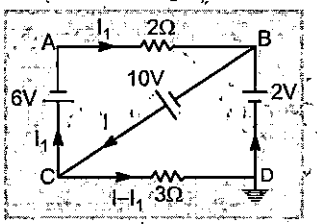
17. [a, b, c]

$$V_C - 6 - 2I_1 - 2 + 3(I - I_1) = V_C$$

$$8 = 3I - 5I_1 \quad \ldots(i)$$

$$V_C - 3(I - I_1) + 2 + 10 = V_C$$

$$3I - 3I_1 = 12 \quad \ldots(ii)$$



From eqn. (i) and (ii),

$$I_1 = 2\ \text{A}$$

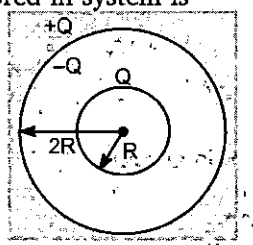
$$I = 6\ \text{A}$$

$$I - I_1 = 4\ \text{A}$$

Also current in wire D to earth is 0.

18. [a, b, c]

Initially the system is shown in figure. Initial energy stored in system is



$$U_1 = \frac{Q^2}{8\pi\varepsilon_0 R} = \frac{KQ^2}{2R}$$

As the two shell are connected, the entire charge Q passes through wire and resides on outer shell, so that both the shells have same potential.

Final energy stored in system is

$$U_f = \frac{Q^2}{8\pi\varepsilon_0 \times 2R} = \frac{KQ^2}{4R}$$

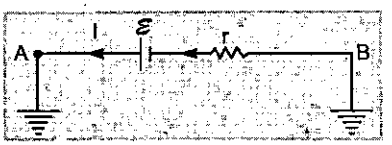
So, amount of heat generated in wire is,

$$\Delta H = U_i - U_f = \frac{KQ^2}{4R}$$

19. [a, d] $V_A = 0, V_B = 0$ (as A and B are earthed)

$$V_A - V_B = 0$$

From Kirchhoff's law



$$V_A - \mathcal{E} + Ir = V_B$$

$$V_A - V_B = \mathcal{E} - Ir$$

$$0 = \mathcal{E} - Ir$$

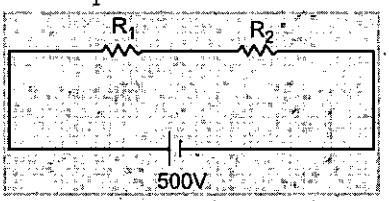
$$I = \frac{\mathcal{E}}{r}$$

31. [b, c]

Let resistance of bulbs are R_1 and R_2

$$\Rightarrow \quad \frac{(200)^2}{R_1} = \frac{(300)^2}{R_2}$$

$$\frac{R_2}{R_1} = \frac{9}{4} \quad \ldots(i)$$



$$V_1 = IR_1 = \left(\frac{500}{R_1 + R_2}\right) \times R_1$$

$$= \frac{500}{1 + \frac{R_2}{R_1}} = \frac{500}{1 + \frac{9}{4}} = \frac{2000}{13}$$

$$V_2 = \frac{500}{R_1 + R_2} \times R_2$$

$$= \frac{500}{1 + \frac{4}{9}} = \frac{4500}{3}$$

$$\frac{V_1}{V_2} = \frac{2000}{4500} = \frac{4}{9}$$

$$\frac{P_1}{P_2} = \frac{(V_1)^2 R_2}{R_1 (V_2)^2} = \left(\frac{V_1}{V_2}\right)^2 \times \frac{R_2}{R_1}$$

$$= \left(\frac{4}{9}\right)^2 \times \frac{9}{4} = \frac{4}{9}$$

32. [b, c]

For a bulb $R = \frac{V^2}{W}$

$$\Rightarrow \quad R_B \le R_A$$

when switch is open $I_A = I_B$

$$P_A = R_A I_A^2$$

$$P_B = R_B I_B^2$$

$$\Rightarrow \quad P_B < P_A \quad \text{and} \quad V_A > V_B$$

$$\Rightarrow \quad V_A > 12\text{V} \quad \text{and} \quad V_B < 12\text{V}$$

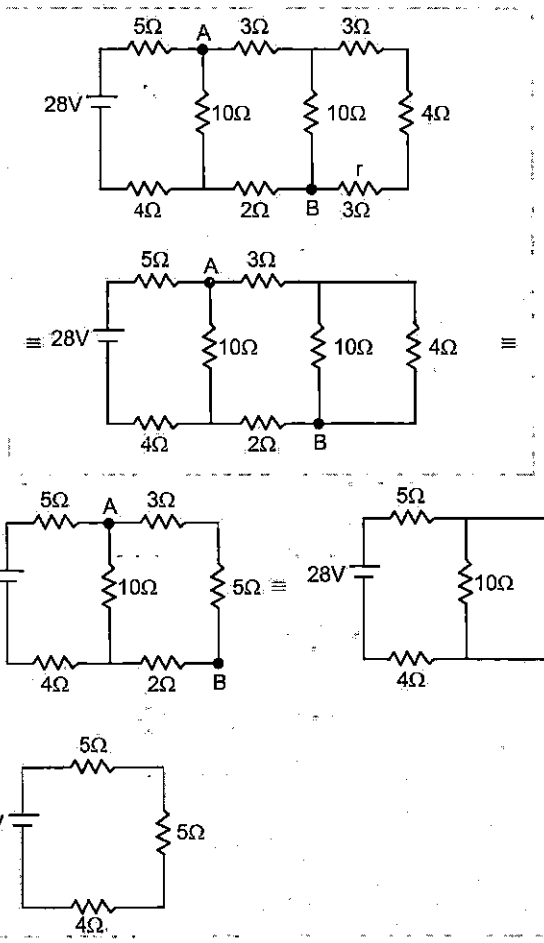
After closing the switch

$$V = V_B = 12\text{V}$$

$$\Rightarrow \quad P_B = 36\text{W}$$

$$P_A = 24\text{W}$$

33. [a, c]



I = Current through 5Ω resistor $= \dfrac{28}{5+5+4} = 2\text{A}$

$I_1 = \left(\dfrac{10}{10+10}\right) I = \dfrac{1}{2} = 1\text{A}$

$V_A - V_B = 5I_1 + 3I_1 = 8I_1 = 8\text{A}$

34. [b, d]

For circuit (a) $I_A = \dfrac{20}{4} = 5\text{A}$

For circuit (b) $I_A = \dfrac{20-V}{6} = \dfrac{20-8}{6} = 2\text{A}$

For circuit (c) $I_A = 0$ (Balanced Wheatstone's Bridge)

For circuit (d) 6Ω is in parallel with 6 + 6 = 12Ω.

Their effective resistance is $\dfrac{6\times 12}{6+12} = 4\Omega$

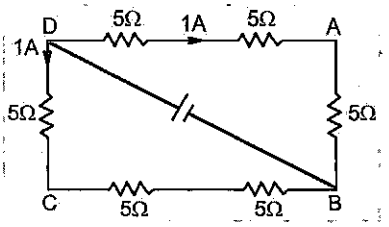
∴ Net resistance of the circuit = 4 + 6 = 10Ω

∴ $I_A = 20/10 = 2\text{A}$

35. [c, d]

Current in the circuit $I = \dfrac{15}{15} \times 2$

$I = 2\text{A}$



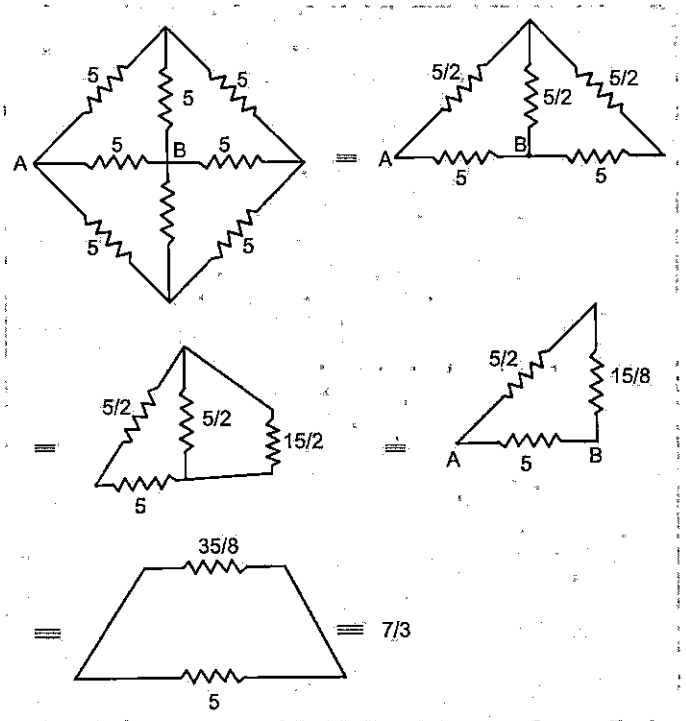
$V_D = +15$

$V_D - 10\times 1 = V_A$

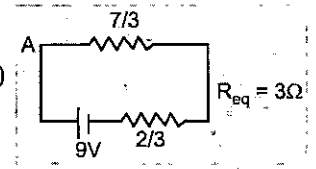
$V_A = 15 - 10 = 5\text{V}$

$V_C = 15 - 5\times 1 = 10\text{V}$

37. [a, b]

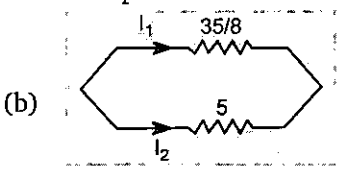


So, eg. circuits diagrams



$I = \dfrac{9}{R_{eq}} = 3\text{A}$

Heat produced in cell $= I^2 r = 9\times(2/3) = 6\text{W}$



$I_2 = \dfrac{R_1 I}{R_1 + R_2}$

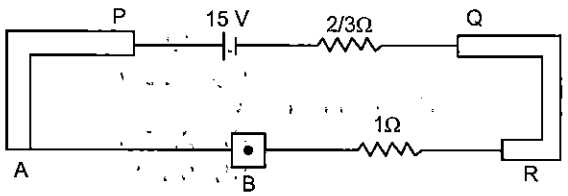
$= \dfrac{3\times(35/8)}{\dfrac{35}{8}+5} = \dfrac{7}{5} = 1.2\text{A}$

So, (a) and (b) are correct.

Level 3 Comprehension Based Problems

Passage 1

A thin uniform wire AB of unknown resistance is connected between points A and B. AP and QR are thick conducting stripes. A battery of unknown e.m.f. $\mathcal{E}$ and a galvanometer (with a sliding jockey connected to it) are available. Connections are to be made to find e.m.f. $\mathcal{E}$, resistance of wire AB and length of AB. The battery with galvanometer is connected between points A and B in two alternative way such that jockey when pressed on wire AB at distance 80 cm and 120 cm respectively from end A, galvanometer shows no deflection.



If one end of battery with galvanometer is connected to pt. R and jockey is pressed on AB at distance 180 cm from end A, the deflection in galvanometer is zero then find :

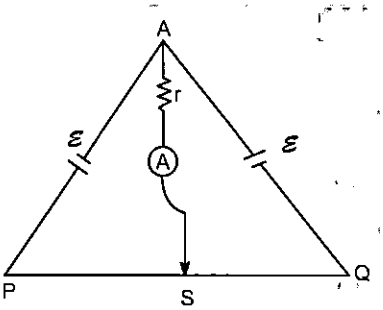
1. The length of wire :
(a) 200 cm (b) 280 cm
(c) 320 cm (d) 300 cm

2. The resistance of wire AB is :
(a) $\frac{1}{3}\Omega$ (b) 3 Ω
(c) $\frac{10}{3}\Omega$ (d) 10 Ω

3. The e.m.f. of battery is $\mathcal{E}$ = :
(a) 2 V (b) 5 V
(c) 4 V (d) 10 V

Passage 2

In the figure, PQ is a wire of uniform cross-section and resistance R_0. A is an ideal ammeter and the cells are of negligible internal resistance. The jockey J can be freely slide over wire PQ making contact on it at S. Length of wire PQ is 'l'.



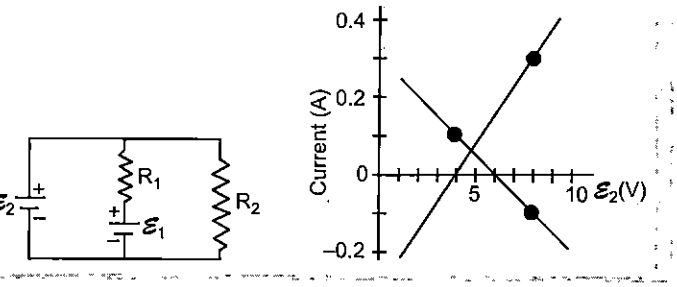
1. Reading of ammeter when $PS = \frac{l}{n}$ is:
(a) $\left(1-\frac{4}{n}\right)\frac{\mathcal{E}}{\left(r+\frac{R_0}{n}\right)}$ (b) $\left(1-\frac{2}{n}\right)\frac{\mathcal{E}}{\left(r+\frac{R_0}{n}\right)}$
(c) $\left(1-\frac{1}{n}\right)\frac{\mathcal{E}}{\left(r+\frac{R_0}{n}\right)}$ (d) $\left(1-\frac{1}{n^2}\right)\frac{\mathcal{E}}{\left(r+\frac{R_0}{n}\right)}$

2. Find the value of PS for which reading of ammeter is min.:
(a) $\frac{2l}{3}$ (b) $\frac{l}{2}$
(c) $\frac{3l}{4}$ (d) l

3. Find the value of PS for which reading of ammeter will be max.:
(a) 0 (b) $\frac{l}{2}$
(c) l (d) $\frac{2l}{3}$

Passage 3

In the circuit shown, both batteries are ideal. EMF $\mathcal{E}_1$ of battery 1 has a fixed value, but emf $\mathcal{E}_2$ of battery 2 can be varied between 1 V and 10 V. The graph gives the currents through the two batteries as a function of $\mathcal{E}_2$, but are not marked as which plot corresponds to which battery. But for both plots, current is assumed to be negative when the direction to the current through the battery is opposite the direction of that battery's emf. (Direction of from negative to positive)



1. The value of emf $\mathcal{E}_1$ is:
   (a) 8 V (b) 6 V
   (c) 4 V (d) 2 V
2. The resistance R_1 has value :
   (a) 10Ω (b) 20 Ω
   (c) 30 Ω (d) 40 Ω
3. The resistance R_2 is equal to :
   (a) 10Ω (b) 20 Ω
   (c) 30 Ω (d) 40 Ω

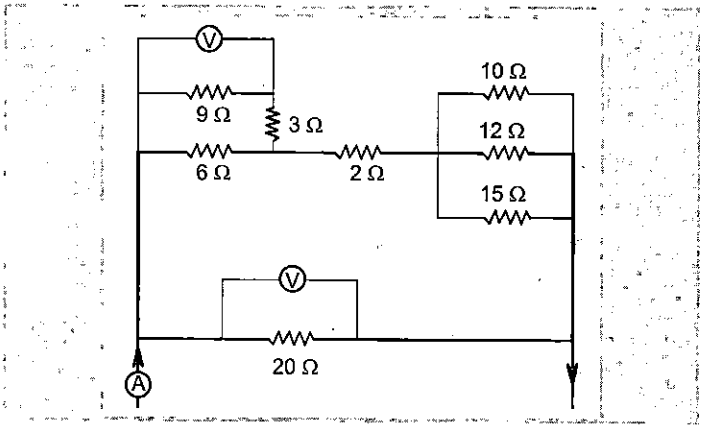
Passage 4

A car battery with a 12 V emf and an internal resistance of 0.04 Ω is being charged with a current of 50 A.

1. The potential difference V across the terminals of the battery are :
   (a) 10 V (b) 12 V
   (c) 14 V (d) 16 V
2. The rate at which energy is being dissipated as heat inside the battery is :
   (a) 100 W (b) 500 W
   (c) 600 W (d) 700 W
3. The rate of energy conversion from electrical to chemical is :
   (a) 100 W (b) 500 W
   (c) 600 W (d) 700 W

Passage 5

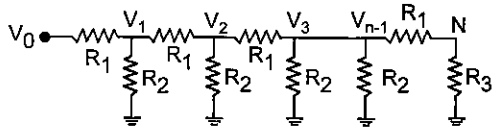
Fig. shows two ideal voltmeters and an ammeter which are connected across the various circuit elements. If the voltmeter connected across 9 Ω resistance reads 4.5 V, then answer the following questions.



1. The current through 12 Ω resistance is :
   (a) 0.1 A (b) 0.75 A
   (c) 0.5 A (d) 1.25 A
2. The reading of the voltmeter connected across 20 Ω resistance
   (a) 15 V (b) 10 V
   (c) 5 V (d) 22.5 V
3. The reading of the ammeter is
   (a) 0.5 A (b) 2.25 A
   (c) 1.5 A (d) 0.1 A

Passage 6

A network of resistance is constructed with R_1 and R_2 as shown in Fig. The potential at the points 1, 2, 3... N are $V_1, V_2, V_3, \ldots V_n$, respectively, each having a potential K times smaller than the previous one.



1. The ratio $\frac{R_1}{R_2}$ is:
   (a) $k^2 - \frac{1}{k}$ (b) $\frac{k}{k-1}$
   (c) $k - \frac{1}{k^2}$ (d) $\frac{(k-1)^2}{k}$
2. The ratio $\frac{R_2}{R_3}$ is:
   (a) $\frac{(k-1)^2}{k}$ (b) $k^2 - \frac{1}{k}$
   (c) $\frac{k}{k-1}$ (d) $k - \frac{1}{k^2}$

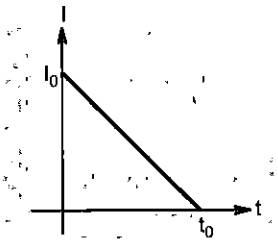
3. The current that passes through the resistance R_2 nearest to the V_0 is:

(a) $\frac{(k-1)^2}{k}\frac{V_0}{R_3}$ (b) $\frac{(k+1)^2}{k}\frac{V_0}{R_3}$

(c) $\left(k+\frac{1}{k^2}\right)\frac{V_0}{R_3}$ (d) $\left(k-\frac{1}{k^2}\right)\frac{V_0}{R_3}$

Passage 7

Relation between current in conductor and time is shown in Fig. then determine



1. Total charge flown through the conductor is

(a) $I_0t_0/2$ (b) I_0t_0

(c) $I_0t_0/4$ (d) $2I_0t_0$

2. Write the expression of current in terms of time :

(a) $I = I_0\frac{t}{t_0}$ (b) $I = I_0\left(1+\frac{t}{t_0}\right)$

(c) $I = I_0\left(\frac{t}{t_0}-1\right)$ (d) $I = I_0\left(1-\frac{t}{t_0}\right)$

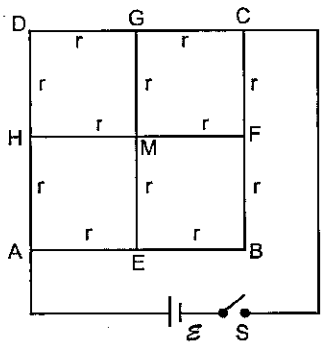
3. If the resistance of conductor is R, then total heat dissipated across resistance R is

(a) $\frac{I_0^2\,Rt_0}{2}$ (b) $\frac{I_0^2\,Rt_0}{4}$

(c) $\frac{I_0^2\,Rt_0}{3}$ (d) $I^2{}_0Rt_0$

Passage 8

In Fig. each of the segments (e.g., AE, GM, etc.) has resistance r. A battery of e.m.f. $\mathcal{E}$ is connected between A and C. Internal resistance of the battery is negligible.



1. What is the equivalent resistance of the system about A and C ?

(a) r (b) $\frac{r}{2}$

(c) $\frac{3r}{2}$ (d) $2r$

2. Find the ratio of the power developed in segment AE to that in segment HM:

(a) 1 (b) 2

(c) 3 (d) 4

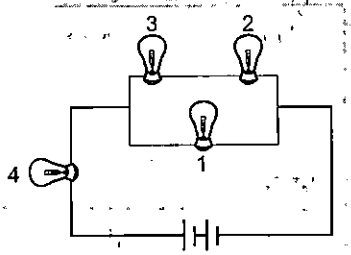
3. If a potentiometer circuit having potential gradient k is connected across the points H and C, find the balancing length shown by the potentiometer:

(a) $\frac{v}{k}$ (b) $\frac{2v}{3k}$

(c) $\frac{3v}{2k}$ (d) none of these

Passage 9

All bulbs consume same power. The resistance of bulb 1 is 36Ω . Answer the following questions



1. What is the resistance of bulb 3 ?

(a) 4 W (b) 9 W

(c) 12 W (d) 18 W

2. What is the resistance of bulb 4?

(a) 4 W (b) 9 W

(c) 12 W (d) 18 W

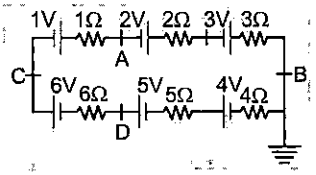
3. What is the voltage output of the battery if the power of each bulb is 4 W?

(a) 12 V (b) 16 V

(c) 24 V (d) none of these

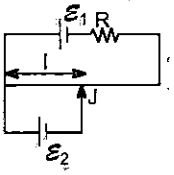
MATCHING TYPE PROBLEMS

1. Six batteries of increasing emf and increasing internal resistance are as shown in figure. Match the following:



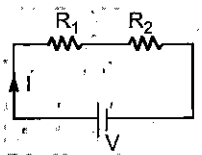
	Column–I		Column–II
(a)	Potential of point A	(p)	Zero
(b)	Potential of point B	(q)	2 V
(c)	Potential of point C	(r)	4 V
(d)	Potential of point D	(s)	6 V
		(t)	None

2. In the potentiometer arrangement shown in figure, null point is obtained at length l. Match the following :



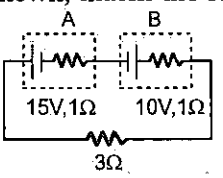
	Column–I		Column–II
(a)	If $\mathcal{E}_1$ is increased	(p)	l should increase
(b)	If R is increase	(q)	l should decrease
(c)	If $\mathcal{E}_2$ is increased	(r)	l should remain the same to again get the null point

3. In the circuit shown in figure, if a resistance R is connected in parallel with R_2, then match the following :



	Column–I		Column–II
(a)	Main current I	(p)	will increase
(b)	Power across R_1	(q)	will decrease
(c)	Power across R_2	(r)	will remain same

4. In the circuit shown, match the following :



	Column–I		Column–II
(a)	Potential difference across battery A	(p)	A
(b)	Potential difference across battery B	(q)	B
(c)	Power is supplied by battery	(r)	14 V
(d)	Power is consumed by battery	(s)	9 V
		(t)	None

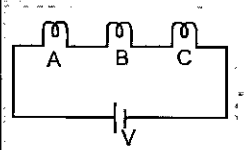
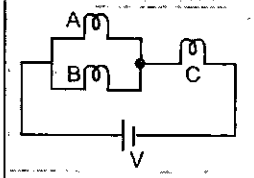
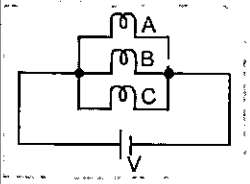
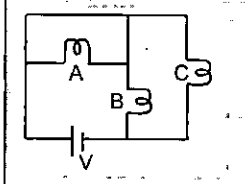
5. Current is flowing through a wire of non-uniform cross-section. Cross-section of wire A is less than the cross-section of wire at B. Then match the following Column:

	Column–I		Column–II
(a)	current at A	(p)	is zero
(b)	drift velocity of electrons at A	(q)	is more than at B
(c)	electric field in the wire at A	(r)	is less than at B
(d)	current density at A	(s)	is equal to that at B

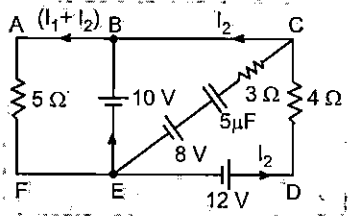
6. A potential difference V is applied across a copper wire of diameter d and length l. In Column I the information about the change in one of the physical quantity is given and the effect(s) of this change are mentioned in Column II. Match the Column I with Column II.

	Column-I		Column-II
(a)	Voltage V is doubled	(p)	Drift speed will decrease
(b)	Length l is doubled	(q)	Drift speed will be halved
(c)	Diameter d is doubled	(r)	Drift speed will be doubled
(d)	Temperature of the wire is increased	(s)	Drift speed will not change

7. Three bulbs A, B and C are having rated powers P_A, P_B and P_C respectively, each bulb is designed to operate at rated voltage V. It is given the $P_A > P_B > P_C$. In Column I the three bulbs are arranged in different configurations, while in Column II the information about intensities of bulbs are mentioned. Match the Column-I with Column-II. Neglect the variation in resistance due to change in temperature.

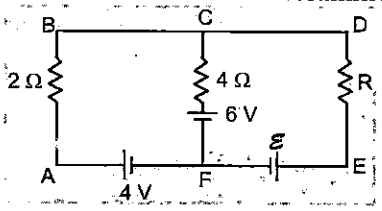
	Column-I		Column- II
(a)		(p)	A is having the minimum intensity
(b)		(q)	C is glowing with maximum brightness
(c)		(r)	B is glowing with minimum brightness
(d)		(s)	A is glowing with maximum brightness.

8. A network consisting of three resistors, three batteries, and a capacitor is shown in Fig.



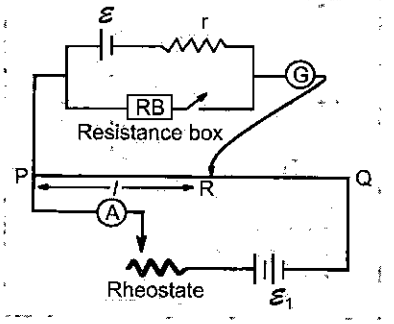
	Column-I		Column-II
(a)	Current in branch EB is	(p)	10 μC
(b)	Current in branch CB is	(q)	0.5 A
(c)	Current in branch ED is	(r)	1.5 A
(d)	Charge on Capacitor is	(s)	5μC

9. A circuit is shown in Fig. R is a non-zero variable with finite resistance. $\mathcal{E}$ is some unknown emf with polarities as shown. Match the columns.



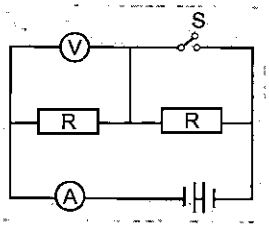
	Column-I		Column-II
(a)	Current passing through 4Ω resistance can be zero	(p)	possible if $\mathcal{E} = 6$ V
(b)	Current passing through 4Ω resistance can be from F to C direction	(q)	possible if $\mathcal{E} > 6$ V
(c)	Current passing through 4Ω resistance can be from C to F direction	(r)	possible if $\mathcal{E} < 6$ V
(d)	Current passing through 2Ω resistance will be from B to A direction	(s)	possible for any value of $\mathcal{E}$ from zero to infinit

10. Fig. shows an experimental setup for a potentiometer, point R is null point [no deflection of galvanometer] $\mathcal{E}_1$ is primary source. Match the following:



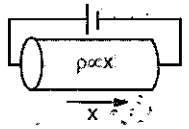
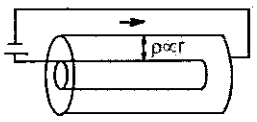
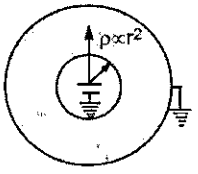
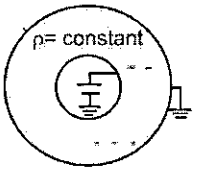
	Column-I		Column-II
(a)	If only emf of battery $\mathcal{E}_1$ is increased	(p)	Point R will shift to left
(b)	If only resistance of rheostatic is increased	(q)	Point R will shift to right
(c)	If resistance box is connected and resistance RB is increased	(r)	Point R may shift to left or right
(d)	If an ideal battery is connected in parallel to $\mathcal{E}$		

11. The diagram shows a circuit with two identical resistors. The battery has a negligible internal resistance.
What will the effect on the ammeter and voltmeter be if the switch S is closed?

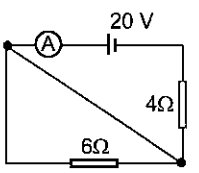
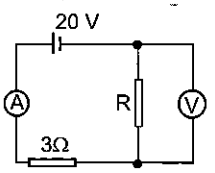
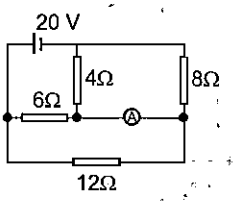
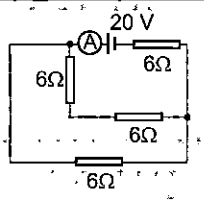


	Column-I		Column-II
(a)	Ammeter reading	(p)	Increases
(b)	Voltmeter reading	(q)	Decreases
(c)	Equivalent resistance of circuit	(r)	does not change
(d)	Power dissipated across R in right branch.	(s)	becomes zero

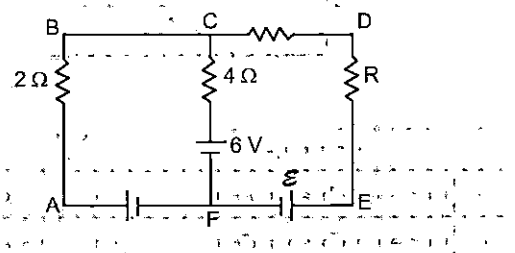
12. Column-I has some conductor across which battery is connected as shown. Variation of resistivity ρ is also indicated. Which of the quantities in column-II remain constant throughout the volume of conductor.

	Column-I		Column-II
(a)		(p)	Magnitude of electric field
(b)		(q)	Magnitude of current density
(c)		(r)	Power dissipated per unit volume
(d)		(s)	Drift speed of free electron
		(t)	Electric current

13. Column-I has four circuits each having an ammeter. Column-II has four values of current in the ammeter. The ammeter has zero resistance. The voltmeter, in (B) has infinite resistance and a reading 8V. The resistance R has not been specified. Match the circuit with its correct ammeter reading.

	Column-I		Column-II
(a)		(p)	0
(b)		(q)	2 ampere
(c)		(r)	4 ampere
(d)		(s)	5 ampere

14. A circuit is shown in Fig. R is a non zero variable variable with finite resistance. $\mathcal{E}$ is some unknown emf with polarities as shown. Match the columns



	Column-I		Column-II
(a)	Current passing through 4Ω resistance can be zero	(p)	possible if $\mathcal{E} = 6$ V
(b)	Current passing through 4Ω resistance can be from F to C direction	(q)	possible if $\mathcal{E} > 6$ V
(c)	Current passing through 4Ω resistance can be from C to F direction	(r)	possible if $\mathcal{E} < 6$ V
(d)	Current passing through 2Ω resistance will be from B to A direction	(s)	Possible for any value of $\mathcal{E}$ from zero to infinity

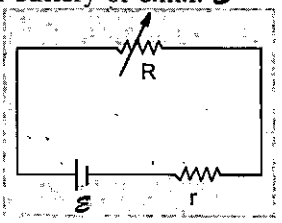
15. In the circuit shown in Fig. battery, ammeter and voltmeter are ideal and the switch S is initially closed as shown. When switch S is opened, match the parameters of column I with the effects in column II

	Column-I		Column-II
(a)	Equivalent resistance across the battery	(p)	remains same
(b)	Power dissipated by left resistance R	(q)	increases
(c)	Voltmeter reading	(r)	decreases
(d)	Ammeter reading	(s)	becomes zero

16. In a potentiometer experiment :

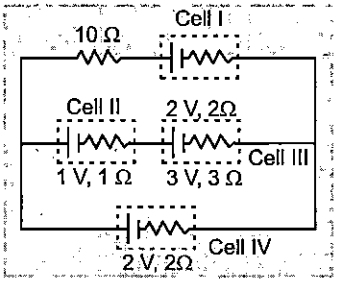
	Column-I		Column-II
(a)	Deflection of galvano- meter is in same direction at the two ends of the wire	(p)	Accuracy in measurement increases
(b)	A protective resistance added in series to the galvanometer	(q)	Accuracy in measurement decreases
(c)	A short wire is used as a potentiometer	(r)	e.m.f. of the battery in the primary circuit is less than the e.m.f. of the cell to be measured
(d)	more length of potentiometer up to null point	(s)	Uncertainty in the location of balance point increases

17. In Fig. the resistance R is variable, r is the internal resistance of battery of e.m.f. $\mathcal{E}$



	Column-I		Column-II
(a)	Terminal potential dif4ference across the cell to be maximum	(p)	$R > r$
(b)	Power transferred to R is less than the maximum possible	(q)	$R < r$
(c)	Power dissipated in the cell is maximum	(r)	$R = \infty$
(d)	Fastest drift of ions in the electrolyte in the cell will be for	(s)	$R = 0$

18. For the circuit shown in Fig., 4 cells are arranged. In Column I, the cell number is given while in Column II, some statement related to cells are given. Match the Column I with Column II.



	Column-I		Column-II
(a)	Cell I	(p)	Chemical energy of cell is decreasing
(b)	Cell II	(q)	Chemical energy of cell is increasing
(c)	Cell III	(r)	Work done by cell is + ve
(d)	Cell IV	(s)	Thermal energy developed in cell is + ve

ASSERTION AND REASON

Direction: In the questions that follows two statements are given. Statement-2 is purported to be the explanation for statement-1. Study both the statements carefully and then select your answers according to the codes given below:

Select your answer as

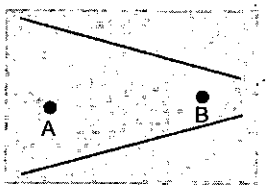
(a) If Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

(b) If Statement-1 is true, Statement-2 is true, Statement-2 is not a correct explanation for Statement-1.

(c) If Statement-1 is true; Statement-2 is false.

(d) If Statement-1 is false; Statement-2 is true.

1. **Statement 1:** A steady current is flowing in a conductor having non-uniform cross-section as shown in figure. The drift speed of the electrons increases as one moves from A to B.



Statement 2: Drift velocity is given by, $v_d = \frac{eE\tau}{m}$ and in above described situation, $\vec{E}$ is increasing as one moves from A to B.

2. **Statement 1:** Potential difference across the terminals of a battery can be greater than its emf.
Statement 2: When current is taken from battery, $V = \mathcal{E} - Ir$ (Symbols have their usual meaning).

3. **Statement 1:** If the length of a conductor is doubled, the drift velocity will become half of the original value (keeping potential difference unchanged).
Statement 2: At constant potential difference drift velocity is inversely proportional to the length of the conductor.

4. **Statement 1:** Current flows in a conductor only when there is an electric field within the conductor.
Statement 2: The drift velocity of electrons in the presence of electric field decreases.

5. **Statement 1:** A piece of copper and other of germanium are cooled from room temperature to 100 K conductivity of copper increases and that of germanium decreases.
Statement 2: Copper has positive temperature coefficient where as germanium has negative temperature coefficient.

6. **Statement 1:** Two unequal resistances are connected in series across a cell, then potential drop across the larger resistance is more.
Statement 2: The current will be same in both unequal resistances.

7. **Statement 1:** Two unequal resistances are connected in parallel across a cell, then current through the smaller resistor is more.
Statement 2: More current will flow through a larger resister.

8. **Statement 1:** If a wire is stretched to increase its length n times then its resistance also become n time.
Statement 2: Resistance of the wire is directly proportional to its length.

9. **Statement 1:** When a wire is stretched so that its diameter is halved then its resistance become 16 times.
Statement 2: Resistance of wire decrease with increase in length.

10. **Statement 1:** The value of temperature coefficient of resistance is positive for metals.
Statement 2: The temperature coefficient of resistance for insulator is also positive.

11. **Statement 1:** When an insulated wire is bent, its resistivity increases.
Statement 2: On bending, the velocity of electron decreases.

12. **Statement 1:** If the radius of copper wire carrying a current is doubled, then the drift velocity of the electrons will become one fourth.
Statement 2: Drift velocity will change according to the relation, $I = neAv_d$.

13. **Statement 1:** A wire of resistance R is bent in the form of a circle. The resistance between two points on circumference or the wire or at the end of diameter is $R/4$.
Statement 2: The resistance between the two points on circumference of the circle will be the parallel combination of two resistances of upper and lower parts of the circle.

14. **Statement 1:** The equivalent resistance in series combination is larger than even the largest individual resistance.
Statement 2: The equivalent resistance of the parallel combination is smaller than even the smallest resistance.

15. **Statement 1:** When a battery is supplying power to a circuit, work done by electrostatic forces on electrolyte ions inside the battery is positive
Statement 2: Electric field is directed from positive to negative electrode inside a battery.

ANSWERS

Level-3: Comprehension Based Problems

Passage-1:	**1.** (a)	**2.** (c)	**3.** (c)	**Passage-2:**	**1.** (b)	**2.** (b)	**3.** (a, c)
Passage-3:	**1.** (b)	**2.** (b)	**3.** (d)	**Passage-4:**	**1.** (c)	**2.** (a)	**3.** (c)
Passage-5:	**1.** (c)	**2.** (a)	**3.** (b)	**Passage-6:**	**1.** (d)	**2.** (c)	**3.** (d)
Passage-7:	**1.** (a)	**2.** (d)	**3.** (c)	**Passage-8:**	**1.** (c)	**2.** (d)	**3.** (b)
Passage-9:	**1.** (b)	**2.** (a)	**3.** (b)				

Matching Type Problem

1. (a)-p; (b)- p; (c)-p; (d)- p
2. (a)-q; (b)- p; (c)- p
3. (a)-p; (b)-p; (c)- q
4. (a)-r; (b)- t; (c)- p; (d)- q
5. (a)-s; (b)- q; (c)- q; (d) -q
6. (a)-r; (b)-p, q; (c)-s; (d)-p
7. (a)-p, q; (b)-q, r; (c)-s; (d)-p
8. (a)-r; (b)-q; (c)-q; (d)-p
9. (a)- q; (b)-p, q, r; (c)-q; (d)-p, q, r, s
10. (a)-p; (b)-q; (c)-q; (d)-r
11. (a)-p; (b)-p; (c)-q; (d)-q, s
12. (a)-q, s, t; (b)-q, s, t; (c)-p, t; (d)-t
13. (a)-s; (b)-r; (c)-p; (d)-q
14. (a)-q; (b)-p, q, r; (c)-q; (d)-p, q, r, s
15. (a)-q; (b)-r; (c)-r; (d)-r
16. (a)-r; (b)-s; (c)-q, s; (d)-p
17. (a)-r; (b)-p, q, r, s; (c)-s; (d)-s
18. (a)-q, s; (b)-p, r, s; (c)-p, r, s; (d)-q, s

Assertion and Reason

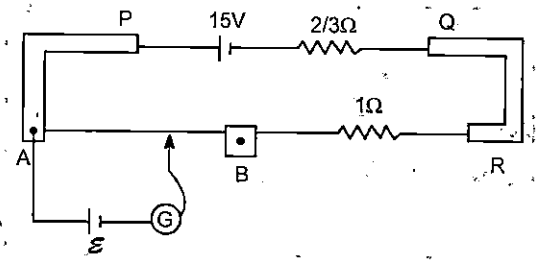
1.	(a)	**2.**	(b)	**3.**	(a)	**4.**	(c)	**5.**	(a)	**6.**	(d)	**7.**	(c)	**8.**	(d)
9.	(c)	**10.**	(c)	**11.**	(d)	**12.**	(a)	**13.**	(a)	**14.**	(b)	**15.**	(d)		

Level-3: Comprehension Based Problems

Passage-1

1. [a] In first case batteries was connected through point A and in second case battery was connected through point B

Let length of wire $AB = l$



Resistance of $AB = R$

Accordingly

When battery was connected through the point A

$$\varepsilon = \frac{15}{\frac{5}{3}+R} \times \frac{R}{l} \times 80 \quad \ldots(i)$$

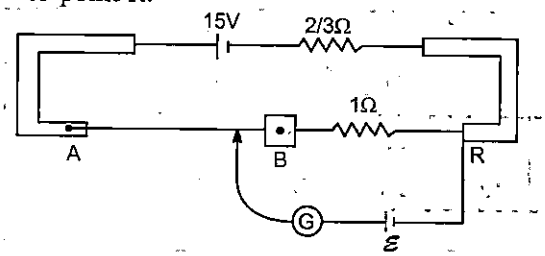
When battery was connected through the point B

$$\varepsilon = \frac{15}{\frac{5}{3}+R} \times \frac{R}{l} \times (l-120) \quad \ldots(ii)$$

Dividing both

$$l = 200 \text{ cm}$$

2. [c] From third condition when battery was connected to point R.



$$\varepsilon = \frac{15}{\frac{5}{3}+R} \times \left[1 + \frac{R}{200} \times (200-180)\right] \quad \ldots(iii)$$

Divide (i) by (iii),

$$1 + \frac{R}{10} = \frac{R \times 80}{200}$$

$$1 = \frac{2R}{5} - \frac{R}{10} = \frac{3R}{10}$$

$$R = \frac{10}{3}\Omega$$

3. [c] Put $R = \frac{10}{3}$ in eqn. (i)

We get

$$\varepsilon = \frac{15}{\frac{5}{3}+\frac{10}{3}} \times \frac{\frac{10}{3}}{200} \times 80$$

$$= 3 \times \frac{16}{3} \times \frac{2}{5}$$

$$\varepsilon = 4\text{ V}$$

Passage-2

1. [b] $V_P - V_Q = 2\varepsilon$

$$V_A = V_P - \varepsilon$$

In length l potential difference 2ε

$\Rightarrow$ In length $\frac{l}{n}$ potential difference $= \frac{2\varepsilon}{n}$

$\Rightarrow$ $V_S = V_P - \frac{2\varepsilon}{n}$

$$V_A = V_P - \varepsilon$$

Resistance of length $\frac{l}{n}$ will be $\frac{R_0}{n}$

$$V_S - V_A = \varepsilon\left(1 - \frac{2}{n}\right)$$

$$I = \text{ammeter reading} = \left(1-\frac{2}{n}\right)\frac{\varepsilon}{\left(r+\frac{R_0}{n}\right)}$$

2. [b] From above $I_{min} = 0$ for $n = 2$

$\Rightarrow$ for $PS = \frac{l}{2}$ reading will be 0

3. [a, c]

For maximum reading potential difference $|V_S - V_A|$ should be maximum

$\therefore$ for $PS = 0, l$ reading is maximum.

Matching Type Problem

6. (a)-r; (b)-p, q; (c)-s; (d)-p

$$v_d = \frac{eE\tau}{m}$$

If V is doubled, then E gets double and hence v_d.

If l is doubled, then E gets half and hence v_d decreases. If d is doubled, then there is no effect on v_d.

If temperature increases then relaxation time $\downarrow$ and hence v_d.

7. (a)-p, q; (b)-q, r; (c)-s; (d)-p

As $P_A > P_B > P_C$ and the rated voltage is same for all, their resistances will satisfy the relation $R_A < R_B < R_C$.

For (a) As all A, B and C are in series, current through them would be same to I^2R is maximum for C and minimum for A. C is brightest and A is dimmest.

For (b) If current through A and B be I_1 and I_2 then current through C is $I_1 + I_2$. So I^2R is maximum for C and as A and B have same applied voltage across them so $\frac{V_1^2}{R_A} > \frac{V_1^2}{R_B}$ so power delivered to B is minimum.

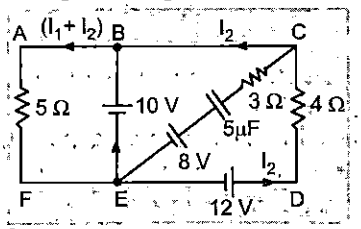
For (c) As voltage across all the bulbs is same so $\frac{V^2}{R}$ is maximum for A and minimum for C.

For (d) Here bulb A is shorted so no current flows through it, while B and C are in parallel so power delivered to B is greater than to C. So, B is glowing brightest and A is dimmest.

8. (a)-r; (b)-q; (c)-q; (d)-p

When a steady state is reached, no current passes through the capacitor or the branch CE.

Considering the loop $ABEFA$,



$$5\times(I_1+I_2)=10$$

or $$I_1+I_2=2\text{A}$$

Considering the loop $BCDEB$:

$$4I_2=12-10=2$$

$\Rightarrow$ $$I_2=\mathbf{0.5A}$$

So, $$I_1=2-0.5=1.5\text{A}$$

To find the charge on capacitor, we must known potential difference across the plates.

Consider the loop $CEDC$:

$$-12+4I_2+3\times0-V_C+8=0$$

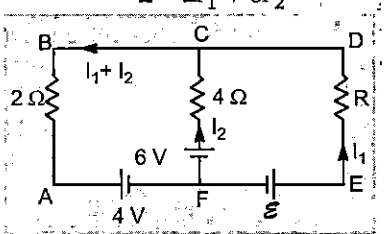
or $V_C=-2$V. So charge on capacitor $Q=CV=10\mu$C

9. (a)-q; (b)-p, q, r; (c)-q; (d)-p, q, r, s

Loop $FEDCF$: $\mathcal{E}-6=RI_1-4I_2$ ...(i)

Loop $AFCBA$: $6-4=4I_2+2(I_1+I_2)$

$$2=2I_1+6I_2 \quad \text{...(ii)}$$



Solving them we get:

$$I_1=\frac{3e-14}{4+3R},\quad I_2=\frac{R+6-\mathcal{E}}{4+3R}$$

(a) $$I_2=0 \quad\Rightarrow\quad \mathcal{E}=R+6$$

$$\mathcal{E}>6\text{V} \qquad (\because\ R\neq0)$$

(b) For current from F to C direction

$$I_2>0 \quad\Rightarrow\quad R+6>\mathcal{E}$$

$\Rightarrow$ $$\mathcal{E}<R+6$$

possible for any finite value of $\mathcal{E}$, because R is finite

(c) For current from F to C direction

$$I_2<0 \quad\Rightarrow\quad \mathcal{E}>R+6$$

(d) For current in 2Ω from B to A direction

$$I_1+I_2=\frac{R-8+2\mathcal{E}}{4+3R}>0$$

$$R-8+2\mathcal{E}>0 \quad\Rightarrow\quad \mathcal{E}>4-\frac{R}{2}$$

Depending upon the value of R, $\mathcal{E}$ can take any value from zero to infinity.

11. (a)-p; (b)-p; (c)-q; (d)-q, s

$R_{eq.}$ decreases $\Rightarrow$ $I\uparrow\Rightarrow V\uparrow$

13. (a)-s; (b)-r; (c)-p; (d)-q

(a) Effective resistance of the circuit $=4\Omega$

(b) Potential difference across $3\Omega=20\text{V}-8\text{V}=12\text{V}$

(c) Find currents in resistors using Ohm's law & series parallel and then use Junction law to find current in ammeter

(d) Effective resistance of the circuit $=10\Omega$

Assertion and Reason

1. (a) Both statement are correct and Statement 2 is correct explanation of Statement 1. Due to decreasing cross-section area, the $\vec{\mathbf{E}}$ increases and hence drift velocity as one moves from A to B.

2. (b) When the battery is undergoing charging processes then,

$$V=\mathcal{E}+Ir>\mathcal{E}$$

So, Statement 1 is correct.

Statement 2 is also correct but not explaining Statement 1.

3. (a) $V=IR\Rightarrow V=neAv_d\frac{\rho l}{A}\Rightarrow v_d=\frac{V}{ne\rho l}\Rightarrow v_d\propto\frac{1}{l}$

4. (c) Drift velocity is directly proportional to electric field. If there is no electric field, then no drifting of electrons in a particular direction, hence no current in the conductor.

5. (a) Copper is a conductor and germanium is a semi-conductor.

6. (a) In series, current in both resistances will be same. For same current, more is resistance more is the potential drop.

7. (c) Smaller is the resistance, more is the current in parallel.

8. (d) It is true that resistance of a wire is directly proportional to its length, but here when length is doubled, area of cross-section decreases as the volume remains constant. Finally, resistance becomes n^2 times.

9. (c) The resistance of a wire is $R=\frac{\rho l}{A}$ ...(1)

Hence, when the diameter is halved the resistance of the wire is

$$R\propto\frac{1}{\left(\frac{r}{2}\right)^2}=16R$$

Hence, its resistance will become 16 times.

Again from eqn. (1),

$$R \propto \frac{l}{A} \quad \text{or} \quad R \propto \frac{l^2}{Al} \quad \text{or} \quad R \propto l^2$$

Therefore, on increasing the length the resistance increases.

10. (c) On increasing the temperature of metals, the resistance of metal increases. Therefore, the temperature coefficient of resistance of metals is positive.

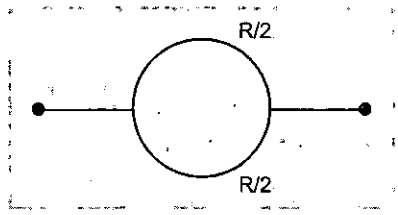
On increasing the temperature of insulators, the resistance decreases. Therefore, temperature coefficient of resistance of insulators is negative.

11. (d) Resistivity or specific resistance is a material property. So, it does not change on bending the insulated wire.

On bending, the cross-sectional area of wire changes but drift velocity of electron does not depend on area of cross-section so it does not change.

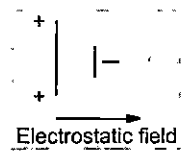
12. (a) $v_d = \frac{I}{neA}$, If radius is doubled, A becomes 4 times and hence v_d becomes one fourth.

13. (a) Both $R/2$ are in parallel, so their equivalent resistance is $R/4$.



14. (b) Both the statements are correct, but independent of each other.

15. (d) When battery is supplying power, inside battery positive charge moves opposite to electric field. So work done by electrostatic forces is negative.



3 CAPACITORS

SOURCES OF EMF

Mechanical energy can be stored in a spinning flywheel (rotational kinetic energy), a compressed or a stretched spring (elastic potential energy). There are devices which store and increase the electric potential energy of the charges. Devices that increase the electric potential energy of the charges are known as **sources of emf** or equivalently **independent voltage sources.** Common examples of sources of emf are battery, electric generators, solar cells, fuel cells.

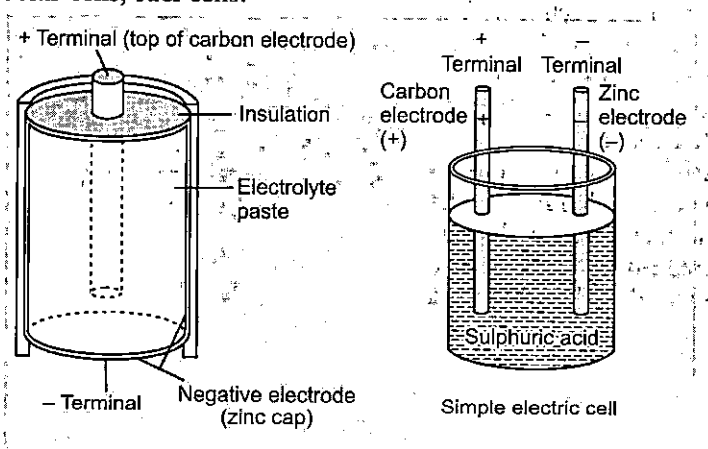
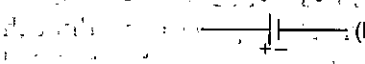


Fig. 3.1

A battery produces electricity by transforming chemical energy into electrical energy. The simplest battery is shown in Fig. 3.1. It contains two rods made of dissimilar metals (one can be carbon) called **electrodes.** The electrodes are immersed in an electrolyte; such a device is called electric cell. That part of each electrode outside the solution is called the **terminal**, connections to wires and circuits are made here.

When a closed conducting path is connected between the terminals of a battery, we have an electric circuit. A battery is represented by a symbol.



When a device (light bulb, a heater, etc.) is connected to a battery, charge can flow through the wires of the circuit, from one terminal of the battery to the other. Such flow of charge is called an electric current. Electric current can flow whenever there is a potential difference between the ends of a conductor, if there are opposite charges at the two ends of the conductor, or even is space.

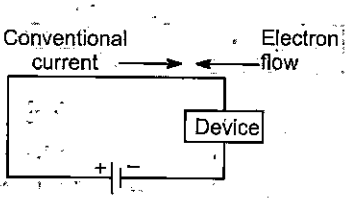


Fig. 3.2

When a circuit is formed, charge can flow through the wires of the circuit, from one terminal of the battery to the other. This ordered **flow of charges is called an electric current.**

The conductors contain large number of free electrons. When a conducting wire is connected to the terminals of a battery, the potential difference between the terminals of the battery sets up an electric field inside the wire and parallel to the wire. The free electrons at one end of the wire are attracted into the positive terminal and at the same time, electrons leave the negative terminal of the battery and enter the wire at the other end. Thus, there is a continuous flow of electrons through the wire. Conventionally it was assumed that positive charge flowed in the wire. The

positive charge flowing in one direction is exactly equivalent to negative charge flowing in the opposite direction. Historical convention of flow of positive charge is still followed. So, by the current in the circuit we mean the direction in which the positive charge would flow, which is referred to as **conventional current.**

A potential difference that can be used to supply energy and thereby sustain a current in an external circuit is an electromotive **force** *or* **emf** *(pronounced ee em ef).* **The emf is the voltage measured across the terminals of a source when no current is being drawn from or delivered to it.**

In order to calculate potential difference between two points A and B, take potential of A and subtract from it potential of B.

Potential difference between A and $B = V_A - V_B$

Potential difference between B and A

$$= V_B - V_A = -(V_A - V_B)$$

Since the choice of reference point is arbitrary, the (+) symbol does not necessarily mean that potential at the point has a positive value. The symbol means that the point is at higher electric potential that the point marked with (–). Similarly (–) symbol does not necessarily mean that the point is at a lower electric potential than the marked point (+).

A battery raises the electric potential energy of positive charges as they move from lower-potential terminal marked (–) to the higher potential terminal marked (+). The energy transformed when an infinitesimal charge dq moves through a potential difference V is $dw = dqV$. So the potential difference is the work done by battery on one coulomb of positive charge as it moves from the negative to positive terminal inside the battery.

An ideal battery maintains the specified potential difference V between the (+) terminal and the (–) terminal irrespective of external circuit element connected to it, and has an infinite lifetime.

CAPACITORS

A capacitor is combination of two conductors (with any geometry) isolated from each other so that they can be given equal but opposite charge. The conductors of a capacitor are called **plates.** (whether they are spherical, cylindrical or even rolled sheets, the conductors are still called plates.)

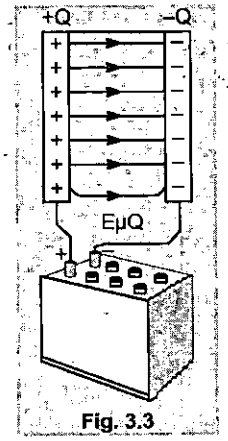


Fig. 3.3

When battery terminals are connected to an initially uncharged capacitor, equal amounts of positive and negative charge, $+Q$ and $-Q$, are acquired by conductors. The capacitor remains neutral overall, but we refer to it as storing a charge Q. When the two conductors have equal and opposite charges, the capacitor is said to be **charged.** If both the conductors of the capacitor have zero charge, the capacitor is said to be **uncharged.** Note that, whether charged or uncharged, the total electric charge on a capacitor as a whole is zero.

A simple capacitor consists of a pair of parallel plates of area A separated by a small distance d. A capacitor is represented by the symbol

⊣⊢

When a capacitor is connected to a battery it gets charged. Terminals of the battery, connecting wires and plate of the capacitor are conductors, therefore any acquire same potential, *i.e.*, plate connected to positive terminal similarly another plate connected to negative terminal, acquires a negative potential.

When we apply a certain potential difference ΔV, a fixed amount of charge q accumulates on each conductor ($+q$ on one plate and $-q$ on the other). When the potential difference between the conductors is doubled, the charge on each conductor is doubled, *i.e.*, the magnitude of the charge on each conductor is proportional to the potential difference between them. Now consider a parallel plate capacitor. Each field line starts on an individual positive charge and ends on a negative one, so that there will be more field lines if there is more charge. The electric field is directly proportional to Q. As we learned earlier that field between two parallel plates is given by

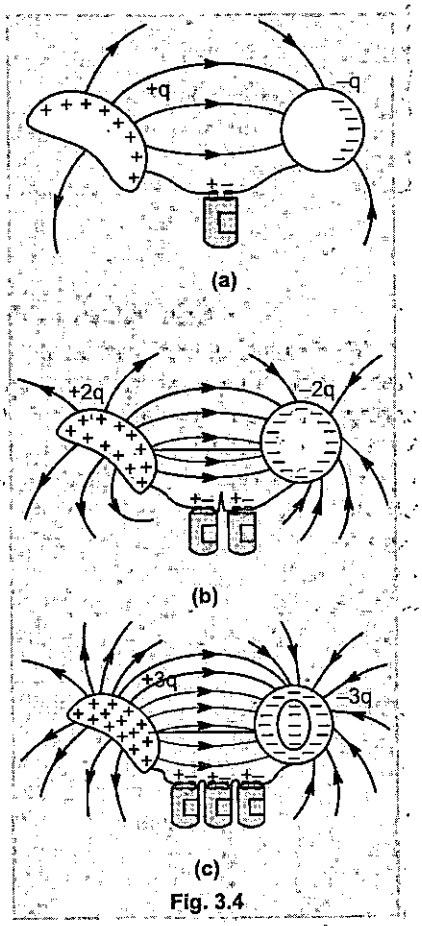


Fig. 3.4

$$E = \frac{Q}{\varepsilon_0 A}$$

$$E \propto Q$$

Since the field is uniform, the potential difference across parallel plates is $V = Ed$, thus

$$V \propto E$$

It follows then that $V \propto Q$ conversely

$$Q \propto V$$

In general, the greater the voltage applied to any capacitor, the greater the charge stored in it. The magnitude

of the charge on each conductor is proportional to the potential difference between them. The proportionality constant is called the **capacitance** C.

The charge stored in a capacitor is given by

$$Q = CV$$

The capacitance depends on the particular geometry of the two conductors constituting the capacitor. Capacitance does not depend on the charge q nor on the potential difference ΔV.

The unit of capacitance is the **Farad** (F) derived from the name of the physicist Michael Faraday.

$$1\text{F} = 1\frac{\text{C}}{\text{V}}$$

1 μF (microfarad) $= 10^{-6}$ F

1 pF (picofarad) $= 10^{-12}$ F

Problem Solving Tactics

To compute the capacitance of a combination of two conductors:

1. Consider charges $\pm q$ on the two conductors.
2. Use Gauss's law or otherwise to compute the electric field between conductors as a function of charge q.
3. Use the relation between electric field and potential

$$\Delta V = V_f - V_i = \int_i^f dV = \int_i^f \vec{\mathbf{E}} \cdot d\vec{\mathbf{r}}$$

to determine the potential difference between the two conductors. Remember to integrate from low potential to high potential when choosing the direction of $d\vec{\mathbf{r}}$, so that ΔV is positive.

4. Apply $C = \dfrac{q}{\Delta V}$

Capacitance of an Ideal, Parallel Plate Capacitor

Consider two parallel, conducting plates each of area A separated by distance d. Fig. 3.5 shows that parallel plate capacitor with charges $\pm q$ on the two plates. The Gaussian

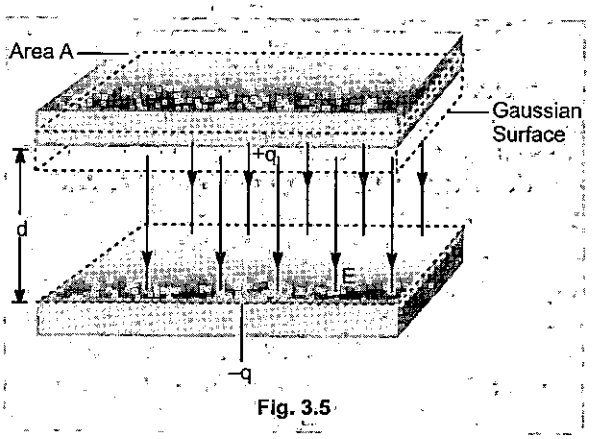


Fig. 3.5

surface is box-shaped, whose top surface is inside the upper conducting plate, where $\vec{\mathbf{E}} = 0$. The bottom face between the plates where $\vec{\mathbf{E}}$ is constant. The electric flux through side faces is zero, as electric field is normal to area vector. Flux linked through top face is zero because $\vec{\mathbf{E}} = 0$. Only bottom surface contributes to the flux. From Gauss's law,

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_{\text{enclosed}}}{\varepsilon_0}$$

$$EA = \frac{q}{\varepsilon_0}$$

Electric field, $E = \dfrac{q}{\varepsilon_0 A}$

Next we determine the potential difference between two plates, with reference to position $v = 0$ at lower plate. Displacement vector $d\vec{\mathbf{r}}$ is directed from lower plate to upper plate. $\vec{\mathbf{E}}$ and $d\vec{\mathbf{r}}$ are in opposite directions.

$$-\vec{\mathbf{E}} \cdot d\vec{\mathbf{r}} = -E\,dr$$

Hence, $$\Delta V = -\int_0^d \vec{\mathbf{E}} \cdot d\vec{\mathbf{r}} = +\int_0^d E\,dr$$

$$= \int_0^d \frac{q}{A\varepsilon_0} dr = \frac{qd}{\varepsilon_0 A}$$

Therefore, capacity

$$C = \frac{q}{\Delta V}$$

$$= \frac{q}{(qd/\varepsilon_0 A)} = \frac{\varepsilon_0 A}{d}$$

The term q or ΔV does not appear in the expression for capacity. The capacitance of an ideal, parallel plate capacitor depends only on the geometrical factors A and d. The capacitance for any geometry is always ε_0 times a combination of geometrical factors with the overall unit of length.

A Spherical Capacitor

Consider two thin, concentric, conducting spheres with radii R_1 and R_2 ($R_1 < R_2$). We assume charges $-q$ on the inner sphere and $+q$ on the outer sphere. The resulting field is radially inward between the spheres. We consider a spherical Gaussian surface for applying Gauss's law

$$\phi_E = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = -E(4\pi r^2)$$

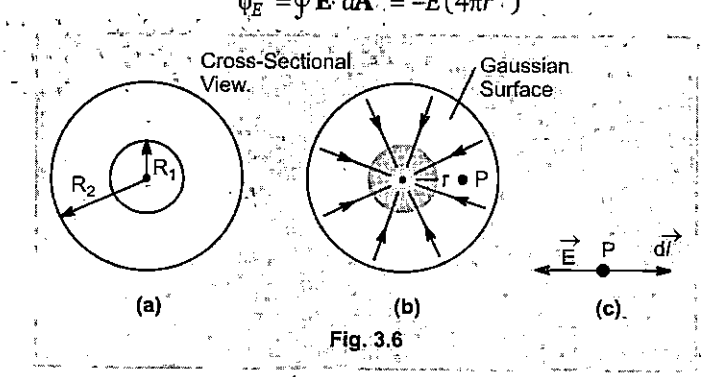


Fig. 3.6

Negative sign shows that electric field $\vec{\mathbf{E}}$ is antiparallel to the area vectors at all points on the Gaussian surface.

Thus, $-E(4\pi r^2) = \frac{Q_{enclosed}}{\varepsilon_0} = -\frac{q}{\varepsilon_0}$

Magnitude of electric field E is given by

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

We assign the potential of negative square $V_1 = 0$ and that of the positive outer surface as $V_2 = V$. The displacement vector $d\vec{\mathbf{r}}$ is directed from inner sphere to outer sphere.

$$\vec{\mathbf{E}}\cdot d\vec{\mathbf{r}} = E\,dr\cos 180° = -E\,dr$$

Therefore, $\Delta V = -\int_{R_1}^{R_2}\vec{\mathbf{E}}\cdot d\vec{\mathbf{r}}$

$$= +\int_{R_1}^{R_2} E\,dr = \frac{q}{4\pi\varepsilon_0}\int_{R_1}^{R_2}\frac{dr}{r^2}$$

So, potential difference between two sphere is

$$\Delta V = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

and $$C = \frac{Q}{\Delta V} = 4\pi\varepsilon_0\left(\frac{R_1R_2}{R_2 - R_1}\right)$$

Note that this result depends on the geometry through R_1 and R_2, and has the form ε_0 times a geometrical factor with overall unit of length.

Concept: $C = \frac{Q}{|\Delta V|} = 4\pi\varepsilon_0\left(\frac{ab}{b-a}\right)$

The capacitance C depends only on the physical dimensions, a and b.

An "isolated" conductor (with the second conductor placed at infinity) also has a capacitance. In the limit where $b \to \infty$, the above equation becomes.

$$\lim_{b\to\infty} C = \lim_{b\to\infty} 4\pi\varepsilon_0\left(\frac{ab}{b-a}\right) \lim_{b\to\infty} 4\pi\varepsilon_0\frac{a}{\left(1-\frac{a}{b}\right)} = 4\pi\varepsilon_0 a$$

Thus, for a single isolated spherical conductor of radius R, the capacitances is $C = 4\pi\varepsilon_0 R$

The above expression can also be obtained by noting that a conducting sphere of radius R with a charge Q uniformly distributed over its surface has $V = Q/4\pi\varepsilon_0 R$, using infinity as the reference point having zero potential, $V(\infty) = 0$. This gives

$$C = \frac{Q}{|\Delta V|} = \frac{Q}{Q/4\pi\varepsilon_0 R} = 4\pi\varepsilon_0 R$$

As expected, the capacitance of an isolated charged sphere only depends on its geometry, namely, the radius R.

A Cylindrical Capacitor

We consider a cylindrical capacitor comprised of two concentric cylindrical conductors of length L. The inner and outer radius are R_1 and R_2 respectively as shown in Fig. 3.7.

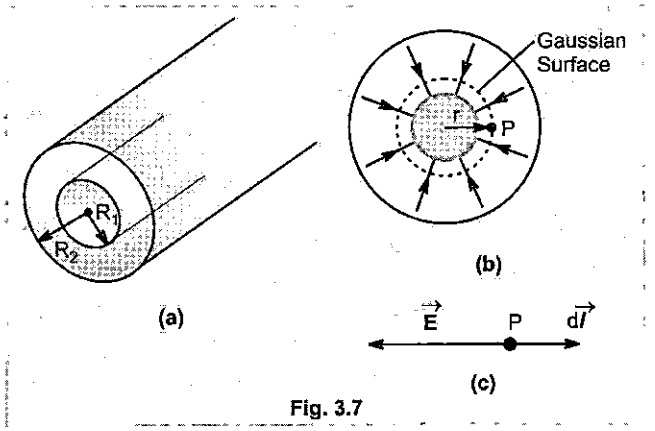


Fig. 3.7

We assume a charge $-q$ on the inner cylinder and $+q$ on the outer one and E is directed radially inward at all points between cylinders, we ignore fringing effect of the electric field at the ends of the cylinders. Applying Gauss's law to a cylindrical Gaussian surface of radius r.

$$\phi_E = \oint\vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = -E\cdot 2\pi rl$$

$$= \frac{Q_{enclosed}}{\varepsilon_0} = -\frac{q}{\varepsilon_0}$$

$$E = \frac{q}{2\pi\varepsilon_0 rL}$$

In order to calculate the potential difference between the cylindrical conductors we must integrate from R_1 to R_2 in order for ΔV to be positive. The displacement vector $d\vec{\mathbf{r}}$ is radially outward and electric field is radially inward.

$$\vec{\mathbf{E}}\cdot d\vec{\mathbf{r}} = E\,dr\cos 180° = -E\,dr$$

Thus, $\Delta V = -\int_{R_1}^{R_2}\vec{\mathbf{E}}\cdot d\vec{\mathbf{r}} = +\int_{R_1}^{R_2} E\,dr$

$$= \frac{q}{2\pi\varepsilon_0 L}\int_{R_1}^{R_2}\frac{dr}{r} = \frac{q}{2\pi\varepsilon_0 L}\ln\left(\frac{R_2}{R_1}\right)$$

Therefore, $C = \frac{q}{\Delta V} = \frac{2\pi\varepsilon_0 L}{\ln(R_2/R_1)}$

Capacitance per unit length $\frac{C}{L} = \frac{2\pi\varepsilon_0}{\ln(R_2/R_1)}$

The capacity is product of ε_0 and a combination of geometric factors with the overall unit of length.

Energy Storage in a Capacitor

Energy stored in a capacitor is electrical potential energy. This energy is related to the charge Q and voltage V on the capacitor. The electrical potential energy, $\Delta PE = q\Delta V$.

The capacitor starts with zero voltage and gradually charged to its full voltage. The first charge placed on the capacitor experiences $\Delta V = 0$, since the capacitor has zero voltage when uncharged. When same charge is on each plate, it requires work to add more charges of same sign because of the electric repulsion. The final charge placed on the capacitor experiences a potential difference $\Delta V = V$, since the capacitor now has full voltage V on it. The average potential during the charging is half of this final value.

Average potential difference $= \frac{1}{2}V$

The total potential energy U_E stored by the charged capacitor equals the sum of the increase in potential energy of all the charge increments Δq, *i.e.*, the increase in potential energy of all the transferred charge Q. This increase in potential energy equals the product of the charge times the average value of the potential difference.

$$U_E = \frac{1}{2}QV$$

Since, $Q = CV$

$$U_E = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C}$$

The net effect of charging a capacitor is to remove charge from one plate and add it to the other plate. When a battery is connected to a capacitor it performs this task. The work needed to add a small amount of charge dq, when a potential difference V is across the plates, is

$$dW = V\,dq = \frac{q\,dq}{C} \qquad (\text{since } V = q/C)$$

The work needed to store a total charge Q from initially uncharged state is

$$W = \int_0^Q V\,dq = \frac{1}{C}\int_0^Q q\,dq = \frac{1}{2}\frac{Q^2}{C}$$
$$= \frac{1}{2}CV^2 = \frac{1}{2}QV$$

When a capacitor is charged by connecting it to a battery having potential difference V between its terminals, the work done by the battery is QV in delivering a charge Q to the capacitor. The work done by the battery is twice the energy stored in the capacitor. The extra work done by the battery is dissipated in heating the connecting wires or by radiation.

Electrostatic Field Energy

An electric field is produced between the plates of a capacitor.

The work done required for charging may be though of as the work required to create an electric field.

The energy stored in the capacitor is the energy stored in the electric field, this energy is called electrostatic field energy.

Energy of this capacitor is stored in its surface to infinity.

Concept:1. *Energy Density of the Electric Field:* *One can think of the energy stored in the capacitor as being stored in the electric field itself. In the case of a parallel-plate capacitor, with $C = \varepsilon_0 A / d$ and $|\Delta V| = Ed$, we have*

$$U_E = \frac{1}{2}C|\Delta V|^2 = \frac{1}{2}\frac{\varepsilon_0 A}{d}(E\cdot d)^2$$
$$= \frac{1}{2}\varepsilon_0 E^2 (A\cdot d)$$

Since the quantity $A\cdot d$ represents the volume between the plates, we can define the electric energy density as

$$u_E = \frac{U_E}{\text{volume}} = \frac{1}{2}\varepsilon_0 E^2$$

Note that u_E is proportional to the square of the electric field. Alternatively, one may obtain the energy stored in the capacitor from the point of view of external work. Since the plates are oppositely charged, force must be applied to maintain a constant separation between them.

We see that a small patch of charge $\Delta q = \sigma(\Delta A)$ experience an attractive force $\Delta G = \sigma^2(\Delta A)/2\varepsilon_0$. If the total area of the plate is A, then an external agent must exert a force $F_{ext} = \sigma^2 A/2\varepsilon_0$ to pull the two plates apart. Since the electric field strength in the region between the plates is given by $E = \sigma/\varepsilon_0$, the external force can be rewritten as

$$F_{ext} = \frac{\varepsilon_0}{2}E^2 A$$

Note that F_{ext} is independent of d. The total amount of work done externally to separate the plates by a distance d is then

$$W_{ext} = \int \vec{F}_{ext}\cdot d\vec{S} = F_{ext}d = \left(\frac{\varepsilon_0 E^2 A}{2}\right)d$$

consistent with Eq. Since the potential energy of the system is equal to the work done by the external agent, we have $u_E = W_{ext}/Ad = \varepsilon_0 E^2/2$. In addition, we note that expression for u_E is identical to Eq. in. Therefore, the electric energy density u_E can also be interpreted as electrostatic pressure P.

2. *Energy Stored in a Spherical Shell*

The electric field associated of a spherical shell of radius a is

$$\vec{E} = \begin{cases} \dfrac{Q}{4\pi\varepsilon_0 r^2}\hat{r}, & r > a \\ \vec{0}, & r < a \end{cases}$$

The corresponding energy density is:

$$u_E = \frac{1}{2}e_0 E^2 = \frac{Q^2}{32\pi^2\varepsilon_0 r^4}$$

outside the sphere, and zero inside. Since the electric field is non-vanishing outside the spherical shell, we must integrate over the entire region of space from $r = a$ to $r = \infty$. In spherical coordinates, with $dV = 4\pi r^2 dr$, we have

$$U_E = \int_a^\infty \left(\frac{Q^2}{32\pi^2\varepsilon_0 r^4}\right)4\pi r^2 dr$$
$$= \frac{Q^2}{8\pi\varepsilon_0}\int_a^\infty \frac{dr}{r^2} = \frac{Q^2}{8\pi\varepsilon_0 a} = \frac{1}{2}QV$$

where $V = Q/4\pi\varepsilon_0 a$ is the electric potential on the surface of the shell, with $V(\infty) = 0$. We can readily verify that the energy of the system is equal to the work done in charging the sphere. To show this, suppose at some instant the sphere has charge q and is at a potential $V = q/4\pi\varepsilon_0 a$. The work required to add an additional charge dq to the system is $dW = Vdq$. Thus, the total work is:

$$W = \int dW = \int Vdq = \int_0^Q dq\left(\frac{q}{4\pi\varepsilon_0 a}\right) = \frac{Q^2}{8\pi\varepsilon_0 a}$$

Combination of Capacitors

Consider a combination of two capacitors as shown in Fig. 3.8(a). The capacitors are connected such that the upper two plates of the capacitor are connected by a conducting wire and are therefore at common potential V_a. The lower plates are also connected together and are at common potential V_b. The potential difference is same across each capacitor. The capacitors are said to be connected in **parallel**. In Fig. 3.8(b), two capacitors are connected so that the magnitude of the charge on the two capacitors is the same. These capacitors are said to be connected is series.

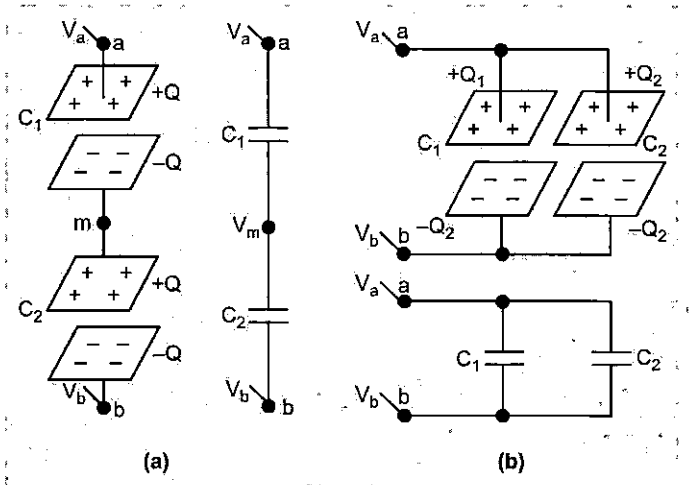


Fig. 3.8

Parallel Combination of Capacitors

Consider a collection of four capacitors in parallel as shown in Fig. 3.9. Since the left plates of all the capacitors are connected to a common point through connected wires, they have same potential, similarly the right plates have same potential. From the definition of capacitance,

$$C_1 = \frac{Q_1}{V}, C_2 = \frac{Q_2}{V}$$

$$C_3 = \frac{Q_3}{V}, C_4 = \frac{Q_4}{V}$$

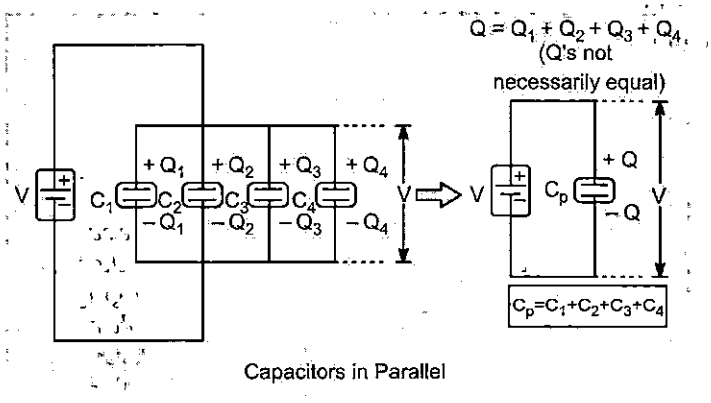


Fig. 3.9

We want to find a single equivalent capacitor of capacitance $C_{eq.}$ that can be placed between the same two terminals (A and B) and maintain the same potential difference

$$i.e., \qquad C_{eq.} = \frac{Q}{V}$$

The total charge on one side of the capacitor must be the sum of charges on the individual capacitor plates, the same is true for the other side of the capacitors. Thus, the magnitude of charge on each plate of the equivalent capacitor must be

$$Q = Q_1 + Q_2 + Q_3 + Q_4$$

$$C_{eq.}V = C_1V + C_2V + C_3V + C_4V$$

$$C_{eq.} = C_1 + C_2 + C_3 + C_4$$

$$C_{eq.} = \sum_{i=1}^{N} C_i$$

The equivalent capacitance is sum of the individual capacitances.

Series Connection of Capacitors

Four capacitors in series are shown in Fig. 3.10. Let their capacitances be C_1, C_2, C_3 and C_4. The total potential difference across V between the point A and B is the sum of potential differences across each capacitor. A charge $+Q$ flows from the battery to one plate of C_1 and $-Q$ flows to one

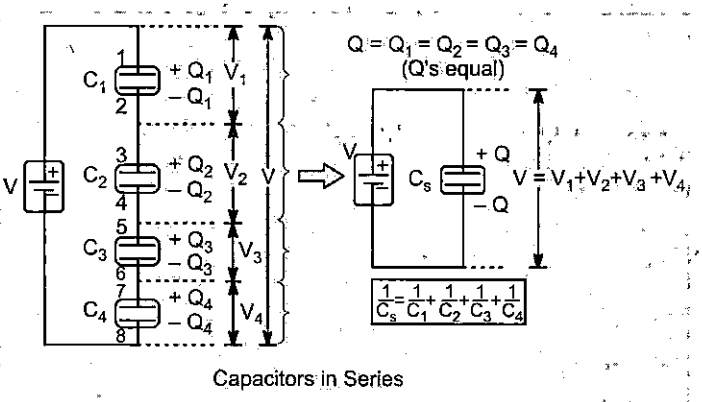


Fig. 3.10

plate of C_4. The plates numbered 2, 3, 4, 5, 6 and 7 were originally neutral, so net charge there must still be zero. The positive charge on plate 1 causes charge separation to take place on the isolated and neutral plate 2. Since plates 2 and 3 have to be neutral, there is a charge $+Q$ on plate 3. Similar considerations apply to other capacitors. So we see the charge on each plate has same magnitude Q. A single equivalent capacitor that can be plated between points A and B to maintain same potential difference would have capacitors $C_{eq.}$ where

$$Q = C_{eq.}V$$

$$V = V_1 + V_2 + V_3 + V_4 \qquad \ldots(1)$$

$Q = C_1V_1 = C_2V_2 = C_3V_3 = C_4V_4$. So we substitute for V_1, V_2, V_3 and V_4 and V into eqn. (1) and get

$$\frac{Q}{C_{eq.}} = \frac{Q}{C_1} + \frac{Q_2}{C_2} + \frac{Q_3}{C_3} + \frac{Q_4}{C_4}$$

$$\frac{1}{C_{eq.}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_4}$$

Note that equivalent capacitance $C_{eq.}$ is smaller than the smallest contributing capacitance.

CAPACITORS AND DIELECTRICS

A dielectric is a technical term for an insulator. Most capacitors in electronic applications have dielectric materials between the two conductors. When a dielectric is placed between the plates of a capacitor, the capacitance increases. The capacitance of a parallel plate capacitor is given by

$$C = \varepsilon_0 \frac{A}{d}$$

The constant $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m is the permittivity of free space. The term free space is used for vacuum. In any insulating medium the permittivity ε is given by

$$\varepsilon = K\varepsilon_0$$

where K is called **dielectric constant**.

The dielectric constant of vacuum is 1 and for air under standard conditions is very close to 1, so that air-filled capacitors act much like those with vacuum between their plates.

The value of dielectric constant depends on environmental factors such as temperature and pressure. The changes in dielectric constant are the result of submicroscopic (atomic or molecular) changes in the material.

Dielectric strength (V/m) is the field above which the material begins to break down and conduct. The dielectric strength imposes a limit on the voltage that can be applied for a given plate separation. Dielectric strength of air is 3×10^6 V/m and that of teflon is 60×10^6 V/m. So the capacitor filled with teflon has a greater capacitance and can be subjected to a much greater voltage.

When the electric field in air exceeds its dielectric strength, air molecules become ionized and are accelerated by field and so the air becomes conducting. This happens, *e.g.*, in an electrical storms or when the electric field around a high-voltage transmission line becomes too great.

Expression for capacitance of a parallel plate capacitor when a dielectric is placed between the plates,

$$C = \frac{K\varepsilon_0 A}{d} = KC_0$$

Polarisation of the insulator is responsible for the increase in capacitance. The more easily a dielectric is polarized, the greater the dielectric constant K. Polarization is separation of charge within an atom or molecule.

If the charge distribution in a molecule is not symmetric, so that it has positive and negative sides or "poles", it is called a **Polar Molecule** (Fig. 3.11).

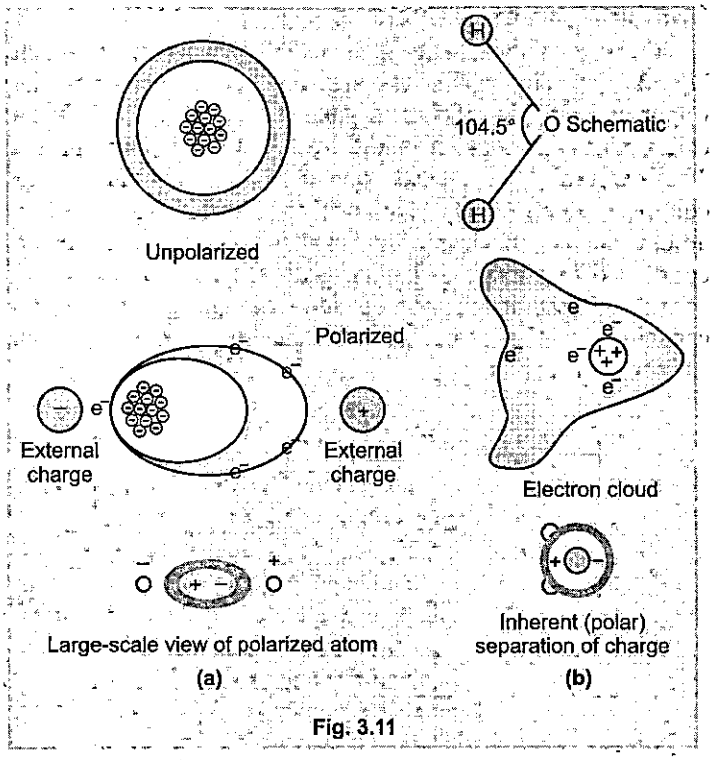


Fig. 3.11

The orbits of electrons are viewed as electron clouds with density of the cloud related to the probability of finding an electron in that location. This cloud is shifted by the Coulomb force so that the atom on average has a separation of charge. Water molecules have an inherent separation of charge and are thus called polar molecules. Fig. 3.11(b) illustrates the separation of charge in a water molecule. The electrons in a water molecule are more concentrated around the highly charged oxygen end is slightly negative and the hydrogen end is slightly negative.

The centre of charge (analogous to centre of mass) of the positive charges and that of negative charges are separated, which means that the molecule acts like a small electric dipole.

The torque experienced by an electric dipole in a uniform electric field is

$$\vec{\tau} = \vec{\mathbf{p}} \times \vec{\mathbf{E}}$$

The potential energy of a dipole in an electric field is

$$pE = -\vec{\mathbf{p}} \cdot \vec{\mathbf{E}}$$

An electric dipole in an electric field tends to align itself so that dipole moment vector is parallel to field and electrical potential energy is minimum. The alignment is not complete due to string intermolecular bonds and thermal motions of molecules, but increasing the field increases the degree of alignment with the field. The inherent separation of charge in polar molecules makes it easier to align them with external fields and charges. *Polar molecules therefore exhibit greater polarization effects and have greater dielectric constants.*

In non-polar molecules, the charge distribution inside each molecule is eclectrically symmetric in the absence of a field. When the material is placed in an electric field, the lighter, negatively charged electrons and their centre of charge are displaced slightly w.r.t. the centre of the positive charge distribution (formed from the much more massive, less mobile, positively charged nuclei). The result is an induced electric dipole moment in the molecules of the nuclei.

Introduction of Dielectric Slab in a Capacitor

Consider an isolated, charged parallel plate capacitor with air or vacuum between its plates. The capacitance of the system is

$$C_0 = \frac{\varepsilon_0 A}{d}$$

The magnitude of the electric field E_0, between the plates, is related to the potential difference by

$$V_0 = E_0 d$$

Now we insert a dielectric slab, dielectric constant k, completely filling the space between the plates of the capacitor as shown in the figure. The capacitor is not connected to the battery so charge cannot flow to or from the plates. This capacitor has a charge $+Q$ on one plate and $-Q$ on the other.

In the absence of electric field the dipole moments of polar molecules are randomly oriented and the net charge on all the dielectric's surface is zero. When the dielectric is placed in the electric field of the capacitor, the polar molecules experience a torque and tend to align with the electric field.

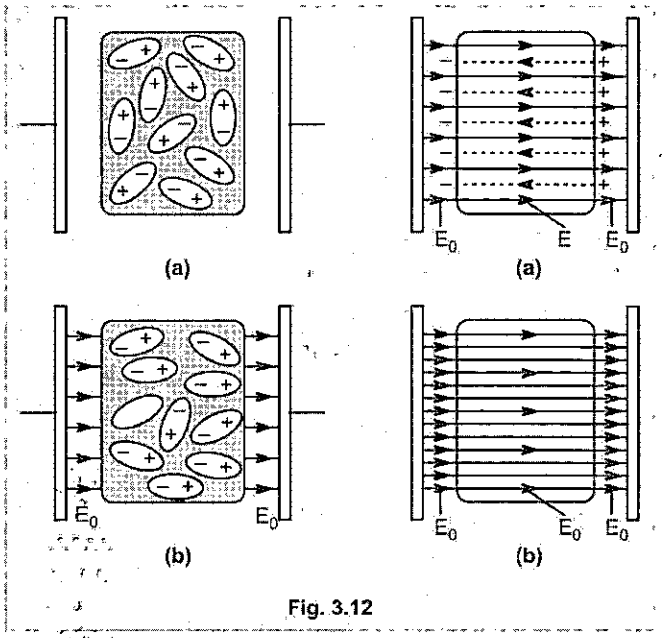


Fig. 3.12

Note that overall effect of the dipole alignment is to make the left surface of the dielectric negative (dielectric surface facing the capacitor plate with charge $+Q_0$) and the right surface positive (dielectric surface facing the capacitor plate with charge $-Q_0$). These unbalanced charges are not free to move (charges are free in conductors only) they are tied to the molecule. These charges on the surface of the dielectric are called **bound charges, induced surface charges.** This bound charge density, σ_{bound}, creates an electric field of magnitude $\sigma_{bound}/\varepsilon_0$, while that due to the free charge on the capacitor plates creates a field of magnitude σ/ε_0. The field created by bound charges is opposite in direction to that created by free charge on the capacitor plates. Note that free charge is that which we place on the conducting capacitor plates by some external source (battery) whereas bound charges appear due to polarization effects.

When the charged capacitor is not connected to a source of additional charge the creation of the bound charge density σ_{bound} means that the electric field strength E between the plates is reduced to a lower value E from its value E_0. The total electric field between the plates of capacitor is reduced by the factor of dielectric constant K.

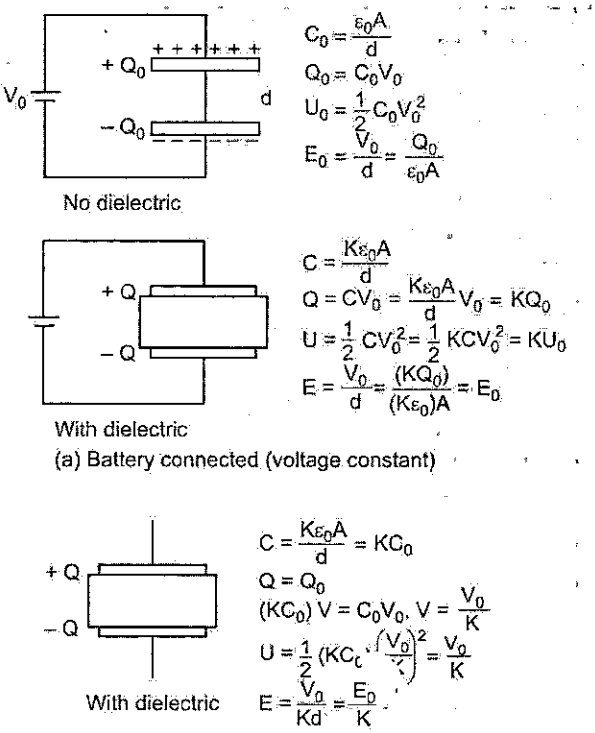
$$E = \frac{E_0}{K}$$

The potential difference between the capacitor plates is decreased by a factor K by the presence of dielectric slab

$$V = \frac{V_0}{K}$$

The charge Q_0 on the conducting plates is unaffected because the capacitor is isolated.

Fig. 3.13 summarizes the effect of inserting a dielectric between the plates of a capacitor.



$C_0 = \frac{\varepsilon_0 A}{d}$, $Q_0 = C_0 V_0$, $U_0 = \frac{1}{2} C_0 V_0^2$, $E_0 = \frac{V_0}{d} = \frac{Q_0}{\varepsilon_0 A}$

$C = \frac{K\varepsilon_0 A}{d}$, $Q = CV_0 = \frac{K\varepsilon_0 A}{d} V_0 = KQ_0$, $U = \frac{1}{2} CV_0^2 = \frac{1}{2} KCV_0^2 = KU_0$, $E = \frac{V_0}{d} = \frac{(KQ_0)}{(K\varepsilon_0)A} = E_0$

$C = \frac{K\varepsilon_0 A}{d} = KC_0$, $Q = Q_0$, $(KC_0)V = C_0 V_0$, $V = \frac{V_0}{K}$, $U = \frac{1}{2}(KC_0)\left(\frac{V_0}{K}\right)^2 = \frac{V_0}{K}$, $E = \frac{V_0}{Kd} = \frac{E_0}{K}$

Fig.3.13

Gauss's Law and the Electric Field Vectors

The Gauss's law in its familiar form is expressed as

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_{\text{enclosed}}}{\varepsilon_0} \quad \ldots(1)$$

Fig. 3.14(a) shows the capacitor with no dielectric present and the resulting field. Fig. 3.14(b) shows the same capacitor with dielectric between the plates. In each case

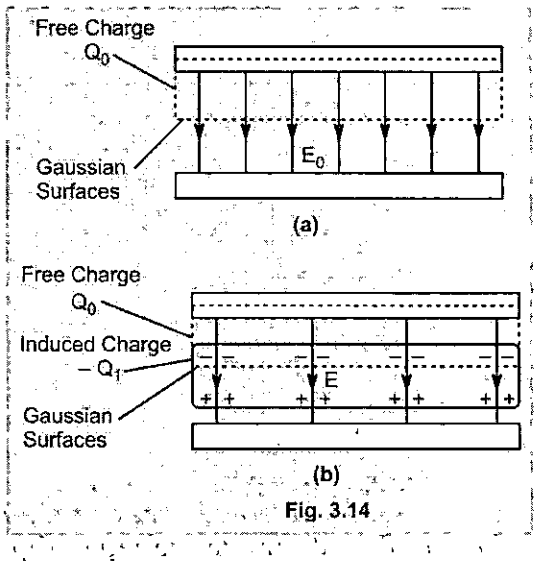


Fig. 3.14

consider a rectangular box-shaped Gaussian surface, the top side of which is in the conductor so that $E = 0$ through this portion of the surface. The flux linked through front, back and sides is zero as the electric field is perpendicular to the area vectors. Flux is linked with bottom surface only, the electric field E_0 in case (a) and E in case (b) is parallel to the area vectors.

In Fig. 3.14 (a) the charge enclosed is Q_0, from Gauss's law,

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \oint E_0 dA$$

$$= E_0 A = \frac{Q_0}{\varepsilon_0}$$

or $$E_0 = \frac{Q_0}{\varepsilon_0 A} \quad \ldots(2)$$

In Fig. 3.14 (b) the net charge enclosed is $Q_0 - Q_{\text{bound}}$

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_0 - Q_{\text{bound}}}{\varepsilon_0} \quad \ldots(3)$$

$$\oint E\, dA = EA$$

$$= \frac{Q_0 - Q_{\text{bound}}}{\varepsilon_0}$$

$$E = \frac{Q_0 - Q_{\text{bound}}}{\varepsilon_0} \quad \ldots(4)$$

As $K = \frac{E_0}{E}$, we substitute expressions for E_0 and E to obtain

$$K = \frac{Q_0/A\varepsilon_0}{(Q_0 - Q_{\text{bound}})/A\varepsilon_0}$$

$$Q_0 - Q_{\text{bound}} = \frac{Q_0}{K} \quad \ldots(5)$$

We put this result in eqn. (3) to obtain Gauss's law when dielectric is present.

$$\oint E \cdot dA = \frac{Q_0}{K\varepsilon_0}$$

In vacuum $K = 1$ and the Gauss's law is reduced to its original form.

An isolated capacitor is connected to a battery so that potential difference is constant. Now a dielectric is inserted into the space between the plates of the capacitor, completely filling it. With dielectric or without it, the battery maintains a constant potential difference between plates. Because $\Delta V = Q_0/C_0$ and C_0 increases to KC_0, the charge on the capacitor must change:

$$\Delta V = \frac{Q_0}{C_0} = \frac{Q}{KC_0}$$

Solving for new charge Q on the capacitor, we have

$$Q = KQ_0 \qquad (\Delta V \text{ held constant})$$

Energy stored by the capacitor has changed.

$$V = \frac{Q^2}{2e} = \frac{(KQ_0)^2}{2(KC_0)} = K\left(\frac{Q_0^2}{2C_0}\right)$$

$$U = KU_0 \qquad (\Delta U \text{ held constant})$$

When the dielectric is inserted the bound charges are produced on the surface facing the capacitor plates. The field produced by the bound charges is opposite to the original field between the plates. The total field is the superposition of the field from the charges on the conducting plates and the field caused by the bound charges on the surface of the dielectric. Additional charge must be supplied to the conducting plates of the capacitor by the battery, in order to maintain the same total field as before.

The capacity, $$C = \frac{Q_0}{V} = \frac{Q_0}{(E_0 d/K)}$$

$$= K\frac{Q_0}{E_0 d} = K\frac{Q_0}{V_0}$$

$$= KC_0$$

The capacity is increased by the factor of the dielectric constant.

Field due to free charge $= \frac{\sigma_0}{\varepsilon_0}$

Field due to bound charge $= \frac{\sigma_{\text{bound}}}{\varepsilon_0}$

Because the field due to free charge and the field due to bound charges are opposite in direction, the magnitude of total field is

$$E = E_0 - E_{\text{bound}}$$

$$= \frac{\sigma_0}{\varepsilon_0} - \frac{\sigma_{\text{bound}}}{\varepsilon_0}$$

As we learned earlier that

$$E = \frac{E_0}{K} = \frac{\sigma_0}{\varepsilon_0}\frac{1}{K}$$

Therefore $$\frac{\sigma_0}{\varepsilon_0}\frac{1}{K} = \frac{\sigma_0}{\varepsilon_0} - \frac{\sigma_{bound}}{\varepsilon_0}$$

Solving for σ_{bound}, we obtain

$$\sigma_{bound} = \left(\frac{K-1}{K}\right)\sigma_0$$

Also $$Q_{bound} = \left(\frac{K-1}{K}\right)Q_0$$

The bound surface charge density is always less than the free surface charge density.

Electrostatic Field Energy of a Spherical Conductor

Consider a spherical conductor of radius R that carries a charge Q. The electric field is radial and given by

$$E = 0, \quad (r < R) \quad \text{(inside the conductor)}$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}, \quad (r > R) \text{(outside the conductor)}$$

The electric field is spherically symmetric. We choose a spherical shell of radius r and thickness dr as our volume element.

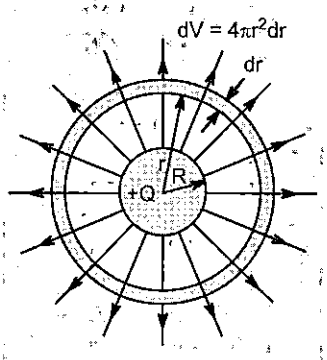


Fig. 3.15

The volume is $dV = 4\pi r^2 dr$

The energy in this volume element is

$$dU = u_E dV = \frac{1}{2}(\varepsilon_0 E^2)4\pi r^2 dr$$

$$= \frac{1}{2}\varepsilon_0\left(\frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\right)^2 4\pi r^2 dr$$

$$= \frac{1}{2}\left(\frac{1}{4\pi\varepsilon_0}\right)Q^2\frac{dr}{r^2}$$

The electric field is zero for $r < R$, we obtain the total energy in the electric field by integrating from $r = R$ to $r = \infty$.

$$U = \int u_E dV$$

$$= \frac{1}{2}\frac{1}{4\pi\varepsilon_0}Q^2\int_R^\infty \frac{dr}{r^2} = \frac{Q^2}{8\pi\varepsilon_0 R}$$

$$= \frac{1}{2}Q\left(\frac{1}{4\pi\varepsilon_0}\frac{Q}{R}\right)$$

$$= \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C}$$

The result could have been obtained directly, since a spherical conductor is capacitor of capacitance $4\pi\varepsilon_0 R$. Consider a parallel plate capacitor.

The capacitance of capacitor,

$$C = \frac{\varepsilon_0 A}{d}$$

where A is the area of the plates and d is the separation between the plates.

Potential difference between the plates, $V = Ed$, where E is the electric field between the plates. The energy stored is

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\left(\frac{\varepsilon_0 A}{d}\right)(Ed)^2$$

$$= \frac{1}{2}\varepsilon_0 E^2(Ad)$$

where Ad is the volume of the space between the plates of the capacitor containing electric field. The energy per unit volume is called the energy density u_E.

Energy density,

$$u_E = \frac{\text{energy}}{\text{volume}} = \frac{1}{2}\varepsilon_0 E^2$$

The electric energy stored per unit volume in any region of space is proportional to the square of the electric field in that region. The result derived is true for any region of space where electric field exists.

CHARGE SHARING BETWEEN CONDUCTORS

When two conductors of capacities C_1 and C_2 having charges q_1 and q_2 respectively are joined together by a conducting wire. Charge redistributes till potential of both the conductors become equal.

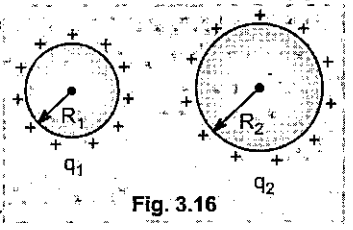


Fig. 3.16

Let q_1' and q_2' be the final charges on them. Charge on a conductor is proportional to its capacity.

Hence $$\frac{q_1'}{q_2'} = \frac{C_1}{C_2}$$

In case of spherical conductors,

$$\frac{C_1}{C_2} = \frac{R_1}{R_2}$$

$$\frac{q_1'}{q_2'} = \frac{C_1}{C_2} = \frac{R_1}{R_2} \quad \ldots(1)$$

From conservation of charge, we have

$$q_1 + q_2 = q_1' + q_2' \quad \ldots(2)$$

Hence from eqn. (1) and eqn. (2), we have

$$q_1' = \left(\frac{C_1}{C_1 + C_2}\right)(q_1 + q_2)$$

$$= \left(\frac{R_1}{R_1 + R_2}\right)(q_1 + q_2)$$

and $$q'_2 = \left(\frac{C_2}{C_1 + C_2}\right)(q_1 + q_2)$$

$$= \frac{R_2}{R_1 + R_2}(q_1 + q_2)$$

If redistribution of charge takes place energy is lost.

Initial potential energy,

$$U_i = \frac{1}{2}\frac{q_1^2}{C_1} + \frac{1}{2}\frac{q_2^2}{C_2}$$

Final potential energy,

$$U_f = \frac{1}{2}\frac{(q_1 + q_2)^2}{C_1 + C_2}$$

$$\Delta U = U_i - U_f = \frac{1}{2}\left[\frac{q_1^2}{C_1} + \frac{q_2^2}{C_2} - \frac{(q_1 + q_2)^2}{C_1 + C_2}\right]$$

or $$\Delta U = \frac{1}{2C_1C_2(C_1 + C_2)}[q_1^2C_1C_2 + q_1^2C_2^2 + q_2^2C_1^2 + q_2^2C_1C_2 - q_1^2C_1C_2 - q_2^2C_1C_2 - 2q_1q_2C_1C_2]$$

$$= \frac{C_1^2C_2^2}{2C_1C_2(C_1 + C_2)}\left[\frac{q_1^2}{C_1^2} + \frac{q_2^2}{C_2^2} - \frac{2q_1q_2}{C_1C_2}\right]$$

$$= \frac{C_1C_2}{2(C_1 + C_2)}[V_1^2 + V_2^2 - 2V_1V_2]$$

or $$\Delta U = \frac{C_1C_2}{2(C_1 + C_2)}(V_1 - V_2)^2$$

Because $(V_1 - V_2)^2$ is positive, $U_i > U_f$, i.e., there is decrease in energy.

Example 1

A parallel plate capacitor completely filled with a dielectric slab, dielectric constant $K = 3.4$, is fully charged with a 100 V battery. When the capacitor is fully charged, the battery is removed. The area of plates $A = 4.0\,m^2$ and separation between plates $d = 4.0$ mm. (a) Find the capacitance, the charge on the capacitor. (b) The dielectric slab is slowly removed without changing the plate separation and any charge transfer from capacitor. Find the new capacitance, electric field strength, voltage between plates and the energy stored in the capacitor.

Solution: (a) The capacitance of a capacitor with dielectric,

$$C = \frac{K\varepsilon_0 A}{d} = \frac{(3.4)(8.85\times10^{-12})(4)}{4\times10^{-3}}$$

$$= 3.0\times10^{-8}\,F$$

The charge on the plates is

$$E = \frac{V}{d} = \frac{100}{4.0\times10^{-3}} = 25\,kV/m$$

The total energy stored in the capacitor is

$$U = \frac{1}{2}CV^2 = \frac{1}{2}(3\times10^{-8})(100)^2$$

$$= 1.5\times10^{-4}\,J$$

(b) The capacitance without dielectric is

$$C_0 = \frac{C}{K} = \frac{3\times10^{-8}}{3.4} = 8.8\times10^{-9}\,F$$

There is no transfer of charge, hence

$$CV = C_0V_0$$

$$(KC_0)V = C_0V_0$$

$$V_0 = KV$$

So potential difference between plates increases by a factor $K = 3.4$

The electric field is

$$E = \frac{V}{d} = \frac{340}{4.0\times10^{-3}} = 85\,kV/m$$

The energy of the capacitor is

$$U = \frac{1}{2}CV^2 = \frac{1}{2}(8.8\times10^{-9})(340)^2$$

$$= 5.1\times10^{-4}\,J$$

The energy of the capacitor has increased because work has to be done by an external agent to remove the dielectric slab. The work is required because of the force of attraction between induced charge on the dielectric and the charges on the plates.

Example 2

A parallel plate capacitor with plates of area A, plate separation d, is partially filled with a dielectric slab of constant K, as shown in Fig. 3E.2. The thickness of the dielectric slab is d.

(a) What is the equivalent capacitance of this arrangement?

(b) If a potential difference of V volt is maintained across the plates, find the induced surface charge on the dielectric.

Solution: (a) The given capacitor arrangement is equivalent to the series combination of the two capacitors as shown in Fig. 3E.2(b) because the electric potential is the same at all points on the lower surface of the dielectric. All the points on the surface of a conductor are equipotential, therefore, all points on the I-shaped conductor connecting the two capacitors are equipotential. Thus we can divide the entire region of capacitors in two parts. The capacitances of the two part capacitors are

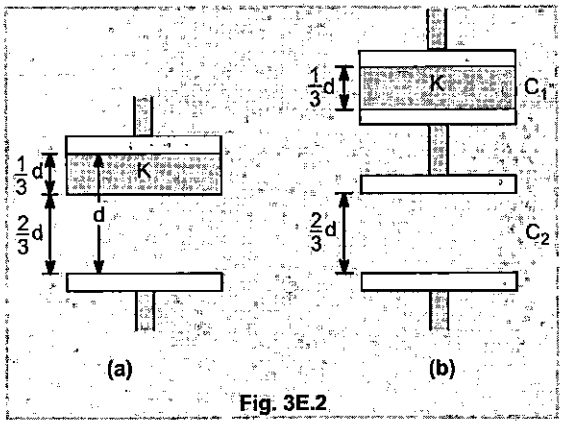


Fig. 3E.2

$$C_1 = \frac{AK\varepsilon_0}{d/4}$$

and $$C_2 = \frac{A\varepsilon_0}{(3d/4)}$$

Equivalent capacity,

$$\frac{1}{C_{eq.}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$= \frac{d/4}{AK\varepsilon_0} + \frac{3d/4}{K\varepsilon_0}$$

$$C_{eq.} = \frac{4A\varepsilon_0}{d}\left[\frac{3K+1}{K}\right]$$

(b) The total charge on the capacitor,

$$Q = \frac{4A\varepsilon_0}{d}\left[\frac{3K+1}{K}\right] \times V$$

Since the two capacitors are in series, they have the same charge Q.

Free charge density,

$$\sigma_0 = \frac{Q}{A} = \frac{4A\varepsilon_0}{d}\left[\frac{3K+1}{K}\right]V$$

Induced charge density on the dielectric,

$$\sigma_i = \left(\frac{K-1}{K}\right)\sigma_0$$

$$= \left(\frac{K-1}{K}\right)\left(\frac{4\varepsilon_0}{d}\right)\left[\frac{3K+1}{K}\right]V$$

$$= \frac{4\varepsilon_0 V(K-1)(3K+1)}{K^2 d}$$

Example 3

A capacitor of capacity C_0 is connected across a battery of emf V_0. How much work must be done in order to double the plate separation (a) with the battery connected, and (b) with it disconnected?

Solution: (a) The capacitance of a parallel-plate capacitor is inversely proportional to the separation of the plates, the final capacitance must be $C_0/2$. The initial energy stored by the capacitor is $C_0V^2/2$, and the final energy stored is $C_0V^2/4$ if the battery is connected (so that the potential difference across the capacitor remains at V). Thus the energy stored by the capacitor has decreased by $C_0V^2/4$. However, the charge stored in the capacitor must change from C_0V to $C_0V/2$, so an amount of charge $C_0V/2$ must be pushed 'backwards' through the battery, increasing its potential energy by $C_0V^2/2$. The increase in the total energy of the system, and thus the work which must be done on it, is thus $C_0V^2/4$.

(b) If the battery is disconnected, the charge on the capacitor must remain constant while the capacitance is being charged. This will change the potential difference across the capacitor. From the relationship $Q = CV$, having the capacitance must double the potential difference. Thus the energy stored by the capacitor after the plates have been separated must be $\frac{1}{2}\left(\frac{C_0}{2}\right)(2V)^2 = C_0V^2$, *i.e.*, an increase of $C_0V^2/2$.

Since the battery is disconnected this is the only energy term, so the work done on the system must be $C_0V^2/2$.

Example 4

A parallel-plate capacitor has a plate separation d and plate area A. An uncharged metallic slab of thickness a is inserted mid-way between the plates. (a) Find the capacitance of the device. (b) Show that the capacitance is unaffected if the metallic slab is infinitesimally thin. (c) Show that the answer to part (a) does not depend on where the slab is inserted.

Solution: (a) Any charge that appears on one plate of the capacitor must induce a charge of equal magnitude but opposite sign on the near of the slab, as shown in Fig.

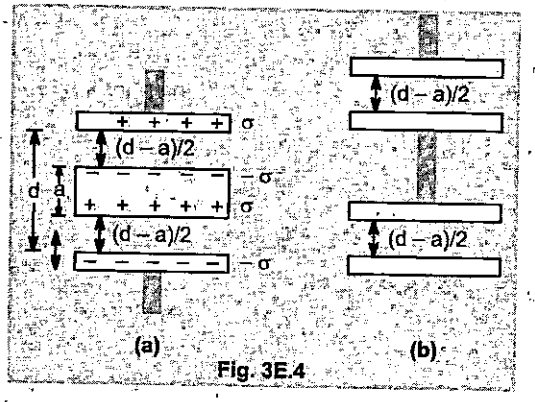


Fig. 3E.4

3E.4(a). Consequently, the net charge on the slab remains zero, and the electric field inside the slab is zero. Hence, the capacitor is equivalent to two capacitors in series, each having a plate separation $(d-a)/2$, as shown in series, we obtain

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$= \frac{1}{\frac{\varepsilon_0 A}{(d-a)/2}} + \frac{1}{\frac{\varepsilon_0 A}{(d-a)/2}}$$

$$C = \frac{\varepsilon_0 A}{d-a}$$

Note that, C approaches infinity as a approaches d. Why?

(b) In the result for part (a), we let $a \to 0$;

$$C = \lim_{a\to 0} \frac{\varepsilon_0 A}{d-a} = \frac{\varepsilon_0 A}{d}$$

which is the original capacitance.

(c) Let the slab in Fig. 3E.4(a) is moved upward so that the distance between the upper edge of the slab and the upper plate is b. Then, the distance between the lower edge

of the slab and the lower plate is $d-b-a$. The total capacitance of the series combination.

$$\frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_2}=\frac{1}{\frac{\varepsilon_0 A}{b}}+\frac{1}{\frac{\varepsilon_0 A}{d-b-a}}$$

$$=\frac{b}{\varepsilon_0 A}+\frac{d-b-a}{\varepsilon_0 A}=\frac{d-a}{\varepsilon_0 A}$$

$$C=\frac{\varepsilon_0 A}{d-a}$$

This is the same result as in part (a). It is independent of the value of b, so it does not matter where the slab is located.

Example 5

A parallel plate capacitor of plate area A and separation x is given a charge Q.

(a) What is the force acting on the plates of the capacitor?

(b) Show that force on plates can be expressed as $F=(1/2)EQ$, where Q is charge on one plate and E is the electric field between the plates.

(c) Now capacitor plates are drawn apart by separation d. Calculate the work done if during the process the charging source is connected and if the charging source is removed.

Solution: (a) Attractive force acts between capacitor plates. Energy of capacitor at separation x is

$$U=\frac{1}{2}\frac{Q^2}{C}=\frac{1}{2}\frac{Q^2 x}{\varepsilon_0 A} \qquad \ldots(1)$$

For a conservative field,

$$F=-\frac{dU}{dx}$$

Hence,

$$F=-\frac{d}{dx}\left[\frac{1}{2}\frac{Q^2}{\varepsilon_0 A}x\right]$$

$$=-\frac{1}{2}\frac{Q^2}{2\varepsilon_0 A} \qquad \ldots(2)$$

Negative sign implies attractive force between plates.

(b) Electric field between plates is given by

$$E=\frac{\sigma}{\varepsilon_0}=\frac{Q}{\varepsilon_0 A}$$

Thus,

$$F=\frac{1}{2}EQ \qquad \ldots(3)$$

(c) From eqn. (2),

$$F=-\frac{1}{2}\frac{Q^2}{\varepsilon_0 A}=-\frac{1}{2}\frac{\varepsilon_0 A}{x^2}V^2$$

As

$$Q=CV=\frac{\varepsilon_0 A}{x}V$$

Work done in increasing separation force is attractive, the external agent acts against it, so

$$W=\int_x^{x+d}F\,dx$$

$$=\frac{1}{2}\varepsilon_0 AV^2\int_x^{x+d}\frac{dx}{x^2}$$

$$=\frac{1}{2}\varepsilon_0 AV^2\left(\frac{1}{x}-\frac{1}{x+d}\right)$$

$$=\frac{1}{2}\frac{\varepsilon_0 Ad}{x(x+d)}V^2$$

The work done by external agent increases energy of the system, however, in this case, overall energy of system is decreased. The capacity of the system decreased by increasing the separation between the plates.

Initial charge of capacitor, $Q_i=\left(\frac{\varepsilon_0 A}{x}\right)V$

Final charge of capacitor, $Q_f=\left(\frac{\varepsilon_0 A}{x+d}\right)V$

Charge transferred to battery $=Q_i-Q_f$

$$=\frac{\varepsilon_0 A}{x}V-\frac{\varepsilon_0 A}{x+d}V$$

$$=\frac{\varepsilon_0 AdV}{x(x+d)}$$

Work done by battery,

$$W_{\text{battery}}=-(Q_i-Q_f)V$$

$$=\frac{-\varepsilon_0 Ad}{x(x+d)}V^2$$

The battery does negative work on these charges as they move from positive to negative terminal. The decrease in energy of the capacitor is one-half the work done by battery.

If the plates are drawn apart after the battery is disconnected, the charge on the capacitor remains constant, therefore force between plates is constant.

Thus,

$$W=\int_x^{x+d}\frac{Q^2}{2\varepsilon_0 A}dx=\frac{Q^2 d}{2\varepsilon_0 A}$$

This positive work done increase the energy of the system.

Concepts: *1. Force on the Plates of a Capacitor:*

The plates of a parallel-plate capacitor have area A and carry total charge +Q (see Fig. 3.17). These plates attract each other with a force given by $F=Q^2/(2\varepsilon_0 A)$.

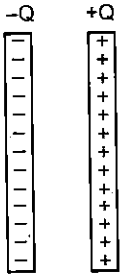


Fig. 3.17

If you pull the plates apart, against their attraction, you are doing work and that work goes directly into creating additional electrostatic energy.

The force per unit area (the electrostatic stress) acting on either capacitor plate is given by $\varepsilon_0 E^2/2$. This result is true for a conductor of any shape with an electric field $\vec{E}$ at its surface E.

2. *Each plate of a parallel-plate air capacitor has an area A. What amount of work has to be performed to slowly increase the distance between the plates from x_1 to x_2 if (a) the charge on the capacitor, which is equal to q. or (b) the voltage across the capacitor, which is equal to V, is kept constant in the process ?*

(a) Sought work is equivalent to the work performed against the electric field created by one plate, holding at rest and to bring the other plate away. Therefore the required work,

$$W_{\text{agent}} = qE(x_2 - x_1),$$

where $E = \dfrac{\sigma}{2\varepsilon_0}$ *is the intensity of the field created by one plate at the location of other.*

So, $$W_{\text{agent}} = q\frac{\sigma}{2\varepsilon_0}(x_1 - x_1) = \frac{q^2}{2\varepsilon_0 A}(x_2 - x_1)$$

Alternate method: $W_{\text{ext}} = \Delta U$

$$= \frac{q^2}{2\varepsilon_0 A}x_2 - \frac{q^2}{2\varepsilon_0 A}x = \frac{q^2}{2\varepsilon_0 A}(x_2 - x_1)$$

(b) When voltage is kept const., the force acting on each plate of capacitor will depend on the distance between the plates.

So, elementary work done by agent, in its displacement over a distance dx, relative to the other,

$$dA = -F_x dx$$

But, $$F_x = -\left(\frac{\sigma(x)}{2\varepsilon_0}\right)A\,\sigma(x)$$

and $$\sigma(x) = \frac{\varepsilon_0 V}{x}$$

Hence, $$A = \int dA \int \frac{1}{2}\varepsilon_0 \frac{AV^2}{x^2}dx$$

$$= \frac{\varepsilon_0 AV^2}{2}\left[\frac{1}{x_1} - \frac{1}{x_2}\right]$$

Alternate: From energy Conservation,

$$U_f - U_i = W_{\text{cell}} + W_{\text{agent}}$$

or $$\frac{1}{2}\frac{\varepsilon_0 A}{x_2}V^2 - \frac{1}{2}\frac{\varepsilon_0 A}{x_1}V^2 = \left[\frac{\varepsilon_0 A}{x_2} - \frac{\varepsilon_0 A}{x_1}\right]V^2 + W_{\text{agent}}$$

$$(\text{as } W_{\text{cell}} = (q_f - q_i)V = (C_f - C_i)V^2)$$

So, $$W_{\text{agent}} = \frac{\varepsilon_0 AV^2}{2}\left[\frac{1}{x_1} - \frac{1}{x_2}\right]$$

Example 6

A parallel plate capacitor has a capacitance C_0 and a plate separation d. Two dielectric slabs of constants K_1 and K_2 each of thickness d/2 and having the same area as the plates are inserted between the plates as shown in Fig. 3E.6. When the charge on the plates is Q, find (a) the electric field in each dielectric, and (b) the potential difference between the plates. (c) Show that the equivalent capacity of the system is $C = \dfrac{2K_1K_2}{K_1 + K_2}C_0$.

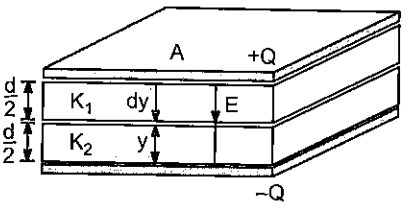


Fig. 3E.6

Solution: (a) The electric field inside any one of the dielectric is superposition of fields due to charge on positive and negative plates, both fields point in the same direction.

$$E_1 = \frac{Q}{2K_1\varepsilon_0 A} + \frac{Q}{2K_1\varepsilon_0 A}$$

$$= \frac{Q}{K_1\varepsilon_0 A} \quad \text{[toward negative plate]}$$

Similarly field in second dielectric is

$$E_2 = \frac{Q}{2K_2\varepsilon_0 A} + \frac{Q}{2K_2\varepsilon_0 A}$$

$$= \frac{Q}{K_2\varepsilon_0 A} \quad \text{[toward negative plate]}$$

(b) If we start at lower plate that is assumed to be negative, the potential difference between capacitor plates is

$$V = -\int_0^{d/2}\left(\frac{Q}{K_2\varepsilon_0 A}\right)dy\cos 180° - \int_{d/2}^{d}\frac{Q}{K_1\varepsilon_0 A}\cos 180°$$

$$= \frac{Q}{K_2\varepsilon_0 A}\int_0^{d/2}dy + \frac{Q}{K_1\varepsilon_0 A}\int_{d/2}^{d}dy$$

$$= \frac{Qd}{2\varepsilon_0 A}\left(\frac{1}{K_1} + \frac{1}{K_2}\right)$$

$$V = \frac{Q}{2C_0}\left(\frac{1}{K_1} + \frac{1}{K_2}\right)$$

The equivalent capacity of system is

$$C_{\text{eq.}} = \frac{Q}{V} = \frac{2K_1K_2}{(K_1 + K_2)}C_0$$

This system can be considered to be series combination of two capacitors of thickness $d/2$ filled with dielectric of constants K_1 and K_2.

$$\frac{1}{C_{eq.}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{(d/2)}{K_1\varepsilon_0 A} + \frac{(d/2)}{K_2\varepsilon_0 A}$$

$$C_{eq.} = \frac{\varepsilon_0 A}{d}\left[\frac{2K_1K_2}{K_1+K_2}\right]$$

Example 7

A parallel plate capacitor is filled with two dielectric of equal size as shown in figure. Find the capacity of the system.

Solution: If Q is the total charge given to the plates and V the potential difference between plates, the capacity of system is $C = \frac{Q}{V}$.

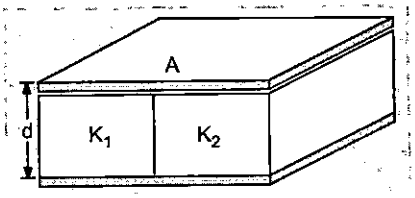


Fig. 3E.7

The separation between plates and potential difference is same for both the parts of the capacitor, therefore electric field must be same in both parts. The total charge Q distributes on the surface of the dielectrics such that

$$Q = Q_1 + Q_2 \qquad \ldots(1)$$

Thus we have

$$E = \frac{Q_1}{K_1\varepsilon_0(A/2)} = \frac{Q_2}{K_2\varepsilon_0(A/2)} \qquad \ldots(2)$$

From eqns. (1) and (2), we have

$$Q = Q_1 + Q_2 = \varepsilon_0 E\left(\frac{K_1A}{2} + \frac{K_2A}{2}\right)$$

Therefore, the capacity is

$$C = \frac{Q}{V} = \frac{Q}{Ed} = \frac{\varepsilon_0}{d}\frac{(K_1A + K_2A)}{2}$$

$$= \frac{C_0}{2}(K_1 + K_2)$$

This configuration is equivalent to two capacitors in parallel combination.

$$C = C_1 + C_2$$

where $C_1 = \frac{K_1\varepsilon_0 A}{2d}$

$$C_2 = \frac{K_2\varepsilon_0 A}{2d}$$

Example 8

A parallel plate capacitor has a plate area A and a separation d. A dielectric slab of thickness t and area A is inserted between the plates.

(a) What is the equivalent capacitance of this system?

(b) What is the equivalent capacity if a metallic slab of thickness t is inserted?

Solution: (a) Let Q be the charge given to the capacitor plates. The electric field between the plates of the capacitor is given by

$$E = \frac{Q}{\varepsilon_0 A}$$

The electric field in the region of dielectric slab is

$$E' = \frac{Q}{K\varepsilon_0 A} = \frac{E}{K}$$

The potential difference between two plates is, beginning from negative plate,

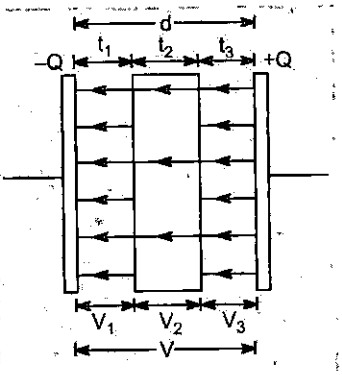


Fig. 3E.8

$$V = V_1 + V_2 + V_3$$

$$= -\int_0^{t_1}\frac{Q}{\varepsilon_0 A}dx\cos 180° - \int_0^{t_2}\frac{Q}{K\varepsilon_0 A}dx\cos 180° - \int_0^{t_3}\frac{Q}{\varepsilon_0 A}dx\cos 180°$$

$$= \frac{Q}{\varepsilon_0 A}t_1 + \frac{Q}{K\varepsilon_0 A}t_2 + \frac{Q}{\varepsilon_0 A}t_3$$

$$= \frac{Q}{\varepsilon_0 A}(t_1 + t_3) + \frac{Q}{K\varepsilon_0 A}t_2$$

$$= \frac{Q}{\varepsilon_0 A}(d - t) + \frac{Q}{K\varepsilon_0 A}t$$

As capacity is $C = \frac{Q}{V}$

$$= \frac{1}{\frac{(d-t)}{\varepsilon_0 A} + \frac{t}{K\varepsilon_0 A}} = \frac{\varepsilon_0 A}{(d-t) + \frac{t}{K}}$$

(b) The dielectric constant of conducting slab is infinity (∞), therefore term t/K reduces to zero. Thus

$$C = \frac{\varepsilon_0 A}{d - t}$$

Concepts: **1.** $C = \frac{Q}{|\Delta V|} = \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{K_e}\right)}$

It is useful to check the following limits:

(i) As $t \to 0$ i.e., the thickness of the dielectric approaches zero, we have $C = \varepsilon_0 A/d = C_0$, which is the expected result for no dielectric.

(ii) As, $K_e \to 1$, we again have $C \to \varepsilon_0 A/d = C_0$, and the situation also correspond to the case where the dielectric is absent.

(iii) In the limit where $t \to d$, the space is filled with dielectric, we have. $C \to K_e \varepsilon_0 A/d = K_e C_0$.

We also comment that the configuration is equivalent to two capacitors connected in series, as shown in Fig. 3.18.

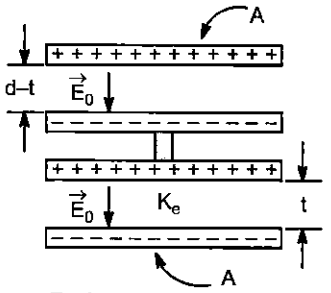


Fig. 3.18

Using Eq. of capacitors connected in series, the equivalent capacitance is

$$\frac{1}{C} = \frac{d-t}{\varepsilon_0 A} + \frac{t}{K_e \varepsilon_0 A}$$

2. Capacitor connected to a Spring: *Consider an air-filled parallel-plate capacitor with one plate connected to a spring having a force constant k, and another plate held fixed. The system rests on a table top as shown in Fig. 3.19.*

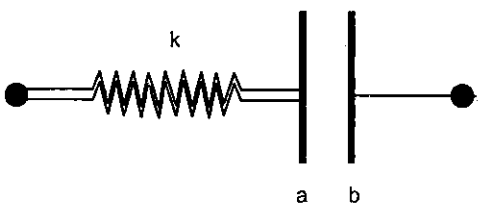


Fig. 3.19

If the charges placed on plates a and b are $+Q$ and $-Q$, respectively. The spring force $\vec{F}_S$ acting on plate a is given by

$$\vec{F}_S = -kx\hat{i}$$

Similarly, the electrostatic force $\vec{F}_e$ due to the electric field created by plate b is

$$\vec{F}_e = QE\hat{i} = Q\left(\frac{\sigma}{2\varepsilon_0}\right)\hat{i} = \frac{Q^2}{2A\varepsilon_0}\hat{i}$$

where A is the area of the plate. Notice that charges on plate a cannot exert a force on itself, as required by Newton's third law. Thus, only the electric field due to plate b is considered. At equilibrium the two forces cancel and we have

$$kx = Q\left(\frac{Q}{2A\varepsilon_0}\right)$$

which gives $$x = \frac{Q^2}{2kA\varepsilon_0}$$

3. *Consider two metallic plates of equal area A separated by a distance d, as shown in Fig. 3.20. The top plate carries a charge $+Q$ while the bottom plate carries a charge $-Q$. The charging of the plates can be accomplished by means of a battery which produces a potential difference.*

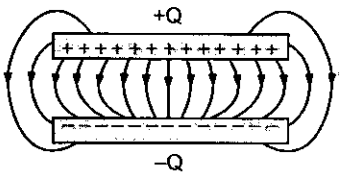


Fig. 3.20

A real capacitor is finite in size. Thus, the electric field lines at the edge of the plates are not straight lines, and the field is not contained entirely between the plates. This is known as edge effects, and the non-uniform fields near the edge are called the fringing fields. In figure the field lines are drawn by taking into consideration edge effects. However, in what follows we shall ignore such effects and assume an idealized situation, where field lines between the plates are straight lines.

4. Forces of dielectrics: *Just a conductor is attracted into an electric field, so too is a dielectric because the bound charge tends to accumulate near the free charge of the opposite sign.*

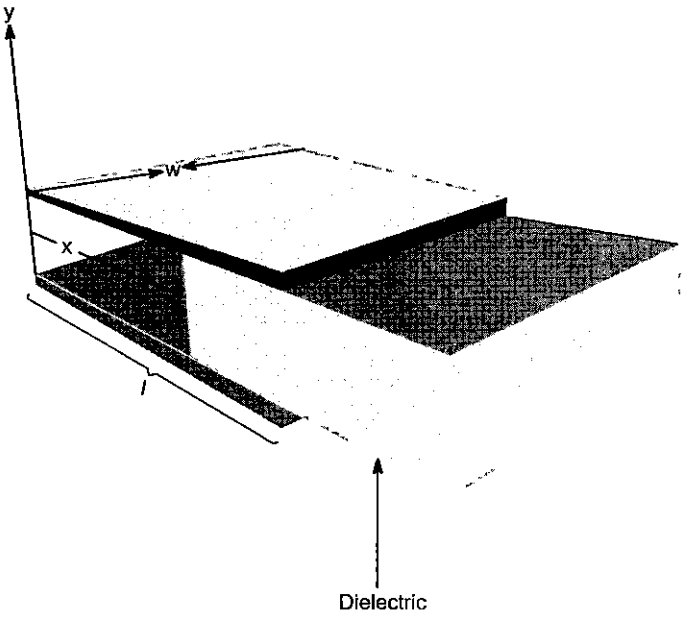


Fig. 3.21

Consider, a slab of linear dielectric material, partially inserted between the plates of a parallel-plate capacitor (Fig. 3.21). We have always considered that the field is uniform inside a parallel-plate capacitor, and zero outside. In this case there would be no net force on the dielectric at all, since the field everywhere would be perpendicular to the plates. There is a ***fringing field*** *around the edges, which but in this case is responsible for the force on dielectric. The field could not terminate abruptly at the edge of the capacitor, for if it did the line integral of E around the closed loop shown in Fig. 3.22 would not be zero. It is this nonuniform fringing field that pulls the dielectric into the capacitor.*

Let U be the energy of the system-it depends on the amount of overlap. If we pull the dielectric out an infinitesimal distance dx, the energy is changed by an amount equal to the work done:

$$dW = F_{\text{external}}\,dx,$$

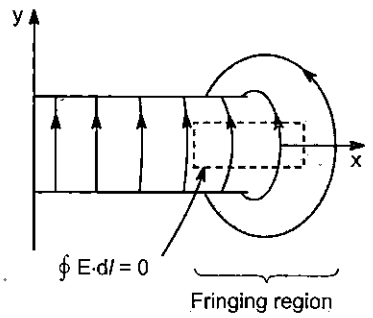


Fig. 3.22

where F_{me} is the force external agent exert, to counteract the electrical force F on the dielectric: $F_{me} = -F$. Thus the electrical force on the slab is

$$F = -\frac{dU}{dx}.$$

Now, the energy stored in the capacitor is

$$W = \frac{1}{2}CV^2,$$

and the capacitance in this case is

$$C = \frac{\varepsilon_0 wx}{d} + \frac{K\varepsilon_0 (l-x)w}{d}$$

where l is the length of the plates (Fig. 3.21). Let's assume that the total charge on the plates ($Q = CV$) is held constant, as the dielectric moves. In terms of Q,

$$W = \frac{1}{2}\frac{Q^2}{C},$$

so

$$F = -\frac{dU}{dx} = \frac{1}{2}\frac{Q^2}{C^2}\frac{dC}{dx}$$
$$= \frac{1}{2}V^2\frac{dC}{dx}$$

The minus sign indicates that the force is in the negative x direction; the dielectric is pulled into the capacitor.

This procedure can not be used with V constant in computing the force. One then obtains

$$F = -\frac{1}{2}V^2\frac{dC}{dx},$$

which is not correct due to minus sign. It is possible to maintain the capacitor at a constant potential, by connecting it up to a battery. But in that case the battery also does work as the dielectric moves; we have

$$dW = F_{\text{external}}\,dx + V\,dQ$$

where V dQ is the work done by the battery. It follows that

$$F = -\frac{dW}{dx} + V\frac{dQ}{dx}$$
$$= -\frac{1}{2}V^2\frac{dC}{dx} + V^2\frac{dC}{dx} = \frac{1}{2}V^2\frac{dC}{dx}$$

The same as before with the correct sign. The force on the dielectric does not depend on whether we hold Q constant or V constant-it is determined entirely by the distribution of charge, free and bound.

Note that we are able to determine the force without knowing about the fringing fields that are ultimately responsible for it! The energy stored in the fringing fields themselves stays constant, as the slab moves; what does change is the energy well inside the capacitor, where the field is uniform.

Example 9

A rectangular parallel plate capacitor of length a and width b has a dielectric of width b partially inserted a distance x between the plates as shown in Fig. 3E.9(a). Find the capacitance of the system as a function of x. (b) If the capacitor carries charges +Q and –Q on the plates, find the force that acts on the dielectric slab. (c) If the capacitor is connected to a constant voltage source V, find the stored energy as a function of x to determine the force that acts on the dielectric slab.

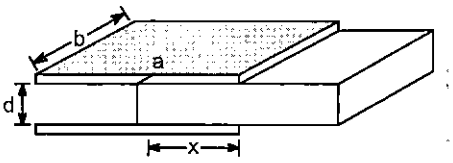


Fig. 3E.9

Solution: In the configuration the capacitor is a parallel combination of two capacitors, one with empty space and the other with dielectric slab. The equivalent capacity is

$$C_{\text{eq.}} = C_{\text{empty}} + C_{\text{dielectric}}$$
$$= \frac{\varepsilon_0 b(a-x)}{d} + \frac{K\varepsilon_0 bx}{d}$$
$$= \frac{\varepsilon_0 b}{d}[a + (K-1)x] \qquad \ldots(1)$$

(a) The energy of the capacitor is

$$U = \frac{Q^2}{2C} = \frac{Q^2 d}{2\varepsilon_0 b}\frac{1}{[a+(K-1)x]} \quad \ldots(2)$$

In the absence of any source charge of the capacitor remains constant.

As electrostatic electric field is conservative, we have

$$F = -\frac{dU}{dx}$$

$$= \frac{Q^2 d}{2\varepsilon_0 b}\frac{d}{dx}\left[\frac{1}{a+(K-1)x}\right]$$

$$= \frac{Q^2 d}{2\varepsilon_0 b}\frac{(K-1)}{[a+(K-1)x]^2} \quad \ldots(3)$$

From eqn. (1), we substitute for $C_{eq.}$ in eqn. (3); we get

$$F = \frac{1}{2}\frac{\varepsilon_0 b}{d}\frac{Q^2}{C_{eq.}^2}(K-1)$$

(b) When constant voltage source is attached,

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\frac{\varepsilon_0 b}{d}[a+(K-1)x]V^2$$

$$F = -\frac{dU}{dx}$$

$$= \frac{1}{2}\frac{\varepsilon_0 b}{d}V^2\frac{d}{dx}[a+(K-1)x]$$

$$= \frac{1}{2}\frac{\varepsilon_0 b}{d}(K-1)V^2$$

Example 10

Two parallel-plate air capacitors, each of capacitance C, were connected in series to a battery with emf $\mathcal{E}$. Then one of the capacitors was filled up with slab of dielectric constant K.
(a) What amount of charge flows through the battery ?
(b) Find the factor by which electric field in each capacitor charges during the process. $\left(\text{i.e., } \frac{E_{after}}{E_{before}}\right)$

Solution: From the symmetry of the problem, the voltage across each capacitor, $\Delta V = V/2$ and charge on each capacitor $q = C\mathcal{E}/2$ in the absence of dielectric.

Now when the dielectric is field up in one of the capacitors, the equivalent capacitance of the system,

$$C_0' = \frac{C\varepsilon}{1+\varepsilon}$$

and the potential difference across the capacitor, which is filled dielectric,

$$\Delta V' = \frac{q'}{\varepsilon C} = \frac{C\varepsilon}{(1+\varepsilon)}\frac{\mathcal{E}}{C\varepsilon} = \frac{\mathcal{E}}{(1+\varepsilon)}$$

but $V \alpha E$

So, as φ decreases $\frac{1}{2}(1+\varepsilon)$ times, the field strength also decreases by the same factor and flow of charge, $\Delta q = q' - q$.

$$= \frac{C\varepsilon}{(1+\varepsilon)}\mathcal{E} - \frac{C}{2}\mathcal{E} = \frac{1}{2}C\mathcal{E}\frac{(\varepsilon-1)}{(\varepsilon+1)}$$

Example 11

Between the plates of a parallel-plate capacitor there is a metallic plate whose thickness takes up $\eta = 0.060$ of the capacitor gap. When that plate is absent the capacitor has a capacity $C = 20nF$. The capacitor is connected to a dc voltage source $V = 100V$. The metallic plate is slowly extracted from the gap. Find (a) the change in the energy of the capacitor; (b) the mechanical work performed in the process of plate extraction.

Solution: When the plates is absent the capacity of the condenser is

$$C = \frac{\varepsilon_0 S}{d}$$

When it is present, the capacity is

$$C' = \frac{\varepsilon_0 S}{d(1-\eta)} = \frac{C}{1-\eta}$$

(a) The energy increment is clearly,

$$\Delta U = \frac{1}{2}CV^2 - \frac{1}{2}C'V^2$$

$$= \frac{C\eta}{2(1-\eta)}V^2$$

(b) The charge on the plate is

$q_i = \frac{CV}{1-\eta}$ initially and $q_f = CV$ finally.

A charge $\frac{CV\eta}{1-\eta}$ has flown through the battery charging it and withdrawing $\frac{CV^2\eta}{1-\eta}$ units of energy from the system into the battery. The energy of the capacitor has decreased by just half of this. The remaining half. *i.e.*, $\frac{1}{2}\frac{CV^2\eta}{1-\eta}$ must be the work done by the external agent in withdrawing the plate. This ensures conservation of energy.

Example 12

A glass plate of dielectric constant K totally fills up the gap between the electrodes of a parallel-plate capacitor whose capacitance in the absence of that glass plate is equal to $C = 20\mu F$. The capacitor is connected to a dc voltage source $V = 100V$. The plate is slowly, and without friction, extracted from the gap. Find the change in energy of capacitor and the mechanical work performed in the process of plate extraction.

Solution: Initially, capacitance of the system $= CV$.

So, initial energy of the system: $U_i = \frac{1}{2}(CK)V^2$

and finally, energy of the capacitor: $U_f = \frac{1}{2}CV^2$

Hence capacitance energy increment,

$$\Delta U = \frac{1}{2}CV^2 - \frac{1}{2}(CK)V^2$$

$$= -\frac{1}{2}CV^2(K-1) = -0.5\,\text{mJ}$$

From energy conservation

$$\Delta U = W_{\text{cell}} + W_{\text{agent}}$$

(as there is no heat liberation)

But $W_{\text{cell}} = (C_f - C_i)V^2 = (C - CK)V^2$

Hence $W_{\text{agent}} = \Delta U - W_{\text{cell}}$

$$= \frac{1}{2}C(1-K)V^2 = 0.5\,\text{mJ}$$

Example 13

A glass plate completely fills the gap between the plates of a parallel-plate capacitor whose capacitance is equal to C_0 when the plate is absent. The capacitor is connected to a source of permanent voltage U. Find the mechanical work which must be done against electric forces for extracting the plate out of the capacitor.

Solution: According to the law of conservation of energy, we can write

$$W_m + W_s = \Delta W,$$

where W_m is the mechanical work accomplished by extraneous forces against electric forces, A_s is the work of the voltage source in this process, and ΔW is the corresponding increment in the energy of the capacitor (we assume that contributions of other forms of energy to the change in the energy of the system is negligible small).

Let us find ΔW and W_s. It follows from the formula $W = CU^2/2 = qU/2$ for the energy of a capacitor that for U = const.

$$\Delta W = \Delta C\,U^2/2 = \Delta qU/2$$

Since the capacitance of the capacitor decreases upon the removal of the plate ($\Delta C < 0$), the charge of the capacitor also decreases ($\Delta Q < 0$). This means that the charge has passed through the source against the direction of the action of extraneous forces, and the source has done negative work.

$$W_s = \Delta q \bullet U$$

Comparing formulas (3) and (2), we obtain

$$W_s = 2\Delta W$$

Substitution of this expression into (1) gives

$$W_m = -\Delta W \quad \text{or} \quad W_m = \frac{1}{2}(\varepsilon - 1)C^0U^2$$

Thus,extracting the plate out of the capacitor, we (extraneous forces) do a positive work (against electric forces). The emf. source in this case accomplishes a negative work, and the energy of the capacitor decreases:

$$W_m > 0, W_s < 0, \Delta W < 0$$

Example 14

A dielectric slab fills the entire space of a parallel plate capacitor. The dielectric constant of the slab varies linearly with distance. Consider two cases: (a) Dielectric constant varies from K_1 to K_2, from one plate to other [see Fig. 3E.14(a) and (b)]. Dielectric constant varies from K_1 to K_2 from one edge to the other. Find the equivalent capacity of the system. Separation between plates is d and area of plates A.

Solution: (a) The dielectric constant varies linearly, hence

$$K(y) = ay + b \qquad ...(1)$$

where a and b are constants.

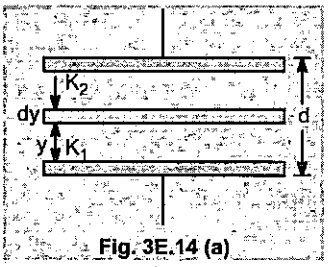


Fig. 3E.14 (a)

These values can be determined from boundary conditions

at $y = 0$, $K = K_1$

at $y = d$, $K = K_2$

From (1) we have

$$K_1 = b$$

or $$a = \frac{K_2 - K_1}{d},$$

$$b = K_1$$

Let charge Q be given to the system, electric field at a distance y from lower plate is

$$E(y) = \frac{Q}{K(y)\varepsilon_0 A}$$

So the potential difference across a differential element dy at a distance y is

$$dV = -E(y)\,dy$$

Thus $$-\int_{v_1}^{v_2} dV = \int_0^d \frac{Q}{K(y)\varepsilon_0 A}\,dy$$

$$= \frac{Q}{\varepsilon_0 A}\int_0^d \frac{dy}{ay+b}$$

$$= \frac{Q}{\varepsilon_0 A}\left[\frac{1}{a}\ln(ay+b)\right]_0^d$$

$$= \frac{Q}{\varepsilon_0 Aa}\ln\left(\frac{ad+b}{b}\right)$$

Thus $$C = \frac{Q}{V} = \frac{\varepsilon_0 Aa}{\ln\left(\frac{ad+b}{b}\right)}$$

On substituting values of a and b, we get

$$C = \frac{\varepsilon_0 A}{d}\,\frac{(K_2 - K_1)}{\ln(K_2/K_1)}$$

(b) In this case we may consider that the capacitor consists of differential capacitors of thickness dx. All these differential capacitors are in parallel arrangement as these

plates are common. Capacity of one such differential capacitor at a distance x from left is

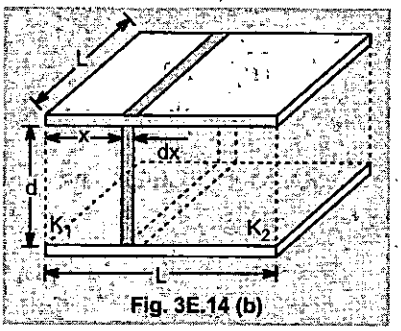


Fig. 3E.14 (b)

$$dl = \frac{(a+bx)\varepsilon_0 (L\,dx)}{d}$$

$$C = \frac{\varepsilon_0 L}{d}\int_0^L (a+bx)\,dx$$

$$= \frac{\varepsilon_0 L}{d}\left[ax + \frac{bx^2}{2}\right]_0^L$$

$$= \frac{\varepsilon_0 L}{d}\left[aL + \frac{bL^2}{2}\right]$$

On substituting values of a and b and $A = L^2$; we get

$$C = \frac{\varepsilon_0 A}{d}\left[\frac{K_1 + K_2}{2}\right]$$

Example 15

Consider two concentric metallic spherical shells as shown in Fig. 3E.15(a) . Now consider three cases: (a) Inner shell is given a charge and outer is earthed. (b) Outer shell is given a charge and inner shell is earthd. (c) Inner shell is taken out and kept at a distance d from outer shell. Both are given same magnitude of charge but opposite in sign. In each of these cases determine the capacity of the system.

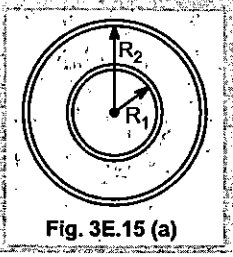


Fig. 3E.15 (a)

Solution: (a) When the inner shell is given a charge it induces equal and opposite charge on the inner surface of the outer shell. Since the outer shell was initially neutral, migration of negative charges to inner surface creates equal amount of positive charges on the outer surface. When outer shell is grounded, its potential is essentially zero. The Earth is considered to be a conductor of infinite capacity and unlimited supply of charge. Some electrons from the Earth are transferred to the outer shell. Let the charge induced on outer shell be q'.

$$\frac{q+q'}{4\pi\varepsilon_0 b} = 0 \quad \text{or} \quad q' = -q$$

Now consider a Gaussian surface between the shells, net charge enclosed by it is q.

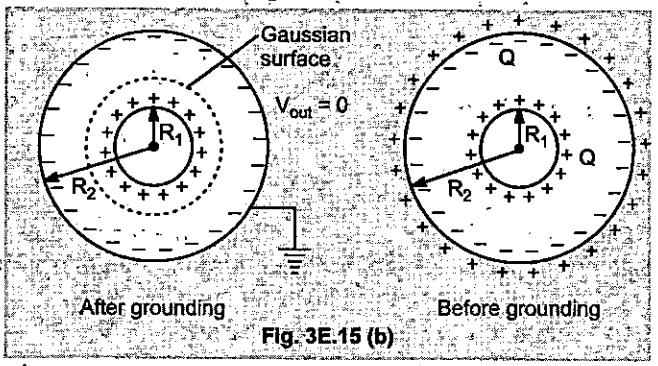


Fig. 3E.15 (b)

From Gauss's law,

$$\varepsilon_0 \oint \vec{E}\cdot d\vec{s} = q_{\text{enclosed}}$$

$$\varepsilon_0 E 4\pi r^2 = q$$

$$E = \frac{q}{4\pi\varepsilon_0 r^2}$$

Note that $E(r < R_1) = 0$

and $E(r > R_1) = 0$

because a Gaussian surface inside the inner shell has net charge zero and a Gaussian surface outside the outer shell also has net charge zero.

As $\int_i^f dV = -\int_i^f \vec{E}\cdot d\vec{r}$

$$\int_V^0 dV = -\int_{R_1}^{R_2} \frac{q}{4\pi\varepsilon_0}\frac{dr}{r^2}$$

so $$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

Capacity of system is

$$C = \frac{q}{V} = \frac{4\pi\varepsilon_0}{\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$

$$= \frac{4\pi\varepsilon_0 R_1 R_2}{R_2 - R_1}$$

(b) When inner surface is grounded its potential is reduced to zero. The outer sphere is at higher potential relative to inner sphere, so an electric field exists, a redistribution of charges takes place so that two conditions must be satisfied.

(1) Net charge enclosed inside a conductor must be zero.

(2) Potential of earthed conductor must be zero. Let q' be the charge induced on the inner shell so that

$$\frac{1}{4\pi\varepsilon_0}\left[\frac{q'}{R_1} + \frac{q}{R_2}\right] = 0$$

or $$q' = -\frac{R_1}{R_2}q$$

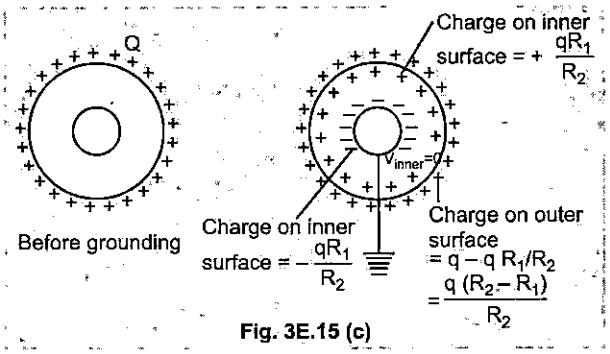


Fig. 3E.15 (c)

The inner surface of outer shell has same magnitude of charge but opposite in sign, in the condition of equilibrium. This makes net charge inside a conducting shell zero.

The charge on the outer surface of outer shell is

$$q-\left(\frac{qR_1}{R_2}\right)=q\left(\frac{R_2-R_1}{R_2}\right)$$

In this situation electric field exists between the shells as well as outside the shells.

Energy density in an electric field is given by

$$U=\frac{1}{2}\varepsilon_0 E^2$$

From Gauss's law,

$$E(r<R_1)=\frac{1}{4\pi\varepsilon_0}\frac{qR_1}{R_2}\frac{1}{r^2}$$

$$E(r>R_2)=\frac{1}{4\pi\varepsilon_0}\frac{q(R_2-R_1)}{R_2}\frac{1}{r^2}$$

Total energy of the system is sum of energy in electric field between inner shells and outside shells.

$$U=\int_{R_1}^{R_2}\frac{1}{2}\varepsilon_0\left[\frac{1}{4\pi\varepsilon_0}\frac{qR_1}{R_2}\frac{1}{r^2}\right]^2 4\pi r^2 dr+\int_{R_2}^{\infty}\frac{1}{2}\varepsilon_0\left[\frac{1}{4\pi\varepsilon_0}\frac{q(R_2-R_1)}{R_2}\frac{1}{r^2}\right]4\pi r^2 dr$$

$$=\frac{q^2}{8\pi\varepsilon_0}\left(\frac{R_1}{R_2}\right)^2\int_{R_1}^{R_2}\frac{dr}{r^2}+\frac{q^2}{8\pi\varepsilon_0}\frac{(R_2-R_1)^2}{R_2^2}\int_{R_2}^{\infty}\frac{dr}{r^2}$$

$$=\frac{q^2}{8\pi\varepsilon_0}\left(\frac{R_1}{R_2}\right)^2\left[\frac{1}{R_1}-\frac{1}{R_2}\right]+\frac{q^2}{8\pi\varepsilon_0}\frac{(R_2-R_1)^2}{R_2^2}\left[\frac{1}{R_2}\right]$$

$$=\frac{q^2}{8\pi\varepsilon_0}\left[\left(\frac{R_1}{R_2}\right)^2\left(\frac{R_2-R_1}{R_1R_2}\right)+\frac{(R_2-R_1)^2}{R_2^3}\right]$$

$$=\frac{q^2}{8\pi\varepsilon_0}\frac{(R_2-R_1)}{R_2^3}[R_1+R_2-R_1]$$

$$=\frac{q^2}{8\pi\varepsilon_0}\frac{(R_2-R_1)}{R_2^2}$$

As energy of a capacitor is given by $U=\frac{1}{2}\frac{q^2}{C_{eq.}}$,

where $C_{eq.}$ is equivalent capacity of the system.

$$\frac{1}{2}\frac{q^2}{C_{eq.}}=\frac{q^2}{8\pi\varepsilon_0}\frac{(R_2-R_1)}{R_2^2}$$

$$C_{eq.}=\frac{4\pi\varepsilon_0 R_2^2}{(R_2-R_1)}$$

This configuration is equivalent to parallel combination of two capacitors.

(1) Spherical capacitor of inner and outer radius R_1 and R_2 respectively.

(2) Spherical conductor of radius R_2

$$C_{eq.}=\frac{4\pi\varepsilon_0 R_1R_2}{(R_2-R_1)}+4\pi\varepsilon_0 R_2$$

$$=\frac{4\pi\varepsilon_0 R_2^2}{(R_2-R_1)}$$

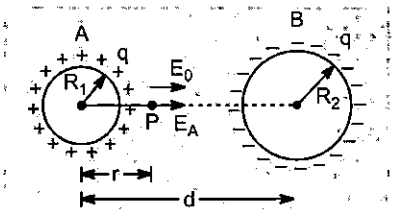


Fig. 3E.15 (d)

(c) Electric field at a point P, distance r from sphere A, is

$$\vec{\mathbf{E}}=\vec{\mathbf{E}}_A+\vec{\mathbf{E}}_B$$

$$=\frac{q}{4\pi\varepsilon_0 r^2}+\frac{q}{4\pi\varepsilon_0(d-r)^2}$$

So $$-\int_{V_A}^{V_B}dV=\frac{q}{4\pi\varepsilon_0}\int_a^{d-b}\left[\frac{1}{r^2}+\frac{1}{(d-r)^2}\right]dr$$

$$V_A-V_B=\frac{q}{4\pi\varepsilon_0}\left[-\frac{1}{r}+\frac{1}{(d-r)}\right]_a^{d-b}$$

$$=\frac{q}{4\pi\varepsilon_0}\left[\frac{1}{a}+\frac{1}{b}-\frac{1}{d-a}-\frac{1}{d-b}\right]$$

If we assume that $d>>a$ and $d>>b$,

$$V_A-V_B=V$$

$$=\frac{q}{4\pi\varepsilon_0}\left[\frac{1}{a}+\frac{1}{b}-\frac{2}{d}\right]$$

Hence $$C_{eq.}=\frac{q}{V}$$

$$=\frac{4\pi\varepsilon_0}{\left(\frac{1}{a}+\frac{1}{b}-\frac{2}{d}\right)}$$

Method 2 :

Electric potential energy of the system

$$U_{total}=U_A+U_B+U_{AB}$$

where U_A = self energy of A

$$=\frac{1}{2}q_1V_1=\frac{q^2}{8\pi\varepsilon_0 R_1}$$

U_B = self energy of B

$$= \frac{1}{2} q_2 V_2 = \frac{q^2}{8\pi\varepsilon_0 R_2}$$

U_{AB} = interaction energy of A and B

$$= \frac{q_1 q_2}{4\pi\varepsilon_0 r} = \frac{-q^2}{4\pi\varepsilon_0 d}$$

$$U_{\text{total}} = \frac{q^2}{8\pi\varepsilon_0}\left[\frac{1}{a} + \frac{1}{b} - \frac{2}{d}\right]$$

As $U_{\text{total}} = \frac{1}{2}\frac{q^2}{C_{\text{eq.}}}$

Hence, $C_{\text{eq.}} = \frac{4\pi\varepsilon_0}{\left[\frac{1}{a} + \frac{1}{b} - \frac{2}{d}\right]}$

Example 16

A small drop of mercury of radius r lies on a shallow metal disc connected to a steady voltage source V. The drop is then broken into n droplets on the dish. Find the extra charge drawn from the voltage source.

Solution: We have capacity of the drop $= 4\pi\varepsilon_0 r$. Charge on the drop $= V(4\pi\varepsilon_0 r)$.

When the drop is broken in the presence of the voltage source, the potential of droplets will remain same and mass will be conserved. Conserving mass we have,

$$\frac{4\pi}{3} r^3 \rho_{\text{mer}} = n\left(\frac{4\pi}{3} r'^3\right)\rho_{\text{mer}}$$

where r' = radius of each droplet $\Rightarrow r' = \frac{r}{n^{1/3}}$.

$\therefore$ Capacity of each other $= 4\pi\varepsilon_0 r' = 4\pi\varepsilon_0 r/n^{1/3}$

$\therefore$ Charge of droplet

$$= n \times \left(\frac{4\pi\varepsilon_0 r}{n^{1/3}} \times V\right) = 4\pi\varepsilon_0 rV \times n^{2/3}$$

$\therefore$ Extra charge drawn from the voltage source

$$= 4\pi\varepsilon_0 rVn^{2/3} - 4\pi\varepsilon_0 rV$$

$$= 4\pi\varepsilon_0 rV(n^{2/3} - 1)$$

Example 17

A soap bubble of radius R and thickness t is charged to a potential V. The bubble bursts and falls as a spherical drop. Determine the potential of the drop.

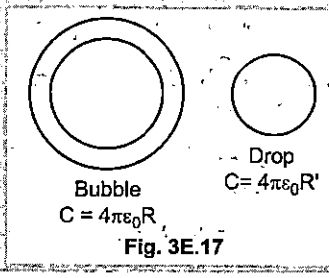


Fig. 3E.17

Solution: The capacitance of a spherical shell of radius R is given by

$$C = 4\pi\varepsilon_0 R$$

The charge on the shell,

$$Q = (4\pi\varepsilon_0 R)V$$

When the bubble bursts a spherical drop of radius R' carrying same charge is formed. The volume of this drop is equal to the volume of the liquid contained in the bubble which is $4\pi R^2 t$, where t is the wall thickness. Thus

$$4\pi R^2 t = \frac{4}{3}\pi R'^3$$

So $R' = (3R^2 t)^{1/3}$

Assuming that soap solution conducts electricity, all of the charge will reside on the surface of the drop and its capacitance will be given by $4\pi\varepsilon_0 R'$. The potential of this drop is

$$V' = \frac{Q}{4\pi\varepsilon_0 R'} = \frac{4\pi\varepsilon_0 RV}{4\pi\varepsilon_0 R'}$$

$$= \frac{RV}{R'} = \frac{R}{(3R^2 t)^{1/3}} V$$

Example 18

A conducting sphere S_1 of radius r is attached to an insulating handle. Another conducting sphere S_2 of radius R is mounted on an insulating stand, S_2 is initially uncharged. S_1 is given a charge Q brought into contact with S_2 and removed. S_1 is recharged such that the charge on it is again Q and it is again brought into contact with S_2 and removed. This procedure is repeated n times:

(a) Find the electrostatic energy of S_2 after n such contacts with S_1.

(b) What is the limiting value of this energy as $n \to \infty$?

Solution: (a) Let C_1 and C_2 be the capacities of S_1 and S_2. Capacity of a spherical conductor is proportional to its capacity.

Hence $\frac{C_2}{C_1} = \frac{R}{r}$

Charge distribution on S_1 and S_2 will take place in the ratio of their capacities. After first contact let charge acquired by S_2 is q_1. Therefore, charge on S_1 will be $Q - q_1$.

$$\frac{q_1}{Q - q_1} = \frac{C_2}{C_1} = \frac{R}{r} \quad \ldots(1)$$

If Q charge is to be distributed in S_2 and S_1 in the ratio of R/r. Then

$$q_1 = Q\left(\frac{R}{R + r}\right) \quad \ldots(2)$$

In the second contact, S_1 again acquires the charge Q, now total charge in S_1 and S_2 will be

$$Q + q_1 = Q\left(1 + \frac{R}{R + r}\right)$$

Charge again distribute in the same ratio. Therefore, charge on S_2 in second contact,

$$q_2 = Q\left(1+\frac{R}{R+r}\right)\left(\frac{R}{R+r}\right)$$

$$= Q\left[\frac{R}{R+r}+\left(\frac{R}{R+r}\right)^2\right]$$

Similarly, $q_3 = Q\left[\frac{R}{R+r}+\left(\frac{R}{R+r}\right)^2+\left(\frac{R}{R+r}\right)^3\right]$

and $q_n = Q\left[\frac{R}{R+r}+\left(\frac{R}{R+r}\right)^2+\cdots\left(\frac{R}{R+r}\right)^n\right]$ ...(3)

Now we apply the formula for sum of n terms of a geometric progression $\left[\because S_n = \frac{a(1-r^n)}{(1-r)}\right]$

or $$q_n = Q\frac{R}{r}\left[1-\left(\frac{R}{R+r}\right)^n\right] \quad ...(4)$$

Therefore, electrostatic energy of S_2 after n such contacts

$$U_n = \frac{q_n^2}{2C_2} = \frac{q_n^2}{2(4\pi\varepsilon_0 R)} \text{ or } U_n = \frac{q_n^2}{8\pi\varepsilon_0 R}$$

(b) From eqn. (3) we can obtain the sum for $n \to \infty$ by using formula $\left[S_\infty = \frac{a}{1-r}\right]$.

Thus $$q_\infty = \frac{QR}{R+r}\left(\frac{1}{1-\frac{R}{R+r}}\right)$$

$$= \frac{QR}{R+r}\left(\frac{R+r}{r}\right) = Q\frac{R}{r}$$

and $$U_\infty = \frac{q_\infty^2}{2C_2} = \frac{Q^2R^2/r^2}{8\pi\varepsilon_0 R}$$

or $$U_\infty = \frac{Q^2R}{8\pi\varepsilon_0 r^2}$$

Example 19

Two identical parallel plate capacitors are first connected in parallel and then in series. In each case, the plates of one capacitor are brought closer by a distance Δd and the plates of the other capacitor are moved away by same distance Δd. How does the total capacity of the system change in both cases by the above displacement of plates?

Solution: (i) Two identical capacitors C_1 and C_2 connected in parallel. Initially the net capacity of the system is

$$C = C_1 + C_2 = 2\frac{\varepsilon_0 A}{d}$$

where, A is the area of the plates, and d the distance between the plates, for both the capacitors.

When the plates are brought closer by Δd in one capacitor, its capacity increase to $\frac{\varepsilon_0 A}{d-\Delta d}$, whereas the capacity of the second capacitor, whose plates are moved away by Δd, decreases to $\frac{\varepsilon_0 A}{d+\Delta d}$. Net capacity of the system becomes

$$C' = \varepsilon_0 A\left(\frac{1}{d-\Delta d}+\frac{1}{d+\Delta d}\right)$$

$$= \frac{2\varepsilon_0 Ad}{d^2-\Delta d^2}$$

or $$C' = \frac{2\varepsilon_0 A}{d-(\Delta d^2/d)}$$

We find the $C' > C$; net capacity increases.

(ii) Capacitor connected in series. Initially capacity C is

$$\frac{1}{C} = \frac{1}{C_1}+\frac{1}{C_2} = \frac{d}{\varepsilon_0 A}+\frac{d}{\varepsilon_0 A}$$

or $$C = \frac{\varepsilon_0 A}{2d}$$

When the plates are moved in the given manner, net capacity C' of the system is given by

$$\frac{1}{C'} = \frac{d-\Delta d}{\varepsilon_0 A}+\frac{d+\Delta d}{\varepsilon_0 A}$$

or $$C' = \frac{\varepsilon_0 A}{2d}$$

Net capacity remains same as before.

Example 20

The figure shows a conducting sphere A radius a is surrounded by a shell B of radius $b (> a)$. Initially switches S_1, S_2 and S_3 are open and the sphere A carries a charge Q. First the switch S_1 is closed to connect the shell B with the ground, then again the switch S_1 is closed to connect the shell B with ground and then opened. Now the switch S_2 is closed so that the sphere A is grounded and then S_2 is opened. Finally the switch S_3 is closed to connect the spheres together. Find the total heat produced after closing the switch S_3.

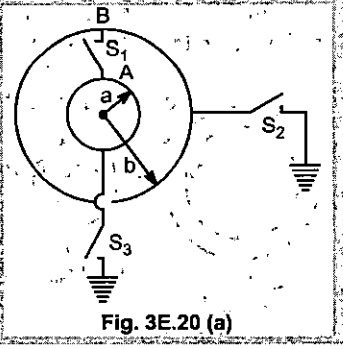


Fig. 3E.20 (a)

Solution: When the outer sphere is connected to ground, charge $-Q$ resides on the inner surface of the sphere B. When it is disconnected from the ground $-Q$ remains on its inner surface of it. Now the sphere A is connected to the earth potential on its surface becomes zero.

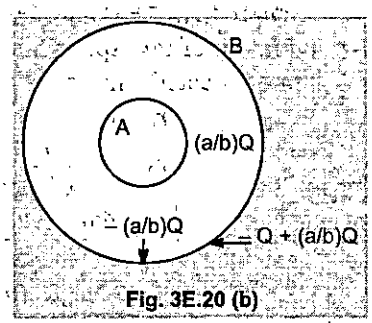


Fig. 3E.20 (b)

Let the charge on the sphere A becomes q

$$\Rightarrow \quad \frac{1}{4\pi\varepsilon_0}\frac{q}{a}-\frac{1}{4\pi\varepsilon_0}\frac{Q}{b}=0$$

$$\Rightarrow \quad q=\frac{a}{b}Q$$

Consider Fig. 3E.20(b). In this position,

Energy stored,

$$U_1=\frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{1}{a}\left(\frac{a}{b}Q\right)^2+\frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{1}{b}Q^2$$

$$+\frac{1}{4\pi\varepsilon_0}\frac{1}{b}\left(\frac{a}{b}Q\right)(-Q) \qquad \ldots(1)$$

When switch S_3 is closed, total charge will appear on the outer surface of the shell B. In this position,

Energy stored, $U_2=\frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{1}{b}\left(\frac{a}{b}-1\right)^2Q^2$

$$\text{Heat produced}=U_1-U_2=\frac{Q^2a(b-a)}{8\pi\varepsilon_0 b^3}$$

Example 21

Find the capacitance of a system of two identical metal balls of radius a if the distance between their centres is equal to b, with $b >> a$. The system is located in a uniform dielectric with permittivity ε.

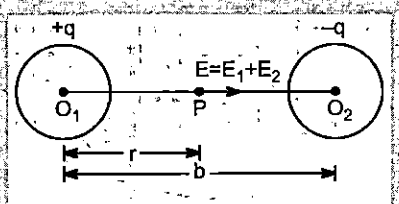


Fig. 3E.21

Solution: The electric field at point P is

$$E=E_1+E_2$$

where E_1 = electric field due to positively charged ball.

$$=\frac{q}{4\pi\varepsilon_0\varepsilon r^2}$$

and E_2 = electric field due to negatively charged ball.

$$=\frac{q}{4\pi\varepsilon_0\varepsilon(b-r)^2}$$

$$E=\frac{q}{4\pi\varepsilon_0\varepsilon r^2}+\frac{q}{4\pi\varepsilon_0\varepsilon(b-r)^2}$$

or $$-\int_{V_+}^{V_-}dV=\int_a^{b-a}\left\{\frac{q}{4\pi\varepsilon_0\varepsilon r^2}+\frac{q}{4\pi\varepsilon_0\varepsilon(b-r)^2}\right\}dr$$

or $$V_+-V_-=\frac{q}{4\pi\varepsilon_0\varepsilon}\left[\frac{2}{a}-\frac{2}{b}\right]$$

Hence capacitance of system is

$$C=\frac{q}{V_+-V_-}=\frac{4\pi\varepsilon_0\varepsilon}{\left[\frac{2}{a}-\frac{2}{b}\right]}=\frac{2\pi\varepsilon_0\varepsilon}{\frac{1}{a}-\frac{1}{b}}=\frac{2\pi\varepsilon_0\varepsilon}{\frac{b-a}{ab}}$$

But $b >> a$, we can write $b-a\approx b$

or $$C\approx\frac{2\pi\varepsilon_0\varepsilon}{\frac{b}{ab}}$$

or $$C\approx 2\pi\varepsilon_0\varepsilon a$$

Method 2 : Total energy of the system is

$$U=U_+ +U_- +U_\pm$$

where U_+ = self potential energy of positively charged balls

$$=\frac{q^2}{8\pi\varepsilon_0\varepsilon a}$$

U_- = self potential energy of negatively charged balls

$$=\frac{(-q)^2}{8\pi\varepsilon_0\varepsilon a}=\frac{q^2}{8\pi\varepsilon_0\varepsilon a}$$

and $U_\pm$ = mutual potential energy of both balls

$$=\frac{(q)(-q)}{4\pi\varepsilon_0\varepsilon b}=\frac{-q^2}{4\pi\varepsilon_0\varepsilon b}$$

Thus, $$U=\frac{q^2}{4\pi\varepsilon_0\varepsilon}\left[\frac{1}{2a}+\frac{1}{2a}-\frac{1}{b}\right]$$

Comparing it with energy of capacitor,

$$U=\frac{q^2}{2C}$$

$$\frac{q^2}{2C}=\frac{q^2}{4\pi\varepsilon_0\varepsilon}\left[\frac{1}{2a}+\frac{1}{2a}-\frac{1}{b}\right]$$

or $$C=\frac{2\pi\varepsilon_0}{\frac{1}{a}-\frac{1}{b}}=\frac{2\pi\varepsilon_0\varepsilon}{\left(\frac{b-a}{ab}\right)}$$

For $b >> a$, $b-a=b$

$$C=2\pi\varepsilon_0\varepsilon a$$

Example 22

Three cylindrical shells of radii a, b and c ($a < b < c$) length L, have charge $+q$, $+q/2$ and $-q$. Determine the capacity of system between inner and outer cylinders.

Solution: Working on the procedure explained before, we must first determine potential difference $V_A - V_C$.

$$V_A - V_C = (V_A - V_B) + (V_B - V_C)$$

From Gauss's law, electric field in the region $a < r < b$ is

$$E = \frac{q}{2\pi\varepsilon_0 rL}$$

$$V_A - V_B = \int \vec{E}\cdot d\vec{r}$$

$$= \int_a^b \frac{q}{2\pi\varepsilon_0 rL} dr$$

$$= \frac{q}{2\pi\varepsilon_0 L} \ln\left(\frac{b}{a}\right)$$

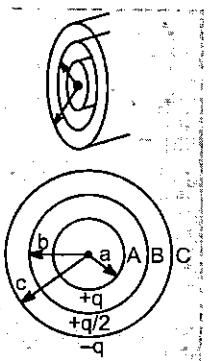


Fig. 3E.22

From Gauss's law, field in the region $b < r < c$ is

$$E = \frac{3q}{4\pi\varepsilon_0 rL}$$

Note that net charge enclosed in a Gaussian cylinder of radius $r\ (a < r < b)$ is $3q/2$.

$$V_B - V_C = \int_b^c \frac{3q}{4\pi\varepsilon_0 rL} dr = \frac{3q}{4\pi\varepsilon_0 L} \ln\left(\frac{c}{b}\right)$$

Thus $$V_A - V_C = \frac{q}{2\pi\varepsilon_0 L} \ln\left(\frac{b}{a}\right) + \frac{3q}{4\pi\varepsilon_0 L}\left(\frac{c}{b}\right)$$

$$= \frac{q}{2\pi\varepsilon_0 L}\left[\ln b - \ln a + \frac{3}{2}\ln c - \frac{3}{2}\ln b\right]$$

$$= \frac{q}{2\pi\varepsilon_0 L}\left[\frac{3}{2}\ln c - \ln a - \frac{1}{2}\ln b\right]$$

Thus the capacity of the system is

$$C = \frac{q}{V_A - V_C} = \frac{2\pi\varepsilon_0 L}{\left[\frac{3}{2}\ln c - \ln a - \frac{1}{2}\ln b\right]}$$

Example 23

Two concentric spherical shells have radii a and b, the charge on the shells is +q and −q respectively. The space between shells is field with two dielectrics of constant K_1 and K_2. Each dielectric occupies half the thickness available. Find the capacity of the system between innermost and outermost spheres.

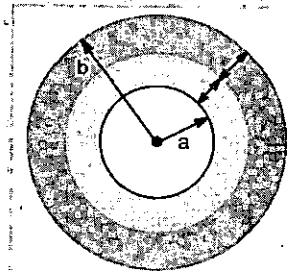


Fig. 3E.23

Solution: The given arrangement is series combination of two spherical capacitors.

Inner capacitor has radii

$$r_1 = a$$

$$r_2 = a + \left(\frac{b-a}{2}\right) = \frac{a+b}{2}$$

Hence, $$C_1 = \frac{4\pi\varepsilon_0 K_1 a[(a+b)/2]}{[(a+b)/2] - a}$$

$$= \frac{4\pi\varepsilon_0 K_1 (a+b)}{(b-a)}$$

Outer capacitor has radii $r_1 = \frac{a+b}{2}, r_2 = b$

Hence, $$C_2 = \frac{4\pi\varepsilon_0 K_2 b[(a+b)/2]}{b - [(a+b)/2]}$$

$$= \frac{4\pi\varepsilon_0 K_2 b(a+b)}{(b-a)}$$

The equivalent capacity of the system is

$$\frac{1}{C_{eq.}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$= \frac{b-a}{4\pi\varepsilon_0 K_1 a(a+b)} + \frac{b-a}{4\pi\varepsilon_0 K_2 b(a+b)}$$

On solving for $C_{eq.}$, we get

$$C_{eq.} = \frac{4\pi\varepsilon_0 K_1 K_2 ab(a+b)}{(b-a)[K_1 a + K_2 b]}$$

Example 24

A cylindrical capacitor of inner and outer radii a and b respectively is filled with two dielectrics of constants K_1 and K_2. Each dielectric occupies half the length of the cylinder. Find the capacity of the system between the inner and outer cylinders.

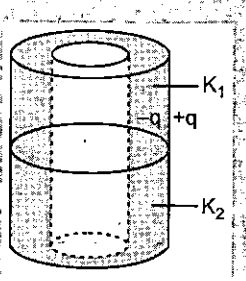


Fig. 3E.24

Solution: The given arrangement is a parallel combination of two capacitors.

The capacity of a cylindrical capacitor is given by

$$C = \frac{2\pi\varepsilon_0 L}{\ln(b/a)}$$

$$C_1 = \frac{2\pi\varepsilon_0 K_1 (L/2)}{\ln(b/a)}$$

$$C_2 = \frac{2\pi\varepsilon_0 K_2 (L/2)}{\ln(b/a)}$$

$$C_{eq.} = C_1 + C_2 = \frac{\pi L}{\ln(b/a)}(K_1 + K_2)$$

Example 25

Figure shows three conducting spherical shells A, B and C with charges $-q, +q/2, +q$ respectively. Determine the capacitance of the system between points A and C.

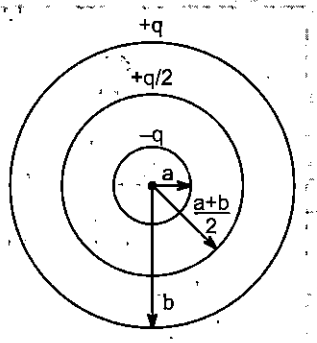


Fig. 3E.25

Solution:

Potential at $A, (V_A)$ = Potential due to charge on sphere A + Potential due to charge on sphere C + Potential due to charge on sphere B

$$= -\frac{1}{4\pi\varepsilon_0}\frac{q}{a} + \frac{1}{4\pi\varepsilon_0}\frac{q}{b} + \frac{q/2}{4\pi\varepsilon_0[(a+b)/2]}$$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{b} - \frac{1}{a} + \frac{1}{(a+b)}\right)$$

Potential at $C, (V_C)$ = Potential due to charge on sphere A + Potential due to charge on sphere C + Potential due to charge on sphere B

$$= -\frac{1}{4\pi\varepsilon_0}\frac{q}{b} + \frac{1}{4\pi\varepsilon_0}\frac{q}{2b} + \frac{1}{4\pi\varepsilon_0}\frac{q}{b} = \frac{q}{8\pi\varepsilon_0 b}$$

Potential difference $= V_A - V_C$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{b} - \frac{1}{a} + \frac{1}{(a+b)}\right) - \frac{q}{8\pi\varepsilon_0 b}$$

$$= \frac{q}{8\pi\varepsilon_0}\left(\frac{a^2 - 2b^2 + ab}{ab(a+b)}\right)$$

Capacitance of the arrangement $= \frac{q}{V_A - V_C}$

$$= \frac{q}{\left\{\frac{q}{8\pi\varepsilon_0}\left(\frac{a^2 - 2b^2 + ab}{ab(a+b)}\right)\right\}} = \frac{8\pi\varepsilon_0 ab(a+b)}{(a^2 - 2b^2 + ab)}$$

Example 26

Three concentric conducting shells A, B and C of radii a, b and c are as shown in Fig. 3E.26.
(a) Find the capacitance of the assembly between A and C.
(b) Suppose a dielectric of dielectric constant K is filled between A and B. Find the capacitance between A and C.

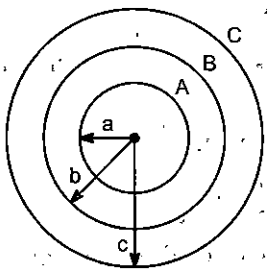


Fig. 3E.26

Solution: (a) Suppose a source of emf $\mathcal{E}$ is connected between shells A and C so that A acquires a charge Q and shell C a charge $-Q$. The potentials of the shells A and C will become

$$V_A = \frac{Q}{4\pi\varepsilon_0 a} - \frac{Q}{4\pi\varepsilon_0 c}$$

$$V_C = \frac{Q}{4\pi\varepsilon_0 c} - \frac{Q}{4\pi\varepsilon_0 c} = 0$$

Note that potential due to induced charges on B is zero. Potential difference between A and C is

$$\Delta V = V_A - V_C = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{c}\right) = \frac{q}{4\pi\varepsilon_0}\left(\frac{c-a}{ac}\right)$$

The capacitance of system is

$$C = \frac{q}{\Delta V} = 4\pi\varepsilon_0\left(\frac{ac}{c-a}\right)$$

(b) After inserting the dielectric, we will have to determine the potential difference between A and C. The electric field from A to C can be determined from Gauss's law. The field is

$$E(r) = \frac{q}{4\pi\varepsilon_0 K r^2}$$

for $a \le r \le b = \frac{q}{4\pi\varepsilon_0 r^2}$

for $b \le r \le c$

From $dV = -\int \vec{\mathbf{E}} \cdot d\vec{\mathbf{r}}$, the potential difference between A and C is

$$\Delta V = V_A - V_C$$

$$= \frac{q}{4\pi\varepsilon_0 Kabc}[c(b-a)+Ka(c-b)]$$

The capacitance system is

$$C = \frac{q}{V} = \frac{4\pi\varepsilon_0 Kabc}{Ka(c-b)+c(b-a)}$$

ANALYSIS OF SIMPLE CAPACITIVE CIRCUITS

- Consider a charged capacitor connected to an uncharged capacitor, redistribution of charges will occur to equalize the potential difference across each capacitor.

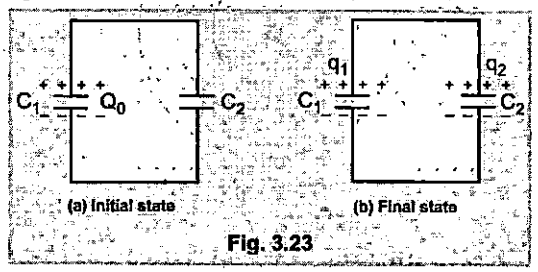


Fig. 3.23

Let q_1 and q_2 be the final charges on capacitors C_1 and C_2, when potential is equalized.

$$\frac{q_1}{C_1} = \frac{q_2}{C_2} = \frac{q_1+q_2}{C_1+C_2} = \frac{q_0}{C_1+C_2}$$

Thus $$q_1 = q_0\left[\frac{C_1}{C_1+C_2}\right]$$

$$q_2 = q_0\left[\frac{C_2}{C_1+C_2}\right]$$

Initial electrostatic energy stored in the capacitor,

$$U_i = \frac{q_0^2}{2C_1}$$

and the final energy is

$$U_f = \frac{q_1^2}{2C_1} + \frac{q_2^2}{2C_2} = \frac{q_0^2}{2(C_1+C_2)}$$

The expression for final energy shows that there is loss of energy given by

$$U_{loss} = U_i - U_f = \left(\frac{C_2}{C_1+C_2}\right)U_1$$

Fractional loss in energy,

$$\frac{U_{loss}}{U_i} = \frac{C_2}{C_1+C_2}$$

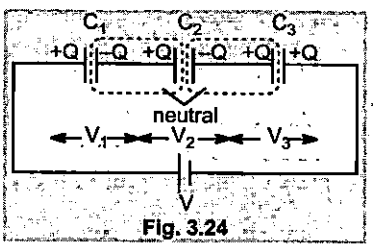


Fig. 3.24

- For a circuit consisting of three capacitors in series, magnitude of charge on each plate is same.

$$Q = C_1V_1 = C_2V_2 = C_3V_3 \quad ...(1)$$

$$V = V_1 + V_2 + V_3 \quad ...(2)$$

$$= \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} \quad ...(3)$$

or $$Q = \frac{V}{\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}}$$

$$= \frac{C_1C_2C_3V}{C_1C_2+C_2C_3+C_1C_3}$$

Hence $$V_1 = \frac{Q}{C_1} = \frac{C_2C_3V}{C_1C_2+C_2C_3+C_1C_3}$$

$$V_2 = \frac{Q}{C_2}$$

$$= \frac{C_1C_3V}{C_1C_2+C_2C_3+C_1C_3}$$

$$V_3 = \frac{C_1C_2V}{C_1C_2+C_2C_3+C_1C_3}$$

- For a circuit having three capacitors in parallel arrangement,

$$Q = Q_1 + Q_2 + Q_3$$

$$= C_1V + C_2V + C_3V$$

or $$V = \frac{Q}{C_1+C_2+C_3}$$

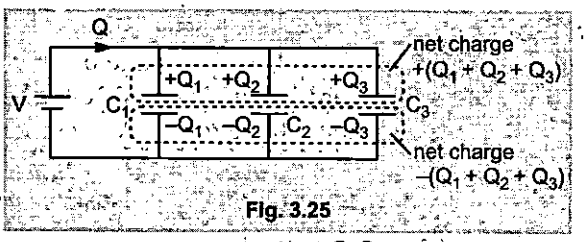


Fig. 3.25

Hence $$Q_1 = C_1V = \frac{C_1Q}{C_1+C_2+C_3}$$

$$Q_2 = C_2V = \frac{C_2Q}{C_1+C_2+C_3}$$

$$Q_3 = C_3V = \frac{C_3Q}{C_1+C_2+C_3}$$

Kirchhoff's current law (KCL) is based on the law of conservation of charge. Net charge at any junction of a circuit must be zero.

Kirchhoff's voltage law (KVL) is based on the fact that the potential of a point is a point junction, electric field is a conservative field, if we traverse through a closed path the net potential drop must be zero.

Sign convention for capacitors and emf are :

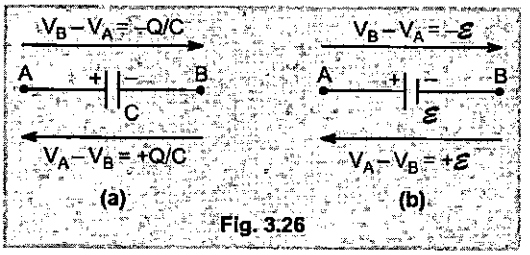


Fig. 3.26

For detailed analysis please read Kirchhoff's law for circuit analysis of next chapter.

We wish to find the potential of the junction point O for the circuit shown in Fig. 3.27

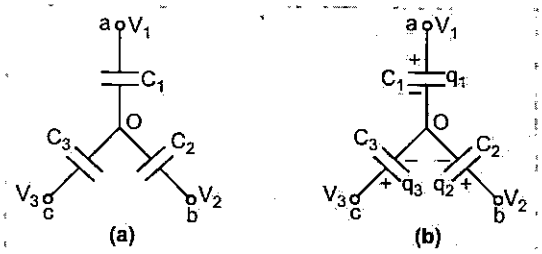


Fig. 3.27

Let V_0 be the potential of the junction O. Then the charge on each capacitor C_1, C_2 and C_3 is

$$q_1 = C_1(V_1 - V_0)$$
$$q_2 = C_2(V_2 - V_0)$$
$$q_3 = C_3(V_3 - V_0)$$

Applying KCL, we have

$$q_1 + q_2 + q_3 = 0$$

$$\therefore \quad C_1(V_1 - V_0) + C_2(V_2 - V_0) + C_3(V_3 - V_0) = 0$$

or
$$V_0 = \frac{C_1V_1 + C_2V_2 + C_3V_3}{C_1 + C_2 + C_3}$$

Example 27

A capacitor of capacitance $C_1 = 1.0\mu F$ with stands the maximum voltage $V_1 = 6.0kV$ while a capacitor of capacitance $C_2 = 2.0\mu F$, the maximum voltage $V_2 = 4.0kV$. What voltage will the system of these two capacitors withstand if they are connected in series ?

Solution: Amount of charge, that the capacitor of capacitance C_1 can withstand, $q_1 = C_1V_1$ and similarly the charge, that the capacitor of capacitance C_2 can withstand, $q_2 = C_2V_2$. But in series combination, charge on both the capacitors will be same, so, $q_{max.}$ that the combination can withstand $= C_1V_1$, as $C_1V_1 < C_2V_2$, from the numerical data, given.

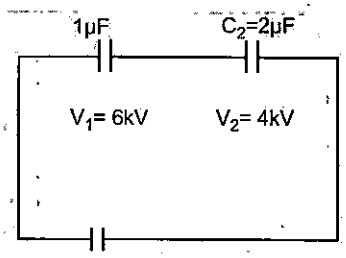


Fig. 3E.27

Now net capacitance of the system,

$$C_0 = \frac{C_1C_2}{C_1 + C_2}$$

and hence,
$$V_{max} = \frac{q_{max}}{C_0}$$

$$= \frac{C_1V_1}{C_1C_2/C_1 + C_2}$$

$$= V_1\left(1 + \frac{C_1}{C_2}\right) = 9kV$$

Example 28

A circuit has a section AB shown in Fig. The emf of the source equals $\mathcal{E} = 10V$, the capacitor capacitances are equal to $C_1 = 1.0\mu F$ and $C_2 = 2.0\mu F$, and the potential difference $V_A - V_B = 5.0V$. Find the voltage across each capacitor.

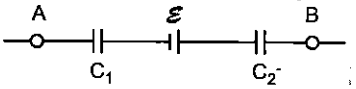


Fig. 3E.28 (a)

Solution: Let us make the charge distribution, as shown in the figure.

Now,
$$V_A - V_B = \frac{q}{C_1} - \mathcal{E} + \frac{q}{C_2}$$

or,
$$q = \frac{(V_A - V_B) + \mathcal{E}}{C_1 + C_2} C_1C_2$$

Hence, voltage across the capacitor C_1

Fig. 3E.28 (b)

$$= \frac{q}{C_1} = \frac{(V_A - V_B) + \mathcal{E}}{C_1 + C_2} C_2 = 10V$$

and voltage across the capacitor, C_2

$$= \frac{q}{C_2} = \frac{(V_A - V_B) + \mathcal{E}}{C_1 + C_2} C_1 = 5V$$

Example 29

In a circuit shown in Fig. find the potential difference between the left and right plates of each capacitor.

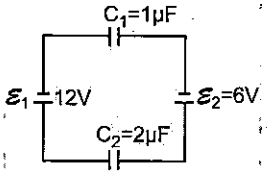


Fig. 3E.29 (a)

Solution: Let $\mathcal{E}_2 > \mathcal{E}_1$, then using $-\Delta\varphi = 0$ in the closed circuit, Fig. 3E.29 (a)

$$\frac{-q}{C_1} + \mathcal{E}_2 - \frac{q}{C_2} - \mathcal{E}_1 = 0$$

or
$$q = \frac{(V_2 - V_1)C_1C_2}{C_1 + C_2}$$

Hence the P.D. across the left and right plates of capacitors,

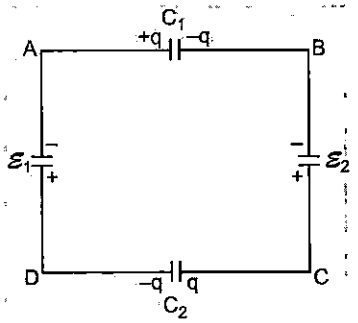


Fig. 3E.29 (b)

$$\varphi_1 = \frac{q}{C_1} = \frac{(\mathcal{E}_2 - \mathcal{E}_1)C_2}{C_1 + C_2}$$

and similarly $\varphi_2 = \frac{-q}{C_2} = \frac{(\mathcal{E}_1 - \mathcal{E}_2)C_1}{C_1 + C_2}$.

Example 30

What charges will flow after the shorting of the switch Sw in the circuit illustrated in Fig.3E.30 (a). Through sections 1 and 2 in the directions indicated by the arrows ?

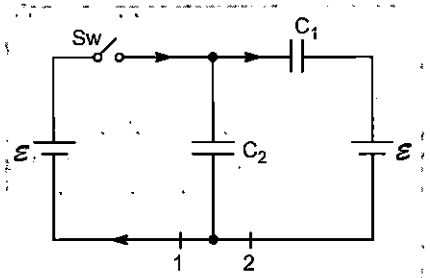


Fig. 3E.30 (a)

Solution: Initially, charge on the capacitor C_1 or C_2, $q = \frac{\mathcal{E}C_1C_2}{C_1 + C_2}$, as they are in series combination [Fig. 3E.30 (a)]

When the switch is closed, in the circuit CDEFC from $-\Delta V = 0$, [Fig. 3E.30 (b)]

$$\mathcal{E} - \frac{q_2}{C_2} = 0 \quad \text{or} \quad q_2 = C_2\mathcal{E} \qquad ...(1)$$

And in the closed loop BCFAB from $-\Delta\varphi = 0$

$$\frac{-q_1}{C_1} + \frac{-q_2}{C_2} - V = 0 \qquad ...(2)$$

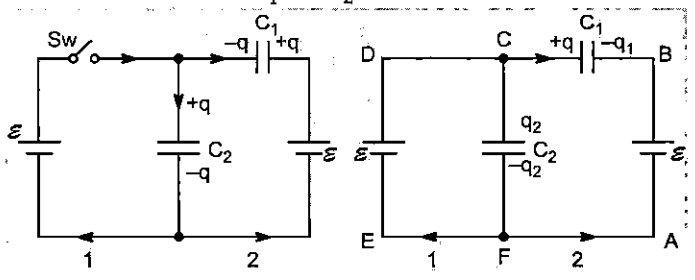


Fig. 3E.30

From eqn. (1) and (2) $q_1 = 0$

Now, charge flown through section 1

$$= (q_1 + q_2) - 0 = C_2\mathcal{E}$$

and charge flown through section 2

$$= -q_1 - q = -\frac{\mathcal{E}C_1C_2}{C_1 + C_2}$$

Example 31

In the circuit shown in Fig. 3E.31 (a). the emf of each battery is equal to V = 60V, and the capacitor capacitances are equal to $C_1 = 2.0\mu F$ and $C_2 = 3.0\mu F$. Find the charges which will flow after the shorting of the switch Sw through the section 1, 2 and 3 in the directions indicated by the arrows.

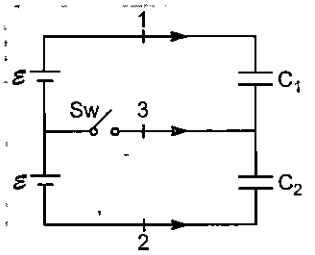


Fig. 3E.31 (a)

Solution: When the switch is open, Fig. 3E.31 (a)

$$q_0 = \frac{2\mathcal{E}C_1C_2}{C_1 + C_2}$$

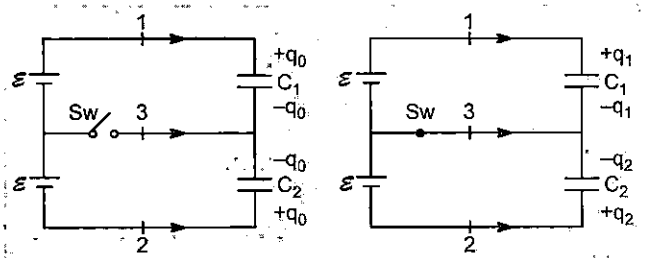


Fig. 3E.31

and when the switch is closed,

$$q_1 = \mathcal{E}C_1 \quad \text{and} \quad q_2 = \mathcal{E}C_2$$

Hence section

$$1 = q_1 - q_0 = \mathcal{E}C_1\left[\frac{C_1 - C_2}{C_1 + C_2}\right] = -24\mu C$$

through the section $2 = -q_2 - (q_0)$

$$= \mathcal{E}C_2\left[\frac{C_1 - C_2}{C_1 + C_2}\right] = -36\mu C$$

through the section $2 = q_2 - (q_2 - q_1) - 0$

$$= \mathcal{E}(C_2 - C_1) = -60\mu C$$

Example 32

Two capacitors C_1 and C_2 (where $C_1 > C_2$) are charged to the same initial potential difference ΔV_i, but with opposite polarity. The charged capacitors are removed from the opposite polarity. The charged capacitors are removed from the battery and their plates are connected as shown in Fig. 3E.32 (a). The witches S_1 and S_2 are then closed, as shown in Fig. 3E.32 (b).

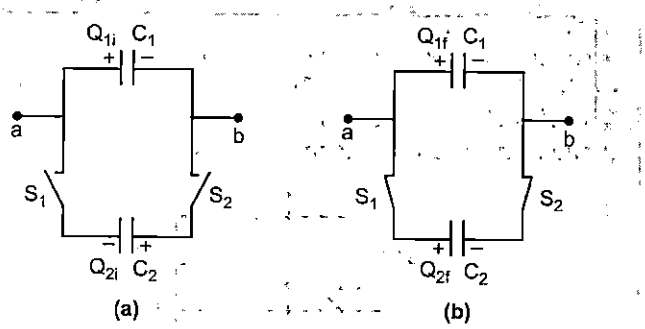


Fig. 3E.32

(a) Find the final potential difference ΔV_f between a and b after the switches are closed.

(b) Find the total energy stored in the capacitors before and after the switches are closed and the ratio of the final energy to the initial energy.

Solution: (a) The left-hand plates of the capacitors act as an isolated system because they are not connected to the right-hand plates by conductors. The charges on the left-hand plates before the switches are closed are

$$Q_{1i} = C_1 \Delta V_i$$

and $$Q_{2i} = -C_2 \Delta V_i$$

The total charge Q in the system is

$$Q = Q_{1i} + Q_{2i} = (C_1 - C_2)\Delta V_i \quad \ldots(1)$$

After the switches are closed, the total charge in the system remains the same.

$$Q = Q_{1f} + Q_{2f} \quad \ldots(2)$$

The charges redistribution until the entire system is at the same potential ΔV_f. Thus, the final potential difference across C_1 must be the same as the final potential difference across C_2. The charges on the capacitors after switches are closed are

$$Q_{1f} = C_1 \Delta V_f$$

and $$Q_{2f} = C_2 \Delta V_f$$

Dividing the first equation by the second, we have

$$\frac{Q_{1f}}{Q_{2f}} = \frac{C_1 \Delta V_f}{C_2 \Delta V_f} = \frac{C_1}{C_2}$$

$$Q_{1f} = \frac{C_1}{C_2} Q_{2f}$$

Combining eqns. (2) and (3), we obtain

$$Q = Q_{1f} + Q_{2f} = \frac{C_1}{C_2} Q_{2f} + Q_{2f}$$

$$= Q_{2f}\left(1 + \frac{C_1}{C_2}\right)$$

$$Q_{2f} = Q\left(\frac{C_2}{C_1 + C_2}\right)$$

Using eqn.(3) to find Q_{1f} in terms of Q, we have

$$Q_{1f} = \frac{C_1}{C_2} Q_{2f}$$

$$= \frac{C_1}{C_2} Q\left(\frac{C_2}{C_1 + C_2}\right) = Q\left(\frac{C_1}{C_1 + C_2}\right)$$

Finally, voltage across each capacitor is,

$$\Delta V_{1f} = \frac{Q_{1f}}{C_1}$$

$$= \frac{Q\left(\frac{C_1}{C_1 + C_2}\right)}{C_1} = \frac{Q}{C_1 + C_2}$$

$$\Delta V_{2f} = \frac{Q_{2f}}{C_2}$$

$$= \frac{Q\left(\frac{C_2}{C_1 + C_2}\right)}{C_2} = \frac{Q}{C_1 + C_2}$$

As noted earlier,

$$\Delta V_{1f} = \Delta V_{2f} = \Delta V_f.$$

To express ΔV_f in terms of the given quantities C_1, C_2 and ΔV_i, we substitute the value of Q from eqn. (1) to obtain

$$\Delta V_f = \left(\frac{C_1 - C_2}{C_1 + C_2}\right)\Delta V_i$$

(b) Before the switches are closed, the total energy stored in the capacitor is

$$U_i = \frac{1}{2} C_i (\Delta V_i)^2 + \frac{1}{2} C_2 (\Delta V_i)^2$$

$$= \frac{1}{2}(C_1 + C_2)(\Delta V_i)^2$$

After the switches are closed, the total energy stored in the capacitor is

$$U_f = \frac{1}{2} C_1 (\Delta V_f)^2 + \frac{1}{2} C_2 (\Delta V_f)^2$$

$$= \frac{1}{2}(C_1 + C_2)(\Delta V_f)^2$$

$$= \frac{1}{2}(C_1 + C_2)\left(\frac{Q}{C_1 + C_2}\right)^2$$

$$= \frac{1}{2}\frac{Q^2}{C_1 + C_2}$$

Using eqn. (1), we can express this as

$$U_f = \frac{1}{2}\frac{Q^2}{(C_1 + C_2)}$$

$$= \frac{1}{2}\frac{(C_1 - C_2)^2 (\Delta V_i)^2}{(C_1 + C_2)}$$

Therefore, the ratio of the final energy stored to the initial energy stored is

$$\frac{U_f}{U_i}=\frac{\frac{1}{2}\frac{(C_1-C_2)^2(\Delta V_i)^2}{(C_1+C_2)}}{\frac{1}{2}(C_1+C_2)(\Delta V_i)^2}=\left(\frac{C_1-C_2}{C_1+C_2}\right)^2$$

This ratio is less than unity, indicating that the final energy is less than the initial energy, the "missing" energy is radiated away in the form of electromagnetic waves.

Nodal Analysis for Capacitive Circuits

Nodal analysis is based on conservation of charge [Kirchhoff's current law]. At any node or junction (an isolated part of circuit) charge is conserved

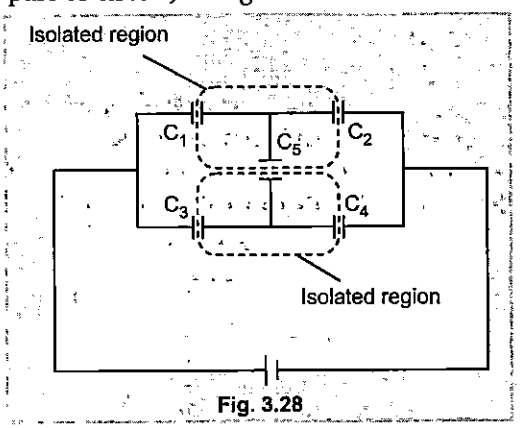


Fig. 3.28

$\Sigma q_i = 0$ if capacitors are initially uncharged

$\Sigma q_i = \Sigma$ initial charge of plates of capacitor connected to node if capacitors have initial charge

Figure illustrates the isolated plates of circuit. Following steps are to be followed in problem solving

Step-I: Assign a node as a reference and assume its potential zero.

Step-II: Assign the potential of every junction of the circuit as unknown S.

Step-III: Apply Kirchhoff's current law at the junction corresponding to unknown potential.

Step-IV: Solve the equations to get the desired unknown potentials.

Example 33

In the circuit shown in Fig. 3E.33 determine charge on each capacitor.

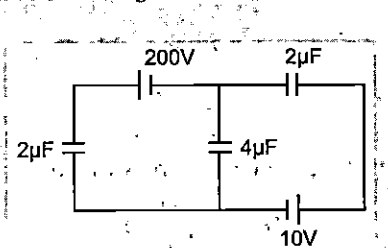


Fig. 3E.33 (a)

Solution: Figure shows two loops which contain isolated plates.

Writing nodal equation at node N_a, we get

$$\underbrace{2(x-20)}_{\substack{\text{charge on left}\\ 2\mu F}}+\underbrace{4x}_{\substack{\text{charge on}\\ 4\mu F}}+\underbrace{(x+10)2}_{\substack{\text{charge on top}\\ 2\mu F}}=0$$

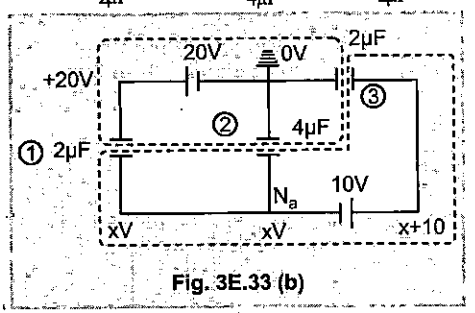


Fig. 3E.33 (b)

Which on solving yields

$$8x = 20$$

or $x = 2.5V$

Thus charge on capacitor (1) = $35\mu C$

Thus charge on capacitor (2) = $10\mu C$

Thus charge on capacitor (3) = $25\mu C$

Example 34

What are charges on all the four capacitors.

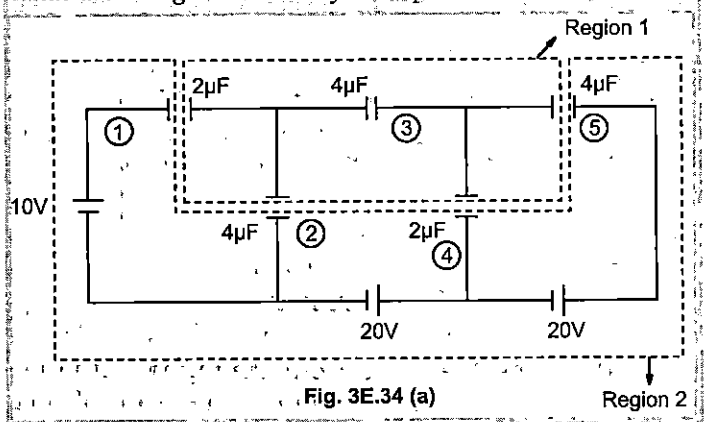


Fig. 3E.34 (a)

Solution:

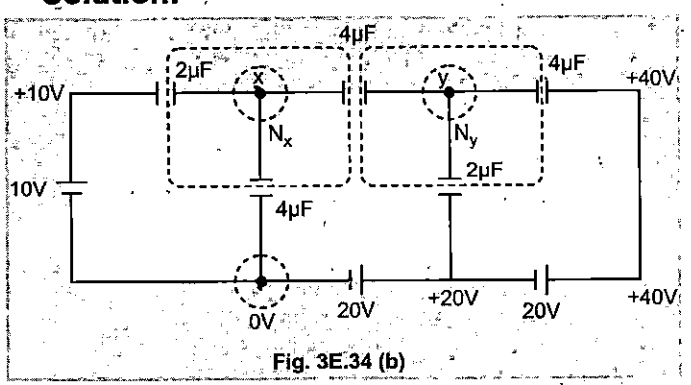


Fig. 3E.34 (b)

Nodal equation at node N_y

$$(y-40)4+(y-x)4+(y-20)2=0$$

Nodal equation at node N_x

$$(x-10)2+(x-y)4+4x=0$$

On solving we get

$$y=\frac{100}{3}\text{V}$$

$$x=\frac{100}{3}\text{V}$$

Thus charge on capacitors (1)

$$|Q_1|=|(x-10)2|=\frac{140}{3}\mu\text{C}$$

$$|Q_2|=\left|\frac{400}{3}\right|\mu\text{C}=4x$$

$$|Q_3|=0$$

$$|Q_4|=(y-40)4=-\frac{80}{3}\mu\text{C}$$

$$|Q_5|=(y-20)y=\frac{40}{3}\mu\text{C}$$

Example 35

Find charge on all the capacitors in circuit.

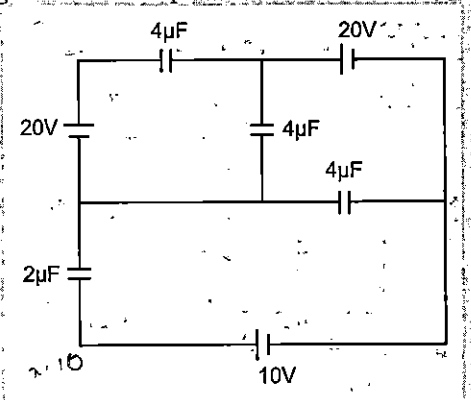


Fig. 3E.35 (a)

Solution: The figure shows the region that includes all the capacitor plates isolated from other parts of circuit. We can include in our isolated region, because battery is net neutral.

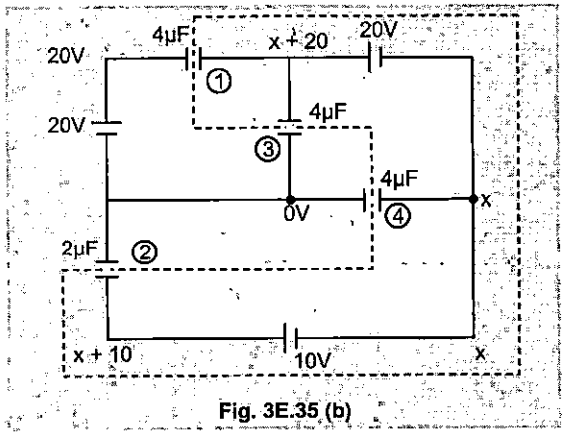


Fig. 3E.35 (b)

Nodal equation is,

$$4x+4(x+20)+4x+2(x+10)=0$$

$$14x=-100$$

$$x=-\frac{50}{7}\text{V}$$

$$|Q_1|=\frac{200}{7}\mu\text{C}$$

$$|Q_2|=\frac{240}{7}\mu\text{C}$$

$$|Q_3|=\frac{190}{7}\mu\text{C}$$

$$|Q_4|=\frac{200}{7}\mu\text{C}$$

Example 36

Fig. 3E.36 shows a capacitive circuit, with a switch S_w. (a) What is potential difference between a and b when switch is open ? (b) What is potential of b after switch is closed ? (c) What charge flows through switch when it is closed ?

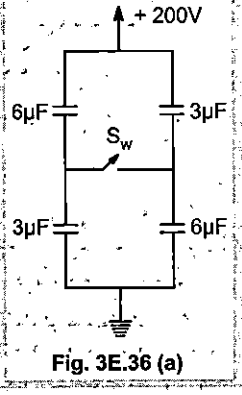


Fig. 3E.36 (a)

Solution: When switch S is open for plates of region 1

$$(x-200)6+3x=0$$

or

$$x=\frac{400}{3}\text{V}$$

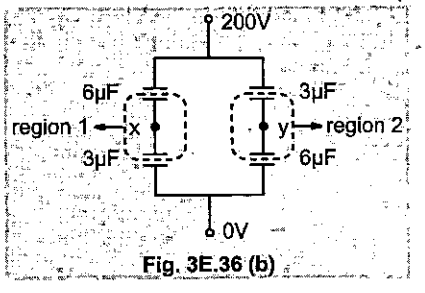


Fig. 3E.36 (b)

Similarly for plates of region 2

$$(y-200)3+6y=0$$

or

$$y=\frac{200}{3}\text{V}$$

Thus

$$V_a-V_b=\frac{200}{3}\text{V}$$

Thus initial charge distribution on all the plates is shown in figure.

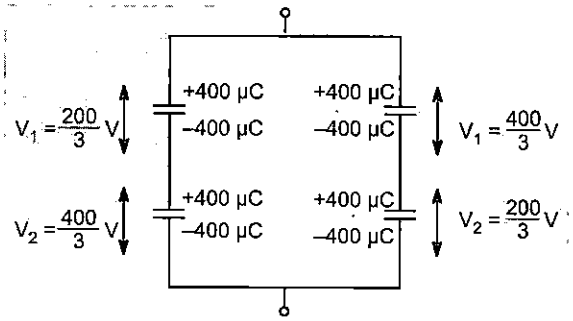


Fig. 3E.36 (c)

When switch is closed,

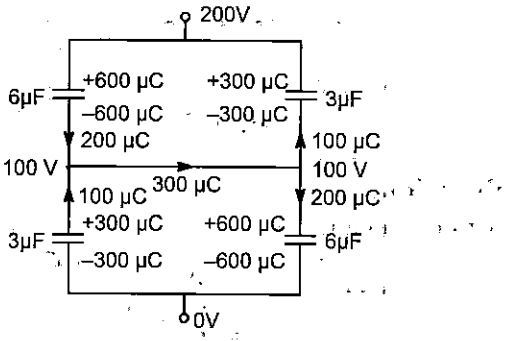


Fig. 3E.36 (d)

Figure shows charge transfer on plates connected by switch.

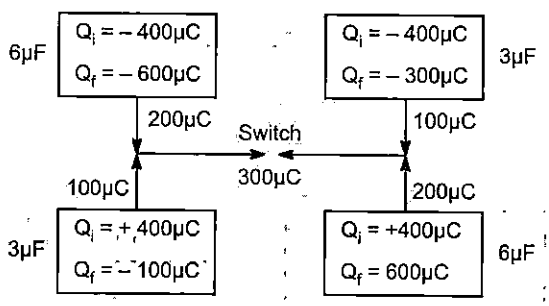


Fig. 3E.36 (e)

Charge flowing through switch = 300μC

Example 37

What is equivalent capacitance of circuit shown in fig. 3E.37 (a) and (b) ?

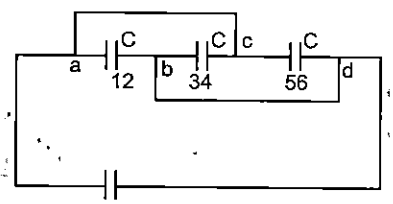


Fig. 3E.37 (a)

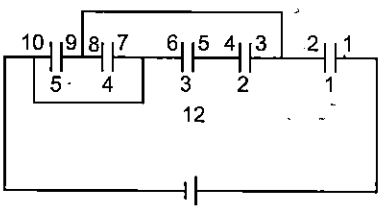


Fig. 3E.37 (b)

Solution: The equivalent circuit of Fig. 3E.37 (a) is three capacitors are in parallel arrangement $C_{eq} = 3C$

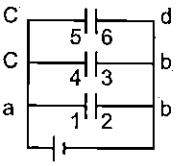


Fig. 3E.37 (c)

The equivalent circuit of Fig. 3E.37 (b) is

Thus equivalent capacity is $\frac{2}{7}C$

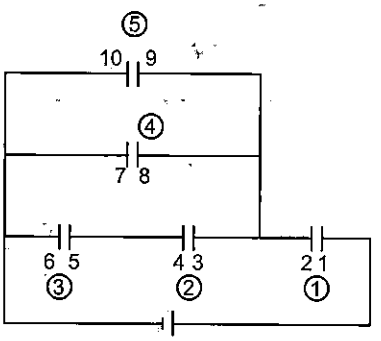


Fig. 3E.37 (d)

Example 38

In the figure shown, all the capacitors are in steady state initially. Now switch is closed (a) What is charge flown through switch when it is closed ? (b) What is charge flowing through section AB, (c) Workdone by battery? (d) Heat produced when S is closed.

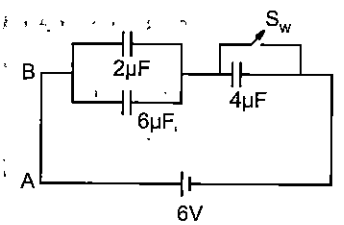


Fig. 3E.38 (a)

Solution: Before closing switch

$$4x + (x-6)2 + (x-6)2 = 0$$

or $$x = 4V$$

$$|Q_{4\mu F}| = 16\mu C; \quad |Q_{2\mu F}| = 4\mu C$$

$$|Q_{6\mu F}| = 12\mu C$$

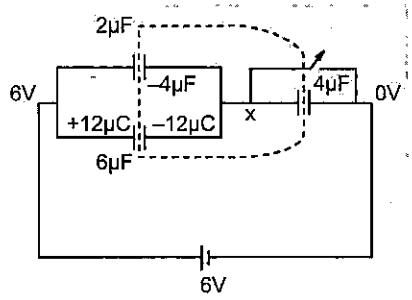
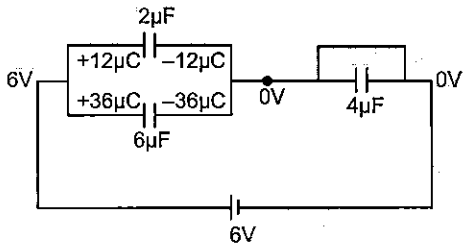
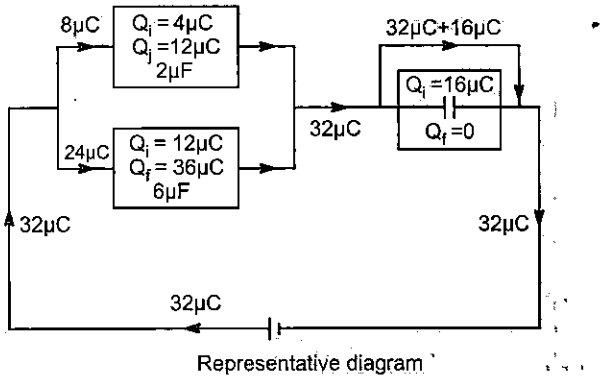


Fig. 3E.38 (b)

When switch S_w is closed, capacitor 4μF is short circuited, p.d. across it is zero.Circuit after closing switch



(c)



(d)

Fig. 3E.38

The representative diagram shows the charges coming and going out of capacitors.

Charge through section $AB = 32\mu C$

Charge through switch $= 48\mu C$

When switch is closed, 16μC charge from left plate goes past switch and neutralises −16μC on the right plate.

Charge passing through battery is $= 32\mu C$

Workdone by battery $= (32\mu C)(6V) = 192\mu J$

Heat produced $= w_b - \Delta U = w_b - (U_f - U_i)$

$$U_f = \frac{1}{2}(2)\times 6^2 + \frac{1}{2}(6)(6)^2 = 144\mu J$$

$$U_i = \frac{1}{2}4(4)^2 + \frac{1}{2}(8)2^2 = 48\mu J$$

$$H = (192 - [144 - 48])\mu J = 96\mu J$$

Example 39

In the circuit shown in Fig. 3E.39 (a) find heat produced when switch S is closed.

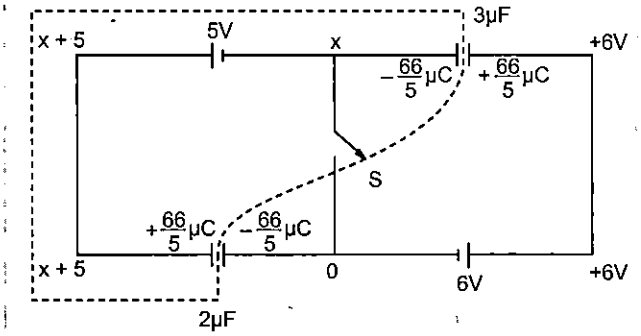
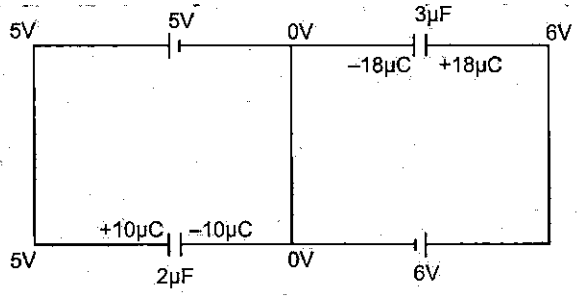


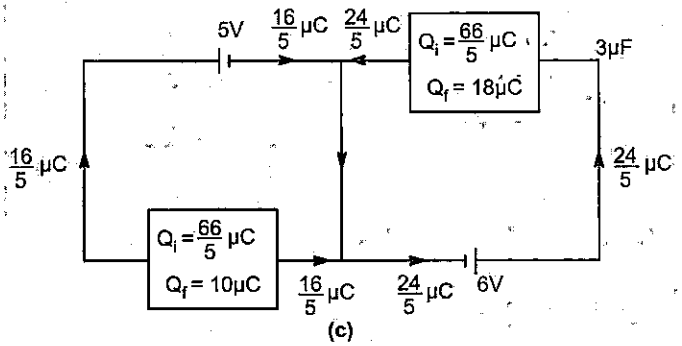
Fig. 3E.39 (a)

Solution:

Before switch is closed



(b)



(c)

Fig. 3E.39

$$(x-6)3 + (x+5)2 = 0$$

or $$x = \frac{8}{5}V$$

When switch is closed

Note that $\frac{16}{5}\mu C$ is charging battery W/D by 5V battery

$$= -\left(\frac{16}{5}\mu C\right)(5V) = -16\mu J$$

W/D by 6V battery $= \left(\frac{29}{5}\mu C\right)(6V)$

$$= \frac{144}{5}\mu J$$

$$\Delta U = U_f - U_i$$

$$= \frac{1}{2}3\left[\frac{6}{5}\right]^2 + \frac{1}{2}2(5)^2 - \left(\frac{1}{2}3\left[\frac{29}{5}\right]^2 + \frac{1}{2}2\left(\frac{33}{5}\right)^2\right)$$

$$= -3.12\mu J$$

$$H = w_b - \Delta U$$

$$= 15.92\mu J$$

Example 40

Find the potential difference between point A and B in the circuit shown in figure.

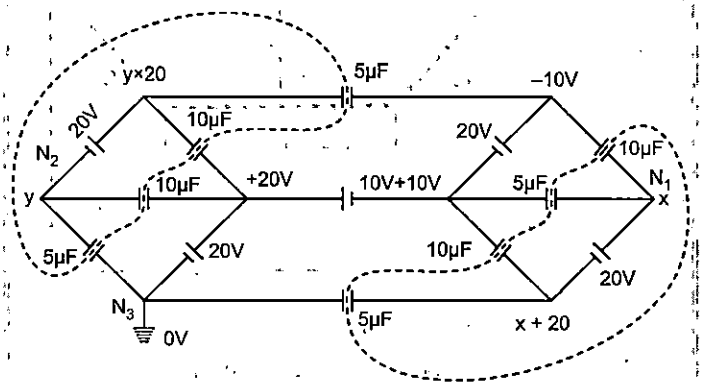


Fig. 3E.40

Solution: We assign node N_3 as 0V and N_1, N_2 as x and y volt respectively.

Writing nodal equation for region 1, we get

$$(y+30)5 + y(10) + (y-20)10 + 5y = 0$$

$$30y = 50$$

$$y = \frac{5}{3}V$$

Nodal equation for region 2, we get

$$(x+10)10 + (x-10)5 + (x+10)10 + (x+20)5 = 0$$

$$30x = -250$$

$$x = \frac{-25}{3}V$$

Thus, $$V_A - V_B = \frac{5}{3} - \left(\frac{-25}{3}\right) = 10V$$

PROBLEMS BASED ON EQUIVALENT CAPACITANCE

In all the following circuits we determine the equivalent, capacitance

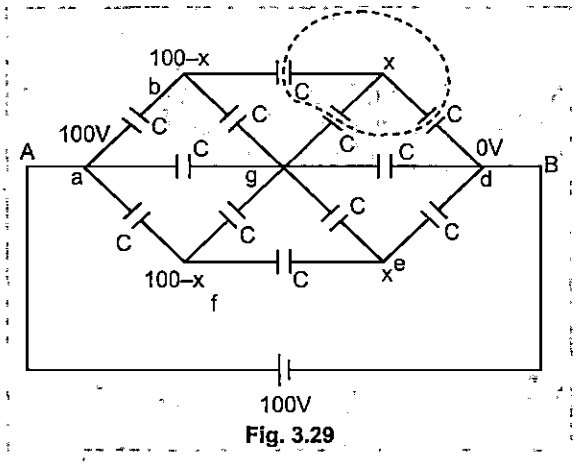


Fig. 3.29

(i) Step-1. Join an imaginary battery across circuit.

Step-2. Assign potentials x, y and z etc. at nodes.

Step-3. In symmetrical circuits, symmetrically placed points have same potential for example node a and e similarly b and f have same potential.

Step-4. Calculate charge that is being transferred through battery, which is $xC + 50C + xC$

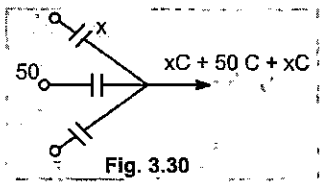


Fig. 3.30

If circuit has equivalent capacitance C_{eq}

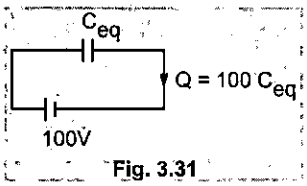


Fig. 3.31

Node g has potential 50V because, $100 - y = y$, where y is potential drop on right and left capacitors.

Writing nodal eqn. at node C, we get

$$xC + (x-50)C + (2x-100)C = 0 \quad \text{or} \quad x = \frac{75}{2}V$$

Step-5. $xC + 50C + xC = 100C_{eq}$

or $$C_{eq} = \frac{xC + 50 + xC}{100} = \frac{(2\times 75/2) + 50)C}{100}$$

$$C_{eq} = \frac{5C}{4}$$

(ii)

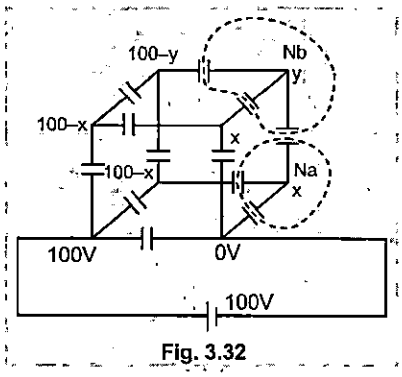


Fig. 3.32

$$C_{eq} = \frac{2Cx + 100C}{100}$$

At node N_a; $xC + (2x - 100)C + (x - y)C = 0$

At node N_b ; $(2y - 100)C + 2(y - x)C = 0$

On solving for x and y, we get

$$x = \frac{250}{7}\text{V}$$

and $$y = \frac{300}{7}\text{V}$$

Thus, $$C_{eq} = \frac{2C\left(\frac{250}{7}\right) + 100C}{100}$$

$$C_{eq} = \frac{12C}{7}$$

(iii)

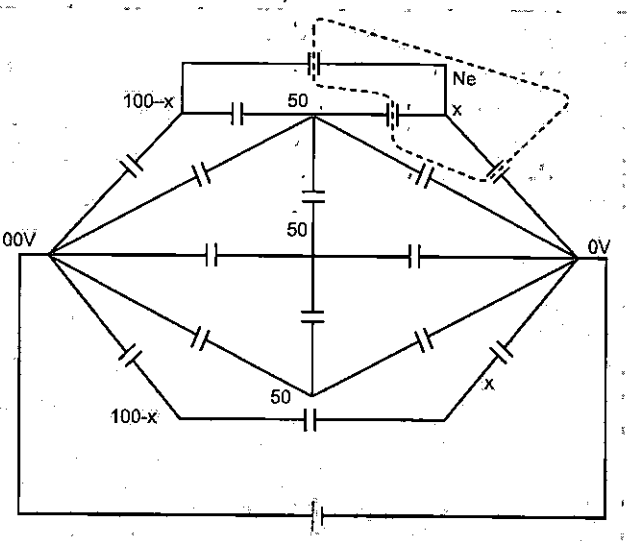


Fig. 3.33

$$(2x - 100)C + (x - 50)C + xC = 0$$

or $$x = \frac{75}{2}\text{V}$$

$$C_{eq} = \frac{(2x + 100)C}{100}$$

$$= \frac{\left(2 \times \frac{75}{2} + 100\right)C}{100} = \frac{5}{2}C$$

(iv)

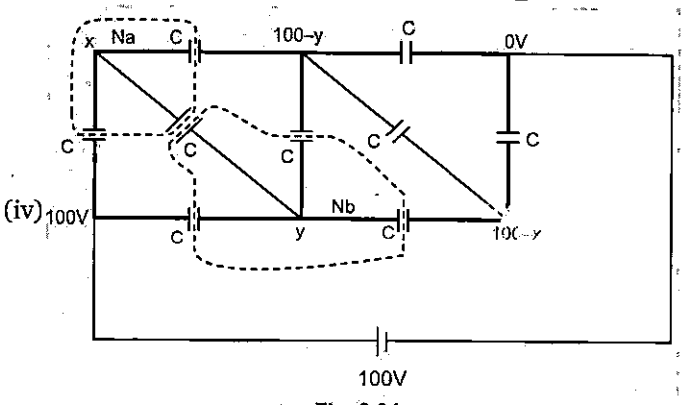


Fig. 3.34

At node N_a we have

$$(x + y - 100)C + (x - y)C + (x - 100)C = 0$$

At node N_b we get

$$(y + x - 100)C + (2y - 100)C + (y - 100)C + (y - x)C = 0$$

On solving for x and y, we get

$$x = \frac{200}{3}\text{V} \quad \text{and} \quad y = 60\text{V}$$

and $$C_{eq} = \frac{(100 - y)C + (100 - x)C}{100} = \frac{11}{15}C$$

(v)

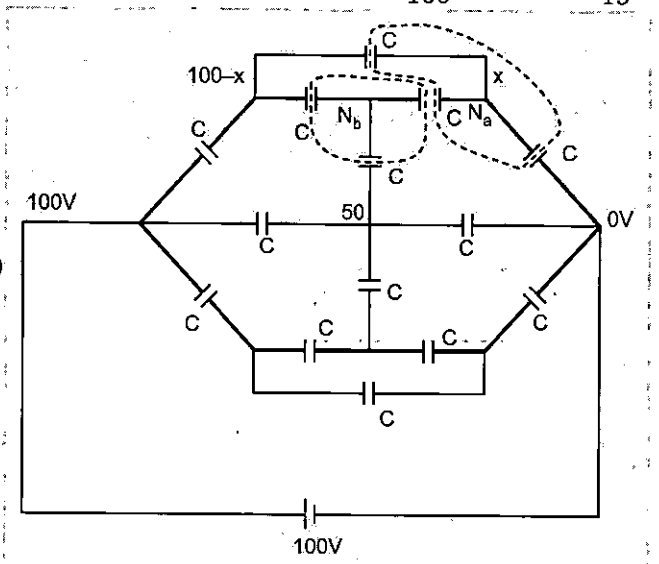


Fig. 3.35

At node N_a and N_b we get the equations

$$(y - x)C + (y - 50)C + (y - x + 100)C = 0$$

and $$xC + (x - 100x)C + (x - y)C = 0$$

or $$x = \frac{75}{2}\text{V and } y = 50\text{V}$$

Thus $$C_{eq} = \frac{2xC + 50C}{100} = \frac{\left(\frac{75}{2}\right)2C + 50C}{100} = \frac{5C}{4}$$

(vi)

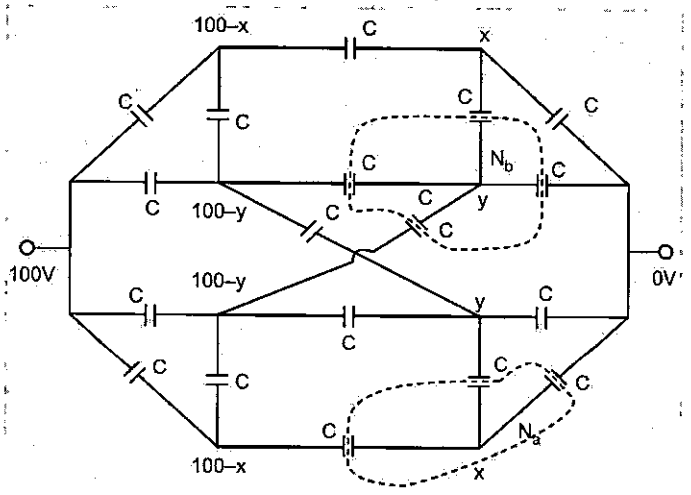


Fig. 3.36

On writing nodal equation at N_a and N_b we get

$$(x-y)C + xC + (2x-100)C = 0$$

and $$(y-x)C + 2(2y-100)C = 0$$

on solving for x and y we get

$$x = \frac{800}{23}$$

$$y = \frac{900}{23}$$

$$C_{eq} = \frac{2xC + 2yC}{100} = \frac{39C}{23}$$

Example 41

Fig. 3E.41 shows two identical parallel plate capacitors connected to a battery with the switch S closed. The switch is now opened and the free space between the plates of the capacitors is filled with a dielectric of dielectric constant (or relative permittivity) 3. Find the total ratio of the total electrostatic energy stored in both capacitors before and after the introduction of the dielectric.

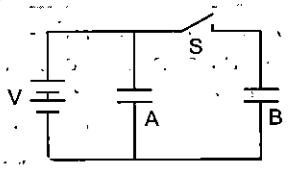


Fig. 3E.41

Solution: Initially when the switch is closed, both the capacitors A and B are in parallel and the energy stored in the capacitors is therefore

$$U_i = 2\times\frac{1}{2}CV^2 = CV^2 \qquad ...(1)$$

When switch S is opened, B gets disconnected from the battery. The capacitor B is now isolated, the charge of an isolated capacitor remains constant, often referred to as bound charge. On the other hand, A remains connected to battery. Hence, potential V remains constant on it.

When the dielectric is filled in the capacitors their capacitance increases to KC.

Therefore, energy stored in B changes to $\frac{1}{2}\frac{Q^2}{KC}$, where $Q = CV$ is the charge on B which remains constant. And energy stored in A charges to $\frac{1}{2}KCV^2$, where V, the potential on A, remains constant. Thus, finally, the total energy stored in the capacitors is,

$$U_f = \frac{1}{2}\frac{(CV)^2}{KC} + \frac{1}{2}KCV^2$$

$$= \frac{1}{2}CV^2\left(K + \frac{1}{K}\right) \qquad ...(2)$$

From eqn. (1) and (2), we find

$$\frac{U_i}{U_f} = \frac{2K}{K^2+1}$$

It is given that $K = 3$. Therefore, we have

$$\frac{U_i}{U_f} = \frac{3}{5}$$

Example 42

Two parallel plate capacitors of capacitance C are connected in series with a battery of emf $\mathcal{E}$. Then one of the capacitors is filled with a dielectric of dielectric constant K. Find the change in electric field in the two capacitors, if any. What amount of charge flows through the battery?

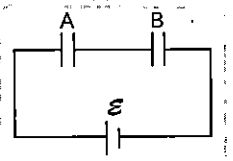


Fig. 3E.42

Solution: (i) Two capacitors A and B initially have same charge Q and potential $V = Q/C$. The electric field between the capacitor plates is given by $E = V/d$.

Since the two capacitors are connected in series with the battery, the sum of potentials across the capacitors must be equal to $\mathcal{E}$,

i.e., $$\mathcal{E} = 2V = \frac{2Q}{C} \qquad ...(1)$$

(ii) When one of the capacitors, say A, is filled with a dielectric, the capacity of A increases to $C' = KC$ while that of B remains unchanged, *i.e.,* C.

Suppose, now charge on the capacitors becomes Q' and potentials across A and B are V'_A and V'_B respectively, with $V'_A + V'_B = \mathcal{E}$. Hence, we have

$$V'_A = \frac{Q'}{C'} = \frac{Q'}{KC}$$

$$V'_B = \frac{Q'}{C}$$

Hence, $$\mathcal{E} = \frac{Q'}{C}\left(1 + \frac{1}{K}\right) \qquad ...(2)$$

From eqn. (1) and (2), we get,

$$Q' = \frac{2K}{1+K}\cdot Q$$

Since $K > 1$, therefore $Q' > Q$. Also,

$$V'_A = \frac{2}{1+K}V$$

and $$V'_B = \frac{2K}{1+K}V$$

Thus, the electric field (or potential difference) in capacitor A decreases by a factor of $\left(\frac{2}{1+K}\right)$ while that in B increases by a factor $\left(\frac{2K}{1+K}\right)$.

The amount of charge that flows into the circuit is given by

$$\Delta Q = Q' - Q = \left(\frac{2K}{1+K} - 1\right) \cdot Q$$

$$= \frac{K-1}{K+1} \cdot Q = \frac{1}{2}\frac{K-1}{K+1} \cdot C\mathcal{E}$$

Initially, the energy is given by

$$U_i = 2 \times \frac{1}{2} CV^2 = CV^2$$

Final energy is,

$$U_f = \frac{1}{2} C' V_A^2 + \frac{1}{2} V_B^2$$

$$= \frac{1}{2} KC \left(\frac{2}{1+K}\right)^2 V^2 + \frac{1}{2} C \left(\frac{2K}{1+K}\right)^2 V^2$$

$$= \frac{4K}{1+K}\left(\frac{1}{2} CV^2\right) = \frac{2K}{1+K} \cdot U_i$$

Thus, the total energy stored in the capacitor increases $\frac{2K}{1+K}$ times. This energy is drawn from the battery.

Example 43

Four identical metal plates are arranged in air at equal distance d from each other. The area of each plate is A. Discuss the charge distribution and find the capacitance of the system between points P and Q if the other two plates are connected by a conducting wire as shown Fig. 3E.43.

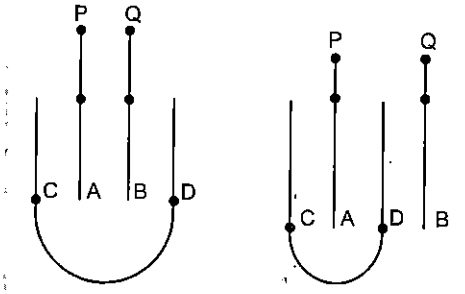


Fig. 3E.43

Solution: (i) The equivalent arrangement is shown in Fig. 3E.43(c) and (d).

The upper two capacitors are in series and the third one is in parallel. Hence the equivalent capacitance is given by

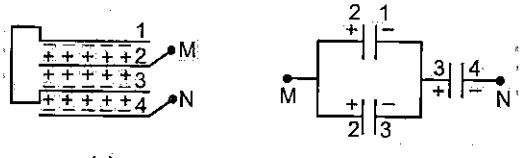


Fig. 3E.43

$$\frac{1}{C'} = \frac{1}{C} + \frac{1}{C} = \frac{2}{C}$$

or $$C' = \frac{C}{2}$$

$$C_0 = \frac{1}{2} C + C = \frac{3C}{2} = \frac{3\varepsilon_0 A}{2d}$$

(ii) In the given arrangement, we have three capacitors of equal capacitance $C = (\varepsilon_0 A/d)$. The equivalent circuit is shown in Fig. 3E.43 (e) and (f).

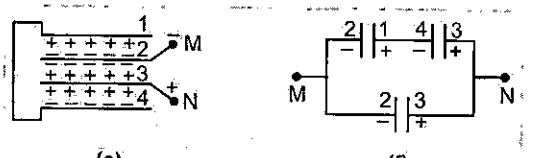


Fig. 3E.43

The first two condensers are in parallel while the third one is in series. Hence the equivalent capacitance C_0 is given by

$$C_0 = \frac{(C+C) \times C}{(C+C)+C} = \frac{2}{3} C = \frac{2\varepsilon_0 A}{3d}$$

Example 44

Five identical conducting plates 1, 2, 3, 4 and 5 are fixed parallel to and equidistant from each other as shown in Fig. 3E.44 (a). Plates 2 and 5 are connected by a conductor while 1 and 3 are joined by another conductor. The junction of 1 and 3 and the plate 4 are connected to a source of constant emf $\mathcal{E}_0$. Find :

(i) the effective capacity of the system between the terminals of the source,

(ii) the charges on plats 3 and 5.

Given d = distance between any two successive plates and

A = area of either face of each plate.

Solution: (i) The equivalent circuit is shown in Fig. 3E.44(b). The system consists of four capacitors, *i.e.*, C_{12}, C_{32}, C_{34} and C_{54}. The capacity of each capacitors is $(K\varepsilon_0 A/d) = C_0$.

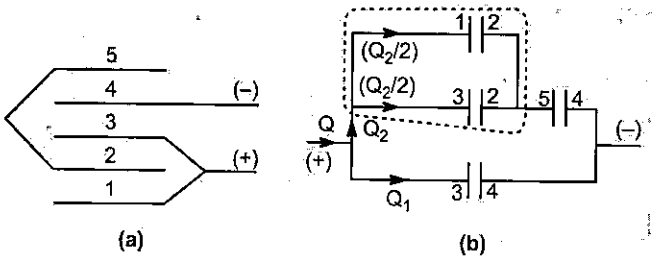


Fig. 3E.44

The capacitors C_{12} and C_{32} are in parallel, their capacity is $C_0 + C_0 = 2C_0$. The capacitor C_{54} is in series with the parallel combination of C_{12} and C_{32}. Hence the resultant capacity will be

$$\frac{C_0 \times 2C_0}{C_0 + 2C_0}.$$

Further, C_{34} is again in parallel with the combination of C_{12}, C_{32}, C_{54}. Hence the effective capacity

$$= C_0 + \frac{C_0 \times 2C_0}{C_0 + 2C_0}$$

$$= \frac{5}{3}C_0 = \frac{5}{3}K\varepsilon_0\frac{A}{d}$$

(ii) Charge on the plate 5 = charge on the upper half of parallel combination

$$\therefore \quad Q_5 = \mathcal{E}_0\left(\frac{2}{3}C_0\right) = \frac{2}{3}\frac{K\varepsilon_0 A\mathcal{E}_0}{d}$$

Charge on plate 3 on the surface facing 4

$$= \mathcal{E}_0 C_0 = \frac{K\varepsilon_0 A\mathcal{E}_0}{d}$$

Charge on plate 3 on the surface facing 4

$$= \mathcal{E}_0 C_0 = \frac{K\varepsilon_0 A\mathcal{E}_0}{d}$$

Charge on plate 3 on the surface facing 2 = [potential difference across $(3-2)]C_0$

$$= \mathcal{E}_0 \frac{C_0}{C_0 + 2C_0} C_0 = K\varepsilon_0 \frac{A\mathcal{E}_0}{3d}$$

$$\therefore \quad Q_3 = \frac{K\varepsilon_0 A\mathcal{E}_0}{d}\left[1 + \frac{1}{3}\right] = \frac{4}{3}K\varepsilon_0\frac{A}{d}\mathcal{E}_0$$

Example 45

What amount of heat will be generated in the circuit shown Fig. 3E.45 after the switch S is shifted from position 1 to position 2?

Solution: Initially, when the switch is closed on position 1, the capacitor C is connected in series with batteries $\mathcal{E}_1$ and $\mathcal{E}_2$.

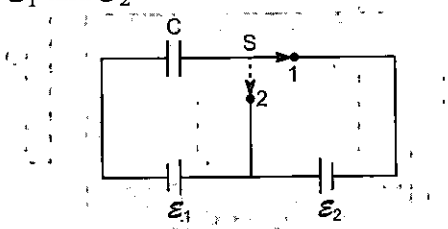


Fig. 3E.45

From KVL we have

$$\frac{Q_i}{C} - \mathcal{E}_2 + \mathcal{E}_1 = 0$$

or $$Q_i = (\mathcal{E}_2 - \mathcal{E}_1)C \qquad \ldots(1)$$

depending upon the sign of $(\mathcal{E}_2 - \mathcal{E}_1)$, charge Q_i on the left plate may be positive (if $\mathcal{E}_2 > \mathcal{E}_1$), or negative (if $\mathcal{E}_2 < \mathcal{E}_1$); charge on right plate would be equal and opposite.

When the switch is moved to position 2, the left plate earlier having charge $+Q_i$, will have charge

$$Q_f = -\mathcal{E}_1 C \qquad \ldots(2)$$

The net charge flow through the circuit is

$$\Delta Q = Q_f - Q_i$$

$$= [-\mathcal{E}_1 - (\mathcal{E}_2 - \mathcal{E}_1)]C = -\mathcal{E}_2 C$$

We can say that a net positive charge equal to $\mathcal{E}_2 C$ is pulled by the battery of emf $\mathcal{E}_1$ from the left plate of the capacitor, which flows through battery $\mathcal{E}_1$ and is transferred to the right plate of the capacitor. Work done by battery $\mathcal{E}_1$ in the process of charge transfer is

$$\Delta W = \mathcal{E}_1\mathcal{E}_2 C \qquad \ldots(3)$$

A part of this work changes the energy of the capacitor:

$$\Delta W_C = \frac{Q_f^2}{2C} - \frac{Q_i^2}{2C}$$

$$= \frac{1}{2}\mathcal{E}_1^2 C - \frac{1}{2}(\mathcal{E}_2 - \mathcal{E}_1)^2 C$$

$$= \frac{1}{2}(2\mathcal{E}_1\mathcal{E}_2 - \mathcal{E}_2^2)C$$

and the remaining part is lost as Joule heating :

$$H = \Delta W - \Delta W_C = \frac{1}{2}\mathcal{E}_2^2 C$$

Example 46

A capacitor of capacitance C is initially charged to a potential difference of V volt. Now it is connected to a battery of 2V with opposite polarity. The ratio of heat generated to the final energy stored in the capacitor is:

(a) 1.75 (b) 2.25
(c) 2.5 (d) 1/2

Solution:

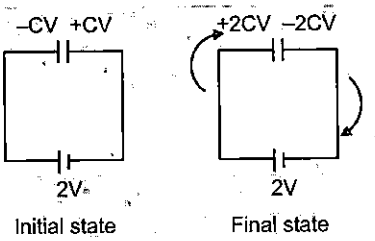


Fig. 3E.46

Concept: *In final steady state p.d. across capacitor becomes 2V and it has charges as +2CV and −2CV. The battery has transferred 3CV charge from right positive plate to left negative plate.*

W/D by battery $= (3CV)(2V) = 6CV^2$

$$\Delta U = U_f - U_i$$

$$= \frac{1}{2}C(2V)^2 - \frac{1}{2}CV^2$$

$$= \frac{3}{2}CV^2$$

Heat developed = W/D by battery $-\Delta U$

$$= 6CV^2 - \frac{3}{2}CV^2$$

$$= \frac{9}{2}CV^2$$

$$\text{Received ratio} = \frac{\frac{9}{2}CV^2}{2CV^2} = \frac{9}{4} = 2.25$$

Example 47

Two identical plates of a capacitor are given charges as shown in figure.Now this capacitor is connected to a battery of emf $\mathcal{E}$, with positive plate of capacitor connected to positive terminal of battery.

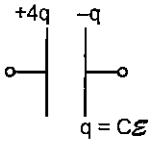


Fig. 3E.47 (a)

(a) What is final charge on capacitor plates ?

(i) $2C\mathcal{E}$ *(ii)* $3C\mathcal{E}$

(iii) $C\mathcal{E}$ *(iv)* $\frac{3}{2}C\mathcal{E}$

(b) In the given connection

(i) Work is being done on battery

(ii) Battery transferes charge from negative plate of capacitor to positive plate of capacitor

(iii) Charge is being transferred from positive plate of capacitor to negative plate of capacitor through battery

(iv) No heat is developed in the circuit.

(c) Work done by battery is

(i) $+2C\mathcal{E}^2$ *(ii)* $-2C\mathcal{E}^2$

(iii) $+C\mathcal{E}^2$ *(iv)* $-C\mathcal{E}^2$

(d) Heat developed is

(i) $C\mathcal{E}^2$ *(ii)* $2C\mathcal{E}^2$

(iii) $\frac{C\mathcal{E}^2}{2}$ *(iv)* $3C\mathcal{E}^2$

Solution:

Concept: *In final steady state charge on inner facing plates of capacitor is $C\mathcal{E}$. Charge is transferred from positive plate of capacitor to negative plate through battery such that final potential difference between plates is $\mathcal{E}$.*

In initial state charge distribution is

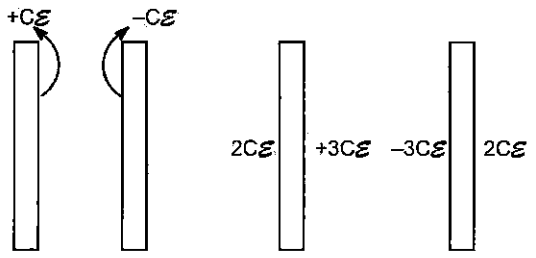


Fig. 3E.47 (b)

p.d. between plates initially 3V

Initial energy in capacitor $= \frac{1}{2}C(3V)^2$

$= \frac{9}{2}CV^2$

When connected with a battery final p.d.is $\mathcal{E}$ *i.e.,*$2C\mathcal{E}$ charge is transferred through battery to the other plate.

W/D on battery $= (-2C\mathcal{E})\mathcal{E} = -2C\mathcal{E}^2$

Final energy in capacitor $= \frac{1}{2}C\mathcal{E}^2$

From work energy theorem we get

W/D by battery $= \Delta H + \Delta U$

$\Delta H = W/D$ by battery $-\Delta U$

$= -2C\mathcal{E}^2 - \left[\frac{1}{2}C\mathcal{E}^2 - \frac{9}{2}C\mathcal{E}^2\right]$

$\Delta H = 2C\mathcal{E}^2$

Example 48

The plates of a parallel plate capacitor are given charges +4Q and –2Q. The capacitor is then connected across are uncharged capacitor of same capacitance as first one. The final potential difference between the plates of the first capacitor is:

(a) $\frac{Q}{C}$ *(b)* $\frac{Q}{2C}$

(c) $\frac{3Q}{2C}$ *(d)* $\frac{5Q}{2C}$

Solution:

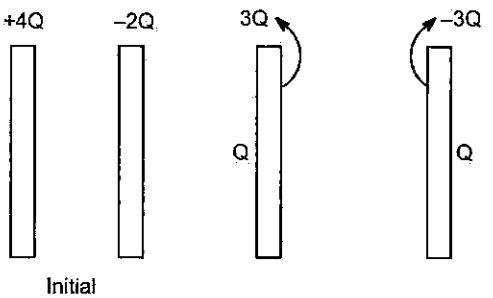


Fig. 3E.48 (a)

Steady state before connection are with second capacitor.

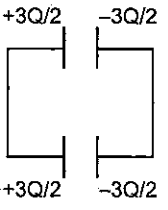


Fig. 3E.48 (b)

After connection with second capacitor the charges on inner facing surface are shared. Finally two capacitors will have same p.d. across them with magnitude of charge on each of the facing surfaces as $\frac{3Q}{2}$ and p.d. $\frac{3Q}{2C}$.

Example 49

What is equivalent capacitance of the circuit shown in the figure ?

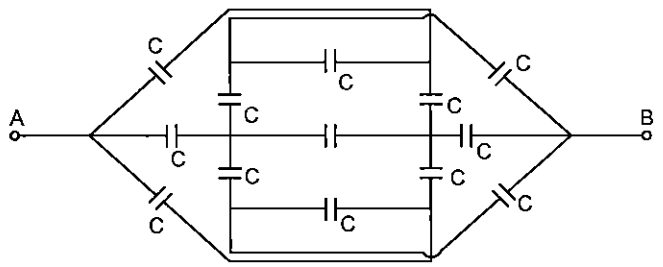


Fig. 3E.49 (a)

Solution:

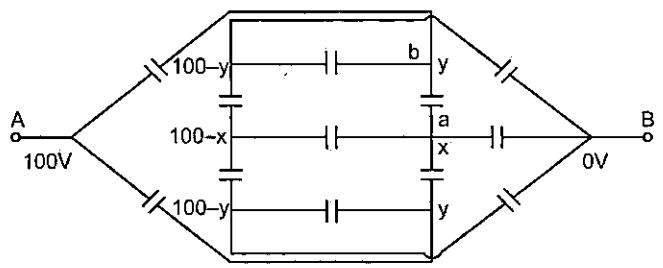


Fig. 3E.49 (b)

Writing nodal equation for above fig. we get

At node *b*:

$$(y-x)C+(2y-100)C+(y-100)C=0$$

At node *a*:

$$2(x-y)C+x+y2xC-100C=0$$

On solving for x and y, we get

$$x=\frac{900}{9};\ y=\frac{550}{9}$$

Also $$xC+2(100-y)C=100C_{eq}$$

or $$C_{eq}=\frac{11C}{9}$$

Example 50

The two plates of a parallel plates capacitor are given charges as shown in the figure. Separation between plates is d and area of plates is A.What is heat produced in the circuit after switch is closed.

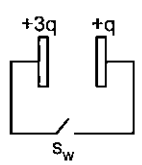


Fig. 3E.50 (a)

Solution: Initial energy of capacitor $=\frac{1}{2}\frac{q^2}{C}$

$$=\frac{1}{2}\frac{q^2d}{(\varepsilon_0 A)}$$

When switch is closed both the plates attain common potential, and energy of capacitor becomes zero

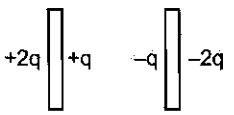


Fig. 3E.50 (b)

W/D by battery $=\Delta H+\Delta U$

as W/D by battery $=0$

$$\Delta H=-\Delta U=U_i-U_f$$

$$=\frac{1}{2}\frac{q^2d}{\varepsilon_0 A}-0=\frac{1}{2}\frac{q^2d}{\varepsilon_0 A}$$

Example 51

In the figure shown all the batteries have emf 20V, circuit is in steady state.

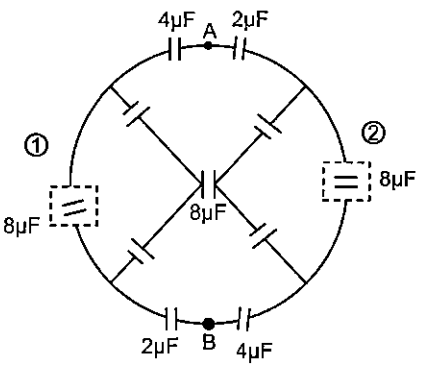


Fig. 3E.51 (a)

(a) Charge on capacitors 8μF labelled (1) and (2) is:

(a) 40μC (b) 20μC

(c) zero (d) 80μC

(b) Potential difference between A and B is:

(a) 10V (b) 20V

(c) 30V (d) 15V

(c) Charge on central 8μF capacitor is:

(a) 40μC (b) 80μC

(c) 20μC (d) 160μC

(d) Charge on 2μF and 4μF capacitors are:

(a) 40μC (b) 80μC

(c) 20μC (d) 160μC

Solution: Potential difference across 8μF in side branches is zero.

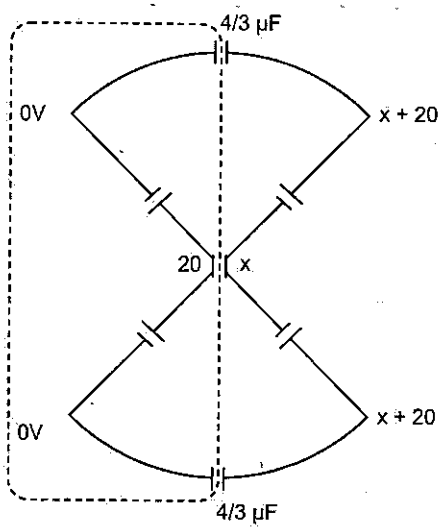


Fig. 3E.51 (b)

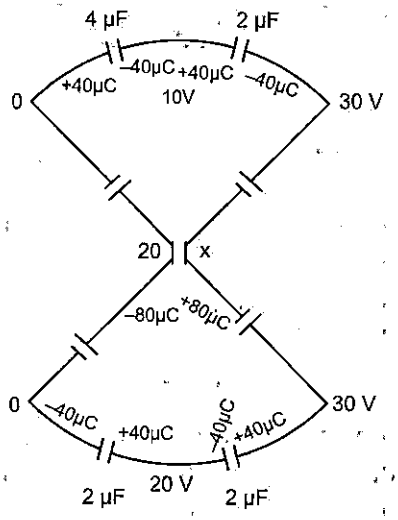


Fig. 3E.51 (c)

or $(20-x)8+(0-(x+20)\frac{4}{3}\times 2=0$

or $x=10\text{V}$

Example 52

The charge on the plates of a capacitor are shown in figure. Capacitor is now connected to an identical uncharged capacitor. What is final p.d. across capacitors ?

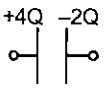


Fig. 3E.52 (a)

Solution: Initial charge on inner facing plates is $3Q$, p.d. between plates of capacitor is $\frac{3Q}{C}$. When connected with uncharged identical capacitor. Charge is shared till both have same potential difference across them.

Thus $3Q = 2CV$

or $V = \frac{3Q}{2C}$

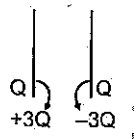


Fig. 3E.52 (b)

Example 53

Five identical capacitor plates, each of area A are arranged such that adjacent plates are a distance d apart, the plates are connected to a source of emf $\mathcal{E}$ as shown in figure.

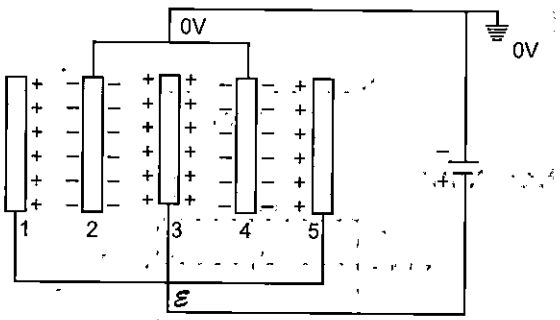


Fig. 3E.53

(a) *The charge on plate 1 is:*

(A) $\frac{\varepsilon_0 A}{d}\mathcal{E}$ (B) $\frac{\varepsilon_0 A}{2d}\mathcal{E}$

(C) $\frac{-\varepsilon_0 A}{d}\mathcal{E}$ (D) $\frac{-\varepsilon_0 A}{2d}\mathcal{E}$

(b) *The charge on plate 4 is:*

(A) $\frac{-2\varepsilon_0 A}{d}\mathcal{E}$ (B) $\frac{e_0 A}{d}\mathcal{E}$

(C) $\frac{-\varepsilon_0 A}{d}\mathcal{E}$ (D) $\frac{+2\varepsilon_0 A}{d}\mathcal{E}$

Solution: Here we have four capacitors in parallel arrangement. If we consider negative terminal of battery at zero potential, plates 2 and 4 have zero potential and 1, 3, 4 have p.d. $\mathcal{E}$. Thus charge on each capacitors is CV. But plate 2 is common to two capacitors, thus it has net charge $-2C\mathcal{E}$, similarly plate 4 has net charge $-2C\mathcal{E}$ and plate 3 has charge $+2C\mathcal{E}$ plate 1, $+C\mathcal{E}$ and 5, $C\mathcal{E}$.

Example 54

Fig. 3E.54(a) shows four plates each of plate area A and separation between plates is d. Two switches Sw_1 and Sw_2 can activate two batteries in the circuit.

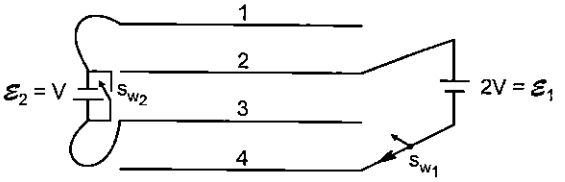


Fig. 3E.54 (a)

(a) *Switch Sw_1 and Sw_2 are closed. What is charge on plate 2 ?*

(A) $\frac{3}{2}\frac{\varepsilon_0 AV}{D}$ (B) $\frac{2}{3}\frac{\varepsilon_0 AV}{d}$

(C) $\frac{1}{3}\frac{\varepsilon_0 AV}{d}$ (D) $\frac{4}{3}\frac{\varepsilon_0 AV}{d}$

(b) *What is charged passing through battery 1 ?*

(A) $\frac{3}{2}\frac{\varepsilon_0 AV}{D}$ (B) $\frac{2}{3}\frac{\varepsilon_0 AV}{d}$

(C) $\frac{1}{3}\frac{\varepsilon_0 AV}{d}$ (D) $\frac{4}{3}\frac{\varepsilon_0 AV}{d}$

(c) *Now switch Sw_2 is opened, what is charge on plate 4?*

(A) $-5/3\frac{\varepsilon_0 AV}{d}$ (b) $+5/3\frac{\varepsilon_0 AV}{d}$

(C) $-\frac{1}{3}\frac{\varepsilon_0 AV}{d}$ (D) $+\frac{1}{3}\frac{\varepsilon_0 AV}{d}$

(d) *What is charge passing through battery negative to positive terminal ?*

(A) $\frac{-\varepsilon_0 AV}{d}$ (b) $\frac{-2\varepsilon_0 AV}{d}$

(C) $\frac{+2\varepsilon_0 AV}{d}$ (D) $\frac{2\varepsilon_0 AV}{d}$

Solution: When both the switches are closed, circuit is:

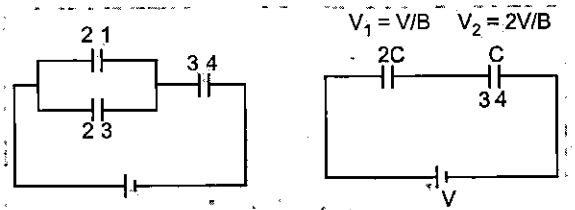


Fig. 3E.54 (b)

Charge on plate 2

$$= \frac{CV}{3} + \frac{CV}{3} = \frac{2CV}{3} = \frac{2\varepsilon_0 AV}{3d}$$

Charge on plate 3

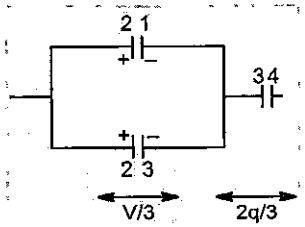


Fig. 3E.54 (c)

$$= \frac{-CV}{3} + \frac{2CV}{3}$$

$$= \frac{CV}{3} = \frac{\varepsilon_0 AV}{3d}$$

Charge on plate 1

$$= \frac{-CV}{3} = \frac{\varepsilon_0 AV}{3d}$$

Where switch Sw_2 is opened; circuit is

$$[(x - V/-2V)]C + (x - 2V)C + xc = 0$$

or $$x = \frac{5}{3}\text{V}$$

Charge on plate 4 is

$$-\frac{5}{3}CV = -\frac{5\varepsilon_0 AV}{3d}$$

Initial charge on plate

$$1 = \frac{-CV}{3}$$

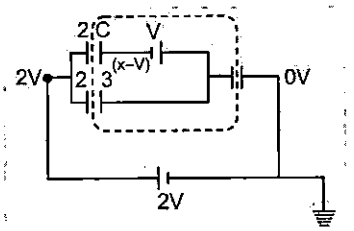


Fig. 3E.54 (d)

Charge on plate after opening switch

$$Sw_2 = \frac{-4}{3}CV$$

Charge passing from negative to positive terminal of battery is

$$= \frac{\varepsilon_0 AV}{d}$$

Example 55

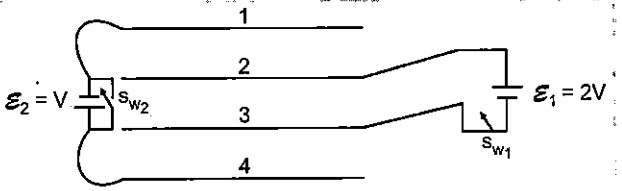


Fig. 3E.55 (a)

Initially switches Sw_1 and Sw_2 both are closed:

(a) What is charge on plate 3 ?

(b) What is charge on plate 1 ?

Now switch Sw_2 is opened.

(c) What is charge on plate 1 ?

(d) What is charge flown through battery 1 ?

Solution: Initially the circuit is

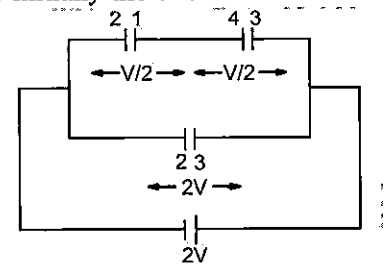


Fig. 3E.55 (b)

Charge on plate $3 = 2CV + CV/2 = +\dfrac{5CV}{2}$

$$= \frac{5}{2}\frac{\varepsilon_0 AV}{d}$$

Charge on plate $4 = -\dfrac{CV}{2} = -\dfrac{\varepsilon_0 AV}{2d}$

Charge on plate $1 = \dfrac{+CV}{2} = +\dfrac{\varepsilon_0 AV}{2d}$

Charge on plate $2 = -2CV - \dfrac{CV}{2} = -\dfrac{5CV}{2}$

$$= -5/2\frac{\varepsilon_0 AV}{d}$$

Where switch Sw_2 is opened circuit becomes

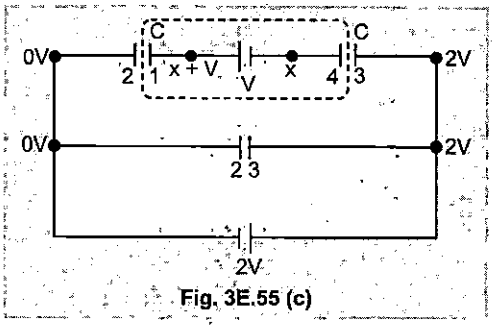


Fig. 3E.55 (c)

$$(x - 2V)C + (x + V)C = 0$$

$$x = V/2$$

Charge on plate $4 = -3CV/2 = -3/2\dfrac{\varepsilon_0 AV}{d}$

Charge on plate $1 = +3CV/2 = +3/2\dfrac{\varepsilon_0 AV}{d}$

Charge on plate $2 = -2CV - 3CV/2$

$$= -7CV/2 = -\frac{7}{2}\frac{\varepsilon_0 AV}{d}$$

Charge on plate $3 = +2CV + 3CV/2$

$$= +\frac{7}{2}\frac{\varepsilon_0 AV}{d}$$

Charge flowing through battery 1

$$= 7/2\frac{\varepsilon_0 AV}{d} - 5/2\frac{\varepsilon_0 AV}{d}$$

$$= \frac{\varepsilon_0 AV}{d}$$

Example 56

Charge flowing through switch when it is closed is:

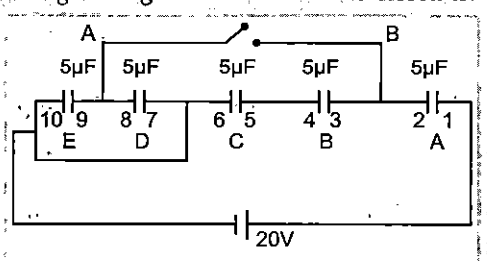


Fig. 3E.56 (a)

Solution:

The equivalent circuit before closing switch, is

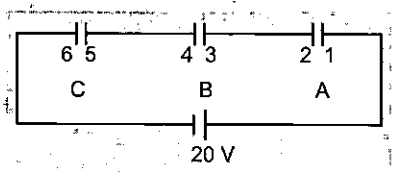


Fig. 3E.56 (b)

Magnitude of charge on each plate is $\dfrac{100}{3}\mu C$ when switch S_w is closed equivalent circuit, is

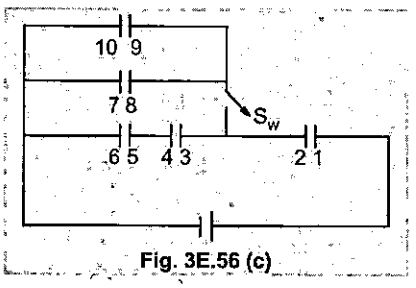


Fig. 3E.56 (c)

When switch S_w is closed plates 8 and 9 are charged which had initially zero charge.

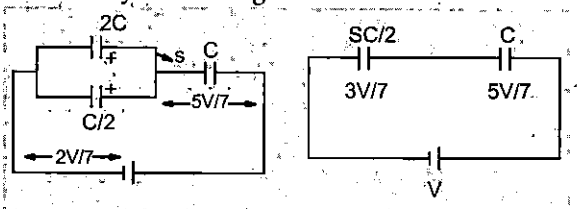


Fig. 3E.56

Charge passing through switch is $\dfrac{4CV}{7}$ towards plates 8 and 9.

Thus $q = \dfrac{4 \times 5 \times 10}{7}\mu C = \dfrac{200}{7}\mu C$

Example 57

Three charged capacitors C_1, C_2, and C_3 as shown in figure

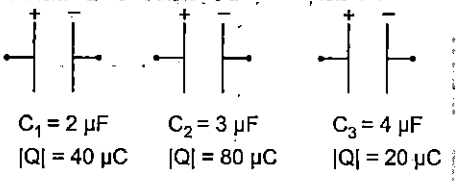


Fig. 3E.57 (a)

are connected in circuit as shown in figure

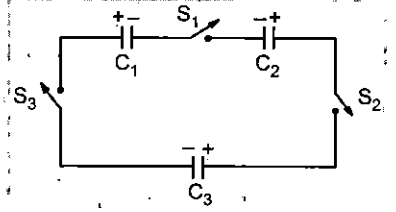


Fig. 3E.57 (b)

What are magnitude charges on all the capacitors when switch S_1, S_2 and S_3 are closed.

Solution: Writing nodal equation for plates 1 and 5 we get

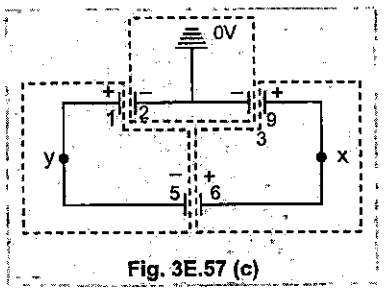


Fig. 3E.57 (c)

$$2y + 4(y - x) = 20 \quad ...(1)$$

For plates 2, 3

$$-2y - 3x = -120 \quad ...(2)$$

on solving we get $x = \frac{340}{13}$ V

and $y = \frac{270}{13}$

Thus $|Q_1| = 2 \times y = \frac{540}{13}\mu C$

$$|Q_2| = 3x = \frac{1020}{3}\mu C$$

Example 58

These charged capacitors are connected as shown in figure to a battery of 20V and an uncharged capacitor of 2μ F.

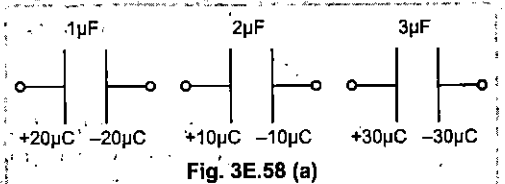


Fig. 3E.58 (a)

What is charge on all the capacitor in steady state ?

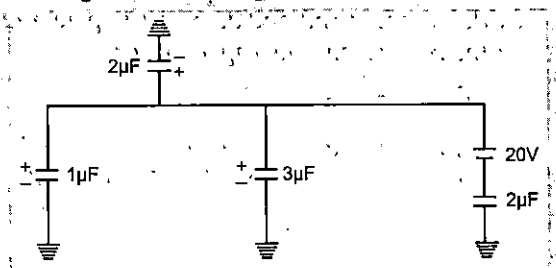


Fig. 3E.58 (b)

Solution: The circuit can be redrawn as

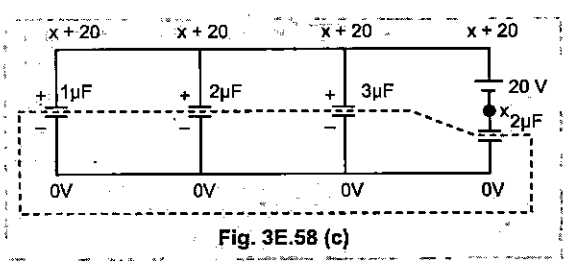


Fig. 3E.58 (c)

Conserving charge on lower plates we get

$$-[(x+20)1 + (x+20)2 + (x+20)3 + 2x] = -60$$

or $x = -\frac{15}{2}$ V

$|Q_{1\mu F}| = 12.5\mu C$

$|Q_{2\mu F}| = 25\mu C$

$|Q_{3\mu F}| = 37.5\mu C$

$|Q_{2\mu F}| = 15\mu C$

[Initially uncharged]

AN INTRODUCTION TO TRANSIENTS IN CIRCUITS

When flash unit of camera is switched on, there is a typical delay before the flash is ready to operate. The delay is caused by the time it takes the battery to fully charge a capacitor, needed to accumulate sufficient electrical potential energy for the lamp. As an example of such a transient we consider a single loop circuit consisting of a battery, a resistor and a capacitor.

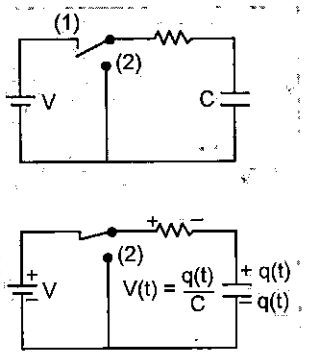


Fig. 3.37

A charging capacitor :

Initially the capacitor shown in Fig. 3.37 is uncharged. At time $t = 0$, the switch is closed to position 1. At time t, let the capacitor have the charges $\pm q$ on its plates.

$$V = \frac{q}{C}$$

We apply the KVL to the single loop. Starting at the lower left corner and traversing the circuit anticlockwise.

$$\frac{q}{C} + IR - V = 0$$

The current I is related to q.

$$I = \frac{dq}{dt}$$

$$\frac{q}{C} + \frac{dq}{dt} - V = 0$$

$$R\frac{dq}{dt} = V - \frac{q}{C} = \frac{CV - q}{q}$$

$$\int_0^q \frac{dq}{CV - q} = \int_0^t \frac{1}{RC} dt$$

$$\left| -\ln(CV - q) \right|_0^q = \frac{t}{RC}$$

$$-\ln\frac{CV - q}{CV} = \frac{t}{RC}$$

$$\frac{CV - q}{CV} = e^{-t/RC}$$

$$q = CV(1 - e^{-t/RC})$$

This is the expression for instantaneous charge on the capacitor.

Immediately after closing the switch, *i.e.,* at $t \to 0$, we have

$$q = CV(1-1) = 0$$

Where V/R is initial current. Apply KVL to the loop at the initial instant, when current I_0 is in the circuit.

$$-V + I_0 R + 0 = 0$$

$$I_0 = \frac{V}{R}$$

Alternatively we can check the limit $t \to 0$ of the expression for current.

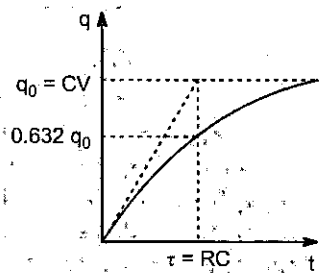


Fig. 3.38

Immediately after closing the switch,

$$I = \lim_{t \to 0} \frac{V}{R} e^{-t/RC} = \frac{V}{R}$$

as $e^0 = 1$

Long time after closing the switch,

$$I = \lim_{t \to \infty} \frac{V}{R} e^{-t/RC} = 0$$

as $\frac{1}{e^\infty} = 0$

Thus we are led to a conclusion that the DC current (the steady, time independent current) through a capacitor is zero. **After being fully charged a capacitor in a DC circuits acts like an open switch in the branch in which it is placed.**

Since $e^{-t/RC} = e^0 = 1$

Long time after closing the switch, *i.e.,* at $t \to \infty$, we have

$$q = CV(1-0)$$

since $$e^{-t/RC} = \frac{1}{e^{t/RC}} = \frac{1}{e^\infty} = 0$$

- The quantity CR is called capacitive time constant (τ) of the circuit. It has dimensions of time charge on the capacitor plates after one time constant.

$$q = q_0(1 - e^{-\tau/\tau})$$

$$= q_0\left(1 - \frac{1}{e}\right) = 0.632\, q_0$$

- On differentiating the expression for charge q we obtain

$$I = \frac{dq}{dt} = \frac{d}{dt}[q_0(1 - e^{-t/CR})]$$

$$= \frac{q_0}{CR} e^{-t/CR} = \frac{CV}{CR} e^{-t/CR}$$

$$= \frac{V}{R} e^{-t/CR} = I_0 e^{-t/CR}$$

The initial slope of the graph of equation, *i.e.,* derivative of I with **respect** to t, at $t = 0$, has the value $-I_0/\tau$.

Hence in Fig. 3.39, the line denoting initial slope of the curve intersects the time axis at the time τ. If the current decreased at this constant rate, the current would be zero at the time τ, but the current decreases exponentially, it is e^{-1} times its initial value after one time constant.

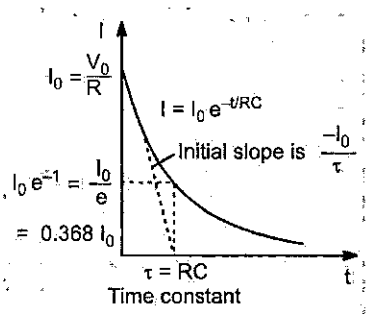


Fig. 3.90

Concept: *In time dt a charge dq (= I dt) moves through any cross-section of the circuit and is deposited on the positive plate of the capacitor. The work (= $\mathcal{E}\,dq$;) done by the source of emf must equal the internal energy (= I^2Rdt) produced in the resistor during time dt, plus the increase dU in the amount of energy $U(= q^2/2C)$ that is stored in the capacitor. Conservation of energy gives:*

$$\mathcal{E}\,dq = I^2Rdt + d\left(\frac{q^2}{2C}\right)$$

or

$$\mathcal{E}\,dq = I^2Rdt + \frac{q}{C}dq.$$

Dividing by dt yields

$$\mathcal{E}\frac{dq}{dt} = I^2R + \frac{q}{C}\frac{dq}{dt}.$$

Since q is the charge on the upper plate, positive I means positive dq/dt. With I = dq/dt, this equation becomes

$$\mathcal{E} = IR + \frac{q}{C}$$

Equation also follows from the loop rule, as it must since the loop rule was derived from the conservation of energy principle. Starting from point x and going around the circuit clockwise, we experience an increase in potential in going through the source of emf and decreases in potential in going through the resistor and the capacitor, or

$$\mathcal{E} - IR - \frac{q}{C} = 0$$

Fig. 3.40 shows the resulting plots of ΔV_R and ΔV_C. Note the following. (1) At $t = 0, \Delta V_R = \mathcal{E}$ (the full potential difference appears across R), and $\Delta V_C = 0$ (the capacitor is not charged). (2) As $t \to \infty$, $\Delta V_C \to \mathcal{E}$ (the capacitor becomes fully charged), and $\Delta V_R \to 0$ (the current stops flowing). (3) At all times, $\Delta V_R + \Delta V_C = \mathcal{E}$.

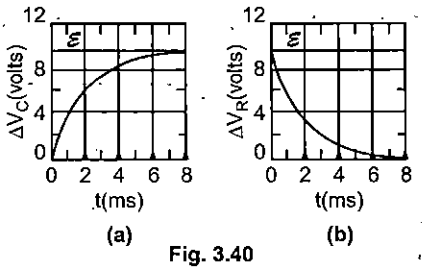


Fig. 3.40

Now consider the way energy is exchanged during the charging of a capacitor. The energy transferred to the charge carriers of the circuit by the emf of the battery during the entire charging process (as $t \to \infty$) is $\mathcal{E}Q_\infty = \mathcal{E}(\mathcal{E}C) = \mathcal{E}^2 C$ because $\mathcal{E}$ is the energy per unit charge transferred by the emf of the battery and a charge $Q_\infty = \mathcal{E}C$ passes through the battery during the entire charging process. Recall that $\frac{1}{2}Q^2/C$ is the energy stored in a capacitor with charge Q. The energy stored in the capacitor after the charging is completed is $\frac{1}{2}Q_\infty^2/C = \frac{1}{2}(\mathcal{E}C)^2/C = \frac{1}{2}\mathcal{E}^2 C$.

Thus half of the energy expended by the emf of the battery is stored in the capacitor. What happened to the other half of this energy ? It is dissipated as heat in the resistor. The rate at which energy is dissipated in the resistor is $P = -(dU/dt) = I^2R$, where U is the electric potential energy of the carriers. The energy $-\Delta U$ dissipated in the resistor during the charging is

$$-\Delta U = -\int dU = \int_0^\infty (I^2 R)\,dt$$

$$= \int_0^\infty \left(\frac{\mathcal{E}}{R}e^{-t/RC}\right)^2 R\,dt = \left(\frac{\mathcal{E}}{R}\right)^2\left(\frac{RC}{2}\right)R\int_0^\infty e^{-x}\,dx$$

where $x = (2/RC)t$ and $dx = (2/RC)\,dt$. Since

$$\int_0^\infty e^{-x}dx = 1$$

we have

$$-\Delta U = \frac{1}{2}\mathcal{E}^2 C$$

Half of the energy expended by the battery's emf is stored in the capacitor and half is dissipated as heat in the resistor.

A DISCHARGING CAPACITOR

When the capacitor in the circuit shown in Fig. 3.41, is fully charged (steady state, current in the circuit has vanished), the capacitor acts as open circuit.

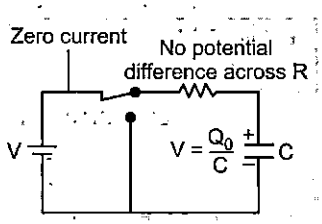


Fig. 3.41

Since current is zero, potential difference across resistor is zero.

Now the switch is opened, the capacitor gets isolated, the charges on it are bound, there is no electrical path for the charges to move from one plate to get to the other plate. Now the switch is moved to position 2, we have a closed loop, the capacitor starts discharging. We apply KVL to closed loop, traversing it clockwise.

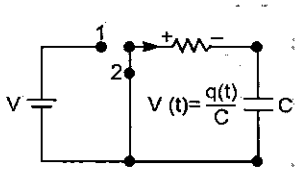


Fig. 3.42

$$IR + \frac{q}{C} = 0$$

$$R\frac{dq}{dt} + \frac{q}{C} = 0$$

$$\frac{dq}{dt} = -\frac{1q}{RC}$$

On rearranging the equation, we have

$$\frac{dq}{q} = -\frac{1}{RC}dt$$

$$\int_{Q_0}^{q}\frac{dq}{q} = -\frac{1}{RC}\int_0^t dt$$

$$\ln q - \ln Q_0 = -\frac{t}{RC}$$

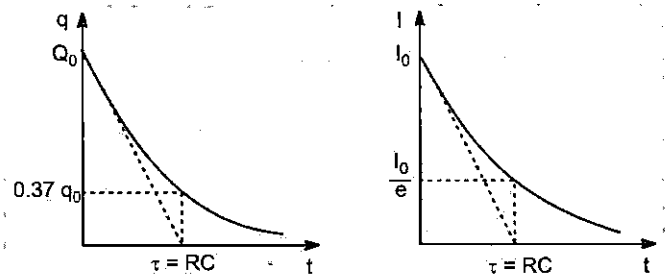


Fig. 3.43 (a)

Taking antilogarithms, we get

$$q(t) = Q_0 e^{-t/RC}$$

where Q_0 is initial charge on the capacitor.

$$Q_0 = CV$$

$$q(t) = CVe^{-t/RC}$$

The charge on the capacitor decreases with time.

The current in the circuit is

$$I = \frac{dq}{dt} = CV_0 e^{-t/RC}\left(-\frac{1}{RC}\right)$$

$$I(t) = -\frac{V_0}{R}e^{-t/RC}$$

The minus sign shows that the current in the circuit actually is in the direction opposite to that shown in Fig. 3.42. After a time $t = 5\tau$, the current in the circuit is only e^{-5} of its initial value. Since $e^{-5} = 0.006738$, the current is negligible.

If $\int_0^x \frac{dx}{a-bx} = \int_0^t c\,dt$

then $x = \frac{a}{b}(1 - e^{-bct})$

And if $\int_{x_0}^x \frac{dx}{a-bx} = \int_0^t c\,dt$

then $x = \frac{a}{b} - \left(\frac{a}{b} - x_0\right)e^{-bct}$

Here a, b and c are constants.

Similarly, if x increases from x_2 to x_1 exponentially, then $x-t$ equation is,

$$x = x_2 + (x_1 - x_2)(1 - e^{-kt});$$

$$q = C\varepsilon\left(1 - e^{-\frac{t}{RC}}\right) + q_0 e^{-t/RC}$$

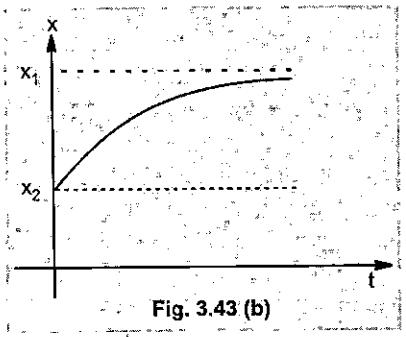


Fig. 3.43 (b)

Another approach to analyse the above condition. The charge further supplied by battery $\Delta q = (C\varepsilon - q_0)$

Hence, transition charge $= \Delta q = (C\varepsilon - q_0)$

Hence, charge at any time

$$q = q_0 + (C\varepsilon - q_0)(1 - e^{-t/RC})$$

$$q = C\varepsilon(1 - e^{-t/RC})q_0 + e^{-t/RC}$$

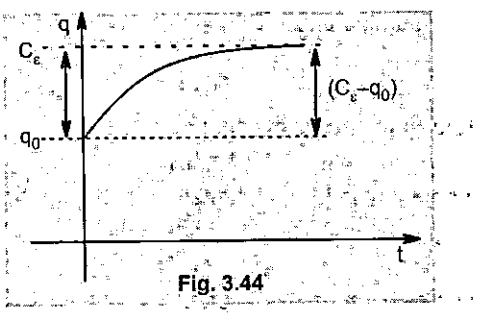


Fig. 3.44

The expression is the same as the one we have calculated in the previous section.

Sometimes a physical quantity x decreases from x_1 to x_2, exponentially, then the x-t equation is like,

$$x = x_2 + (x_1 - x_2)e^{-kt}$$

Here, k is a constant.

A capacitor offers zero resistance in a circuit when it is uncharged, *i.e.*, it can be assumed as short circuited and it offers infinite resistance when it is fully charged.

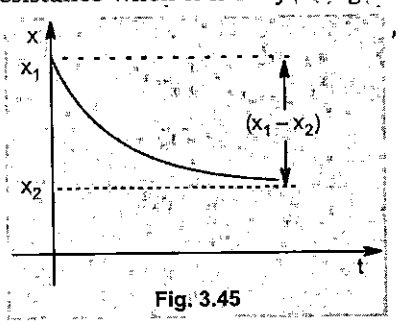


Fig. 3.45

Equivalent Time Constant

To find the equivalent time constant of a circuit, following steps are followed:

(a) Short – circuit the battery.

(b) Find net resistance across the capacitor (say it R_{net})

(c) $\tau_C = (R_{net})C$

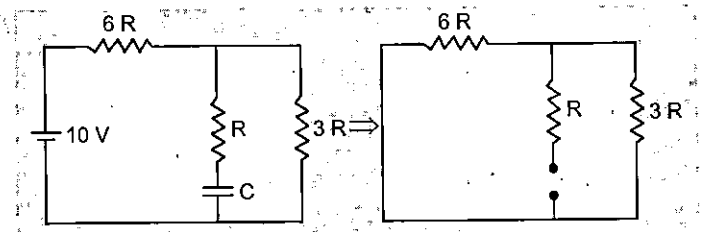


Fig. 3.46

For example, in the circuit shown in Fig. 3.46, after short circuiting the battery $3R$ and $6R$ are in parallel, so their combined resistance is $\frac{(6R)(3R)}{6R + 3R} = 2R$

Now this $2R$ is in series with the remaining R.

Hence, $R_{net} = 2R + R = 3R \Rightarrow \tau_C = (R_{net})C = 3RC$

Alternate method of finding current in the circuit and the charge on the capacitor at any time t.

In a complicated C-R circuit it is easy to find current in the circuit and charge stored in the capacitor at time $t = 0$ and $t = 8$. But to find the current and the charge at time t is $q = q_0(1 - e^{-t/RC})$.

By differentiating it w.r.t. time we can find the current through the capacitor at time t. Then by using Kirchhoff's laws we can calculate currents in other parts of the circuit also.

Example 59

How long does it take for current to decay to half its value, while charging? We can consider this time as half-life of the current in a charging series RC circuit.

Solution: The current in a charging series RC circuit is given by,

$$I = I_0 e^{-t/RC}$$

When the current is halved,

$$\frac{I_0}{2} = I_0 e^{-t/RC} \Rightarrow \frac{1}{2} = e^{-t/RC}$$

Taking the natural logarithms on both sides,

$$\ln 1 - \ln 2 = -\frac{t}{RC}$$

On solving for t, we have

$$t = RC \ln 2 = \tau \ln 2 = 0.693\tau$$

Example 60

At what time the charge on the capacitor in a discharging circuit equal to one half its initial value ?

Solution: The charge at time t in a discharging circuit is given by

$$q = Q_0 e^{-t/RC} = Q_0 e^{-t/\tau}$$

When the charge is halved,

$$\frac{Q_0}{2} = Q_0 e^{-t/\tau}$$

$$\frac{1}{2} = e^{-t/\tau}$$

Taking the natural logarithms on both sides, we get

$$-0.693 = -t/\tau$$

$$t = 0.693\tau$$

Example 61

In a discharging RC circuit, at what time is the electrical potential energy equal to one half of its initial value.?

Solution: The electric potential energy stored in the capacitor is

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2C}Q_0^2 e^{-2t/\tau}$$

Since $U_0 = \frac{Q_0^2}{2C}$ is initial potential energy, we have

$$U = U_0 e^{-2t/\tau}$$

When the potential energy is half its initial value,

$$\frac{1}{2} = e^{-2t/\tau}$$

Taking natural logarithms of both sides, we get

$$-0.693 = -2t/\tau$$

$$t = 0.34\tau$$

The potential energy decreases to half its value in half the time it takes the charge to decrease to half its value.

Example 62

(a) Calculate the power absorbed by a charging capacitor in the series RC circuit as a function of time. (b) Integrate P(t) from $t = 0$ to $t = \infty$ to show that the energy stored in the capacitor is $\left(\frac{1}{2}\right)CV_0^2$, when it is fully charged.

Solution: Rate at which energy is dissipated in the resistor is

$$P(t) = I^2 R$$

$$= \left(\frac{V_0}{R}e^{-t/RC}\right)^2 R$$

$$= \frac{V^2}{R}e^{-2t/RC}$$

We find total joule heat by integrating from $t = 0$ to $t = \infty$

$$P = \int_0^\infty \frac{V_0^2}{R}e^{-2t/RC}dt$$

$$= \frac{V_0^2 C}{2}\left(e^{-2t/RC}\right)_0^\infty$$

$$= \frac{V_0^2 C}{2}[-0+1] = \frac{1}{2}CV_0^2$$

- The result is independent of R. Therefore, when a capacitor is charged by a battery with a constant emf, half the energy provided by the battery is stored in the capacitor and half goes into thermal energy, independent of resistance; the thermal energy includes the energy that goes into the internal resistance of the battery.

While charging, the charge flowing through battery $= CV_0$

Work done by battery $= (CV_0)V_0 = CV_0^2$

Power dissipated in the resistor $= \frac{1}{2}CV_0^2$

Energy stored in the capacitor $= \frac{1}{2}CV_0^2$

Example 63

A leaky parallel plate capacitor is filled completely with a material having dielectric constant $K = 5$ and electrical conductivity $\sigma = 7.4\times10^{-12}\Omega^{-1}m^{-1}$. If the charge on the plate at the instant $t = 0$ is $q = 8.85\mu C$, then calculate the leakage current at $t = 12s$.

Solution: Fig. 3E.63 shows a model of a real capacitor, it consists of a resistance R in parallel with an ideal capacitor. The electrical path through R provides a way to explain the observation that all capacitors gradually discharge. The opposite charges on each conducting plate of capacitor are gradually neutralized as electrons migrate through the material separating the two conductors of the capacitor. The capacitor discharges through the resistor, hence

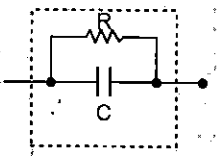


Fig. 3E.63

$$q = q_0 e^{-t/CR}$$

$$I = \frac{dq}{dt} = -\frac{q_0}{CR}e^{-t/CR}$$

Time constant $CR = \left(\frac{\varepsilon_0 A}{d}\right)\left(e\frac{d}{A}\right) = \frac{\varepsilon_0 K}{\sigma}$

Example 64

A $10\mu F$ capacitor is charged through a resistance of $0.1M\Omega$ from a battery of $1.5V$. Calculate the time required for the capacitor to get charged up to $0.75V$ for situations shown in Figs. 3E.64(a) and (b).

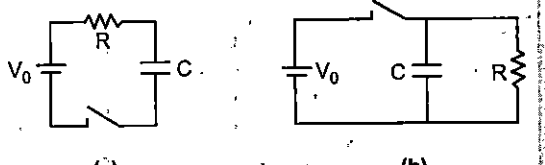


Fig. 3E.64

Solution: In Fig. 3E.64 (a) capacitor and resistor form a series *RC* circuit. Charge at time *t* is

$$q = q_0[1 - e^{-t/CR}]$$

$$CV = CV_0[1 - e^{-t/CR}]$$

$$\frac{V}{V_0} = [1 - e^{-t/CR}]$$

$$\frac{0.75}{1.5} = [1 - e^{-t/CR}]$$

$$e^{-t/CR} = 1/2$$

Taking natural logarithm on both sides, we have

$$t = CR \ln 2$$

$$= (10 \times 10^{-6})(0.1 \times 10^{6}) \ln 2$$

$$= \ln 2 = 0.693 \text{ s}$$

In Fig. 3E.64 (b) capacitor is in parallel with the battery, initially it acts like a short circuit, it will be charged instantaneously.

$$CR = \frac{8.846 \times 10^{-12} \times 5}{7.4 \times 10^{-12}} = 6$$

Hence $$I = \frac{8.85 \times 10^{-6}}{6} e^{-12/6} = 0.24 \mu A$$

where $e = 2.718$ and $e^2 = 7.39$

Example 65

Fig. 3E.65 shows a circuit in which time interval of flashes of a neon gas tube is controlled by a capacitor. When cool, the gas is a good insulator. The tube "fires" (ionizes and emits light) when the potential difference across it reaches the firing value V_f. Its resistance becomes very small and so the capacitor rapidly discharges through it.

As the potential difference drops, the gas cools down and becomes an insulator. The extension potential difference is V_e. At this stage the capacitor again starts to charge.

The variation of potential difference is shown in Fig. 3E.65. Determine the time period T of flashing of the neon gas tube.

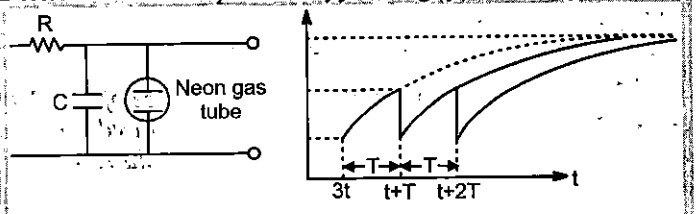


Fig. 3E.65

Solution: The charge on a capacitor at time *t* is

$$q = q_0(1 - e^{-t/RC})$$

$$CV = CV_0(1 - e^{-t/RC})$$

Thus the potential difference at time *t* is

$$V = V_0(1 - e^{-t/RC}) \quad \ldots(1)$$

Let the potential difference V_e be attained at time *t* and V_f at time $t + T$, we have

$$V_e = V_0(1 - e^{-t/RC}) \quad \ldots(2)$$

$$V_f = V_0(1 - e^{-(t+T)/RC}) \quad \ldots(3)$$

From eqn. (2),

$$e^{-t/RC} = V_0 - V_e \quad \ldots(4)$$

From eqn. (3),

$$e^{-(t+T)/RC} = V_0 - V_f \quad \ldots(5)$$

Dividing eqn. (5) by (4), we get

$$\frac{V_0 - V_f}{V_0 - V_e} = e^{-(t+T)/RC + \frac{t}{RC}} = e^{-T/RC} \quad \ldots(6)$$

Now taking logarithm of both sides of eqn. (6), we get

$$T = RC \ln\left(\frac{V_0 - V_e}{V_0 - V_f}\right)$$

Example 66

In the given circuit, the switch is closed in the position 1 at $t = 0$ and then moved to 2 after 250μs. Derive an expression for current as a function of time for $t > 0$. Also, plot the variation of current with time.

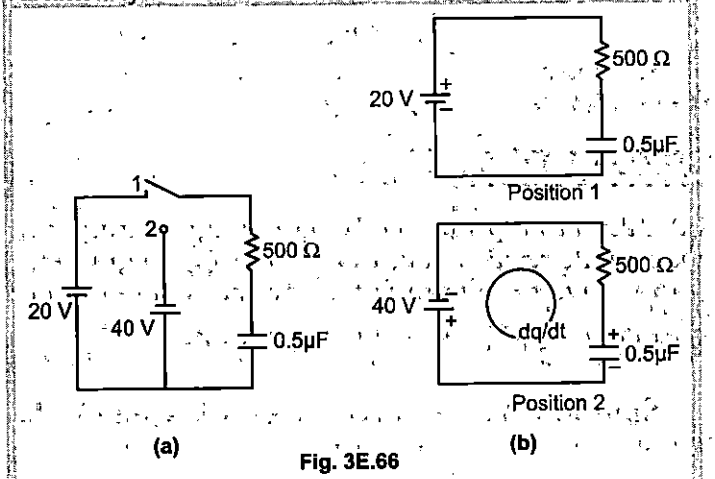


Fig. 3E.66

Solution: When the switch is shifted to position 1, the capacitor gets charged and the instantaneous charge is

$$q = (0.5 \times 10^{-6})(20)(1 - e^{-4000t})$$

or $$q = 10(1 - e^{-4000t}) \mu C$$

because time constant,

$$\tau = RC$$

$$= (500)(0.5 \times 10^{-6}) = 250 \mu s$$

or $$\frac{1}{\tau} = \frac{10^6}{250} = 4000$$

At $t = 250\mu s$, $q = 10(1 - e^{-1}) = 6.3\mu C$

When the switch is shifted to position 2 at $t = 250\mu C$. Applying KVL, we get

$$-R\frac{dq}{dt} - \frac{q}{C} - 40 = 0$$

Putting $R = 500\Omega, C = 0.5\mu F$

and $\frac{1}{\tau} = 4000 s^{-1}$, we get

$$\int_{6.3\mu C}^{q} \frac{dq}{q+20} = -4000\int_{250\mu s}^{t} dt$$

or $$\ln\left|\frac{q+20}{26.3}\right| = -(4000t - 1)$$

or $$q = 26.3e^{-(4000t-1)} - 20\mu C$$

or $$q = 71.5e^{-4000t} - 20\mu C$$

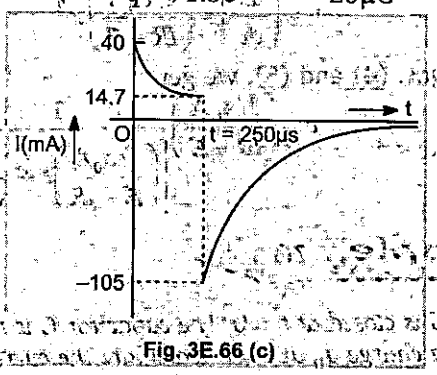


Fig. 3E.66 (c)

Differentiating eqn. (1) and (2), we get

$$I_1 = \frac{dq}{dt} = 40e^{-4000t}(mA) \quad 0 < t < 250\mu s$$

$$I_2 = \frac{dq}{dt} = -286e^{-4000t}(mA) \quad t > 250\mu s$$

Example 67

Consider a parallel plate capacitor of capacitance C with partially conducting medium between its plates having a resistance R_C. If this capacitor is connected to a battery of emf $\mathcal{E}$ and a resistor R as shown in Fig. 3E.67, find:

(i) charge on the capacitor as a function of time.

(ii) current through 'R' as a function of time.

Solution: The equivalent circuit can be drawn as shown. Applying Kirchhoff's law for the two loops,

$$(I - I_1)R_C + IR = \mathcal{E} \quad ...(1)$$

$$\frac{q}{C} = R_C(I - I_1) \quad ...(2)$$

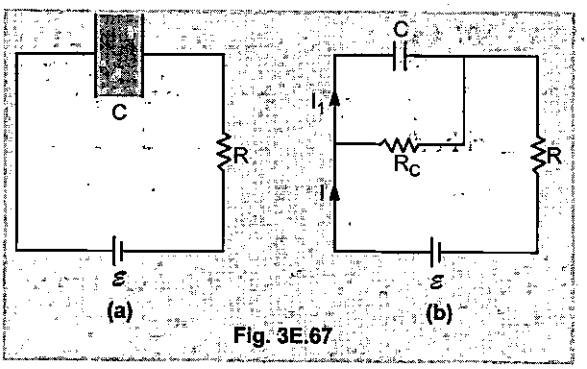


Fig. 3E.67

(i) Eliminating I between eqn. (1) and (2), we get

$$\frac{q}{C}\left(1 + \frac{R}{R_C}\right) + I_1 R = \mathcal{E}$$

For capacitor C, $\frac{dq}{dt} = I_1$

$$\Rightarrow \quad \frac{dq}{dt} = \frac{C\mathcal{E} - q(1+\alpha)}{RC} \text{ (where } \alpha = R/R_C)$$

(ii) Solving, we get

$$q = \frac{\mathcal{E}C}{1+\alpha}(1 - e^{-t((1+\alpha)/RC)})$$

Adding eqn. (1) and (2), we get

$$\frac{Q}{C} + IR = \mathcal{E}$$

or $$I = \frac{C\mathcal{E} - q}{RC}$$

Substituting for q, we get

$$I = \frac{\mathcal{E}}{R}\left[1 - \frac{1}{1+\alpha}(1 - e^{-t(1+\alpha)/RC})\right] \quad (\text{where } \alpha = R/R_C)$$

Example 68

A capacitor initially given a charge Q_0 is connected across a resistor R at $t = 0$. The separation between the plates changes according to

$$d = \frac{d_0}{(1+t)}(0 \le t < 1)$$

A small bulb is connected across the plates of the capacitor which lights when potential difference across the plates of the capacitor reaches V_0. Find

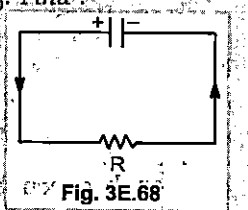


Fig. 3E.68

(a) the variation of charge with time

(b) when the bulb will light.

Solution: Capacitance at $t = 0$,

$$C_0 = \frac{\varepsilon_0 A}{d_0},\ C = C_0(1+t).$$

Using Kirchhoff's law,

$$\frac{q}{C} - RI = 0$$

$$\frac{q}{C_0(1+t)} + R\frac{dq}{dt} = 0$$

$$\frac{dq}{q} = -\frac{1}{RC_0}\frac{dt}{(1+t)}$$

$$\ln q\Big|_{Q_0}^{q} = -\frac{1}{RC_0}\ln(1+t)\Big|_0^t$$

$$\ln\left(\frac{q}{Q_0}\right) = -\frac{1}{RC_0}\ln(1+t)$$

$$\ln\left(\frac{q}{Q_0}\right) = \ln(1+t)^{-1/RC_0}$$

$$q = Q_0(1+t)^{-1/RC_0}$$

(b) $$V = \frac{q}{C} = \frac{Q_0(1+t)^{-1/RC_0}}{C_0(1+t)}$$

$$= \frac{Q_0}{C_0}(1+t)^{-(1/RC_0)+1}$$

which gives $$t = 1 - \left(\frac{V_0 C_0}{Q_0}\right)^{-RC_0/(RC_0+1)}$$

Example 69

In the following R-C circuit, the capacitor is in the steady state. The initial separation of the capacitor plates is x_0. If at $t = 0$, the separation between the plates starts changing so that a constant current flows through R, find the velocity of the moving plates as a function of time. The plate area is A.

Solution: Let q be the instantaneous charge on the capacitor when a steady current i flows through the circuit. Applying KVL on the circuit, we have.

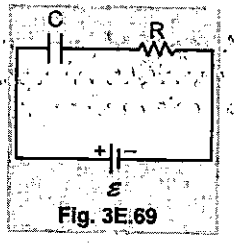


Fig. 3E.69

$$\mathcal{E} = \frac{q}{C} + IR$$

or $$\mathcal{E} = \frac{qx}{\varepsilon_0 A} + IR \quad \ldots(1)$$

At the instant separation between the plates of the capacitor is x, its capacity $C = \varepsilon_0 A/x$.

Differentiating eqn. (1) with respect to time, we get

$$0 = \frac{q}{\varepsilon_0 A}\left(\frac{dx}{dt}\right) + \frac{Ix}{\varepsilon_0 A} \qquad (\because q = It)$$

or $$q = -\frac{Ix}{(dx/dt)} \qquad \left(\text{where}\frac{dx}{dt} = \text{velocity}\right) \quad \ldots(2)$$

From eqn. (2), substituting the value of q in eqn. (1), we have

$$\mathcal{E} = -I\frac{x^2}{\varepsilon_0 A\,(dx/dt)} + IR$$

or $$\frac{dx}{dt} = v = \left(\frac{-I/\varepsilon_0 A}{\mathcal{E} - IR}\right)x^2$$

or $$-\frac{dx}{x^2} = \left(\frac{I/\varepsilon_0 A}{\mathcal{E} - IR}\right)dt \quad \ldots(3)$$

Integrating the above expression w.r.t. time, we get

$$\frac{1}{x} - \frac{1}{x_0} = \left[\frac{I/\varepsilon_0 A}{\mathcal{E} - IR}\right]t \quad \ldots(4)$$

Again differentiating w.r.t. time

$$-\frac{1}{x^2}\frac{dx}{dt} = \frac{I/\varepsilon_0 A}{\mathcal{E} - IR}$$

or $$v = \left(\frac{dx}{dt}\right) = \left(\frac{I/\varepsilon_0 A}{IR - \mathcal{E}}\right)x^2 \quad \ldots(5)$$

From eqns. (4) and (5), we get

$$v = \left(\frac{I/\varepsilon_0 A}{IR - \mathcal{E}}\right)\frac{1}{\left[\left(\frac{I/\varepsilon_0 A}{\mathcal{E} - IR}\right)t + \frac{1}{x_0}\right]^2}$$

Example 70

The switch S is closed at $t = 0$. The capacitor C is uncharged but C_0 has a charge q_0 at $t = 0$. Calculate the current $I(t)$ in the circuit.

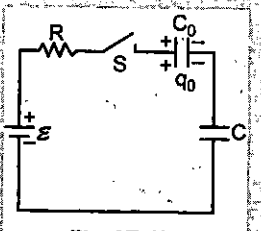


Fig. 3E.70

Solution: Let q_0 and q be the instantaneous charges on C_0 and C respectively. Applying KVL to the circuit, we have

$$\frac{q_0}{C_0} + \frac{q}{C} + IR = \mathcal{E} \quad \ldots(1)$$

Differentiating this equation, we get

$$\frac{1}{C_0}\frac{dq_0}{dt} + \frac{1}{C}\frac{dq}{dt} + R\frac{dI}{dt} = 0$$

or $$I\left[\frac{1}{C_0} + \frac{1}{C}\right] = -R\frac{dI}{dt} \qquad \left(\text{where}I = \frac{dq_0}{dt} = \frac{dq}{dt}\right)$$

or $$\frac{dI}{I} = -\frac{dt}{RC_{eq.}} \qquad \left(\text{where}C_{eq.} = \frac{C_0 C}{C + C_0}\right)$$

Integrating this expression, we have

$$\int_{I_0}^{I(t)}\frac{dI}{I} = -\int_0^t \frac{dt}{RC_{eq.}}$$

or $$[\log_e I]_{I_0}^{I(t)} = -\frac{t}{RC_{eq.}}$$

or $I(t) = I_0 e^{-(t/R)C_{eq}}$ ...(2)

where I_0 is the initial current.

Further, $I_0 R + \frac{q_0}{C_0} = \mathcal{E}$

or $I_0 = \left(\mathcal{E} - \frac{q_0}{C_0}\right)\Big/ R$ ...(3)

Substituting I_0 from eqn. (3) into eqn. (2), we get

$$I(t) = \left[\frac{\mathcal{E} - (q_0/C_0)}{R}\right] e^{-(t/R)C_{eq}}$$

Example 71

Plates of a parallel plate capacitor initially charged to Q_0 is connected to a variable resistance R where resistance is given by $R = R_0 + \alpha t$. At time $t = 0$, $R = R_0$.

Find the current through the capacitor as function of time.

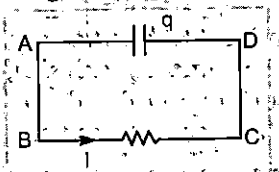


Fig. 3E.71

Solution: $R = R_0 + \alpha t$.

Applying Kirchhoff's law to close circuit *ABCDA* at time *t*, when charge on capacitor is *q*,

$$\frac{q}{C} - RI = 0 \quad ...(1)$$

$$\frac{q}{C} - R\frac{dq}{dt} = 0$$

$$\Rightarrow \int_{Q_0}^{q} \frac{dq}{q} = \int_0^t \frac{dt}{RC}$$

$$\ln\frac{q}{Q_0} = \int_0^t \frac{dt}{(R_0 + \alpha t)C}$$

$$\ln\frac{q}{Q_0} = \frac{1}{\alpha C}\ln(R_0 + \alpha t)$$

$$\frac{q}{Q_0} = e^{\ln(R_0+\alpha t)^{1/\alpha C}}$$

$$q = Q_0(R_0 + \alpha t)^{1/\alpha C}$$

From eqn. (1), $I = \frac{q}{CR}$

$$I = \frac{Q_0 e^{(1/\alpha C)\ln(R_0+\alpha t)}}{C(R_0 + \alpha t)}$$

Example 72

A conductor of capacitance $10^{-2}\mu F$ to the ground is insulated from the ground by a silica plate 2.5 mm thick and 5 cm^2 in cross-sectional area. What is the minimum resistivity of the silica if the rate of decrease of potential is to be no greater than 0.1% per min of its instantaneous value?

Solution: In this problem, the capacitor is initially charged to its maximum value, and then discharges to ground through the silica plate [see Fig. 3E.72 (a)]. The latter may be considered to be an effective resistance [see Fig. 3E.72 (b)]. Before solving the problem, we must know the voltage across the capacitor as a function of time.

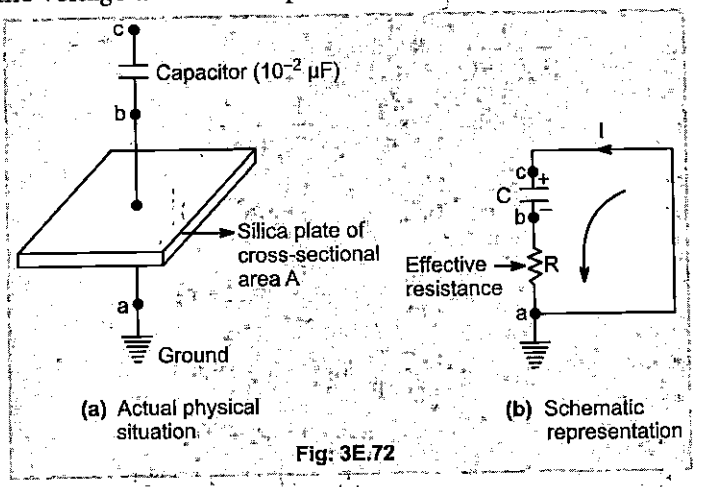


Fig: 3E.72

Kirchhoff's voltage law states that the net voltage around a closed circuit loop is zero. Hence,

$$V_{cb} + V_{ba} = 0 \quad ...(1)$$

Starting at *c*, and traversing the loop in the given direction [see Fig. 3E.72 (b)], we note that *c* is at a higher potential than *b*. Hence $V_c > V_b$ or $V_b - V_c = V_{cb} < 0$. But, by definition of capacitance,

$$C = \frac{|Q|}{V_{bc}}$$

or $$V_{bc} = \frac{|Q|}{C}$$

where $|Q|$ is the absolute value of the charge on 1 plate of the capacitor. Therefore,

$$V_{cb} = -V_{bc} = \frac{-Q}{C} \quad ...(2)$$

where $Q > 0$.

Furthermore, $V_b > V_a$

or $V_a - V_b = V_{ba} < 0$

By Ohm's Law, $V_{ba} = -IR$ ...(3)

Using (3) and (2) in (1),

$$\frac{-Q}{C} - IR = 0 \quad ...(4)$$

But $I = \frac{dQ}{dt}$, where *Q* is the net charge passing a point of the circuit per unit time. Eqn. (4) then becomes

$$\frac{1}{C}Q + R\frac{dQ}{dt} = 0$$

or $$Q + RC\frac{dQ}{dt} = 0 \quad ...(5)$$

Solving (5), $$\frac{dQ}{Q} = -\frac{1}{RC}dt$$

$$\int_{Q_0}^{Q}\frac{dQ}{Q}=-\frac{1}{RC}\int_0^t dt$$

Hence, $$Q(t)=Q_0e^{-t/RC} \quad ...(6)$$

The voltage across C is then, by definition of C,

$$V=\frac{|Q|}{C}=\frac{Q_0}{C}e^{-t/RC}$$

Example 73

The variable capacitor in Fig. 3E.73 is connected to a battery of emf V and internal resistance r. The current in the circuit is kept constant by changing the capacitance. (Assume that we are able to increase the capacitance indefinitely in order to accomplish this).

(a) Calculate the power supplied by the battery.

(b) Compere the rate at which energy is supplied by the battery with the rate of change of the energy stored in the capacitor.

Solution: (a) The current in the circuit is related to the voltage across the capacitor as

$$C\frac{dV_c}{dt}=I$$

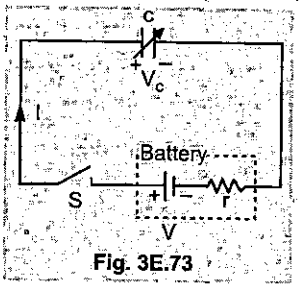


Fig. 3E.73

As the capacitor charges up, V_c rises toward its limiting value and its rate of change goes to zero. Therefore, in order to keep I constant, capacitance, C, should compensate for this drop in $\frac{dV_c}{dt}$ (observe that, as time $t\to\infty$, $\frac{dV_c}{dt}\to 0$ and we should have $C_\infty\to\infty$). Ignoring the practical difficulties of maintaining a constant I, asymptotically we have a continuous charge flow in the circuit

$$\frac{dq}{dt}=I \quad \text{and} \quad V_c=V$$

The battery supplies a constant power since its emf is constant,

$$P_B=VI$$

(b) The capacitor is being continuously charged. Once the voltage across the capacitor stabilizes at $t\to\infty$, we have

$$V=Ir+V_{c,\infty}$$

or $$V_{c,\infty}=V-Ir$$

The energy stored in the capacitor becomes

$$W=\frac{1}{2}q_\infty V_{c,\infty}$$

$$=\frac{1}{2}q_\infty(V-Ir)$$

The rate of change of W with time is

$$P_c=\frac{dE}{dt}=\frac{1}{2}\frac{dq_\infty}{dt}V_{c,\infty}$$

$$=\frac{1}{2}I(V-Ir)$$

The rate of dissipation of energy in the resistor r is

$$P_r=I^2r,$$

Therefore the power supplied directly to the capacitor is

$$P=P_B-P_r=IV-I^2r=I(V-Ir),$$

which is twice as large as that used in charging the capacitor:

$$P=2P_c$$

The battery supplies twice as much energy to the capacitor as is stored in the capacitor. The difference is the work done by the capacitor against the external agent that is changing the capacitance.

Example 74

The capacitor C_1 in Fig. 3E.74 initially carries a charge q_0. When the switches S_1 and S_2 are shut, capacitor C_1 is connected in series to a resistor R and a second capacitor C_2, which initially does not carry any charge.

(i) Find the charges deposited on the capacitors and the current through R as a function of time.

(ii) What is the heat lost in the resistor after a long time of closing the switch?

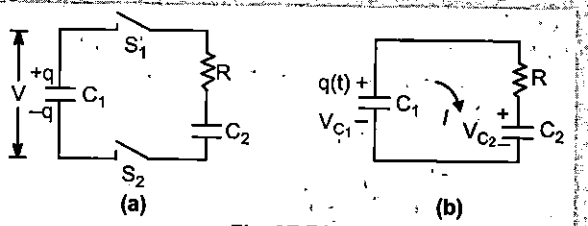


Fig. 3E.74

Solution: (i) Suppose at a moment 't' the charge deposited on C_1 is $q(t)$.

$$\therefore \quad V_{C_1}=\frac{q(t)}{C_1} \quad \text{and} \quad V_{C_2}=\frac{q_0-q(t)}{C_2}$$

$$V_R=IR, \quad \text{and} \quad I=-\frac{dq}{dt}$$

Applying KVL,

$$\frac{q}{C_1}-\frac{(q_0-q)}{C_2}=IR=-R\frac{dq}{dt}$$

$$\therefore \quad R\frac{dq}{dt}+\left(\frac{1}{C_1}+\frac{1}{C_2}\right)q=\frac{q_0}{C_2}$$

Put $$\frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_2}$$

$$\therefore \quad q(t)=\left(1-\frac{C}{C_2}\right)q_0e^{-t/RC}+\frac{C}{C_2}q_0$$

or $$q(t)=\frac{C}{C_1}q_0e^{-t/RC}+\frac{C}{C_2}q_0$$

$$\therefore \quad I(t) = -\frac{dq}{dt} = \frac{q_0}{RC_1} e^{-t/RC}$$

Charge on C_2,

$$q_0 - q(t) = q_0 \frac{C}{C_1}(1 - e^{-t/RC})$$

(ii) Electrostatic energy at $t = 0$ is

$$U(0) = \frac{q_0^2}{2C_1}$$

$$\text{Final energy} = U(\infty) = \frac{q_0^2}{2(C_1 + C_2)}$$

$$\Delta U = U(0) - U(\infty) = \frac{q_0^2 C_2}{2C_1(C_1 + C_2)}$$

Example 75

Two large conducting plates are arranged to form a parallel-plate capacitor of capacitance C. The plates are given charges Q_1 and Q_2 respectively. The plates are then connected to the terminals of a battery of emf $\mathcal{E}$ and internal resistance r at $t = 0$. Find the charges on the four surfaces as function of time.

Before the battery was connected, the four surfaces will have charges as shown.

When the battery is connected, the charges on the inner surfaces will always be equal and opposite. Since the electric field inside the plates will be zero, the cancellation of electric field must be accomplished by the charges on the outer surfaces. For this, they have to be of same magnitude and nature. It thus follows that the charges on the outer surfaces will always be what they initially were, i.e., $\frac{Q_1 + Q_2}{2}$.

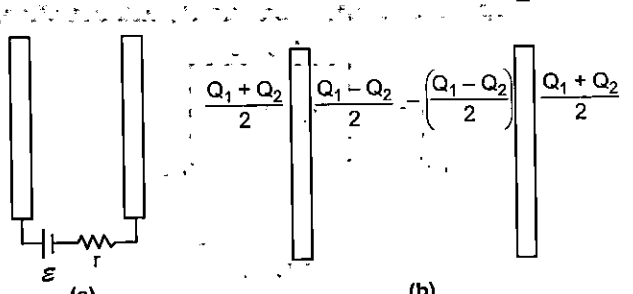


Fig. 3E.75

Solution: Let q be the charges at time t on the inner surfaces. Applying KVL,

$$\frac{-q}{C} - Ir + \mathcal{E} = 0 \quad \Rightarrow \quad \frac{dq}{\mathcal{E} - \frac{q}{C}} = \frac{dt}{r}$$

Integrating, $\quad -C \ln\left|\mathcal{E} - \frac{q}{C}\right| = \frac{t}{r} + C^*$

At $t = 0$, $\quad q = \frac{Q_1 - Q_2}{2}$

$$\Rightarrow \quad C^* = -C \ln\left|\mathcal{E} - \frac{Q_1 - Q_2}{2C}\right|$$

$$\therefore \quad q = \frac{Q_1 - Q_2}{2} e^{-t/rC} + C\mathcal{E}(1 - e^{-t/rC})$$

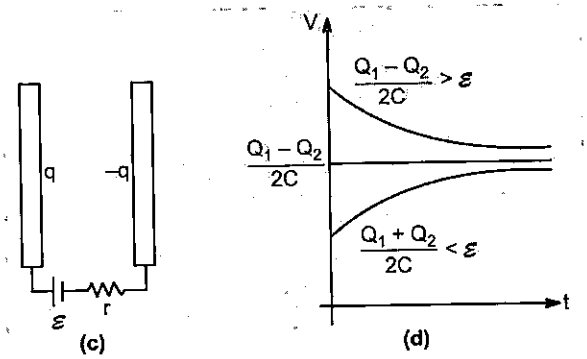


Fig. 3E.75

Remark :

We can interpret this result as a superposition of discharging and charging of a capacitor. Also note that in steady state, the charge on the inner surfaces will be $C\mathcal{E}$ and $-C\mathcal{E}$ respectively, regardless of the initial value. In particular if $\frac{Q_1 - Q_2}{2C}$ (initial potential difference between the plates) is equal to $\mathcal{E}$, there is no charge flow through the battery. If $\frac{Q_1 - Q_2}{2C} > \mathcal{E}$, an extra charge is given to the battery and if $\frac{Q_1 - Q_2}{2C} < \mathcal{E}$, the battery supplies the charge to the capacitor. This all is shown in the graph shown in Fig. 3E.75 (d) above, wherein potential difference across the plates has been plotted as function of time.

TIME CONSTANT

Product "RC" which has dimension of time is called time constant of CR circuit

$$Q = Q_0(1 - e^{-t/RC})$$

If $\quad t = RC$

$$Q = Q_0(1 - e^{-1})$$

$$\frac{Q}{C} = IR$$

$$\frac{Q}{C} = \frac{-dQ}{dt} R$$

$$Q = Q_0 e^{-t/RC}$$

where $\quad Q_0 = C\mathcal{E}$

$$I = -I_0 e^{-t/RC}$$

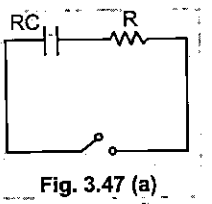


Fig. 3.47 (a)

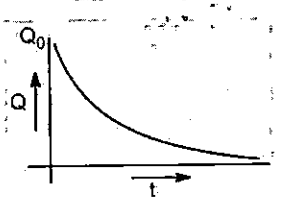


Fig. 3.47 (b)

Concept: *In all problem involving more than one resistance and batteries thevenin theorem is to be used for transisent analysis when charge is a function of time.*

Steps of thevenin theorem

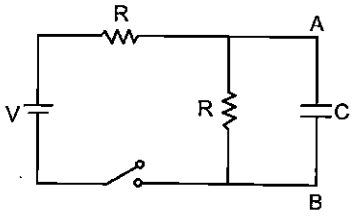


Fig. 3.48

Step-1: *Short all the battery, make capacitor an open branch as shown in figure. Find* R_{eq} *across open points of capacitor this is* R_{thevenin} *or* R_{th}.

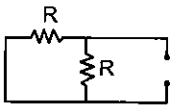


Fig. 3.49

Step-2: *Now connect battery and calculate p.d. across open branch of capacitor we call it thevenin voltage* $V_{\text{thevenin}} = V_{\text{th}}$

Step-3: $q = CV_{\text{th}}(1 - e^{-t/R_{\text{th}}C})$

Example 76

Find charge on capacitor as a function of time when switch is closed at $t = 0$ *assuming capacitor is initially uncharged.*

Solution: $V_{th} = \frac{V}{2}$, $R_{th} = \frac{R}{2}$

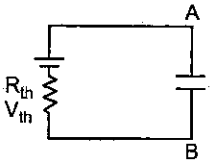


Fig. 3E.76

$$Q = CV_{th}\left(1 - e^{\frac{-t}{RC}}\right)$$

$$Q = \frac{CV}{2}(1 - e^{-t/RC})$$

Example 77

Calculate the constant of a circuit as shown in Fig. 3E.77.

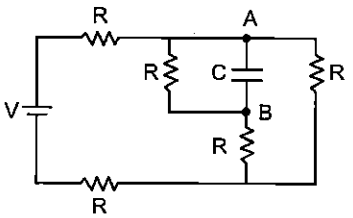


Fig. 3E.77 (a)

Solution: Time constant $R_{th}C$

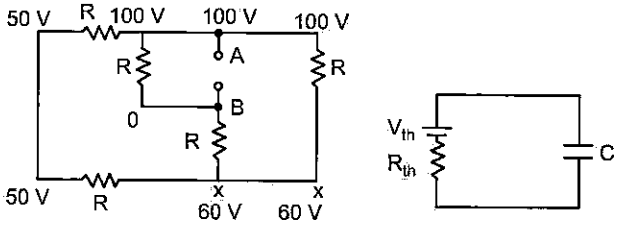


Fig. 3E.77

$$R_{AB} = \frac{100}{\frac{100-60}{R} + \frac{100}{R} + \frac{100-80}{R}}$$

$$\Rightarrow \quad R_{AB} = \frac{5R}{8}$$

Thus, $\tau = \frac{5RC}{8}$

Example 78

$R_1 = R_2 = R_3 = R$

Find Q as function of time.

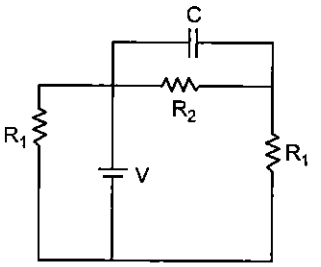


Fig. 3E.78 (a)

Solution: ∵ on shorting battery R_3 will not affect

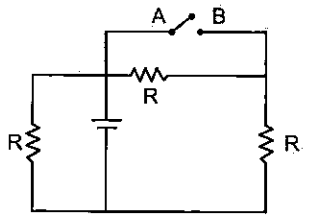


Fig. 3E.78 (b)

$$R_{th} = \frac{R}{2} \quad \Rightarrow \quad V_{th} = \frac{V}{2}$$

$$Q = \frac{C\mathcal{E}}{2}(1 - e^{-2b/RC})$$

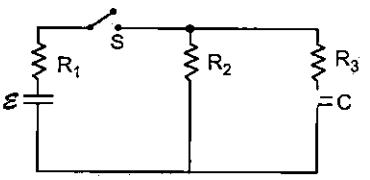


Fig. 3E.78 (c)

In Fig. 3E.78 (c)

$$R_1 = R_2 = 6R_3$$

Find Q as function of time

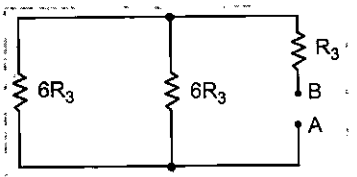


Fig. 3E.78 (d)

$$R_{th} = 4R_3$$

$$V_{th} = \frac{V}{2}$$

$$Q = \frac{CV}{2}(1 - e^{\frac{-t}{4R_3C}})$$

Example 79

Calculate charge as a function of time when S_1 and S_2 are closed simultaneously given that initial charge on capacitor is zero.

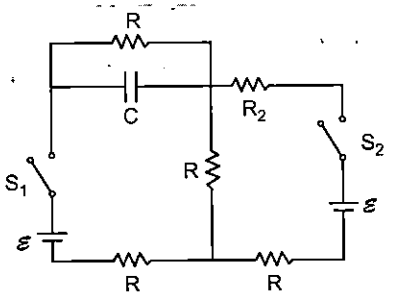


Fig. 3E.79 (a)

Solution: $Q = C\mathcal{E}(1 - e^{\frac{-t}{CR_{th}}})$

Step-1: $R_{th} = ?$ from Fig. 3E.79 (b)

$$R_{th} = \frac{\frac{3R}{2} \times \frac{R}{2}}{\frac{3R}{2} \times \frac{R}{2}} = \frac{3R}{8}$$

Step-2: For V_{th} apply KVL

$$IR + (I - I_1)R + IR = \mathcal{E}$$

$$\Rightarrow \quad 3IR - I_1R = \mathcal{E}, \mathcal{E} + I_1R - (I - I_1)R = 0$$

$$\Rightarrow \quad 2I_1R - IR + \mathcal{E} = 0$$

$$3IR - I_1R = \mathcal{E}, 6I_1R - 3IR + 3\mathcal{E} = 0$$

$$5I_1R = -2\mathcal{E}$$

$$\Rightarrow \quad I_1 = -\frac{2\mathcal{E}}{5R}$$

$$I = \frac{\mathcal{E}}{5R}$$

$$\therefore \quad V_{AB} = IR = \frac{\mathcal{E}}{5}$$

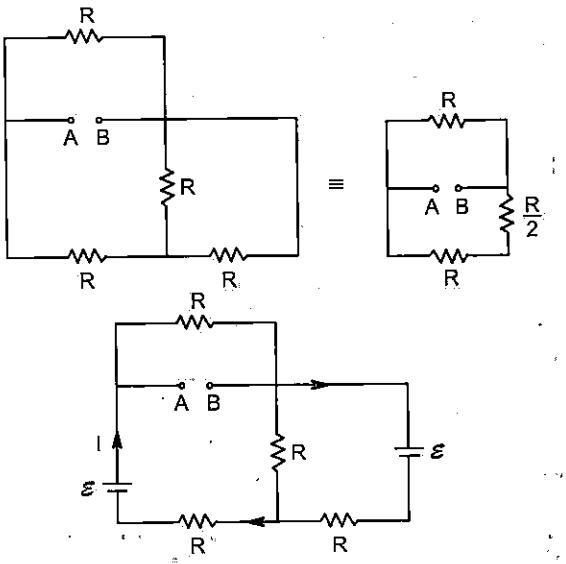


Fig. 3E.79 (b)

Example 80

Two plates of a capacitor of capacitance C are given charges Q_1 and Q_2. This capacitor is connected across a resistance R as shown. Key is closed at $t = 0$. Find the charges on the plates after time t.

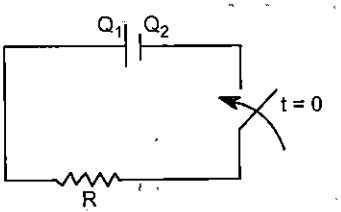


Fig. 3E.80 (a)

Solution: Initially the charges on the plates will be distributed as shown. Charge on outer surface $\frac{Q_1 + Q_2}{2}$ is constant because $Q_1 + Q_2$ is constant. *P.D.* across the plates is given by $\frac{Q_1 - Q_2}{2C}$ and is independent of the charge on the outer surfaces. Thus, when capacitor is discharged, the charges on the outer surface do not change. Charge on the inner surface decreases according to the equation.

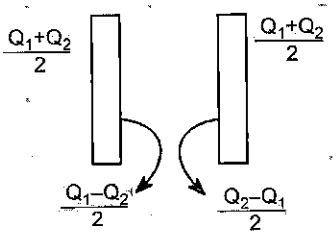


Fig. 3E.80 (b)

$$q=\left(\frac{Q_1-Q_2}{2}\right)e^{-\frac{t}{RC}}$$

∴ Total charge on the left plate

$$q_1=\frac{Q_1+Q_2}{2}+\left(\frac{Q_1-Q_2}{2}\right)e^{-\frac{t}{RC}}$$

And total charge of the right plate

$$q_2=\frac{Q_1+Q_2}{2}-\left(\frac{Q_1-Q_2}{2}\right)e^{\frac{t}{RC}}$$

Example 81

In the circuit diagram shown in Fig. 3E.81. The capacitor of capacitance C is uncharged when the key k is open. The key is closed for time during which the capacitor becomes charged to a voltage V. Determine the amount of heat Q_2 liberated during this time in the resistor of resistance R_2 if the emf of the source is $\mathcal{E}$, and its internal resistance can be neglected.

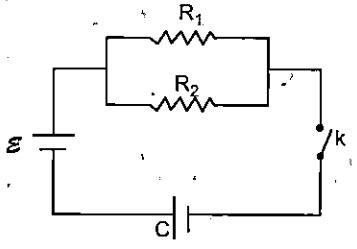


Fig. 3E.81

Solution: Charge stored in capacitor $q=CV$. It is the charge provided by battery. Thus the work done by battery

$$W=q\mathcal{E}=C\mathcal{E}V$$

Energy stored in capacitor $=\frac{1}{2}CV^2$

Thus the total energy loss in the form of heat

$$Q=C\mathcal{E}V-CV^2=\frac{CV(2\mathcal{E}-V)}{2}$$

$$\therefore\quad Q_2=\frac{R_1}{R_1+R_2}\times Q$$

$$=\frac{R_1CV}{2(R_1+R_2)}(\mathcal{E}-V)$$

Example 82

Initially switch S was closed for long time. Switch is opened at $t=0$. Find the current (in amperes) through battery at time $RC\ln 2$. Take $V=12V$, $R=4$ ohm, $C=1.0\mu F$.

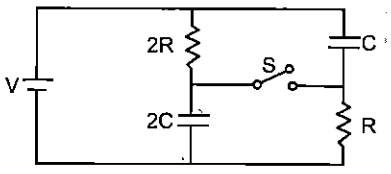


Fig. 3E.82 (a)

Solution: At time t

$$V-I_1(2R)\frac{-q_1}{2C}=0$$

$$\left[\text{At } t=0, q_1=\frac{2CV}{3}; I_1(2R)=\frac{2V}{3}; I_1=\frac{V}{3R}\right]$$

$$0-\frac{dI_1}{dt}(2R)-\frac{I_1}{2C}=0$$

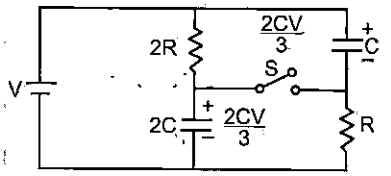


Fig. 3E.82 (b)

$$\int_{\frac{V}{3R}}^{I_1}\frac{dI_1}{I_1}=\int_0^t-\frac{dt}{4RC}$$

$$\ln\frac{I_1}{V/3R}=\frac{-t}{4RC}\Rightarrow I_1=\frac{V}{3R}e^{-\frac{t}{RC}}$$

Similarly $\quad I_2=\frac{V}{3R}e^{-\frac{t}{RC}}$

At $t=RC\ln 2$

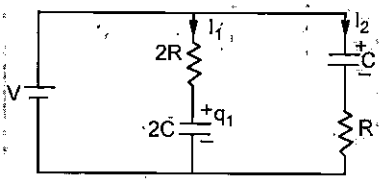


Fig. 3E.82 (c)

$$I_1=\frac{V}{3R}e^{-\frac{RC\ln 2}{4RC}}$$

$$=\frac{V}{3R}\left(\frac{1}{2^{1/4}}\right)=\frac{12}{12}\left(\frac{1}{2^{1/4}}\right)$$

$$I_2=\frac{V}{3R}e^{-\frac{RC\ln 2}{RC}}=\frac{V}{3R}\left(\frac{1}{2}\right)=\frac{1}{2}$$

$$I=I_1+I_2=0.84+0.50=1.34\text{A}$$

Example 83

In the Fig. 3E.83 (a) current in the galvanometer I is 10 mA, while the voltmeter reads a p.d of 3 V. What is the current in galvanometer II (in mA)? The galvanometers are identical, the internal resistance of the battery is negligible.

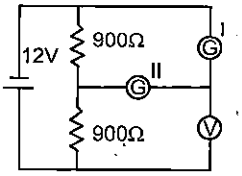


Fig. 3E.83 (a)

Solution: $9 = 10\times10^{-3} R_G$

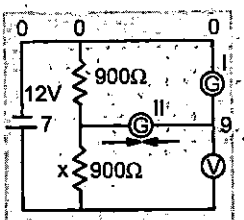


Fig. 3E.83 (b)

$$900 = R_G$$

$$\frac{x-0}{900}+\frac{x-12}{900}+\frac{x-9}{900}=0 \Rightarrow x=7$$

$$\therefore \quad I_2 = \frac{9-7}{900} = \frac{20}{9}\text{ mA} = 2.22\text{ mA}$$

Example 84

Find the ratio of charges on 5 F and 10μF capacitor when circuit is in steady state.

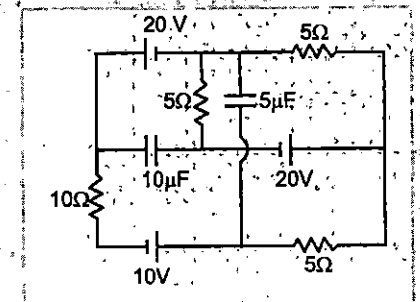


Fig. 3E.84 (a)

Solution:

In loop *ABCEA*

$$20-10I+10-5I-5I_1=0$$

$$\Rightarrow \quad 30-15I-5I_1=0 \qquad \ldots(1)$$

In loop *DEFGD*

$$20-5I_1+5(I-I_1)=0$$

$$\Rightarrow \quad 20-10I_1+5I=0 \qquad \ldots(2)$$

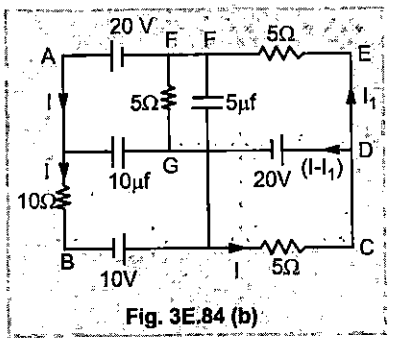


Fig. 3E.84 (b)

Solving eqn. (1) and (2), we get

$$I=\frac{8}{7}\text{A}$$

$$I_1=\frac{36}{14}\text{A}$$

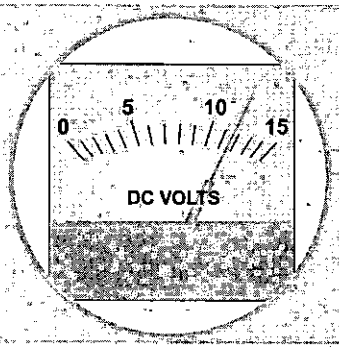
Potential difference across 5μF capacitor

$$=20-10I+10=\frac{130}{7}\text{V}$$

Potential difference across 10μF capacitor

$$=20-(I-I_1)5=\frac{190}{7}\text{V}$$

$$\frac{\text{Charge on 5μF capacitor}}{\text{Charge on 10μF capacitor}}=\frac{\left(\frac{130}{7}\times5\right)\mu\text{C}}{\frac{190}{7}\times10\mu\text{C}}=\frac{13}{38}$$



PROBLEMS

Level 1 Only One Alternative is Correct

1. A capacitor of capacitance C is charged to a potential difference V from a cell and then disconnected from it. A charge $+Q$ is now given to its positive plate. The potential difference across the capacitor is now:

(a) V (b) $V+\frac{Q}{C}$

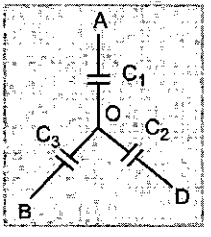
(c) $V+\frac{Q}{2C}$ (d) $V-\frac{Q}{C}$, if $Q<CV$

2. The space between the plates of a parallel plate capacitor is filled completely with a dielectric substance having dielectric constant 4 and thickness 3 mm. The distance between the plates is now increased by inserting a second sheet of thickness 5 mm and dielectric constant K.If the capacitance of the capacitor so formed is one half of the original capacitance, the value of K is :

(a) $\frac{10}{3}$ (b) $\frac{20}{3}$

(c) $\frac{5}{3}$ (d) $\frac{15}{3}$

3. Three uncharged capacitors of capacitances C_1, C_2 and C_3 are connected as shown in the figure to one another and to the points A, B and D at potentials V_A, V_B and V_D. Then the potential at O will be:



(a) $\frac{V_A+V_B+V_D}{C_1+C_2+C_3}$

(b) $\frac{V_AC_1+V_BC_3+V_DC_2}{C_1+C_2+C_3}$

(c) $\frac{V_AC_1^2+V_BC_2^2+V_DC_3^2}{C_1^2+C_2^2+C_3^2}$

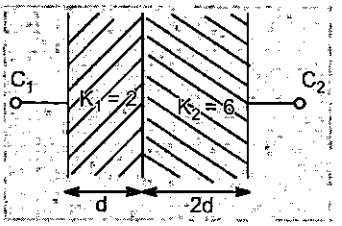
(d) $\frac{V_AV_BV_D}{C_1C_2+C_2C_3+C_3C_1}$

4. A capacitor is filled with an insulator and a certain potential difference is applied to its plates. The energy stored in the capacitor is U. Now the capacitor is disconnected from the source and the insulator is pulled out of the capacitor. The work performed against the forces of the electric field in pulling out the insulator is $4U$. Then dielectric constant of the insulator is:

(a) 4 (b) 8

(c) 5 (d) 3

5. A parallel plate capacitor has two layers of dielectrics as shown in figure. This capacitor is connected across a battery, then the ratio of potential difference across the dielectric layers is:



(a) $\frac{4}{3}$ (b) $\frac{1}{2}$

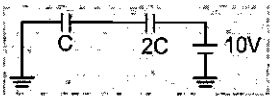
(c) $\frac{5}{2}$ (d) $\frac{3}{2}$

6. Capacity of a spherical capacitor is C_1 when inner sphere is charged and outer sphere is earthed and C_2 when inner sphere is earthed and outer sphere is charged. Then $\frac{C_1}{C_2}$ is (a = radius of inner sphere, b = radius of outer sphere):

(a) 1 (b) $\frac{a}{b}$

(c) $\frac{b}{a}$ (d) $\frac{a+b}{a-b}$

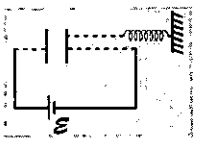
7. In the circuit shown in the figure $C = 6\ \mu$ F. The charge stored in capacitor of capacity C is:



(a) zero (b) 90 μ C

(c) 40 μ C (d) 60 μ C

8. One plate of a capacitor is connected with a spring as shown in figure. Area of both the plates is A. In steady state separation between the plates is $0.8d$ (spring was unstretched and the distance between the plates was d when the capacitor was uncharged). The force constant of the spring is approximately:



(a) $\frac{2.5\varepsilon_0 A\mathcal{E}^2}{d^3}$ (b) $\frac{2\varepsilon_0 A\mathcal{E}}{d^2}$

(c) $\frac{6\varepsilon_0\mathcal{E}^2}{Ad^3}$ (d) $\frac{\varepsilon_0 A\mathcal{E}^3}{2d^3}$

9. A capacitor of capacity $C_1 = 1\mu F$ is charged to a potential of 100 V. The charging battery is then removed and it is connected to another capacitor of capacity $C_2 = 2\ \mu$ F. One plate of C_2 is earthed as shown in figure. The charges on C_1 and C_2 in steady state will be:

(a) 50 μC, 50 μC (b) 100 μC, zero

(c) $\frac{100}{3}$ μC, $\frac{200}{3}$ μC (d) zero, zero

10. Capacity of an isolated sphere is increased n times when it is enclosed by an earthed concentric sphere. The ratio of their radii is:

(a) $\frac{n^2}{n-1}$ (b) $\frac{n}{n-1}$

(c) $\frac{2n}{n+1}$ (d) $\frac{2n+1}{n+1}$

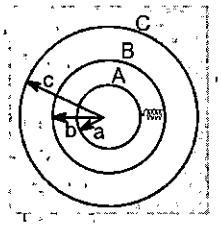
11. Three conducting spheres A, B and C are as shown in figure. The radii of the spheres are a, b and c respectively. A and B are connected by a conducting wire. The capacity of the system is:

(a) $4\pi\varepsilon_0(a+b+c)$

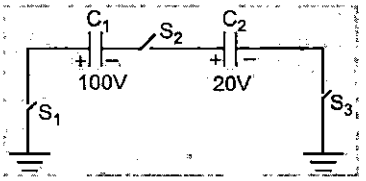
(b) $4\pi\varepsilon_0\left[\frac{bc}{c-b}\right]$

(c) $4\pi\varepsilon_0\left[\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right]$

(d) $4\pi\varepsilon_0\left[\frac{abc}{ab+bc+ca}\right]$



12. In the circuit shown in figure $C_1 = 1\ \mu F$ and $C_2 = 2\ \mu F$. Capacitor C_1 is charged to 100 V and C_2 to 20 volts. After charging they are connected as shown. When the switches S_1, S_2 and S_3 all are closed:



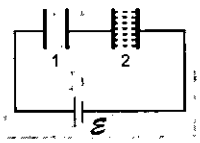
(a) No charge will flow through S_2

(b) 80 μC will flow through S_1

(c) 40 μC will flow through S_2

(d) 60 μC will flow through S_2

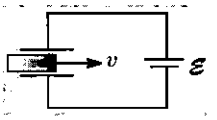
13. Two identical capacitors 1 and 2 are connected in series to a battery as shown in figure. Capacitor 2 contains a dielectric slab of dielectric constant K as shown in figure. Q_1 and Q_2 are the charges stored in the capacitors. Now the dielectric slab is removed and the corresponding charges are Q_1' and Q_2', then



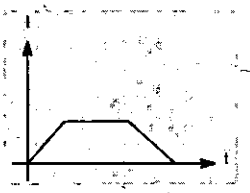
(a) $\frac{Q_1'}{Q_1}=\frac{K+1}{K}$ (b) $\frac{Q_2'}{Q_2}=\frac{K+1}{2}$

(c) $\frac{Q_2'}{Q_2}=\frac{K+1}{2K}$ (d) $\frac{Q_1'}{Q_1}=\frac{K}{2}$

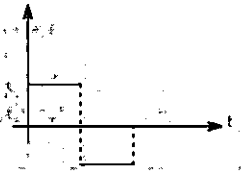
14. A dielectric slab of area A and thickness d is inserted between the plates of capacitor of area $2A$ and distance between the plates d with a constant speed v as shown in figure. The capacitor is connected to a battery of emf $\mathcal{E}$. The current in the circuit varies with time as:



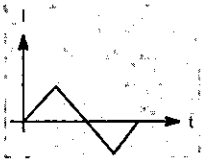
(a)



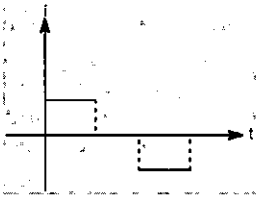
(b)



(c)



(d)

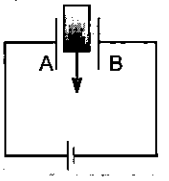


15. Three charged particles are initially in position 1. They are free to move and they come in position 2 after sometime. Let U_1 and U_2 be the electrostatic potential energies in position 1 and 2. Then:

(a) $U_1 > U_2$ (b) $U_2 > U_1$

(c) $U_1 = U_2$ (d) $U_2 \geq U_1$

16. An insulator plate is passed between the plates of a capacitor. Then current:



(a) Always flows from A to B

(b) Always flows from B to A

(c) First flows from A to B and then from B to A

(d) First flows from B to A and then from A to B

17. The capacitance of capacitor of plate areas A_1 and A_2 ($A_1 < A_2$) at a distance d is:

(a) $\frac{\varepsilon_0 A_1}{d}$

(b) $\frac{\varepsilon_0 A_2}{d}$

(c) $\frac{\varepsilon_0 (A_1 + A_2)}{2d}$

(d) $\frac{\varepsilon_0 \sqrt{A_1 A_2}}{d}$

18. If an electron enters into a space between the plates of a parallel plate capacitor at an angle α with the plates and leaves at angle β to the plates. The ratio of its kinetic energy while entering the capacitor to that while leaving will be:

(a) $\left[\frac{\cos\alpha}{\cos\beta}\right]^2$ (b) $\left[\frac{\cos\beta}{\cos\alpha}\right]^2$

(c) $\left[\frac{\sin\alpha}{\sin\beta}\right]^2$ (d) $\left[\frac{\sin\beta}{\sin\alpha}\right]^2$

19. Capacitance of a capacitor becomes $\frac{4}{3}$ times its original value if a dielectric slab of a thickness $t = \frac{d}{2}$ is inserted between the plates (d = the separation between the plates). The dielectric constant of the slab is:

(a) 2 (b) 4

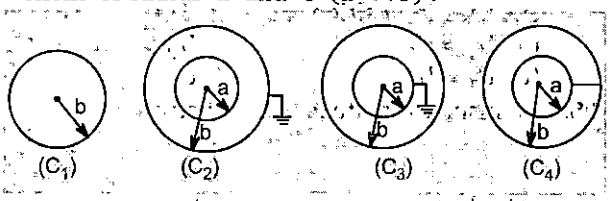
(c) 6 (d) 8

20. The plates of a parallel plate capacitor are charged up to 100 V. A 2 mm thick insulator sheet is inserted between the plates. Then to maintain the same potential difference, the distance between capacitor plates is increased by 1.6 mm. The dielectric constant of the insulator is:

(a) 6 (b) 8

(c) 5 (d) 4

21. Consider the following four arrangement of spherical shells of radius 'a' and 'b' ($a << b$)



Then which of the following holds good for the value of their capacitances:

(a) $C_3 > C_2 > C_1 = C_4$ (b) $C_3 > C_1 = C_4 > C_2$

(c) $C_2 > C_3 > C_1 > C_4$ (d) $C_2 > C_1 = C_4 > C_3$

22. In the shown circuit, if the charge on 6 μF capacitor is 10 μC, then the charge on 4 μF capacitor will be:

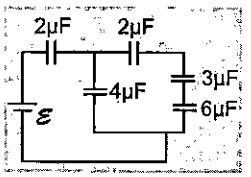
(a) 40 μC

(b) 10 μC

(c) 20 μC

(d) 30 μC

23. In the above question, what is the emf of the cell?



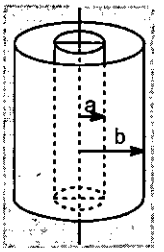
(a) 35 V

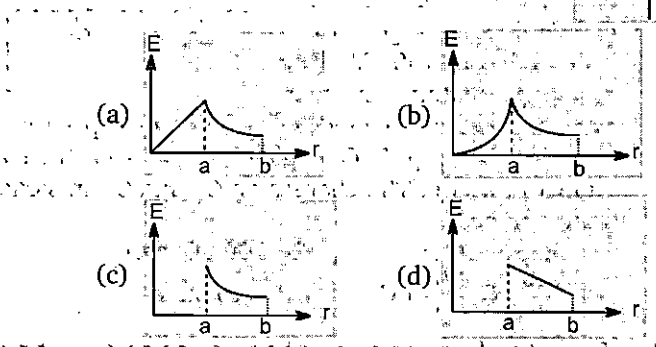
(b) 25 V

(c) 10 V

(d) can't be found

24. In case of cylindrical capacitor (see figure), choose the correct graph between the electric field and radial distance 'r' from the axis. Assume positive charge on inner plate.





25. For a uniform spherical positive distribution of charge over volume of radius R, the potential will be maximum:

(a) on the surface

(b) at the center

(c) at a distance $\frac{R}{2}$ from the surface

(d) at a distance $\frac{3R}{2}$ from the surface

26. A student charges a capacitor in such a manner that it stores energy of 1 J. Now he wants to increase the potential energy to 4 J. He should:

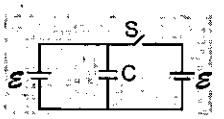
(a) quadruple the potential difference across the capacitor without changing the charge

(b) double the potential difference across the capacitor without changing the charge

(c) double both the potential difference and charge

(d) double the charge without changing the potential difference

27. Initially the circuit is in steady state. When the switch S is closed, the heat generated in the circuit will be:



(a) $\frac{\varepsilon^2 C}{2}$ (b) $2\varepsilon^2 C$

(c) $\frac{3\varepsilon^2 C}{2}$ (d) Zero

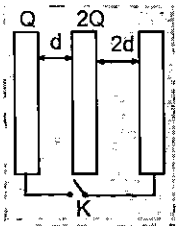
28. Three large plates are arranged as shown. How much charge will flow through the key K if it is closed?

(a) $\frac{5Q}{6}$

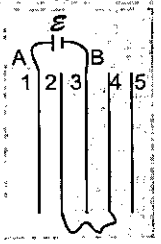
(b) $\frac{4Q}{3}$

(c) $\frac{3Q}{2}$

(d) none of these



29. Five conducting parallel plates having area A and separation between them d, are placed as shown in the figure. Plate number 2 and 4 are connected wire and between point A and B, a cell of emf $\mathcal{E}$ is connected. The charge flown through the cell is:



(a) $\frac{3}{4}\frac{\varepsilon_0 A\mathcal{E}}{d}$ (b) $\frac{2}{3}\frac{\varepsilon_0 A\mathcal{E}}{d}$

(c) $\frac{4\varepsilon_0 A\mathcal{E}}{d}$ (d) $\frac{\varepsilon_0 A\mathcal{E}}{2d}$

30. When a dielectric slab is inserted inside a charged parallel plate capacitor (the charging cell being disconnected), it experiences a force dragging it inside the capacitor. This happens because of:

(a) The uniform electric field inside the capacitor

(b) The fringing of the electric field near the edges

(c) Variation of charge on the plates

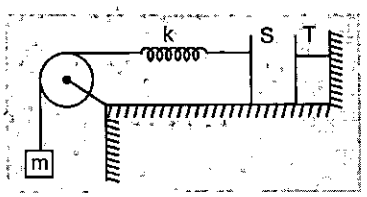
(d) All of the above

31. A conducting body 1 has some initial charge Q, and its capacitance is C. There are two other conducting bodies, 2 and 3 having capacitances: $C_2 = 2C$ and $C_3 \to \infty$. Bodies 2 and 3 are initially uncharged. "Body 2 is touched with body 1. Then, body 2 is removed from body 1 and touched with body 3 and then removed." This process is repeated N times. Then, the charge on body 1 at the end must be:

(a) $Q/3^N$ (b) $Q/3^{N-1}$

(c) Q/N^3 (d) None of these

32. The plates S and T of an uncharged parallel plate capacitor are connected across a battery. The battery is then disconnected and the charged plates are now connected in a system as shown in the figure. The system shown is in equilibrium. All the strings are insulating and massless. The magnitude of charge on one of the capacitor plates is: [Area of plates = A]



(a) $\sqrt{2mgA\varepsilon_0}$ (b) $\sqrt{\frac{4mgA\varepsilon_0}{k}}$

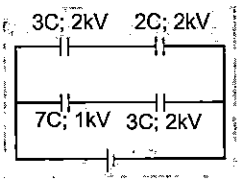
(c) $\sqrt{mgA\varepsilon_0}$ (d) $\sqrt{\frac{2mgA\varepsilon_0}{k}}$

33. A capacitor of capacitance 1μF withstands the maximum voltage 6 kV while a capacitor of 2μF withstands the maximum voltage 4 kV. What maximum voltage will the system of these two capacitor withstands of if they are connected in series?

(a) 10 kV (b) 12 kV

(c) 8 kV (d) 9 kV

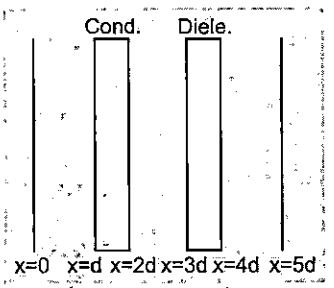
34. The diagram shows four capacitors with capacitances and break down voltages as mentioned. What should be the maximum value of the external emf source such that no capacitor breaks down? **[Hint:** First of all find out the break down voltages of each branch. After that compare them.]

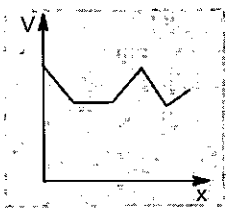


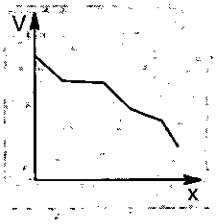
(a) 2.5 kV (b) 10/3 kV

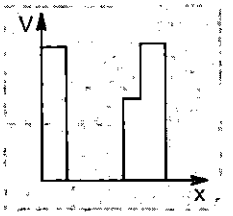
(c) 3 kV (d) 1 kV

35. The distance between plates of a parallel plate capacitor is $5d$. Let the positively charged plate is at $x = 0$ and negatively charged plate is at $x = 5d$. Two slabs one of conductor and other of a dielectric of equal thickness d are inserted between the plates as shown in figure. Potential *versus* distance graph will look like.



(a)


(b)


(c)


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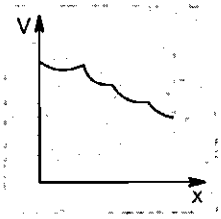
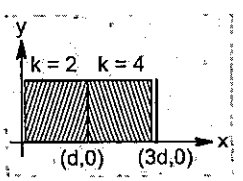
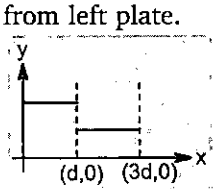
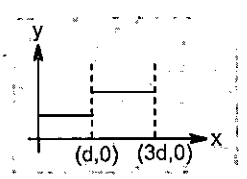
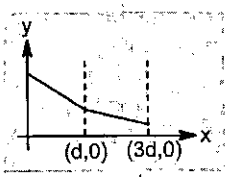


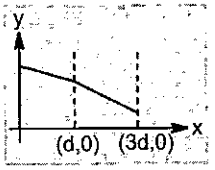
36. A parallel plate capacitor has two layers of dielectric as shown in figure. This capacitor is connected across a battery. The graph which shows the variation of electric field (E) and distance (x) from left plate.



(a)


(b)


(c) 

(d) 

37. A parallel plate capacitor is charged up to voltage V_0 and then detached from the charger. Now, the distance between the plates is varied periodically according to the law $d = d_0 + d_1 \sin \omega t$. The value of d_1 if the potential difference across the capacitor changes according to the law $V = V_0\left(1 + \frac{1}{2}\sin \omega t\right)$ is:

(a) d_0 (b) $\frac{d_0}{2}$

(c) $\frac{3d_0}{2}$ (d) $2d_0$

38. In the above question, the electric field between the plates will

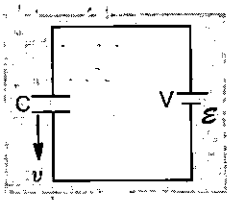
(a) vary according to the law $E = \frac{V_0}{d_0}(1 + \sin \omega t)$

(b) vary according to the law $E = \frac{V_0}{d_0}\left(1 + \frac{1}{2}\sin \omega t\right)$

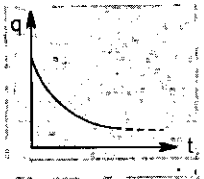
(c) will not vary

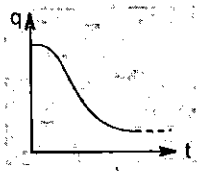
(d) will vary arbitrarily

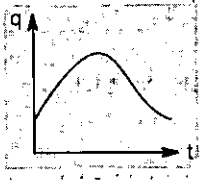
39. An ideal cell is connected across a capacitor as shown in Fig. The initial separation between the plates of a parallel plate capacitor is d. The lower plate is pulled down with a uniform velocity v. Neglect the resistance of the circuit. Then the variation of charge on capacitor with time is given by:



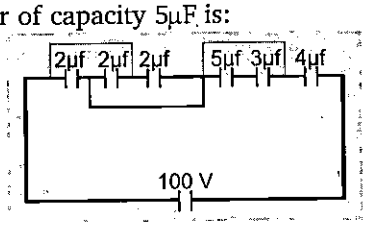
(a)

(b) 

(c) 

(d) 

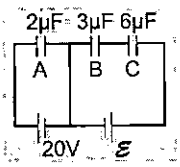
40. In the circuit shown in figure charge stored in the capacitor of capacity 5μF is:



(a) 60μC (b) 20μC

(c) 30μC (d) zero

41. In the given circuit if emf $\mathcal{E}$ is *increased the steady state charge:*



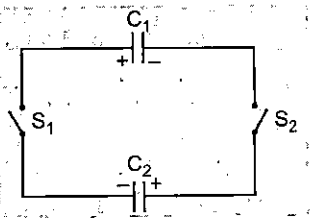
(a) on each capacitor will decrease

(b) will increase for capacitors B and C and decrease for capacitor A

(c) on each capacitor will increase

(d) remains constant for A and increases for capacitors B and C

42. Two capacitors $C_1 = 2\mu F$ and $C_2 = 1\mu F$ are charged to same potential $V = 100V$, but with opposite polarity as shown in the Fig. The switches S_1 and S_2 are closed. The ratio of final energy to the initial energy of the system is:



(a) 1 (b) $\frac{1}{2}$

(c) 1/9 (d) $\frac{1}{4}$

43. A parallel plate capacitor of capacitance C is connected to a battery and is charged to a potential difference V. Another capacitor of capacitance $2C$ is similarly charged to a potential difference $2V$. The charging battery is now disconnected and the capacitors are connect in parallel to each other in such a way that the positive terminal of one is connected to the negative terminal of the other. The final energy of the configuration is:

(a) zero (b) $\frac{3}{2}CV^2$

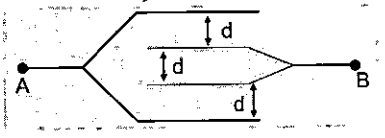
(c) $\frac{25}{6}CV^2$ (d) $\frac{9}{2}CV^2$

44. The capacitance (C) for an isolated conducting sphere of radius (a) is given by $4\pi\varepsilon_0 a$. If the sphere is enclosed with an earthed concentric sphere. The ratio of the radii of the spheres being $\frac{n}{(n-1)}$ then the capacitance of such a sphere will be increased by a factor:

(a) n (b) $\frac{n}{(n-1)}$

(c) $\frac{(n-1)}{n}$ (d) $a.\ n$

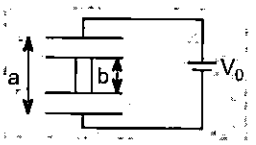
45. Four metallic plates are arranged as shown in the figure. If the distance between each plate then capacitance of the given system between points A and B is: (Given $d << A$)



(a) $\frac{\varepsilon_0 A}{d}$ (b) $\frac{2\varepsilon_0 A}{d}$

(c) $\frac{3\varepsilon_0 A}{d}$ (d) $\frac{4\varepsilon_0 A}{d}$

46. Figure shows two capacitors in series. The rigid center section of length 'b' is movable. The area of each plate is S. If the voltage difference between the plates is maintained constant at V_0. The change in stored energy if the center section is removed is:



(a) $\frac{S\varepsilon_0 V_0^2 a}{2b(a-b)}$ (b) $\frac{S\varepsilon_0 V_0^2 b}{2a(a-b)}$

(c) $\frac{S\varepsilon_0 V_0^2 b}{a(a-b)}$ (d) $\frac{S\varepsilon_0 V_0^2 a}{b(a-b)}$

47. In above figure, if the center section is moved up or down, then:

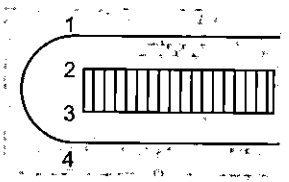
(a) Stored energy will increase

(b) Stored energy will decrease

(c) Stored energy will remain same

(d) Stored energy will vary between a maximum and minimum value

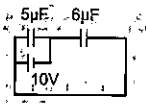
48. Four identical plates 1, 2, 3 and 4 placed parallel to each other at equal distance as shown in the figure. Plates 1 and 4 are joined together and the space between 2 and 3 is filled with a dielectric of dielectric constant $k = 2$. The capacitance of the system between 1 and 3 & 2 and 4 are C_1 and C_2 respectively. The ratio $\frac{C_1}{C_2}$ is:



(a) $\frac{5}{3}$ (b) 1

(c) $\frac{3}{5}$ (d) $\frac{5}{7}$

49. The charge supplied by the battery in the circuit shown in the figure :



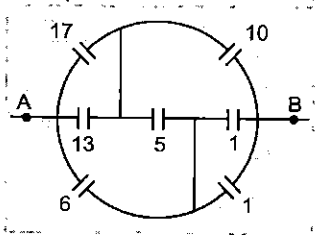
(a) $\frac{300}{11}\mu C$

(b) 60 μC

(c) 50 μC

(d) 110 μC

50. Find equivalent capacitance across AB. (all capacitances in μF)



(a) $\frac{20}{3}\mu F$ (b) 9μF

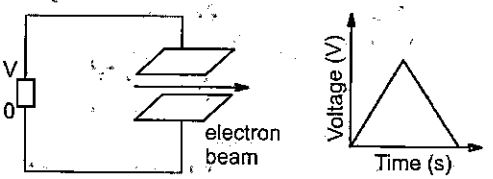
(c) 48μF (d) None

51. The potential across a 3μF capacitor is 12 V when it is not connected to anything. It is then connected in parallel with an uncharged 6μF capacitor. At equilibrium, the charge q on the 3μF capacitor and the potential difference V across it are :

(a) $q=12\mu C$, $V=4V$ (b) $q=24\mu C$, $V=8$ V

(c) $q=36\mu C$, $V=12$ V (d) $q=12\mu C$, $V=6$ V

52. An electron beam passes between two parallel plate electrodes as shown in the diagram. The bottom plate is kept at zero potential, while a slowly varying positive voltage is applied to the upper plate, as shown in the graph. After passing between the plates the beam hits a screen and makes spot. Ignoring gravity as the potential varies the spot is:



(a) deflected up

(b) deflected down

(c) deflected up then down

(d) deflected down then up

53. A capacitor of capacitance of 2μF is charged to a potential difference of 200 V, after disconnecting from the battery, it is connected in parallel with another uncharged capacitor. The final common potential is 20 V, the capacitance of second capacitor is:

(a) 2μF (b) 4μF

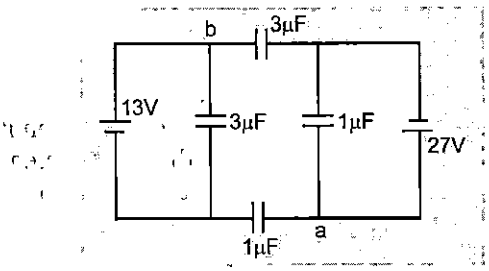
(c) 18μF (d) 16μF

54. A parallel-plate capacitor is connected to a resistanceless circuit with a battery having emf $\mathcal{E}$ until the capacitor is fully charged. The battery is then disconnected from the circuit and the plates of the capacitor are moved to half of their original separation using insulated gloves. Let V_{new} be the potential difference across the capacitor plates after the plates had been moved together and V_{old} be the potential difference across the capacitor plates when connected to the battery. $V_{new} / V_{old} =$

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$

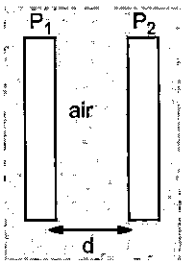
(c) 1 (d) 2

55. Four capacitors and two sources of emf are connected as shown in the Fig. The p.d. in volts between the points a and b is :



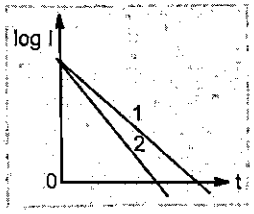
(a) zero (b) 13
(c) 17 (d) 27

56. Two conducting large plates P_1 & P_2 are placed parallel to each other at very small separation 'd'. The plate area of either face of plate is A. A charge $+2Q$ is given to plate P_1 & $-Q$ to the plate P_2 (neglect ends effects). If plate P_1 & P_2 are now connected by conducting wire, then total amount of heat produced is:



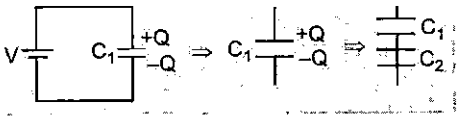
(a) $\frac{4Q^2d}{3\varepsilon_0 A}$ (b) $\frac{9}{8}\frac{Q^2d}{8\varepsilon_0 A}$
(c) $\frac{3Q^2d}{4\varepsilon_0 A}$ (d) None of these

57. A capacitor of capacitance C is charged to a potential difference V_0 and is then discharged through a resistance R. The discharge current gradually decreases, with a straight line 1 corresponding to this process, as shown in Fig where time is along x-axis and the logarithm of the current on y-axis. Later on, one of the three parameters V_0, R or C, is changed (keeping the other two unchanged) in such a manner than the ln I v/s t dependence is represented by the straight line 2. Which option correctly represents the change?



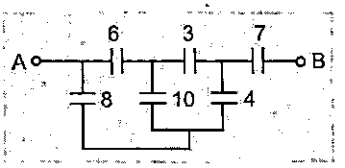
(a) V_0 is decreased (b) R is decreased
(c) R is increased (d) C is decreased

58. A capacitor, C_1 is connected to a battery until charged completely, and then disconnected from the battery. A second capacitor, C_2, is then connected to the first capacitor C_1 as shown in Fig. Let U, V and Q be potential energy of, magnitude of potential difference and charge on capacitor C_1 respectively. What changes occur in capacitor C_1 after C_2 is connected as shown.



(a) V same, Q increases, U increases
(b) V same, Q decreases, U increases
(c) V decrease, Q same, U decreases
(d) None of these

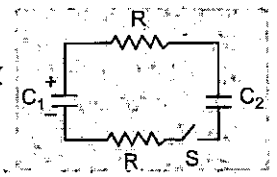
59. In the circuit diagram shown all the capacitors are in μF. The equivalent capacitance between points A and B is: (in μF)



(a) 14/5 (b) 7/5
(c) 3/7 (d) None of these

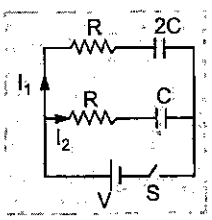
60. The time constant of the CR circuit is $\frac{2}{\ln 2}$ seconds. Capacitor is discharged at time $t = 0$. The ratio of the charge on the capacitor at time $t = 2$ s and 6s is:
(a) 3 : 1 (b) 8 : 1
(c) 4 : 1 (d) 2 : 1

61. In the circuit shown in the figure $C_1 = 2C_2$. Capacitor C_1 is charged to a potential of V. The current in the circuit just after the switch is closed is:

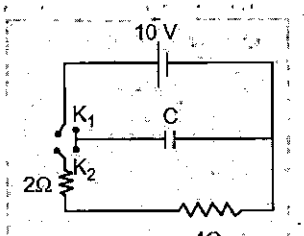


(a) zero
(b) $\frac{2V}{R}$
(c) Infinite
(d) $\frac{V}{2R}$

62. In the circuit shown in the figure switch S is closed at time $t = 0$. Let I_1 and I_2 be the currents at any time t then the ratio $\frac{I_1}{I_2}$:
(a) is constant
(b) increases with time
(c) decreases with time
(d) first increases and then decreases

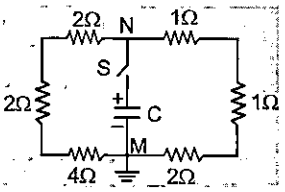


63. A capacitor of capacitance 3μF is first charged by connecting it across a 10 V battery by closing the key K_1. Then it is allowed to get discharged through 2 Ω and 4 Ω resistors by closing the key K_2. The total energy dissipated in 4 Ω resistor is equal to :



(a) 10 μ J (b) 100 μ J
(c) 0.1 μ J (d) zero

64. A capacitor of capacity 6 μ F and initial charge 160 μ *C* is connected with key *S* and resistance as shown in figure. Point *M* is earthed. If key is closed at $t = 0$; then the current through resistance *R* (= 1Ω) at $t = 16$ μ s is:



(a) $\frac{10}{3e}$ A (b) $\frac{10}{e}$ A

(c) $\frac{20}{3e}$ A (d) none

65. Consider a capacitor charging circuit. Let Q_1 be the charge given to the capacitor in a time interval of 10 μs and Q_2 be the charge given in the next time interval of 10 ms. Let 10 μ C charge be deposited in time interval t_1 and the next 10 μ C charge deposited in next time interval t_2, then :

(a) $Q_1 > Q_2, t_1 > t_2$

(b) $Q_1 > Q_2, t_1 < t_2$

(c) $Q_1 < Q_2, t_1 > t_2$

(d) $Q_1 < Q_2, t_1 < t_2$

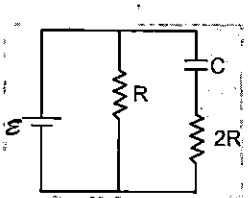
66. The time constant of charging of the circuit shown in the figure is:

(a) $\frac{2RC}{3}$

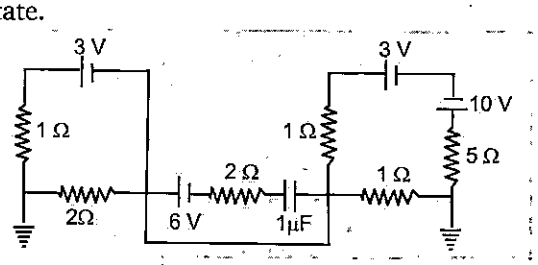
(b) $2RC$

(c) $3RC$

(d) $\frac{3RC}{2}$



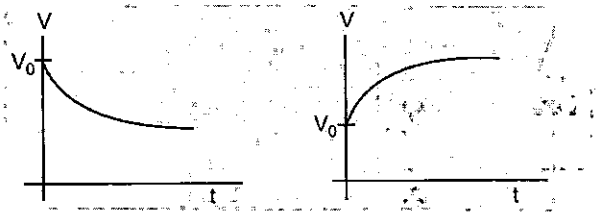
67. For the circuit shown in the figure, determine the charge of capacitor in steady state.



(a) 4μC (b) 6μC

(c) 1μC (d) zero

68. A capacitor is charged upto a potential V_0. It is then connected to a resistance *R* and a battery of emf $\mathcal{E}$. Two possible graphs of potentials across capacitor *vs* time are shown.



What is the most reasonable explanation of these graphs ?

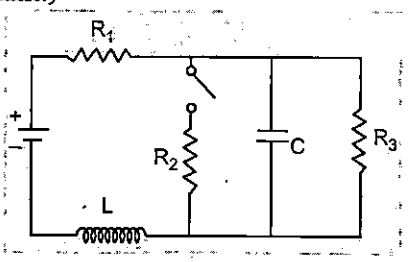
(a) The first graph shows what happens when the capacitor has a less than $\mathcal{E}$ potential initially and the second shows what happens when it has a greater than $\mathcal{E}$ potential initially.

(b) The first graph shows what happens when the capacitor has a greater than *E* potential initially and the second shows what happens when it has a less than *E* potential initially.

(c) The first graph is the correct qualitative shape for any initial potential, but the second is not possible.

(d) The second graph is the correct qualitative shape for any initial potential, but the first is not possible

69. The circuit shown in the Fig. is in a steady state with the switch open. When the switch is closed, which of the following will change immediately? (All of them eventually change, but three of them stay the same for an instant.)



(a) the potential difference across the capacitor *C*

(b) the current through the inductor *L*

(c) the potential difference around the resistor R_1

(d) the current through the resistor R_2

70. In an *RC* circuit, the time required for the charge on a capacitor to build up to a given fraction of its equilibrium value, is independent of:

(a) the value of the applied emf to the circuit

(b) the value of *C*

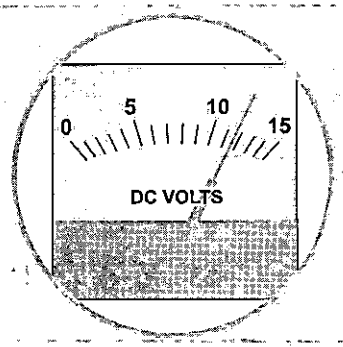
(c) the value of *R*

(d) none of the above

ANSWERS

Level-1: Only One Alternative is Correct

1.	(c)	**2.**	(b)	**3.**	(b)	**4.**	(c)	**5.**	(d)	**6.**	(b)	**7.**	(c)	**8.**	(a)
9.	(b)	**10.**	(b)	**11.**	(b)	**12.**	(b)	**13.**	(c)	**14.**	(d)	**15.**	(a)	**16.**	(d)
17.	(a)	**18.**	(b)	**19.**	(a)	**20.**	(c)	**21.**	(b)	**22.**	(a)	**23.**	(a)	**24**	(c)
25.	(b)	**26.**	(c)	**27.**	(d)	**28.**	(a)	**29.**	(b)	**30.**	(b)	**31.**	(a)	**32.**	(a)
33.	(d)	**34.**	(a)	**35.**	(b)	**36.**	(a)	**37.**	(b)	**38.**	(c)	**39.**	(b)	**40.**	(d)
41.	(d)	**42.**	(c)	**43.**	(b)	**44.**	(a)	**45.**	(b)	**46.**	(b)	**47.**	(c)	**48.**	(b)
49.	(d)	**50.**	(b)	**51.**	(a)	**52.**	(a)	**53.**	(c)	**54.**	(b)	**55.**	(c)	**56.**	(b)
57.	(d)	**58.**	(d)	**59.**	(a)	**60.**	(c)	**61.**	(d)	**62.**	(b)	**63.**	(b)	**64.**	(c)
65.	(b)	**66.**	(b)	**67.**	(d)	**68.**	(b)	**69.**	(d)	**70.**	(a)				



SOLUTION

Level-1: Only One Alternative is Correct

1. [c] Final net charge on positive plate $= CV + Q = Q_1$

Final net charge on negative plate $= -CV = Q_2$

So, final charge on capacitor

$$= \frac{Q_1 - Q_2}{2} = \frac{2CV + Q}{2}$$

So, potential difference between the plates of capacitor $= \frac{2CV + Q}{2C} = V + \frac{Q}{2C}$

2. [b] $d_0 = 3$ mm

$$C_0 = \frac{A\varepsilon_0 \times 4}{d_0} = \frac{A\varepsilon_0 \times 4}{3}$$

$$C_1 = \frac{A\varepsilon_0}{\frac{d_1}{K_1} + \frac{d_2}{4}} = \frac{A\varepsilon_0}{\frac{3}{4} + \frac{5}{K}}$$

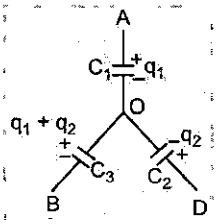
Given $C_1 = \frac{C_0}{2}$

$$\Rightarrow \quad \frac{A\varepsilon_0}{\frac{3}{4} + \frac{5}{K}} = \frac{1}{2}\frac{A\varepsilon_0}{\frac{3}{4}}$$

$$\Rightarrow \quad \frac{3}{2} = \frac{5}{K} + \frac{3}{4}$$

$$\Rightarrow \quad \frac{5}{K} = \frac{3}{4} \Rightarrow K = \frac{20}{3}$$

3. [b] Let the distribution of charges be as shown in figure.



Using Kirchhoff's law from A to B

$$V_A - \frac{q_1}{C_1} - \frac{(q_1 + q_2)}{C_3} = V_B$$

$$\Rightarrow \quad q_1(C_1 + C_3) + q_2 C_1 = (V_A - V_B)C_1 C_3 \qquad ...(i)$$

From A to D

$$V_A - \frac{q_1}{C_1} + \frac{q_2}{C_2} = V_D$$

$$\Rightarrow \quad q_1 C_2 - q_2 C_2 = C_1 C_2 (V_A - V_D) \qquad ...(ii)$$

From equation (i) + (ii), we get

$$q_1(C_1 + C_2 + C_3) = C_1[(C_1 + C_3)V_A - C_2 V_D - C_3 V_D]$$

$$\Rightarrow \frac{q_1}{C_1} = \frac{1}{(C_1 + C_2 + C_3)}[(C_2 + C_3)V_A - C_2 V_D - C_3 V_B] \qquad ...(iii)$$

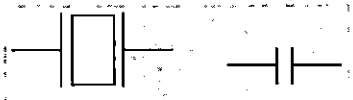
Now from A to O

$$V_A - \frac{q_1}{C_1} = V_O$$

$$\Rightarrow \quad V_O = V_A - \frac{q_1}{C_1}$$

$$V_O = \frac{V_A C_1 + V_D C_2 + V_B C_3}{C_1 + C_2 + C_3} \text{ (using eqn.(iii)}$$

4. [c]



Charge $q_0 = C_0 V_0$ $\quad q_0 = C_0 V_0$

(charge is conserved)

Capacitance C_0 $\quad C' = \frac{C_0}{K}$

Energy $\quad U = \frac{1}{2}\frac{q_0^2}{C_0}$ $\quad V' = \frac{1}{2}\frac{q_0^2}{C'}$

$$= \frac{1}{2}\frac{q_0^2}{C_0} K$$

$$U' = KU$$

Work done = Final energy – Initial energy

$$= KU - U$$

$$= (K - 1)U$$

Given $(K-1)U = 4U$

$\Rightarrow$ $K-1=4$

$K=5$

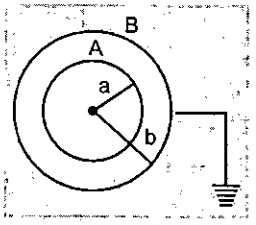
5. [d] Capacitance of left section $= \frac{2\varepsilon_0 A}{d} = 2\,C_0$ (say)

Capacitance of right section $= \frac{6\varepsilon_0 A}{2d} = 3\,C_0$

Since charge on each section will be same so

$$V \propto \frac{1}{C} \Rightarrow \frac{V_1}{V_2} = \frac{C_2}{C_1} = \frac{3}{2}$$

6. [b] When outer sphere 'B' is earthed only



$$C_{net} = C_1 = \frac{4\pi\varepsilon_0 ab}{b-a}$$

When inner sphere is earthed there will be two capacitors

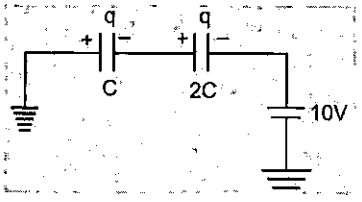
$$C_2 = C_1 + C_1^1 = \frac{4\pi\varepsilon_0 ab}{b-a} + 4\pi\varepsilon_0 b$$

$$C_2 = \frac{4\pi\varepsilon_0 b^2}{b-a}$$

$$\frac{C_1}{C_2} = \frac{ab}{b^2} = \frac{a}{b}$$

7. [c] Let the charge distribution be as shown in figure.

From Kirchhoff's law



$\Rightarrow$ $0 - \frac{q}{C} - \frac{q}{2C} - 10 = 0$

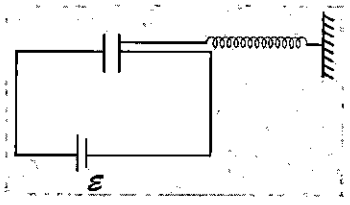
$$\frac{3q}{2C} = -10,\ q = \frac{-20C}{3}$$

$$= -\frac{20 \times 6\mu C}{3}$$

$q = 40\mu C$ *i.e.* charge on capacitor is $40\mu C$

8. [a] Stretch in the spring $= d - 0.8d = 0.2d$

force due to spring $= Kx = K \times 0.2d$



Electric field due to one plate of capacitor $= \frac{q}{2\varepsilon_0 A}$

$\therefore$ force on plate $= \frac{q^2}{2\varepsilon_0 A}$

In steady state $\frac{q^2}{2\varepsilon_0 A} = K \times 0.2d$ ...(i)

Here $q = C\mathcal{E}, C = \frac{A\varepsilon_0}{d}$

Putting the values in equation (i), we get

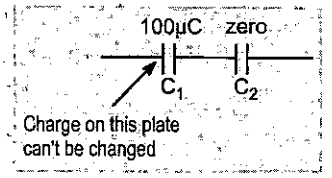
$$\frac{C^2\mathcal{E}^2}{2\varepsilon_0 A} = K \times 0.2d$$

$$K = \frac{A\varepsilon_0 \mathcal{E}^2}{0.4d^3}$$

$$K = \frac{2.5\,\varepsilon_0 A\mathcal{E}^2}{d^3}$$

9. [b] Initially charge on capacitor $C_1 = 100\mu C$

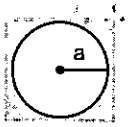
Now C_1 is attached to C_2 through one of the plates but still the other plate isolated.



Due to this charge on one plate of capacitor C_1 can't change. This means that charge on other plate will also not be affected.

$\Rightarrow$ charge on $C_2 = 0$

10. [b] Let initially one shell of radius 'a'

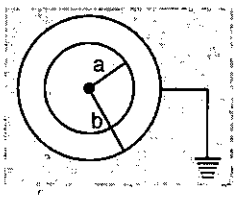


$\Rightarrow$ $C_1 = 4\pi\varepsilon_0 a$

Now when it is enclosed by a earthed concentric shell

$$C_2 = \frac{4\pi\varepsilon_0 ab}{b-a}$$

Given $C_2 = nC_1$



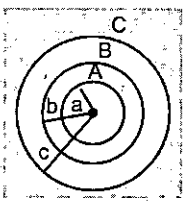
$\Rightarrow$ $\frac{4\pi\varepsilon_0 ab}{b-a} = n4\pi\varepsilon_0 a$

$\Rightarrow$ $b = nb - na$

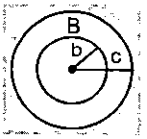
$\Rightarrow$ $na = (n-1)b$

$$\frac{b}{a} = \frac{n}{n-1}$$

11. [b] When A, B are connected then the capacitor formed in between shells A and B is short-circuited *i.e.*, all the charge is transferred to shell B.

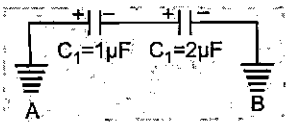


$\therefore$



Now capacitance $= \frac{4\pi\varepsilon_0 bc}{c-b}$

12. [b]

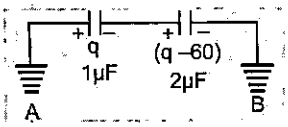


Initially charge on $C_1 = 100\times 1\mu C = 100\mu C$

Initially charge on $C_2 = 20\times 2\mu C = 40\mu C$

$\Rightarrow$ sum of charges on –ve plate of C_1 and +ve plate of C_2 will be constant (since isolated)

$= -100 + 40 = -60\mu C$



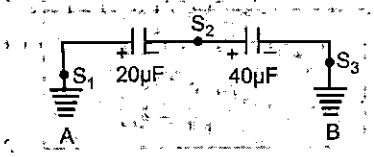
Now let the distribution of charge is as shown in figure.

$$V_B - \frac{q}{1} - \frac{(q-60)}{2} = V_B$$

$\Rightarrow$ $$\frac{-3q}{2} + \frac{60}{2} = 0$$

$\Rightarrow$ $$q = 20\mu C$$

$\Rightarrow$ final distribution is



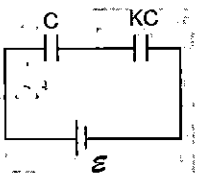
Charge flown through $S_1 = 20 - 100 = -80\mu C$

Charge flown through $S_2 = -20 - (-100) = 80\mu C$

Charge flown through $S_3 = 40 - (-40) = 80\mu C$

13. [c] **Case I** $C_{equivalent} = \frac{KC}{K+1}$

$$Q_1 = Q_2 = K\frac{C\mathcal{E}}{K+1} \quad ...(i)$$



(since charge on capacitor in series is same)

Case II When dielectric is removed

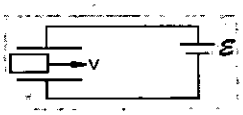
$\Rightarrow$ $$Q_1' = Q_2' = \frac{C\mathcal{E}}{2} \quad ...(ii)$$

From eqn. (i) and (ii),

$$\frac{Q_1'}{Q_1} = \frac{Q_2'}{Q_2} = \frac{K+1}{2K}$$

14. [d] Area of dielectric slab $= A$

Area of plate of capacitor $= 2A$



after time 't' slab will cover distance 'vt' inside the capacitor let $A = l\times b$

Now capacitance at time 't'

$$C(t) = \frac{A\varepsilon_0}{d} + \frac{K(vt\times b)\varepsilon_0}{d} + \frac{(l-vt)\times b\varepsilon_0}{d}$$

$$Q(t) = CV = C\mathcal{E} = C(t)\times \mathcal{E}$$

$$I = \frac{dQ(t)}{dt} = \frac{Kv\times b\varepsilon_0}{d} - \frac{vb\varepsilon_0}{d}$$

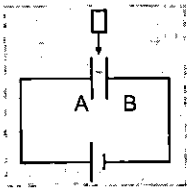
$$I = \frac{(K-1)vb\varepsilon_0}{d} = \text{const.}$$

i.e., $I = \text{const.}$

Also when slab totally enters the capacitor after that it goes out from the other side, it will move inside the capacitor for some time with C remaining constant and than C will start decreasing. So current will reverse its direction.

15. [a] Because part of initial potential energy will be converted into kinetic energy of the charge system.

16. [d] Same as question 14



Initially as slab enters the plate +ve charge on A increases as capacitance increases –ve charge on B increases

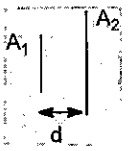
$\Rightarrow$ initially +ve charge flow from B to A

$\Rightarrow$ current flow from B to A

Now when slab emerges from the other side charge on A and B decreases (since capacitance decreases)

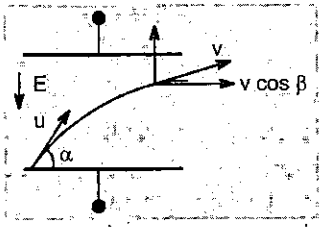
$\therefore$ direction of current is from A to B.

17. [a] Capacitance depends on area common to both plates *i.e.*, area of smaller plate



$\Rightarrow$ $$C = \frac{A_1\varepsilon_0}{d}$$

18. [b] Since there is no horizontal force

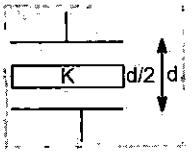


$\Rightarrow$ horizontal velocity will remains same

$\Rightarrow$ $$u\cos\alpha = v\cos\beta \quad ...(i)$$

$$\frac{\text{K. E.}_{\text{initial}}}{\text{K. E.}_{\text{final}}} = \frac{\frac{1}{2}mu^2}{\frac{1}{2}mv^2} = \left(\frac{\cos\beta}{\cos\alpha}\right)^2 \quad \text{[using egn.(i)]}$$

19. [a] Initially $C = \frac{A\varepsilon_0}{d}$



Now after inserting slab

$$C' = \frac{A\varepsilon_0}{d - \frac{d}{2} + \frac{d}{2K}}$$

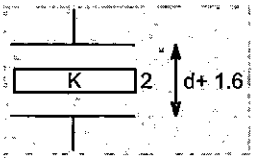
$$= \frac{2A\varepsilon_0}{d\left(1+\frac{1}{K}\right)} = \frac{2C}{1+\frac{1}{K}}$$

Given $C' = \frac{4}{3}C$

$\Rightarrow \quad \frac{2C}{1+\frac{1}{K}} = \frac{4}{3}C$

$\Rightarrow \quad 1+\frac{1}{K} = \frac{3}{2} \Rightarrow K = 2$

20. [c] Charge will remain constant and for same potential difference *i.e.* 100 V, capacitance should be same (since $q = eV$)



$\Rightarrow \quad \frac{A\varepsilon_0}{d} = C'$

$C' = \frac{A\varepsilon_0}{d+1.6-2+\frac{2}{K}} = \frac{A\varepsilon_0}{d}$

$\frac{1}{d-0.4} = \frac{1}{d} \Rightarrow \frac{2}{K} = 0.4 \Rightarrow K = 5$

21. [b] $C_1 = 4\pi\varepsilon_0 b, \quad C_2 = \frac{4\pi\varepsilon_0 ab}{b-a}$

$C_3 = \frac{4\pi\varepsilon_0 b^2}{b-a}, \quad C_4 = 4\pi\varepsilon_0 b$

Clearly $C_1 = C_4$

also $C_3 > C_1 = C_4 \left(\because \frac{b^2}{b-a} > b\right)$

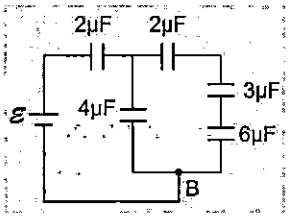
Now $C_2 = \frac{4\pi\varepsilon_0 ab}{b-a} \approx 4\pi\varepsilon_0 a$

$\Rightarrow \quad C_3 < C_1 = C_4 < C_2$

$\therefore$ '*b*' is correct

22. [a] Charge on 6μF = 10μC

$\Rightarrow$ charge on each of 6, 3, 2 μF each will be 10 μC equivalent of 6, 3 and 2 μF = 1μF



$\Rightarrow \quad V_A - V_B = \frac{10}{1} = 10\ \text{V}$

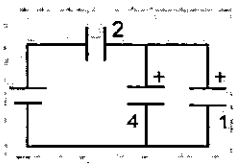
$\therefore$ charge on 4μF

$= 4 \times 10\mu\text{C}$

$= 40\ \mu\text{C}$

23. [a] Charge on 2μF = 50 μC

Potential drop across $2\mu\text{F} = \frac{50\mu\text{C}}{2\mu\text{F}} = 25\ \text{V}$

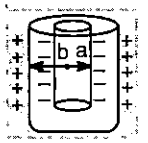


So total drop = emf of cell

$= 25 + 10 = 35\ \text{V}$

24. [c] $\int \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = \frac{q}{\varepsilon_0}$

Inside the inner cylinder *i.e.*, of radius '*a*'



There is no charge

$\therefore \quad E = 0$

From radius *a* to *b*, $q_{\text{enclosed}} \neq 0$

i.e., it encloses charge on inner cylinder

$\int \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = E \cdot 2\pi r \times l + 0 + 0 = \frac{q}{\varepsilon_0}$

$E = \frac{q}{2\pi\varepsilon_0 l r}$

$E = \frac{K}{r}$

Again for $r > b$

$q_{\text{enclosed}} = 0$

$E = 0$

25. [b] Electric field will be radially outwards and in direction of field potential decreases

$\therefore$ maximum potential will be at centre.

26. [c] $U_{\text{initial}} = 1\ \text{J}$

$U_{\text{final}} = 4\ \text{J}$

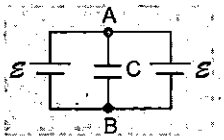
$V = \frac{1}{2}CV^2 = \frac{1}{2}\frac{q^2}{C}$

$q = CV$

C is fixed $\Rightarrow$ if we change '*V*' then *q* will be changed

$\therefore$ charge and potential both should be doubled

27. [d] As switch is closed potential difference across capacitor is still '*E*'



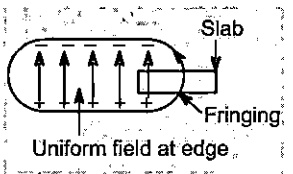
i.e., $V_A - V_B = E$ (constant *t*.)

{from Kirchhoff's law}

$\therefore$ charge on capacitor does not change

$\therefore$ heat dissipated = 0

30. [b] The electric field near the edges of capacitor plates is not uniform. A component of field exist parallel to dielectric slab surface which exert force on it towards the capacitor.



37. [b] As charge on capacitor is conserved initially charge

$= C_0 V_0 = \frac{A\varepsilon_0 V}{d}$ ($\because$ charge at $t = 0$ = charge at '*t*')

Putting value of '*V*'

We get $d = \frac{d_0}{2}$

38. [c]

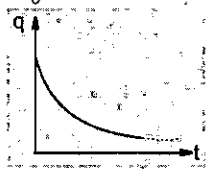
$$E = \frac{V}{d}$$

On putting values

$$E = \frac{V_0}{d_0} = \text{const.}$$

39. (b) Let the area of parallel plate be A. Then the capacitance of the capacitor at any time t is $C = \frac{A\varepsilon_0}{d + vt}$. Hence the charge on capacitor at any instant t is $q = C\mathcal{E} \frac{A\varepsilon_0}{d+vt}\mathcal{E}$.

or $q(d + vt) = A\varepsilon_0\mathcal{E} = \text{constant}$



Therefore the graph showing the variation of q with t

41. [d] As $\mathcal{E}$ is increased

Potential difference across capacitor A is not changed *i.e.*, it remains 20 V

$\therefore$ charge on capacitor A does not change.

Now as $\mathcal{E}$ is increased

Potential difference across B and C increases (since charge on equivalent of B and C increases)

$\therefore$ charge on B and C increases.

42. [c] Q = total charge shared by the capacitors

$$= C_1V - C_2V = (C_1 - C_2)V$$

C = affective capacitance of the circuit

V_1 = Voltage across each capacitor

$$= \frac{Q}{C_1 + C_2} = \left[\frac{C_1 - C_2}{C_1 + C_2}\right]V$$

$$= \left(\frac{2-1}{2+1}\right) \times 100 = \frac{100}{3}\text{V}$$

E_i = Initial energy of the system

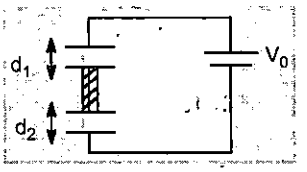
$$= \frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2$$

$$= \frac{1}{2}(C_1 + C_2)V^2$$

E_f = Final energy of the system $= \frac{1}{2}(C_1 + C_2)V_1^2$

$$\frac{E_f}{E_i} = \frac{V_1^2}{V^2} = \left(\frac{100}{3}\right)^2 \frac{1}{(100)^2} = \frac{1}{9}$$

46. [b] Let 'd_1' be the distance between the two upper plates and 'd_2' be the distance between the two lower plates



$\therefore$ $d_1 + d_2 = a - b$

$\therefore$ $C_1 = \frac{\varepsilon_0 S}{d_1}$, $C_2 = \frac{\varepsilon_0 S}{d_2}$

$\Rightarrow$ $C_{net} = \frac{C_1C_2}{C_1 + C_2} = \frac{S\varepsilon_0}{d_1 + d_2} = \frac{S\varepsilon_0}{a-b}$

As C_{net} is independent of d_1 and d_2

$\Rightarrow$ energy stored is independent of position of centre section

$$u = \frac{1}{2}CV_0^2 = \frac{S\varepsilon_0 V_0^2}{a-b}$$

If centre section is removed energy stored

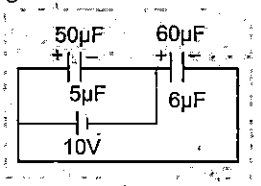
$$= u' = \frac{1}{2}\frac{S\varepsilon_0 V_0^2}{a}$$

$$\Delta u = u - u' = \frac{S\varepsilon_0 V_0^2 b}{2(a-b)a}$$

47. [c] C_{net} is constant *i.e.*, independent of d_1 and d_2

$\therefore$ stored energy $= \frac{1}{2}C_{net}V_0^2 = \text{constant}$

49. [d] Total charge



Supplied by battery = 50 + 60

= 110 μC

51. [a]

3μF

$$Q = CV = 3 \times 10^{-6} \times 12$$

$$Q = 36\mu\text{C}$$

$$\frac{36-q}{3} = \frac{q}{6}$$

$$72 - 2q = q$$

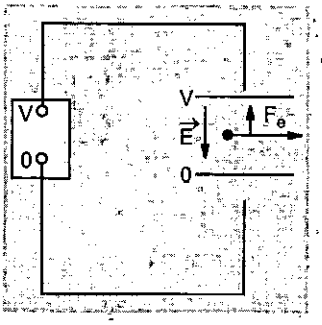
$$q = \frac{72}{3} = 24\mu\text{C}$$

$\therefore$ Charge on 3μF

$\Rightarrow$ $36 - q = 12\mu\text{C}$

$$V = \frac{36-q}{C} = \frac{12}{3} = 4\text{Volt}$$

52. [a]



electrostatic force electrons is opposite to direction of electric field.

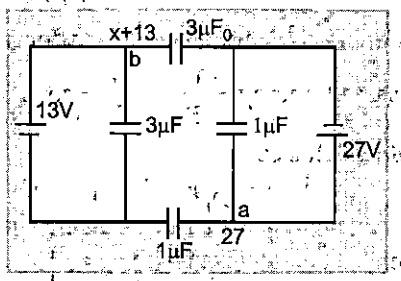
53. [c] $V' = \frac{CV}{C+C'}$

54. [b] Since process is done after disconnecting the battery, thus charge remain conserved.

So, $\frac{V_{new}}{V_{old}} = \frac{q/C_{new}}{q/C_{old}} = \frac{C_{old}}{C_{new}}$

$= \frac{\varepsilon_0 A(d/2)}{d\varepsilon_0 A} = \frac{1}{2}$

55. [c]



$(x+13)\times 3 = (27-x)\times 1$

$3x+39 = -x+27$

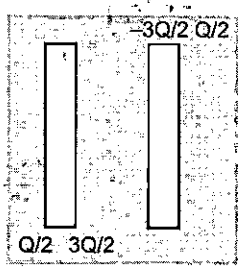
$x = -3$

So $V_a - V_b = 27-(x+13) = 17$

56. [b]

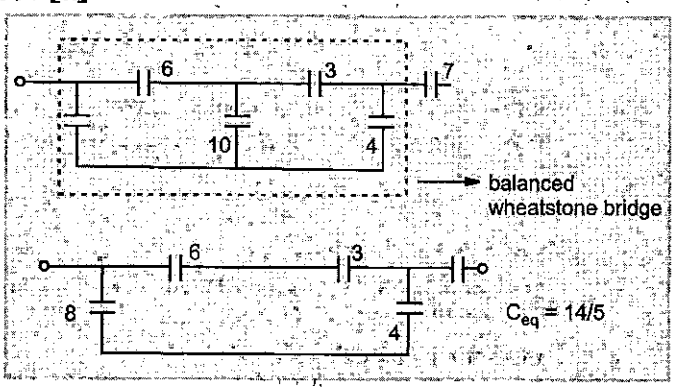
$E_{in} = \frac{q^2}{2C} = \frac{(3Q/2)^2}{2\left(\frac{A\varepsilon_0}{d}\right)}$

$= \frac{9Q^2 d}{8A\varepsilon_0}$



58. [d] All the quantities remain same as no charge transfer occurs after C_2 is connected to C_1.

59. [a]



60. [c] $q = q_0 e^{-t/RC}$

$q = q_0 e^{-t/(2/\ln 2)}$

$q(t) = q_0 e^{\frac{-t\ln 2}{2}}$

$q(2) = q_0 e^{\frac{-2}{2}\times \ln 2}$

$= q_0 e^{-\ln 2}$

$q(2) = \frac{q_0}{2}$

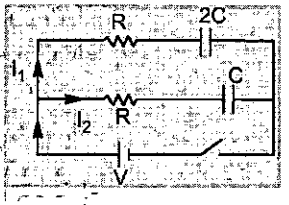
$q(6) = q_0 e^{-3\ln 2} = \frac{q_0}{8}$

$\frac{q(2)}{q(6)} = \frac{4}{1}$

61. [d] An uncharged capacitor initially offers O resistance $\therefore R_{equivalent}$ across $2C$ is $2R$. As capacitor is charged to potential V initial current $I = \frac{V}{2R}$

62. [b] For $2C$

$I_1 = \frac{V}{R} e^{-t/2RC}$



For C $I_2 = \frac{V}{R} e^{-t/RC}$

$\frac{I_1}{I_2} = e^{-\frac{t}{2RC}+\frac{t}{RC}} = e^{\frac{t}{2RC}}$

$\Rightarrow \frac{I_1}{I_2}$ increases with time.

63. [b] Total energy stored in capacitor

$$U = \frac{1}{2} \times 3 \times 10^2 = 150\ \mu\text{J}$$

Some energy will be dissipated through 4 Ω and some energy will be dissipated through 2 Ω.

Heat dissipated $H = \frac{1}{2} I^2 R t$

Since, $H \propto R$

$\therefore$ Heat dissipated through 4 Ω will be twice of heat dissipated through 2 Ω.

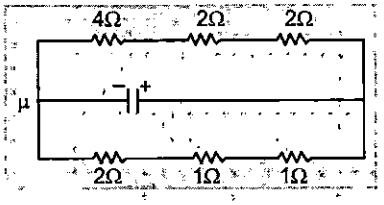
$\therefore$ $U = H + \frac{H}{2}$

$$150 = \frac{3H}{2}; \quad H = 100$$

$\therefore$ Heat dissipated through 4 Ω is 100 μ J.

64. [c] $q_0 = 160\ \mu\text{C}$

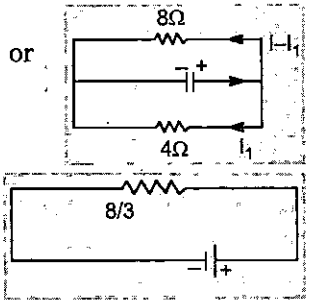
$C = 6\ \mu\text{F}$



$$q = 160 e^{-t/\frac{8}{3}\times 6}$$

$$q = 160 \times e^{-t/16}$$

or



$$I = \frac{160}{16} e^{-t/16}$$

$$I = 10 e^{-t/16}$$

$$I(16) = 10 e^{-1} = \frac{10}{e}$$

$$I_1 \times 4 = \left(\frac{10}{e} - I_1\right) \times 8$$

$$12 I_1 = \frac{80}{e}$$

$$\Rightarrow \quad I_1 = \frac{80}{12e} = \frac{20}{3e}\ \text{A}$$

65. [b] In charging circuit

$$q = q_0(1 - e^{-t/RC})$$

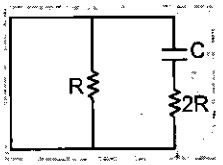
$$\frac{dq}{dt} = \frac{q_0}{R_C} e^{-t/RC}$$

i.e., decreases with time rate of charging decreases with time *i.e.*, for same time interval

$$Q_1 > Q_2$$

Also for depositing same charge time taken by later will be more *i.e.*, $t_1 > t_2$.

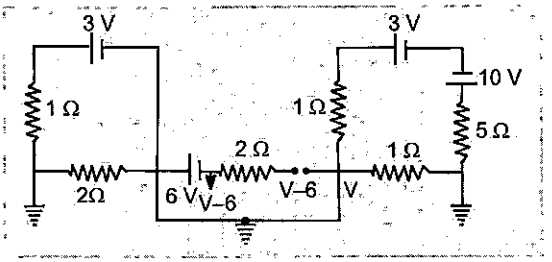
66. [b] While calculating time constant, imagine battery as a conducting wire and the figure will be as shown here.



Equivalent resistance across $C = 2R$

So, time constant $\tau = 2R \times C$

67. [b] In steady state the capacitor is fully charged and is treated as open circuit, so no current flows through branch containing capacitor in steady state. So, the circuit can be redrawn as.



Potential difference across capacitor in steady state

$$= V - 6 - V = -6V$$

(–ve sign signifies that left hand plate is of negative polarity)

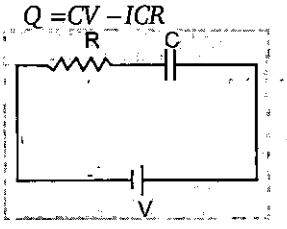
$$\text{Charge} = CV = 1 \times 6 = 6\mu\text{C}$$

68. [b] If the capacitor has greater than $\mathcal{E}$ potential initially its potential will decrease and graph is first one. When capacitor has less than $\mathcal{E}$ potential initially it will follow the second graph.

69. [d] On closing the circuit the current through R_2 will suddenly charge only.

70. [a] $Q = C(V - IR)$

$$Q = CV - ICR$$



$$\frac{t}{CR} = -\ln\left(\frac{CV - nCV}{CV}\right)$$

$$CV - Q = \frac{CR\,dQ}{dt} - \frac{t}{CR} = \ln(1 - n)$$

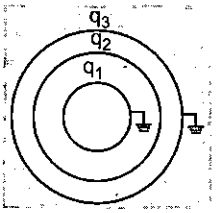
$$\Rightarrow \quad \int_0^t CR = \int_0^{nCV} \frac{dQ}{CV - Q}$$

$$t = -CR \ln(1 - n)$$

independent of V.

Level 2 More than One Alternative is/are Correct

1. Three concentric conducting spherical shells have radii r, $2r$ and $3r$ and charges q_1, q_2 and q_3 respectively. Innermost and outermost shells are earthed as shown in figure. Select the correct alternatives:



(a) $q_1 + q_3 = -q_2$

(b) $q_1 = \frac{-q_2}{4}$

(c) $\frac{q_3}{q_1} = 3$

(d) $\frac{q_3}{q_2} = -\frac{1}{3}$

2. A positive charge q is given to each plate of a parallel plate capacitor having area of each plate A and separation between them, d :

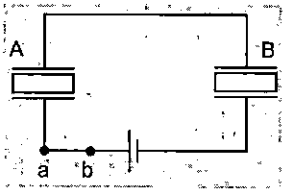
(a) Since both plates are identically charged, therefore, capacitance becomes equal to zero

(b) Energy stored in the space between the capacitor plate is equal to $\frac{q^2 d}{\varepsilon_0 A}$

(c) No charge appears on the inner surface of the plates

(d) Potential difference between the plates is $\frac{2qd}{\varepsilon_0 A}$

3. Identical dielectric slabs are inserted into two identical capacitors A and B. These capacitors and a battery are connected as shown in the figure. Now the slab of capacitor B is pulled out with battery remaining connected :



(a) During the process, charge flow from a to b

(b) Finally charge on capacitor B will be less than that on A

(c) During the process work is done by the external force F, part of which appears as heat in the circuit

(d) During the process internal energy of the battery increases

4. A capacitor of capacitance C is charged to a potential V_0. The charging battery is disconnected and the capacitor is connected to a capacitor of unknown capacitance C_x. The potential difference across the combination is V, then the value of C_x should be :

(a) $\frac{C(V_0 - V)}{V}$ (b) $\frac{C(V - V_0)}{V}$

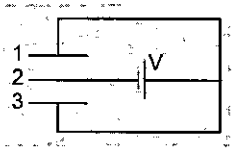
(c) $\frac{CV}{V_0}$ (d) $\frac{CV_0}{V}$

5. Two parallel plate capacitors of capacitances C and $2C$ are connected in parallel and charged to a potential difference V. The battery is then disconnected and the region between the plates of the capacitance C is completely filled with a material of dielectric constant K, then the potential difference across the capacitors now becomes:

(a) $\frac{V}{3K+1}$ (b) $\frac{3V}{K+2}$

(c) $\frac{2V}{K}$ (d) $\frac{V}{2K+1}$

6. Three plates 1, 2 and 3 each of area A and separation between two consecutive plates d, are connected as shown in the figure. The energy stored when the capacitors are fully charged is:



(a) $\frac{\varepsilon_0 AV^2}{2d}$ (b) $\frac{\varepsilon_0 AV^2}{d}$

(c) $\frac{2\varepsilon_0 AV}{d}$ (d) $\frac{\varepsilon_0 AV^2}{d}$

7. A capacitor of capacitance C_0 is charged to a potential V_0 and isolated. A small capacitor C is then charged from C_0, discharged and charged again; the process being repeated n times. Due to this, potential of the larger capacitor is decreased to V. Value of C is:

(a) $C_0\left[\frac{V_0}{V}\right]^{1/n}$ (b) $C_0\left[\left(\frac{V_0}{V}\right)^{1/n} - 1\right]$

(c) $C_0\left[\left(\frac{V_0}{V}\right) - 1\right]^n$ (d) $C_0\left[\left(\frac{V_0}{V}\right)^n - 1\right]$

8. A parallel plate air capacitor is connected to a battery. The quantities charge, voltage, electric field and electrostatic energy associated with this capacitor are given by Q_0, V_0, E_0 and U_0 respectively. A dielectric slab is now introduced to fill the space between the plates with battery still in connection. The corresponding quantities are now given by Q, V, E and U related to the previous ones as :

(a) $Q > Q_0$ (b) $V > V_0$

(c) $E > E_0$ (d) $U > U_0$

9. When a charge of amount Q is given to an isolated metal plate X of surface area A its surface charge density becomes σ_1. When an isolated identical plate Y is brought close to X the surface charge density on X becomes σ_2. When Y is earthed the surface charge density becomes σ_3:

(a) $\sigma_1 = \frac{Q}{A}$ (b) $\sigma_1 = \frac{Q}{2A}$

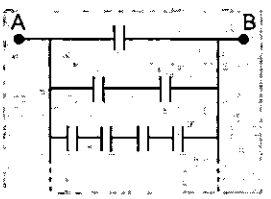
(c) $\sigma_1 = \sigma_2$ (d) $\sigma_3 = \frac{Q}{A}$

10. A parallel plate capacitor of plate area A and separation d is charged to a potential V and then battery is removed. Now a slab of dielectric constant K is introduced between the plates of the capacitor. If Q, E and W denote the charge, electric field between the plates (after introducing the slab) and the work done on the system in the process of introducing the slab then :

(a) $Q = \frac{\varepsilon_0 AV}{d}$ (b) $Q = \frac{\varepsilon_0 KAV}{d}$

(c) $E = \frac{V}{kd}$ (d) $W = \frac{\varepsilon_0 AV^2}{2d}\left[1 - \frac{1}{K}\right]$

11. Rows of capacitor containing 1, 2, 4, 8, ... ∞ capacitors each of capacitance 2F are connected in parallel as shown in the figure. The potential difference AB is 10 V.



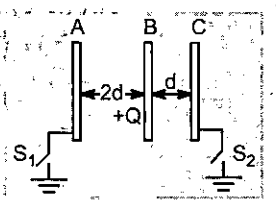
(a) Total capacitance across AB is 4F

(b) Charge on each capacitor is same

(c) Charge on capacitor in first row is more than that on any other capacitor

(d) Energy of all the capacitors is 50 J

12. Three identical parallel conducting plates A, B and C are placed as shown. Switches S_1 and S_2 are opened and connect A and C to earth when closed. $+Q$ charge is given to B.



(a) If S_1 is closed with S_2 open, a charge of amount Q will pass through S_1

(b) If S_2 is closed with S_1 open, a charge of amount Q will pass through S_2

(c) If S_1 and S_2 are closed together, a charge of amount $\frac{Q}{3}$ will pass through S_1 and a charge of amount $\frac{2Q}{3}$ will pass through S_2

(d) All the above statements are incorrect

13. A parallel plate capacitor is charged from a cell and then isolated from it. The separation between the plates is now increased.

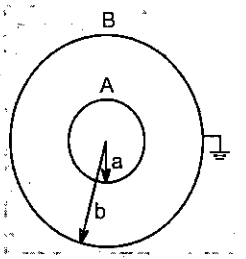
(a) the force of attraction between the plates will increased

(b) the field in the region between the plates will not change

(c) the energy stored in the capacitor will increase

(d) the potential difference between the plates will decrease

14. A conducting sphere A of radius a, with charge Q, is placed concentrically inside a conducting shell B of radius b. B is earthed. C is the common center of A and B.



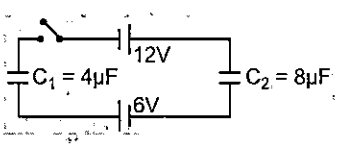
(a) the field at a distance r from C where $a \le r \le b$ is $\frac{KQ}{r^2}$

(b) the potential at the distance r from C where $a \le r \le b$ is $\frac{KQ}{r}$

(c) The potential difference between A and B is $KQ\left[\frac{1}{a} - \frac{1}{b}\right]$

(d) The potential at a distance r from C where $a \le r \le b$ is $kQ\left[\frac{1}{r} - \frac{1}{b}\right]$

15. In the circuit shown initially C_1, C_2 are uncharged. After closing the switch:



(a) The charge on C_2 is grater than on C_1

(b) The charge on C_1 and C_2 are the same

(c) The potential drops across C_1 and C_2 are the same

(d) The potential drops across C_2 is greater than that across C_1

16. A parallel plate capacitor has a parallel sheet of copper inserted between and parallel to the two plates without touching the plates. The capacity of the capacitor after the introduction of the copper sheet is:

(a) minimum when the copper sheet touches one of the plates.

(b) maximum when the copper sheet touches one of the plates

(c) invariant for all positions of the sheet between the plates.

(d) greater than that before introducing the sheet.

17. A parallel plate capacitor is charged by connecting it to a battery. The battery is disconnected and the plates of the capacitor are pulled apart to make the separation between the plates twice. Again the capacitor is connected to the battery (with same polarity) then:

(a) Charge from the battery flows into the capacitor after reconnection

(b) Charge from capacitor flows into the battery after reconnection.

(c) The potential difference between the plates increases when the plates are pulled apart.

(d) After reconnection of battery potential difference between the plate will immediately becomes half of the initial potential difference. (Just after disconnecting the battery)

18. A capacitor C is charged to a potential difference V and battery is disconnected. Now if the capacitor plates are brought close slowly by some distance:

(a) some +ve work is done by external agent

(b) energy of capacitor will decrease

(c) energy of capacitor will increase

(d) none of the above

19. The separation between the plates of a isolated charged parallel plate capacitor is increased. Which of the following quantities will change?

(a) charge on the capacitor

(b) potential difference across the capacitor

(c) energy of the capacitor

(d) energy density between the plates.

20. When two identical capacitors are charged individually to different potentials and connected parallel to each other, after disconnecting them from the source:

(a) net charge on connected plates is less than the sum of initial individual charges.

(b) net charge on connected plates equals the sum of initial charges.

(c) the net potential difference across them is different from the sum of the individual initial potential differences.

(d) the net energy stored in the two capacitors is less than the sum of the initial individual energies.

21. An isolated metallic sphere of radius R has an electric charge. It is connected by means of a conducting wire to a distant uncharged metallic sphere of radius r ($r < R$). Which of the following statements is/are correct?

(a) Interaction energy of two spheres will be developed

(b) Total energy of the system must decrease

(c) Final surface charge densities on two spheres may be equal to each other

(d) None of these

22. A parallel plate capacitor is charged and then disconnected from the source of potential difference. If the plates of the condenser are then moved farther apart by the use of insulated handle, which one of the following is true?

(a) the charge on the capacitor increases

(b) the charge on the capacitor decreases

(c) the capacitance of the capacitor increases

(d) the potential difference across the plate increases

23. A parallel plate capacitor A is filled with a dielectric whose dielectric constant varies with applied voltage as $K = V$. An identical capacitor B of capacitance C_0 with air as dielectric is connected to voltage source $V_0 = 30V$ and then connected to the first capacitor after disconnecting the voltage source. The charge and voltage on capacitor.

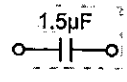
(a) A are 25 C_0 and 25V

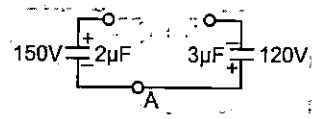
(b) A are $25C_0$ and 5 V

(c) B are $5C_0$ and 5V

(d) B are $5C_0$ and 25 V

24. Two capacitors of 2μF and 3μF are charged to 150 volt and 120 volt respectively. The plates of capacitor are connected as shown in the figure. A discharged capacitor of capacity 1.5μF falls to the free ends of the wire. Then:





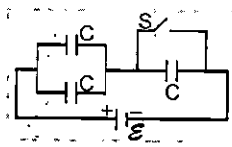
(a) charge on the 1.5μF capacitors is 180μC

(b) charge on the 2μF capacitor is 120μC

(c) charge flows through A from right to left.

(d) charge flows through A from left to right.

25. In the circuit shown, each capacitor has a capacitance C. The emf of the cell is $\mathcal{E}$. If the switch S is closed:



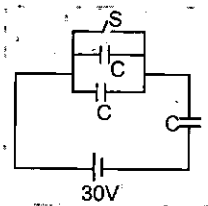
(a) positive charge will flow out of the positive terminal of the cell

(b) positive charge will enter the positive terminal of the cell

(c) the amount of charge flowing through the cell with be $C\mathcal{E}$

(d) the amount of charge flowing through the cell will be $4/3\ C\mathcal{E}$.

26. Three capacitors each having capacitance $C = 2\ \mu F$ are connected with a battery of emf = 30 V as shown in figure. When the switch S is closed:



(a) The amount of the charge flown through the battery is 20 μC

(b) The heat generated in the circuit is 0.6 mJ

(c) The energy supplied by the battery is 0.6 mJ

(d) The amount of charge flown through the switch S is 60 μC

27. A parallel plate air-core capacitor is connected across a source of constant potential difference. When a dielectric plate is introduced between the two plates then:

(a) some charge from the capacitor will flow back into the source.

(b) some extra charge from the source will flow back into the capacitor.

(c) the electric field intensity between the two plate does not change.

(d) the electric field intensity between the two plates will decrease.

28. Two concentric conducting shells A and B of radii 'a' and 'b' ($a < b$) respectively carry some charge. If the outer shell is earthed, then:

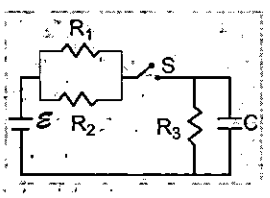
(a) Potential difference between them remains same

(b) Electric field between them remains same

(c) Energy stored between them remains same

(d) Total energy also remains same

29. The circuit shown in the figure consists of a battery of emf $\mathcal{E} = 10V$; a capacitor of capacitance $C = 1.0\mu F$ and three resistor of values $R_1 = 2\Omega$, $R_2 = 2\Omega$ and $R_3 = 1\Omega$. Initially the capacitor is completely uncharged and the switch S is open. The switch S is closed at $t = 0$.



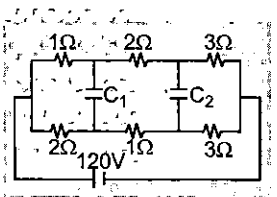
(a) The current through resistor R_3 at the moment the switch closed is zero.

(b) The current through resistor R_3 a long time after the switch closed is 5A

(c) The ratio of current through R_1 and R_2 is always constant.

(d) The maximum charge on the capacitor during the operation is $5\mu C$.

30. In the circuit shown in figure $C_1 = C_2 = 2\mu F$. Then charge stored in:



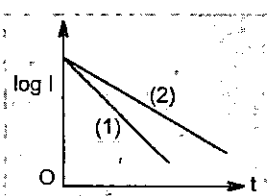
(a) capacitor C_1 is zero

(b) capacitor C_2 is zero

(c) both capacitor is zero

(d) capacitor C_1 is $40\mu C$

31. A capacitor of capacity C is charged to a steady potential difference V and connected in series with an open key and a pure resistor 'R'. At time $t = 0$, the key is closed. If I-current at time t, a plot of log I against 't' is as shown in (1) in the graph. Later one of the parameters *i.e.*, V, R or C is changed keeping the other two constant, and the graph (2) is recorded. Then:



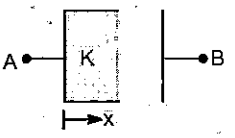
(a) C is reduced

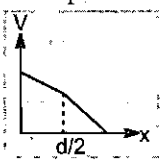
(b) C is increased

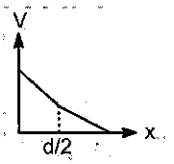
(c) R is reduced

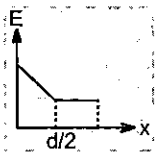
(d) R is increased

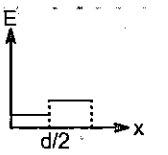
32. In the adjacent figure, half of the space between the plates of a parallel plate capacitor is filled with dielectric material of constant K. Then which of the plots are possible :



(a)


(b)


(c)


(d)


33. The plates of a parallel plate capacitor with no dielectric are connected to a voltage source.Now a dielectric of dielectric constant K is inserted to fill the whole space between the plates with voltage source remaining connected to the capacitor.

(a) the energy stored in the capacitor will become K–times

(b) the electric field inside the capacitor will decrease to K–times

(c) the force of attraction between the plates will increase to K^2–times

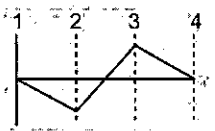
(d) the charge on the capacitor will increase to K–times

34. Four identical metallic plates (1, 2, 3 and 4) are arranged in air at same distance 'd' from each other, with their outer plates being connected together and earthed. If the plates 2 and 3 are connected with a cell of constant emf E, then:

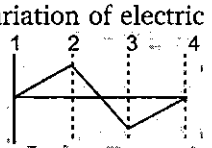
(a) $E_1 : E_2 : E_3 = \frac{2}{3} : 1 : \frac{2}{3}$

(b) $E_1 : E_2 : E_3 = \frac{1}{2} : 1 : \frac{1}{2}$

(c) Variation of electric potential will be



(d) Variation of electric potential will be



35. The capacitance of a parallel plate capacitor is C when the region between the plate has air. This region is now filled with a dielectric slab of dielectric constant K. The capacitor is connected to a cell of emf $\mathcal{E}$, and the slab is taken out:

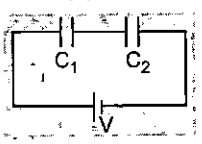
(a) charge $C\mathcal{E}(K-1)$ flows through the cell
(b) energy $\mathcal{E}^2C(K-1)$ is absorbed by the cell.
(c) the energy stored in the capacitor is reduced by $\mathcal{E}^2C(K-1)$
(d) the external agent has to do $\frac{1}{2}\mathcal{E}^2C(K-1)$ amount of work to take the slab out.

36. Two capacitors of capacitances 1μF and 3μF are charged to the same voltages 5V. They are connected in parallel with oppositely charged plates connected together. Then:
(a) Final common voltage will be 5V
(b) Final common voltage will be 2.5 V
(c) Heat produced in the circuit will be zero
(d) Heat produced in the circuit will be 37.5μJ

37. The two plates X and Y of a parallel plate capacitor of capacitance C are given a charge of amount Q each. X is now joined to the positive terminal and Y to the negative terminal of a cell of emf $\mathcal{E} = Q/C$.
(a) Charge of amount Q will flow from the negative terminal to the positive terminal of the cell inside it.
(b) The total charge on the plate X will be $2Q$.
(c) The total charge on the plate Y will be zero.
(d) The cell will supply $C\mathcal{E}^2$ amount of energy.

38. A dielectric slab is inserted between the plates of an isolated charged capacitor. Which of the following quantities will remain the same?
(a) the electric field in the capacitor
(b) the charge on the capacitor
(c) the potential difference between the plates
(d) the stored energy in the capacitor

39. Two parallel plate capacitors of capacitance C_1 and C_2 ($C_1 < C_2$) and having same plate area are connected across a battery as shown in the figure. Then choose the incorrect statement:



(a) Charge on each capacitor is same
(b) If V is increased then increase in energy of C_1 will be higher than that of C_2
(c) Potential drop across C_1 is more than C_2
(d) Electric field in C_1 is higher than that in C_2

40. Each plate of a parallel plate capacitor has a charge q on it. The capacitor is now connected to a battery. Now,
(a) the facing surfaces of the capacitor have equal and opposite charges.
(b) the two plates of the capacitor have equal and opposite charges.
(c) the battery supplies equal and opposite charges to the two plates.
(d) the outer surfaces of the plates have equal charges.

41. Following operations can be performed on a capacitor:
X – connect the capacitor to a battery of emf $\mathcal{E}$.
Y – disconnect the battery
Z – reconnect the battery with polarity reversed.
W – insert a dielectric slab in the capacitor
(a) In XYZ (perform X, then Y, then Z) the stored electric energy remains unchanged and no thermal energy is developed.
(b) The charge appearing on the capacitor is greater after the action XWY than after the action XYW.
(c) The electric energy stored in the capacitor is greater after the action WXY than after the action XYW.
(d) The electric field in the capacitor after the action XW is the same as that after WX.

42. An uncharged capacitor having capacitance C is connected across a battery of emf $\mathcal{E}$. Now the capacitor is disconnected and then reconnected across the same battery but with reversed polarity. Then :
(a) after reconnection, thermal energy produced in the circuit will be equal to $2C\mathcal{E}^2$
(b) after reconnection, thermal energy produced in the circuit will be equal to two-third of the total energy supplied by the battery
(c) after reconnection, no energy is supplied by the battery
(d) after reconnection, whole of the energy supplied by the battery is converted into heat

43. A parallel plate capacitor is charged and then disconnected from the source steady emf. The plates are then drawn apart farther. Again it is connected to the same source. Then:
(a) the potential difference across the plate increases, while the plates are being drawn apart.
(b) the charge from the capacitor flows into the source, when the capacitor is reconnected.
(c) more charge is drawn to the capacitor from the source, during the reconnection.
(d) the electric intensity between the plates remains constant during the drawing apart of plates.

44. When a parallel plates capacitor is connected to a source of constant potential difference
(a) all the charge drawn from the source is stored in the capacitor.
(b) all the energy drawn from the source is stored in the capacitor.
(c) the potential difference across the capacitor grows very rapidly initially and this rate decreases to zero eventually.
(d) the capacity of the capacitor increases with the increase of the charge in the capacitor.

45. Two conducting plates form a parallel plate capacitor of capacitance C_0. The two plates are given charges Q_1 and Q_2. If they are connected by a conducting wire, then

(a) charge transfer through this wire will be $\frac{Q_1+Q_2}{2}$

(b) charge transfer through this wire will be $\frac{Q_1-Q_2}{2}$

(c) heat produced in the connecting wire will be $\frac{(Q_1-Q_2)^2}{4C_0}$

(d) heat produced in the connecting wire will be $\frac{(Q_1-Q_2)^2}{8C_0}$

46. A parallel-plate capacitor is connected to a cell. Its positive plate A and its negative plate B have charges $+Q$ and $-Q$ respectively. A third plate C, identical to A and B, with charge $+Q$, is now introduced midway between A and B, parallel to them. Which of the following are correct?

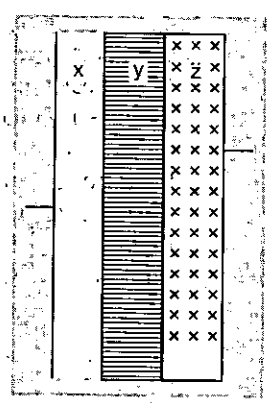
(a) The charge on the inner face of B is now $-\frac{3Q}{2}$

(b) There is no change in the potential difference between A and B.

(c) The potential difference between A and C is one-third of the potential difference between B and C.

(d) The charge on the inner face of A is now $Q/2$.

47. The figure shows a capacitor having three layers between its plates. Layer x is vacuum, y is conductor and z is a dielectric. Which of the following change (s) will result in increase in capacitance?



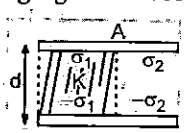
(a) Replace x by conductor

(b) Replace y by dielectric

(c) Replace z by conductor

(d) Replace x by dielectric

48. A parallel plate capacitor of area A and separation d is charged to potential difference V and removed from the charging source. A dielectric slab of constant $K=2$, thickness d and area $A/2$ is inserted, as shown in the Fig. Let σ_1 be free charge density at the conductor-dielectric surface and σ_2 be the charge density at the conductor-vacuum surface:



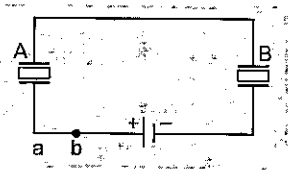
(a) The electric field have the same value inside the dielectric as in the free space between the plates.

(b) The ratio σ_1/σ_2 is equal to 2:1

(c) The new capacitance is $3\varepsilon_0 A/(2d)$

(d) The new potential difference is (2/3)V

49. Identical dielectric slabs are inserted into two identical capacitors A and B. These capacitors and a battery are connected as shown in Fig. Now the slab of capacitor B is pulled out with battery remaining connected:



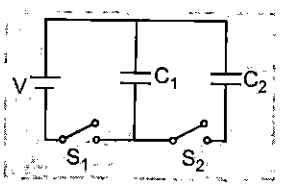
(a) During the process positive charge flows from a to d

(b) Finally charge on capacitor B will be less than that on capacitor A

(c) During the process,a work is bone by the external force F, which appears as heat in the circuit

(d) During the process, internal energy of the battery increases

50. Consider the circuit shown where $C_1=6\mu F, C_2=3\mu F$ and $V=20$ V. Capacitor C_1 is first charged by closing the switch S_1. Switch S_1 is then opened and the charged capacitor is connected to the uncharged capacitor C_2 by closing S_2:



(a) Total charge that has flown through the battery is $120\mu C$

(b) Final charge on C_1 after opening switch S_1 and closing switch S_2 is $80\mu C$

(c) Final charge on C_2 after opening switch S_1 and closing switch S_2 is $40\mu C$

(d) Total heat produced after closing switch S_2 1.8 mJ

51. Which of the following statements is/are true?

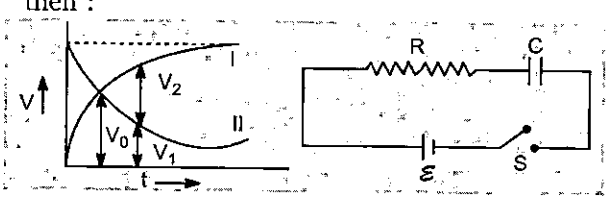
(a) The capacitance of a capacitor is the magnitude of net charge present on the two plates divided by the potential difference between the plates.

(b) The equivalent capacitance of two capacitors in parallel is always the sum of the two capacitances.

(c) A dielectric material inserted between the conductors of a capacitor increases its capacitance.

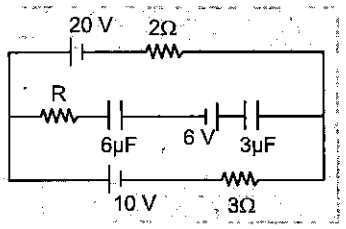
(d) The electrostatic energy stored in a capacitor is always equal to energy lost as heat when it is connected across a battery.

52. In the given $R-C$ circuit switch S is closed at $t=0$ and the variation of potential drop across the resistor (V_R) and across capacitor (V_c) is represented by given plot, then :



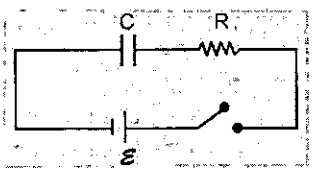
(a) $2V_0 = 2V_1 + V_2$
(b) $2V_0 = V_1 + V_2$
(c) curve I is for V_C and II is for V_R
(d) curve I is for V_R and II is for V_C

53. Consider the circuit shown in figure. Then which of the following statements are true in steady state.



(a) Charge on 6 μ F capacitor is same as on 3 μ F capacitor, equal to 44 μ C
(b) Charge on each capacitor is zero
(c) Current in 'R' resistor is zero
(d) Charge on capacitors depend on 'R'

54. Consider the following statement and answer which of them are true (capacitor was initially uncharged):



(a) capacitor in an circuit initially offers zero resistance
(b) capacitors offers ∞ resistance at $t \to \infty$
(c) initial current flowing through the capacitor is $\frac{\varepsilon}{R}$
(d) no current flows through the capacitor at any instant

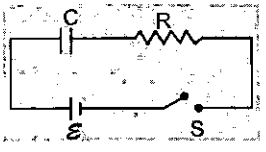
55. The differential equation of charging of a capacitor is as given below:

$$\frac{1}{K_1}\frac{dq}{dt} + K_2 q = K_3$$

The time constant and steady state charge are respectively:

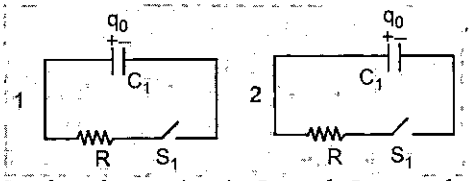
(a) $\frac{1}{K_1}$ and K_3 (b) $\frac{1}{K_1 K_2}$ and $\frac{K_3}{K_1}$
(c) $\frac{K_2}{K_1}$ and $K_2 K_3$ (d) $\frac{1}{K_1 K_2}$ and $\frac{K_3}{K_1}$

56. In the given circuit if switch 'S' is closed at $t = 0$. Then mark the correct statements :



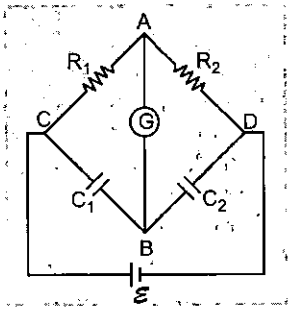
(a) Rate of energy stored in capacitor will first increase then decrease
(b) Rate of energy stored in capacitor always decreases
(c) Rate of energy dissipated in resistor first increase then decrease
(d) Rate of energy dissipated in resistor decreases with time

57.



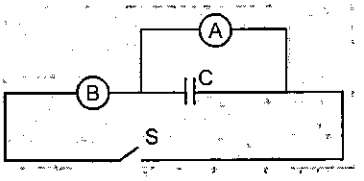
In the above circuit S_1 and S_2 are closed at $t = 0$ if $C_1 > C_2$ then choose incorrect statement :
(a) C_1 will loose 50% of initial charge sooner than C_2.
(b) Initial current in circuit 1 will be more then 2
(c) Total amount of heat dissipated in both the circuit will be same.
(d) Initial rate of heat dissipation in 1 will be less than in 2.

58. In the given circuit galvanometer shows zero deflection in steady state. Then the condition for zero deflection in steady state is :



(a) $\frac{R_1}{R_2} = \frac{C_1}{C_2}$
(b) $\frac{R_1}{R_2} = \frac{C_2}{C_1}$
(c) $R_1^2 C_1 = R_2^2 C_2$
(d) $R_1 C_1^2 = R_2 C_2^2$

59. A capacitor of capacitance C is connected to two voltmeters A and B. A is ideal having infinite resistance while B has resistance R. The capacitor is charged and then S is closed. The reading of A and B will be equal:



(a) at all times
(b) after time RC
(c) after time $RC \ln 2$
(d) only after a very long time

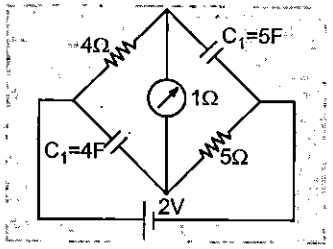
60. A capacitor is charged to certain potential difference and then discharged through a resistor R. It takes 2 μs for current through R to become half its initial value. It would take 4 μs for current to become half its initial value if :
(a) C is doubled
(b) R is doubled
(c) either (a) or (b)
(d) both R and C are doubled

61. A parallel plate capacitor, filled with a dielectric of dielectric constant K, is charged to a potential V_0. It is now disconnected from the cell and the slab is removed. If it now discharges, with time constant τ, through a resistance, the potential difference across it will be V_0 after time :

(a) $K\tau$ (b) $\tau \ln(K-1)$

(c) $\tau \ln\left(1-\frac{1}{K}\right)$ (d) $\tau \ln K$

62. In the circuit shown below, the cell is ideal, with emf = 2 V. The resistance of the coil of the galvanometer G is 1Ω:



(a) No current flows in G

(b) 0.2 A current flows in G

(c) Potential difference across C_1 is 1 V

(d) Potential difference across C_2 is 1.2 V

63. Two identical capacitors A and B are charged to the same potential and then made to discharge through resistances R_A and R_B respectively, with $R_A > R_B$:

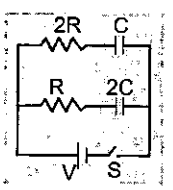
(a) A will require greater time than B to discharge completely.

(b) More heat will be produced in A than in B.

(c) More heat will be produced in B than in A.

(d) All the above options are incorrect.

64. In the shown circuit, switch S is closed at time $t = 0$. Select the correct statement(s) :



(a) rate of increase of charge is same in both the capacitors

(b) ratio of charge stored in capacitors C and $2C$ at any time t would be 1 : 2

(c) time constant of both the capacitors are equal

(d) steady state charge in capacitors C and $2C$ are in ratio 1 : 2

65. A capacitor with no dielectric is connected to a battery at $t = 0$. Consider a point A in the connecting wire and point B in between the plates :

(a) There is no current through A

(b) There is no current through B

(c) There will be current through A as long as the charging is not complete

(d) There is no current through B as long as the charging is not complete

66. A capacitor $C_1 = 1\ \mu$ F and capacitor of $C_2 = 2\ \mu$ F are separately charged by a common battery for a long time. The two capacitors are then separately discharged through equal resistors. Both the discharge circuits are connected at $t = 0$.

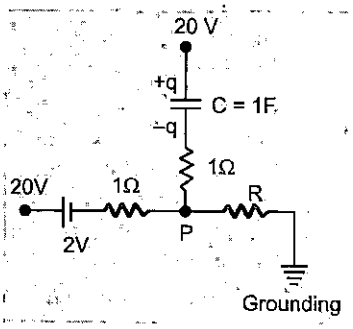
(a) The current in each of the two discharging circuits is zero at $t = 0$.

(b) The current in each of the two discharging circuits at $t = 0$ is equal but not zero.

(c) The current in each of the two discharging circuits at $t = 0$ is unequal.

(d) C_1 loses 50% of its initial charge sooner than C_2 loses 50% of its initial charge.

67. Figure shows a part of complete circuit at $t = 0$ charge on capacitor as a function of time is given by $q = 3(1 - e^{-t})$ coulombs. At $t = 0$, mark the correct statement(s).



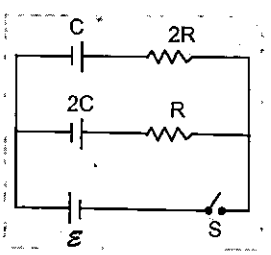
(a) Current through branch containing capacitor is 3A.

(b) Current through R is 7 A.

(c) The potential of P is 14 V.

(d) The value of R is 2 Ω.

68. In the circuit shown switch S is closed at $t = 0$. Select the correct statement(s).



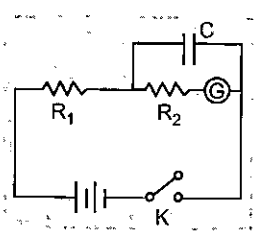
(a) Rate of increase of charge is same in both the capacitors.

(b) Time constant for both the capacitors would be same.

(c) Ratio of charge stored in capacitors C and 2C at any time t would be 1 : 2.

(d) Steady state charge in capacitors C and 2C are in the ratio 1 : 2.

69. In the connection shown in the figure the switch K is open and the capacitor is uncharged. Then we close the switch and let the capacitor charge upto the maximum and open the switch again. The values indicated by the galvanometer:



(a) increases after closing the switch and then decreases after opening the switch

(b) direction of current is same through the galvanometer before and after opening the switch again

(c) after opening the switch again energy dissipation in R_1 is zero

(d) after opening the switch again energy dissipation in R_2 is zero

70. In the charging process of R-C circuit, the time constant may be defined as the time during which:

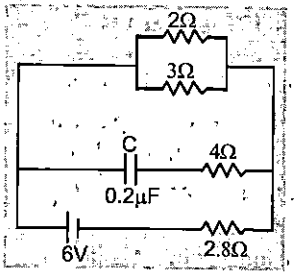
(a) capacitor voltage rises to 0.632 of its final steady value

(b) charging current falls to 0.632 of its initial maximum value

(c) charging current falls to 0.37 of its initial maximum value

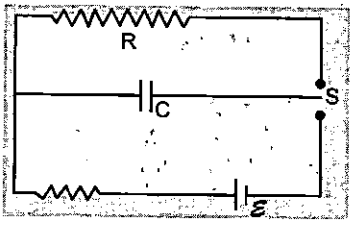
(d) capacitor voltage rises to 0.37 of its final steady value

71. In the circuit shown



(a) a current of 0.9 A flows through 2Ω resistor when steady state is reached.
(b) a potential drop of 4.2V appears across the resistance of 2.8Ω.
(c) a potential drop of 1.8V appears across the capacitor C.
(d) a potential of 4.2 V appears across the capacitor C.

72. A capacitor charged to a p.d. $\mathcal{E}$ is discharged through a resistor R. When discharge takes place, 2 microseconds elapses for the current through R to become half its initial value (when the discharge begins). For the current through R to become half its initial value.



(a) it takes 4 microseconds if the capacitance is doubled
(b) it takes 4 microseconds if R is doubled
(c) it takes 2 microseconds if both C and R are doubled
(d) it takes 16 microseconds if both C and R are doubled

ANSWERS

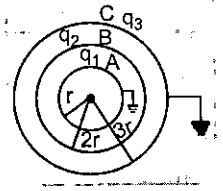
Level-2: More than One Alternatives are Correct

1.	(a, b, c)	**2.**	(c)	**3.**	(a, c, d)	**4.**	(a)	**5.**	(b)	**6.**	(b)
7.	(b)	**8.**	(a, d)	**9.**	(b, c, d)	**10.**	(a, c, d)	**11.**	(a, c)	**12.**	(a, b, c)
13.	(b, c)	**14.**	(a, c, d)	**15.**	(b)	**16.**	(c, d)	**17.**	(b,c)	**18.**	(b)
19.	(b, c)	**20.**	(b, c, d)	**21.**	(a, b)	**22.**	(d)	**23.**	(b, c)	**24.**	(a, b, c)
25.	(a, c, d)	**26.**	(a, c, d)	**27.**	(b, c)	**28.**	(a, b, c)	**29.**	(a, b, c, d)	**30.**	(b, d)
31.	(b)	**32.**	(a, d)	**33.**	(a, c, d)	**34.**	(b, d)	**35.**	(a, b, d)	**36.**	(b, d)
37.	(a, b, c, d)	**38.**	(b)	**39.**	(b, c, d)	**40.**	(a, c, d)	**41.**	(b, c, d)	**42.**	(a, d)
43.	(a, b, d)	**44.**	(a, c)	**45.**	(b, d)	**46.**	(a, b, c, d)	**47.**	(a, c, d)	**48.**	(c, d)
49.	(a, d)	**50.**	(a, c, d)	**51.**	(b, c)	**52.**	(a, c)	**53.**	(a, c)	**54.**	(a, b, d)
55.	(b)	**56.**	(a, d)	**57.**	(a, b, c)	**58.**	(b)	**59.**	(a)	**60.**	(c)
61.	(d)	**62.**	(b, c, d)	**63.**	( )	**64.**	(b, c, d)	**65.**	(b, c)	**66.**	(b, d)
67.	(a, b, c, d)	**68.**	(b, c, d)	**69.**	(a, b, c)	**70.**	(a, c)	**71.**	(a, b, d)	**72.**	(a, b)

Level-2: More than One Alternatives are Correct

1. [a, b, c]

As shell A is earthed



$$\Rightarrow \quad V_A = 0 = \frac{q_1}{Kr} + \frac{q_2}{K \times 2r} + \frac{q_3}{K \cdot 3r} = 0 \qquad \text{... (i)}$$

Also $V_C = 0 = \frac{q_1}{K \cdot 3r} + \frac{q_2}{K \cdot 3r} + \frac{q_3}{K \cdot 3r} = 0$

$$\Rightarrow \quad q_1 + q_2 + q_3 = 0$$

$$\Rightarrow \quad q_1 + q_3 = -q_2 \qquad \text{...(ii)}$$

equation (i) is

$$q_1 + \frac{q_2}{2} + \frac{q_3}{3} = 0 \qquad \text{...(iii)}$$

Solving (ii) and (iii),

We get $q_2 = \frac{-4q_3}{3}$

$$\frac{q_3}{q_2} = \frac{-3}{4} \qquad \text{...(iv)}$$

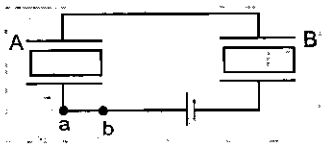
Similarly, getting other ratios by putting value of (iv) in (ii)

$$\frac{q_3}{q_1} = 3 \Rightarrow \frac{q_1}{q_2} = \frac{-1}{4}$$

2. [c] Capacitance does not depend on charges on the plates. Since both plates have same charges so, all the charge will reside at outer surface. So potential difference is zero, and have no energy is stored between the plates.

3. [a, c, d]

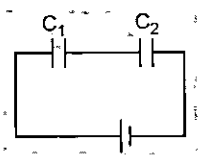
As slab is pulled out of capacitor 'B' capacitance of B decreases.



Now consider given circuit

$\therefore$ as capacitance of B decreases net capacitance also decreases

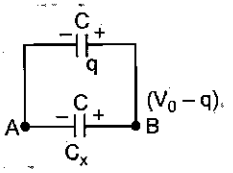
$\Rightarrow$ $q = CV$ *i.e.*, charge on A decreases *i.e.*, +ve charge moves from a to b but still A and B are in series



$\therefore \quad Q_A = Q_B$

external force 'F' do work while taking slab out. This work (energies) is partly absorbed by battery since it does –ve work and rest appears as heat in circuit since charge is flown.

4. [a] Initially charge on capacitor C was CV_0 after it has been connected to capacitor C_x charge distribution is as shown from Kirchhoff's law



$$V_A + \frac{q}{C} - \frac{(CV_0 - q)}{C_x} = V_A$$

$$\Rightarrow \quad C_x = \frac{(CV_0 - q)C}{q} \qquad \text{...(i)}$$

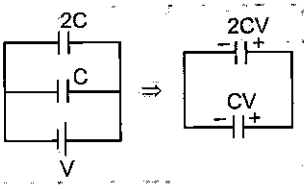
Given $|V_A - V_B| = V = \frac{q}{C}$

$$\Rightarrow \quad q = CV \qquad \text{...(ii)}$$

Put value of (ii) in (i)

We get $C_x = \frac{C(V_0 - V)}{V}$

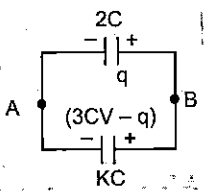
5. [b]



After slab is introduced in capacitor 'C' charge distribution is

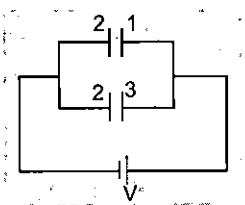
$$\Rightarrow \quad \frac{q}{2C} = \frac{(3CV - q)}{KC}$$

$$\Rightarrow \quad q = \frac{6CV}{2 + K}$$



$$V_A - V_B = \frac{q}{2C} = \frac{3V}{2 + K}$$

6. [b] Given circuit can be redraw as



Both capacitor are of capacitance

$$C = \frac{A\varepsilon_0}{d}$$

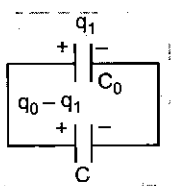
$$E = \frac{1}{2}CV^2 + \frac{1}{2}CV^2$$

$$= CV^2 = \frac{A\varepsilon_0}{d}V^2$$

7. [b] Initially capacitor C_0 was charged to

$$q_0 = C_0 V_0$$

Let distribution of charge after 'C' is joined to C_0 is as shown



From figure $\frac{q_1}{C_0} = \frac{q_0 - q_1}{C}$

$\Rightarrow$ $$q_1 = \frac{C_0 q_0}{C + C_0} = \left(\frac{C_0}{C + C_0}\right) q_0$$

i.e., charge left on capacitor C_0 after 'C' is charged from it first time is $q_1 = \frac{C_0 q_0}{C + C_0}$ after 'C' is discharged.

This charge q_1 is now initial charge on C_0 and again charge left on C_0 after it is being joined to 'C'

$$q_2 = \frac{C_0}{C + C_0}$$

$$q_1 = \left(\frac{C_0}{C + C_0}\right)^2 q_0$$

$$q_n = \left(\frac{C_0}{C + C_0}\right)^n q_0$$

i.e., charge left on capacitor C_0 after it is being discharged through capacitor 'C' n times is q_n.

Now potential across C_0 is 'V' (given)

$\Rightarrow$ $$V = \frac{q_n}{C_0}$$

$\Rightarrow$ $$V = \left(\frac{C_0}{C + C_0}\right)^n \frac{q_0}{C_0}$$

$\Rightarrow$ $$V = \left(\frac{C_0}{C + C_0}\right)^n V_0 \qquad (q_0 = C_0 V_0)$$

$\Rightarrow$ $$C = C_0\left[\left(\frac{V_0}{V}\right)^{\frac{1}{n}} - 1\right]$$

8. [a, d]

Initially

Capacitance	$C_0 = \frac{A\varepsilon_0}{d}$
Charge	$Q_0 = C_0 V$
Potential	$V_0 = V$
Electric field	$E_0 = \frac{V}{d}$
Energy	$U_0 = \frac{1}{2} C_0 V^2$

Finally

$$C = \frac{AK\varepsilon_0}{d} = KC_0$$

$$Q = KC_0 \times V (C'V)$$

$$V = V$$

(since battery is still connected)

$$E = \frac{V}{d} = E_0$$

$$U = \frac{1}{2} CV^2 = \frac{1}{2} KC_0 V^2$$

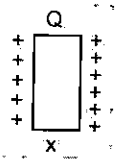
From above we find that

$$Q > Q_0, \qquad U > U_0$$

$$E = E_0, \qquad V = V_0$$

9. [b, c, d]

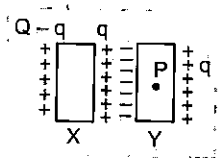
Initially charge 'Q' will be distributed uniformly on plate of surface area A, a plate has 2 surface area.



$\therefore$ $$q_1 = \frac{Q}{2A}$$

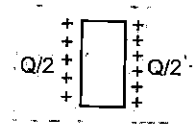
As plate Y is brought closer

Charge on outer surface of $X = \frac{Q - 0}{2} = \frac{Q}{2}$



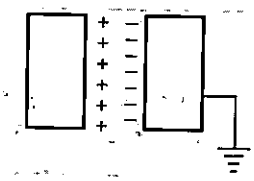
$\Rightarrow$ distribution of charges on X is

$$\sigma_2 = \frac{Q}{2A} = \sigma_1$$



When Y is earthed the new charge distribution will be as shown in the figure

$\therefore$ $$\sigma_3 = \frac{Q}{A}$$



10. [a, c, d]

Initially, charge on capacitor $= \frac{\varepsilon_0 A}{d} V$

Since battery is removed charge will remain conserved.

i.e., $$Q = q_0 = \frac{A\varepsilon_0 V}{d} \qquad \text{...(i)}$$

$$E = \frac{\text{Potential difference}}{d}$$

$$E = \frac{q_0}{C'd} = \frac{CV}{C'd} = \frac{CV}{KCd} = \frac{V}{Kd} \qquad \text{...(ii)}$$

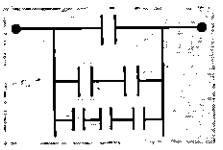
Work done = change on energy

$$= \frac{1}{2}\frac{q_0^2}{C} - \frac{1}{2}\frac{q_0^2}{C'}$$

$$= \frac{1}{2}\left(\frac{A\varepsilon_0 V}{d}\right)^2\left[\frac{1}{C} - \frac{1}{KC}\right]$$

$$W = \frac{1}{2}\frac{A\varepsilon_0 V^2}{d}\left[1 - \frac{1}{K}\right] \left(\text{since } C = \frac{A\varepsilon_0}{d}\right)$$

11. [a, c]



$$C_{net} = C + \frac{C}{2} + \frac{C}{4} + \ldots.$$

$$= C\left[1 + \frac{1}{2} + \frac{1}{4} + \ldots.\right]$$

$$C = C \times \frac{1}{1 - \frac{1}{2}} = 2C$$

$$C_{net} = 2 \times 2F = 4F$$

$$V_A - V_B = 10V$$

Energy of all the capacitors $= \frac{1}{2} C_{net} \times 10^2$

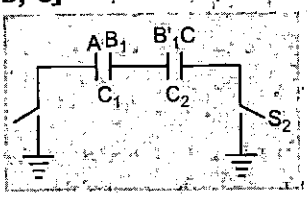
Energy $= 200$ J

Capacitors of each now are in series

$\Rightarrow$ charge on capacitors of one row is same

Since capacitance of first row is maximum so charge will be maximum because same potential is applied on each row.

12. [a, b, c]



$$C_1 = \frac{A\varepsilon_0}{2d}, \; C_2 = \frac{A\varepsilon_0}{d}$$

Charge on plate B_1' + charge on plate $B_1 = Q$

I. Now if only S_1 is closed then charge on plate 'C' will still be zero

$\therefore$ charge on plate $B_1 = 0$

$\therefore$ total charge $+Q$ is on plate B_1'

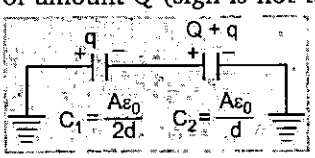
$\therefore$ $-Q$ amount of charge will flow through S_1

So as to form a capacitor C_1

II. Similarly, if only S_2 is closed $-Q$ amount of charges will flow through S_2

$\Rightarrow$ in both of above cases

charge of amount Q (sign is not taken) is flown



Now, if S_1 and S_2 both are closed let the distribution charges on the capacitors are

Now $\frac{-q}{C_1} - \frac{(Q+q)}{C_2} = 0$

$\Rightarrow$ $q = \frac{-Q}{3}$

$\Rightarrow$ $\frac{-Q}{3}$ amount of charge is flown through S_1 and $\left(Q - \frac{Q}{3}\right)$ *i.e.*, $\frac{2Q}{3}$, amount of charge is flown through S_2.

13. [b, c]

Initially

after capacitor is detached from battery and distance between plate is increased

new capacitance $C' = \frac{A\varepsilon_0}{d'} < C = \frac{A\varepsilon_0}{d}$ since $d' > d$

charge remains conserved

$\Rightarrow$ $CV = C'V'$

$\Rightarrow$ $V' = \frac{C}{C'}V$

$\Rightarrow$ $V' > V$ $\left(\text{since } C > C' \Rightarrow \frac{C}{C'} > 1\right)$

i.e., potential difference between plates increases.

$E = \frac{q_0}{\varepsilon_0 A}$ which independent of separation between plates.

$\Rightarrow$ $E = E'$

Since field and charge are same.

Force of attraction $= \frac{q^2}{2A\varepsilon_0}$ is same.

14. [a, c, d]

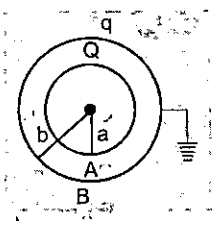
Let charge on shell B is 'q'

$$V_B = 0$$

$\Rightarrow$ $\frac{q}{4\pi\varepsilon_0 b} + \frac{Q}{4\pi\varepsilon_0 b} = 0$

$$q = -Q$$

For $a \le r \le b$



Electric field is due to shell A only since point will be inside shell B

$\therefore$ electric field due to shell $B = 0$

$\therefore$ $E_r = \frac{KQ}{r^2}$ $\quad a \le r \le b$

Potentially at distance r $\quad a \le r \le b$

$$V = \frac{KQ}{r} - \frac{KQ}{b}$$

$$V_A - V_B = \frac{KQ}{a} + \frac{K(-Q)}{b} - \frac{KQ}{b} - \frac{K(-Q)}{b}$$

$$V_A - V_B = KQ\left[\frac{1}{a} - \frac{1}{b}\right]$$

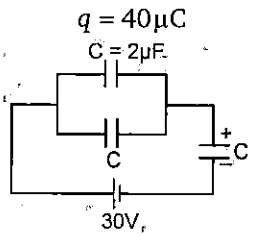
21. [a, b]

Since both spheres will get charged so interaction energy be developed. Also since charge will flow so energy will be lost.

26. [a, c, d]

Initially equivalent capacitance $= \frac{4}{3}\mu F$

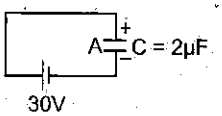
Charge on capacitor A will be $q = \frac{4}{3} \times 30\ V$



Now when switch S is closed

Charge $q' = 30 \times 2 = 60\mu C$

The charge on –ve plate of A has been charged from –40 to –60



$\Rightarrow$ 20μC amount of charge has been flown through battery

Work done by cell $= +20(\mu C) \times 20V = 600\mu J$

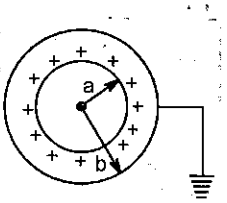
Initial energy stored in $CKt = \frac{1}{2} \times \frac{4}{3} \times 30^2 = 600\mu J$

Final energy stored in $CKt = \frac{1}{2} \times 2 \times 30^2 = 900\mu J$

Heat produced $= (600 + 600) - 900 = 300\mu J$

28. [a, b, c]

The outer shell does not contribute in field between the two shell



$\therefore$ electric field in between them remains same. Therefore, energy stored between them will also be same.

$\Delta V = \int \vec{E} \cdot d\vec{r}$ field does not change means potential difference between them will also be same.

Initially there were two capacitor, *i.e.,* in between shell A to B and form B to infinite. After earthlings of B one capacitor is short circuited.

$\therefore$ total energy will not be same.

32. [a, d]

Suppose field in air gap is E then in dielectric it is $\frac{E}{K}$

For dielectric medium

$$dV = -\frac{E}{K}dx \Rightarrow V = -\frac{E}{K}x$$

For air gap medium

$$V = -Ex$$

34. [b, d]

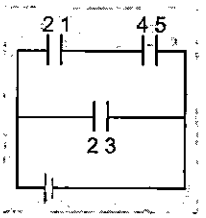


Plate 2 acquires a net +ve charge and 3 acquire a net –ve charge

$\therefore$ due to induction plate 1 acquire –ve charge

and due to induction plate 4 acquire +ve charge 1

and 4 are equipotential (since they are joined).

Due to symmetry

$$V_{1-2} = V_{3-4}$$

and $V_{2-3} = 2V_{2-1} = 2V_{4-3}$

because charge capacitor 2–3 is double.

$$\text{Electric field} = \frac{\text{potential}}{d}$$

$\Rightarrow$ $E_1 : E_2 : E_3 = 1 : 2 : 1$

$$= \frac{1}{2} : 1 : \frac{1}{2}$$

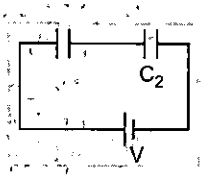
and from plate 1 to 4

Potential first increases then decreases and again increases.

Since we are going first in direction opposite of E and then in direction of E again in direction opposite of E.

39. [b, c, d]

The capacitor are connected in series



$$C_1 = \frac{A\varepsilon_0}{d_1}$$

$$C_2 = \frac{A\varepsilon_0}{d_2}$$

$C_1 < C_2 \Rightarrow d_1 < d_2$ (i)

Initial charge $= Q_0 = \frac{C_1 C_2 V}{C_1 + C_2}$

Energy stored in capacitors

$\frac{1}{2}\frac{Q_0^2}{C_1}$ and $\frac{1}{2}\frac{Q_0^2}{C_2}$ respectively

Finally let charge $= Q_1$

$\therefore$ energy stored in $C_1 = \frac{1}{2}\frac{Q_1^2}{C_1}$

Energy stored in $C_2 = \frac{1}{2}\frac{Q_1^2}{C_2}$

Change in energy

$$\Delta V_1 = \frac{1}{2}\frac{(Q_1^2 - Q_0^2)}{C_1}$$

$$\Delta V_2 = \frac{1}{2}\frac{(Q_1^2 - Q_0^2)}{C_2}$$

$$C_1 < C_2$$

$$\Rightarrow \quad \Delta V_1 > \Delta V_2$$

i.e., increase in energy of C_1 > increase in energy of C_2

potential across $C_1 = \frac{Q_0}{C_1}$

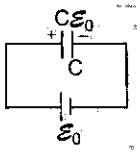
potential across $C_2 = \frac{Q_0}{C_2}$

clearly $\frac{Q_0}{C_1} > \frac{Q_0}{C_2}$

electric field $= \frac{Q}{A\varepsilon_0}$ will be same for both.

42. [a, d]

Initially energy stored = final energy stored $= \frac{1}{2}C\mathcal{E}_0^2$



i.e., there is no change in stored energy work done by battery

$$= \mathcal{E}_0(2C\mathcal{E}_0)$$
$$= 2C\mathcal{E}_0^2$$

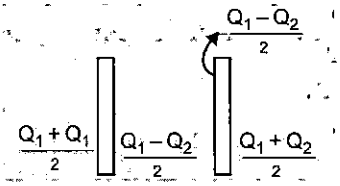
$\Rightarrow$ total amount of work will be lost as heat.

45. [b, d]

Initial charge distribution is shown in figure.

If the two plates are connected the charge on capacitor (*i.e.*, on facing surface) will become zero.

Also total energy stored between the plates will become zero.



Heat produced

$$= \frac{1}{2}\frac{Q^2}{C_0} = \frac{1}{2}\frac{\left(\frac{Q_1-Q_2}{2}\right)^2}{C_0} = \frac{(Q_1-Q_2)^2}{8C_0}$$

47. [a, c, d]

Here effective dielectric constant of conductor can be taken as infinity.

51. [b, c]

Option (a) is wrong because amount of charge present on facing sides of the plates is used in calculation of capacitance.

Option (c) is correct because $C = \frac{\varepsilon_0 AK}{d}$

Option (d) is wrong because the statement holds only when uncharged capacitor is charged using a battery.

52. [a, c]

Charge on capacitor increases with time

$\therefore$ potential $V_C = \frac{q}{C}$ increases too

$\therefore$ Curve I is for V_C

and curve II is for V_R as current will decrease with time.

also $V_R + V_C = \mathcal{E}$

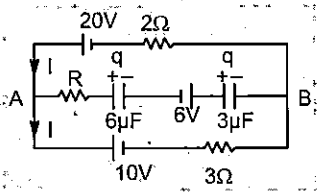
$V_R + V_C = V_0 + V_0 = 2V_0$ (when both were equal)

$V_1 + (V_1 + V_2) = 2V_0$

$2V_1 + V_2 = 2V_0$

53. [a, c]

Current in 'R' will be zero in steady state



$$-20 + 2I + 3I + 10 = 0$$

$$\Rightarrow \quad I = 2\text{ A}$$

$$V_A - 20 + 4 = V_B$$

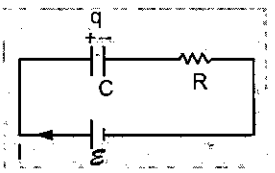
$$V_A - V_B = 16\text{ V}$$

$$\therefore \quad 16 = -\left(\frac{q}{6} + 6 - \frac{q}{3}\right)$$

$$q = 22 \times 2 = 44\ \mu\text{C}$$

55. [b]

Consider the circuit



$$\mathcal{E} - \frac{q}{C} - IR = 0$$

and $I = \frac{dq}{dt}$

$$\Rightarrow \quad \mathcal{E}C = \frac{dq}{dt} \cdot RC + q$$

$$\Rightarrow \quad RC\frac{dq}{dt} + q = \mathcal{E}C \qquad \ldots(1)$$

$$\Rightarrow \quad q = q_0(1 - e^{-t/RC})$$

$$\Rightarrow \quad q = \mathcal{E}C(1 - e^{-t/RC})$$

Comparing eqn. (1) and given eqn., we get

$$\tau = \frac{1}{K_1K_2},\ q_0 = \frac{K_3}{K_2}$$

56. [a, d]

Rate of heat dissipated in Resistor $= I^2R$

$$= \frac{\mathcal{E}^2}{R}e^{-2t/RC}$$

$\Rightarrow$ always decreases w.r.t. time

Rate of energy stored in capacitor

$$P_C = \frac{du}{dt} = \frac{d}{dt}\left(\frac{1}{2}\frac{q^2}{C}\right)$$

$$= \frac{q}{C} \times \frac{dq}{dt} = V_C \times \hat{I}$$

$$= \frac{\mathcal{E}^2}{R}[e^{-t/RC} - e^{-2t/RC}]$$

$$\left[\frac{dP_C}{dt} = \frac{e^2}{R^2C}[2e^{-2t/RC} - e^{-2/RC}] = 0 \Rightarrow t = RC\ln 2\right]$$

So, P_C increases upto $t = RC\ln 2$ and then decreases.

57. [a, b, c] $t_{1/2} = RC\ln 2$

$$t_1 = RC_1\ln 2 > t_2RC_2\ln 2$$

$$I = \frac{q_0}{RC}e^{-t/RC}$$

$$I_1 = \frac{q_0}{RC_1}, I_2 = \frac{q_0}{RC_2}$$

$$C_1 > C_2$$

$\Rightarrow$ $I_2 > I_1$

Total amount of heat dissipated = amount of energy stored

$$U_1 = \frac{1}{2}\frac{q_0^2}{C_1}$$

$$U_2 = \frac{1}{2}\frac{q_0^2}{C_2}$$

$\Rightarrow$ U_2 is not equal to U_1 (since C_1 is not equal to C_2)

Initial rate of heat dissipation

$$H_1 = I_1^2R,\ H_2 = I_2^2R$$

From above $I_2 > I_1$

$\Rightarrow$ $H_2 > H_1$

58. [b] In steady state current through capacitor is zero.

For zero deflection $V_A = V_B$

So, $IR_1 = \frac{q}{C_1}$

and $IR_2 = \frac{q}{C_2} \Rightarrow \frac{R_1}{R_2} = \frac{C_2}{C_1}$

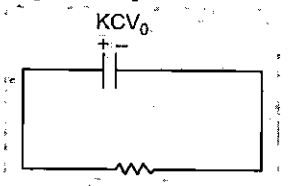
59. [a] A and B are in parallel

$\Rightarrow$ potential drop across each will be same so, they will show same reading.

60. [c] Time constant = RC

So, if either R or C is doubled, time will be doubled

61. [d] Initial charge on capacitor



$$Q_0 = KC \times V_0$$

where $C = \frac{A\varepsilon_0}{d}$

$\therefore$ $Q = Q_0e^{-t/\tau}$

Charge when potential difference across capacitor is V_0 is given by

$$C \times V_0 = KC \times V_0e^{-t/\tau}$$

$$t = \tau\ln K$$

62. [b, c, d]

In steady state, no current will flow through capacitor

$\Rightarrow$ current flowing through resistors

$$= \frac{2}{4+1+5} = \frac{2}{10} = 0.2\text{ A}$$

i.e., 0.2 A flow through G

Potential difference across $C_1 = (4+1)0.2 = 1\text{ V}$

Across $C_2 = (1+5) \times 0.2 = 1.2\text{ V}$

63. [d] $q_0 = CV_0$ = same for both capacitors

$q_1 = q_0e^{-t/R_AC}$, $q_2 = q_0e^{-t/R_BC}$

Both capacitor require infinite time to discharge completely

Total heat produced in R_A or R_B

= Total energy stored in capacitors

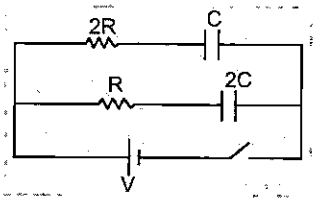
$= \frac{1}{2}CV_0^2$ = same for both R_A and R_B

64. [b, c, d]

For capacitor 'C'

$$q_1 = CV(1 - e^{-t/2RC})$$

for capacitor '$2C$'



$$q_2 = 2CV(1 - e^{-t/2RC})$$

$$\frac{q_1}{q_2} = \frac{1}{2}$$

$\tau = 2RC$ for both capacitors

Steady state charges are in ratio $\frac{CV}{2CV} = \frac{1}{2}$

65. [b, c]

There can't be current in between plates of capacitor.

Also during charging current flows in the circuit of capacitor and battery.

66. [b, d]

$$q = q_0 e^{-t/RC}$$

$$q_1 = C_1 V e^{-t/RC_1}$$

$$q_2 = C_2 V e^{-t/RC_2}$$

$$I_1(t) = \frac{V}{R} e^{-t/RC_1}$$

$$I_2(t) = \frac{V}{R} e^{-t/RC_1}$$

$$I_1(0) = I_2(0) = \frac{V}{R}$$

$$t_{1/2} = RC \ln 2$$

$t_{1/2}$ for 'C_1' $< t_{1/2}$ 'C_2'

$\therefore$ C_1 will loose 50% charge sooner then C_2.

67. [a, b, c, d]

Current through branch containing capacitor is,

$$\frac{dq}{dt} = I = 3e^{-t} \text{ amp.}$$

At $t = 0$, $I = 3$ A in downward direction, *i.e.*, towards P. Let V is the potential of point P then

$$20 - V = \frac{q}{C} + 3 \times 1 = \frac{3}{1} + 3 = 6$$

$$\Rightarrow \quad V = 14 \text{ volt}$$

Current through branch containing capacitor is,

$$I_2 = \frac{18 - 14}{1} = 4A \text{ again towards } P.$$

Applying KCL at P, current through R is 7 A potential difference across R is $V = 14$ V $= 7R \Rightarrow R = 2\Omega$

68. [b, c, d]

Charge on capacitor C as a function of time is,

$$q_1 = C\mathcal{E}[1 - e^{-\frac{t}{2RC}}]$$

Charge on capacitor $2C$ as a function of time is,

$$q_2 = 2C\mathcal{E}[1 - e^{-\frac{t}{2RC}}]$$

72. [a, b]

$$I = I_0 e^{-t/RC}$$

$$\frac{I_0}{2} = I_0 e^{-t/RC}$$

$$\Rightarrow \quad 2 = e^{t/RC}$$

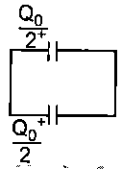
$$\ln 2 = \frac{t}{RC}$$

$$t = (\ln 2)RC$$

Level 3 Comprehension Based Problems

Passage 1

Two capacitors each having area A and plate separation 'd' are connected as shown in the circuit. Each capacitor carries charge $\frac{Q_0}{2}$.



The plates of one capacitor are slowly pulled apart by an external agent till the separation between them becomes $2d$. The other capacitor is not disturbed.

1. The work done by the external agent in this process will be:

(a) $\frac{Q_0^2 d}{12A\varepsilon_0}$ (b) $\frac{Q_0^2 d}{6A\varepsilon_0}$

(c) $\frac{Q_0^2 d}{18A\varepsilon_0}$ (d) $\frac{Q_0^2 d}{9A\varepsilon_0}$

2. The force applied by the external agent when the separation between the plates is 'x' is given by:

(a) $\frac{Q_0^2}{4\left(\frac{x}{d}+1\right)^2 A\varepsilon_0}$ (b) $\frac{Q_0^2}{2\left(\frac{x}{d}+1\right)^2 A\varepsilon_0}$

(c) $\frac{Q_0^2}{2A\varepsilon_0}$ (d) $\frac{Q_0^2}{\left(\frac{x}{d}+1\right)^2 A\varepsilon_0}$

3. At this position the potential difference across the capacitors is given by:

(a) $\frac{Q_0 xd}{2A\varepsilon_0(x+d)}$ (b) $\frac{Q_0 xd}{A\varepsilon_0(x+d)}$

(c) $\frac{Q_0 xd}{4A\varepsilon_0(x+d)}$ (d) $\frac{Q_0 d}{2A\varepsilon_0 x}$

4. Heat produced in this process will be equal to:

(a) $\frac{5}{72}\frac{Q_0^2}{C}$ (b) $\frac{5}{36}\frac{Q_0^2}{C}$

(c) $\frac{1}{2}\frac{Q_0^2}{C}$ (d) $\frac{1}{4}\frac{Q_0^2}{C}$

Passage 2

While assembling a parallel plate capacitor, the space between two conducting plates is filled with some insulating material called dielectric material. If the capacitor is charged, electric field is created inside the dielectric. Due to this field the electrons (which are not free), experience force in opposite direction of the field. If a very high field is applied in the dielectric, the outer electrons may get detached from the atoms and then the dielectric behaves like a conductor. This phenomenon is called *dielectric breakdown*. The minimum field at which the breakdown occurs is called the *dielectric strength* of the material and corresponding potential is called *breakdown potential*.

There are two capacitors of capacitances C and $2C$. The breakdown potential of each capacitor is V_0.

1. If they are joined in series, then the maximum potential difference that can be applied across the combination for their safely use will be:

(a) V_0 (b) $\frac{3}{2}V_0$

(c) $2V_0$ (d) $3V_0$

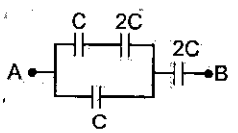
2. If the voltage across the series combination is increased which capacitor will undergo breakdown first?

(a) C
(b) $2C$
(c) Both at same moment
(d) None of these

3. If the voltage across the parallel combination of these two capacitors is increased, which capacitor will undergo breakdown first?

(a) C
(b) $2C$
(c) Both at same moment
(d) None of these

4. Consider the shown circuit, which is framed using the same two capacitors. What can be the maximum voltage which can be applied across A and B so that this combination may be safely used?

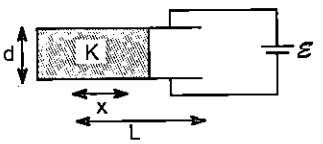


(a) $\frac{11V_0}{4}$ (b) $\frac{11V_0}{5}$

(c) $\frac{11V_0}{6}$ (d) $\frac{11V_0}{8}$

Passage 3

Figure shows a parallel plate capacitor with plates of width b and length L. The separation between the plates is d. The plates are rigidly clamped and connected to a battery of emf $\mathcal{E}$. A dielectric constant K is slowly inserted between the plates.



It is found that the dielectric slab is being attracted by the capacitor. Hence external agent has to apply equal and opposite force so that the slab is slowly inserted. The magnitude of the attractive force applied by the capacitor on the dielectric slab can be obtained using energy considerations. Suppose x length of the dielectric slab enters the capacitor then the plate area of the part with the dielectric is bx. Its capacitance is

$$C_1 = \frac{K\varepsilon_0 bx}{d}$$

Similarly, the capacitance of the part without the dielectric is

$$C_1 = \frac{\varepsilon_0 b(L-x)}{d}$$

These two are connected in parallel. The capacitance of the system is, therefore,

$$C = C_1 + C_2 = \frac{\varepsilon_0 b}{d}[L + x(K-1)]$$

The energy of the capacitor

$$U = \frac{1}{2}C\mathcal{E}^2 = \frac{\varepsilon_0 b\mathcal{E}^2}{2d}[L + x(K-1)]$$

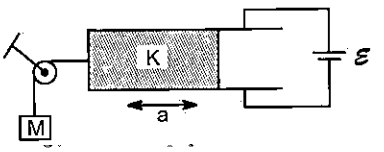
Suppose the electric field attracts the dielectric slab with a force F. An external force of equal magnitude F should be applied in opposite direction so that the plate moves slowly (no acceleration).

The attractive force F will be given by

$$F\frac{dU}{dx} = \frac{\varepsilon_0 b\mathcal{E}^2(K-1)}{2d}$$

1. The capacitor attracts the dielectric slab because :
(a) the plates of capacitor induce oppose charge on the dielectric slab
(b) the fringing of lines of force at the edges of capacitor plates
(c) of (a) and (b) both
(d) none of these

2. Suppose length 'a' of the dielectric slab is inside the capacitor, then to keep the dielectric slab in equilibrium, a block of mass M is attached to the dielectric slab via a light in-extensible string passing over an ideal pulley as shown in figure (2). The value of mass M is equal to :



(a) $\frac{\varepsilon_0 b\mathcal{E}^2(K-1)}{2dg}$ (b) $\frac{\varepsilon_0 b\mathcal{E}^2(K-1)}{dg}$

(c) $\frac{\varepsilon_0 b\mathcal{E}^2(K-1)}{4dg}$ (d) $\frac{\varepsilon_0 b\mathcal{E}^2 K}{2dg}$

3. If the string connecting the block of mass M with the dielectric slab in figure (2) is burnt then :
(a) the motion of dielectric slab will be SHM with time period $T = 8\sqrt{\frac{d(L-a)M}{\varepsilon_0 b\mathcal{E}^2(K-1)}}$
(b) the motion of dielectric slab will be SHM with time period $T = 4\sqrt{\frac{d(L-a)M}{\varepsilon_0 b\mathcal{E}^2(K-1)}}$
(c) the motion of the dielectric slab will be only oscillatory but not SHM with time period $T = 8\sqrt{\frac{d(L-a)M}{\varepsilon_0 b\mathcal{E}^2(K-1)}}$
(d) the motion of the dielectric slab will be only oscillatory but not SHM with time period $T = 4\sqrt{\frac{d(L-a)M}{\varepsilon_0 b\mathcal{E}^2(K-1)}}$

4. If the block of mass M in figure (2) is detached from the dielectric slab and the dielectric slab is allowed to move inside the capacitor with constant velocity v_0, then neglecting the resistance of the circuit, the current through the battery will be:

(a) $\frac{\varepsilon_0 bKv_0\mathcal{E}}{d}$ (b) $\frac{\varepsilon_0 b(K-1)v_0\mathcal{E}}{d}$

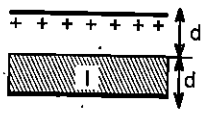
(c) $\frac{\varepsilon_0 bv_0\mathcal{E}}{d}$ (d) $\frac{\varepsilon_0 bv_0\mathcal{E}}{d(K-1)}$

5. If E, Q and U refer to the electric field between the plates, charge on capacitor and energy stored in the capacitor and scripts 1 and 2 respectively refer to conditions in figure (1) before the dielectric slab in introduced and after the dielectric slab completely fills the space between the plates of capacitor, with battery remaining connected then :

(a) $E_2 < E_1$ $Q_2 > Q_1$ $U_2 > U_1$
(b) $E_2 < E_1$ $Q_2 > Q_1$ $U_2 < U_1$
(c) $E_2 = E_1$ $Q_2 > Q_1$ $U_2 > U_1$
(d) $E_2 = E_1$ $Q_2 > Q_1$ $U_2 < U_1$

6. In the figure (1), when the slab fills the space between the plates completely, the battery is disconnected and then separation between the plates is doubled as shown in the figure (3). If V_0, E_0, U_0 and V, E, U

respectively be the potential difference between the plates, electric field in the dielectric slab and total energy stored in the capacitor before and after the separation between the plates is increased then :

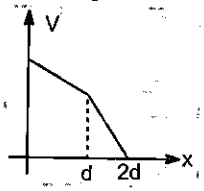


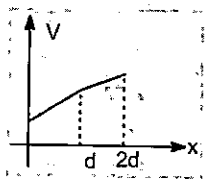
(a) $V > V_0, U > U_0, E = E_0$
(b) $V > V_0, U > U_0, E > E_0$
(c) $V < V_0, U > U_0, E = E_0$
(d) $V > V_0, U < U_0, E < E_0$

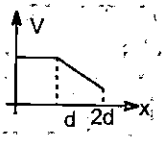
7. Suppose dielectric strength of air is E_1 and that of dielectric slab is $E_2(E_1 < E_2)$ then in the figure (3), maximum potential difference that can exist across the plates of capacitor without any electrical breakdown, will be:

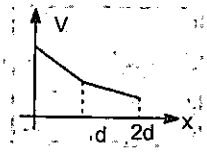
(a) $E_1\left(1+\frac{1}{K}\right)d$ (b) $E_2\left(1+\frac{1}{K}\right)d$
(c) $E_1(K+1)d$. (d) $E_2(K+1)d$

8. In the figure (3), if upper plate ($x = 0$) is positive, then the variation of potential (V) with depth (x) will be correctly represented by :

(a) 

(b) 

(c) 

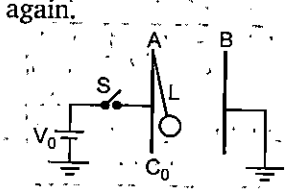
(d) 

9. In reference to figure (3) consider the following statements for region I and II
(I) Energy stored in the region I is less than that in region II
(II) Electric field in region I is less than that in region II
(III) Direction of electric field in region I and region II will be opposite to each other
Correct statements are :
(a) (I) and (III) (b) (II) and (III)
(c) (I) and (II) (d) only (II)

10. Suppose in the figure (3), a particle carrying a negative charge is placed in equilibrium between the plates in air (region II). Now upper plate is removed then:
(a) particle will remain in equilibrium
(b) particle will accelerate upwards with acceleration $\frac{g}{2}$
(c) particle will accelerate downwards with acceleration $\frac{g}{2}$
(d) particle will accelerate downwards with acceleration g

Passage 4

In the given setup the parallel plate capacitor AB has vertical plates with separation, $d = 50$ mm and capacitance 'C_0'. From the plate 'A' a small conducting ball hangs on a non-conducting silk thread of length $L = 100$ mm. The mass of the ball is 'm' and capacitance is 'C_1'. It initially touches the plate A, as shown in the figure. The plate B is grounded while plate A is connected to a power supply of potential 'V_0' for a short time by closing the switch 'S' and then opening it again.



The motion of the conducting ball was observed. It was found that due to the charge deposited on the plates and the ball, the ball swings across, touches the plate B, swings back, touches A and finally swings out again such that it almost touches plate B. Take $g = 10\ m/s^2$.

1. The angle made by the ball and string with the horizontal when the ball is in its final position is:
(a) 30° (b) 45°
(c) 60° (d) 0°

2. The ball initially touches plate B, it loses its charge to B, swings back and touches A and then finally again swings out and almost touches plate B. The new potential 'V' is given by :
(a) $\frac{C_1V_0}{C_0+C_1}$ (b) $\frac{C_0V_0}{C_0+C_1}$
(c) V_0 (d) none of these

3. In the final position the charge carried by the small ball is given by:
(a) $\frac{C_1^2V_0}{C_0+C_1}$
(b) $\frac{C_1C_0V_0}{C_0+C_1}$
(c) C_1V_0
(d) can't be calculated

4. The ratio of the charge carried by the plates and the ball finally is:
(a) $C_1:C_0$ (b) $C_1^2:C_0^2$
(c) $C_0^2:C_1^2$ (d) $C_0:C_1$

5. Based on the description given in the passage, the required power supply voltage (V_0) is given by:
(a) $V_0 = \left(1+\frac{C_1}{C_0}\right)\sqrt{\frac{m}{2\sqrt{3}\,C_1}}$

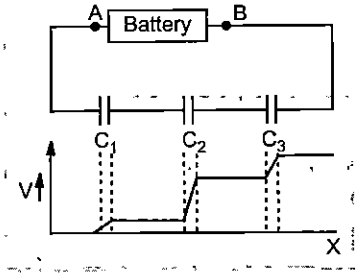
(b) $V_0 = \left(1+\frac{C_0}{C_1}\right)\sqrt{\frac{m}{2\sqrt{3}\,C_1}}$

(c) $V_0 = \left(1+\frac{C_0}{C_1}\right)\sqrt{\frac{5m}{\sqrt{3}\,C_1}}$

(d) $V_0 = \left(1+\frac{C_1}{C_0}\right)\sqrt{\frac{5m}{\sqrt{3}\,C_1}}$

Passage 5

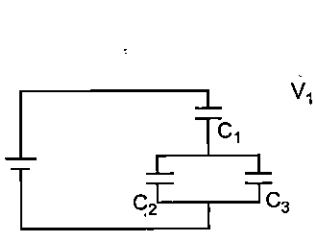
Fig. shows a circuit containing a battery and three parallel plate capacitors with identical plate separation (filled with air). The capacitors lie along x-axis and a graph of the electric potential V along that axis is shown.

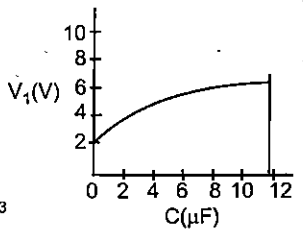


1. The charge on capacitor $C_1, C_2 \& C_3$ are $q_1, q_2 \& q_3$ respectively then
   (a) $q_1 > q_2 > q_3$ (b) $q_1 < q_2 > q_3$
   (c) $q_1 = q_2 = q_3$ (d) $q_1 > q_2 < q_3$
2. If the area of the plates of capacitors are $A_1, = A_2 \& A_3$ respectively then
   (a) $A_1 < A_2 > A_3$ (b) $A_1 > A_3 > A_2$
   (c) $A_1 = A_2 = A_3$ (d) $A_1 > A_2 > A_3$
3. If the magnitude of electric field between their plates are $E_1, E_2 \& E_3$ respectively then
   (a) $E_1 > E_2 > E_3$ (b) $E_1 < E_2 > E_3$
   (c) $E_1 > E_2 < E_3$ (d) $E_1 < E_2 < E_3$

Passage 6

Capacitor C_3 in the circuit is a variable capacitor (its capacitance can be varied). Graph is plotted between potential difference V_1 (across capacitor C_1) *versus* C_3. Electric potential V_1 approaches on asymtote of 10 V as $C_3 \to \infty$.

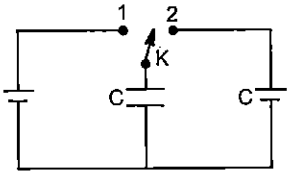




1. The electric potential V across the battery is equal to :
   (a) 10 V (b) 12 V
   (c) 16 V (d) 20 V
2. The capacitance of the capacitor C_1 has value:
   (a) 2μF (b) 6μF
   (c) 8μF (d) 12μF
3. The capacitance of C_2 is equal to:
   (a) 2μF (b) 6μF
   (c) 8μF (d) 12μF

Passage 7

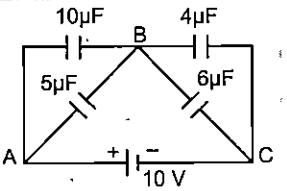
A circuit consists of two capacitors of capacitances C(μF) each and ideal cell of emf $\mathcal{E}$ volts as shown in figure. Initially both the capacitors are uncharged. The key is pushed to position 1 and then to position 2. This operation is repeated infinite times



1. The magnitude of net charge (in μC) that crossess the cell is:
   (a) $C\mathcal{E}$ (b) 1.5 $C\mathcal{E}$
   (c) 2 $C\mathcal{E}$ (d) zero
2. The magnitude of net work done (in μJ) by the cell is:
   (a) $C\mathcal{E}^2$ (b) $1.5C\mathcal{E}^2$
   (c) $2C\mathcal{E}^2$ (d) zero
3. The net heat produced in the circuit (in μ J) is:
   (a) $C\mathcal{E}^2$ (b) $1.5C\mathcal{E}^2$
   (c) $2C\mathcal{E}^2$ (d) $0.5C\mathcal{E}^2$

Passage 8

In the circuit shown in figure, four capacitors are connected to a battery.

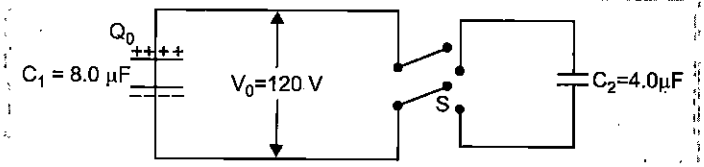


1. The equivalent capacitance of the circuit is:
   (a) 25μF (b) 6μF
   (c) 8.4μF (d) none of these
2. The charge on the 5μF capacitor is:
   (a) 60μF (b) 24μC
   (c) 12μC (d) 20μC
3. The potential difference across the 6μF capacitor is:
   (a) 6 V (b) 4 V
   (c) 5 V (d) none

4. The maximum energy is stored in the capacitor of:
(a) 10μF (b) 6μF
(c) 5μF (d) 4μF

Passage 9

We charge a capacitor of capacitance $C_1 = 8.0\mu F$ by connecting it to a source of potential difference $V_0 = 120\,V$ (not shown in the figure). The switch S is initially open. Once C_1 is charged the source of potential difference is disconnected.



1. The charge Q_0 on C_1, if switch S is left open, is :
(a) 960μC (b) 360μC
(c) 720μC (d) 520μC

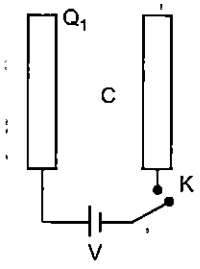
2. The energy stored in C_1, if switch S is left open, is :
(a) 36 mJ (b) 96 mJ
(c) 57.6 mJ (d) 24 mJ

3. The capacitor of capacitance $C_2 = 4.0\mu F$ is initially uncharged. After we close switch S, the potential difference across C_2 is :
(a) 80 V (b) 48 V
(c) 36 V (d) 72 V

4. The total energy of the system after we close switch S is :
(a) 72.6 mJ (b) 48 mJ
(c) 38.4 mJ (d) 12 mJ

Passage 10

For the system shown in Fig., capacitance is C. Left plate is given a charge Q_1 and right plate is uncharged. Now, switch is closed.

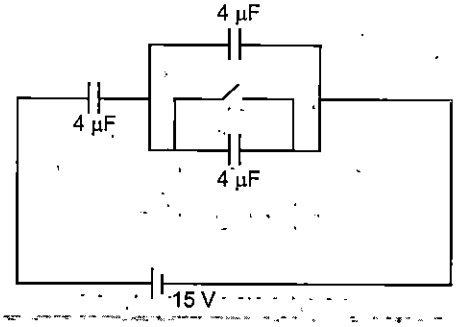


1. Find the amount of charge that will flow through the battery before the steady state is achieved:
(a) CV (b) $CV - Q_1$
(c) $CV + \frac{Q_1}{2}$ (d) $CV - \frac{Q_1}{2}$

2. Find the charge appearing on the inner face of the left plate:
(a) $CV - \frac{Q_1}{2}$ (b) $CV + Q_1$
(c) $CV + \frac{Q_1}{2}$ (d) CV

Passage 11

Consider the circuit shown in Fig., after switch S is closed.

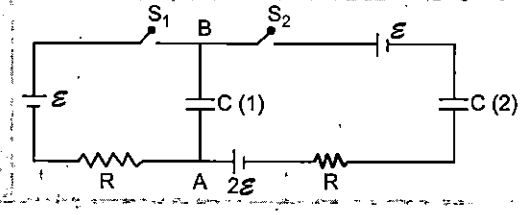


1. What amount of charge will flow through the battery?
(a) 20μC
(b) 60μC
(c) 40μC
(d) No charge will flow

2. What amount of charge will flow through the switch ?
(a) 20μC
(b) 60μC
(c) 40μC
(d) No charge will flow

Passage 12

In the given circuit first switch S_1 is closed and when steady state is reached for $C(1)$ switch S_1 is opened and switch S_2 is closed at $t = 0$. Now answer the following question :



1. Just after closing S_2 current through S_2 is :
(a) $\frac{\varepsilon}{R}$ (b) $\frac{4\varepsilon}{R}$
(c) $\frac{2\varepsilon}{R}$ (d) $\frac{3\varepsilon}{R}$

2. Time constant for the charging capacitor is :
(a) RC (b) $\frac{RC}{2}$
(c) $2RC$ (d) $\frac{RC}{4}$

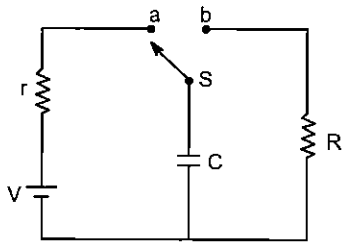
3. Time at which charge on both capacitor will be equal, is :
(a) RC (b) $RC \ln 2$

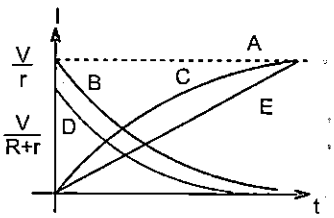
(c) $\frac{RC}{2}\ln 2$ (d) $2RC \ln 2$

4. Time at which rate of energy dissipating in capacitor 1 is equal to rate of energy stored in capacitor 2 in :
(a) RC (b) $RC \ln 2$
(c) $\frac{RC}{2}\ln 2$ (d) $2RC \ln 2$

Passage 13

In the circuit shown in figure capacitor is initially charged to steady state by connecting switch to a. Switch is transferred to b at time $t = 0$. Corresponding curve represents variation I *vs* t.



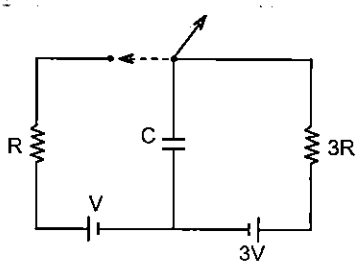


1. Which is correct curve ?
(a) A (b) B
(c) C (d) D

2. If graph is plotted between q and I Which curve best describes relation ?
(a) A (b) C
(c) D (d) E

Passage 14

In the circuit shown in figure initially switch is at position 1, till capacitor is fully charged. Now switch is transferred to position 2 at $t = 0$.



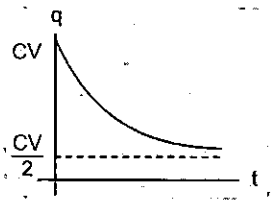
1. What is initial current in circuit at $t = 0$?
(a) $\frac{2V}{3R}$ (b) $\frac{3V}{2R}$
(c) $\frac{V}{3R}$ (d) $\frac{V}{R}$

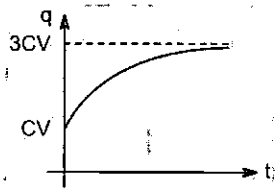
2. What is steady state charge on capacitor ?
(a) $3CV$ (b) $\frac{CV}{3}$
(c) $2CV$ (d) CV

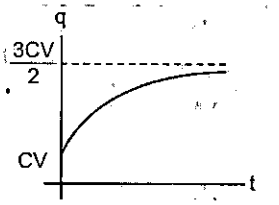
3. What is heat developed in circuit ?
(a) CV^2 (b) $2CV^2$
(c) $4CV^2$ (d) $3CV^2$

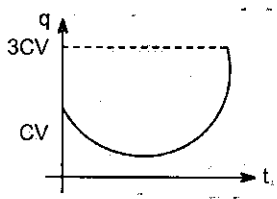
4. What is workdone by $3V$ battery ?
(a) $0CV^2$ (b) $2CV^2$
(c) $6CV^2$ (d) $4CV^2$

5. The graph between charge on capacitor C and time t is:

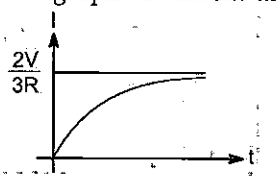
(a)


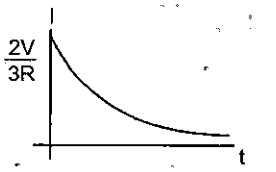
(b)


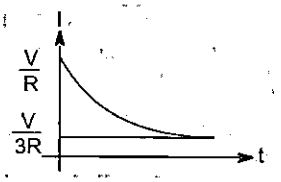
(c)


(d)


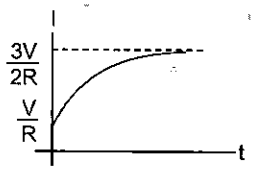
6. What is graph of current as function of time:

(a)


(b)


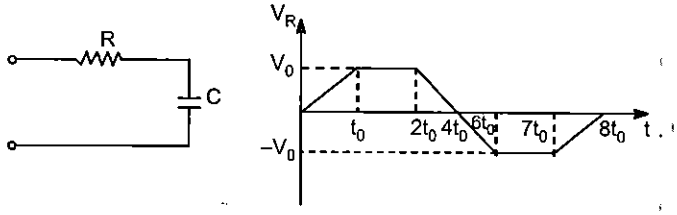
(c)


(d)



Passage 15

Figure shows a part of an RC circuit, in which potential drop across resistor is V_R. Variation of potential difference across resistor is shown in figure. Initially ($t = 0$) capacitor is uncharged.



1. What is charge an capacitor at $t = t_0$?

(a) $\frac{2V_0t_0}{R}$ (b) $\frac{3V_0t_0}{2R}$

(c) $\frac{V_0t_0}{2R}$ (d) $\frac{5V_0t_0}{2R}$

2. What is charge at $t = 2t_0$?

(a) $\frac{2V_0t_0}{R}$ (b) $\frac{3V_0t_0}{2R}$

(c) $\frac{V_0t_0}{2R}$ (d) $\frac{5V_0t_0}{2R}$

3. What is charge at $t = 4t_0$?

(a) $\frac{2V_0t_0}{R}$ (b) $\frac{3V_0T_0}{2R}$

(c) $\frac{V_0T_0}{2R}$ (d) $\frac{5V_0T_0}{2R}$

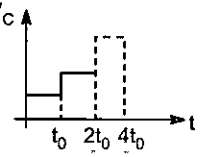
4. What is a current as function of time for $2t_0 \le t \le 4t_0$?

(a) $\frac{2V_0}{R} - \frac{V_0}{2t_0R}t$ (b) $\frac{V_0}{2R} - \frac{3V_0}{2t_0R}t$

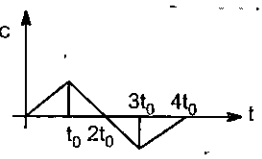
(c) $\frac{V_0}{4R} - \frac{2V_0}{t_0R}t$ (d) $\frac{3V_0}{R} - \frac{V_0}{3t_0R}$

5. Most appropriate graph of potential difference across capacitor *versus* time graphs for $0 \le t \le 4t_0$

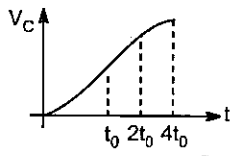
(a)



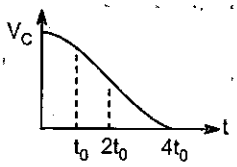
(b)



(c)

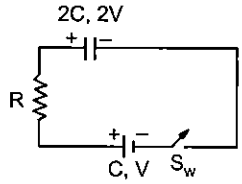


(d)



Passage 16

Two capacitors of capacitance is $2C$ and C are charged to potential difference $2V$ and V respectively. Now the charged capacitors are connected with a resistor R as shown in figure. Switch is closed at $t = 0$.



1. What is initial current in circuit ?

(a) $\frac{V}{R}$ (b) $\frac{2V}{R}$

(c) $\frac{V}{2R}$ (d) $\frac{3V}{R}$

2. What is potential difference across each capacitor in steady state ?

(a) $\frac{2V}{5}$ (b) $\frac{2}{3}V$

(c) $\frac{5V}{3}$ (d) $\frac{3}{5}V$

3. What is total heat produced in circuit till steady state is obtained ?

(a) $\frac{1}{3}CV^2$ (b) $\frac{CV^2}{2}$

(c) $\frac{2}{3}CV^2$ (d) $\frac{3}{5}CV^2$

4. What is current in circuit as function of time ?

(a) $\frac{V}{R}e^{-3t/RC}$ (b) $\frac{2V}{R}e^{-2t/3RC}$

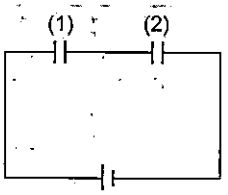
(c) $\frac{V}{3R}e^{-t/2RC}$ (d) $\frac{V}{R}e^{-3t/2RC}$

MATCHING TYPE PROBLEMS

1. On a capacitor of capacitance C_0 following steps are performed in the order as given in column I.
(A) Capacitor is charged by connecting it across a battery of emf $\mathcal{E}_0$.
(B) Dielectric of dielectric constant K and thickness d is inserted
(C) Capacitor is disconnected from battery
(D) Separation between plates is doubled

	Column–I (Steps performed)		Column–II Final value of Quantity (Symbols have usual meaning)
(a)	(A) (D) (C) (B)	(p)	$Q=\frac{C_0\mathcal{E}_0}{2}$
(b)	(D) (A) (C) (B)	(q)	$Q=\frac{KC_0\mathcal{E}_0}{K+1}$
(c)	(B) (A) (C) (D)	(r)	$C=\frac{KC_0}{K+1}$
(d)	(A) (B) (D) (C)	(s)	$V=\frac{\mathcal{E}_0(K+1)}{2K}$

2. In the circuit, both capacitors are identical. Column I indicates action done on capacitor 1 and Column II indicates effect on capacitor 2.

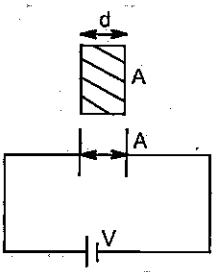


	Column-I		Column-II
(a)	Plates are moved further apart.	(p)	Amount of charge on left plate increases
(b)	Area increased	(q)	Potential difference increases
(c)	Left plate is earthed	(r)	Amount of charge on right plate decreases
(d)	It's plates are short circuited	(s)	None of the above effects

3. The potential across a 3μF capacitor is 12 V when it is not connected to anything. It is then connected in parallel with an uncharged 6μF capacitor. At equilibrium, the charge and potential difference across the capacitor 3μF and 6μF are listed in column I. Match it with column. II.

	Column-I		Column-II
(a)	charge on 3 μF capacitor	(p)	12μC
(b)	charge on 6 μF capacitor	(q)	24μC
(c)	potential difference across 3μF capacitor	(r)	8 V
(d)	potential difference across 6μF capacitor	(s)	4 V

4. Some events related to a capacitor are listed in column-I. Match these events with their effect(s) is column-II.

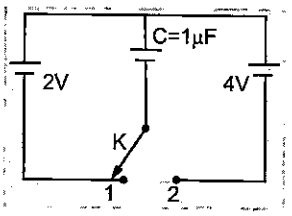


	Column-I (Events)		Column-II (Effects)
(a)	Insertion of dielectric while battery remain attached	(p)	Electric field between plates changes
(b)	Removal of dielectric while battery is not present	(q)	Charge present on plates changes
(c)	Slow decrease in separation between plates while battery is attached	(r)	Energy stored in capacitor increases
(d)	Slow increase of separation between plates while battery is not present.	(s)	Work done by external agent is positive.

5. Column I gives certain situations in which capacitance of a capacitor is changed by different means. Column II gives resulting effect under different conditions. Match the column I with the in column II.

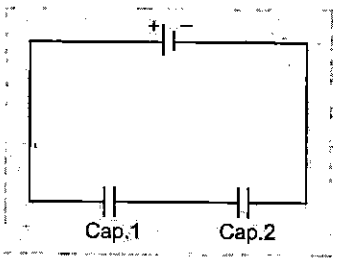
	Column-I		Column-II
(a)	The plates of a plane parallel capacitor are slowly pulled apart. Then the magnitude of electric field intensity inside the capacitor.	(p)	Increases if the capacitor is maintained at constant charge.
(b)	The plates of a plane parallel plate capacitor are slowly pulled apart. Then the potential energy stored in the capacitor.	(q)	Decreases if the capacitor is maintained at constant charge.
(c)	The capacitance of an air filled plane parallel plate capacitor on insertion of dielectric.	(r)	Increases if the capacitor is maintained at constant potential difference
(d)	A dielectric slab is inserted inside an air filled plane parallel plate capacitor. The potential energy stored in the capacitor.	(s)	Decreases if the capacitor is maintained at constant potential difference.

6. The circuit involves two ideal cells connected to a 1 μF capacitor via a key K. Initially the key K is in position 1 and the capacitor is charged fully by 2V cell. The key is pushed to position 2. Column I gives physical quantities involving the circuit after the key is pushed from position 1. Column II gives corresponding results. Match the Column I with Column II.



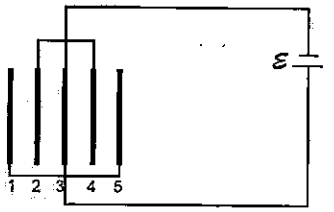
	Column-I		Column-II
(a)	The net charge crossing the 4 volt cell in μC is	(p)	2
(b)	The magnitude of work done by 4 volt cell in μJ is	(q)	6
(c)	The gain in potential energy of capacitor in μJ is	(r)	8
(d)	The net heat produced in circuit in μJ is	(s)	16

7. Two identical capacitors are connected in series and the combination is connected with a battery, as shown in Fig. Some changes in capacitor 1 are now made independently after the steady state is achieved, listed in column I. Some effects which may occur in new steady state due to these changes on the capacitor 2 are listed in column II. Match the changes on capacitor 1 in column I with corresponding effect on capacitor 2 in column II.



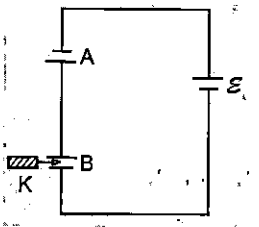
	Column-I		Column-II
(a)	A dielectric slab is inserted	(p)	Charge on the capacitor increases
(b)	Separation between plates increased	(q)	Charge on the capacitor decrease
(c)	A metal plate is inserted connecting both plates	(r)	Energy stored in the capacitor increases.
(d)	The left plate is grounded	(t)	No change occurs

8. Five identical capacitor plates, each of area A, are arranged such that adjacent plates are at d distance apart. Plates are connected to a source of e.m.f. $\mathcal{E}$ as shown in Fig. Match the following :



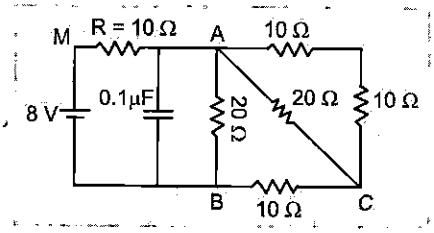
	Column-I		Column-II
(a)	Charge on plate 1	(p)	$-2\varepsilon_0 A\mathcal{E}/d$
(b)	Charge on plate 4	(q)	$+\varepsilon_0 A\mathcal{E}/d$
(c)	Potential difference between plates 2 and 3	(r)	zero
(d)	Potential difference between plates 1 and 5	(s)	$\mathcal{E}$

9. Two identical capacitors A and B are connected to a battery of emf $\mathcal{E}$ as shown in figure. Now a dielectric slab is inserted between the plates of capacitor B while battery remains connected. Due to this insertion some physical quantities may change which are mentioned in Column-I and the effect is mentioned in Column-II. Match the Column I with Column-II.



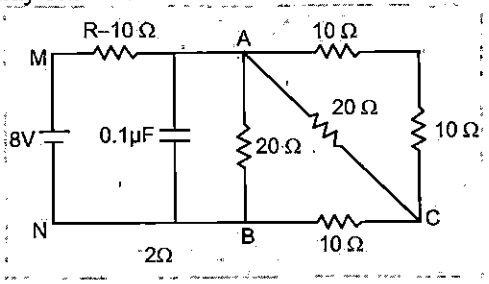
	Column-I		Column-II
(a)	Charge on A	(p)	Increases
(b)	Charge on B	(q)	Decreases
(c)	Potential difference across A	(r)	Remains constant
(d)	Potential difference across B	(s)	Will change

10. A capacitor of capacitance 0.1μF is connected to a battery of e.m.f. 8V (as shown in Fig. under the steady state condition.



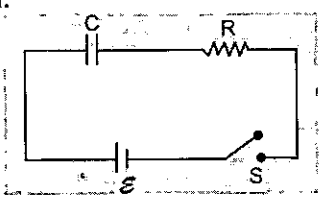
	Column-I		Column-II
(a)	Charge on the capacitor	(p)	0.4 μ C
(b)	Current in branch AC	(q)	0.2 A
(c)	Current in branch AB	(r)	0.1 A
		(s)	0.6 μ C

11. A capacitor of capacitance 0.1μF is connected to a battery of e.m.f. 8V (as shown in Fig.) under the steady state condition.



	Column-I		Column-II
(a)	Charge on the capacitor	**(p)**	0.4μC
(b)	Charge in *AC* branch	**(q)**	0.2 A
(c)	Current in *AB* branch	**(r)**	0.1 A
(d)	Current in *R* connected between *M* and *N*	**(s)**	0.4 A

12. Fig. shows a charging circuit of a capacitor. At $t = 0$, *S* is closed.



	Column-I		Column-II
(a)	When the charging rate of the capacitor is maximum, the current through *R* is	**(p)**	maximum
(b)	When charge on the capacitor is maximum, then current through *R* is	**(q)**	minimum but not zero
(c)	When power supplied by the battery is maximum, then charge on the capacitor is	**(r)**	zero
(d)	The difference in the power supplied by battery and power consumed in *R* at $t = 0$ is	**(s)**	not equal to zero

ASSERTION AND REASON

Direction: In the questions that follows two statements are given. Statement-2 is purported to be the explanation for statement-1. Study both the statements carefully and then select your answers according to the codes given below:

Select your answer as

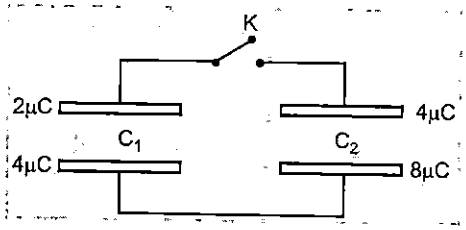
(a) If Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

(b) If Statement-1 is true, Statement-2 is true, Statement-2 is not a correct explanation for Statement-1.

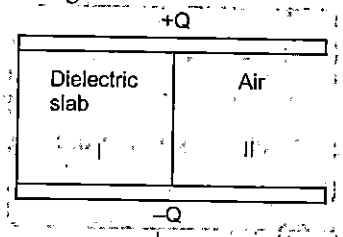
(c) If Statement-1 is true; Statement-2 is false.

(d) If Statement-1 is false; Statement-2 is true.

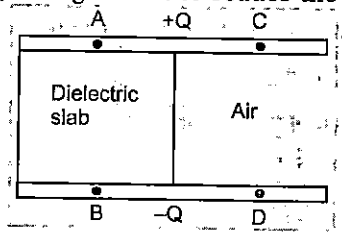
1. **Statement-1:** If temperature is increased, the dielectric constant of a polar dielectric decreases whereas that of a non-polar dielectric does not change significantly.
   Statement-2: The magnitude of dipole moment of individual polar molecule decreases significantly with increase in temperature.
2. **Statement-1:** The heat produced by a resistor in any time t during the charging of a capacitor in a series circuit is half the energy stored in the capacitor by that time.
   Statement-2: Current in the circuit is equal to the rate of increase in charge on the capacitor.
3. **Statement-1:** Charges are given to plates of two plane parallel plate capacitors C_1 and C_2 (such that $C_2 = 2C_1$) as shown in Fig. Then the key K is pressed to complete the circuit. Finally the net charge on upper plate and net charge on lower plate of capacitor C_1 is positive.



   Statement-2: In a parallel plate capacitor both plates always carry equal and opposite charge.
4. **Statement-1:** A dielectric is inserted between the plates of an isolated fully-charged capacitor. The dielectric completely fills the space between the plates. The magnitude of electrostatic force on either metal plate decreases, as it was before the insertion of dielectric medium.
   Statement-2: Due to insertion of dielectric slab in an isolated parallel plate capacitor (the dielectric completely fills the space between the plates), the electrostatic potential energy of the capacitor decreases.
5. **Statement-1:** If the potential difference across a plane parallel plate capacitor is doubled then the potential energy of the capacitor becomes four times under all conditions.
   Statement-2: The potential energy U stored in the capacitor is $U = \frac{1}{2}CV^2$, where C and V have usual meaning.
6. **Statement-1:** A parallel plate capacitor is charged to a potential difference of 100V, and disconnected from the voltage source. A slab of dielectric is then slowly inserted between the plates. Compared to the energy before the slab was inserted, the energy stored in the capacitor with the dielectric is decreased.
   Statement-2: When we insert a dielectric between the plates of a capacitor, the induced charges tend to draw in the dielectric into the field (just as neutral objects are attracted by charged objects due to induction). We resist this force while slowly inserting the dielectric, and thus do negative work on the system, removing electrostatic energy from the system.
7. **Statement-1:** A charged plane parallel plate capacitor has half interplanar region (I) filled with dielectric slab. The other half region II has air. Then the magnitude of net electric field in region I is less than that in region II.



   Statement-2: In a dielectric medium induced (or polarised) charges tend to reduce the electric field.



ANSWERS

Level-3: Comprehension Based Problems

Passage-1:	**1.** (a)	**2.** (b)	**3.** (b)	**4.** (a)	**Passage-2:**	**1.** (b)	**2.** (a)	**3.** (c)	**4.** (c)
Passage-3:	**1.** (b)	**2.** (a)	**3.** (c)	**4.** (b)	**Passage-4:**	**1.** (c)	**2.** (b)	**3.** (b)	**4.** (d)
	5. (c)	**6.** (a)	**7.** (a)	**8.** (a)		**5.** (a)			
	9. (c)	**10.** (c)							
Passage-5:	**1.** (c)	**2.** (b)	**3.** (b)		**Passage-6:**	**1.** (a)	**2.** (c)	**3.** (a)	
Passage-7:	**1.** (c)	**2.** (c)	**3.** (a)		**Passage-8:**	**1.** (b)	**2.** (d)	**3.** (a)	**4.** (b)
Passage-9:	**1.** (a)	**2.** (c)	**3.** (a)	**4.** (c)	**Passage-10:**	**1.** (d)	**2.** (d)		
Passage-11:	**1.** (a)	**2.** (b)			**Passage-12:**	**1.** (a)	**2.** (b)	**3.** (c)	**4.** (c)
Passage-13:	**1.** (b)	**2.** (d)			**Passage-14:**	**1.** (b)	**2.** (a)	**3.** (b)	**4.** (c)
						5. (b)	**6.** (b)		
Passage-15:	**1.** (c)	**2.** (b)	**3.** (d)	**4.** (a)	**Passage-16:**	**1.** (a)	**2.** (c)	**3.** (a)	**4.** (d)
	5. (c)								

Matching Type Problem

1. (a)-p, r, s; (b)-p, r, s; (c)- r; (d)-q, r

2. (a)-r; (b)-p, q; (c)-s; (d)-p, q

3. (a)-p; (b)-q; (c)-s; (d)-s

4. (a)-q, r; (b)-p, r, s; (c)-p, q, r; (d)-r, s

5. (a)-s; (b)-p, s; (c)-p, r; (d)-q, r

6. (a)-p; (b)-r; (c)-q; (d)-p

7. (a)-p, r; (b)-q; (c)-p, r; (d)-s

8. (a)-q; (b)-p; (c)-s; (d)-r

9. (a)-p, s; (b)-q, s; (c)-q, s; (d)-q, s

10. (a)-q; (b)-r; (c)-q

11. (a)-p; (b)-r; (c)-q; (d)-s

12. (a)-p, s; (b)-r; (c)-r; (d)-r

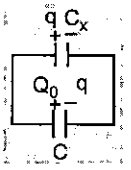
Assertion and Reason

1.	(c)	**2.**	(d)	**3.**	(c)	**4.**	(d)	**5.**	(d)	**6.**	(a)	**7.**	(d)

Level-3: Comprehension Based Problems

Passage-1

1. [a] When distance between plates of one capacitor is 'x'



$$C_x = \frac{A\varepsilon_0}{x},\ C = \frac{A\varepsilon_0}{d}$$

Charge on C_x is q

$$q = Q_x = \frac{C_x Q_0}{C + C_x}$$

$$\left(\text{since } \frac{q}{C_x} = \frac{Q_0 - q}{C} \Rightarrow q = \frac{C_x Q_0}{C + C_x}\right)$$

$$F_x = \frac{Q_x^2}{2A\varepsilon_0} = \frac{C_x^2 Q_0^2}{(C + C_x)^2 \times 2A\varepsilon_0}$$

$$dW = F_x . dx$$

$$W = \int_d^{2d} F_x . dx$$

Putting value of F_x and C_x

We get $W = \dfrac{Q_0^2 d}{12A\varepsilon_0}$

2. [b] From above question

$$F_x = \frac{C_x^2 Q_0^2}{(C + C_x)^2 \times 2A\varepsilon_0}$$

$$C_x = \frac{A\varepsilon_0}{x}$$

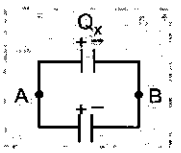
This is also equal to force by external agent since acceleration of plates = 0 (*i.e.,* they are moved slowly)

$$\therefore \quad F_{\text{external}} = F_x$$

$$= \frac{\left(\frac{A\varepsilon_0}{x}\right)^2 Q_0^2}{\left(\frac{A\varepsilon_0}{d} + \frac{A\varepsilon_0}{x}\right)^2 \cdot 2A\varepsilon_0}$$

$$F = \frac{Q_0^2}{2\left(\frac{x}{d} + 1\right)^2 A\varepsilon_0}$$

3. [b] From previous question

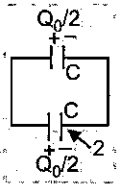


$$V = \frac{Q_x}{C_x}$$

$$V = \frac{C_x Q_0}{(C + C_x) C_x} = \frac{Q_0}{C + C_x}$$

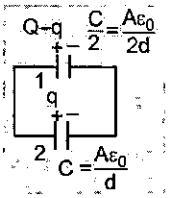
$$V = \frac{Q_0}{\frac{A\varepsilon_0}{d} + \frac{A\varepsilon_0}{x}} = \frac{Q_0 x d}{A\varepsilon_0 (x + d)}$$

4. [a] Initially energy stored in capacitor



$$V_0 = \frac{1}{2C}\left(\frac{Q_0}{2}\right)^2 = \frac{Q_0^2}{8C} \qquad \ldots(i)$$

Finally charge distribution is as shown



$$\Rightarrow \quad \frac{Q - q}{\frac{C}{2}} = \frac{q}{C}$$

$$Q - q = 2q$$

$$Q = \frac{Q}{3}$$

$\Rightarrow$ Finally energy stored in capacitor 2

$$U = \frac{1}{2}\left(\frac{Q}{3}\right)^2 \times \frac{1}{C} = \frac{Q^2}{18C}$$

$$\Delta U = U_0 - U = \frac{Q^2}{C}\left(\frac{-1}{18} + \frac{1}{8}\right)$$

$$= \frac{5}{72}\frac{Q^2}{C}, \textit{ i.e.,} \text{ decrease in energy.}$$

Passage-2

1. [b] Maximum possible charge on $C = CV_0$ and that on $2C = 2CV_0$

Since same charge will be on both capacitors so potential difference $= \dfrac{CV_0}{C_{\text{eqv}}} = \dfrac{CV_0}{\frac{C \times 2C}{C + 2C}} = \dfrac{3V_0}{2}$

2. [a] Since capacitors C undergoes break down at lower charge, so, it will breakdown first.

3. [c]

Both the capacitors are in parallel means voltage across each is equal. Breakdown potential of both is also equal. Therefore, they will undergo breakdown at the same moment.

4. [c] Maximum voltage across AD for safe use $= V_0$

Charge on $C = CV_0$

Charge on C or $2C = \dfrac{2CV_0}{3}$

So, charge on $2C$ (between D and B)

$$= \frac{2CV_0}{3} + CV_0 = \frac{5CV_0}{3}$$

So potential difference across this capacitor

$$= \frac{q}{2C} = \frac{5V_0}{6}$$

So net potential difference across A and B

$$= V_0 + \frac{5V_0}{6} = \frac{11V_0}{6}$$

Passage-3

1. [b]

2. [a]

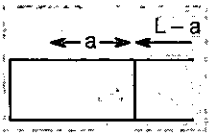
$T = Mg$ $\longleftarrow$ $\longrightarrow$ $F_{\text{inwards}} = \dfrac{\varepsilon_0 b \mathcal{E}^2 (K-1)}{2d}$

For equilibrium,

$$Mg = \frac{\varepsilon_0 b \mathcal{E}^2 (K-1)}{2d}$$

$$M = \frac{\varepsilon_0 b \mathcal{E}^2 (K-1)}{2dg}$$

3. [c] The force on slab is always const.



$\therefore$ It's motion can't be SHM.

Motion will be periodic and it's time period $T = 4t$

also $\dfrac{1}{2} \times \dfrac{F}{M} \times t^2 = (L-a)$

$\Rightarrow$ $t = \sqrt{\dfrac{2(L-a)M}{F}}$

$\Rightarrow$ $t = \sqrt{\dfrac{4d(L-a)M}{\varepsilon_0 b \mathcal{E}^2 (K-1)}}$

$\Rightarrow$ $T = 8\sqrt{\dfrac{d(L-a)M}{\varepsilon_0 b \mathcal{E}^2 (K-1)}}$

4. [b] $C = \dfrac{\varepsilon_0 b}{d}[L + x(K-1)]$

given $x = v_0 t$

$\therefore$ $C = \dfrac{\varepsilon_0 b}{a}[L + v_0 t(K-1)]$

$$Q = C\mathcal{E} = \frac{\varepsilon_0 b \mathcal{E}}{d}[L + v_0 t(K-1)]$$

$$I = \frac{dQ}{dt} = \frac{\varepsilon_0 b \mathcal{E} v_0 (K-1)}{d}$$

5. [c]

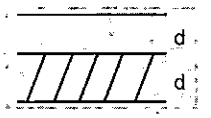
	Initially	**Finally**
Capacitance	$C = \dfrac{bL\varepsilon_0}{d}$	KC
Potential	V = const.	V = const.
Charge	$Q_1 = CV$	$Q_2 = KCV$
Energy stored	$U_1 = \dfrac{1}{2}CV^2$	$U_2 = \dfrac{1}{2}KCV^2$

Electric field strength

$E_1 = \dfrac{V}{d}$ = const. $E_2 = \dfrac{V}{d}$ = const.

$\therefore$ $E_1 = E_2$ $Q_2 > Q_1$, $U_2 > U_1$

6. [a]



	Initially	**Finally**
Capacitance	$C_0 = \dfrac{KA\varepsilon_0}{d}$	$C = \dfrac{A\varepsilon_0}{\frac{d}{K} + d}$
Charge	$Q_0 = C_0 V_0$	$Q_0 = C_0 V_0 = CV$ (charge will be same)
Potential	$V_0 = \dfrac{Q_0}{C_0}$	$V = \dfrac{Q_0}{C}$
	$V_0 = \dfrac{Q_0 d}{KA\varepsilon_0}$	$V = \dfrac{Q_0\left(d + \frac{d}{K}\right)}{A\varepsilon_0}$
Energy	$V_0 = \dfrac{1}{2}\dfrac{Q_0^2}{C_0}$	$U = \dfrac{1}{2}\dfrac{Q_0^2}{C}$

Clearly $C_0 > C$

$\Rightarrow$ $V_0 < V$, $U_0 < U$

also electric field onside slab

initially $E_0 = \dfrac{V_0}{d} = \dfrac{Q_0}{KA\varepsilon_0}$

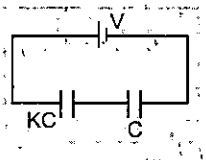
Finally (assuming 2 capacitors in series both will have same charge)

$$E = \frac{Q_0}{KA\varepsilon_0}$$

$\Rightarrow$ $E = E_0$

$\therefore$ $V > V_0, U > U_0, E = E_0$

7. [a]



max. $\rightarrow V_2 = E_2 d - V_1 = E_1 d$ potentials

$$C_{\text{net}} = \frac{KC}{K+1} \Rightarrow Q = \frac{KCV}{K+1}$$

$\therefore$ potential difference across 'KC' capacitor

$$= \frac{Q}{KC} = \frac{V}{K+1} \Rightarrow \frac{V}{K+1} = V_2(\text{max.})$$

$$\frac{V}{K+1} = E_2 d \Rightarrow V = E_2 d(K+1) \quad \text{...(i)}$$

Similarly potential across capacitor 'c' $= \dfrac{KV}{K+1}$

$\Rightarrow$ $V = E_1 d\dfrac{(K+1)}{K}$ ...(ii)

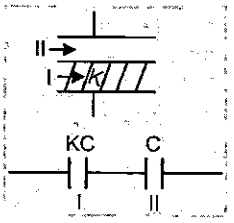
From (i) and (ii)

V_{min} will be taken

$$\therefore \quad V = E_1 d\left(1+\frac{1}{K}\right)$$

$$\left[\because E_1 < E_2 \& \frac{d(K+1)}{K} < d(K+1)\right]$$

9. [c] Suppose two capacitors



In series

Charge on both capacitors will be same

$$U_I = \frac{1}{2}\frac{Q^2}{KC}$$

$$U_{II} = \frac{1}{2}\frac{Q^2}{C}$$

Clearly $U_{II} > U_I$

$$V_I = \frac{Q}{KC},\ V_{II} = \frac{Q}{C}$$

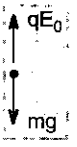
$\Rightarrow$ $E_{II} > E_I$ direction of field is same in capacitors in series

10. [c] $$mg = qE_0 \qquad ...(i)$$

As upper plate was removed. The electric field

$$E = \frac{E_0}{2} \qquad ...(ii)$$

Now $mg - qE = ma$

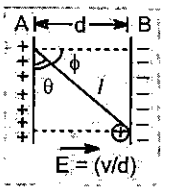


$$\Rightarrow \quad mg - \frac{mg}{2} = ma$$

$$a = \frac{g}{2} \text{ downwards.}$$

Passage-4

1. [c] Consider the ball in its final position.



$$\sin\theta = \frac{d}{l} = \frac{50}{100} = \frac{1}{2}$$

$\therefore$ $\theta = (30°)$, with vertical

$\therefore$ Angle made with horizontal,

$\phi = (60°)$

2. [b] When the ball again touches the plate A the charge on plate $A, i.e., Q_0 = (C_0V_0)$, redistributes. Let charge on plate be Q_1 and on ball be q_1 after redistribution such that the common potential is 'V'. Then we know,

$$Q_1 : q_1 = C_0 : C_1$$

Now, $$q_1 = \left(\frac{C_1}{C_0 + C_1}\right)Q_0$$

$$= \left(\frac{C_1}{C_0 + C_1}\right)C_0V_0$$

The new potential is,

$$V = \frac{q_1}{C_1} = \left(\frac{C_0V_0}{C_0 + C_1}\right)$$

3. [b] The charge carried by ball in final position is,

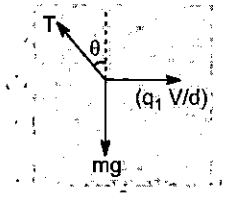
$$q_1 = C_1V = \left(\frac{C_1C_0V_0}{C_0 + C_1}\right)$$

4. [d] $$Q_1 = C_0V = \left(\frac{C_0^2V_0}{C_0 + C_1}\right)$$

$$\Rightarrow \quad q_1 = C_1V = \left(\frac{C_1C_0V_0}{C_0 + C_1}\right)$$

$$\therefore \quad Q_1 : q_1 = (C_0 : C_1)$$

5. [a] Considering the F.B.D. of ball in its final position,



$$mg\tan\theta = \frac{q_1V}{d} \Rightarrow mg\tan\theta = \frac{C_1V^2}{d}$$

$$\Rightarrow \quad mg\tan\theta = \frac{C_1}{d}\left(\frac{C_0V_0}{C_0 + C_1}\right)^2$$

$$\Rightarrow \quad V_0 = \left(\frac{C_0 + C_1}{C_0}\right)\left[\frac{mg\tan\theta}{C_1}\right]^{1/2}$$

$$\Rightarrow \quad V_0 = \left(1 + \frac{C_1}{C_0}\right)\sqrt{\frac{5m}{2\sqrt{3}C_1}}$$

Passage-7 1. (c) 2. (c) 3. (a)

Finally both capacitor have charge $C\mathcal{E}$ each. The net charge crossing the cell is $= 2C\mathcal{E}$. The work done by cell $= W = 2C\mathcal{E} \times \mathcal{E} = 2C\mathcal{E}^2$.

Energy stored in cells $= U = \frac{1}{2}C\mathcal{E}^2 + \frac{1}{2}C\mathcal{E}^2 = C\mathcal{E}^2$.

$\therefore$ Heat produced $= W - U = C\mathcal{E}^2$

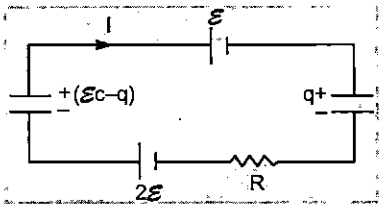
Passage-12

1. [c] Initially potential across capacitor = E

$$\Rightarrow \quad V_A + \mathcal{E} - \mathcal{E} - IR + 2\mathcal{E} = V_A$$

$$I = \frac{2\mathcal{E}}{R}$$

2. [b] $\frac{\mathcal{E}C-q}{C}-\mathcal{E}-\frac{q}{C}-IR+2\mathcal{E}=0\ I=\frac{dq}{dt}$



$$\frac{2(\mathcal{E}C-q)}{C}=IR$$

Solving $q=\mathcal{E}C(1-e^{-2t/RC})$

$\Rightarrow$ $\tau=\frac{RC}{2}$

Alternately :

Equivalent capacitance $=\frac{C}{2}$

So, time constant $=\frac{C}{2}R$

3. [c] Charge will be equal

$\Rightarrow$ $q=\mathcal{E}C\to q\Rightarrow q=\mathcal{E}\frac{C}{2}$

$\therefore$ $t=\tau\ln 2=\frac{RC}{2}\ln 2$

4. [c] Rate of energy stored in capacitor $2=\frac{q_1}{C}\times I$

Rate of energy dissipated in capacitor $1=\frac{q_2}{C}\times I$

For equality $q_1=q_2$

$\Rightarrow$ $q_1=q_2=\frac{\mathcal{E}C}{2}$

$\therefore$ $t=\tau\ln 2=\frac{RC}{2}\ln 2$

Passage-13

When switch is transferred to *b*, initial potential difference across resistor is *V*. Therefore initial current is $\frac{V}{R}$. In discharging circuit current is

$$I=I_0e^{-t/RC}$$
$$=\frac{V}{R}e^{-t/RC}$$

Charge as function of time is

$$q=qe^{-t/RC}$$
$$=CVe^{-t/RC}$$

Thus $\frac{q}{I}=RC$; graph of *E* is *I* is straight line.

Passage-14

Apply KVL on right loop in initial condition. Initially capacitor has charge *CV*.

$$V-3IR-3V=0$$

or $I=\frac{2V}{3R}$

In steady state charge on capacitor is $3CV$;

$$\Delta U=\frac{1}{2}C(3V)^2-\frac{1}{2}CV^2=4CV^2$$

Charge transferred by battery

$$\Delta q=2CV$$
$$(w_b)\text{battery}=6CV^2$$

Also $w_b=H+\Delta U$

$\therefore$ heat developed $H=w_b-\Delta U$

$$H=6CV^2-(4CV^2)$$
$$=2CV^2$$

Now we apply *KVL* in right loop in transient state

$$3V-3IR-\frac{q}{C}=0$$
$$3V-\frac{q}{C}=\frac{3dq}{dt}R$$
$$\int_{CV}^{q}\frac{dq}{3CV-q}=\int\frac{dt}{3RC}$$
$$-\ln(3CV-q)\Big|_{CV}^{q}=\frac{t}{3RC}$$
$$\frac{3CV-q}{2CV}=e^{-t/3RC}$$

or $q=3CV-2CVe^{-t/3RC}$

Current in circuit as function of time is

$$I=\frac{2V}{3R}e^{-t/3RC}$$

Passage-15

Potential difference on resistor as function of time is described below.

$$V_R=\frac{V_0}{t_0}t \qquad 0\le t\le t_0$$
$$=-V_0 \qquad t_0\le t\le 2t_0$$
$$=2V_0-\left(\frac{V_0}{2t_0}\right)t \qquad 2t_0\le t\le 6t_0$$
$$=-V_0 \qquad 6t_0\le t\le 7t_0$$
$$=\left(\frac{V_0}{t_0}\right)t-8V_0 \qquad 7t_0\le t\le 8t_0$$

Current as function of time is $\frac{V_R}{R}$.

For $0\le t\le t_0$

$$I=\frac{V_0}{Rt_0}t$$

or $\frac{dq}{dt}=\frac{V_0}{Rt_0}t$

or $\int_0^q dq = \frac{V_0}{Rt_0}\int_0^t t\,dt$

or $q = \frac{V_0}{2Rt_0}t^2$ (Parabolic curve)

For $t_0 \le t \le 2t_0$

$I = \frac{V_0}{R}$

or $\int_{\frac{V_0t_0}{2R}}^{q} dq = \frac{V_0}{R}\int_{t_0}^{2t_0} dt$

or $q = \frac{V_0t_0}{2R} + \frac{V_0}{R}t_0$

or $[q]_{t=2t_0} = \frac{3V_0t_0}{2R}$

$[V_c]_{t=2t_0} = \frac{q}{C} = \frac{3V_0t_0}{2RC}$

For $2t_0 \le t \le 4t_0$

$I = \frac{2V_0}{R} - \left(\frac{V_0}{2t_0R}\right)t$

$\int_{\frac{3V_0t_0}{2R}}^{q} dq = \int_{2t_0}^{4t_0}\left(\frac{2V_0}{R} - \frac{V_0}{2t_0R}t\right)dt$

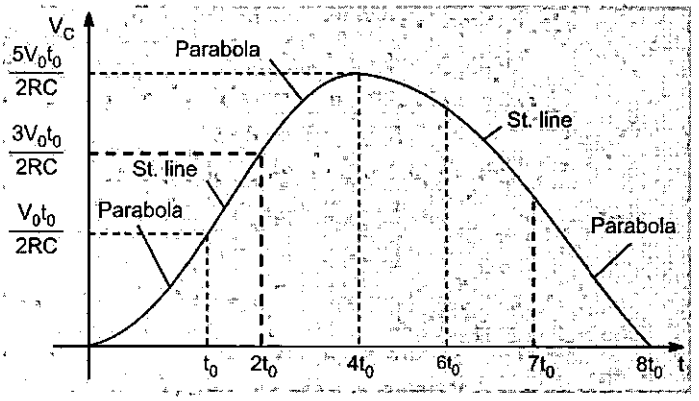
$q = \frac{3V_0t_0}{2R} + \frac{2V_0}{R}[t]_{2t_0}^{4t_0} - \frac{V_0}{4t_0R}|t^2|_{2t_0}^{4t_0}$

$= \frac{3V_0t_0}{2R} + \frac{4V_0t_0}{R} - \frac{3V_0t_0}{R}$

$= \frac{5V_0t_0}{2R}$

$(V)_{t=4t_0} = \frac{q}{C} = \frac{5V_0t_0}{2R}$

In similar manner charge in other part can also be derived. Thus graph of V_C *versus* time t is shown below

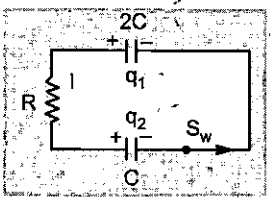


Passage-16

Let Instantaneous charges on capacitors are q_1 and q_2 respectively.

$q_1 + q_2 = 5CV$ ...(1)

and $\frac{dq_1}{dt} + \frac{dq_2}{dt} = 0$



From KVL we get

$\frac{q_1}{2C} - IR - \frac{q_2}{C} = 0$ ...(2)

Note that $I = \frac{-dq_1}{dt}$ as q_1 is decreasing

Thus $\frac{q_1}{2C} + \frac{dq_1}{dt}R - \frac{(5CV - q_1)}{C} = 0$ ...(3)

On differentiating equation (3), we get

$\frac{1}{2C}\frac{dq_1}{dt} + \frac{d^2q_1}{dt^2}R + \frac{dq_1}{dt}\frac{1}{C} = 0$

or $\left(\frac{I}{2C} + \frac{I}{C}\right) = -\frac{dI}{dt}R$

or $\int_0^I \frac{dI}{I} = -\int_0^t \frac{3dt}{2RC}$

or $I = I_0e^{-(3t/2RC)}$

where i_0 is initial current i which can be obtained from eqn. (2) by substituting $q_1 = 4CV$ and $q_2 = CV$; we get

$\frac{4CV}{2C} - I_0R - \frac{CV}{C} = 0$

or $I_0 = V/R$

In steady state potential difference across both the capacitor is same, total charge is conserved.

$5CV = 3CV'$

or $V' = \frac{5}{3}V$

Heat developed in circuit is

$H = U_i - U_F$

$= \frac{1}{2}(2C)(2V^2) + \frac{1}{2}CV^2$

$-\left[\frac{1}{2}(3C)(5/3V)^2\right]$

$= \frac{1}{3}CV^2$

Matching Type Problem

1. (a)-p, r, s; (b)-p, r, s; (c)-r; (d)-q, r

	Q	C	V
(a)	C_0V_0	C_0	V_0
	$\frac{C_0V_0}{2}$	$\frac{C_0}{2}$	V_0
	$\frac{C_0V_0}{2}$	$\frac{C_0}{2}$	V_0
	$\frac{C_0V_0}{2}$	$\frac{KC_0}{K+1}$	$\frac{V_0(K+1)}{2K}$
(b)	0	$\frac{C_0}{2d}$	0
	$V_0\frac{C_0}{2d}$	$\frac{C_0}{2d}$	V_0
	$V_0\frac{C_0}{2d}$	$\frac{C_0}{2d}$	V_0
	$\frac{C_0V_0}{2}$	$\frac{KC_0}{K+1}$	$\frac{V_0(K+1)}{2K}$
(c)	0	$\frac{KA\varepsilon_0}{d}$	0
	KC_0V_0	KC_0	V_0
	KC_0V_0	KC_0	V_0
	KC_0V_0	$\frac{KC_0}{K+1}$	$(K+1)V_0$
(d)	C_0V_0	C_0	V_0
	KC_0V_0	KC_0	V_0
	$\frac{KC_0V_0}{K+1}$	$\frac{K+C_0}{K+1}$	V_0
	$\frac{KC_0V_0}{K+1}$	$\frac{KC_0}{K+1}$	V_0

2. (a)-r; (b) -p, q; (c)-s; (d)-p, q

(a) $C_1=\frac{\varepsilon_0A}{d}$, $d\uparrow \Rightarrow C_1\downarrow \Rightarrow C_{eq}=\frac{C_1C}{C_1+C}\downarrow$

$\Rightarrow Q=C_{eq}V\downarrow$

$\Rightarrow V$ on 2^{nd} cap.$\downarrow$

$a\to r$ $\Rightarrow Q\downarrow$

(b) $C_1=\frac{\varepsilon_0A}{d}$ $A\uparrow \Rightarrow C_1\uparrow \Rightarrow C_{eq}\uparrow \Rightarrow Q\uparrow \Rightarrow V\uparrow$

(c) no effect

(d) $C_{eq}=C \Rightarrow C_{eq}\uparrow \Rightarrow V\uparrow$ and $Q\uparrow$

3. (a)-p; (b)-q; (c)-s; (d)-s

$C_1=3\mu F, V_1=12V$

$\Rightarrow$ $Q_1=36\mu C$

Total charge on the system $=36\mu C$

Total capacitance $=3+6=9\mu F$

Voltage across each $=\frac{36}{9}=4V$

$\therefore$ Charge on $3\mu F=3\times4=12\mu C$

Charge on $6\mu F=6\times4=24\mu C$.

5. (a)-s; (b)-p, s; (c)-p, r; (d)-q, r

(a) At constant charge, the electric field within the capacitor remains same when plate separation is changed. The electric field in capacitor is $E=\frac{V}{d}$.

Hence at constant potential difference the electric field decreases with increases in d.

(b) $U=\frac{1}{2}\frac{Q^2}{C}$. Hence at constant charge U increases with decreases in C.

$U=\frac{1}{2}CV^2$. Hence at constant potential difference U decreases with decreases in C.

(c) Capacitance increases on insertion of dielectric.

(d) As a result of insertion of dielectric the capacitance increases.

$U=\frac{1}{2}\frac{Q^2}{C}$. Hence at constant charge U decreases with increase in C.

$U=\frac{1}{2}CV^2$. Hence at constant potential difference U increases with increase in C.

6. (a)-p; (b)-r; (c)-q; (d)-p

The initial charge on capacitor

$=CV_i=1\times2\mu C=2\mu C$

The final charge on capacitor

$=CV_f=1\times4\mu C=4\mu C$

$\therefore$ Net charge crossing the cell of emf 4V is

$q_f-q_i=4-2=2\mu C$

The magnitude of work done by cell of emf 4 V is

$W=(q_f-q_i)4=8\mu J$

The gain in potential energy of capacitor is

$$\Delta U=\frac{1}{2}C(V_f^2-V_i^2)$$

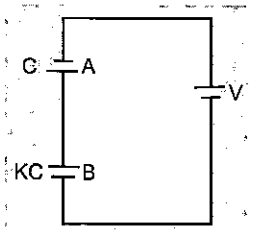
$$=\frac{1}{2}\times[4^2-2^2]\mu J=6\mu J$$

Net heat produced in circuit is

$$\Delta H=W-\Delta U=8-6=2\mu J$$

9. (a)-p, s; (b)-q, s; (c)-q, s; (d)-q, s

As slab is inserted, capacitance of B increases and hence total capacitance of system decreases as a result change on both $\downarrow$.



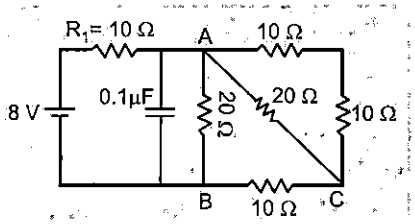
$$Q = \frac{KC}{1+K} \times V$$

$$V_A = \frac{Q}{C} = \frac{KV}{1+K} > \frac{V}{2}$$

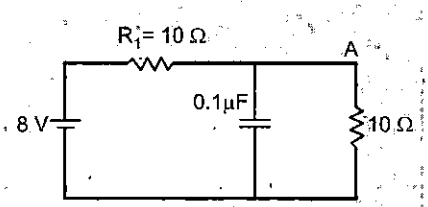
$$V_B = \frac{Q}{KC} = \frac{V}{1+K} < \frac{V}{2}$$

10. (a)-p; (b)-r; (c)-q

The equivalent circuit is as shown in Fig.



(a)



(b)

The current through $R_1 = \frac{8}{20} = 0.4A$

In the steady state, the potential difference across *AB* is 4V.

Charge on capacitor in steady state is

$$q = CV = 0.4\mu C$$

Current through resistor *R* is $I = \frac{V}{R} = \frac{4}{20} = 0.2A$

ASSERTION AND REASON

1. (d) Dipole moment of individual polar molecule does not change with temperature. Due to thermal agitation dielectric constant of polar dielectric decreases.

2. (d) $$I = \frac{V}{R} e^{-t/(RC)}$$

$$\text{Heat} = \int_0^t \frac{V^2}{R^2} e^{-2t/(RC)} . R\, dt$$

$$= \frac{1}{2} CV^2 (1 - e^{-2t/(RC)})$$

$$\text{Energy stored} = \frac{q^2}{2C} = \frac{V^2}{2C}(1 - e^{-t/(RC)})^2$$

$\therefore$ Not equal

$$I = \frac{dq}{dt}$$

4. (d) The electrostatic force on metal of capacitor is

$$= \text{pressure} \times \text{area of plate} = \frac{\sigma^2}{2\varepsilon_0} A$$

σ = charge per unit area on plate.

Since charge on metal plate of an "isolated" capacitor does not change, force on metal plate remains same.

Electric field decreases due to induced charges in dielectric, but this does not effect the charge distribution on isolated metal plate.

5. (d) If potential difference across an isolated charged capacitor is doubled by doubling separation between plates, the energy stored in capacitor from $U = \frac{Q^2}{2C}$ becomes double of previous value.

Hence statement-1 is false.

6. (a) In an isolated charged capacitor the potential energy stored is decreased on insertion of dielectric because in expression $U = \frac{Q^2}{2C}$, C increases on insertion of dielectric.

Statement-2 is correct and correct explanation of statement-1.

7. (d) Let the electric field in region I and II be E_1 and E_2. The potential difference across left half capacitor and right half capacitor is same. Therefore $E_1 d = E_2 d$, where d = inter planar gap.

$\therefore$ $$E_1 = E_2$$

Hence statement-1 is false, statement-2 is correct by definition.

4 THE MAGNETIC FIELD

INTRODUCTION

Using the properties of magnets as a starting point, we develop the concept of *magnetic field B.* We then consider the *magnetic force* experienced by an electric charge when it moves through a magnetic field.

MAGNETS AND MAGNETIC POLES

Magnetism, like electricity, has been known to human since long back. The ancient Greeks knew that certain minerals called *lodestones* could attract iron objects. Chinese navigators were using compasses by the year 1000. In about 1600, William Gilbert recognized that compasses work because the earth itself is a magnet.

Fig. 4.1 shows a **bar magnet**—a permanent magnet in the shape of a long, thin bar. Any bar magnet suspended from a thread soon assumes a roughly northward-southward orientation. The end that points northward is called the **north pole.** The southward-pointing end is called **south pole.**

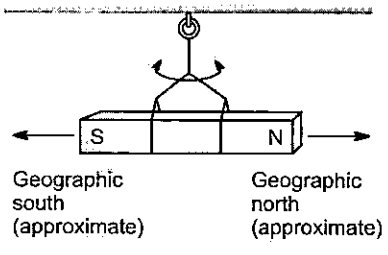


Fig. 4.1

Simple experimentation with magnets leads to the following general observations.

1. When two magnets are brought close together, there is an *attractive* force between the north pole of one and the south pole of the other. We say that **unlike magnetic poles attract one another** Fig. 4.2 (a).

2. When two magnets are brought close together, there is a *repulsive* force between the two north poles and also between the two south poles. We say that **like magnetic poles repel one another** Fig. 4.2 (b).

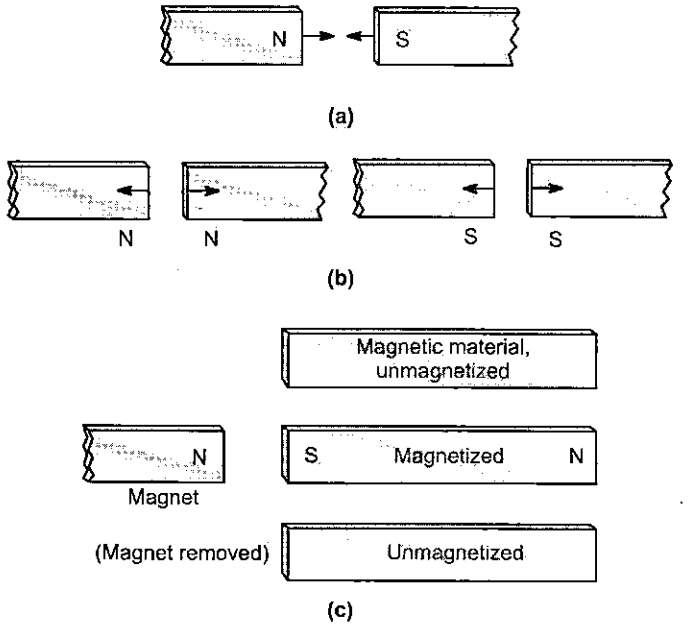


Fig. 4.2

3. Most objects are not obviously affected by magnets. But there is a class of materials, called **ferromagnetic materials** or **magnetic materials** for short, on which a magnet exerts an *attractive* force. The most familiar ferromagnetic materials are iron, nickel, and many of their alloys, but there are others as well.

(a) Any bar of magnetic material brought close to a permanent magnet acts like a magnet itself, with poles as shown in Fig. 4.2 (c). The bar is then said to be **magnetized,** and it exhibits **induced magnetization.** For most magnetic materials, such as soft iron, the induced magnetization vanishes when the permanent magnet is removed.

(b) Permanent magnets are made of a special subclass of ferromagnetic materials that are said to possess **magnetic remanence.** When a bar of such a material is magnetized by induction under proper conditions, it retains its magnetic properties even when the inducing magnet is removed.

4. No force is observed between a magnet and an electrically charged body when the two are placed near each other.

5. When a dielectric is charged by induction, it becomes an *electric dipole*, as shown in Fig. 4.3 (a). If it is then cut into two pieces, as in Fig. 4.3 (b), each piece becomes an **electric monopole,** containing an excess of charge of one sign. In the presence of the inducing monopole, each piece carries an electric dipole moment as well as excess charge of one sign. In Fig. 4.3 (c), the monopole is removed and the two cut pieces separated. Each piece is now an electric monopole. In Fig. 4.3 (d-f), the corresponding process is carried out with a bar of iron magnetized by induction. If Fig. 4.3 (d), the bar acquires a pair of opposite poles; we say it becomes a **magnetic dipole.** When the dipole is cut [Fig. 4.3 (e)]. Each piece of the original magnetic dipole becomes a smaller magnetic dipole. But, when the inducing magnet is removed, each piece reverts to the unmagnetized state [Fig. 4.3 (f)].

When a permanent magnet is cut into two, each piece becomes a smaller magnet [Fig. 4.3 (g, h)].

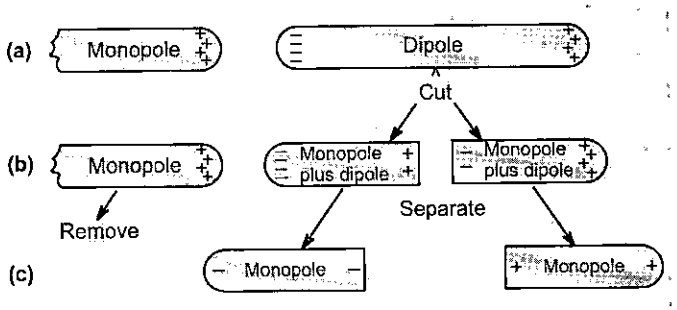


Fig. 4.3

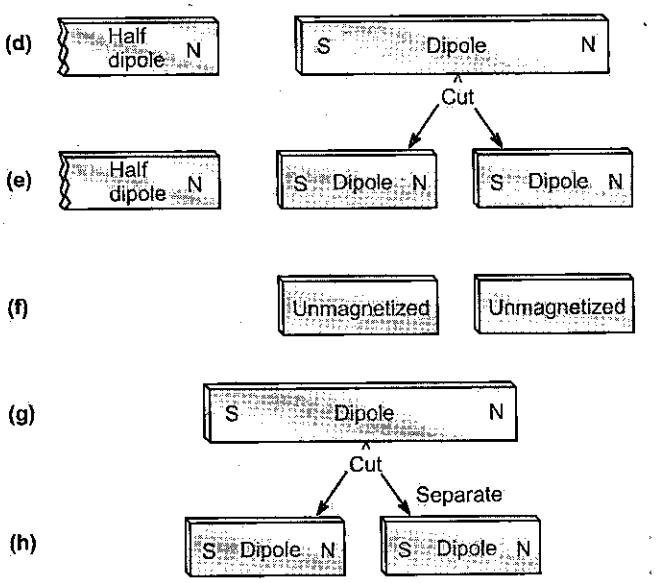


Fig. 4.3

On the basis of experiments, we make the generalization that magnetic monopoles are never observed. The north pole of a compass needle is attracted toward the geographic north pole of the earth and repelled by the earth's geographic south pole. Apparently the earth itself is a large magnet, as shown in Fig. 4.4. Geophysicists generally agree that the earth's magnetic poles arise from currents in its molten iron core. Two interesting facts about the earth's magnetic field are one, that the magnetic poles are offset slightly from the geographic poles of the earth's rotation axis, and two, that the geographic north pole is actually a *south* magnetic pole.

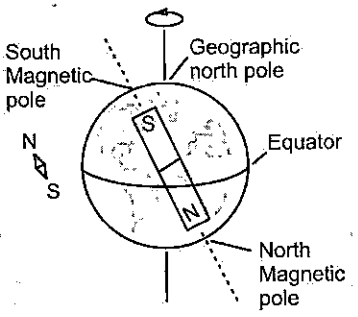


Fig. 4.4

The link between electricity and magnetism was discovered during *a classroom lecture demonstration* in 1819 by the Danish scientist Hans Christian Oersted. Oersted was using a battery to produce a large current in a wire. By chance, a compass was sitting next to the wire, and Oersted noticed that the current caused the compass needle to turn.

Oersted's discovery led to fact that **magnetism** is caused by an electric current.

How can we determine the sense of magnetic field lines? As already noted, a tiny compass will align itself with the tangent to the magnetic field line at its location. By convention, *the orientation of the north pole of the compass determines the sense of the magnetic line.* The north pole of the compass is repelled by the north pole of the bar magnet, while the south pole is attracted. Hence, the sense of the

magnetic field line is the one that leads from the north to the south pole of the bar magnet (Fig. 4.5).

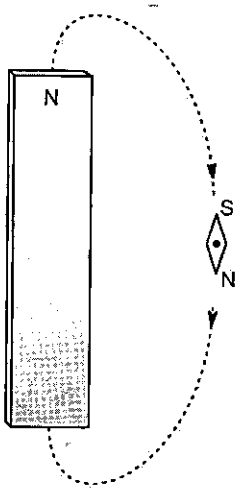


Fig. 4.5: A tiny compass needle orients itself tangent to the magnetic field line passing through its position in the vicinity of a bar magnet. By convention, the north pole of the compass points out the sense of the flux line. That its the flux line emerges from the magnet near its north pole and reenters the magnet near its south pole.

We need experimental results. The necessary experiment can be done as shown in Fig. 4.6.

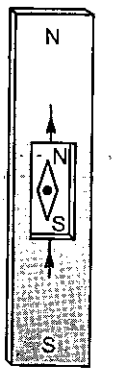


Fig. 4.6: A small cavity is made, and the compass needle is inserted in it. The north pole of the compass indicates that the sense of the flux line within the magnet is south to north.

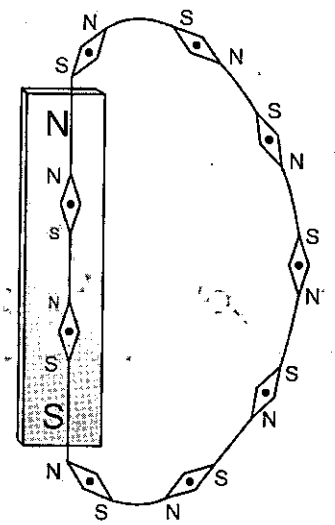


Fig. 4.7: Tiny compass needles are placed at various locations on the same magnetic field line, both outside and inside a bar magnet. The orientation of the north poles of the compass needles determines the sense of the flux line, which is a closed curve.

Within the magnet the magnetic field line runs from south pole to north pole. Magnetic field line are closed curves having no ends, as shown in Fig. 4.7.

A magnetic field exerts no force on an electric charge *at rest. An electric charge does experience a force when it* ***moves*** *in a magnetic field.* This force is called the **magnetic force.** The magnetic force is more complicated than the electric force because the magnetic force is **velocity dependent.** That is, its magnitude F and direction $\hat{\mathbf{F}}$ depend on the magnitude v and direction $\hat{\mathbf{v}}$ of the velocity of the charge as well as on the magnitude B and direction $\hat{\mathbf{B}}$ of the field.

THE MAGNETIC FIELD

The idea of a field is a way to understand the long-range electric force. A charge alters the space around it by creating an electric field. A second charge then experiences a force due to the presence of the electric field. The electric field is the means by which charges interact with each other. Although this idea appeared rather far-fetched, it turned out to be very useful. We need a similar idea to understand the long-range force exerted by a current on a compass needle.

Concept: *The* ***magnetic field*** $\vec{\mathbf{B}}$ *as having the following properties:*

1. A magnetic field is created at all points in space surrounding a current-carrying wire.

2. The magnetic field at each point is a vector. It has both a magnitude, which we call the magnetic field strength B, and a direction.

3. The magnetic field exerts forces on magnetic poles. The force on a north pole is parallel to $\vec{\mathbf{B}}$*; the force on a south pole is opposite* $\vec{\mathbf{B}}$.

Fig. 4.8 shows a compass needle in a magnetic field. The field vectors are shown at several points, the field is present at *all* points in space. A magnetic force is exerted on each of the two poles of the compass, parallel to $\vec{\mathbf{B}}$ for the north pole and opposite $\vec{\mathbf{B}}$ for the south pole. This pair of opposite forces exerts a torque on the needle, rotating the needle until it is parallel to the magnetic field at that point. Magnetic forces cause a compass needle to become aligned parallel to a magnetic field, with the north pole of the compass showing the direction of the magnetic field at that point.

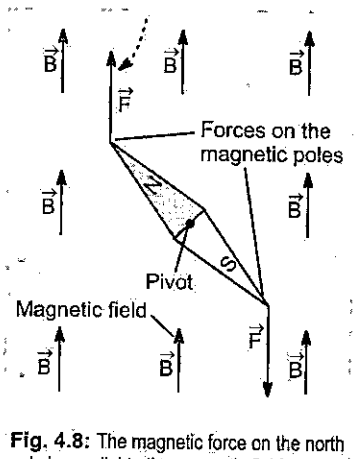


Fig. 4.8: The magnetic force on the north pole is parallel to the magnetic field.

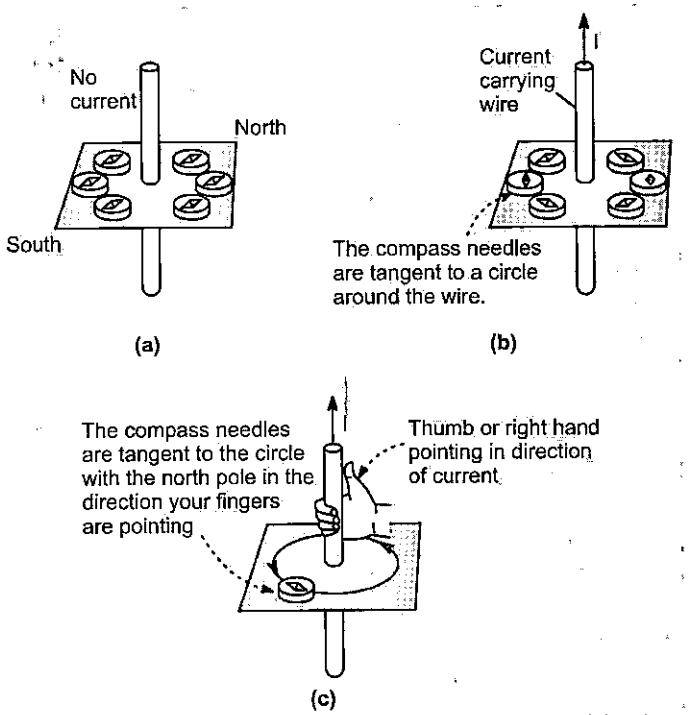


Fig. 4.9: Response of compass needles to a current in a straight wire.

We will often need to indicate field vectors or currents that are perpendicular to the page. Fig. 4.10 (a) shows the notation we will use. Fig. 4.10 (b) demonstrates this notation by showing the compasses around a current that is directed into the page. To use the right-hand rule with this drawing, point your right thumb into the page. Your fingers will curl cw, and that is the direction in which the north poles of the compass needles point.

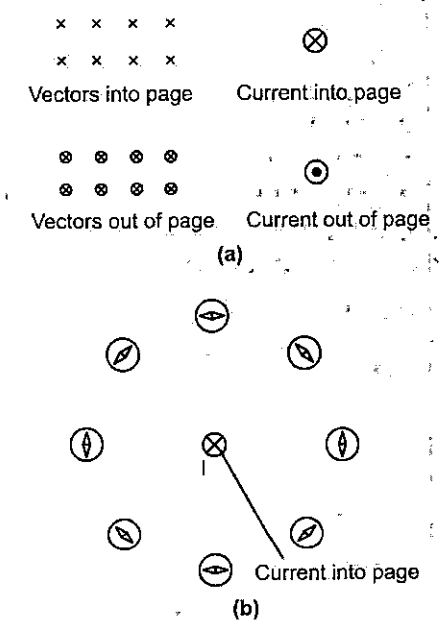


Fig. 4.10: The notation for vectors and currents perpendicular to the page.

FORCE AND FIELD FOR $v \perp B$

We summarize the results of experiments:

1. When a charge q moves *parallel* or *antiparallel* to the magnetic field B, the charge experiences no force.

2. When the direction of motion $\hat{\mathbf{v}}$ of the charge does not coincide with $\hat{\mathbf{B}}$, a force F is exerted on the charge. The direction of F is always *perpendicular* to the plane defined by v and B.

3. The magnitude of F depends on the angle θ between v and B. Specifically, F is proportional to $\sin\theta$ and reaches a maximum when $\theta = 90°$.

4. The magnitude of F is proportional to the charge q.

5. The magnitude of F is proportional to the speed v of the moving charge.

THE FORCE EXERTED BY MOVING CHARGE IN A MAGNETIC FIELD

The experiments show that when a charge q moves with velocity $\vec{\mathbf{v}}$ in magnetic field $\vec{\mathbf{B}}$, the magnetic force $\vec{\mathbf{F}}$ on the charge is

$$\vec{\mathbf{F}} = q_0 \vec{\mathbf{v}} \times \vec{\mathbf{B}}$$

$$|\vec{\mathbf{F}}| = q_0 |\vec{\mathbf{v}}| |\vec{\mathbf{B}}| \sin\theta$$

The magnetic force $\vec{\mathbf{F}}$ is perpendicular to the plane defined by vectors $\vec{\mathbf{v}}$ and $\vec{\mathbf{B}}$. The direction of $\vec{\mathbf{F}}$ is given by the right-hand thumb rule. Point your fingers in the direction of $\vec{\mathbf{v}}$ such that they can be curled into $\vec{\mathbf{B}}$, the thumb points in the direction of $\vec{\mathbf{F}}$. Analyse Figs. 4.11 (a), (b), (c), (d) and (e).

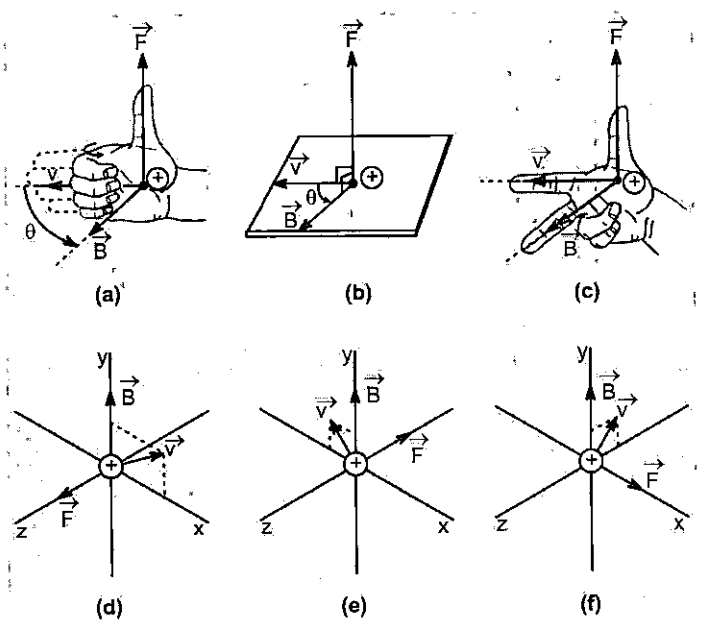


Fig. 4.11

The SI unit of magnetic field is **tesla (T).** A charge of one coulomb moving with a velocity of one metre per second

perpendicular to a magnetic field of one tesla experiences a force of one newton.

$$1\,\text{T} = 1\frac{(\text{N/C})}{\text{m/s}} = 1\,\text{N/A-m}.$$

The C.G.S unit 1 gauss $= 10^{-4}$ tesla

Several important features of magnetic force are:

1. The force is proportional to magnitude of charge q and velocity v, and to the sine of the angle between $\vec{\mathbf{v}}$ and $\vec{\mathbf{B}}$. **A stationary charge has no magnetic force on it.**

2. When the velocity of the charge is parallel or antiparallel to the magnetic field, the force is zero as

$$|F| = qvB\sin(0°) = 0$$

and $$|F| = qvB\sin(180°) = 0$$

3. No work is performed by the magnetic force on a moving charge. From the definition of work,

$$W = \int \vec{\mathbf{F}}\cdot d\vec{\mathbf{r}} = \int q\vec{\mathbf{v}}\times\vec{\mathbf{B}}\cdot d\vec{\mathbf{r}}$$

$$= \int q\vec{\mathbf{v}}\times\vec{\mathbf{B}}\cdot(\vec{\mathbf{v}}\,dt)$$

Since $(\vec{\mathbf{v}}\times\vec{\mathbf{B}})$ is perpendicular to $\vec{\mathbf{v}}$, consequently

$$(\vec{\mathbf{v}}\times\vec{\mathbf{B}})\cdot\vec{\mathbf{v}} = 0,$$

Because dot product of two perpendicular vectors is zero. **Therefore magnetic forces do not work on particles and do not change their kinetic energy.**

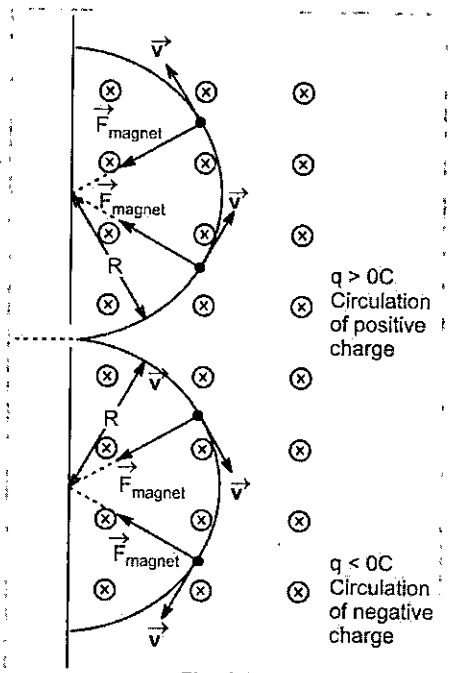


Fig. 4.12

4. A charge q is travelling with a velocity in a uniform magnetic field B perpendicular to v, as shown in Fig. 4.12.

Field out of the plane of page is indicated by a dot and into page by a cross.

The magnetic force on a charged particle is always perpendicular to velocity of the particle, as you have learned that a particle exhibits uniform circular motion when a constant magnitude force directed perpendicular to the velocity acts on it. Magnetic force thus changes the direction of velocity but not its magnitude.

The magnitude of magnetic force is

$$F = q|\vec{\mathbf{v}}\times\vec{\mathbf{B}}| = qvB\sin(90°) = qvB$$

According to Newton's second law,

$$F = qvB = \frac{mv^2}{r}$$

$$r = \frac{mv}{qB}$$

The period of the circular motion, called **cyclotron period,** is the time taken to travel once around the circumference of the circle.

$$T = \frac{2\pi r}{v} = \frac{2\pi(m\,v/qB)}{v} = \frac{2\pi m}{qB}$$

The frequency of the circular motion, called as **cyclotron frequency,** is the reciprocal of the period:

$$\nu = \frac{1}{T} = \frac{qB}{2\pi m}$$

Concept: *An ion with mass m and charge q enters the magnetic field with kinetic energy gained by crossing a potential difference ΔV. Its kinetic energy is $\frac{1}{2}mv^2 = q\Delta V$, so its speed is:*

$$v = \sqrt{\frac{2q\Delta V}{m}}.$$

Its Larmor radius in the magnetic field is:

$$r_L = \frac{mv}{qB} = \frac{(2m\,\Delta V/q)^{1/2}}{B}.$$

For a given ΔV and $\vec{\mathbf{B}}$, an ion's arrival point at the detector depends only on the ion's charge to-mass ratio, q/m: $r_L \propto \sqrt{m/q}$.

5. When the velocity of charge is not perpendicular to the direction of the uniform magnetic field, the path traced by the charge is a **helix,** as shown in Fig. 4.13. We can resolve the velocity of the particle into components $v_{\parallel}$ parallel to $\vec{\mathbf{B}}$ and $v_{\perp}$ perpendicular to $\vec{\mathbf{B}}$. The component of the velocity $v_{\parallel}$ is not affected by the magnetic field and therefore remains constant, *i.e.*, $a_x = 0$. The circular motion is due to the components of velocity that are perpendicular to $\vec{\mathbf{B}} = B_0\hat{\mathbf{i}}$. The magnitude of the velocity of the motion projected on the yz-plane is constant and is given by

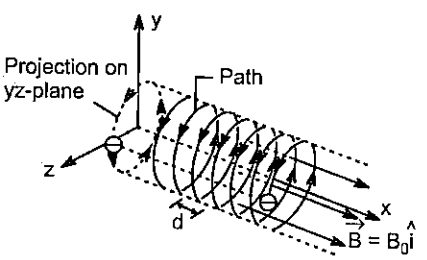


Fig. 4.13

$$v_\perp = \sqrt{v_x^2 + v_y^2}$$

$$F = qv_\perp B$$

If velocity of charge is

$$\vec{\mathbf{v}} = v\cos\theta\,\hat{\mathbf{i}} + v\sin\theta\,\hat{\mathbf{j}},$$

$$r = \frac{m(v\sin\theta)}{qB}$$

$$T = \frac{2\pi r}{v\sin\theta} = \frac{2\pi m}{qB}$$

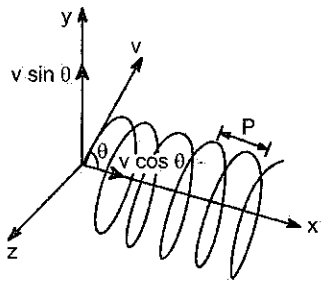


Fig. 4.14

$v \sin\theta$ is tangential velocity, $v\sin\theta = r\omega$

The distance, the charge travels parallel to magnetic field lines during one revolution is called the **pitch of the helix.**

$$p = (v\cos\theta)T = \frac{2\pi m}{qB}(v\cos\theta)$$

THE CYCLOTRON

The time T required for a charged particle to complete one revolution around its circular orbit in a uniform magnetic field is given by the simple relation

$$T = \frac{2\pi m}{qB}$$

For a particle of fixed mass and charge, revolving in a fixed magnetic field, this period of revolution is a constant, called the **cyclotron period.** The corresponding angular frequency,

$$\omega_c = \frac{2\pi}{T} = \frac{q}{m}B$$

is called the **cyclotron frequency.** If the speed of the particle is increased by some external means (the magnetic field can do no work on the particle), the orbit radius will increase proportionately and the period will not change. This isochronous property is often called the **cyclotron effect.**

The principle of operation of the cyclotron is illustrated in Fig. 4.15. The two hollow D-shaped sheet-metal structures, called "dees," are located in a vacuum chamber having the form of a squat cylinder. The vacuum chamber lies between the poles of a large electromagnet so that the flux lines are normal to the dees. Charged particles (usually protons or deuterons—the nuclei of deuterium, or "heavy hydrogen") are injected at low energy near the center of the device.

Consider a particle of charge q and mass m, moving in a circular orbit in a plane normal to the magnetic field. The particle will move in a circular orbit of constant radius. But suppose that, just as it traverses the space between the dees, an electric potential is imposed between the dees. If the resulting electric field in the space is in the right direction, the particle will accelerate. The radius of its orbit will increase. Now suppose that, in the time it takes for the particle to complete a semicircle, $T/2 = \pi m/qB$, the polarity of the potential between the dees is reversed. The particle is again accelerated when it crosses the space. If the applied potential oscillates with a frequency $\nu = \omega_c/2\pi$, the particle

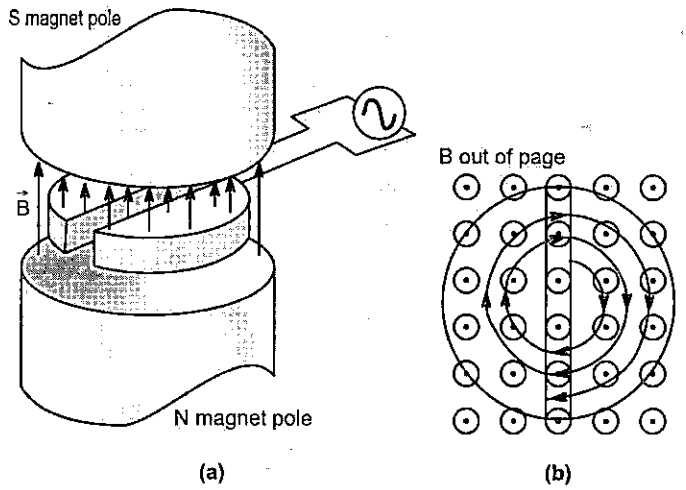


Fig. 4.15

will receive accelerate every time it traverses the space. It can thus acquire a large energy by means of repeated traverses through a relatively small potential difference, typically a few thousand volts. The acceleration process repeats until the radius of the orbit approaches that of the dees.

Cyclotron can produce a continuous stream of energetic particles. The process gives upto about 100 MeV energy. At this energy, the mass m of the particles begins to increase appreciably as a function of their energy, in accordance with the special theory of relativity. As m changes, the cyclotron frequency changes as well, drifting away from the frequency of the potential difference applied between the dees. The most energetic particles, in the outermost orbits, no longer arrive at the space between the dees at the proper time for further acceleration.

BUBBLE CHAMBER

Magnetic field are also important in the analysis of the elementary particles produced in these high-energy collisions. Radius of circulation r_{cir} can be written $mv = p = r_{cir}qB$. The momentum of a charged particle can be determined by measuring the radius of its orbit in a known magnetic field. This is done inside a device called a *bubble chamber,* where the particles leave a trace of their trajectories in the form of a string of tiny bubbles in liquid hydrogen. This bubble pattern is photographed, and the radius of the particle's orbit is measured from the photograph. Fig. 4.16 is a photograph of a collision in which an electron and a positron (an *antielectron,* having the mass of an electron but charge $+e$) were created. Notice that they spiral in opposite directions, because of their opposite charges, with slowly decreasing radii as they lose energy in collisions with the hydrogen atoms.

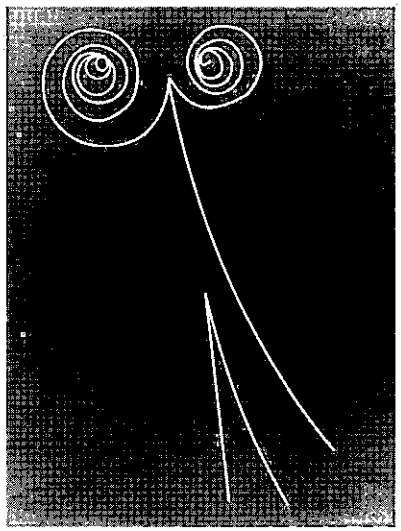


Fig. 4.16

MOVING CHARGES IN NON-UNIFORM MAGNETIC FIELDS

When the B-field is non-uniform, the motion of a charged particle within it is usually complicated. In order to achieve nuclear fusion, ionized gases with temperatures measured in millions of kelvins are necessary. Magnetic fields provide the only practical method of containing such states of matter called **plasmas.** One field orientation, called the **magnetic bottle,** is shown in Fig. 4.17. Two parallel, current-carrying coils can be used to create such a field. In the following paragraphs we will see that many ions that spiral toward one end of the magnetic bottle are reflected by the increased magnetic field strength at that end.

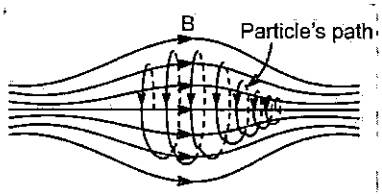


Fig. 4.17

In Fig. 4.18 shows a charged particle's orbit in a region of space where the magnetic field strength increases slightly along the positive x-axis. Consider a charged particle moving in this field with a velocity that has a small component along the x-axis. If the magnetic field were uniform, the particle's trajectory would be a helix. Let's assume that the change in magnetic field is small enough that the particle must move through many turns of the helix before there is a noticeable change in the magnetic field strength. Because the radius of the helix is approximately the same as that for circular orbits ($r = mv/qB$), we except the radius of the orbit to decrease as the field strength B increases.

Second, if we apply the right-hand rule and draw the resulting magnetic force, we see, as in Fig. 4.18, that throughout the particle's orbit, the magnetic force has a small component in the negative x-direction. This small

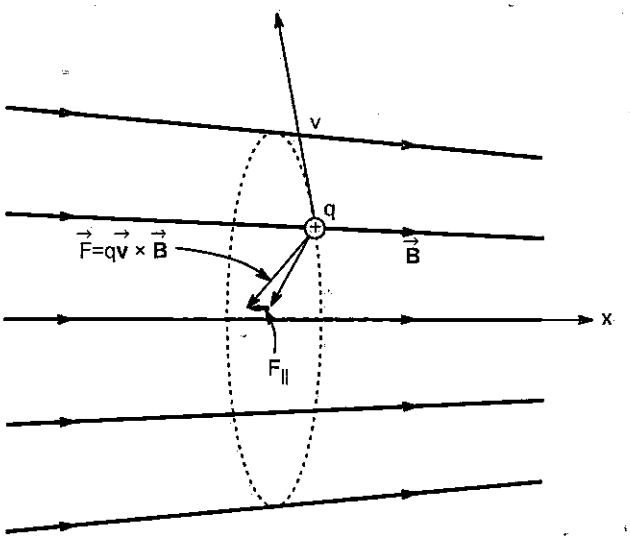


Fig. 4.18: Motion of a charge +q in a converging B-field. Because the direction of B is tilted slightly toward the x-axis, a component $F_{\|}$ of $\vec{F} = q\vec{v} \times \vec{B}$ is directed along the negative x-axis.

component $F_{\|}$ arises *not* because the charge has an x-component to its velocity but rather because the B-field lines converge slightly. This component of the magnetic force accelerates the particle in the negative x-direction, ultimately reversing its direction. Note, however, that the magnitude of the particle's total velocity cannot change because the magnetic force is always perpendicular to the velocity. Nonetheless, the magnetic force can, of course, change the direction of the particle's velocity.

The earth's magnetic field traps charged particles, in a region called the Van Allen belt (Fig. 4.19). These particles originate in distant stars but most come from our sun; all are called **cosmic rays.** When these particles spiral around the earth's magnetic field lines, they collide with air molecules in the upper atmosphere, causing them to emit light. This phenomenon can be observed in the latitudes near the north and south poles. In the north, this phenomenon is called the aurora borealis, or northern lights, and in the south it is known as the aurora australis.

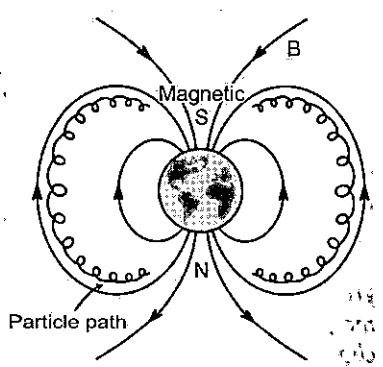


Fig. 4.19

Concept: ***The Magnetic Mirror:*** *In a non-uniform magnetic field the field lines converge or diverge. A particle spiraling along converging field lines toward a region of greater* $|\vec{B}|$ *encounters a force component that reduces the particle's parallel velocity (Fig. 4.20).*

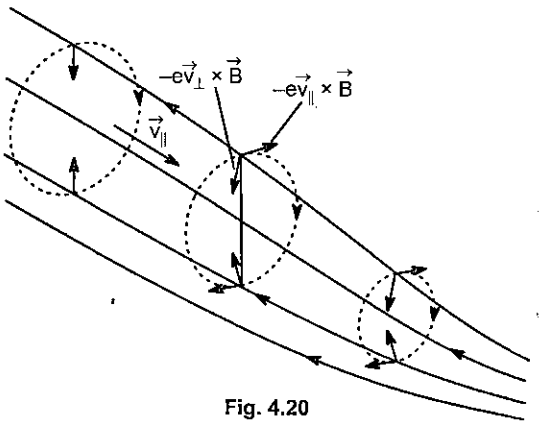


Fig. 4.20

In a strongly converging field, this force reverses the particle's velocity parallel to the field, reflecting it from the region of intense field. The converging field lines form a magnetic mirror. Only particles originally moving nearly parallel to the field can penetrate the mirror.

The magnetic force has two components due to the two components of $\vec{v}$: $v_\perp$ *(the circular motion) and* $v_\parallel$ *(along the field line). The force component* $-e\vec{v}_\perp \times \vec{B}$ *does not point exactly at the center of the circle but has a component along the field line which reduces* $v_\parallel$. *The force component* $-e\vec{v}_\parallel \times \vec{B}$ *points along the circumference of the circle and increases* $v_\perp$. *At the mirror point,* $v_\parallel = 0$ *and all the particle energy is in the circular motion.*

Example 1

An electron accelerated through a potential difference of 2.5 kV, moves horizontally into a region of space in which there is a downward directed uniform electric field of magnitude 10 kV/m. (a) In what direction must a magnetic field be applied so that the electron moves undeflected ? Ignore the gravitational force. What is the magnitude of the smallest magnetic field possible in this case ? (b) What happens if the charge is on a proton that passes through the same combination of fields ?

Solution: The Lorentz force experienced by the electron is $F = -e(\vec{E} + \vec{v} \times \vec{B})$, where $\vec{E}$ is the electric field and $\vec{B}$ is magnetic field, and v is the velocity of electron. The magnitude of magnetic force is $evB \sin\theta$, where θ is the angle between the velocity and the field. The total force vanishes, hence

$$B = E/v \sin\theta \qquad ...(1)$$

The magnetic field is to be smallest, therefore $\theta = 90°$; then $B = E/v$.

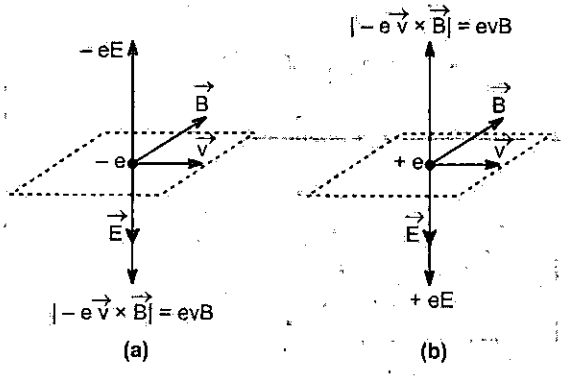


Fig. 4E.1

Fig. 4E.1 (a) shows forces on an electron, in case of proton both electric and magnetic forces in reverse direction, but they still cancel; see Fig. 4E.1(b).

Kinetic energy of electron = eV = 2.5 keV

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(2.5\times10^3)(1.60\times10^{-19})}{9.11\times10^{-31}}}$$

$$= 2.96\times10^7 \text{ m/s}$$

Thus $$B = \frac{E}{v} = \frac{10\times10^3}{2.96\times10^7} = 3.37\times10^{-4} \text{ J}$$

Example 2

Kinetic energy of each electron in a beam of television picture tube is 12.0 keV. The electron are emitted horizontally from geomagnetic south to geomagnetic north. The vertical component of earth's magnetic field points down and has a magnitude 55.0 μ J . (a) In which direction will the electrons deflect ? (b) How far will the beam deflect in moving 20.0 cm through the television tube ?

Solution: (a) Fig. 4E.2 shows the electron travelling north. The magnetic field points inward. From right hand rule $\vec{v} \times \vec{B}$ vector is toward west. Since electron is negative, the force on it is toward east.

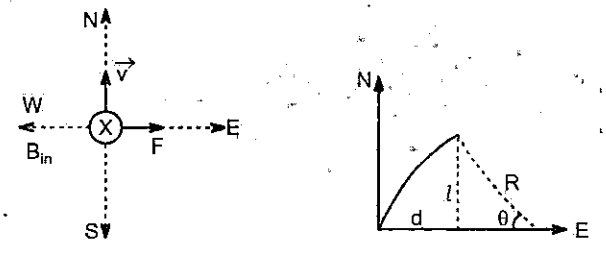


Fig. 4E.2

(b) The electron speed can be determined from kinetic energy.

$$K = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2K}{m}}$$

$$= \sqrt{\frac{2(12.0\times10^3)(1.60\times10^{-19})}{9.11\times10^{-31}}}$$

$$= 6.49\times10^7 \text{ m/s}$$

The magnetic force on the electron is $F_B = evB\sin\theta$ where θ is angle between the electron velocity and the magnetic field. In this case, velocity v and magnetic field B are perpendicular, $\theta = 90°$. The magnetic force provides centripetal force for circular motion, therefore

$$evB = \frac{mv^2}{R}$$

$$R = \frac{mv}{eB} = \frac{9.11\times10^{-31}\times6.49\times10^7}{1.6\times10^{-19}\times(55\times10^{-6})} = 6.72\text{ m}$$

Let the arc traced by the electron beam subtend an angle θ at the centre, l be the length of the tube, and d the deflection.

From figure, $l = R\sin\theta$

$$d = R - R\cos\theta$$

or $R\cos\theta = R - d$

On squaring and adding these two eqns., we have

$$R^2 = (R-d)^2 + l^2$$

or $d^2 - 2Rd + l^2 = 0$

On solving the above quadratic eqn., we get

$$d = R \pm \sqrt{R^2 - l^2}$$

The plus sign corresponds to angle 180°–θ and minus sign corresponds to our case.

For $l \ll R$, $(R^2 - l^2)^{1/2} \simeq R - \frac{1}{2}\frac{l^2}{R}$

Hence $d = \frac{1}{2}\frac{l^2}{R} = \frac{1}{2}\times\frac{(0.200)^2}{6.72} = 0.00298\text{ m}$

Example 3

A particle of mass 1×10^{-26} kg and charge $+1.6\times10^{-19}$ C travelling with a velocity 1.28×10^6 m/s in the +x-direction enters a region in which uniform electric field E and a uniform magnetic field of induction B are present such that $E_x = E_y = 0, E_z = -102.4$ kV/m, and $B_x = B_z = 0, B_y = 8\times10^{-2}$. The particle enters this region at time $t = 0$. Determine the location (x, y, z coordinates) of the particle at $t = 5\times10^{-6}$ s. If the electric field is switched off at this instant (with the magnetic field present), what will be the position of the particle at $t = 7.45\times10^{-6}$ s ?

Solution: The Lorentz force on the charged particle is

$$F = q(\vec{\mathbf{E}} + \vec{\mathbf{v}}\times\vec{\mathbf{B}})$$

The electric force on the charged particle, $F_E = qE_z$ which acts toward negative direction z in the direction of electric field.

The magnetic force on the charged particle,

$$F_B = qv_xB_y$$

as velocity of charge is in $+x$-direction and magnetic field is along $+y$-direction. From right hand rule the magnetic force acts along positive z-direction.

The resultant force

$$F = F_E + F_B = q(E_z + v_xB_y)$$

$$= q[-102.4\times10^3 + 1.28\times10^6\times8\times10^{-2}]$$

$$= 0$$

During time $t = 0$ to $t_1 = 5\times10^{-6}$ the resultant force on the particle is zero, it moves with uniform velocity v_x. The position of the particle (X_1, Y_1, Z_1) after time t_1 is

$$X_1 = v_xt_1 = (1.28\times10^6)\times(5\times10^{-6}) = 6.4\text{ m}$$

When electric field is switched off, the particle circulates in xz-plane under the influence of magnetic field.

Radius R of circulation is

$$R = \frac{mv_x}{qB_y} = \frac{10^{-26}\times1.28\times10^6}{1.6\times10^{-19}\times8\times10^{-2}} = 1\text{ m}$$

Let the particle rotate by an angle $\theta = \omega(t_2 - t_1)$. The arc length $P_1P_2 = R\theta = v_x(t_2 - t_1)$, as the particle circulates for

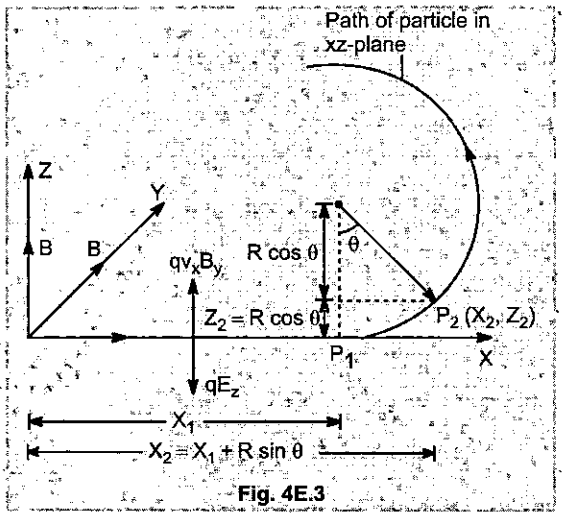


Fig. 4E.3

$$t_2 - t_1 = (7.45 - 5)\times10^{-6} = 2.45\times10^{-6}\text{ s}$$

$$\theta = \frac{v_x(t_2 - t_1)}{R} = \frac{(1.28\times10^6)\times(2.45\times10^{-6})}{1}$$

$$= 3.136 \simeq \pi \text{ radians.}$$

The coordinates of the particle are

$$X_2 = X_1 + R\sin\theta$$

$$= X_1 + R\sin\pi$$

$$= X_1$$

$$= 6.4\text{ m}$$

and $Z_2 = R - R\cos\theta$

$$= R - R\cos\pi$$

$$= 2R = 2\text{ m}$$

Note that $\theta = \pi$ implies that $t_2 = T/2$ where T is time period of circulation. We could have written the result directly.

Example 4

A particle of mass m and charge +q enters a region of magnetic field with a uniform velocity v, see Fig. 4E.4
(a) Find the angle subtended by the circular arc described by it in the magnetic field.
(b) How long does the particle say inside the magnetic field ?
(c) If the particle enters at E, what is the intercept EF ?

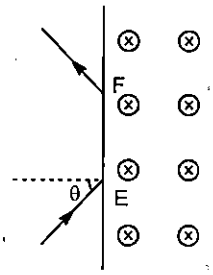


Fig. 4E.4 (a)

Solution: (a) The particle circulates under the influence of magnetic field. As the magnetic field is uniform the charge comes out symmetrically. The angle subtended at the centre is (180°–2θ).

(b) The length of the arc traced by the particle,

$$l = R(2\pi - \theta)$$

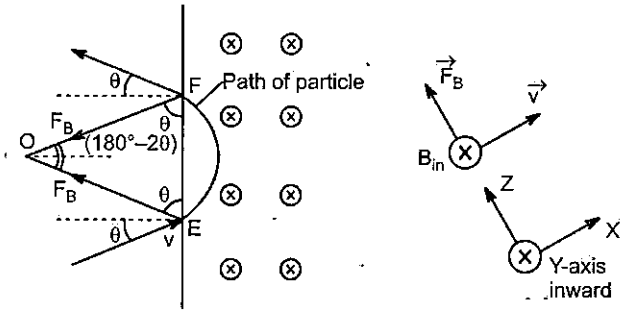


Fig. 4E.4 (b)

Time spent in the field,

$$t = \frac{l}{v} = \frac{R(2\pi - \theta)}{v}$$

As $$R = \frac{mv}{Bq}$$

$$t = \frac{m}{Bq}(2\pi - \theta)$$

At time period, $$T = \frac{2\pi m}{Bq}$$

$$t = \frac{T}{2\pi}(2\pi - \theta)$$

We can generalise this result. If ϕ is the angle subtended by the arc traced by the charged particle in the magnetic field, the time spent is

$$t = T\left(\frac{\phi}{2\pi}\right)$$

(c) Intercept $EF = 2R\cos\theta$.

* The negative charge circulates in a clockwise sense.

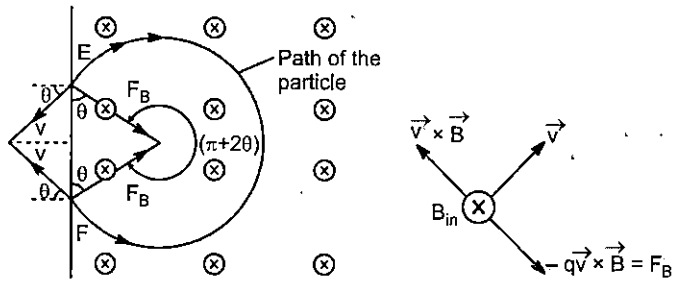


Fig. 4E.4 (c)

Angle subtended by arc at the centre $= (\pi + 2\theta)$.

Time spent by particle in the magnetic field,

$$t = T\frac{(\pi + 2\theta)}{2\pi}$$

Where $$T = \frac{2\pi m}{Bq}$$

Intercept $$EF = 2R\cos\theta$$

Example 5

A proton and an alpha particle are projected in a magnetic field which exists in the width of region d. Compare the angles of deviation suffered by the proton and the alpha particle if before entering the magnetic field both the particles (a) have the same momentum (b) have the same kinetic energy (c) are accelerated through the same potential difference.

Take $m_\alpha = 4m_p, q_\alpha = 2q_p$.

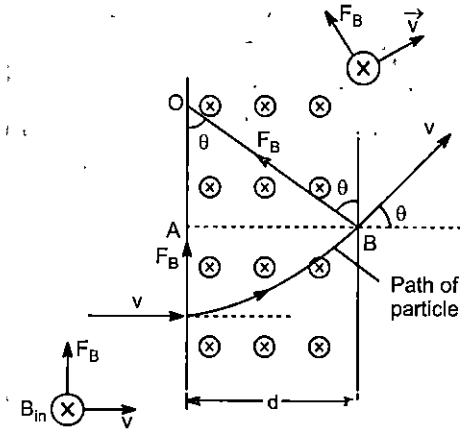


Fig. 4E.5

Solution: The radius of circulation is

$$R = \frac{mv}{Bq} = \frac{p}{Bq} = \frac{\sqrt{2mK}}{Bq}$$

From Fig. 4E.5, angle of deviation is θ; it is the angle between directions of final velocity and initial velocity.

In ΔOAB, $\sin\theta = \frac{d}{R} = \frac{dBq}{mv}$

$$= \frac{dBq}{p} = \frac{dBq}{\sqrt{2mK}}$$

Case (i): $p_\alpha = p_p$

$$\frac{\sin\theta_p}{\sin\theta_\alpha} = \frac{q_p}{q_\alpha} = \frac{1}{2}$$

Case (ii): $K_\alpha = K_p$

$$\frac{\sin\theta_p}{\sin\theta_\alpha} = \frac{q_p}{q_\alpha} \times \sqrt{\frac{m_\alpha}{m_p}}$$

$$= \frac{1}{2} \times \sqrt{\frac{4}{1}} = 1$$

Case (iii): When a charged particle is accelerated through a potential difference V, gain of kinetic energy is qV.

$$K_p = q_\alpha V, \quad K_\alpha = q_\alpha V$$

Now, $\sin\theta = \frac{dBq}{\sqrt{2mqV}} = \frac{dB\sqrt{q}}{\sqrt{2mV}}$

Hence, $\frac{\sin\theta_p}{\sin\theta_\alpha} = \sqrt{\frac{q_p}{q_\alpha}} \times \sqrt{\frac{m_\alpha}{m_p}}$

$$= \sqrt{\frac{1}{2}} \times \sqrt{\frac{4}{1}} = \sqrt{2}$$

Example 6

An electron gun G emits electrons of energy 2 keV travelling in the positive x-direction. The electrons are required to hit the spot S where GS = 0.1 m and the line GS makes an angle 60° with the x-axis, as shown in Fig. 4E.6. A uniform magnetic field $\vec{B}$ parallel to GS exists in the region outside the electron gun. Find the minimum value of B needed to make the electron hit S.

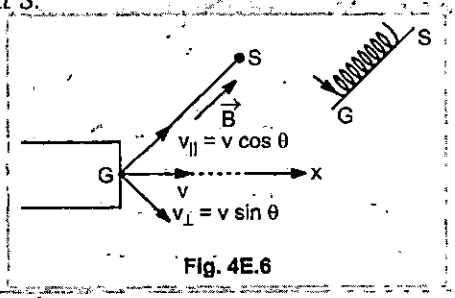


Fig. 4E.6

Solution: The initial velocity of electrons is along x-axis. We can resolve the velocity of electron parallel and perpendicular to magnetic field.

$$v_\parallel = v\cos\theta$$

$$v_\perp = v\sin\theta$$

The electron circulates due to $v_\perp$ and pitch of the helical path is determined by $v_\parallel$. Magnetic force

$$F_B = qv_\perp B = \frac{mv_\perp^2}{R}$$

$$R = \frac{mv_\perp}{Bq} = \frac{mv\sin\theta}{Bq}$$

The helix traced by the electron lies above line GS. The electron will hit the point S if the distance GS is an integral multiple of the pitch of the helix.

$$GS = n(v_\parallel T) = nv\cos\theta T$$

Time period of circulation

$$T = \frac{2\pi R}{v_\perp} = \frac{2\pi R}{v\sin\theta} = \frac{2\pi m}{Bq}$$

$$GS = n\left(v\cos\theta \times \frac{2\pi m}{Bq}\right)$$

$$B = \frac{n(2\pi mv\cos\theta)}{q(GS)}$$

For B to be minimum, $n = 1$

$$B_{min} = \frac{2\pi mv\cos\theta}{q(GS)}$$

Kinetic energy of electron is

$$\frac{1}{2}mv^2 = (2\times10^3)(1.6\times10^{-19})$$

$$v = \sqrt{\frac{4\times10^3\times1.6\times10^{-19}}{1.66\times10^{-27}}}$$

On substituting v, $GS = 0.1$ m and $\cos\theta = 1/2$, we obtain

$$B_{min} \simeq 15\pi\times10^{-4}\ \text{T}$$

Example 7

A velocity selector allows only particles with well known velocities. This device provides perpendicular electric and magnetic fields as shown in Fig. 4E.7. Compute the velocities of particles with charge q that pass undeflected through the velocity selector if the electric field has magnitude E and the magnetic field strength is B.

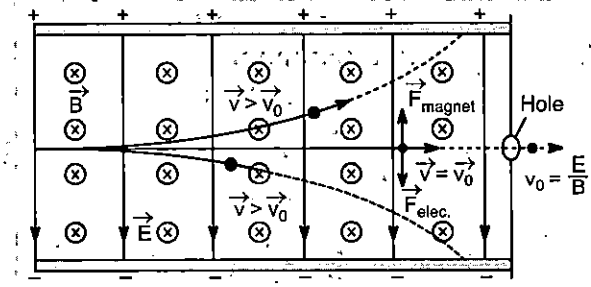


Fig. 4E.7

Solution: With the coordinate system shown in Fig. 4E.7, the magnetic field is

$$\vec{B} = -B\hat{k}$$

and the velocity is initially

$$\vec{\mathbf{v}} = v_0\hat{\mathbf{i}}$$

Thus the initial magnetic force on the charge is

$$\vec{\mathbf{F}} = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$$
$$= qv_0\hat{\mathbf{i}} \times -(B\hat{\mathbf{k}})$$
$$= qv_0B\hat{\mathbf{j}}$$

If the charge q is positive the charged particle experiences force in $\hat{\mathbf{j}}$ direction; if q is negative the force is towards $-\hat{\mathbf{j}}$.

The uniform electric fields is directed so that the electric force on the charge is opposite to the direction of magnetic force on it. When the electric field is in downward direction the electric force on q is

$$\vec{\mathbf{F}}_{\text{electric}} = qE(-\hat{\mathbf{j}})$$

Then the resultant force on the charge is

$$\vec{\mathbf{F}}_{\text{magnetic}} + \vec{\mathbf{F}}_{\text{electric}} = qv_0B\hat{\mathbf{j}} - qE\hat{\mathbf{j}}$$

For a particle to pass through the velocity selector undeviated,

$$qv_0B\hat{\mathbf{j}} - qE\,\hat{\mathbf{j}} = 0$$

or
$$qv_0B = qE$$
$$v_0 = \frac{E}{B}$$

Note that particle with greater velocity than this value are deflected upward by the stronger magnetic force, while those with smaller velocities are deflected downward by the stronger electric force. A small aperture is placed on the far right end of the selector to allow only undeviated charges through.

THE LORENTZ FORCE

Two kinds of forces that can be exerted on an electrically charged particle are the electric force $F_e = qE$ and the magnetic force $F_m = qv \times B$. Taken together, they constitute *all* of the forces that can be exerted on a particle *by virtue of its electric charge* q. Their sum F is called the **Lorentz force:**

$$F = F_e + F_m = q(E + v \times B)$$

The Thomson *e/m* Experiment

In 1897, Thomson (1856-1940) measured the charge-to-mass ratio of the electron for the first time. He used the apparatus sketched in Fig. 4.21 called **cathode-ray tubes.** A high vacuum is essential to the performance of the experiment because electrons must traverse the length of the tube without colliding with gas molecules. Thereby ionising them.

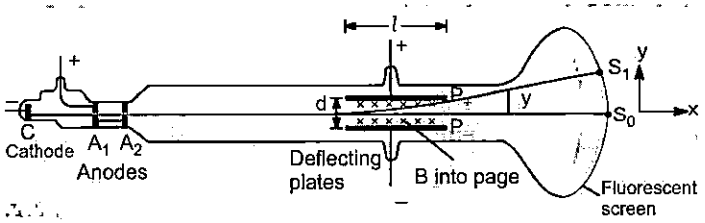


Fig. 4.21

Electrons are emitted from the **cathode** C. A large potential difference between C and the metal plate A_1 accelerates the electrons, and they arrive at A_1 with a speed v. There is a small hole in A_1 and a similar hole in A_2. Only those electrons headed straight down the axis of the tube pass into the space to the right of A_2. The electrons strike the fluorescent zinc sulfide screen at the right end of the tube at the point S_0. The impact produces a spot of light.

The parallel plates P_+ and P_- lie a distance d apart. If a potential difference V is applied between them, the resulting transverse electric field is $E = V/d$. While passing between the plates, the electrons experience a force $F = eE$, and the electron stream is bent into a parabolic trajectory. The transverse acceleration is

$$a_y = \frac{F_e}{m} = \frac{e}{m}E.$$

Once the electrons pass the plates, they continue in a straight line because their is no force and strike the fluorescent screen at S_1. The displacement y of the electrons as they emerge from the plates can be calculated.

If the plates P are of length l, the time required for an electron to pass through the electric field is $t = l/v$. Because the acceleration a_y is constant, we can write

$$y = \frac{1}{2}a_yt^2 = \frac{1}{2}\frac{e}{m}\frac{El^2}{v^2} \qquad ...(1)$$

By knowing the value of v, we could evaluate the ratio e/m.

The value of v is obtained by applying **crossed fields** as in case of velocity selector.

Let us assume that the region in which the electrons are exposed to the magnetic field coincides with the region in which they are exposed to the electric field. If the magnetic field is adjusted the Lorentz force—the total force on the electrons—must be zero:

$$F = F_e + F_m = 0,$$

or
$$F_e = -F_m$$

We use the definition of the Lorentz force to write

$$E = vB$$

We thus have a value for the speed v of the electron as they pass between the plates:

$$v = \frac{E}{B}$$

We substitute this value into eqn. (1) to obtain

$$y = \frac{1}{2}a_yt^2 = \frac{1}{2}\frac{e}{m}\frac{B^2l^2}{E}$$

Finally, we solve for e/m, obtaining

$$\frac{e}{m} = \frac{2yE}{(Bl)^2} \qquad ...(2)$$

All the quantities on the right side of the equation can be measured and the charge-to-mass ratio thus determined.

Thomson's 1897 value, expressed in SI units, was $e/m = 10^{11}$ C/kg. The best modern value is

$$\frac{e}{m} = 1.75881962 \times 10^{11} \text{ C/kg}$$

We can obtain a value of the electron mass m by combining the value of e/m with the value of e. To three significant figures we have

$$m = \frac{e}{e/m} = \frac{1.602 \times 10^{-19}\text{C}}{1.759 \times 10^{11}\text{ C/kg}} = 9.11 \times 10^{-31}\text{ kg}$$

Example 8

A beam of singly ionized atoms of carbon (each charge +e) all have the same speed and enter a mass spectrometer, as shown in Fig. 4E.8. The ions strike the photographic plate in two different locations 5.00 cm apart. The $^{12}C_6$ isotope traces a path of smaller radius, 15.0 cm. What is the atomic mass number of other isotope ?

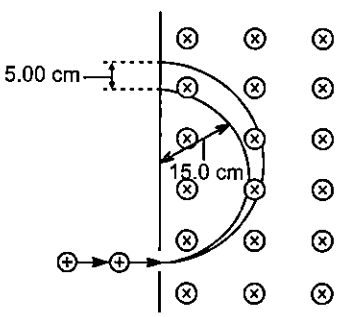


Fig. 4E.8

Solution: Let R_1 be the radius of trajectory of the isotope $^{12}C_6$ and R_2 that of the unknown isotope. The trajectory of the unknown isotope has a greater radius, and so the mass of the unknown isotope is greater than that of the $^{12}C_6$ isotope. From Fig. 4E.8,

$$2R_2 - 2R_1 = 5.00 \times 10^{-12}\text{m}$$

$$R_2 - R_1 = 2.5 \times 10^{-12}\text{m}$$

Since $R_1 = 15.0 \times 10^{-2}\text{m}$

$$R_2 = 15.0 \times 10^{-2}\text{m} + 2.50 \times 10^{-2}\text{m} = 17.5 \times 10^{-2}\text{m}$$

For each isotope, $R_1 = \frac{m_1 v}{Bq}, R_2 = \frac{m_2 v}{Bq}$

Taking the ratio of these expressions,

$$\frac{R_2}{R_1} = \frac{m_2}{m_1}$$

Let A be the atomic mass number of the unknown isotope. The atomic mass number of $^{12}C_6$ is 12. The masses of the isotopes are proportional to the mass number.

$$m_1 = \alpha(12), \quad m_2 = \alpha A$$

Where α is a constant.

$$\frac{m_2}{m_1} = \frac{A}{12}$$

or

$$\frac{R_2}{R_1} = \frac{m_2}{m_1} = \frac{A}{12}$$

$$\frac{17.5 \times 10^{-12}\text{m}}{15.0 \times 10^{-12}\text{m}} = \frac{A}{12}$$

Solving for unknown atomic mass number, we obtain

$$A = 14$$

Thus the unknown isotope is $^{14}C_6$.

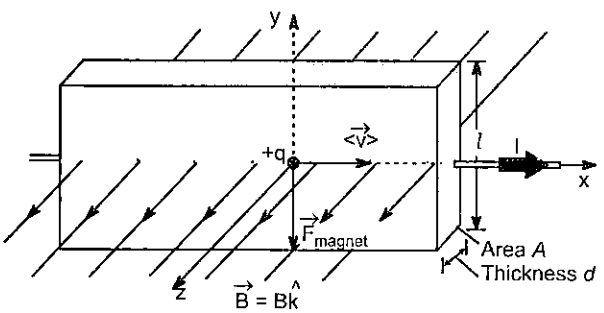
Example 9

Consider a conducting rectangular strip carrying a current. The strip is kept perpendicular to the direction of motion of the charged particles creating the current. Discuss how this experiment leads to the conclusion that charge carriers are negative ? If I, B, n, q, l are current, magnetic field, number of charge carriers per unit volume, charge, thickness of strip respectively, determine the potential established in transverse direction of current.

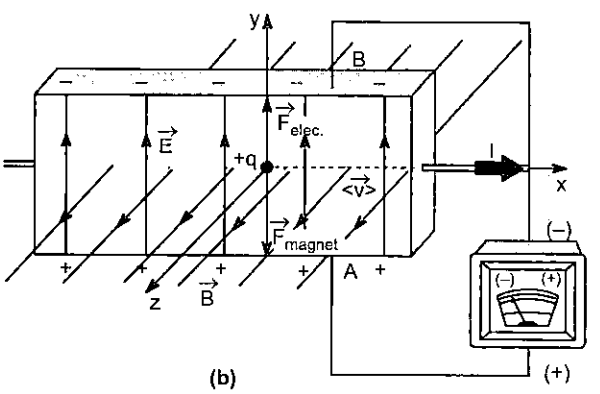
Solution: Scenario 1: Positive charge carriers:

If the current arises due to positive charge carriers $q > 0$,

$$F_{\text{magnet}} = q\vec{\mathbf{v}}_d \times \vec{\mathbf{B}} = q(v_d\hat{\mathbf{i}}) \times (B\hat{\mathbf{k}}) = -qv_dB\hat{\mathbf{j}}$$



(a)



(b)

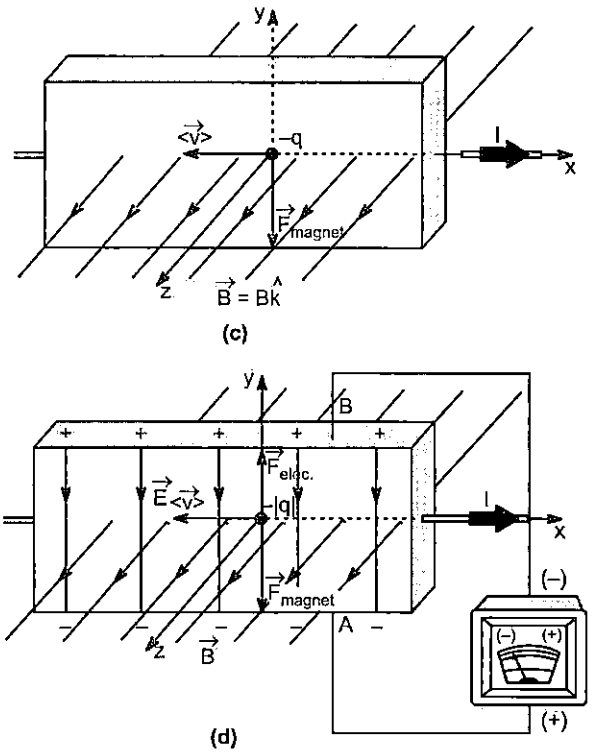


Fig. 4E.9

Where v_d is the drift velocity of charge carriers. Due to magnetic force the positive charges move downward and accumulate near the lower edge of the strip, equivalent amount of negative charge appears near the upper edge of the strip. The charge separation produces an electric field transverse to the current. This electric field produces an electrical force on the positive charge carriers

$$\vec{\mathbf{F}}_{\text{elect}} = qE\hat{\mathbf{j}}$$

As the charge accumulates the electric field increases; the charge accumulation stops when the electric and magnetic force are of the same magnitude.

The direction of the electric field (toward $+\hat{\mathbf{j}}$) indicates that the lower edge is at higher potential than the upper edge.

Scenario 2: Negative charge carriers:

If the current is due to negative charge carriers, then the magnetic force

$$\vec{\mathbf{F}}_{\text{magnet}} = (-q)(-v_d\hat{\mathbf{i}}) \times (B\hat{\mathbf{k}})$$
$$= -qvB\hat{\mathbf{j}}$$

The magnetic force is again in the downward direction. Now negative charge carriers move downward; equivalent positive charge appears on the top edge.

The electric force

$$\vec{\mathbf{F}}_{\text{elect}} = (-q)(-E\hat{\mathbf{j}})$$
$$= +qE\hat{\mathbf{j}}$$

The charge separation continues and the electric field grows in magnitude until electric force is equal in magnitude to magnetic force. The direction of the electric field indicates that the upper edge is at high potential, reverse of scenario 1.

If the voltmeter (see figure) indicates positive potential difference between edges, the charge carriers are positive; if the voltmeter indicates a negative potential difference, the charge carriers are negative. The actual experiments show the latter situation, *i.e.,* the charge carriers are negative.

Equating electric and magnetic forces, we obtain

$$qE = qv_dB$$
$$E = v_dB$$
$$V_B - V_A = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{r}} = -\int_A^B E\hat{\mathbf{j}} \cdot dy\,\hat{\mathbf{j}}$$
$$= -El$$

Where l is the separation of the edges of the bar.

$V_{\text{Hall}} = V_A - V_B$ is referred to as **Hall voltage.**

$$V_{\text{Hall}} = El$$
$$V_{\text{Hall}} = v_dlB$$

As current $I = nqAv_d$,

$$v_d = \frac{I}{nqA}$$
$$V_{\text{Hall}} = \frac{IlB}{nqA} = \frac{IlB}{nq(dl)}$$
$$= \frac{IB}{nqd}$$

Note that the Hall voltage is measured across the width l; d is the thickness.

Concept: 1. *Current in a metal strip is driven by an applied electric field* $\vec{\mathbf{E}}$. *Each conduction electron in a stationary strip has an average drift velocity* $\vec{\mathbf{v}}_d$. *In a region of uniform magnetic field, the magnetic force* $-e\vec{\mathbf{v}}_d \times \vec{\mathbf{E}}$ *causes electrons to drift toward one side of the strip (side a in the figure), leaving excess positive charge on side b. These charge concentrations produce an electric field* $\vec{\mathbf{E}}_H$. *The sideways motion ceases when the electric force* $-e\vec{\mathbf{E}}_H$ *on each electron balances the magnetic force* $-e\vec{\mathbf{v}}_d \times \vec{\mathbf{E}}$. *The resulting potential difference across the strip.*

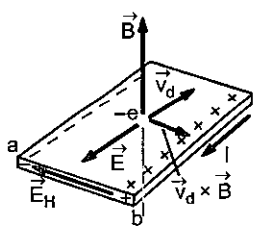


Fig. 4.22

2. *The sign of the Hall potential would be reversed if current were carried by positive instead of negative charges.*

To produce the same current, the positive charges would have to drift in the opposite direction. With both charge and velocity changing sign, the magnetic force would be in the same direction as with negative charge carriers. Thus positive charge carriers would drift to the same edge as negative carriers do and give rise to Hall electric field in the opposite direction. The sign of the potential difference would be reversed. Certain materials (e.g., beryllium) do have positive charge carriers.

3. *In equilibrium, the Hall electric force balance the magnetic force on moving electrons, so the component across the strip of the net force on each electron is zero. However, the force on each stationary, positively charged ion is due only to the Hall field and is not balanced by any magnetic force. The Hall electric field transmits the magnetic force to the ions and thus to the strip as a whole.*

Example 10

In a certain region there are a uniform electric field $\vec{\mathbf{E}}$ and a uniform magnetic field $\vec{\mathbf{B}}$ both directed along z-axis. A particle of charge Q and mass m is projected at time t=0 with a velocity v_0 along x-axis. Find the velocity at any time t. What would be the motion of the particle if the initial velocity v_0 were parallel to z-axis.

Solution: The Lorentz force on the particle is

$$\vec{\mathbf{F}} = Q(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}})$$

where $\vec{\mathbf{E}} = E\hat{\mathbf{k}}, \qquad \vec{\mathbf{B}} = B\hat{\mathbf{k}}$

Let $\vec{\mathbf{v}} = v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}} + v_z\hat{\mathbf{k}}$

$$\vec{\mathbf{F}} = Q[E\hat{\mathbf{k}} + (v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}} + v_z\hat{\mathbf{k}}) \times B\hat{\mathbf{k}}]$$

$$m\left[\frac{dv_x}{dt}\hat{\mathbf{i}} + \frac{dv_y}{dt}\hat{\mathbf{j}} + \frac{dv_z}{dt}\hat{\mathbf{k}}\right] = Q[v_yB\hat{\mathbf{i}} - v_xB\hat{\mathbf{j}} + E\hat{\mathbf{k}}]$$

First we consider z-component; we have

$$\frac{dv_z}{dt} = \frac{QE}{m}$$

Hence $v_z = \frac{qEt}{m}$

as $v_z = 0$ when $t = 0$

Now considering motion in the x and y-direction. We have

$$\frac{dv_x}{dt} = \frac{QBv_y}{m}$$

and $$\frac{dv_y}{dt} = -\frac{QBv_x}{m}$$

This is a pair of coupled equations. It can be decoupled by differentiating one of them w.r.t. time and substituting the result into the other equation. We get

$$\frac{d^2v_x}{dt^2} = -\frac{Q^2B^2}{m^2}.v_x$$

and $$\frac{d^2v_y}{dt^2} = -\frac{Q^2B^2v_y}{m^2}$$

These equations are similar to equation of S.H.M.,with the general solutions

$$v_x = a\cos\omega t + b\sin\omega t$$

$$v_y = c\cos\omega t + d\sin\omega t$$

where $\omega = \frac{QB}{m}$.

Now at time $t = 0, v_x = v_0$

and $v_y = 0$.

Also $\frac{dv_x}{dt} = 0$

and $\frac{dv_y}{dt} = -\frac{QBv_0}{m} = -\omega v_0$

thus $a = v_0, \quad b = 0$

$c = 0, \quad d = -v_0$

We can thus finally write the velocity at time t as

$$v = v_0\cos\left(\frac{QBt}{m}\right)\hat{\mathbf{i}} - v_0\sin\left(\frac{QBt}{m}\right)\hat{\mathbf{j}} + \frac{QEt}{m}\hat{\mathbf{k}}$$

The motion is equivalent to circular motion in xy-plane combined with uniform acceleration in the z-direction.

If the initial velocity is in the z-direction, then at $t = 0$ we have

$$v_x = v_y = \frac{dv_x}{dt}$$

$$= \frac{dv_y}{dt} = 0$$

Thus $a = b = c = d = 0$. The particle has acceleration only in the z-direction with

$$v_z = v_0 + \frac{QEt}{m}$$

Example 11

A particle of charge +q and mass m is released from rest in the crossed electric field E and magnetic field B as shown in Fig. 4E.11. Find the trajectory of the particle.

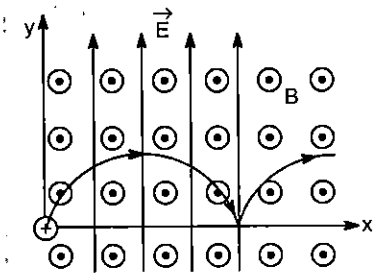


Fig. 4E.11

Solution: The given electric and magnetic fields can be expressed as

$$\vec{\mathbf{E}} = E\,\hat{\mathbf{j}}, \qquad \vec{\mathbf{B}} = B\hat{\mathbf{k}}$$

The electric force is always along the electric field, *i.e.*, *y*-axis in this case. The magnetic force will be in *xy*-plane. Thus the particle can never come out of the *xy*-plane. Let the velocity of particle at any time t be

$$\vec{\mathbf{v}} = v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}}$$

The Lorentz force on the particle is

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}} + q(\vec{\mathbf{v}} \times \vec{\mathbf{B}})$$

$$= q[E\hat{\mathbf{j}} + (v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}})B\hat{\mathbf{k}}]$$

$$m\left[\frac{dv_x}{dt}\hat{\mathbf{i}} + \frac{dv_y}{dt}\hat{\mathbf{j}}\right] = q[(E - v_xB)\hat{\mathbf{j}} + Bv_y\hat{\mathbf{i}}]$$

Component wise we can write

$$\frac{dv_x}{dt} = \frac{q}{m}Bv_y \qquad ...(1)$$

$$\frac{dv_y}{dt} = \frac{q}{m}(E - v_xB) \qquad ...(2)$$

This is a coupled differential equations. We can decouple the equations by differentiating one of them w.r.t. time and substituting the result into the other equation.

Differentiating eqn. (2) w.r.t. time,

$$\frac{d^2v_y}{dt^2} = -\frac{q}{m}B\frac{dv_x}{dt} \qquad ...(3)$$

Now substituting $\frac{dv_x}{dt}$ from eqn. (1) in (3), we get

$$\frac{d^2v_y}{dt^2} = -\left(\frac{qB}{m}\right)^2 v_y = -\omega^2 v_y \qquad ...(4)$$

where $\omega = \frac{qB}{m}$

Similarly by differentiating eqn. (1), we get

$$\frac{d^2v_x}{dt^2} = \frac{q}{m}B\frac{dv_y}{dt} \qquad ...(5)$$

On substituting $\frac{dv_y}{dt}$ from eqn. (2) we get

$$\frac{d^2v_x}{dt^2} = \frac{q^2B^2}{m^2}(E - v_xB) \qquad ...(6)$$

The eqn. (4) is similar to equation of particle executing simple harmonic motion. Hence its solution is of the form

$$v_y = v_0\sin\omega t \qquad ...(7)$$

$$\frac{dv_y}{dt} = v_0\omega\cos\omega t \qquad ...(8)$$

From eqn. (8) at $t = 0$,

$$\frac{dv_y}{dt} = v_0\omega$$

Also $v_x = 0$

at $t = 0$

Hence from eqn. (2),

$$\frac{dv_y}{dt} = \frac{qE}{m}$$

Thus $v_0\omega = \frac{qE}{m}$

or $v_0 = \frac{E}{B}$

and $v_y = \frac{E}{B}\sin\omega t = \frac{dy}{dt}$

or $\int_0^y dy = \frac{E}{B}\int_0^t \sin\omega t\,dt$

$$y = \frac{E}{B\omega}(1 - \cos\omega t) \qquad ...(9)$$

On substituting the expression for $\frac{dy}{dt}$ in eqn. (1), we get

$$\frac{dv_x}{dt} = \left(\frac{q}{m}B\right)\left(\frac{E}{B}\sin\omega t\right)$$

$$\int_0^{v_x} dv_x = \frac{qE}{m}\int_0^t \sin\omega t\,dt$$

$$v_x = \frac{E}{B}(1 - \cos\omega t) = \frac{dx}{dt}$$

$$\int_0^x dx = \frac{E}{B}\int_0^t (1 - \cos\omega t)\,dt$$

$$x = \frac{E}{B\omega}(\omega t - \sin\omega t) \qquad ...(10)$$

The equations (9) and (10) are the parametric equations of a cycloid.

* The maximum value of y is $y_{max} = \frac{2E}{\omega B}$.
* If a circle of radius $R = \frac{E}{\omega B}$ were to roll on the *x*-axis, a fixed point on the circle would generate this cycloid.
* An observer at the centre of this circle will be moving with velocity $R\omega = E/B$ along *x*-axis. The particle will appear to move along the circumference of a circle with speed $R\omega = E/B$.

$$v^2 = v_x^2 + v_y^2$$

$$\Rightarrow \quad v = \frac{E}{B}\sqrt{(1 - \cos\omega t)^2 + \sin^2\omega t}$$

$$= \frac{E}{B}.2\sin\frac{\omega t}{2} = \frac{2E}{B}\sin\frac{\omega t}{2}.$$

Now $v = 0$ when $\frac{\omega t}{2} = 0, \pi, 2\pi, t = 0, \frac{2\pi}{\omega}, \frac{4\pi}{\omega}, \ldots$

Thus the two nearest instants at which velocity is zero differ by $2\pi/\omega$.

Thus, $S = \int_0^{2\pi/\omega} v\,dt = \int_0^{2\pi/\omega} \frac{2E}{B}\sin\frac{\omega t}{2}\,dt$

$$= \frac{8E}{\omega B} = \frac{8E}{\frac{qB}{m}B} = \frac{8mE}{qB^2}.$$

Example 12

Uniform electric and magnetic fields with strength E and induction B respectively are along y-axis as shown in Fig. 4E.12. A particle with specific charge q/m leaves the origin O in the direction of x-axis with an initial non-relativistic velocity v_0. Find (a) the coordinate y_n of the particle when it crosses the y-axis for the n^{th} time; (b) the angle α between the particle's velocity vector and y-axis at that moment.

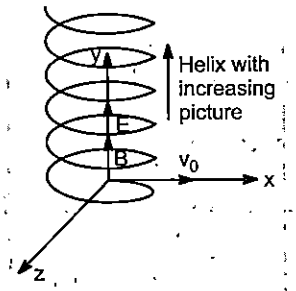


Fig. 4E.12

Solution: (a) Proceeding similar to example we can see that particle's acceleration in y-direction.

$$\frac{dv_y}{dt} = \frac{qE}{m} = \text{constant}$$

The motion of the particle is equivalent to circular motion in xz-plane with uniform acceleration in y-direction.

Hence,

$$v_0^2 = v_x^2 + v_z^2 = \text{constant}$$

The magnetic force cannot change the magnitude of v_0.

The y-component of velocity, $v_y = \frac{qE}{m}t$

The y-coordinate at time t, $y = \frac{1}{2}\frac{qE}{m}t^2$

The time period of circular motion in xz-plane,

$$T = \frac{2\pi m}{Bq}$$

Let the particle cross y-axis after n rotations; then

$$t = nT = \frac{2\pi mn}{qB}$$

Thus $$y_n = \frac{qE}{2m} \times \left(\frac{2\pi mn}{qB}\right)^2$$

$$= \frac{2\pi^2 mn^2 E}{qB^2}$$

(b) As $$v_y = a_y t = \left(\frac{qE}{m}\right)\left(\frac{2\pi mn}{qB}\right)$$

$$= \frac{2\pi nE}{B}$$

Thus $$\tan\alpha = \frac{v_0}{v_y} = \frac{v_0 B}{2\pi nE}$$

$$\alpha = \tan^{-1}\left(\frac{v_0 B}{2\pi nE}\right)$$

Example 13

A particle of mass m and charge q is moving in a region where uniform constant electric and magnetic fields $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ are present. $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ are parallel to each other. At time t = 0 the velocity $\vec{\mathbf{v}}_0$ of the particles is perpendicular to $\vec{\mathbf{E}}$. (Assume that its speed is always ≤ c, the speed of light in vacuum.) Find the velocity $\vec{\mathbf{v}}$ of the particle at time t. you must express your answer in terms of t,q, m, their magnitudes v_0, E and B.

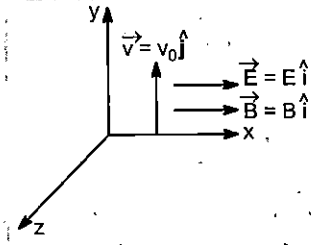


Fig. 4E.13

Solution: Let the velocity v, E and B as shown in Fig. 4E.14.The Lorentz force on the charged particle is

$$q(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}}) = m\frac{d\vec{\mathbf{v}}}{dt}$$

$$q[\vec{\mathbf{E}}\hat{\mathbf{i}} + v_0\hat{\mathbf{j}} \times B\hat{\mathbf{i}}] = m\frac{d}{dt}[v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}} + v_z\hat{\mathbf{k}}]$$

$$q\vec{\mathbf{E}}\hat{\mathbf{i}} + qv_0B(-\hat{\mathbf{k}}) = m\frac{d}{dt}[v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}} + v_z\hat{\mathbf{k}}]$$

On comparing the components on both sides, we get

$$m\frac{dv_x}{dt} = qE \quad ...(1)$$

$$m\frac{dv_y}{dt} = qBv_z \quad ...(2)$$

$$m\frac{dv_z}{dt} = -qBv_y \quad ...(3)$$

From eqn. (1), we have

$$v_x = \frac{qE}{m}t \quad [a_x \text{ is constant}] \quad ...(4)$$

From eqn. (2), we have

$$\frac{d^2v_y}{dt^2} = \frac{qB}{m}\frac{dv_z}{dt} \quad ...(5)$$

We can substitute $\frac{dv_z}{dt}$ from eqn. (3) in eqn. (5); we get,

$$\frac{dv_y}{dt^2} = -\left(\frac{qB}{m}\right)^2 v_y \quad ...(6)$$

Similarly from eqn. (3), we have

$$\frac{d^2v_z}{dt^2} = -qB\frac{dv_y}{dt} \quad ...(7)$$

Now we substitute dv_y/dt from eqn. (2) in (7) to get

$$\frac{d^2v_z}{dt^2} = -\left(\frac{qB}{m}\right)^2 v_z \qquad ...(8)$$

Eqns. (6) and (8) are second order differential equations, similar to that encountered in simple harmonic motion. Solution of eqn. (6),

$$\frac{d^2v_y}{dt^2} = -\omega^2 v_y, \left[\omega = \frac{qB}{m}\right]$$

is $v_y = A\cos(\omega t + \phi)$

Hence from eqn. (2),

$$v_z = \frac{m}{qB}\frac{dv_y}{dt} = -A\sin(\omega t + \phi)$$

At $t = 0, v_y = v_0$ and $v_z = 0$; therefore we get

$$\phi = 0 \quad \text{and} \quad A = v_0$$

Thus the final solutions are

$$v_x = \frac{qE}{m}t,\ v_y = v_0\cos\omega t,\ \ v_z = -v_0\sin\omega t$$

In vector notation,

$$\vec{\mathbf{v}} = \left(\frac{qE}{m}t\right)\hat{\mathbf{i}} + (v_0\cos\omega t)\hat{\mathbf{j}} + (-v_0\sin\omega t)\hat{\mathbf{k}}$$

$$= \left(\frac{qE}{m}t\right)\frac{\vec{\mathbf{E}}}{E} + (v_0\cos\omega t)\frac{\vec{\mathbf{v}}_0}{v} + (-v_0\sin\omega t)\frac{\vec{\mathbf{B}}\times\vec{\mathbf{v}}_0}{Bv_0}$$

$$= \frac{qt}{m}\vec{\mathbf{E}} + v_0\cos\omega t\,\vec{\mathbf{v}}_0 + \frac{\sin\omega t}{B}\vec{\mathbf{v}}_0\times\vec{\mathbf{B}}$$

Example 14

Two long concentric cylindrical conductors of radii a and b ($b < a$) are maintained at a potential difference V and carry equal and opposite currents I. Show that an electron with a particular velocity u parallel to the axis may travel undeviated in the evacuated region between the conductors.

Solution: The electric field E in the region between the conductors varies with the distance r from the axis as

$$E = \frac{\lambda}{2\pi\varepsilon_0 r},$$

Where λ is the charge per unit length on the inner cylinder, and we also know that the potential difference V between the conductors is given by

$$V = \frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{a}{b}\right).$$

Combining these expressions to eliminate λ gives

$$E = \frac{V}{r\ln\left(\frac{a}{b}\right)}.$$

The magnetic field B varies with r as

$$B = \frac{\mu_0 I}{2\pi r}$$

If the inner cylinder is at a positive potential w.r.t. the outer cylinder, the electric field E is directed radially outwards. If we assume that current flows into the page along the inner cylinder and out of the page along the outer cylinder, the magnetic field will be directed clockwise as shown.

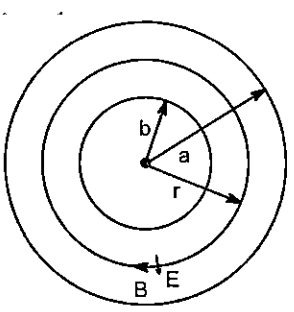


Fig. 4E.14

Consider an electron travelling with velocity u into the page, at a radial distance r from the axis of the cylinders. It experiences an electric force Ee radially inwards, and a magnetic force Beu radially outwards. In order for the electron to be undeviated, these forces must balance so that $u = E/B$. Substituting our expressions for E and B gives

$$u = \frac{\frac{V}{r\ln(a/b)}}{\frac{\mu_0 I}{2\pi r}} = \frac{2\pi V}{\mu_0 I\ln(a/b)}.$$

This expression is independent of r, so the electron's position does not matter.

Example 15

A charged particle of mass m and charge q is projected on a rough horizontal xy-plane surface with z-axis in the vertically upward direction. Both electric and magnetic fields are acting in the region and given by $\vec{\mathbf{E}} = -E_0\hat{\mathbf{k}}$ and $\vec{\mathbf{B}} = -B_0\hat{\mathbf{k}}$ respectively. The particle enters into the field at (a,0,0) with velocity $\vec{\mathbf{v}} = v_0\hat{\mathbf{j}}$. The particle states moving into a circular path on the plane. If the coefficient of friction between the particle and the plane is μ. Then calculate the:

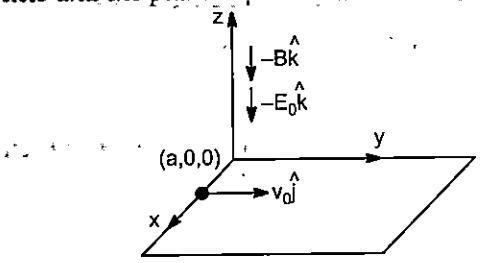


Fig. 4E.15

(a) time when the particle will come to rest
(b) time when the particle will hit the centre
(c) distance travelled by the particle when it comes to rest.

Solution: (a) $N = mg + qE_0$ ...(1)

$$qB_0 v = \frac{mv^2}{R} \qquad ...(2)$$

and $$-m\frac{dv}{dt} = \mu N \qquad ...(3)$$

From eqn. (2), $$R = \frac{mv}{B_0 q} \qquad ...(4)$$

From eqns. (1) and (3),

$$-m\frac{dv}{dt}=\mu(mg+qE_0)$$

or $$-m\int_{v_0}^{0}dv=\mu(mg+qE_0)\int_0^t dt$$

Thus $$t=\frac{mv_0}{\mu(mg+qE_0)}$$

(b) from eqn. (4),

$$dR=\frac{m}{B_0q}dv=-\frac{\mu(mg+qE_0)dt}{qB_0}$$

$$\int_{R_i}^{0}dR=\frac{-\mu(mg+qE_0)}{qB_0}\int_0^t dt$$

or $$t=\frac{qB_0R_i}{\mu(mg+qE_0)}$$

Here $$R_i=\frac{mv_0}{B_0q}$$

Thus $$t=\frac{mv_0}{\mu(mg+qE_0)}$$

(c) $$-mv\frac{dv}{dl}=\mu(mg+qE_0)$$

or, $$-m\int_{v_0}^{0}v\,dv=\mu(mg+qE_0)\int_0^t dl$$

or $$l=\frac{mv_0^2}{2\mu(mg+qE_0)}$$

Example 16

A slightly divergent beam of charged particles accelerated by a P.D. V propagates from a point A along the axis of a solenoid. The beam is brought into focus at a distance l from the point A at two successive values of magnetic induction B_1 and B_2. Find the specific charge q/m of the particle.

Solution: Let us first calculate the velocity of the particles from the energy equation,

$$\frac{1}{2}mv^2=Vq\ \Rightarrow\ v=\sqrt{\frac{2Vq}{m}}$$

Since the charged particles are slightly divergent, they will follow a helical path. Let θ be the small angle made by a particle with B.

Then $$v_{\parallel}=v\cos\theta\quad v_{\perp}=v\sin\theta.$$

$$F_c=\text{centripetal force}=qv_{\perp}B=\frac{mv_{\perp}^2}{R}$$

or $$qB=\frac{mv_{\perp}}{R}$$

$\Rightarrow$ $$qB=\frac{m\omega R}{R}$$

or $$\omega=\frac{qB}{m}$$

$\Rightarrow$ $$T=\frac{2\pi m}{qB}.$$

$\therefore p$ (pitch of the particle) $= v_{\parallel}\times T=v\cos\theta\times\frac{2\pi m}{qB}$

$$=\frac{2\pi vm}{qB}$$

($\because$ for small θ's, $\cos\theta=1$)

Particles are focussed if l contain integral number of pitches.

i.e., $$l=np\ \Rightarrow\ p=l/n=l,l/2,l/3\ldots$$

$\therefore$ For two consecutive focussings,

$$l=p=\frac{2\pi mv}{qB_1}$$

and $$\frac{l}{2}=\frac{2\pi mv}{qB_2}$$

or $$B_1=\frac{2\pi mv}{ql}$$

and $$B_2=\frac{4\pi mv}{ql}$$

or $$B_2-B_1=\frac{2\pi mv}{ql}$$

or $$B_2-B_1=\frac{2\pi m}{ql}\sqrt{\frac{2Vq}{m}}$$

or $$\frac{q}{m}=\frac{8\pi^2V}{l^2(B_2-B_1)^2}$$

Example 17

A particle of charge q and mass m is projected from the origin with velocity $\vec{\mathbf{v}}=v_0\hat{\mathbf{i}}$ in a nonuniform magnetic field $\vec{\mathbf{B}}=-B_0x\hat{\mathbf{k}}$. Here v_0 and B_0 are positive constants of proper dimensions. Find the maximum positive x-coordinate of the particle during its motion.

Solution: Magnetic force on the particle at origin is along positive y-direction, so its trajectory will be concave upwards. As the magnetic field is nonuniform, trajectory is not circular. Magnetic force is always perpendicular to velocity. So its magnitude remains constant. Let at point $P(x, y)$ its velocity vector make an angle θ with positive x-axis. Then the magnetic force $\vec{\mathbf{F}}_B$ will be at an angle θ with the positive y-direction. So,

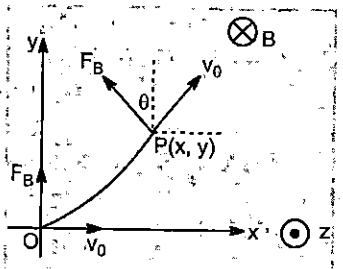


Fig. 4E.17

$$a_y=\left(\frac{F_B}{m}\right)\cos\theta$$

or $$\frac{dv_y}{dt}=\frac{(B_0x)(qv_0\cos\theta)}{m}$$

$[F_B=Bqv_0\sin 90°]$

or $$\left(\frac{dv_y}{dx}\right)\left(\frac{dx}{dt}\right)=\left(\frac{B_0qx}{m}\right)(v_0\cos\theta)$$

where $$\frac{dx}{dt}=v_x=v_0\cos\theta$$

Thus $$\frac{dv_y}{dx}=\left(\frac{B_0q}{m}\right)x$$

At maximum x-displacement, velocity is along positive y-direction

or $$\int_0^{v_0}dv_y=\left(\frac{B_0q}{m}\right)\int_0^{x_{max}}x\,dx$$

or $$v_0=\left(\frac{B_0q}{m}\right)\left(\frac{x_{max}^2}{2}\right)$$

or $$x_{max}=\sqrt{\frac{2mv_0}{B_0q}}$$

Example 18

A long straight wire carries a current I. A particle having a positive charge q and mass m, kept at a distance x_0 from the wire is projected towards it with speed v. Find the closest distance of approach of charged particle to the wire.

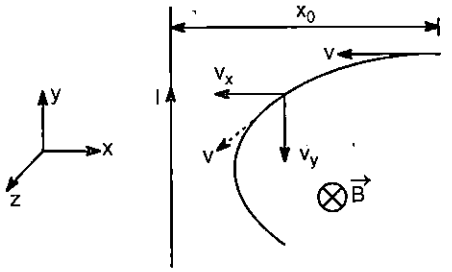


Fig. 4E.18

Solution: Magnetic field due to long wire is

$$\vec{\mathbf{B}}=-\frac{\mu_0 I\hat{\mathbf{k}}}{2\pi x}$$

Let the velocity of the charged particle at any instant be

$$\vec{\mathbf{v}}=(v_x\hat{\mathbf{i}}+v_y\hat{\mathbf{j}})$$

Magnetic force on the charge,

$$\vec{\mathbf{F}}=q(\vec{\mathbf{v}}\times\vec{\mathbf{B}})=-\frac{\mu_0 qIv_y}{2\pi x}\hat{\mathbf{i}}+\frac{\mu_0 qIv_x}{2\pi x}\hat{\mathbf{j}} \quad ...(1)$$

From eqn. (1), y-component of acceleration is

Now $$a_y=\frac{dv_y}{dt}=\frac{\mu_0 qIv_x}{2\pi mx}=\frac{\mu_0 q}{2\pi mx}I\frac{dx}{dt}$$

At the minimum separation, velocity of particle is $-v\hat{\mathbf{j}}$.

or $$\int_0^{-v}dv_y=\frac{\mu_0 qI}{2\pi m}\int_{x_0}^{x_{min}}\frac{dx}{x}$$

or $$-v=\frac{\mu_0 qI}{2\pi m}\ln\left|\frac{x_{min}}{x_0}\right|$$

Thus $$x_{min}=x_0e^{-2\pi mv/\mu_0 qI}$$

FORCE ON A CURRENT CARRYING CONDUCTOR

Fig. 4.23 shows a differential element $d\vec{\mathbf{l}}$ of a current carrying wire. A small element of charge dQ travels through displacement $d\vec{\mathbf{l}}$; if $\vec{\mathbf{v}}_d$ is drift velocity of this charge, then $d\vec{\mathbf{l}}=\vec{\mathbf{v}}_d\,dt$. The force $d\vec{\mathbf{F}}$ on the charge dQ is given by

$$d\vec{\mathbf{F}}=dQ\,\vec{\mathbf{v}}_d\times\vec{\mathbf{B}}$$
$$=(Idt)\vec{\mathbf{v}}_d\times\vec{\mathbf{B}}$$
$$=I(\vec{\mathbf{v}}_d\,dt)\times\vec{\mathbf{B}}$$
$$=Id\vec{\mathbf{l}}\times\vec{\mathbf{B}}$$

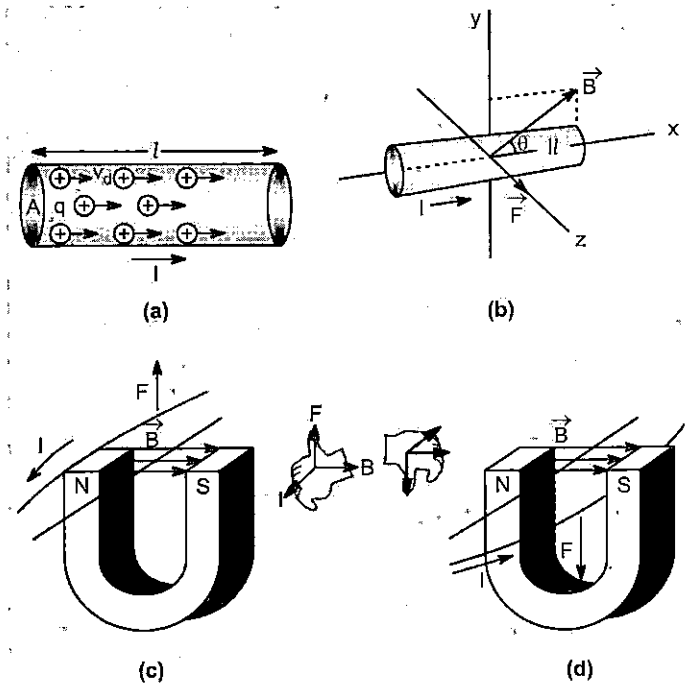


Fig. 4.23

The displacement vector $d\vec{\mathbf{l}}$ is along the direction of the current. The total force on the wire is

$$\vec{\mathbf{F}}=\int d\vec{\mathbf{F}}=I\int d\vec{\mathbf{l}}\times\vec{\mathbf{B}}$$

The direction of magnetic force is determined from Fleming's left hand rule illustrated in Figs. 4.23 (c) and (d).

Concept: *Force acting on a wire segment of arbitrary shape carrying current I in a uniform magnetic field $\vec{\mathbf{B}}$ (Fig. 4.24). The force on any differential piece of the wire is given by eqn.*

$$d\vec{\mathbf{F}}=Id\vec{\mathbf{l}}\times\vec{\mathbf{B}}$$

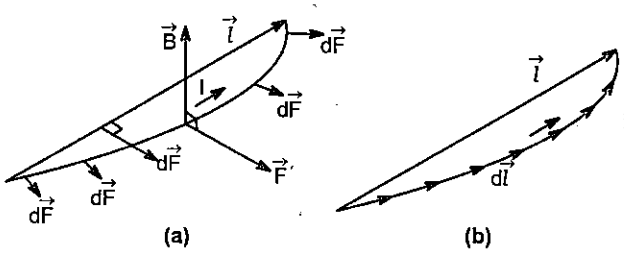


Fig. 4.24

The current I and the magnetic field $\vec{B}$ *are the same for each element, so the total force is:*

$$\vec{F}=\int d\vec{F}=\int Id\vec{l}\times\vec{B}=I(\int d\vec{l})\times\vec{B}$$

The integral is the vector sum of the line element vectors and equals the vector $\vec{l}$ *joining the end points of the segment Fig. 4.24 (b). Thus,* $\vec{F}=I\vec{l}\times\vec{B}$

The net torque about the center of a straight wire segment carrying current I in a uniform field is zero.

Two symmetrically located elements, equidistant from the center, experience equal forces $d\vec{F}_1=d\vec{F}_2$ *(Fig. 4.25). These forces produce equal and opposite torques* $d\vec{\tau}_1=-d\vec{\tau}_2$ *about the center. The total torque is the sum of such balancing pairs and so is zero.*

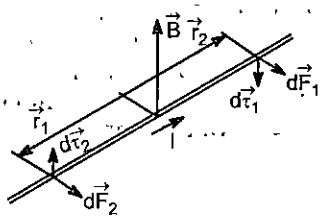


Fig. 4.25: The net torque about the center of a straight wire segment in a uniform magnetic field is zero: the line of action of the net magnetic force passes through the center of the wire.

The total force acts as if it were all applied at the center of the segment. We assumed that all the electrons have the same drift velocity and experience the same magnetic field. Finite lengths of wire usually are not straight, or they pass through regions of variable field. To calculate forces and torques acting on a finite wire, we assume it as a collection of differential segments and add the forces acting on the individual elements.

Example 19

A straight wire lies along a body diagonal of an imaginary cube of side a = 20 cm, and carries a current of 5 A (see Fig. 4E. 19). Find the force on it due to a uniform field $\vec{B}=0.6\hat{j}T$.

Solution: In order to use eqn. $\vec{F}=I\vec{l}\times\vec{B}$ we would need to find the angle between $\vec{l}$ and $\vec{B}$. We avoid this task by using unit vector notation. From Fig. 4E.19, we see that

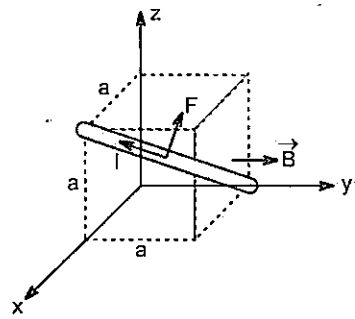


Fig. 4E.19

$$\vec{l}=a\hat{i}-a\hat{j}+a\hat{k}$$

Thus, the force is,

$$\vec{F}=I\vec{l}\times\vec{B}$$
$$=IaB(\hat{i}-\hat{j}+\hat{k})\times(\hat{j})$$
$$=IaB(-\hat{i}+\hat{k})$$

The force lies in the *xz*-plane and has a magnitude

$$F=\sqrt{2}IaB=0.85\,N$$

Example 20

A conducting wire of length 'l' is placed on a rough horizontal surface, where a uniform horizontal magnetic field B perpendicular to the length of the wire exists. Least values of the forces required to move the rod when a current 'I' is established in the rod are observed to be F_1 *and* $F_2\,(<F_1)$ *for the two possible directions of the current through the rod respectively. Find the weight of the rod and the coefficient of friction between the rod and the surface.*

Solution: Changing the direction of current in the wire, we can change the normal reaction on the wire by the surface. In one case magnetic force on the wire will be in upward direction while in the other case it will be in the downward direction. Hence

normal reaction, $N=mg\pm BIl$.

$\Rightarrow$ $f_{(\text{friction limiting})}=\mu(mg\pm BIl)$ as $F_1>F_2$

$\Rightarrow$ $F_1=\mu(mg+BIl)$ ...(1)

and $F_2=\mu(mg-BIl)$ ...(2)

From eqns. (1) and (2),

$$\frac{F_1}{F_2}=\frac{mg+BIl}{mg-BIl}$$

$\Rightarrow$ $$mg=BIl\left[\frac{F_1+F_2}{F_1-F_2}\right]$$ ...(3)

From eqns. (1) and (3), we get

$$\mu=\frac{F_1-F_2}{2BIl}$$

Concept: *We observed that the force acting on a particle with charge q moving with velocity* $\vec{\mathbf{v}}$ *in a magnetic field* $\vec{\mathbf{B}}$ *is*

$$\vec{\mathbf{F}} = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$$

The force is always perpendicular to the particle's velocity and so does no work on the particle:

$dW = \vec{\mathbf{F}} \cdot d\vec{\mathbf{s}} = \vec{\mathbf{F}} \cdot \vec{\mathbf{v}}\, dt = 0$, *since* $\vec{\mathbf{F}}$ *is perpendicular to* $\vec{\mathbf{v}}$.

Magnetic forces can alter the direction of a particle's motion but cannot alter its energy. Since the force is also perpendicular to $\vec{\mathbf{B}}$, *it cannot change a particle's velocity component parallel to the field.*

CAN MAGNETIC FORCE PERFORM WORK

Illustration: A rectangular loop of wire has a block of mass m hanging from it in a uniform magnetic field B, which points into the page in the shaded region of Fig. 4.26. For what current I, in the loop, would the magnetic force upward exactly balance the gravitational force downward ?

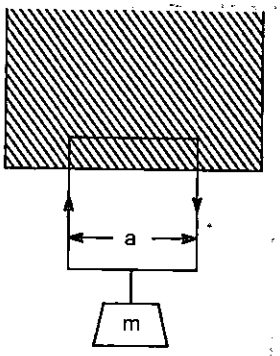


Fig. 4.26

The current must circulate clockwise, in order for $(I \times B)$ in the horizontal segment to point upward. The force is

$$F_{mag} = IBa,$$

where a is the width of the loop. For F_{mag} to balance the weight (mg), we must therefore have

$$I = \frac{mg}{Ba}.$$

If we now increase the current ? Then the upward magnetic force exceeds the downward force of gravity, and the loop rises, lifting the weight. But magnetic force does not perform work, then which agent is doing it and we write.

$$W_{mag} = F_{mag}h = IBah,$$

where h is the distance the loop rises. But we know that magnetic force never do work.

When the loop begins to rise, the charges in the wire are not moving horizontally—their velocity now acquires an upward component u, the speed of the loop (Fig. 4.27), in addition to the horizontal component w associated with the current ($I = \lambda w$ where λ is linear charge density on it). The magnetic force, which is always perpendicular to the velocity and direction of v and therefore it does no work on q. The net vertical force on all the charge (λa) in the upper segment of the loop is

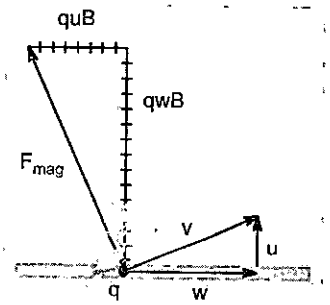


Fig. 4.27

$$F_{vert} = \lambda awB = IBa$$

But it additionally has a horizontal component (quB), which opposes the flow of current. The agent that maintains current, therefore, must now push those charges against the backward component of the magnetic force.

The total horizontal force on the top segment is thus

$$F_{horiz} = \lambda auB$$

In a time dt the charges move a (horizontal) distance wdt, so the work done by battery is:

$$W_{battery} = \lambda aB \int uwdt = IBah,$$

Which is precisely equal to that we calculated from the magnetic force. Was work done in this process ? Yes it was done by the battery. The magnetic force redirected the horizontal force of the battery into the vertical motion of the loop and the weight.

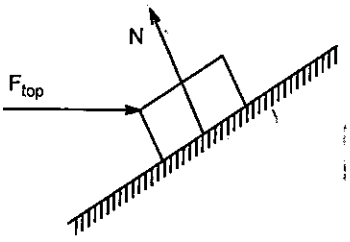


Fig. 4.28

Consider a box pushed up a frictionless incline by pushing on it horizontally Fig. 4.28. The normal force (N) does no work, because it is perpendicular to the displacement. But it does have a vertical component (that lifts the trunk), and a (backward) horizontal component (that we overcome by pushing). Who is doing the work here? External agent which is purely horizontal is not directly responsible for lifting box. The normal force plays the same role as the magnetic force. While doing no work itself, it redirects the efforts of the external agent from horizontal to vertical.

Example 21

Find the work and power required to move the conductor shown in Fig. 4E.21 (a) one full turn in the positive direction at a rotational frequency N revolutions per minute, if $\vec{\mathbf{B}} = B_0\hat{\mathbf{a}}_r$ *(B_0 is positive constant and* $\hat{\mathbf{a}}_r$ *is a unit vector in radial direction).*

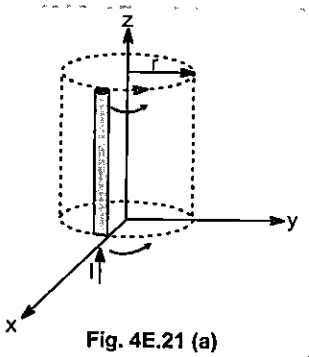


Fig. 4E.21 (a)

Solution: The magnetic force on the conductor is

$$\vec{\mathbf{F}}_B = I\vec{\mathbf{L}} \times \vec{\mathbf{B}}$$
$$= IL\hat{\mathbf{k}} \times B_0\hat{\mathbf{a}}_r$$
$$= B_0IL(\hat{\mathbf{a}}_\phi)$$

External force is

$$\vec{\mathbf{F}}_{ext} = B_0IL(-\hat{\mathbf{a}}_\phi)$$

Where $\hat{\mathbf{a}}_\phi$ is a unit vector in tangential direction.

Therefore work done required to turn the conductor in one full revolution is:

$$W = \int_0^{2\pi} B_0IL(-\hat{\mathbf{a}}_\phi) \cdot rd\phi\hat{\mathbf{a}}_\phi$$
$$= -2\pi rB_0IL$$

The power is $$P = \frac{2\pi rB_0IL}{60}N$$

where $\frac{N}{60}$ is number of revolutions per second.

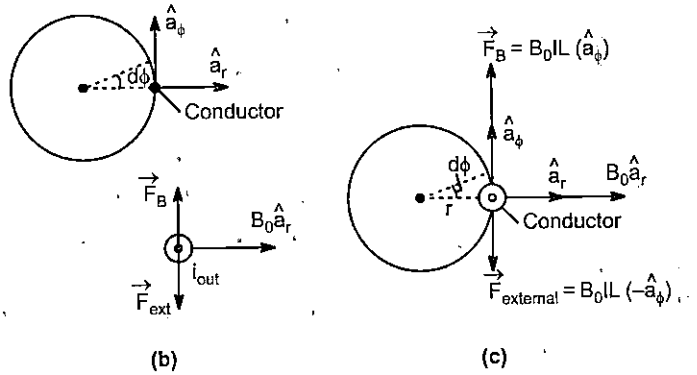


Fig. 4E.21

The negative sign shows that field does work. The fact that work is done around a closed path shows that the force is non-conservative in this case.

Example 22

A wire formed in the shape of a semicircular loop of radius R lies in the xy-plane. It carries an anticlockwise current as shown in Fig. 4E.22. There is a uniform magnetic field $\vec{\mathbf{B}} = B\hat{\mathbf{k}}$ *perpendicular to the plane of the loop. Find the magnetic force on the semicircular loop.*

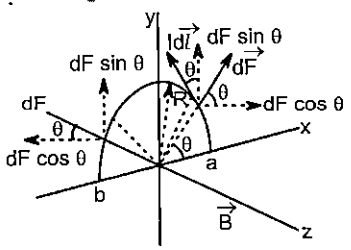


Fig. 4E.22

Solution: We consider a differential element on the loop; it lies in the *xy*-plane and subtends an angle $d\theta$ at the centre.

Force $d\vec{\mathbf{F}}$ on the current element $d\vec{\mathbf{l}}$,

$$d\vec{\mathbf{F}} = Id\vec{\mathbf{l}} \times \vec{\mathbf{B}}$$

Now we express $d\vec{\mathbf{l}}$ in terms of the unit vectors $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$.

$$d\vec{\mathbf{l}} = -dl\sin\theta\hat{\mathbf{i}} + dl\cos\theta\hat{\mathbf{j}}$$
$$= -(Rd\theta)\sin\theta\hat{\mathbf{i}} + R(d\theta)\sin\theta\hat{\mathbf{j}}$$
$$d\vec{\mathbf{F}} = (-IR\sin\theta\,d\theta\,\hat{\mathbf{i}} + IR\cos\theta\,d\theta\,\hat{\mathbf{j}}) \times B\hat{\mathbf{k}}$$
$$= IRB\sin\theta\,d\theta\,\hat{\mathbf{j}} + IRB\cos\theta\,d\theta\,\hat{\mathbf{i}}$$
$$\vec{\mathbf{F}} = IRB\,\hat{\mathbf{i}}\int_0^\pi \cos\theta\,d\theta + IRB\,\hat{\mathbf{j}}\int_0^\pi \sin\theta\,d\theta$$
$$= IRB\,\hat{\mathbf{i}}(\sin\theta)_0^\pi + IRB\,\hat{\mathbf{j}}(-\cos\theta)_0^\pi$$
$$= 0 + 2IRB\hat{\mathbf{j}}$$

Thus the resultant force on the loop has magnitude $|\vec{\mathbf{F}}| = 2IRB$ and directed along positive *y*-axis.

* If we resolve $d\vec{\mathbf{F}}$ into its *x* and *y*-components, the *x*-component is zero from symmetry. For the right half of the loop, $dF\cos\theta$ point to the right and for the left half $dF\cos\theta$ points to the left.

Thus $$F = IRB\int_0^\pi \sin\theta\,d\theta$$
$$= 2IRB$$

* The net force on the semicircular wire is the same as if the semicircle is replaced by a straight line segment of length 2*R* connecting points *a* and *b*.

Example 23

A circular loop of radius R is bent along a diameter and given a shape as shown in Fig. 4E.23 (a). One of the semicircles (KNM) lies in the xz-plane and the other one (KLM) in the yz-plane with their centres at the origin. Current I is flowing through each of the semicircles as shown in Fig. 4E. 23(a).

(a) A particle of charge q is released at the origin with a velocity $\vec{\mathbf{v}} = \vec{\mathbf{v}}_0\hat{\mathbf{i}}$. *Find the instantaneous force* $\mathbf{F}$ *on the particle. Assume that space is gravity free.*

(b) If an external uniform magnetic field $\vec{\mathbf{B}}\hat{\mathbf{j}}$ *is applied, determine the forces* $\vec{\mathbf{F}}_1$ *and* $\vec{\mathbf{F}}_2$ *on the semicircles KLM and KNM due to this field and the net force* $\vec{\mathbf{F}}$ *on the loop.*

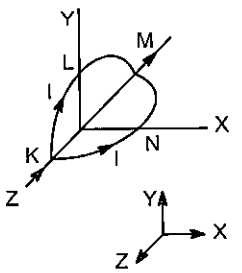


Fig. 4E.23 (a)

Solution: (a) The magnetic field of semicircular arc *KLM*, at its centre

$$\vec{B}_1 = \frac{\mu_0 I}{4R}(-\hat{i})$$

The magnetic field of semicircular arc *KNM* at its centre

$$\vec{B}_2 = \frac{\mu_0 I}{4R}(\hat{j})$$

The resultant magnetic field

$$\vec{B}_R = \frac{\mu_0 I}{4R}(\hat{j} - \hat{i})$$

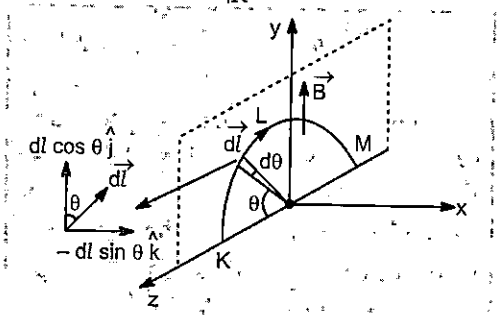


Fig. 4E.23 (b)

The magnetic force on a moving charge is

$$\vec{F} = q(\vec{v} \times \vec{B})$$

$$= \frac{\mu_0 qI}{4R}(-\vec{v}_0 \hat{i}) \times (\hat{j} - \hat{i})$$

$$= \frac{\mu_0 qI}{4R}\vec{v}_0(-\hat{k})$$

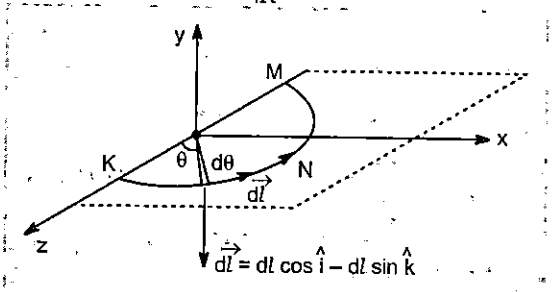


Fig. 4E.23 (c)

(b) We take a differential element on arc *KLM* as shown in Fig. 4E.23(c); length $dl = R\theta$

In vector form, $d\vec{l} = dl\cos\theta\,\hat{j} - dl\sin\theta\,\hat{k}$

The magnetic force on a differential element $d\vec{l}$.

$$d\vec{F} = Id\vec{l} \times \vec{B}$$

$$= I(dl\cos\theta\,\hat{j} - dl\sin\theta\,\hat{k}) \times B\hat{j}$$

$$= (Idl\sin\theta B)\hat{i} = (IRB\sin\theta\,d\theta)(\hat{i})$$

Resultant force

$$\vec{F}_{KLM} = IRB\int_0^{\pi}\sin\theta\,d\theta(\hat{i})$$

$$= 2IRB(\hat{i})$$

Proceeding similar to force on arc *KLM* we have for *KNM*

$$d\vec{l} = dl\cos\theta\,\hat{i} - dl\sin\theta\,\hat{k}$$

where $dl = Rd\theta$, we get

$$dF = I(dl\cos\theta\,\hat{i} - dl\sin\theta\hat{k}) \times B\hat{j}$$

$$= I(dl\cos\theta\,\hat{k} + dl\sin\theta\,\hat{i})B$$

$$= I(Rd\theta)(\cos\theta\hat{k} + \sin\theta\hat{i})B$$

$$F_{KNM} = IRB\left[\int_0^{\pi}\cos\theta\,d\theta\,\hat{k} + \int_0^{\pi}\sin\theta\,d\theta\,\hat{i}\right]$$

$$= IRB[0 + 2\hat{i}]$$

$$= 2IRB\hat{i}$$

Thus resultant force,

$$\vec{F} = \vec{F}_{KLM} + \vec{F}_{KNM} = 4IRB\hat{i}$$

Example 24

A wire bent in some arbitrary shape carries a current I in a uniform magnetic field $\vec{B}$. *Show that the total force from some point a to some point b is* $\vec{F} = I\vec{l} \times \vec{B}$, *where* $\vec{l}$ *is a vector from a to b; hence show that force on a current carrying loop of any arbitrary shape placed in a uniform magnetic field is zero.*

Solution: The force on a current carrying conductor is given by

$$\vec{F} = I\int d\vec{l} \times \vec{B}$$

$$= I(\int d\vec{l}) \times \vec{B}$$

The $\int d\vec{l}$ represents the vector sum of all the length elements from initial point to final point.

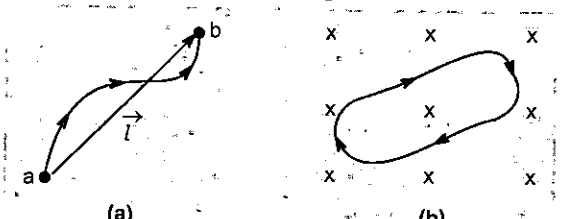


Fig. 4E.24

Therefore, $\int d\vec{l} = \vec{l}$

$\int d\vec{l}$ represents total displacement of charges from point *a* to *b*.

For a closed loop the line integral is replaced by integral along closed path.

$$\vec{F} = I\left(\oint d\vec{l}\right) \times \vec{B}$$

The vector sum of $d\vec{l}$ for a closed loop will be zero.

$$\oint d\vec{l} = 0$$

Therefore $\vec{F} = 0$

TORQUE ON A CURRENT LOOP IN A MAGNETIC FIELD

Consider a rectangular loop of area A and dimensions a and b in a uniform magnetic field that makes an angle θ with a line perpendicular to the plane of the loop. For convenience in describing the orientation of a current carrying loop, we define an **area vector** $\vec{A}$. The magnitude of the area vector is $|\vec{A}| = ab$. The direction of the area vector is perpendicular to the plane of the loop, governed by right hand thumb rule. Curl your fingers in the sense of current; then the extended thumb of your right hand gives the direction of the area vector of the loop. In Fig. 4.29 area vector $\vec{A}$ makes an angle θ with the direction of magnetic field.

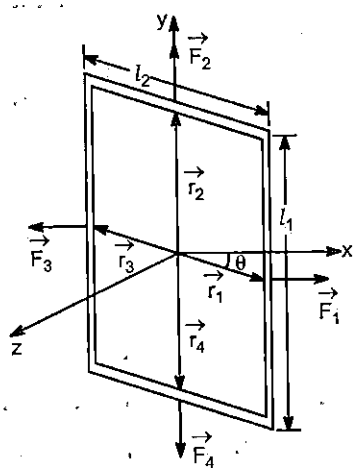


Fig. 4.29

Although magnetic force on a closed loop of any arbitrary shape in a uniform magnetic field is zero nevertheless there may be a non zero torque.

The segment (1) is.

$$\vec{l}_1 = a\hat{j}$$

The force on this segment is

$$\vec{F}_1 = I\vec{l}_1 \times \vec{B}$$
$$= I(a\hat{j}) \times (B\hat{k})$$
$$= IaB\hat{i}$$

The segment (2) is.

$$\vec{l}_2 = b[(-\cos\theta\,\hat{i}) - \sin\theta\,\hat{k}]$$

Thus the magnetic force on this segment is

$$\vec{F}_2 = I\vec{l}_2 \times \vec{B}$$
$$= Ib[-\cos\theta\,\hat{i} - \sin\theta\,\hat{k}] \times B\hat{k}$$
$$= IbB\cos\theta\,\hat{j}$$

The current in segment (3) is opposite to segment (1); therefore

$$\vec{l}_3 = -a\hat{j}$$
$$\vec{F}_3 = I\vec{l}_3 \times \vec{B}$$
$$= I(-a\hat{j}) \times (B\hat{k})$$
$$= -IaB\hat{i}$$

Similarly the current in segment (4) is opposite to segment (2).

$$\vec{l}_4 = b(\cos\theta\,\hat{i} + \sin\theta\,\hat{j})$$

The force on this segment is

$$\vec{F}_4 = I\vec{l}_4 \times \vec{B}$$
$$= Ib[\cos\theta\,\hat{i} + \sin\theta\,\hat{j}] \times B\hat{k}$$
$$= (-IbB\cos\theta)\hat{j}$$

The total force on the current loop is the vector sum of the forces on each segment.

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4$$
$$= IaB\hat{i} + IbB\cos\theta\,\hat{j} - IaB\hat{i} - IbB\cos\theta\,\hat{j}$$
$$= 0$$

This is not a new result, we have already shown it ! (see example 28) Now we find the torque of each force about the centre of the loop.

Forces $\vec{F}_2$ and $\vec{F}_4$ pass through the point of application (there moment arm is zero); these forces have no torque. The moment arms of the forces F_1 and F_3 are

$$\vec{r}_1 = \frac{b}{2}[(\cos\theta)\,\hat{i} + (\sin\theta)\,\hat{k}]$$
$$\vec{r}_3 = \frac{b}{2}[(-\cos\theta)\,\hat{i} - (\sin\theta)\,\hat{k}]$$

The torque of forces F_1 and F_2 are

$$\tau_1 = \vec{r}_1 \times \vec{F}_1$$
$$= \frac{b}{2}[(\cos\theta)\,\hat{i} + (\sin\theta)\,\hat{k}] \times IaB\hat{i}$$
$$= \frac{I}{2}abB\sin\theta\,\hat{j}$$
$$\tau_3 = \vec{r}_3 \times \vec{F}_3$$
$$= \frac{b}{2}[(-\cos\theta)\,\hat{i} - (\sin\theta)\,\hat{k}] \times (-Il_1B)\,\hat{i}$$
$$= \left(\frac{I}{2}l_1l_2B\sin\theta\right)\hat{j}$$

The total torque on the loop is the vector sum of the torques due to $\vec{F}_1$ and $\vec{F}_3$.

$$\vec{\tau} = IaB\sin\theta\,\hat{j}$$
$$= IAB\sin\theta\,\hat{j}$$

where ab is the area of the loop. The total torque is along $\hat{j}$ direction; hence if the loop is free to rotate, it will spin about the $\hat{j}$ direction (y-axis). The sense of rotation can be determined from right-hand thumb rule. Align the thumb of your right hand in the direction of torque; the right hand fingers will curl in the sense of rotation.

The area vector can be expressed as

$$\vec{A} = (-A\sin\theta)\,\hat{i} + (A\cos\theta)\,\hat{k}$$
$$\vec{A} \times \vec{B} = [(-A\sin\theta)\,\hat{i} + (A\cos\theta)\,\hat{k}] \times B\hat{k}$$
$$= (AB\sin\theta)\,\hat{j}$$

Thus $\vec{\tau} = I\vec{A} \times \vec{B}$

* The vector quantity $I\vec{A}$ is called **magnetic dipole moment** of the current loop. Direction of magnetic dipole moment vector is same as that of area vector, and determined from right-hand thumb rule. Hence we can get the expression for the torque on the current loop as

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

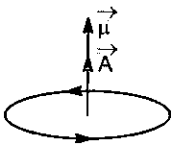
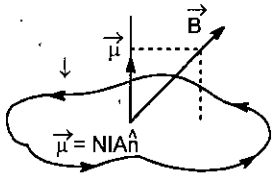


Fig. 4.30



A current loop of arbitrary shape is described by its magnetic moment $\vec{\mu} = NI\hat{A}\hat{n}$. In a magnetic field $\vec{B}$, it experiences a torque $\vec{\mu} \times \vec{B}$.

Fig. 4.31

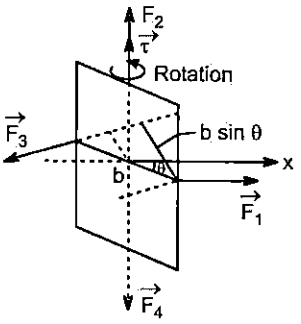


Fig. 4.32

* We have taken a longer route to derive the expression for torque in order of illustrate torque explicitly. We can arrive at the result by a simple reasoning: $\vec{F}_2$ and $\vec{F}_4$ have same magnitude but are opposite in direction; they cancel $\vec{F}_1$ and $\vec{F}_3$ form a couple with $b\sin\theta$ as perpendicular distance between line of action of two forces. Hence magnitude of torque

$$= (IaB)b\sin\theta$$
$$= IAB\sin\theta$$

MAGNETIC MOMENT OF A CIRCULAR COIL

The on-axis magnetic field of a current loop, is

$$B_{loop} = \frac{\mu_0}{2}\frac{IR^2}{(z^2+R^2)^{3/2}}$$

If z is much larger than the diameter of the current loop, $z \gg R$, we can make the approximation $(z^2+R^2)^{3/2} \to z^3$. Then the loop's field is

$$B_{loop} \approx \frac{\mu_0}{2}\frac{IR^2}{z^3} = \frac{\mu_0}{4\pi}\frac{2(\pi R^2)I}{z^3} = \frac{\mu_0}{4\pi}\frac{2AI}{z^3} \quad ...(1)$$

where $A = \pi R^2$ is the area of the loop.

Let's define the **magnetic dipole moment** $\vec{\mu}$ of a current loop enclosing area A to be

$\vec{\mu}$ = (AI, from the south pole to the north pole)

The SI units of the magnetic dipole moment are Am^2.

The magnetic dipole moment, like the electric dipole moment, is a vector. It has the same direction as the on-axis magnetic field. Thus the right-hand rule for determining the direction of $\vec{B}$ also shows the direction of $\vec{\mu}$. Fig. 4.33 shows the magnetic dipole moment of a circular current loop.

The magnetic dipole moment is perpendicular to the loop in the direction of the right hand rule. The magnitude of $\vec{\mu}$ is AI.

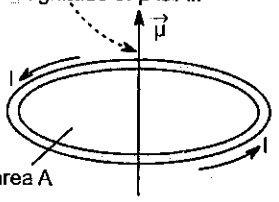


Fig. 4.33

Because the on-axis magnetic field of a current loop points in the same direction as $\vec{\mu}$, we can combine equation (1) and the definition of $\vec{\mu}$ to write the on-axis field of a magnetic dipole as

$$\vec{B}_{dipole} = \frac{\mu_0}{4\pi}\frac{2\vec{\mu}}{z^3} \text{ (on the axis of a magnetic dipole)} \quad ...(2)$$

If you compare $\vec{B}_{dipole}$ to $\vec{E}_{dipole}$, you can see that the magnetic field of a magnetic dipole has the same basic shape as the electric field of an electric dipole.

Concept: *At large distances from any distribution of current, the magnetic field it produces resembles an ideal dipole field.*

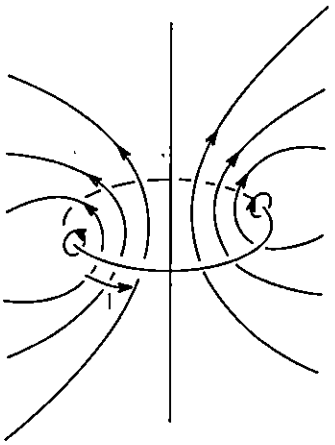


Fig. 4.34: The magnetic field on the axis of a circular current loop. Magnetic field lines cl the wire. At large distances from the loop, the eld pattern resembles that of a dipole.

Magnetic Moment of an Arbitrary Planar Loop

We use the Biot-Savart law to calculate the magnetic field at a large distance from an arbitrary current loop that lies in a single plane. We choose the origin at a point inside the loop (Fig. 4.35) and the z-axis perpendicular to the plane of the loop. Then the magnetic field at a point on the z-axis due to the element $d\vec{\mathbf{l}}$ is:

$$d\vec{\mathbf{B}} = k_m I \frac{d\vec{\mathbf{l}} \times \hat{\mathbf{r}}}{r^2} = k_m I \frac{d\vec{\mathbf{l}} \times \vec{\mathbf{r}}}{r^3}$$

Writing $\vec{\mathbf{r}} = \vec{\mathbf{r}}_\perp + z\hat{\mathbf{k}}$ and assuming $z \gg |\vec{\mathbf{r}}_\perp|$, we have:

$$d\vec{\mathbf{B}} \approx \frac{k_m I}{z^3} d\vec{\mathbf{l}} \times (\vec{\mathbf{r}}_\perp + z\hat{\mathbf{k}})$$

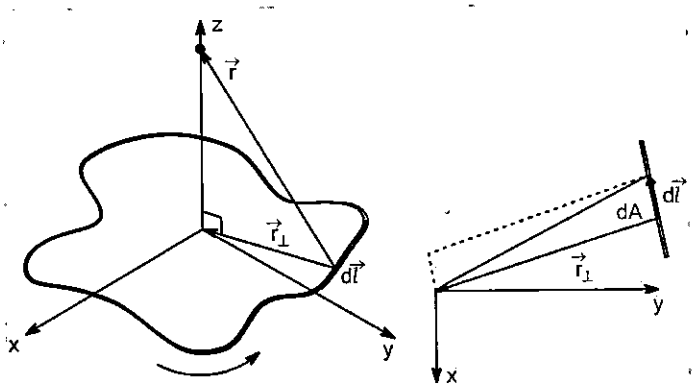


Fig. 4.35

The product $d\vec{\mathbf{l}} \times \vec{\mathbf{r}}_1$ is in the z-direction, while $d\vec{\mathbf{l}} \times \hat{\mathbf{k}}$ lies in the xy-plane. To find the loop's dipole moment, we use the z-component of the field. (At this large distance from the loop, B_x and B_y are zero.

$$dB_z \approx \frac{k_m I}{z^3} |d\vec{\mathbf{l}} \times \vec{\mathbf{r}}_1|.$$

From Fig. 4.35 (b), $\frac{1}{2}|d\vec{\mathbf{l}} \times \vec{\mathbf{r}}_\perp|$ is the differential area dA. The cross product of two vectors represents the area of the parallelogram formed by the two vectors. Thus, $\frac{1}{2}|d\vec{\mathbf{l}} \times \vec{\mathbf{r}}_\perp| = dA$, the triangular area element formed by the line element and the center of the loop. Thus:

$$B_z = \int dB_z = \int \frac{k_m I}{z^3} 2dA = \frac{2k_m IA}{z^3}$$

So $m = IA$ is the correct expression for the magnetic dipole moment of any arbitrary current loop.

* The expression for torque on an electric dipole in an electric field was similar to that for magnetic dipole in a magnetic field.

$$\vec{\tau}_E = \vec{\mathbf{p}} \times \vec{\mathbf{E}} \quad \text{(electric dipole)}$$

$$\vec{\tau}_B = \vec{\mu} \times \vec{\mathbf{B}} \quad \text{(magnetic dipole)}$$

An electric dipole has potential energy given by

$$U = -\vec{\mathbf{p}} \cdot \vec{\mathbf{E}}$$

with $\theta = 90°$ as reference position.

Similarly, in order to rotate a current loop in a magnetic field so as to increase θ, we must do work against torque due to magnetic field. From definition of potential energy,

$$U = \int \tau\, d\theta = \int \mu B \sin\theta$$

$$= -\mu B \cos\theta + C$$

If we choose $U = 0$ at $\theta = \pi/2$, then the constant C is zero and the potential energy is

$$U = -\mu B \cos\theta = -\vec{\mu} \cdot \vec{\mathbf{B}}$$

Example 25

The square loop in Fig. 4E. 25(a) has sides of length 20 cm. It has 5 turns and carries a current of 2A. The normal to the loop is at 37° to a uniform field $\vec{\mathbf{B}} = 0.5\hat{\mathbf{j}}$T. Find: (a) the magnetic moment; (b) the torque on the loop; (c) the work needed to rotate the loop from its position of minimum energy to the given orientation. (d) What are the forces on the horizontal sections of the loop?

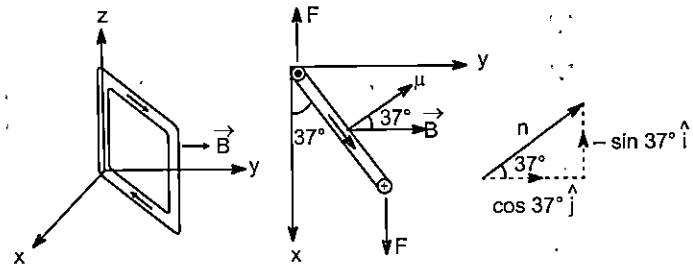


Fig. 4E.25

Solution: We redraw the diagram from a different perspective, the top view shown in Fig. 4E.25(b).

(a) From Fig. 4E.25(b) we see that the unit vector normal to loop $\hat{\mathbf{n}} = -\sin 37°\,\hat{\mathbf{i}} + \cos 37°\,\hat{\mathbf{j}} = -0.6\hat{\mathbf{i}} + 0.8\hat{\mathbf{j}}$. The magnetic moment is

$$\mu = NIA\hat{\mathbf{n}} = (5)(2)(0.2)^2(-0.6\hat{\mathbf{i}} + 0.8\hat{\mathbf{j}})$$

$$= -0.24\hat{\mathbf{i}} + 0.32\hat{\mathbf{j}}\ \text{A-m}^2$$

(b) The torque,

$$\tau = \mu \times \vec{\mathbf{B}} = (-0.24\ \hat{\mathbf{i}} + 0.32\hat{\mathbf{j}}) \times (0.5\hat{\mathbf{j}})$$

$$= -0.12\hat{\mathbf{k}}\ \text{N-m}$$

Looking in the direction of the negative z-axis, as in Fig. 4E.25(b), we see that the (clockwise) torque tends to align $\vec{\mu}$ along $\vec{\mathbf{B}}$.

(c) The potential energy of the loop is $U = -\mu B \cos\theta$ where $\mu = NIA = 0.4\,\text{A m}^2$ and $\vec{\mathbf{B}} = 0.5\hat{\mathbf{j}}$ T. The position of

minimum energy is $\theta = 0$. Thus, the external work, $W_{ext} = +\Delta U$, needed to rotate it to the given orientation is

$$U_f - U_i = (-\mu B\cos 37°) - (-\mu B\cos 0°)$$
$$= (0.4)(0.5)(1 - 0.8) = 0.04\,\text{J}$$

The external work is positive since the dipole moment is rotated away from alignment with the field.

(d) We use $\vec{F} = i\vec{l} \times \vec{B}$ for the top wire:

$$\vec{l} = a\cos 37°\,\hat{i} + a\sin 37°\,\hat{j}$$

so $$\vec{F} = (2)(0.16\,\hat{i} + 0.12\hat{j}) \times (0.5\hat{j})$$
$$= 0.16\hat{k}\,\text{N}$$

For the bottom wire the force is

$$\vec{F} = -0.16\hat{k}\,\text{N}$$

We can use $\vec{F} = IlB\sin\theta, \theta = 53°$, and right hand thumb rule to determine the magnitude and direction of force.

Example 26

A circular wire loop of radius R, mass m, and current I lies on a rough surface [Fig. 4E.26 (a)]. There is a horizontal magnetic field $\vec{B}$. How large can the current I be before one edge of the loop will lift off the surface ?

Solution: The loop will start to lift off when the magnetic torque equals the gravitational torque [Fig. 4.30 (b)].

The magnetic torque acting on the loop,

$$\tau_m = \mu B = I\pi R^2 B$$

The gravitational torque exerted on the loop,

$$\tau_g = mgR$$

torque of magnetic force acts in such a way that $\vec{\mu}$ vector aligns with the magnetic field vector $\vec{B}$.

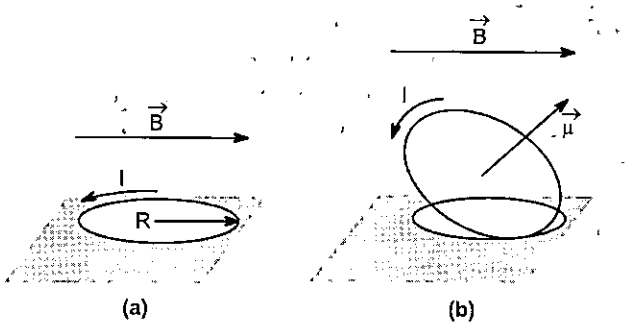


Fig. 4E.26

We equate the torques and solve for I, to get

$$I = \frac{mg}{\pi RB}$$

Example 27

An electron is in a circular orbit about the nucleus of an atom. Find a relationship between the orbital magnetic dipole moment $\vec{\mu}$ and the angular momentum $\vec{L}$ of the electron about the centre of its orbit.

Solution: Let v be the speed of the electron and r the radius of its orbit. The orbiting electron creates an electric current opposite to the velocity. The current is the charge that passes any given point along the circumferential path during one second. Thus current is charge e times the frequency ν of the orbital motion.

$$I = ev = \frac{e}{T} = \frac{ev}{2\pi r}$$

The magnitude of magnetic moment,

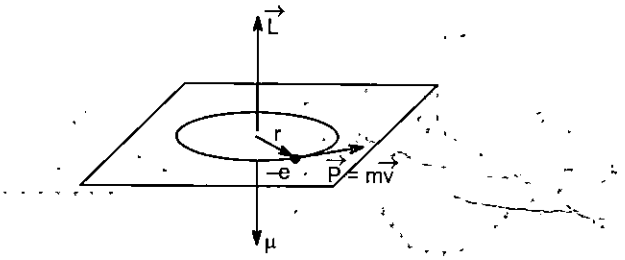


Fig. 4E.27

$$\mu = IA$$
$$= \frac{ev}{2\pi r}\pi r^2 = \frac{evr}{2} \quad ...(1)$$

The direction of the magnetic dipole moment is perpendicular to the loop from right hand thumb rule.

The orbital angular momentum

$$\vec{L}_{\text{orbital}} = \vec{r} \times \vec{p}$$

$$|\vec{L}_{\text{orbital}}| = rp\sin 90° = mvr \quad ...(2)$$

The direction of the orbital angular momentum is determined from the vector product right hand rule. The direction of angular momentum and magnetic dipole moment is opposite to each other. Comparing eqn. (2) and (1), we obtain

$$\vec{\mu} = -\frac{e}{2m}\vec{L}$$

Example 28

A non-conducting disc of mass M and radius R has a surface charge density σ and rotates with an angular velocity ω about its axis. Show that magnetic dipole moment and angular momentum are related as $\vec{\mu} = \left(\frac{Q}{2M}\right)\vec{L}$.

Solution: The charge is distributed on the surface of the disc. We consider a differential ring of radius r, thickness dr.

The charge on the element is

$$dq = \sigma dA = \sigma(2\pi r dr)$$

The magnetic moment of the ring

$$d\mu = (dI)A = (dI)\pi r^2$$

The current in the differential ring

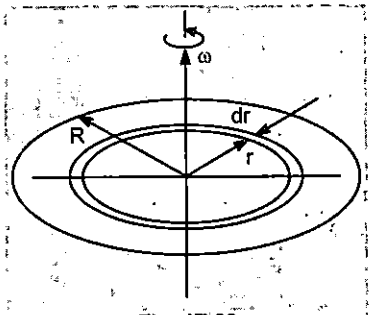


Fig. 4E.28

$$= (dq)v$$

$$= (\sigma dA)\frac{\omega}{2\pi}$$

$$= (\sigma 2\pi r\, dr)\frac{\omega}{2\pi}$$

$$= \sigma\omega r\, dr$$

The magnetic moment of differential ring,

$$d\mu = (\sigma\omega r\, dr)\pi r^2$$

$$= \pi\sigma\omega r^3 dr$$

$$\mu = \int d\mu$$

$$= \int_0^R \pi\sigma\omega r^3 dr$$

$$= 1/4\,\pi\sigma\omega R^4$$

The magnetic moment vector $\vec{\mu}$ is parallel to $\vec{\omega}$ if σ is positive.

$$\vec{\mu} = \frac{1}{4}\pi\sigma R^4 \vec{\omega}$$

In terms of total charge $Q = \sigma\pi R^2$, the magnetic moment is

$$\vec{\mu} = \frac{1}{4}QR^2\vec{\omega}$$

The angular momentum of disc is

$$\vec{\mathbf{L}} = \left(\frac{1}{2}MR^2\right)\vec{\omega}$$

so

$$\vec{\mu} = \left(\frac{Q}{2M}\right)\vec{\mathbf{L}}$$

* This is a general result for any rigid body of any arbitrary shape, with mass M and charge Q.

Example 29

A rod has a total charge Q uniformly distributed along its length L. If the rod rotates with angular velocity ω about its end, compute its magnetic moment.

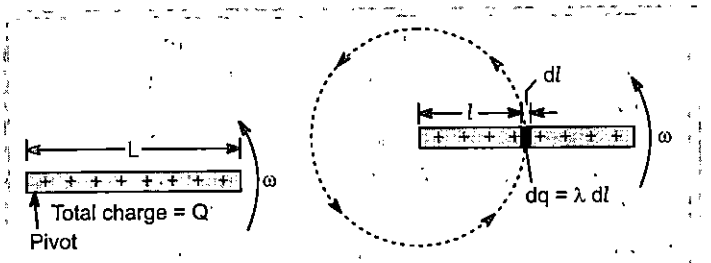


Fig. 4E.29

Solution: We can visualize the rod to consist of differential elements dQ, which constitute a series of concentric current loops. The charge per unit length of the rod λ.

$$\lambda = \frac{Q}{L}$$

So the charge on a differential element of length dl,

$$dq = \lambda dl$$

The current dI due to rotation of this charge is given by

$$dI = \frac{dq}{(2\pi/\omega)}$$

$$= \frac{\omega}{2\pi}dq = \frac{\omega}{2\pi}\lambda dl$$

The magnetic moment of this differential current loop,

$$d\mu = dI(\pi l^2)$$

$$= \left(\frac{\omega}{2\pi}\lambda dl\right)\pi l^2$$

$$= \frac{\omega\lambda}{2}l^2\, dl$$

To find total magnetic moment, we integrate

$$\mu = \frac{\omega\lambda}{2}\int_0^L l^2 dl$$

$$= \frac{\omega\lambda L^3}{6}$$

Substituting for λ, we obtain

$$\mu = \frac{Q\omega L^2}{6}$$

Example 30

(a) A hollow cylinder has length L and inner and outer radii R_1 and R_2 respectively. The cylinder carries a uniform charge density ρ. Derive an expression for the magnetic moment as a function of ω, the angular velocity of rotation of the cylinder about its axis.

(b) A solid cylinder of radius R and length L carries a uniform charge density +ρ between $r = 0$ and $r = R_s$ and an equal charge density of opposite sign −ρ between $r = R_s$ and $r = R$. What must be the radius R_s so that on rotation of the cylinder about its axis the magnetic moment is zero ?

Solution: (a) The cylinder can be visualized to consist of differential cylindrical shells of charge dQ. One such

cylindrical shell has radius r, thickness dr. Charge dQ of differential cylindrical shell is

$$dQ = \rho(2\pi r\, dr)L$$

The current due to rotation of this charge is given by

$$dI = \frac{dQ}{(2\pi/\omega)} = \rho(2\pi r\, dr)L\left(\frac{\omega}{2\pi}\right)$$

The magnetic moment of this differential current loop

$$= (dI)(\pi r^2)$$

$$d\mu = \left[\rho(2\pi r dr)L\frac{\omega}{2\pi}\right]\pi r^2$$

To find total magnetic moment we integrate

$$\mu = \int d\mu = \int_{R_1}^{R_2}\left[\rho(2\pi r\, dr)L\frac{\omega}{2\pi}\right]\pi r^2$$

$$= \pi\rho L\omega\int_{R_1}^{R_2} r^3 dr$$

$$\mu = \frac{1}{4}\pi\rho L\omega(R_2^4 - R_1^4)$$

(b) Proceeding in a similar way as part (a), the magnetic moment of positively charged section of cylinder

$$\mu_+ = \int_0^{R_s}(\rho \times 2\pi r\, dr \times L)\frac{\omega}{2\pi}\times \pi r^2$$

$$= \pi\rho\omega L\int_0^{R_s} r^3 dr$$

$$= \frac{1}{4}\pi\rho\omega L R^4$$

Magnetic moment of negatively charged section

$$\mu_- = -\int_{R_s}^{R}(\rho \times 2\pi r\, dr\, L)\frac{\omega}{2\pi}\pi r^2$$

$$= -\pi\rho\omega L\int_{R_s}^{R} r^3 dr$$

$$= -\frac{1}{4}\pi\rho\omega L(R^4 - R_s^4)$$

Both the magnetic moments point in the opposite direction.

$$\vec{\mu}_{net} = \vec{\mu}_1 + \vec{\mu}_2$$

$$|\mu_{net}| = \mu_1 - \mu_2 = 0$$

$$\mu_1 = \mu_2$$

$$\frac{\pi\rho\omega L R^4}{4} = -\frac{1}{4}\pi\rho\omega L(R^4 - R_s^4)$$

$$R^4 = -R^4 + R_s^4$$

$$R_s = 2^{-1/4}R$$

Example 31

'Q' charge is uniformly distributed over the same surface of a right circular cone of semi-vertical angle θ and height h. The cone is uniformly rotated about its axis at angular velocity ω. Calculate the associated magnetic dipole moment.

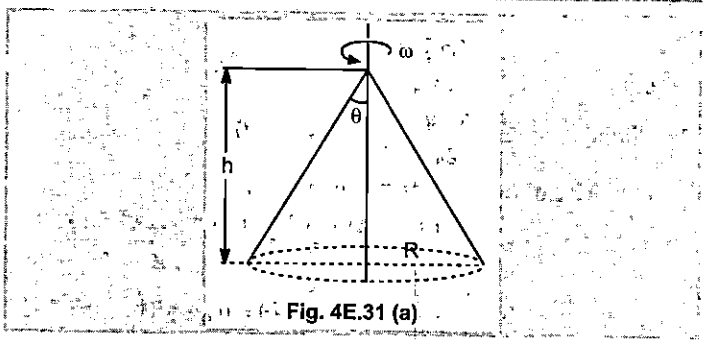


Fig. 4E.31 (a)

Solution: A ring of radius r with charge q rotating with angular velocity ω is equivalent to a dipole of dipole moment

$$\mu = NIA = qfA = q\frac{\omega}{2\pi}A$$

Dipole moment of differential ring,

$$d\mu = \frac{Q_1}{\pi Rl}2\pi r\frac{dx}{\cos\theta}\cdot\frac{\omega}{2\pi}\pi r^2$$

From similar triangles [see Fig. 4E.31 (b)] we have

$$\frac{R}{r} = \frac{h}{h-x}$$

$$d\mu = \frac{Q\cos\theta}{\pi Rh}\frac{dx}{\cos\theta}\omega\pi\left(\frac{h-x}{h}R\right)^3$$

$$\Rightarrow \mu = \frac{Q\omega R^2}{h^4}\int_0^h (h-x)^3 dx$$

$$= \frac{Q\omega R^2}{h^4}\left|\frac{(h-x)^4}{4}\right|_0^h$$

$$= \frac{\omega Q R^2}{h^4}\frac{h^4}{4} = \frac{Q\omega R^2}{4}$$

$$= \frac{Q\omega}{4}h^2\tan^2\theta$$

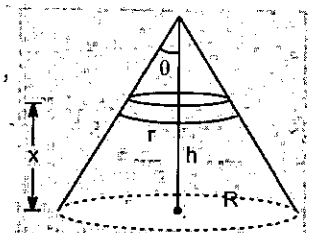


Fig. 4E.31 (b)

Example 32

A sphere of radius R, uniformly charged with the surface charge density σ, rotates around the axis passing through its centre at an angular velocity ω. Find the magnetic induction at the centre of the rotating sphere. Also find its magnetic moment.

Solution: Charge on the differential circular strip is

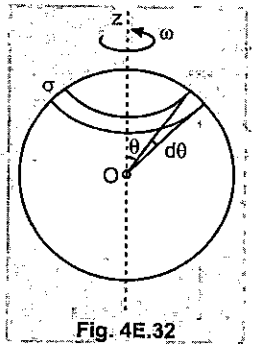


Fig. 4E.32

$$dq = (2\pi R\sin\theta)(R\,d\theta)\sigma$$

or $$dq = 2\pi\sigma R^2\sin\theta\,d\theta$$

Then $$dI = \frac{\omega dq}{2\pi} = \omega\sigma R^2\sin\theta\,d\theta$$

(a) Magnetic field at the centre is

$$dB = \frac{\mu_0 dI(R^2\sin^2\theta)}{2R^3} = \frac{\mu_0\omega\sigma R\sin^3\theta\,d\theta}{2}$$

$$B = \frac{\mu_0\omega\sigma R}{2}\int_0^{\pi}\sin^3\theta\,d\theta = \mu_0\omega\sigma R\int_0^{\pi/2}\sin^3\theta\,d\theta$$

or $$\vec{\mathbf{B}} = \frac{2}{3}\mu_0\omega\sigma R(\hat{\mathbf{k}})$$

(b) Magnetic moment due to elementary ring, $d\mu = (dI)\pi r^2$

$$\therefore \quad \mu = \int(dI)\pi r^2$$

$$= 2\int_0^{\pi/2}(\omega\sigma R^2\sin\theta)\pi R^2\sin^2\theta\,d\theta$$

$$= 2\pi R^4\omega\sigma\int_0^{\pi/2}\sin^3\theta\,d\theta$$

$$= \frac{4}{3}\pi\sigma R^4\omega$$

Example 33

Calculate the magnetic moment of a thin wire with a current $I = 0.8$ A, wound tightly on half a tore [Fig. 4E.33(a)]. The diameter of the cross-section of the tore is equal to $d = 5.0$ cm, the number of turns is $N = 500$.

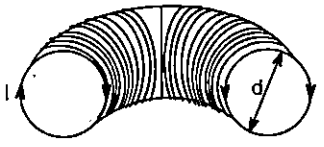


Fig. 4E.33 (a)

Solution: We consider a differential element making an angle $d\theta$ at O.

The number of turns in the considered element is

$$dN = \frac{N}{\pi}d\theta$$

$$dP_m = dNIS = dNI\frac{\pi d^2}{4}$$

From Fig. 4E.33 (c),

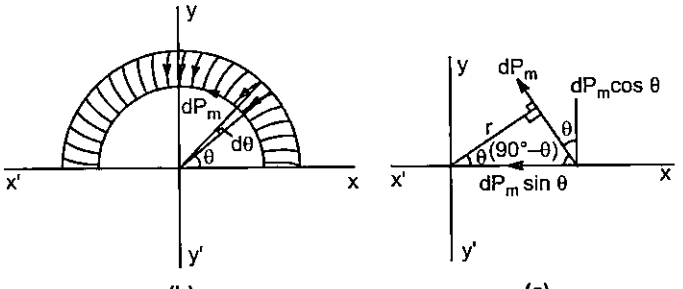


Fig. 4E.33

$$d\vec{\mu}_m = d\mu_m\cos\theta\,\hat{\mathbf{j}} - d\mu_m\sin\theta\,\hat{\mathbf{i}}$$

$$\mu_{mx} = \int_{\theta=0}^{\theta=\pi} -d\mu_m\sin\theta$$

$$= -\int_{\theta=0}^{\theta=\pi} dNI\frac{\pi d^2}{4}\sin\theta$$

$$= -\int_0^{\pi}\frac{NId^2}{4}\sin\theta\,d\theta$$

$$= -\frac{NId^2}{4}\int_0^{\pi}\sin\theta\,d\theta$$

$$= \frac{NId^2}{4}[\cos\theta]_0^{\pi}$$

$$= \frac{NId^2}{4}[\cos\pi - \cos 0] = -\frac{NId^2}{2}$$

Similarly, $$\mu_{my} = \int_0^{\pi}\frac{NId^2}{4}\cos\theta\,d\theta$$

$$= \frac{NId^2}{4}[\sin\theta]_0^{\pi} = 0$$

$$\mu_m = \mu_{mx} = \frac{NId^2}{2} = 0.5\,\text{A-m}^2$$

Example 34

A conductor carries a constant current I along the closed path abcdefgha involving 8 of the 12 edges of length l. Find the magnetic dipole moment of the closed path.

Solution: The closed path is a superposition of three loops: *bcfgb, abgha* and *cdefc*.

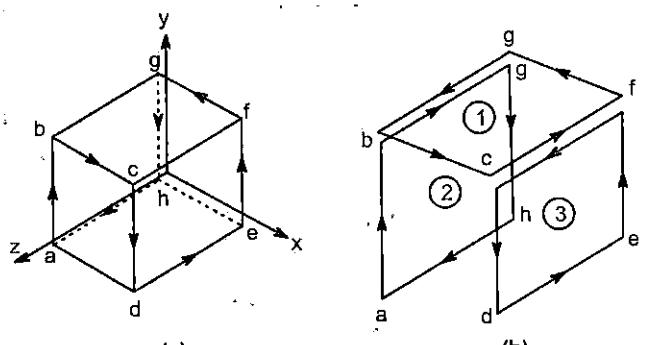


Fig. 4E.34

The magnetic moment of the each loops are :

Loop	1	*bcfgb*	$\vec{\mu}_1 = l^2 I\hat{\mathbf{j}}$
	2	*abgha*	$\vec{\mu}_2 = -l^2 I\hat{\mathbf{i}}$
	3	*cdefc*	$\vec{\mu}_3 = l^2 I\,\hat{\mathbf{i}}$

The total magnetic moment is

$$\vec{\mu} = \vec{\mu}_1 + \vec{\mu}_2 + \vec{\mu}_3$$

or $$\vec{\mu} = l^2 I\,\hat{\mathbf{j}}$$

Example 35

A solid sphere of radius R carries a uniform charge density $+\rho_0$, between $r = 0$ and $r = R_s$ and an equal change density of opposite sign, $-\rho_0$ between $r = R_s$ and $r = R$. The sphere rotates about its diameter with angular velocity ω. Find R_s such that magnetic moment of the sphere is zero. What is the net charge carried by the sphere ?

Solution: The solid sphere can be visualized to consist of differential concentric spherical shells of charge dQ. As we have learned earlier that magnetic moment of a spherical shell of charge Q rotating with angular velocity ω is given by

$$\mu = \frac{1}{3}Q\omega R^2$$

For any rigid body $$\frac{|\vec{\mu}|}{|\vec{L}|} = \frac{Q}{2M}$$

or $$|\vec{\mu}| = \left(\frac{Q}{2M}\right)|\vec{L}| = \frac{Q}{2M}\left(\frac{2}{3}Me^2\omega\right)$$

$$= \frac{1}{3}Q\omega R^2$$

The charge of differential spherical shell is

$$dQ = \rho \times 4\pi r^2 dr$$

Differential magnetic moment,

$$d\mu = \frac{1}{3}(\rho \times 4\pi r^2 dr)\omega r^2$$

Magnetic moment of solid portion between $r = 0$ to $r = R/2$,

$$\mu_1 = \int d\mu = \int_0^{R/2}\frac{1}{3}(\rho \times 4\pi r^2\, dr)\,\omega r^2$$

$$= \frac{1}{3}4\pi\rho\omega^2\int_0^{R/2} r^4\, dr$$

$$= \frac{1}{3}4\pi\rho\omega^2\left|\frac{r^5}{5}\right|_0^{R/2}$$

$$= \frac{4}{32\times 5\times 3}\pi\rho\omega^2 R^5$$

Similarly magnetic moment of solid portion of charge density $-\rho$ between $r = \frac{R}{2}$ to $r = R$,

$$\mu_2 = \int d\mu = -\int_{R/2}^{R}\frac{1}{3}(\rho \times 4\pi r^2 dr)\,\omega r^2$$

$$= -\frac{4}{3}\pi\rho\omega^2\left|\frac{r^5}{5}\right|_{R/2}^{R}$$

$$= -\frac{4}{3}\pi\rho\omega^2\frac{R^5}{5}\left(1 - \frac{1}{2^5}\right)$$

$$= -\frac{31\times 4}{3\times 5\times 32}\pi\rho\omega^2 R^5$$

$$\vec{\mu}_{net} = \vec{\mu}_1 + \vec{\mu}_2$$

$$|\vec{\mu}_{net}| = \mu_2 - \mu_1$$

$$= \left(\frac{\pi}{4}\right)\omega^2\rho R^5, \text{ opposite the direction of } \vec{\omega}$$

Example 36

A rectangular loop PQRS made from a uniform wire has length a, width b and mass m. It is free to rotate about the arm PQ, which remains hinged along a horizontal line taken as the y-axis (see Fig. 4E.36). Take the vertically upward direction as the z-axis. A uniform magnetic field $\vec{B} = (3\hat{i} + 4\hat{k})B_0$ exists in the region. The loop is now released and is found to stay in the horizontal position in equilibrium.

(a) What is the direction of the current I in PQ ?

(b) Find the magnetic force on the arm RS.

(c) Find the expression for I in terms of B_0, a, b and m.

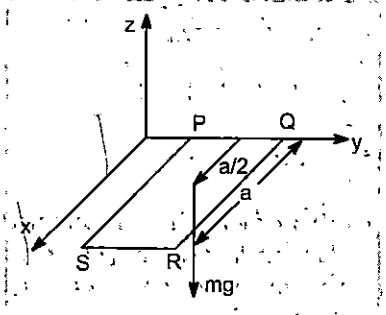


Fig. 4E.36

Solution: (a) Torque due to weight of coil

$$\vec{\tau} = \left(\frac{a}{2}\hat{i}\right) \times (-mg\hat{k})$$

$$= mg\frac{a}{2}(\hat{j})$$

For the equilibrium of loop, torque on it must be along negative y-axis. Let the magnetic moment of loop be $M\hat{k}$. As the loop lies in xy-plane, its magnetic moment vector (from right-hand thumb rule) either points up or down.

Torque due to magnetic force,

$$\vec{\tau}_B = \vec{\mu} \times \vec{B}$$

$$= \mu\hat{k} \times (3\hat{i} + 4\hat{k})B_0$$

$$= 3\mu B_0\hat{j}$$

If $\vec{\tau}_B$ is to be in negative direction, μ must point downward. So the current in the coil must be from P to Q.

(b) Force acting on arm

$$RS = I(\vec{l} \times \vec{B})$$

$$= I[(-b\hat{j}) \times (3\hat{i} + 4\hat{k})B_0]$$

$$= IB_0 b(3\hat{k} - 4\hat{i})$$

(c) In equilibrium,

$$\vec{\tau}_{gravity} + \vec{\tau}_B = 0$$

Hence $$3(abI)B_0 = \frac{mga}{2}$$

or $$I = \frac{mg}{6B_0 b}$$

Example 37

A uniform constant magnetic field $\vec{B}$ is directed at an angle of 45° to the x-axis in the x-y plane. PQRS is a rigid, square wire frame carrying a steady current I_0, with its centre at origin O. At time t = 0, the frame is at rest in the position shown in Fig. 4E.37, with its sides parallel to the x and y-axes. Each side of the frame is mass M and length L. (a) What is the torque $\vec{\tau}$ about O acting on the frame due to magnetic field ? (b) Find the angle by which the frame rotates under the action of this torque in a short interval of time Δt, and the axis about which this rotation occurs. (Δt is short, so that any variation in the torque during this interval may be neglected.) Given: the moment of inertia of the frame about an axis through its centre perpendicular to its plane is $(4/3)ML^2$.

Solution: The magnetic field is directed at an angle 45° with x-axis.

$$\vec{B} = B\cos 45°\,\hat{i} + B\sin 45°\,\hat{j}$$

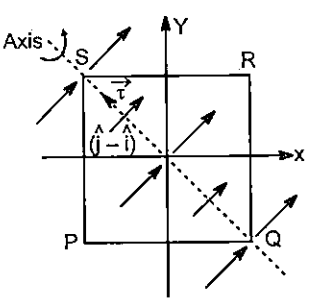


Fig. 4E.37 (a)

The magnetic moment of the loop is

$$\vec{\mu} = I_0 L^2 \hat{k}$$

The torque on the loop is

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

$$= (I_0 L^2 \hat{k}) \times (B\cos 45°\,\hat{i} + B\sin 45°\,\hat{j})$$

$$= \frac{I_0 L^2 B}{\sqrt{2}}(\hat{j} - \hat{i})$$

Magnitude of torque vector is

$$|\vec{\tau}| = I_0 L^2 B$$

* The direction of vector $\hat{j} - \hat{i}$ is diagonal *QS* of the loop, and *QS* is axis of rotation.

Torque is an axial vector, it points along axis of rotation. Point your right thumb in the direction of torque $\vec{\tau}$, the body rotates in the sense of curl of fingers.

From Newton's second law for rotation,

$$\tau = I\alpha \qquad ...(1)$$

From perpendicular axis theorem,

$$I_1 + I_2 = 2I = \frac{4}{3}ML^2$$

$$I = \frac{4}{6}ML^2$$

So $$\alpha = \frac{\tau}{I} = \frac{3}{2}\frac{I_0 B}{M}$$

The angle turned by loop

$$\Delta\theta = \frac{1}{2}\alpha(\Delta t)^2$$

$$= \frac{3}{4}\frac{I_0 B}{M}(\Delta t)^2$$

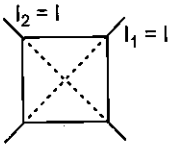


Fig. 4E.37 (b)

Example 38

A ring of radius R having uniformly distributed charge Q is mounted on a rod suspended by two identical strings. The tension in strings in equilibrium is T_0. Now a vertical magnetic field is switched on and the ring is rotated at constant angular velocity ω. Find the maximum ω with which the ring can be rotated if the strings can withstand a maximum tension of $\frac{3T_0}{2}$.

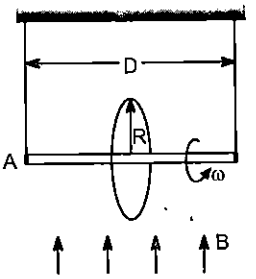


Fig. 4E.38

Solution: In equilibrium,

$$2T_0 = mg$$

or $$T_0 = \frac{mg}{2} \qquad ...(1)$$

Magnetic moment,

$$M = IA = \left(\frac{\omega}{2\pi}Q\right)(\pi R^2)$$

$$\therefore \tau = MB\sin 90° = \frac{\omega BQR^2}{2}$$

Let T_1 and T_2 be the tensions in the two strings when magnetic field is switched on ($T_1 > T_2$).

For translation equilibrium of ring in vertical direction,

$$T_1 + T_2 = mg \qquad ...(2)$$

For rotational equilibrium,

$$(T_1 - T_2)\frac{D}{2} = \tau = \frac{\omega BQR^2}{2}$$

or $$T_1 - T_2 = \frac{\omega BQR^2}{D} \qquad ...(3)$$

Solving equations (2) and (3), we have

$$T_1 = \frac{mg}{2} + \frac{\omega BQR^2}{2D}$$

As $T_1 > T_2$ and maximum values of T_1 can be $\frac{3T_0}{2}$, we have

$$\frac{3T_0}{2} = T_0 + \frac{\omega_{max} BQR^2}{2D} \qquad \left(\frac{mg}{2} = T_0\right)$$

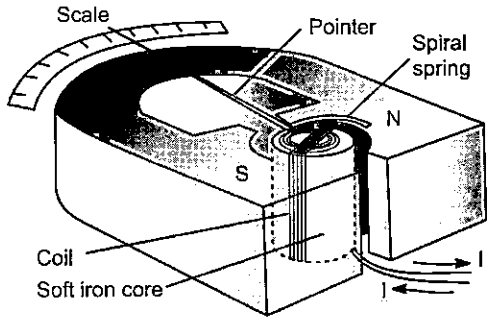
$$\therefore \quad \omega_{max} = \frac{DT_0}{BQR^2}$$

GALVANOMETER

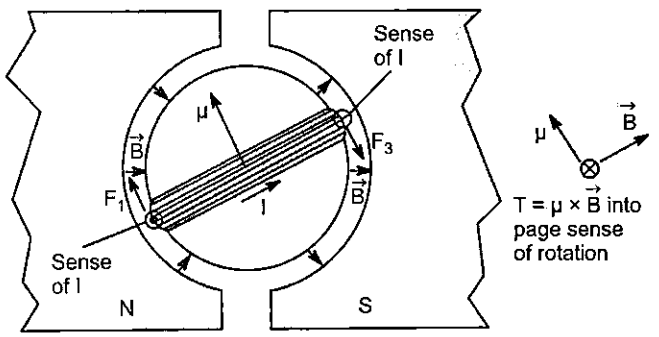
According to equation $T = IA \cdot \vec{B} \sin\theta$, the torque exerted on a current on a current-carrying loop by a magnetic field is proportional to the current through the loop. For a coil of N turns, the corresponding expression is

$$T = NIA\vec{B}\sin\theta \qquad ...(1)$$

Any device that measures current by measuring the torque on a current-carrying loop situated in a known magnetic field is called a **galvanometer.**



(a)



(b)

Fig. 4.36

Because of (1) the high permeability of both the permanent magnet and the soft iron core and (2) the low permeability of air the magnetic flux lines follow paths that minimize their length in air. Nearly all the flux lines therefore pass radially through the air gap between the magnet and the core. Within the limit of travel of the coil (typically about 100°), the magnetic torque exerted on it is independent of its position.

As Fig. 4.36 shows, a current passing through the coil in the sense shown tends to turn the coil clockwise. This turning is opposed by the small spiral spring, which obeys the rotational form of Hooke's law. The coil (and the attached pointer) comes to rest when the spring and the magnetic field exert equal and opposite torques on the coil.

If we assume that the magnetic field through the coil is perpendicular to the normal to the plane of the coil, equation of torque is

$$\tau_m = \mu B$$

Because the circular cross-section of the magnet ensures radial magnetic field lines. This magnetic torque is opposed by the torque due to the spring, which is given by the Hooke's law, $\tau_s = -\kappa\phi$, where κ is the torsional spring constant and ϕ is the angle through which the spring turns. Because the coil does not have an angular acceleration when the pointer is at rest, the sum of these torques must be zero:

$$\tau_m + \tau_s = \mu B - \kappa\phi = 0 \qquad ...(1)$$

The magnetic moment of the N turns of wire to the current through them,

$$\mu = NIA$$

we can substitute this expression for μ in eqn. (1) to obtain

$$(NIA)B - \kappa\phi = 0$$

$$\phi = \frac{NAB}{\kappa} I$$

Thus, the angle of deflection of the pointer is directly proportional to the current in the loop. The factor NAB/κ tells us that deflection also depends on the design of the meter.

The DC Motor

As Fig. 4.37(a) shows, the operation of the **direct-current (dc) motor.** A coil is wound as shown around a shaft-mounted cylindrical iron core, called the **armature** or **rotor.** In the **permanent-magnet motor** shown, the armature lies in the field of a magnet. Current is made to pass through the coil from an external source of emf through contacts called **brushes** that slide on the two

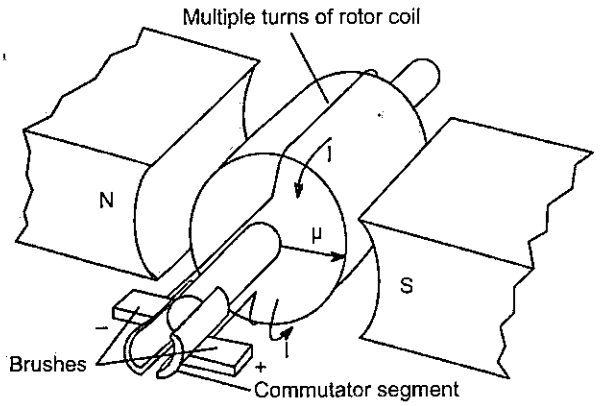


Fig. 4.37 (a)

shaft-mounted segments that constitute the **split-ring commutator.** The sense of the current and the corresponding magnetic moment vector μ of the coil are shown. A torque is exerted on the coil by the external magnetic field in such a way as to align μ with the north -south direction, and the armature turns counterclockwise. Just as the coil reaches its equilibrium orientation (with the

plane of the coil vertical and μ horizontal), the rotation of the commutator results in reversal of the connection.

As Fig. 4.37 (b) shows, the armature of a motor is a coil of wire wound on an axle. When a current passes through the coil, the magnetic field exerts a torque on the armature and causes it to rotate. If the current were steady, the armature would oscillate back and forth around the equilibrium position until it stopped with the plane of the coil perpendicular to the field. A device called a commutator reverses the current direction in the coils every 180°. The current reversal prevents the armature from ever reaching an equilibrium position, so the magnetic torque keeps the motor spinning as long as there is a current.

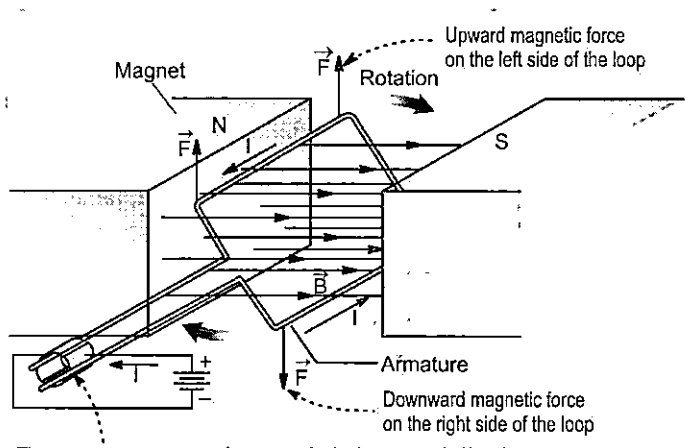


Fig. 4.37 (b)

THE BIOT-SAVART LAW

Consider a small differential length $d\vec{\mathbf{l}}$ of a current carrying wire; the vector $d\vec{\mathbf{l}}$ is directed along the current. This element will create a differential field $d\vec{\mathbf{B}}$ at point P_1 located at a distance r away. According to Biot-Savart law,

$$d\vec{\mathbf{B}} = \frac{\mu_0 I}{4\pi}\frac{d\vec{\mathbf{l}}\times\hat{\mathbf{r}}}{r^2}$$

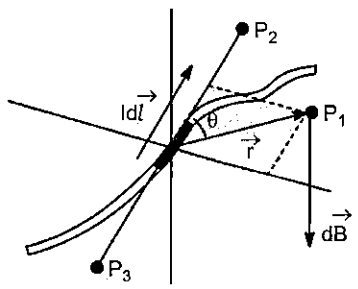


Fig. 4.38

Where $\hat{\mathbf{r}}$ is a unit vector in the direction of $\vec{\mathbf{r}}$. the magnitude of dB is

$$dB = \frac{\mu_0 I dl \sin\theta}{4\pi r^2}$$

The total magnetic field is

$$B = \int dB$$

The magnetic field is perpendicular to $\vec{\mathbf{r}}$ and to the direction of motion of charges, which is along the current. At a point along the line of a current element such as P_2 and P_3, the magnetic field due to that element is zero, because $d\vec{\mathbf{l}}$ is parallel to the vector $\vec{\mathbf{r}}$.

Concept:

Steady currents– *Stationary charge produce electric fields that are constant in time; hence the term* ***electrostatics.*** *Steady currents produce magnetic fields that are constant in time.*

Stationary charges ⇒ constant electric fields: electrostatics.

Steady currents ⇒ constant magnetic fields: magnetostatics.

Steady current implies a continuous flow that has been going on for very long time, without change and without charge accumulation up anywhere.

A moving point charge cannot possibly constitute a steady current. A moving point charge does not produce a static field in the first place.

When a steady current flows in a wire, its magnitude I must be the same all along the line; otherwise, charge would be accumulating up somewhere, and it wouldn't be a steady current.

The magnetic field of a steady current: *The magnetic field of a steady line current is given by the* ***Biot-Savart law:***

$$B(r) = \frac{\mu_0}{4\pi}\int\frac{I\times\hat{\mathbf{r}}}{r^2}dl' = \frac{\mu_0}{4\pi}I\int\frac{dl'\times\hat{\mathbf{r}}}{r^2}.$$

The magnetic field due to a longer piece of wire is found from the principle of superposition:

Principle of superposition: *The magnetic field produced by an extended current distribution is the vector sum of contributions due to its individual elements:*

$$\vec{\mathbf{B}} = \oint d\vec{\mathbf{B}} = \oint\frac{\mu_0}{4\pi}I\frac{d\vec{\mathbf{l}}\times\hat{\mathbf{r}}}{r^2}$$

Strictly speaking, the integral is always over a closed path (hence $\oint$), since steady current always follows closed path. If it did not, charge would build up at the end of the path.

The simplest static electric field is that of a point charge. But a point charge in motion does not produce a static magnetic field. As the charge approaches a given point the magnetic field increases to a maximum and then dies away as the particle recedes. Static magnetic field is produced only by steady electric current.

1. The source term $I\,d\vec{\mathbf{l}}$ in the Biot-Savart law equals the number of electrons in the element of wire multiplied by their drift velocity $\vec{\mathbf{v}}_d$ and their charge $(-e)$.

2. The magnetic field vector $d\vec{\mathbf{B}}$ vanishes at points along the line of the wire element and has maximum magnitude in the plane perpendicular to the element.

3. The Biot-Savart law describes the magnetic field produced by a small segment of current-carrying wire. The vector $d\vec{\mathbf{l}}$ describes the direction of the wire and the sense of the current in it. The vector $\hat{\mathbf{r}}$ points from the source point toward the field point.

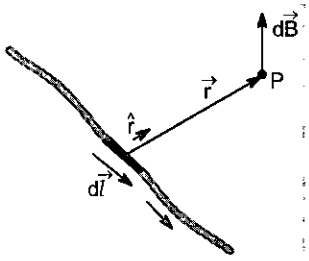


Fig. 4.39

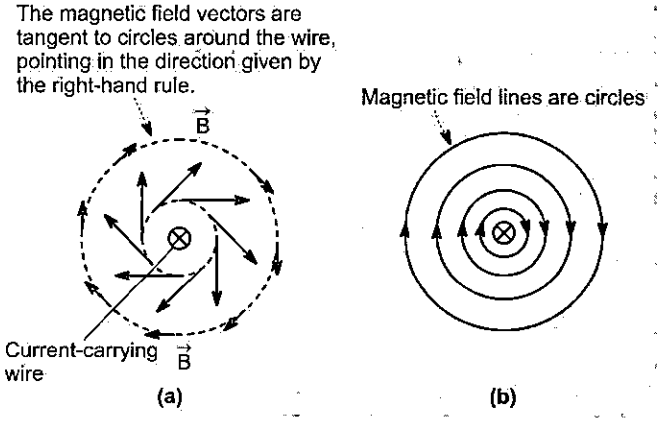
The direction of the magnetic field at P is perpendicular to both $d\vec{\mathbf{l}}$ and $\hat{\mathbf{r}}$, in the direction of their cross product.

DIRECTION OF MAGNETIC FIELD DUE TO CURRENT CARRYING WIRE

Because compass needles align with the magnetic field, the magnetic field at each point must be tangent to a circle around the wire. Fig. 4.40 (a) shows the magnetic field by drawing field vectors.

To picture the field we use **magnetic field lines.** These are imaginary lines drawn through a region of space so that

* A tangent to a field line is in the direction of the magnetic field, and
* The field lines are closer together where the magnetic field strength is larger.



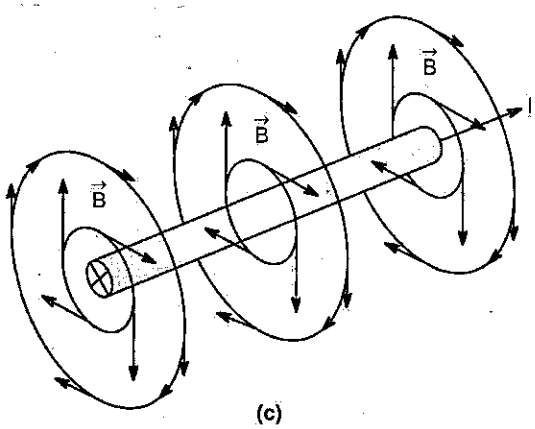


Fig. 4.40

Fig. 4.40 (b) shows the magnetic field lines around a current-carrying wire. Notice that magnetic field lines form loops, with no beginning or ending point. This is in contrast to electric field lines, which stop and start on charges. We see the same circular pattern in the photo of Fig. 4.40 (c).

MAGNETIC FIELD OF MOVING POINT CHARGES

Biot-Savart law gives the magnetic field produced by a current element $I\,dl$. Since $I = dq/dt$, we rewrite

$$I\,dl = \frac{dq}{dt}\,dl = dq\frac{dl}{dt} = dq\,v$$

When a point charge q moves with velocity $\vec{\mathbf{v}}$, it produces a magnetic field $\vec{\mathbf{B}}$ in space given by

$$\vec{\mathbf{B}} = \frac{\mu_0}{4\pi}\frac{q\vec{\mathbf{v}}\times\hat{\mathbf{r}}}{r^2}$$

where $\hat{\mathbf{r}}$ is a unit vector that points from the charge q to the field point P (Fig. 4.41) and μ_0 is a constant of proportionality called the **permeability of free space,** which has the value

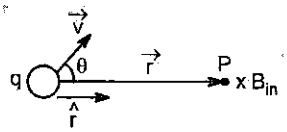


Fig. 4.41

$$\mu_0 = 4\pi\times10^{-7}\ \mathrm{T\cdot m/A}$$
$$= 4\pi\times10^{-7}\ \mathrm{N/A^2}$$

The units of μ_0 are such that B is in tesla when q is in coulomb, v is in metre per second and r is in metre. The unit $\mathrm{N/A^2}$ comes from the fact that $1\,\mathrm{T} = 1\,\mathrm{N/A\text{-}m}$. The constant $1/4\pi$ is arbitrarily included so that the factor 4π will not appear in Ampere's law.

MAGNETIC FORCE AND CONSERVATION OF MOMENTUM

The force $\vec{\mathbf{F}}_{1,2}$ exerted by a charge q_1 moving with velocity v_1 on a charge q_2 moving with velocity v_2 is given by

$$\vec{\mathbf{F}}_{1,2} = q_2\,\vec{\mathbf{v}}_2\times\vec{\mathbf{B}}_1$$

$$= q_2 \vec{v}_2 \times \left(\frac{\mu_0}{4\pi} \frac{q_1 \vec{v}_1 \times \hat{r}_{1,2}}{r_{1,2}^2} \right)$$

where $\vec{B}_1$ is the magnetic field at the position of charge q_2 due to charge q_1, and $\hat{r}_{1,2}$ is the unit vector pointing from q_1 to q_2. Similarly, the force $\vec{F}_{2,1}$ exerted by a charge q_2 moving with velocity v_2 on a charge q_1 moving with velocity v_1 is given by

$$\vec{F}_{2,1} = q_1 \vec{v}_1 \times \vec{B}_2$$

$$= q_1 \vec{v}_1 \times \left(\frac{\mu_0}{4\pi} \frac{q_1 \vec{v}_1 \times \hat{r}_{2,1}}{r_{2,1}^2} \right)$$

The force exerted by charge q_1 on charge q_2 is generally not equal and opposite to that exerted by charge q_2 on charge q_1. That is, these force do not obey Newton's third law. Consider the special case illustrated in Fig. 4.42. Here the magnetic field $\vec{B}_1$ due to charge q_1 at charge q_2 is in the negative z-direction, and the force on q_2 is to the left in the negative x-direction. However, the magnetic field $\vec{B}_2$ due to q_2 at q_1 is zero because q_1 lies along the line of motion of q_2. Thus, there is no magnetic force exerted by q_2 or q_1. There is thus a net force $\vec{F}_{1,2}$ acting on the two-charge system. The system will accelerate in the direction of this force, and linear momentum will not conserved.

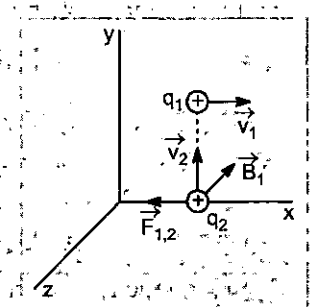


Fig. 4.42

This apparent violation of the law of conservation of linear momentum results from neglecting the momentum carried by the electric and magnetic fields of the moving charges. There is energy associated with an electric field, and there is also energy associated with a magnetic field. Advanced studies of the electric and magnetic fields of moving charges show that there is also momentum associated with these fields. When the charges move, as in Fig. 4.42, the linear momentum produced when the system accelerates to the left is balanced by momentum in the opposite direction carried by the fields. Thus, when we include the momentum of the fields, the total momentum of the system is conserved.

Angular momentum is also carried by the electromagnetic fields produced by moving charges. Consider Fig. 4.43 (a), which shows a point charge q_1, $\vec{R} = x\hat{i} + y\hat{j}$, moving parallel to the x-axis with velocity $\vec{v}_1 = v_1\hat{i}$, and a second point charge q_2 at the origin moving along the x-axis with velocity $\vec{v}_2 = v_2\hat{i}$.

We first find the force on charge q_1. We note that vector $\vec{r}_{2,1}$ from charge q_2 to charge q_1 is just $\vec{R}$. Using $\hat{r}_{2,1} = \vec{R}/R$ we have

$$\frac{\vec{v}_2 \times \hat{r}_{2,1}}{r_{2,1}^2} = \frac{\vec{v}_2 \times \hat{R}}{R^3}$$

$$= \frac{v_2\hat{i} \times (x\hat{i} + y\hat{j})}{R^3} = \frac{yv_2}{R^3}\hat{k}$$

so the magnetic field produced by charge q_2 at the position of charge q_1 is

$$\vec{B}_2 = \frac{\mu_0}{4\pi} \frac{q_2 \vec{v}_2 \times \hat{r}_{2,1}}{r_{2,1}^2}$$

$$= \frac{\mu_0}{4\pi} \frac{q_2 y v_2}{R^3}\hat{k}$$

The magnetic force exerted by charge q_2 on charge q_1 is then

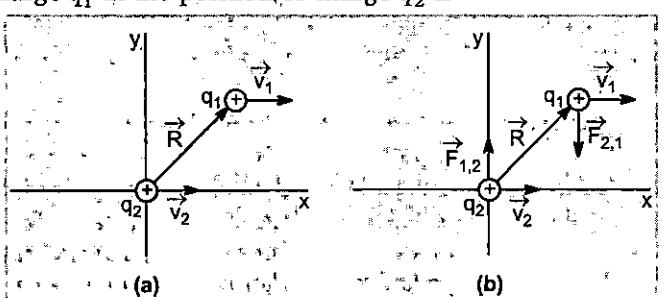
$$\vec{F}_{2,1} = q_1 \vec{v}_1 \times \vec{B}_2$$

$$= q_1 v_1 \hat{i} \times \frac{\mu_0}{4\pi} \frac{q_2 y v_2}{R^3}\hat{k}$$

$$= -\frac{\mu_0}{4\pi} \frac{q_1 q_2 v_1 v_2 y}{R^3}\hat{j}$$

This force is downward and parallel to the y-axis, as shown.

To find the magnetic force exerted by charge q_1 on charge q_2, we note that the vector $\vec{r}_{1,2}$ from charge q_1 to charge q_2 is $-\vec{R}$. Then the magnetic field produced by charge q_1 at the position of charge q_2 is



(a) Two charges moving in parallel directions. (b) The magnetic forces exerted by the charges on each other are equal and opposite, but they are not along the line joining the charges.

Fig. 4.43

$$\vec{B}_1 = \frac{\mu_0}{4\pi} \frac{q_1 \vec{v}_1 \times \hat{r}_{1,2}}{r_{1,2}^2}$$

$$= \frac{\mu_0}{4\pi} \frac{q_1 \vec{v}_1 \times (-\vec{R})}{R^3}$$

$$= -\frac{\mu_0}{4\pi} \frac{q_1 y v_1}{R^3}\hat{k}$$

The magnetic force exerted by charge q_1 on charge q_2 is then

$$\vec{\mathbf{F}}_{1,2} = q_2 \vec{\mathbf{v}}_2 \times \vec{\mathbf{B}}_1$$
$$= q_2 v_2 \hat{\mathbf{i}} \times \left(-\frac{\mu_0}{4\pi} \frac{q_1 y v_1}{R^3} \hat{\mathbf{k}} \right)$$
$$= +\frac{\mu_0}{4\pi} \frac{q_1 q_2 v_1 v_2 y}{R^3} \hat{\mathbf{j}}$$

In this case, the force are equal and opposite, as shown in Fig. 4.43 (b) but they are not along the line joining the two particles. The magnetic forces thus exert a torque on the two particle system. Here the apparent violation of conservation of angular momentum due to the existence of this torque is resolved by the consideration of the angular momentum carried by the electromagnetic field.

Concept: *Magnetic Forces and Newton's Law*–*Fig. 4.44 (a) shows two separate elements of wire and the magnetic forces they exert on each other. The forces are not equal and opposite; the magnetic field of element 1 exerts an upward force on element 2, but element 2 exerts no force at all on element 1! The magnetic field due to element 2 is zero at the position of element 1. The mutual magnetic forces on two point charges in motion display the same behaviour Fig. 4.44 (b).*

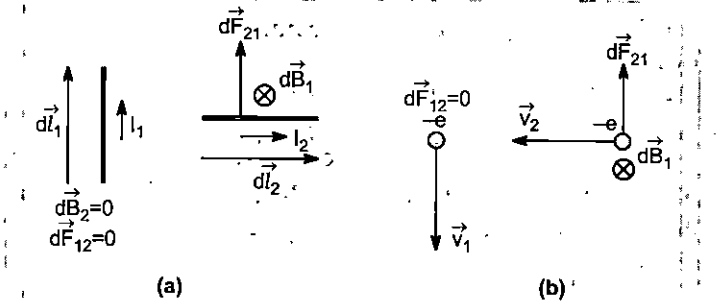


Fig. 4.44

What has happened to Newton's third law ? A proper analysis of momentum conservation for a system of moving charges must include the electric and magnetic fields, which extend over all space. Electric field stores energy. The electromagnetic field can also store momentum. The apparent failure of Newton's third law tells us that momentum is being exchanged between particles and fields in the system.

Momentum of the entire system of particles and field is conserved. The forces between two finite, closed current loops do obey Newton's third law, since the net momentum exchanged with the fields is zero in that case.

Example 39

Point charges Q and Q' are constrained to move along the x-and y-axes, respectively, with the same uniform speed v. At time t = 0 both charges are at the origin. At time t calculate the Lorentz force $\vec{\mathbf{F}}$ on Q' due to the magnetic field of Q.

Solution: The expression for the field $\vec{\mathbf{B}}$ due to a charge Q moving at velocity $\vec{\mathbf{v}}$ is:

$$\vec{\mathbf{B}} = \frac{\mu_0 Q}{4\pi} \frac{\vec{\mathbf{v}} \times r}{r^3}$$

Since both charges start at the origin at $t = 0$, we may write the position $\vec{\mathbf{r}}_1$ of the charge Q as $\vec{\mathbf{v}}_1 t$, and the position $\vec{\mathbf{r}}_2$ of the charge Q' as $\vec{\mathbf{v}}_2 t$. The vector displacement $\vec{\mathbf{r}}$ of Q' w.r.t. Q is then $(\vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1)\, t$, and the magnetic field at Q' due to Q is thus

$$\frac{\mu_0 Q}{4\pi} \frac{\vec{\mathbf{v}}_1 \times (\vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1)}{t^2 |\vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1|^3}.$$

The force on a charge Q' moving at velocity $\vec{\mathbf{v}}_2$ in a magnetic field $\vec{\mathbf{B}}$ is $Q' \vec{\mathbf{v}}_2 \times \vec{\mathbf{B}}$, so the force $\vec{\mathbf{F}}$ acting on Q' is

$$\vec{\mathbf{F}} = \frac{\mu_0 Q Q'}{4\pi t^2} \frac{v_2 \times [\vec{\mathbf{v}}_1 \times (\vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1)]}{|\vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1|^3}$$

Since $\vec{\mathbf{v}}_1 \times \vec{\mathbf{v}}_1 = 0$, this can be simplified to

$$\vec{\mathbf{F}} = \frac{\mu_0 Q Q'}{4\pi t^2} \frac{\vec{\mathbf{v}}_2 \times (\vec{\mathbf{v}}_1 \times \vec{\mathbf{v}}_2)}{|\vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1|^3}$$

Taking $\vec{\mathbf{v}}_1 = (v, 0, 0)$,

and $\vec{\mathbf{v}}_2 = (0, v, 0)$ gives

$$\vec{\mathbf{v}}_1 \times \vec{\mathbf{v}}_2 = (0, 0, v^2),$$
$$\vec{\mathbf{v}}_2 \times (\vec{\mathbf{v}}_1 \times \vec{\mathbf{v}}_2) = (v^3, 0, 0)$$

and $|\vec{\mathbf{v}}_1 - \vec{\mathbf{v}}_2|^3 = 2^{3/2} v^3$.

The magnetic force is thus,

$$\frac{\mu_0 Q Q'}{8\sqrt{2}\pi t^2}$$ parallel to the x-axis.

MAGNETIC FIELD LINES

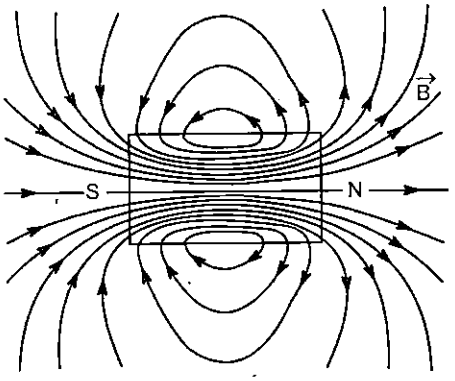
Just as the electric field $\vec{\mathbf{E}}$ can be represented by electric field lines, the magnetic field $\vec{\mathbf{B}}$ can be represented by **magnetic field lines.** In both cases, the direction of the field is indicated by the direction of the field lines and the magnitude of the field is indicated by their density. There are, however, two important differences between electric field lines and magnetic field lines:

1. Electric field lines are in the direction of the electric force on a positive charge, but the magnetic field lines are perpendicular to the magnetic force on a moving charge.

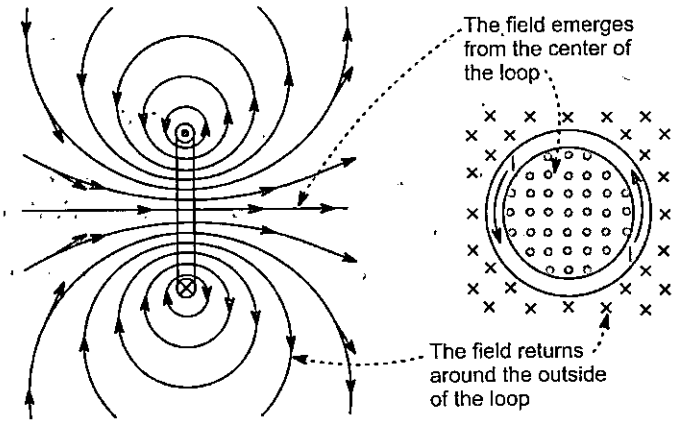
2. Electric field lines begin on positive charges and end on negative charges; magnetic field lines form closed loops. Since isolated magnetic poles apparently do not exist; there

are no points in space where magnetic field lines begin or end.

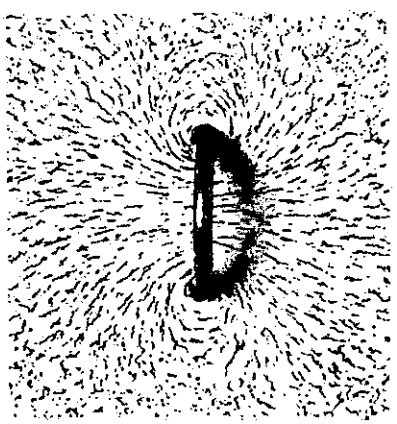
Fig. 4.45 shows the magnetic field lines both inside and outside a bar magnet.



(a)



(b) Cross-section through the current loop (c) The current loop seen from the right



(d) A photo of iron fillings

Fig. 4.45: The magnetic field of a current loop

Example 40

A circular wire loop of radius R carries a current I. (a) Use the Biot-Savart law to find the magnetic field at the centre of the loop. (b) Find the field at a point on the axis a distance x from its centre. (c) Show that the field on the axis for large distance reduces to $B_x = \frac{\mu_0}{4\pi}\frac{\mu m}{|x^3|}$, *where* $\mu = I(\pi R^2)$ *is the magnitude of the magnetic moment of the loop.*

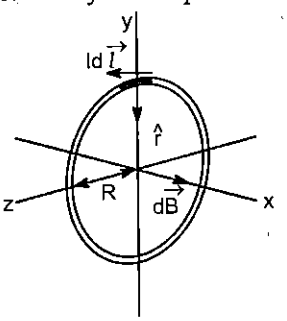


Fig. 4E.40 (a)

Solution: (a) We take a current element $Id\vec{l}$ and the unit vector $\hat{\mathbf{r}}$ directed from the element to the centre of the loop. The magnetic field due to this element is directed along the axis of the ring; the magnitude is

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2}$$

Where θ is the angle between $I\,d\vec{l}$ and $\hat{\mathbf{r}}$, which is 90° in this case. The magnitude due to the entire current is found by integrating over all the current elements in the loop.For each element the field is directed along the *x*-axis.

$$B = \int dB = \frac{\mu_0 I}{4\pi R^2}\oint dt$$

$$= \frac{\mu_0 I}{4\pi R^2}(2\pi R) = \frac{\mu_0 I}{2R}$$

If we consider an arc of the coil that subtends an angle θ at the centre, the contribution to total magnetic field due to this element is

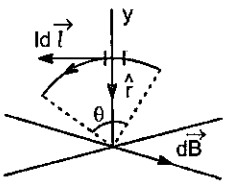
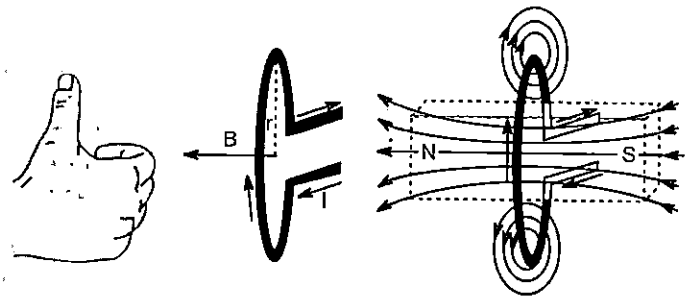


Fig. 4E.40 (b)

$$B = \frac{\mu_0 I}{4\pi R^2}\oint dl$$

$$= \frac{\mu_0 I}{4\pi R^2}(R\theta)$$

$$= \frac{\mu_0 I}{2R}\left(\frac{\theta}{2\pi}\right)$$



(c) (d)

Fig. 4E.40

Fig. 4E.40(c) shows magnetic field at the centre of a circular current-carrying loop. The magnetic field is perpendicular to the plane of the loop. The direction of the field is given by the right-hand thumb rule. With the thumb of the right hand in the direction of the conventional current, the curled fingers indicate the direction of $\vec{\mathbf{B}}$. The overall magnetic field of a current-carrying circular loop is similar to that of the bar magnet.

(b) We consider the current element at the top of the loop as shown in Fig. 4E.40 (e). The magnitude of $d\vec{\mathbf{B}}$ is

$$|d\vec{\mathbf{B}}| = \frac{\mu_0}{4\pi}\frac{I(d\vec{\mathbf{l}}\times\hat{\mathbf{r}})}{r^2}$$

$$= \frac{\mu_0}{4\pi}\frac{I\,dl}{x^2+R^2}$$

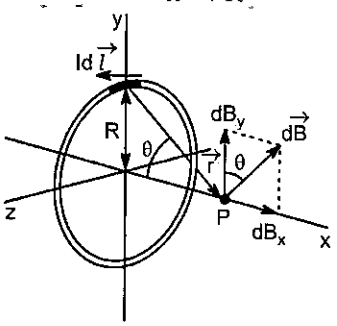


Fig. 4E.40 (e)

where $d\vec{\mathbf{l}}\times\hat{\mathbf{r}} = dl$, as $d\vec{\mathbf{l}}$ and vector $\hat{\mathbf{r}}$ are perpendicular and $r^2 = x^2 + R^2$. We make use of symmetry, the components of $d\vec{\mathbf{B}}$ perpendicular to the axis of the loop, such as dB_y cancel out, leaving only the components parallel to x-axis.

$$dB_x = dB\sin\theta = dB\left(\frac{R}{\sqrt{x^2+R^2}}\right)$$

$$= \frac{\mu_0}{4\pi}\frac{I\,dl\,R}{(x^2+R^2)^{3/2}}$$

$$B_x = \oint dB_x = \oint \frac{\mu_0}{4\pi}\frac{IR}{(x^2+R^2)^{3/2}}dl$$

$$= \frac{\mu_0 IR}{4\pi(x^2+R^2)^{3/2}}\oint dl$$

$$= \frac{\mu_0 IR(2\pi R)}{4\pi(x^2+R^2)^{3/2}}$$

$$= \frac{\mu_0 IR^2}{2(x^2+R^3)^{3/2}}$$

(c) At points far from the loop, when $x \gg R$

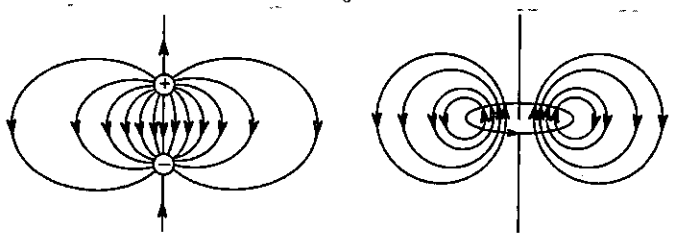
$$(x^2+R^2)^{3/2} \approx x^3$$

and expression for magnetic field reduces to

$$B_x \approx \frac{\mu_0}{4\pi}\frac{2\mu}{x^3}$$

Equation has the same form as for the "far field" on the axis of an electric dipole:

$$E = \frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3}$$



(f) (g)

Fig. 4E.40

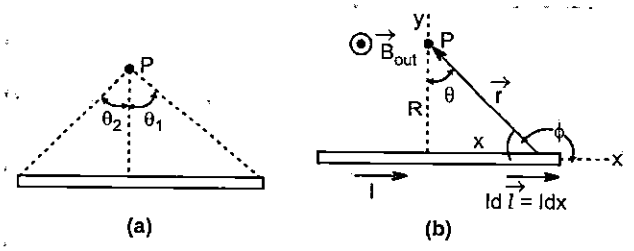
This similarity in form of the far field expressions prompts us to refer to the current loop as a magnetic dipole. However, as Fig. 4E.40(g) shows, the magnetic field pattern near the loop is quite different from that of the electric field due to a dipole. In fact, the fields are in opposite directions.

Example 41

Find the magnetic field $\vec{\mathbf{B}}$ at a point P due to current in the straight wire segment shown in Fig. 4E.41 (a).

Solution: We take a current element $I\,d\vec{\mathbf{l}}$ at a distance x from origin; the vector $\vec{\mathbf{r}}$ points from the element to the field point P. The direction of $I\,d\vec{\mathbf{l}}\times\vec{\mathbf{r}}$ can be obtained from the right-hand thumb rule; it is out of the paper normal to plane of $d\vec{\mathbf{l}}$ vector and $\vec{\mathbf{r}}$ vector; field due to all the elements is in the same direction. Field due to this element,

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dx}{r^2}\sin\phi$$



(a) (b)

Fig. 4E.41

$$= \frac{\mu_0}{4\pi}\frac{Idx}{r^2}\cos\theta$$

We have three variables, r, x and θ which are related as

$$x = R\tan\theta$$

$$dx = R\sec^2\theta\, d\theta = R\frac{r^2}{R^2}d\theta = \frac{r^2}{R}d\theta \text{ (As } \sec\theta = r/R),$$

$$dB = \frac{\mu_0}{4\pi}\frac{I}{r^2}\frac{r^2 d\theta}{R}\cos\theta = \frac{\mu_0}{4\pi}\frac{I}{R}\cos\theta\, d\theta$$

The contribution due to the current elements is to the right of the point $x = 0$; we integrate from $\theta = 0$ to $\theta = \theta_1$, where θ_1 is the angle between the line perpendicular to the wire and the line from P to the right end of the wire.

$$B_1 = \frac{\mu_0 I}{4\pi R}\int_0^{\theta_1}\cos\theta\, d\theta$$

$$= \frac{\mu_0 I}{4\pi R}\sin\theta_1$$

Similarly the contribution from the elements to the left of the point $x = 0$ is

$$B_2 = \frac{\mu_0}{4\pi}\frac{I}{R}\sin\theta_2$$

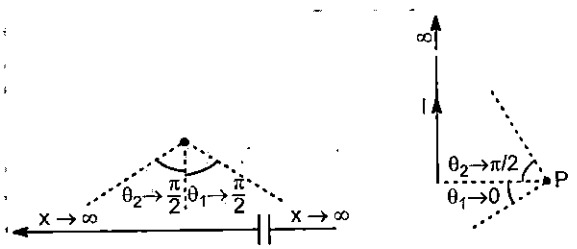
Total magnetic field,

$$B = B_1 + B_2$$

$$= \frac{\mu_0 I}{4\pi R}(\sin\theta_1 + \sin\theta_2)$$

* **Field of an infinite wire:** In this case, $\theta_1 = \theta_2 = \pi/2$; as the length of the wire increase the angles θ_1 and θ_2 approach 90°.

$$B = \frac{\mu_0 I}{2\pi R}$$



(c) (d)

Fig. 4E.41

* **Field at the end of an infinite wire:** In this case $\theta_1 = 0$ and $\theta_2 = \pi/2$.

$$B = \frac{\mu_0 I}{4\pi R}$$

* **Field at the perpendicular bisector of a wire:** In this case $\theta_1 = \theta_2$ and $\sin\theta_1 = \sin\theta_2$

$$= \frac{L/2}{\sqrt{L^2/4 + R^2}} = \frac{L}{\sqrt{L^2 + 4R^2}}$$

$$B = \frac{\mu_0 I}{4\pi R}\frac{2L}{\sqrt{L^2 + 4R^2}}$$

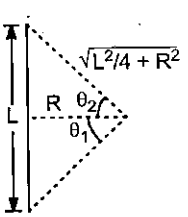
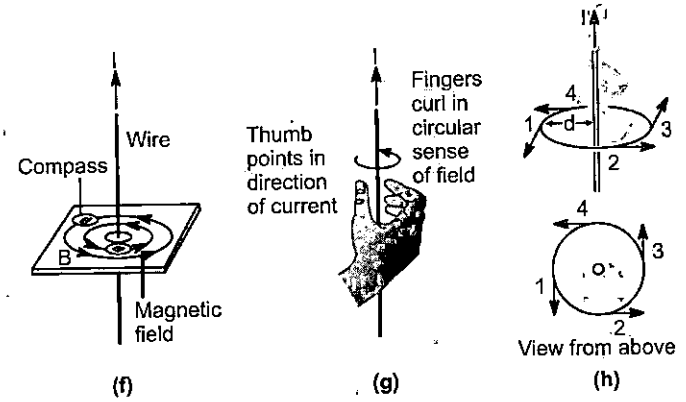


Fig. 4E.41 (e)



(f) (g) (h)

Fig. 4E.41: The field lines form concentric circles around the wire. The circular sense of the field lines is given by the right-hand thumb rule and the magnetic field vector is tangent to the circular field lines at any point.

Example 42

A square loop of wire, edge length a, carries a current I. Compute the magnitude of the magnetic field produced at a point on the axis of the loop at a distance x from the centre.

Solution: A point on the axis of the loop is on the perpendicular bisector of each of the loop sides.

The figure shows field of a single wire.

$$B = \frac{\mu_0 Ia}{\pi\sqrt{4x^2 + a^2}\sqrt{4x^2 + 2a^2}}$$

When the fields of all the four sides are considered, the horizontal components add to zero; so the total field is given by

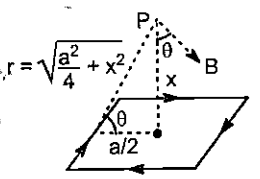


Fig. 4E.42

$$B_R = 4B\cos\theta = \frac{4Ba}{2r}$$

$$= \frac{4Ba}{\sqrt{4x^2 + a^2}}$$

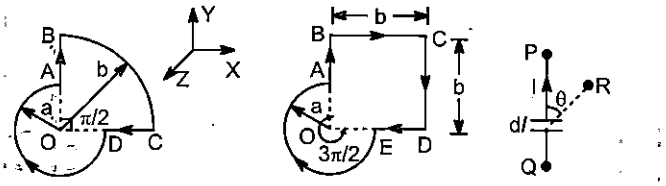
$$= \frac{4\mu_0 Ia^2}{\pi(4x^2 + a^2)\sqrt{4x^2 + 2a^2}}$$

For $x = 0$ the expression reduces to

$$B_R = \frac{4\mu_0 Ia^2}{\pi a^2\sqrt{2a}} = \frac{2\sqrt{2}\,\mu_0 I}{\pi a}$$

Example 43

Find the magnetic induction of the currents coils shown in Fig. 4E.43(a), (b) and (c), at point O, the centre of circular sections.



(a) (b) (c)

Fig. 4E.43

Solution: (a) We require two basic formulae to determine magnetic induction.

Magnetic induction of a finite length wire

$$= \frac{\mu_0 I}{4\pi d}(\sin\alpha + \sin\beta)$$

Magnetic induction at the centre of a sector of a circular coil

$$B = \frac{\mu_0 I}{2R} \times \frac{\alpha}{2\pi}$$

where α is the angle subtended by the sector at the centre.

According to Biot-Savart law, the magnetic induction at R due to current element dl is

$$dB = \frac{\mu_0 I\, dl \sin\theta}{4\pi r^2}$$

Magnetic induction at points P and Q is zero because for these points $\theta = 0$ and π respectively and $\sin 0 = \sin\pi = 0$

Thus the magnetic induction due to segments ab and cd is zero. Magnetic induction due to sector BC, that subtends an angle $\pi/2$ is

$$B_1 = \frac{\mu_0 I}{2b} \times \frac{\pi/2}{2\pi} = \frac{\mu_0 I}{8b}(-\hat{\mathbf{k}})$$

It points inward from right-hand thumb rule.

Magnetic induction due to segment DA, that subtends an angle $3\pi/2$ at the centre, is

$$B_2 = \frac{\mu_0 I}{2a} \times \frac{3\pi/2}{2\pi} = \frac{3\mu_0 I}{8a}(-\hat{\mathbf{k}})$$

It points inward from right-hand thumb rule.

Resultant magnetic induction,

$$\vec{\mathbf{B}}_R = \vec{\mathbf{B}}_1 + \vec{\mathbf{B}}_2 = \frac{\mu_0 i}{8}\left[\frac{1}{b} + \frac{3}{a}\right](-\hat{\mathbf{k}})$$

(b) Magnetic induction due to segments AB and ED is zero. Magnetic induction due to segment BC,

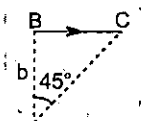


Fig. 4E.43 (d)

$$B_1 = \frac{\mu_0 I}{4\pi b}(\sin 45° + \sin 0°)(-\hat{\mathbf{k}})$$

Magnetic induction due to segment CD,

$$B_2 = \frac{\mu_0 I}{4\pi b}(\sin 45° + \sin 0°)$$

$$= \frac{\mu_0 I}{4\sqrt{2}\pi b}(-\hat{\mathbf{k}})$$

Magnetic induction due to circular segment BC,

$$B_3 = \frac{\mu_0 I}{2a} \times \frac{(3\pi/2)}{2\pi}(-\hat{\mathbf{k}})$$

$$= \frac{3\mu_0 I}{8a}(-\hat{\mathbf{k}})$$

Resultant magnetic induction,

$$\vec{\mathbf{B}}_R = \vec{\mathbf{B}}_1 + \vec{\mathbf{B}}_2 + \vec{\mathbf{B}}_3$$

$$\vec{\mathbf{B}}_R = \frac{\mu_0 I}{4\pi}\left[\frac{3\pi}{2a} + \frac{\sqrt{2}}{b}\right](-\hat{\mathbf{k}})$$

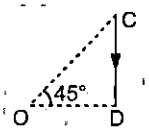
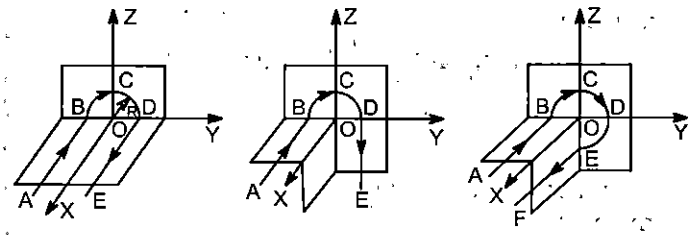


Fig. 4E.43(e)

Example 44

A long current wire is bent in the shapes shown in Fig. 4E.44 (a), (b) and (c). The circular portion has radius R. Find the magnetic induction at the center of the circular segment.



(a) (b) (c)

Fig. 4E.44

Solution: (a) Fig. 4E.44 (d) shows magnetic field lines of a long wire. Note the direction of magnetic induction at points P, Q, R and S. From right-hand thumb rule the magnetic fields at points P, Q, R and S lie along the unit vectors $(\hat{\mathbf{j}})$, $(-\hat{\mathbf{k}})$, $(-\hat{\mathbf{j}})$ and $(\hat{\mathbf{k}})$ respectively.

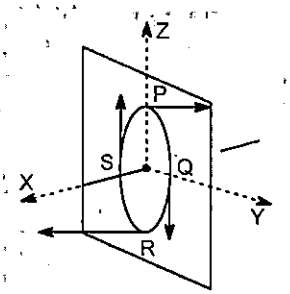


Fig. 4E.44 (d)

Magnetic field due to segment, AB,

$$\vec{\mathbf{B}}_1 = \frac{\mu_0 I}{4\pi R}(-\hat{\mathbf{k}})$$

Magnetic field due to circular segment BCD,

$$\vec{\mathbf{B}}_2 = \frac{\mu_0 I}{2R} \times \left(\frac{\pi}{2\pi}\right)(-\hat{\mathbf{i}})$$

Magnetic field due to segment DE,

$$\vec{\mathbf{B}}_3 = \frac{\mu_0 I}{4\pi R}(-\hat{\mathbf{k}})$$

Resultant magnetic field,

$$\vec{\mathbf{B}}_R = \vec{\mathbf{B}}_1 + \vec{\mathbf{B}}_2 + \vec{\mathbf{B}}_3$$

$$= -\frac{\mu_0 I}{4\pi R}[2(\hat{\mathbf{k}}) + \pi(\hat{\mathbf{i}})]$$

(b) Magnetic field due to segment AB,

$$\vec{\mathbf{B}}_1 = \frac{\mu_0 I}{4\pi R}(-\hat{\mathbf{k}})$$

Magnetic field due to circular segment BCD,

$$\vec{B}_2 = \frac{\mu_0 I}{2R}\left(\frac{\pi}{2\pi}\right)(-\hat{i})$$

Magnetic field due to segment *DE*,

$$\vec{B}_3 = \frac{\mu_0 I}{4\pi r}(-\hat{k})$$

Resultant magnetic field,

$$\vec{B}_R = \vec{B}_1 + \vec{B}_2 + \vec{B}_3$$

$$= -\frac{\mu_0 I}{4\pi R}[\hat{k} + (\pi+1)\hat{i}]$$

(c) Magnetic field due to segment *AB*,

$$\vec{B}_1 = \frac{\mu_0 I}{4\pi R}(-\hat{k})$$

Magnetic field due to circular segment *BCDE*,

$$\vec{B}_2 = \frac{\mu_0 I}{2R} \times \left(\frac{3\pi/2}{2\pi}\right)(-\hat{i})$$

Magnetic field due to segment *EF*,

$$\vec{B}_3 = \frac{\mu_0 I}{4\pi R} \times (-\hat{j})$$

Resultant magnetic field,

$$\vec{B}_R = \vec{B}_1 + \vec{B}_2 + \vec{B}_3$$

$$= -\frac{\mu_0 I}{4\pi R}\left[\hat{k} + \hat{j} + \frac{3\pi}{2}\hat{i}\right]$$

Example 45

A battery is connected between two points A and B on the circumference of a uniform conducting ring of radius r and resistance R as shown in Fig. 4E.45. One of the arc AB of the ring subtends angle θ at the centre. Show that the magnetic field at the centre of the coil is zero and independent of θ.

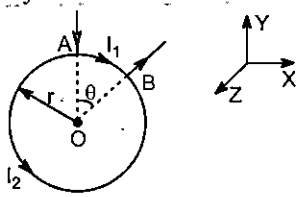


Fig. 4E.45

Solution: Magnetic field at the centre of an arc is given by

$$B = \frac{\mu_0 I}{2R} \times \frac{\theta}{2\pi}$$

Magnetic field due to a smaller arc,

$$\vec{B}_1 = \frac{\mu_0 I_1}{2r} \times \frac{\theta}{2\pi}(-\hat{k})$$

Magnetic field due to larger arc,

$$\vec{B}_2 = \frac{\mu_0 I_2}{2r} \times \frac{(2\pi-\theta)}{2\pi}(+\hat{k})$$

Resultant magnetic field

$$= \left[-\frac{\mu_0 I_1 \theta}{4\pi r} + \frac{\mu_0 I_2 (2\pi-\theta)}{4\pi r}\right](\hat{k}) \quad ...(1)$$

Two arcs form a parallel combination of resistors.

Thus $I_1 R_1 = I_2 R_2$, where R_1 and R_2 are resistances of respective segments.

As the wire is uniform,

$$\frac{R_1}{R_2} = \frac{L_1}{L_2} = \frac{R\theta}{R(2\pi-\theta)}$$

Thus $$\frac{I_1}{I_2} = \frac{\theta}{(2\pi-\theta)}, \quad I_2 = \frac{I_1\theta}{(2\pi-\theta)} \quad ...(2)$$

On substituting I_2 in eqn. (1), we get

$$\vec{B}_R = \left(\frac{\mu_0 I\theta}{4\pi r} - \frac{\mu_0 I\theta}{4\pi r}\right)\hat{k}$$

$$= 0$$

Hence the field is independent of θ.

MAGNETIC FIELD DUE TO REGULAR POLYGON AT ITS CENTRE

To calculate the magnetic induction at the centre of a polygon with *n*-sides which can be inscribed into a circle of radius '*r*'.

From the figure,

$$x = r\cos(\pi/n)$$

Magnetic field at the centre due to the one current carrying side (wire),

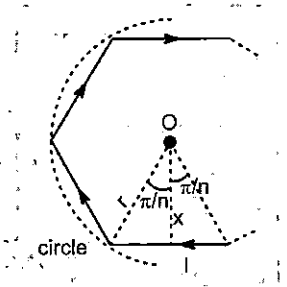


Fig. 4.46

$$B = \frac{\mu_0}{4\pi}\frac{I}{x}\left(\sin\frac{\pi}{n} + \sin\frac{\pi}{n}\right)$$

$$B = \frac{\mu_0}{4\pi}\frac{2I\sin(\pi/n)}{r\cos(\pi/n)}$$

$$= \frac{\mu_0}{2\pi}\frac{I}{r}\tan\frac{\pi}{n}$$

Since the direction of magnetic field due to all the sides is same (normal to the plane and inward), therefore the net magnetic field at the centre due to entire polygon,

$$B_{net} = nB$$

$$B_{net} = \frac{\mu_0}{2\pi}\frac{nI}{r}\tan\frac{\pi}{n}$$

Magnetic Field due to a Circular Coil at the Centre (with battery connected at any two points of coil)

The net magnetic field at the centre due to the two arcs *ABC* and *ADC* is,

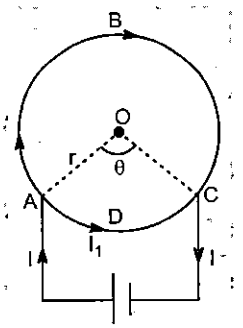


Fig. 4.47

$$B = B_1 - B_2$$
$$= \frac{\mu_0}{4\pi}\frac{I_1}{r} \times \theta - \frac{\mu_0}{4\pi}\frac{(I - I_1)}{r} \times (2\pi - \theta)$$
$$= \frac{\mu_0}{4\pi}\left[\frac{I_1}{r}\theta - \frac{I - I_1}{r}(2\pi - \theta)\right] \quad ...(1)$$

Since the potential difference across both the segment is same, therefore,

$$I_1 R_1 = (I - I_1)R_2$$

Where R_1 and R_2 are the resistances of the two segments. If l_1 and l_2 are the lengths of the two segments, then,

$$R_1 \propto l_1$$
$$R_2 \propto l_2 \quad \text{(Since wire is uniform)}$$
$$\frac{R_1}{R_2} = \frac{l_1}{l_2}$$
$$\frac{I - I_1}{I_1} = \frac{l_1}{l_2}$$
$$I_1 l_1 = (I - I_1)l_2$$
$$I_1 r\theta = (I - I_1)r(2\pi - \theta)$$

Putting in eqn. (1) we get,

$$B = 0$$

Magnetic Induction at the centre of a Spiral

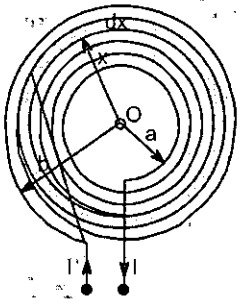


Fig. 4.48

Total number of turns equals N, the number of turns per unit length,

$$n = \frac{N}{(b-a)}$$

To calculate the magnetic induction at the centre O, let us consider an elemental ring of radius 'x' and thickness 'dx'.

Magnetic induction at the cenre due to the ring element,

$$dB = \frac{\mu_0 I}{2x}(n\,dx)$$

where '$n\,dx$' represents the number of turns in the thickness 'dx',

$$dB = \frac{\mu_0 NI}{2(b-a)}\frac{1}{x}dx$$

Hence the net magnetic field at the centre,

$$B = \int dB$$
$$= \frac{\mu_0 NI}{2(b-a)}\ln\left(\frac{b}{a}\right)$$

Magnetic Field due to a Long Straight Wire with Cross-section in the form of a Thin Half Ring

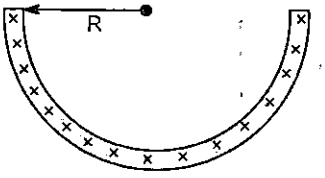


Fig. 4.49

Let us consider an element of thickness 'dx' in the form of a wire at an angle θ as shown in the figure.

The current flowing through the elemental wire along the length of the wire,

$$dI = \frac{I}{\pi R}dx = \frac{I}{\pi R}Rd\theta$$
$$= \frac{I}{\pi}d\theta$$

The net magnetic field at the centre can be obtained as,

$$B = \int dB\cos\theta$$

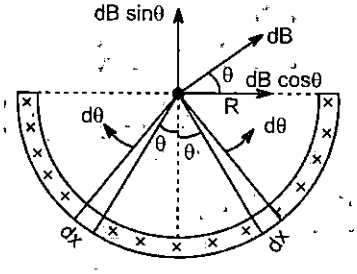


Fig. 4.50

(As the component $dB\sin\theta$ is cancelled by the corresponding component of the opposite element)

$$= \int_{-\pi/2}^{+\pi/2}\frac{\mu_0}{2\pi}\frac{dI}{R}\cos\theta$$
$$= \frac{\mu_0}{2\pi R}\int_{-\pi/2}^{+\pi/2}\cos\theta\frac{I}{\pi}d\theta$$
$$= \frac{\mu_0}{2\pi^2}\frac{I}{R}\int_{-\pi/2}^{+\pi/2}\cos\theta d\theta$$
$$B = \frac{\mu_0}{\pi^2}\frac{I}{R}$$

Magnetic Field Due to a Moving Charge

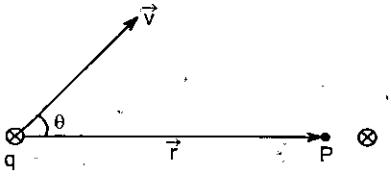


Fig. 4.51

A static charge produce electric field only, while a moving charge produces the electric as well as the magnetic

field. Magnetic field at a point produced by a moving charge is given by,

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q(\vec{v}\times\vec{r})}{r^3}$$

or $$B = \frac{\mu_0}{4\pi}\frac{qv\sin\theta}{r^2}$$

The direction of the magnetic field will be in the direction of $(\vec{v}\times\vec{r})$ if the charge 'q' is positive and opposite to $(\vec{v}\times\vec{r})$ if the charge 'q' is negative.

Electric field produced at the same point, at the same instant by the moving charge is given by,

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

Therefore $$\frac{B}{E} = \mu_0\varepsilon v\sin\theta$$

but $$\frac{1}{\sqrt{\mu_0\varepsilon_0}} = c$$

or $$\mu_0\varepsilon_0 = \frac{1}{c^2}$$

where c is the velocity of light in vacuum,

Hence $$\frac{B}{E} = \frac{v\sin\theta}{c^2}$$

Concept: *Electric field and magnetic field produced by moving charge at any instant are always perpendicular to each other.*

Magnetic Field Due to a Circulating Charge

Magnetic field at the centre O produced by a circulating point charge, is given by,

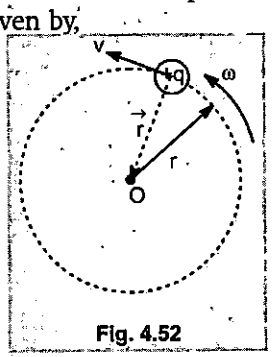


Fig. 4.52

$$\vec{B}_O = \frac{\mu_0}{4\pi}\frac{q(\vec{v}\times\vec{r})}{r^3}$$

$$B_O = \frac{\mu_0}{4\pi}\frac{qvr\sin 90^\circ}{r^3}$$

$$B_O = \frac{\mu_0}{4\pi}\frac{qv}{r^2} \quad \text{or} \quad B_O = \frac{\mu_0}{4\pi}\frac{q\omega}{r}$$

Magnetic Field on the Axis of a Spinning Charged Ring

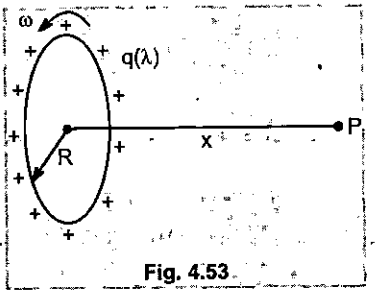


Fig. 4.53

The moving charge caused by the rotation of the ring constitutes the current and is given by,

$$I = fq$$

$$= \frac{\omega}{2\pi}q$$

If the linear charge density of the ring is λ then,

$$q = (2\pi R)\lambda$$

Hence $$I = \frac{\omega}{2\pi}2\pi R\lambda$$

$$= \omega R\lambda$$

$$B = \frac{\mu_0}{2}\frac{IR^2}{(R^2+x^2)^{3/2}}$$

$$= \frac{\mu_0}{2}\frac{\omega R\lambda R^2}{(R^2+x^2)^{3/2}}$$

$$B = \frac{\mu_0\omega\lambda}{2}\frac{R^3}{(R^2+x^2)^{3/2}}$$

For centre of the ring, $x = 0$,

therefore $$B = \frac{\mu_0\omega\lambda}{2}$$

Magnetic Field on the Axis of a Spinning Charged Non-conducting thin Disc

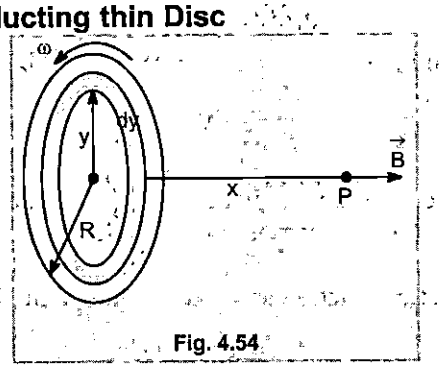


Fig. 4.54

If the disc is assumed to be divided into large number of concentric rings. The ring shaped portion of the disc of radius 'y', and radial thickness 'dy' thus constitutes a current ring of current,

$$di = (2\pi y dy\sigma)\frac{\omega}{2\pi}$$

$$= \sigma\omega\, ydy \qquad (\text{as } I = fq)$$

The magnetic field at the axial point P due to this ring,

$$dB = \frac{\mu_0}{2}\frac{dIy^2}{(y^2+x^2)^{3/2}}$$

$$dB = \frac{\mu_0}{2}\frac{(\sigma\omega\, ydy)y^2}{(y^2+x^2)^{3/2}}$$

$$= \frac{\mu_0\sigma\omega}{2}\frac{y^3}{(y^2+x^2)^{3/2}}dy$$

Therefore the net magnetic field,

$$B = \int_0^R dB$$

$$= \frac{\mu_0\sigma\omega}{2}\int_0^R \frac{y^3}{(y^2+x^2)^{3/2}}dy$$

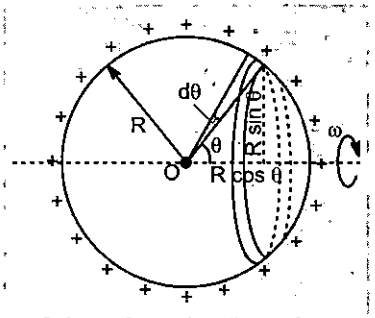
$$B = \frac{\mu_0\sigma\omega}{2}\left[\frac{(R^2+2x^2)}{\sqrt{(R^2+x^2)}} - 2(x)\right]$$

For the centre, $x = 0$ therefore,

$$B_{centre} = \frac{\mu_0\sigma\omega R}{2}$$

Magnetic Field on the Axis of a Spinning Non-conducting Sphere Charged on the Surface with Density σ

Let us consider an elemental ring at an angle θ, the radius of the ring is $R\sin\theta$ and its thickness is $R\,d\theta$.



Sphere charged on the surface

Fig. 4.55

The current constituted due to this ring element,

$$dI = \frac{\omega}{2\pi}(2\pi R\sin\theta)Rd\theta\sigma$$

$$= \omega R^2\sigma\sin\theta\, d\theta \qquad (\text{Since } I = fq)$$

Hence magnetic field at the centre due to the ring element,

$$dB = \frac{\mu_0}{2}\frac{dI(R\sin\theta)^2}{[(R\sin\theta)^2+(R\cos\theta)^2]^{3/2}}$$

$$= \frac{\mu_0}{2}\frac{\omega R^2\sigma\sin\theta\, d\theta R^2\sin^2\theta}{R^3}$$

$$= \frac{\mu_0\omega\sigma R}{2}\sin^3\theta d\theta$$

Therefore the net magnetic field at the centre,

$$B = 2\int_0^{\pi/2}\frac{\mu_0\omega\sigma R}{2}\sin^2\theta\, d\theta$$

$$B = \frac{2\mu_0\omega\sigma R}{3}$$

Example 46

Two straight infinitely long and thin parallel wires are spaced 0.1 m apart and carry a current of 10 ampere each. Find the magnetic field at a point distant 0.1 m from both wires in the two cases when the current are in the (a) same and (b) opposite directions.

Solution: The magnetic lines of force circulate around a long wire. Sense of a magnetic field is determined from the right-hand thumb rule. Magnetic field at point P is shown in the figure; it is tangential to the field line at that point.

Point P is equidistant from the wires; hence

$$B_1 = B_2 = B$$

$$= \frac{\mu_0 I}{2\pi d}$$

$$= \frac{10^{-7}\times 2\times 10}{0.1}$$

$$= 2\times 10^{-5}\ \text{T}$$

The resultant magnetic field,

$$B_R = 2B\cos 30°(-\hat{\mathbf{i}})$$

$$= (2\sqrt{3}\times 10^{-5}\ \text{T})(-\hat{\mathbf{i}})$$

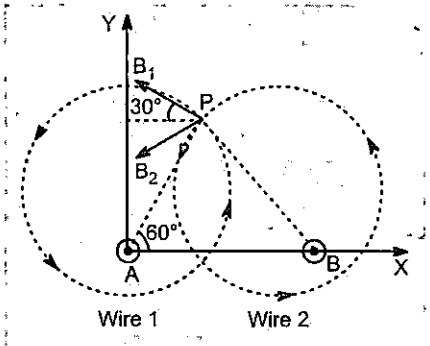


Fig. 4E.46 (a)

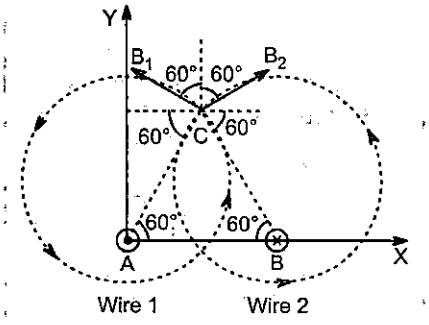


Fig. 4E.46 (b)

(b) magnetic field of wire 1 is normal to AC in anticlockwise sense, and of wire 2 is normal to BC in clockwise sense.

The resultant magnetic field,

$$\vec{B}_R = 2B\cos 60^\circ\,(\hat{j})$$
$$= (2\times 10^{-5}\ \text{T})(\hat{j})$$

Example 47

Two long parallel wires carrying currents 2.5 ampere and 1 ampere in the same direction (directed into the plane of the paper) are held at P and Q respectively such that they are perpendicular to the plane of the paper. The points P and Q are located at a distance of 5 m and 2 m respectively from a collinear point R.

(a) An electron moving with velocity of 4×10^5 m/s along the positive x-direction experiences a force of magnitude 3.2×10^{-20} N at the point R. Find the value of I.

(b) Find all the positions at which a third long parallel wire carrying a current of magnitude 2.5 ampere may be placed so that the magnetic induction at R is zero.

Solution: The magnetic field of both the wires P and Q is along negative y-axis.

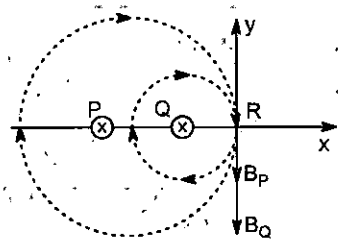


Fig. 4E.47 (a)

Resultant magnetic field,

$$B_R = \frac{\mu_0 I_P}{2\pi d_P}(-\hat{j}) + \frac{\mu_0 I_Q}{2\pi d_Q}(-\hat{j})$$
$$= \frac{\mu_0}{2\pi}\left(\frac{I_P}{d_P} + \frac{I_Q}{d_Q}\right)(-\hat{j}) \qquad ...(1)$$

The magnetic force experienced by a moving charge is given by

$$\vec{F} = q\vec{v}\times\vec{B}, \qquad |\vec{F}| = qvB\sin\theta$$

The particle as moving along positive x-axis and the magnetic field is along negative x-axis, so $\theta = \pi/2$

$$F = qvB,$$
$$B = \frac{F}{qv} = \frac{3.2\times 10^{-20}}{1.6\times 10^{-19}\times 4\times 10^5}$$
$$= 5\times 10^{-7}\ \text{T} \qquad ...(2)$$

From eqns. (1) and (2),

$$2\times 10^{-7}\left[\frac{2.5}{5} + \frac{I_Q}{2}\right] = 5\times 10^{-7}\ \text{T}$$

or $I_Q = 4\text{A}$

(b) If we place the third wire in the regions 1 and 2, the resultant magnetic field cannot be zero, because current in wire Q is greater than current in third wire. The resultant magnetic field to wires P and Q is along negative y-axis, therefore magnetic field of third wire must be along positive y-axis. If third wire is kept in region 3, the current in it must be upward. If the distance of point R from third wire is x, then

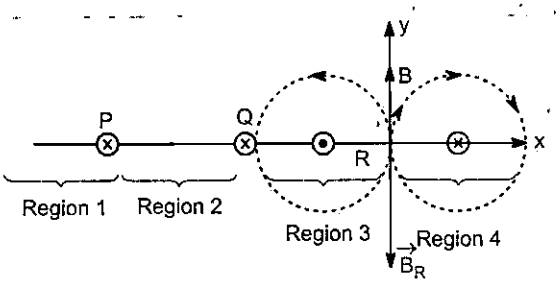


Fig. 4E.47 (b)

$$\frac{\mu_0}{4\pi}\frac{2\times 2.5}{x} + 5\times 10^{-7} = 0$$

Hence $x = -1$ if current is into the page, third wire is to right of R; and $x = +1$ if current is out of page, wire is to the left of R.

FORCE BETWEEN CURRENT CARRYING PARALLEL WIRES

Fig. 4.56 shows two long parallel wires carrying currents in the same direction. We consider the force on a segment $d\vec{l}_2$ carrying current I_2 as shown. The magnetic field $\vec{B}_1$ at this segment due to current I_1 is perpendicular to the segment $I_2\,d\vec{l}_2$ as shown. This is true for all current elements along the wire. The magnetic force $d\vec{F}_2$ on current segment $I_2 d\vec{l}_2$ is directed toward current I_1. Similarly a current segment $I_1 d\vec{l}_1$ will experience a magnetic force directed toward current I_2 due to a magnetic field arising from current I_2. Thus, two parallel currents attract each other. If one of the currents is reversed, the force will be reversed, so two antiparallel currents will repel each other.

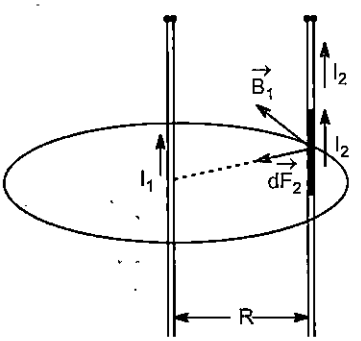


Fig. 4.56

The magnitude of the magnetic force on the segment $I_2\,d\vec{l}_2$ is

$$dF_2 = |I\,d\vec{l}_2\times\vec{B}_1|$$

Since the magnetic field at segment $I_2\,d\vec{l}_2$ is perpendicular to the current segment, we have

$$dF_2 = I_2 dl_2 B_1$$

If the distance R between the wires is much less than their length, the field at $I_2\,d\vec{l}_2$ due to current I_1 will approximate the field due to an infinitely long, current-carrying wire. The magnitude of the force on the segment $I_2\,d\vec{l}_2$ is, therefore,

$$dF_2 = I_2\,dl_2\,\frac{\mu_0 I_1}{2\pi R}$$

The force per unit length is

$$\frac{dF_2}{dl_2} = I_2\,\frac{\mu_0 I_1}{2\pi R}$$

$$= 2\frac{\mu_0}{4\pi}\,\frac{I_1 I_2}{R}$$

Example 48

A conductor carries a current I parallel to a current strip of current per unit width j and width w, as shown in Fig. 4E.48. Find an expression for the force per unit length on the conductor. Discuss the result when the width w approaches infinity.

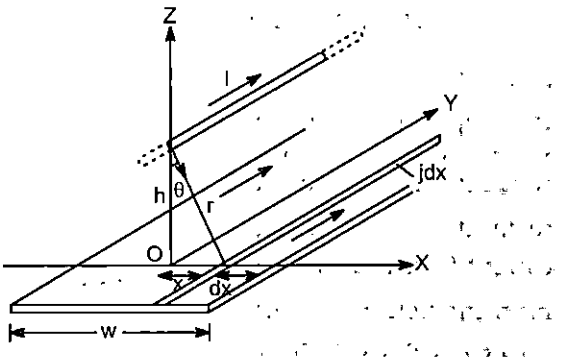


Fig. 4E.48

Solution: We consider a differential segment of thickness dx, carrying current $j\,dx$. The attractive force per unit length is

$$\frac{dF}{L} = \frac{\mu_0 I(j\,dx)}{2\pi r}$$

A symmetrical segment at $-x$ exerts same magnitude of force, the x-components cancel and the resultant force is

$$\frac{dF}{L} = \frac{\mu_0 I(j\,dx)}{2\pi r}\cos\theta$$

$$= \frac{\mu_0 Ij\,dx}{2\pi\sqrt{h^2+x^2}} \times \frac{h}{\sqrt{h^2+x^2}}$$

Integrating over half width of strip,

$$\frac{F}{L} = \frac{\mu_0 Ijh}{\pi}\int_0^{w/2}\frac{dx}{h^2+x^2}$$

$$= \frac{\mu_0 Ij}{\pi}\tan^{-1}\left(\frac{w}{2h}\right)$$

When $w \to \infty, \frac{F}{L} \to \frac{\mu_0 Ij}{2}$, the force is attractive as expected for parallel currents.

Example 49

Two long conducting rods suspended by means of two insulating threads as shown in Fig. 4E.49 are connected to a charged capacitor through a switch which is initially open. At the other end, they are connected by a loose wire.

The capacitor has a charge Q and mass per unit length of the rod is λ. The effective resistance of the circuit after closing the switch is R.

Find the velocity of each rod when the capacitor is discharged after closing the switch. Assume that the displacement of rods during the discharging time is negligible.

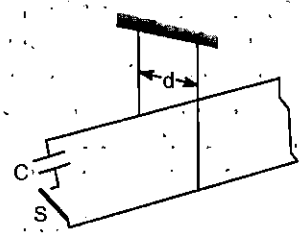


Fig. 4E.49

Solution: The impulsive force due to (antiparallel) discharging currents is equal to the charge in momentum or

$$\lambda v = \int_0^\infty F\,dt = \int_0^\infty \frac{\mu_0 I^2}{2\pi d}\,dt$$

where I = discharging current

Now $q = Qe^{-t/RC}$

$$\Rightarrow \quad |I| = \frac{Q}{RC}e^{-t/RC}$$

Magnetic force per unit length between two current carrying wires is

$$\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi d} = \frac{\mu_0}{2\pi d}\left(\frac{Q}{RC}e^{-t/RC}\right)^2$$

Impulse due to this force

$$= \int_0^\infty \frac{\mu_0}{2\pi d}\,\frac{Q^2}{R^2C^2}\,e^{-2t/RC}\,dt.$$

Hence $$\lambda v = \left|\int_0^\infty \frac{\mu_0}{2\pi d}\,\frac{Q^2}{R^2C^2}\,e^{-2t/RC}\,dt\right| = \frac{\mu_0 Q^2}{4\pi dRC}$$

or $$v = \frac{\mu_0 Q^2}{4\pi\lambda dRC}$$

Example 50

A long horizontal wire AB which is free to move in a vertical plane and carries a steady current of 20 A, is in equilibrium at a height of 0.01 m over another parallel long wire CD, which is fixed in a horizontal plane and carries a steady current of 30 A as shown in Fig. 4E.50. Show that when AB is slightly depressed and released, it executes simple harmonic motion. Find the period of oscillations.

Solution: The force per unit length between two parallel wires is given by

$$\frac{dF}{dl} = \frac{\mu_0 I_1 I_2}{4\pi d},$$

for antiparallel currents force is repulsive.

In equilibrium position,

$$\frac{mg}{L}(-\hat{\mathbf{j}}) + \frac{\mu_0 I_1 I_2}{2\pi d}(\hat{\mathbf{j}}) = 0$$

or
$$\frac{\mu_0 I_1 I_2}{2\pi d} = \frac{mg}{L} \quad ...(1)$$

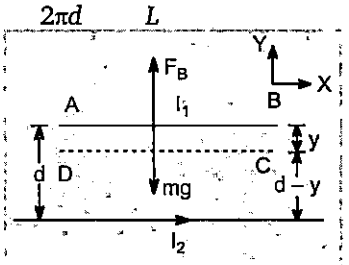


Fig. 4E.50

If upper wire is displaced downward from its equilibrium position, the resultant force on it,

$$\vec{\mathbf{F}}_R = \frac{\mu_0 I_1 I_2}{2\pi(d-y)}\hat{\mathbf{j}} - \frac{mg\hat{\mathbf{j}}}{L} \quad ...(2)$$

From eqn. (1), substituting expression for mg/L, we have

$$\vec{\mathbf{F}}_R = \frac{\mu_0 I_1 I_2}{2\pi}\left[\frac{1}{(d-y)} - \frac{1}{d}\right]\hat{\mathbf{j}}$$

$$= \frac{\mu_0 I_1 I_2}{2\pi}\frac{y}{(d-y)d}\hat{\mathbf{j}}$$

For small displacement y, $y \ll d$,

$$F_R = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d^2} y\,\hat{\mathbf{j}}$$

Note that displacement of wire is along negative y-direction and resultant magnetic force is along positive y-direction; thus motion is simple harmonic.

From eqn. (1),

$$m = \frac{(\mu_0 I_1 I_2)L}{2\pi d g}$$

Acceleration of wire,

$$a = \frac{F_R}{m} = -\frac{g}{d}y \quad [\text{as } y \text{ is opposite to } a]$$

Thus
$$\omega = \sqrt{\frac{g}{d}} = \frac{2\pi}{T}$$

or
$$T = 2\pi\sqrt{\frac{d}{g}}$$

Example 51

Three infinitely long thin wires, each carrying current in the same direction, are in the xy-plane of a gravity-free space. The central wire is along y-axis while the other two are along $x = \pm d$. (a) Find the locus of the points for which the magnetic field B is zero. (b) If the central wire is displaced along the z-direction by a small amount and released, show that it will execute simple harmonic motion. If the linear density of the wire is λ, find the frequency of oscillation.

Solution: (a) To the left and right of all the three wires the magnetic field due to each is in the same direction, it cannot be zero. The resultant magnetic field can be zero if the point P lies between wires. Let point P lie a distance x from the central wire.

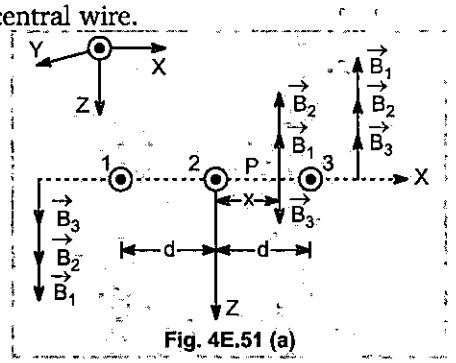


Fig. 4E.51 (a)

$$\vec{\mathbf{B}}_R = \frac{\mu_0 I}{2\pi}\left[\frac{1}{d+x} + \frac{1}{x} - \frac{1}{(d-x)}\right](-\hat{\mathbf{k}})$$

As $\vec{\mathbf{B}}_R = 0$, $\quad \frac{1}{d+x} - \frac{1}{x} + \frac{1}{d-x} = 0$

or $\quad x = \pm d/\sqrt{3}$

So the loci of points for which the resultant magnetic field is zero are straight lines parallel to Y-axis at distance $\pm(d/\sqrt{3})$ from it.

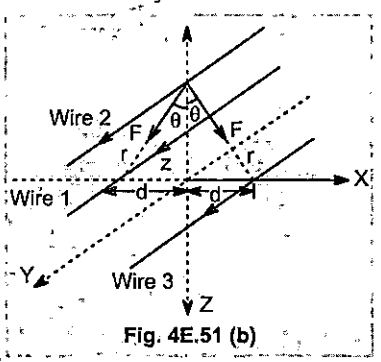


Fig. 4E.51 (b)

(b) Let the wire 2 be displaced along Z-axis by a small distance z. As gravity is absent, the resultant attractive force per unit length on wire 2 is

$$F_R = 2F\cos\theta$$

$$= 2\frac{\mu_0 I_1 I_2}{2\pi r} \times \frac{z}{r}$$

$$= \frac{\mu_0 I_1 I_2}{\pi}\frac{z}{(d^2+z^2)} \qquad [r^2 = d^2 + z^2]$$

For very small displacement, $z \ll d$,

$$F_R = -\frac{\mu_0 I_1 I_2}{\pi} z$$

The minus sign shows that force is opposite to displacement. Acceleration of the wire,

$$a = -\frac{\mu_0 I_1 I_2}{\pi\lambda} z$$

Thus, $$\omega = \sqrt{\frac{\mu_0 I_1 I_2}{\pi\lambda}} = \frac{2\pi}{T} = 2\pi\nu$$

$$\nu = \frac{T}{2\pi d}\sqrt{\frac{\mu_0}{\pi\lambda}}$$

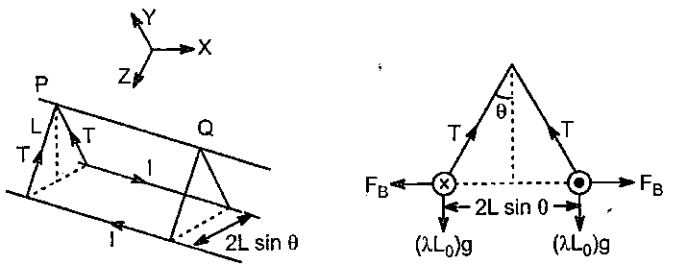
Example 52

Two long parallel wires carry currents of equal magnitude but in opposite directions. These wires are suspended from rod PQ by four chords of same length L as shown in the figure. The mass per unit length of the wire is λ. Determine the value of θ assuming it to be small.

Solution: The force per unit length between current carrying parallel wires is

$$\frac{dF}{dL} = \frac{\mu_0 I_1 I_2}{2\pi d}$$

If two wires carry current in opposite directions the magnetic force is repulsive, due to which the parallel wires in Fig. 4E.52 (a) have moved out so that equilibrium is reached. Fig. 4E.52 (b) shows free body diagram of each wire. In equilibrium,



(a) (b)

Fig. 4E.52

$$\Sigma F_y = 0, \quad 2T\cos\theta = (\lambda L_0)g \qquad ...(1)$$

$$\Sigma F_z = 0, \quad 2T\sin\theta = F_B \qquad ...(2)$$

Now dividing eqn. (2) by eqn. (1) we get

$$\tan\theta = \frac{F_B}{L_0\lambda g} \qquad ...(3)$$

The magnetic force

$$F_B = \left(\frac{dF}{dL}\right) \times L_0 = \frac{\mu_0 I^2}{4\pi\sin\theta} L_0 \qquad ...(4)$$

For small θ, $\tan\theta \simeq \sin\theta \simeq \theta$

On substituting eqn. (4) in eqn. (3), we get

$$\theta = I\sqrt{\frac{\mu_0}{4\pi\lambda g}}$$

Example 53

A straight segment OC (of length L) of a circuit carrying current I ampere is placed along the x-axis. Two infinitely long straight wires A and B each extending from $z = -\infty$ to $+\infty$ are fixed at $y = -a$ m and $y = +a$ m respectively as shown in Fig. 4E.53. If the wires A and B each carry a current I ampere into the plane of the paper, obtain the expression for the force acting on the segment OC. What will be the force on OC if the current in the wire B is reversed ?

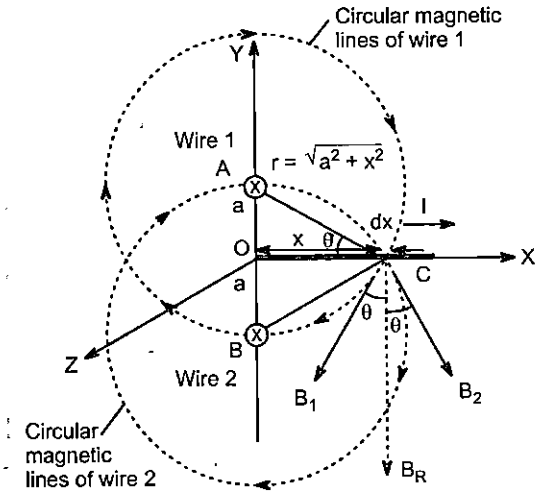


Fig. 4E.53

Solution: The magnetic field due to the two wires 1 and 2 is variable along *X*-axis; therefore each point of the wire experiences different magnetic field. So we consider a differential element *dx* at a distance *x* from origin. The resultant magnetic field.

$$\vec{\mathbf{B}}_R = 2B\cos\theta(-\hat{\mathbf{j}})$$

$$= 2\frac{\mu_0 I}{2\pi r} \times \frac{x}{r}(-\hat{\mathbf{j}})$$

$$= 2\frac{\mu_0 I}{2\pi\sqrt{a^2+x^2}} \times \frac{x}{\sqrt{a^2+x^2}}(-\hat{\mathbf{j}})$$

$$= \frac{\mu_0 I x}{\pi(a^2+x^2)}(-\hat{\mathbf{j}})$$

The magnetic force on current element $dx\,\hat{\mathbf{i}}$ is

$$d\vec{\mathbf{F}} = I\,d\vec{\mathbf{L}} \times \vec{\mathbf{B}} = I\left[dx(\hat{\mathbf{i}}) \times \frac{\mu_0 I x}{\pi(a^2+x^2)}(-\hat{\mathbf{j}})\right]$$

$$= \frac{\mu_0 I^2 x\,dx}{\pi(a^2+x^2)}(-\hat{\mathbf{k}})$$

The resultant force is

$$F = \frac{\mu_0 I^2}{\pi}\int_0^L \frac{x\,dx}{(a^2+x^2)}(-\hat{\mathbf{k}})$$

$$= \frac{2\mu_0 I^2}{\pi}\left|\log_e(a^2+x^2)\right|_0^L(-\hat{\mathbf{k}})$$

$$= \frac{\mu_0 I^2}{2\pi} \log_e \left(1 + \frac{L^2}{a^2}\right)(-\hat{k})$$

Example 54

A current of 10 A flows around a closed path in a circuit which is in the horizontal plane as shown in Fig. 4E.54(c). The circuit consists of eight alternating arcs of radii $r_1 = 0.08\,m$ and $r_2 = 0.12\,m$. Each arc subtends the same angle at the centre.

(a) Find a magnetic field produced by this circuit at the centre.

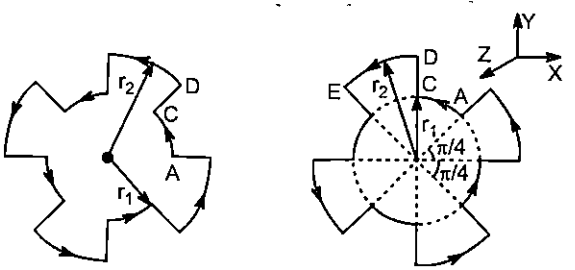


Fig. 4E.54

(b) An infinitely long straight wire carrying a current of 10 A is passing through the centre of the above circuit vertically with the direction of the current being into the plane of the circuit. What is the force acting on the wire at the centre due to the current in the circuit ? What is the force acting on the arc AC and the straight segment CD due to the current at the centre ?

Solution: (a) The magnetic field to straight segment like *CD* is zero. The magnetic field due to an arc subtending angle θ at the centre is given by

$$B = \frac{\mu_0 I}{2R} \times \frac{\theta}{2\pi}$$

The resultant magnetic field due to all the arcs of radius r_1 is

$$\vec{\mathbf{B}}_1 = 4 \times \frac{\mu_0 I}{2r_1} \times \frac{1}{8}(\hat{k}) = \frac{\mu_0 I}{4r_1}(\hat{k})$$

The resultant magnetic field due to all the arcs of radius r_2 is

$$\vec{\mathbf{B}}_2 = \frac{\mu_0 I}{4r_2}(\hat{k})$$

Thus the resultant magnetic field,

$$\vec{\mathbf{B}}_R = \vec{\mathbf{B}}_1 + \vec{\mathbf{B}}_2$$

$$= \frac{\mu_0 I}{4}\left[\frac{1}{r_1} + \frac{1}{r_2}\right]\hat{k}$$

On substituting numerical values we get $B = 6.54 \times 10^{-5}$ T.

(b) (i) Resultant magnetic field at the centre is antiparallel to current; hence in the expression $|d\vec{\mathbf{F}}| = I\,dl\,B\sin\theta$, θ is π, so $|d\vec{\mathbf{F}}| = 0$

The wire does not expression magnetic force.

(ii) Magnetic field due to wire at the centre circulates around it; hence magnetic field is tangential to arc *AC;* so it will not experience magnetic force.

(iii) Section *CD* lies in a variable magnetic field. Magnetic force experienced by a differential element *dx* at a distance *x* from long wire at centre $= \frac{\mu_0 I}{2\pi} I\,dx$.

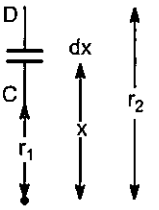


Fig. 4E.54 (c)

Resultant force on *CD* is

$$F = \int_{r_1}^{r_2} \frac{\mu_0 I^2}{2\pi x}\,dx$$

$$= \frac{\mu_0 I^2}{2\pi} \log_e \left[\frac{r_2}{r_1}\right]$$

On substituting numerical values, we get

$$F = 8.11 \times 10^{-6}\ \text{N} \quad \text{(downward)}$$

Example 55

A wire loop carrying a current is placed in the xy-plane as shown in Fig. 4E.55. If a particle with charge q and mass m is placed at the centre P and given a velocity v along NP, find its instantaneous acceleration. If an external uniform magnetic induction $B = B\hat{i}$ is applied, find the force and torque acting on the loop.

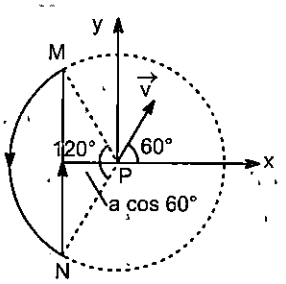


Fig. 4E.55

Solution: The magnetic field at *P* due to arc

$$\vec{\mathbf{B}}_1 = \frac{\mu_0 I}{2a} \times \frac{(2\pi/3)}{2\pi}(\hat{k})$$

The magnetic field at *P* due to straight segment *MN*

$$\vec{\mathbf{B}}_2 = \frac{\mu_0}{4\pi} \frac{I}{a\cos 60°} \times (\sin 60° + \sin 60°)(-\hat{k})$$

The resultant magnetic field at *P* due to the given loop

$$\vec{\mathbf{B}}_R = \vec{\mathbf{B}}_1 + \vec{\mathbf{B}}_2$$

$$= \frac{\mu_0 I}{2\pi a}\left[\sqrt{3} - \frac{\pi}{3}\right](-\hat{k})$$

Velocity of charge at P, $v = v\cos 60°\,\hat{i} + v\sin 60°\,\hat{j}$

Force experienced by a moving charge in a magnetic field is given by

$$\vec{\mathbf{F}} = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$$

$$= q\left(\frac{v}{2}\hat{\mathbf{i}} + \frac{\sqrt{3}}{2}v\hat{\mathbf{j}}\right) \times \frac{\mu_0 I}{2\pi a}\left(\sqrt{3} - \frac{\pi}{3}\right)(-\hat{\mathbf{k}})$$

$$= \frac{\mu_0 qvI}{4\pi a}\left(\sqrt{3} - \frac{\pi}{3}\right)(\hat{\mathbf{i}} + \sqrt{3}\,\hat{\mathbf{j}})(-\hat{\mathbf{k}})$$

$$= \frac{\mu_0 qvI(\sqrt{3} - \pi/3)}{4\pi a}(\hat{\mathbf{j}} - \sqrt{3}\,\hat{\mathbf{i}})$$

Thus acceleration of charges is,

$$\vec{\mathbf{a}} = \frac{\vec{\mathbf{F}}}{m} = \frac{\mu_0 qvI(\sqrt{3} - \pi/3)}{4\pi am}(\hat{\mathbf{j}} - \sqrt{3}\hat{\mathbf{i}})$$

* Net magnetic force on a current carrying coil of any arbitrary shape is zero.

Area of loop $= \frac{1}{3}$ (area of circle) − area of triangle *MNP*

$$= \frac{1}{3}\pi a^2 - \frac{1}{2} \times \frac{a}{2} \times \sqrt{3}\,a$$

$$= 0.6136a^2$$

In vector notation

$$\vec{\mathbf{A}} = 0.6136\hat{\mathbf{k}}$$

Torque $\quad \vec{\tau} = \vec{\mu} \times \vec{\mathbf{B}} = I(\vec{\mathbf{A}} \times \vec{\mathbf{B}})$

$$= 0.6136a^2 IB\hat{\mathbf{j}}$$

Example 56

A square frame carrying a current I_1 is located in the same plane as a long straight wire carrying a current I_2. The frame side has length 2a; the axis of frame is at a distance b from wire. (a) Find the force acting on the frame. (b) If the wire is moved to the other side of wire as shown in Fig. 4E.56, find the work done. (c) Find the work done in turning the frame through 180° about axis of symmetry.

Solution: (a) We take symmetrical differential elements on wires *PQ* and *RS*. From Fleming's left-hand rule force on them is in opposite direction, they cancel out. Thus the force on *PQ* is cancelled out by force on *RS*.

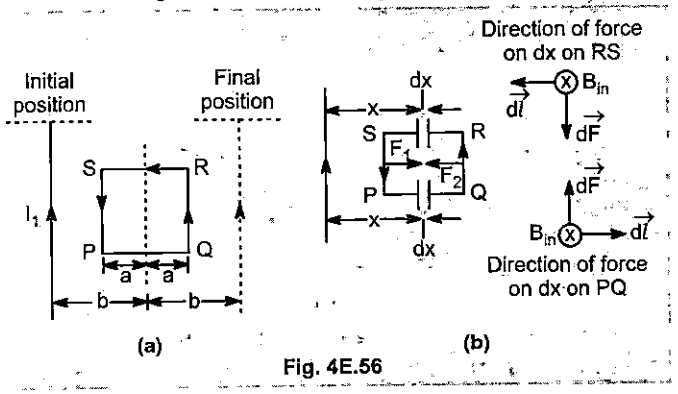


Fig. 4E.56

Resultant force on the frame = Repulsive force on *SP* − attractive force on *QR*

$$= \frac{\mu_0 I_1 I_2}{2\pi}\left[\frac{2a}{b-a} - \frac{2a}{b+a}\right] = \frac{\mu_0 I_1 I_2}{\pi}\frac{2a^2}{(b^2 - a^2)}$$

(b) The repulsive force on the wire at a distance x is

$$F = \frac{\mu_0 I_1 I_2}{\pi}\left[\frac{1}{(x-a)} - \frac{1}{(x+a)}\right]$$

The required work done

$$= -\int_{x=-b}^{x=+b} F\,dx$$

$$= -\frac{\mu_0 I_1 I_2 a}{\pi}\int_{-b}^{+b}\left[\frac{dx}{x-a} - \frac{dx}{x+a}\right]$$

$$= \frac{2\mu_0 I_1 I_2 a}{\pi}\ln\frac{(b+a)}{(b-a)}$$

Example 57

Fig. 4E.57(a) shows a flat strip of width a and negligible thickness carrying a current I. Find the magnetic field (a) in the plane of the strip at a distance R from the edge centre of the strip, (b) at a distance R from the centre of the strip at right angles to the strip.

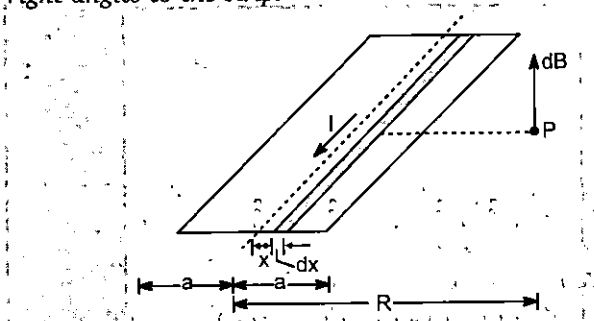


Fig: 4E.57 (a)

Solution: (a) We sub-divide the strip into long differential elements of width dx, each of which may be treated as a wire carrying current dI, where

$$dI = \left(\frac{I}{2a}\right)dx$$

The field contribution dB at point P is

$$dB = \frac{\mu_0(dI)}{2\pi(R-x)}$$

$$= \frac{\mu_0 I dx}{2\pi(2a)(R-x)}$$

The magnetic field contributions due to each element is in the same direction; so we can integrate dB.

$$B = \frac{\mu_0 I}{4\pi a}\int_{-a}^{+a}\frac{dx}{(R-x)}$$

$$= \frac{\mu_0 I}{4\pi a}\ln\left[\frac{R+a}{R-a}\right]$$

(b) Once again we take a differential element at a distance x, as shown in Fig. 4E.57 (b). The field contribution

$$dB = \frac{\mu_0 dI}{2\pi r} = \frac{\mu_0(I\,dx/2a)}{2\pi(R\sec\theta)}$$

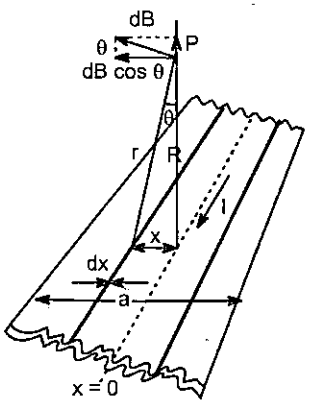


Fig. 4E.57 (b)

We take a symmetrically located differential element on the other side of $x = 0$ and note that, only the horizontal component survives; the vertical component is cancelled. Thus B at point P is given by the scalar integral

$$B = \int dB\cos\theta = \int \frac{\mu_0 (I\,dx/2a)}{2\pi R\sec\theta}\cos\theta$$

$$= \frac{\mu_0 I}{4\pi a R}\int \frac{dx}{\sec^2\theta}$$

From figure,

$$x = R\tan\theta, \qquad dx = \sec^2\theta\, d\theta.$$

The limits on θ are $\pm\tan^{-1}(a/R)$.

$$B = \frac{\mu_0 I}{2\pi a R}\int_{-\tan^{-1}a/R}^{+\tan^{-1}a/R} \frac{R\sec^2\theta d\theta}{\sec^2\theta}$$

$$= \frac{\mu_0 I}{2\pi a}\tan^{-1}\left(\frac{a}{R}\right).$$

At large distance from strip a/R will be very small and for small angles, $\tan\theta \simeq \theta$; so we have

$$B \simeq \frac{\mu_0 I}{2\pi a}\left(\frac{a}{R}\right) = \frac{\mu_0 I}{2\pi R}$$

Example 58

Fig. 4E.58 (a) shows a square loop, 20 cm on each side in the xy-plane with its centre at the origin. The loop carries a current of 7 A. Above it at y = 0, z = 12 cm is an infinitely long wire parallel to the x-axis carrying a current of 10 A. Find the (a) force on each side of loop, (b) torque on the loop, (c) net force on the loop.

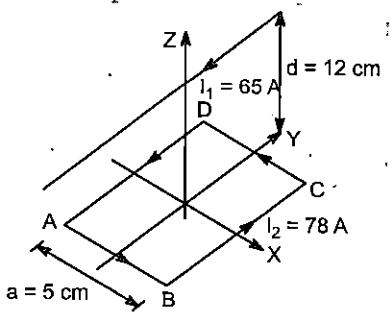


Fig. 4E.58 (a)

Solution: The figure shows side view, looking toward positive Y-axis.

(a) **Force on wire *AB*, *CD*:** We take a differential element dx on the wire, see Fig. 4E.58(b).

$$x = d\tan\theta$$

$$dx = d\sec^2\theta\, d\theta$$

Magnetic force on it,

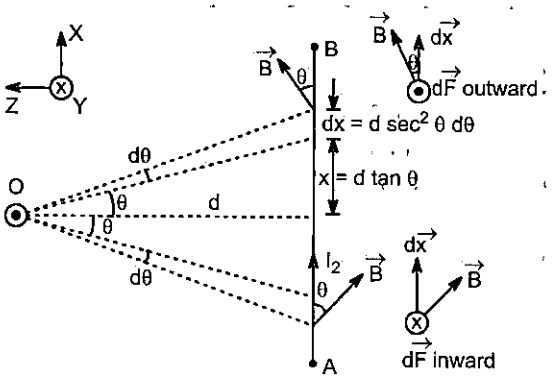


Fig. 4E.58 (b)

$$|d\vec{F}| = I_2 dx B\sin\theta$$

$$= I_2(d\sec^2\theta\, d\theta)\times\frac{\mu_0 I_1}{2\pi(d/\cos\theta)}\sin\theta$$

or

$$|\vec{F}| = \int_{-\theta}^{+\theta}\left(\frac{\mu_0 I_1 I_2}{2\pi}\right)\frac{\sin\theta}{\cos\theta}d\theta$$

$$|\vec{F}| = \frac{\mu_0 I_1 I_2}{2\pi}\int_{-\theta}^{+\theta}\tan\theta\, d\theta = 0$$

Thus resultant force on wire AB is zero; similarly force on CD is also zero. We can check this result by symmetry. We take two similar differential elements at angles θ and $-\theta$ and note the direction of magnetic force: they cancel out.

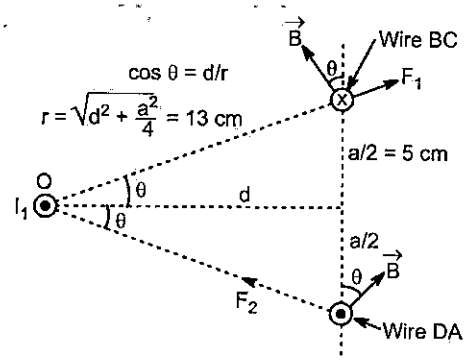


Fig. 4E.58 (c)

Force on wires *BC*, *DA*

Force on wire $BC = \frac{\mu_0 I_1 I_2}{2\pi r}\times a$, along OB

Force on wire $DA = \frac{\mu_0 I_1 I_2}{2\pi r}\times a$, along DO

(b) Torque on the coil

$$= \left(\frac{\mu_0 I_1 I_2}{2\pi r}a\cos\theta\right)a$$

$$= 2\times10^{-7}\times65\times78\times(10\times10^{-2})^2\times\frac{12\times100}{(13)^2}$$

$$= 7.2\times10^{-5}\text{ N-m}$$

The magnitudes of forces $\vec{F}_1$ and $\vec{F}_2$ are same.

$$|\vec{F}_1| = |\vec{F}_2| = F = \frac{\mu_0 I_1 I_2}{2\pi r}\times a$$

The components $F_1 \sin\theta$ and $F_2 \sin\theta$ do not exert torque; torque is due to components $F_1\cos\theta$ and $F_2\cos\theta$.

(c) Resultant force on wires *AB* and *CD* is zero. The components $F_1 \sin\theta$ and $F_2 \sin\theta$ on wires *BC* and *DA* add, whereas components $F_1\cos\theta$ and $F_2\cos\theta$ cancel.

Thus the resultant force acting on the coil

$$= 2F\cos\theta$$

$$= \frac{\mu_0 I_1 I_2}{\pi r}\sin\theta$$

$$= \frac{4\times10^{-7}\times65\times78\times0.1\times(5\times10^{-2})}{13^2\times10^{-4}}$$

$$= 6\times10^{-4}\text{ N}$$

Example 59

A rectangular polygon of n sides is formed by bending a wire of total length $2\pi R$ which carries a current I.
(a) Find the magnetic field at the centre of the polygon.
(b) Deduce the expression for the magnetic field at the centre of a circular coil by letting $n \to \infty$.

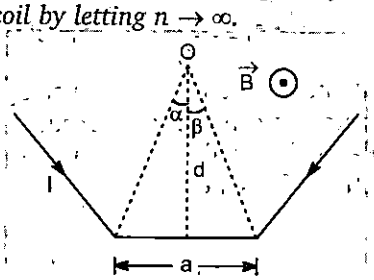


Fig. 4E.59

Solution: Angle subtended by each side of polygon at the centre is

$$\alpha = \beta = \frac{\pi}{n}$$

Perpendicular distance of *O* from the wire is

$$d = \left(\frac{a}{2}\right)\cot\alpha = \left(\frac{\pi R}{n}\right)\cot\left(\frac{\pi}{n}\right)$$

Each side of the polygon creates the same magnetic field at the centre. In accordance with right-hand thumb rule, magnetic field at centre is outward, Hence, net magnetic field,

$$B = (n) \quad \text{(magnetic field due to each side)}$$

$$= n\left\{\frac{\mu_0}{4\pi}\frac{I}{d}(\sin\alpha + \sin\beta)\right\}$$

or $$B = n\left(\frac{\mu_0}{4\pi}\right)\left(\frac{In}{\pi R}\right)\left(\tan\frac{\pi}{n}\right)\left(2\sin\frac{\pi}{n}\right)$$

or $$B = \left(\frac{\mu_0 I}{2R}\right)\frac{\left\{\frac{\sin(\pi/n)}{\pi/n}\right\}^2}{\cos(\pi/n)}$$

(b) As $n \to \infty, \frac{\pi}{n} \to 0$ and the polygon becomes a circle.

and $$\sin(\pi/n)/(\pi/n) \to 1$$

$$\cos(\pi/n) \to 1$$

Thus $$B = \frac{\mu_0 I}{2R}$$

which is the expression for the magnetic field at the centre of a circular coil.

AMPERE'S LAW

As in the case of electric field we used Gauss' law to calculate the electric field for highly symmetric charge distributions; a similar law exists in magnetism. The ampere's law states that

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{encl}$$

The left hand side of Ampere's law is a line integral around a closed path. $\oint \vec{B}\,d\vec{l} = \oint (B\cos\theta)dl$ simply means to integrate (sum up) the product of the path element *dl* and the component of magnetic field *B* that is parallel to this element, $B\cos\theta$. The integral must be performed along a closed path. In order to apply Ampere's law we consider a symmetrical closed curve around the current carrying wire. The right side equation has I_{encl}, which is the net current enclosed by the closed curve under consideration.

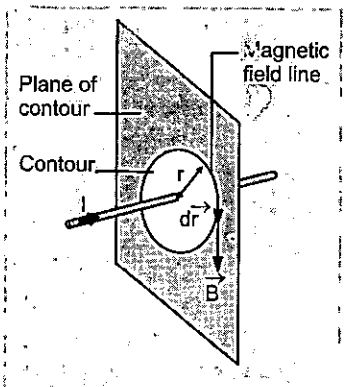


Fig. 4.57

The simplest application of Ampere's law is to find the magnetic field of an infinitely long, straight current carrying wire. By symmetry the magnitude of the magnetic field at a fixed distance *r* from the wire is constant. We choose a circular curve of radius *r* centred on the wire coincident with a magnetic field line. Integrate around the curve in the same sense as the direction of the magnetic field as shown in Fig. 4.57. Magnetic field is tangent to circle; hence $\vec{B}\cdot d\vec{l} = B\,dl$. Ampere's law then gives

$$\oint \vec{B}\cdot d\vec{l} = B\oint dl = \mu_0 I_{encl}$$

The integral of *dl* around the circle equals $2\pi r$, the circumference of the circle; and $I_{encl} = I$.

$$B(2\pi r) = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$

Concept: 1. *The circulation $\mathscr{C}$ of a vector field $\vec{\mathbf{v}}$ around a closed curve C is:*

$$\mathscr{C} \equiv \oint_C \vec{\mathbf{v}} \cdot d\vec{\mathbf{l}} \qquad ...(1)$$

The circulation of $\vec{\mathbf{B}}$ around C describes the extent to which field lines follow the direction of the curve.

2. *The sign convention for current is determined by a right-hand rule: curl the fingers of your right-hand around the curve C in the sense of the vectors $d\vec{\mathbf{l}}$. The current is positive if it is in the direction of your thumb Fig. 4.58. Ampere's law contains the same information as the Biot-Savart law but expressed in a different form.*

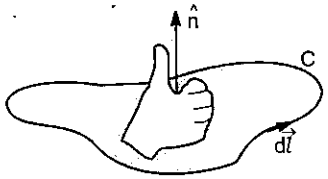


Fig. 4.58: A right-hand rule defines the positive sense of current in Ampere's law. First choose a direction to travel around the curve C. This defines the direction of each element $d\vec{\mathbf{l}}$. Curl the fingers of your right-hand around the curve in the chosen direction; your thumb defines the positive sense for I, or for $\hat{\mathbf{n}}$ in eqn. (2).

3. *Find the circulation of $\vec{\mathbf{B}}$ around a circle of radius a centered on a wire carrying current I Fig. 4.59.*

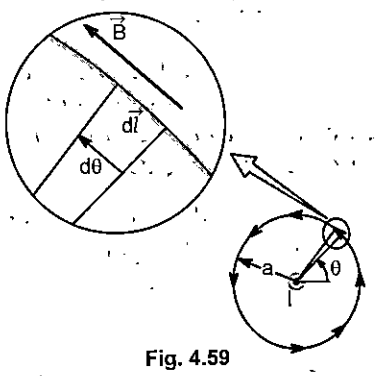


Fig. 4.59

The magnetic field lines form circles around the wire, everywhere parallel to the curve C. We move around C counterclockwise, so the circulation is be positive.

The magnetic field has constant magnitude around the circle and is given by

$$\vec{\mathbf{B}} = \frac{\mu_0 I}{2\pi a}\hat{\theta}$$

We use the angle θ as the coordinate to describe each element of the circle:

$$d\vec{\mathbf{l}} = a\,d\theta\,\hat{\theta}$$

Thus,

$$d\mathscr{C} = \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = \frac{\mu_0 I}{2\pi a} a\,d\theta = \frac{\mu_0 I}{2\pi} d\theta$$

$$\mathscr{C} = \oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = \frac{\mu_0 I}{2\pi}\oint_0^{2\pi} d\theta = \mu_0 I$$

4. *Sometimes it is useful to express Ampere's law in terms of current density, $\vec{\mathbf{j}}$ = current per unit area. Imagine a surface S whose edge is the curve C (Fig. 4.60). The direction of the normal to the surface at any point is chosen according to the right-hand rule, as above. Then we may write Ampere's law as:*

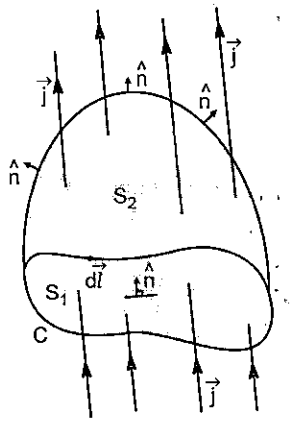


Fig. 4.60

$$\oint_C \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = \mu_0 I = \mu_0 \int_S \vec{\mathbf{j}} \cdot \hat{\mathbf{n}} dA \qquad ...(2)$$

The circulation of $\vec{\mathbf{B}}$ around C is equal to μ_0 multiplied by the flux of $\vec{\mathbf{j}}$ through any surface S spanning the curve C.

5. *Demonstration of Ampere's law:* *Fig. 4.61 shows two arbitrary closed curves near a single long straight wire. Curve (a) surrounds the wire, whereas curve (b) does not. The z-axis is taken along the wire, and polar coordinates r and θ are used in the xy-plane. A typical element $d\vec{\mathbf{l}}$ of the curve has components:*

$$d\vec{\mathbf{l}} = dr\,\hat{\mathbf{r}} + r\,d\theta + dz\,\hat{\mathbf{z}}$$

and the magnetic field produced by the wire is:

$$\vec{\mathbf{B}} = \frac{\mu_0 I}{2\pi r}\hat{\theta}$$

So, since $\hat{\theta}\cdot\hat{\mathbf{r}}$ and $\hat{\theta}\cdot\hat{\mathbf{z}}$ are both zero,

$$\vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = \frac{\mu_0 I}{2\pi} d\theta,$$

and the circulation is

$$\mathscr{C} \equiv \oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} \frac{\mu_0 I}{2\pi}\oint d\theta$$

Around curve (a) the angle θ increases uniformly from 0 to 2π:

$$\oint_a d\theta = 2\pi,$$

and

$$\oint_a \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = \mu_0 I$$

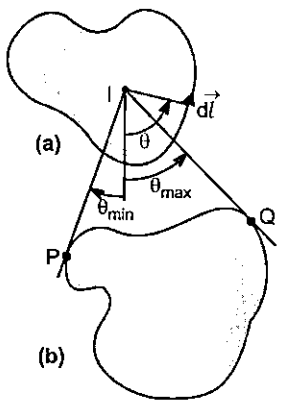


Fig. 4.61

Around curve (b) the angle θ increases from a minimum value to a maximum and back again. The integral is zero. Around curve (b) θ increases from θ_{min} (which is negative in this case) to θ_{max} and back again as we go from P to Q to P:

$$\oint_b d\theta = 0$$

The circulation around a curve from any combination of long straight wires follows from the principle of superposition. A wire passing through the curve contribution $\mu_0 I$; a wire that does not pass through the curve makes no net contribution.

The line integral of $\vec{\mathbf{B}}$ around the current-carrying wire is independent of the radius of the circle. The integral depends only on the amount of current passing through the circle that we integrated around.

Circulation of magnetic field is–

- *independent of the shape of the curve around the current.*
- *independent of where the current passes through the curve.*
- *depends only on the total amount of current through the area enclosed by the integration path.*

Thus whenever total current $I_{through}$ passes through an area bounded by a closed curve, the line integral of the magnetic field around the curve is

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I_{through}$$

6. Gauss' law for the magnetic field: *Magnetic field lines form loops that neither begin nor end. Any field line that enters a volume also leaves it (Fig. 4.62), and no net magnetic flux emerges. Gauss' law for the magnetic field is quite simple:*

The net magnetic flux Φ_B emerging from any volume is zero.

$$\Phi_B = \oint \vec{\mathbf{B}} \cdot \hat{\mathbf{n}}\, dA = 0$$

The integral is taken over the closed surface surrounding the volume.

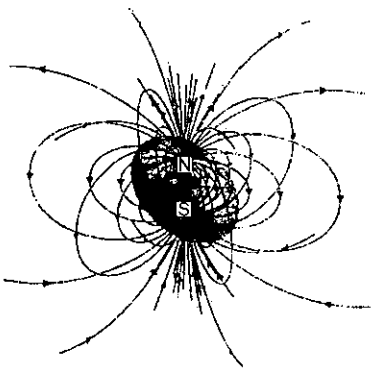


Fig. 4.62: Gauss' law for the magnetic field. The flux through any closed surface is zero: every field line entering the volume through surface S leaves again.

The currents that are the sources of $\vec{\mathbf{B}}$ do not appear in law , so Gauss' law is not helpful for calculating magnetic field. It is very useful for understanding the different behaviour of electric and magnetic fields at a boundary. The electric field changes abruptly at the surface of a conductor because surface charge is present. Whereas since there can be no magnetic charge, the normal component of $\vec{\mathbf{B}}$ is continuous across any boundary.

Example 60

A long, straight wire of radius R carries a current I that is uniformly distributed over the cross-sectional area of the wire. Find the magnetic field outside the wire, B(r > R); inside the wire B(r < R); draw B versus r graph.

Solution: (a) Since the wire is long, straight and cylindrical, we expect from the symmetry of the situation that the magnetic field must be same at points equidistant from the centre of the conductor. The points are equivalent; we do not have reason to give any preference to a point. At a distance r, B is tangent to the circle of radius r about the wire and constant. We choose a circular curve outside wire ($r > R$) coincident with the magnetic field; then $I_{encl} = I$; so

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = B(2\pi r) = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$

which is same result as for a thin wire.

(b) Inside the wire ($r < R$), we again choose a circular curve concentric with the cylinder. Due to symmetry magnetic field is tangent to circle.

$$\text{Current density} = \frac{I}{\pi R^2}$$

$$\text{Hence current enclosed by curve} = \frac{I}{\pi R^2} \times \pi r^2$$

From Ampere's law, we have

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

$$B(2\pi r) = \mu_0 I\left(\frac{\pi r^2}{\pi R^2}\right)$$

$$B = \frac{\mu_0 I r}{2\pi R^2}$$

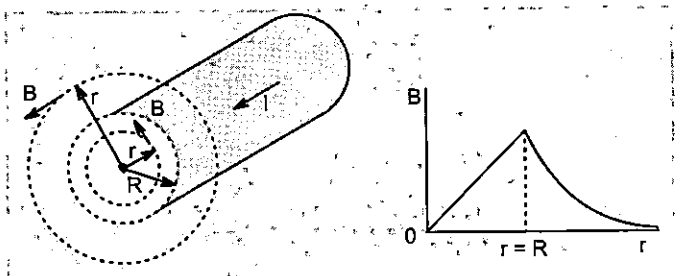


Fig. 4E.60

The field is zero at the centre of the conductor and increase linearly until $r = R$; beyond $r = R$, B decrease as $1/r$.

MAGNETIC FIELD DUE TO AN INFINITELY LONG CYLINDRICAL WIRE

(Current density is constant)

Case-I:

Field inside the wire ($r < R$),

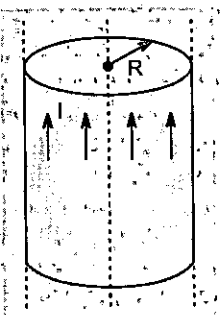


Fig. 4.63

If current density is uniform, then

$$I = j\pi r^2$$

Therefore $$B_{(r<R)} = \frac{\mu_0}{2\pi}\frac{j\pi\pi^2}{r}$$

$$B_{(r<R)} = \frac{\mu_0}{2} jr$$

In vector form,

$$\vec{B}_{(r<R)} = \frac{\mu_0}{2}(\vec{j} \times \vec{r})$$

Where $\vec{r}$ is the position vector of the point with respect to the axis of the cylinder.

Case-II:

Field on or Outside the Cylinder ($r \geq R$)

If current density is uniform then,

$$I = j\pi R^2$$

Therefore $$B_{(r\geq R)} = \frac{\mu_0}{2\pi}\frac{j\pi R^2}{r}$$

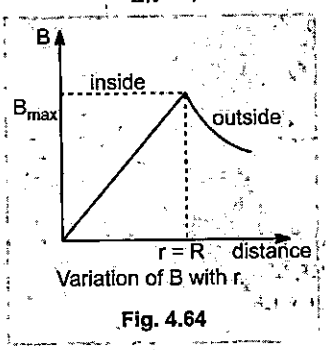


Fig. 4.64

or $$B_{(r\geq R)} = \frac{\mu_0}{2}\frac{jR^2}{r}$$

In vector form,

$$\vec{B}_{(r\geq R)} = \frac{\mu_0 R^2}{2}\frac{\vec{j} \times \vec{r}}{r^2}$$

Intersecting Cylinders

The two intersecting cylinders have their axes parallel. The current density vector has the same magnitude but is in opposite directions in the two cylinders.

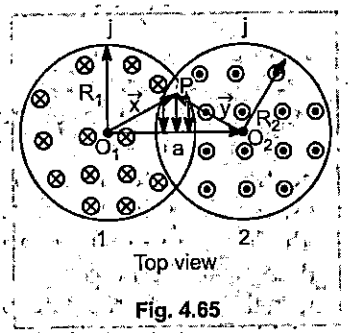


Fig. 4.65

The magnetic field at point P having radius vectors $\vec{x}$ and $\vec{y}$ with respect to the axes of the two cylinders, is given by,

$$\vec{B} = \vec{B}_1 + \vec{B}_2$$

$$= \frac{\mu_0}{2}(\vec{j} \times \vec{x}) + \frac{\mu_0}{2} + (-\vec{j} \times \vec{y})$$

$$= \frac{\mu_0}{2}\vec{j} \times (\vec{x} - \vec{y})$$

$$\vec{B} = \frac{\mu_0}{2}\vec{j} \times \vec{a}$$

The direction of the magnetic field will be perpendicular to the vector $\vec{a}$.

In the common space, the magnetic field is uniform and it is independent of the radii of the two cylinders.

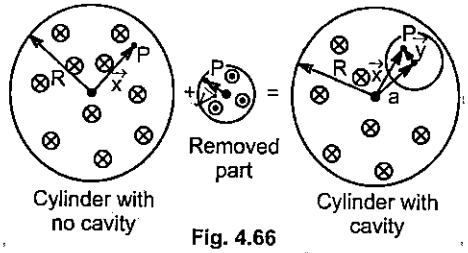


Fig. 4.66

A cavity can be created either by removing the smaller cylinder from larger one or by placing smaller cylinder with the current density having the same magnitude but opposite nature.

Magnetic field at any point P inside the cavity,

$$\vec{\mathbf{B}} = \vec{\mathbf{B}}_1 + \vec{\mathbf{B}}_2$$

$$= \frac{\mu_0}{2}(\vec{\mathbf{j}} \times \vec{\mathbf{x}}) + \frac{\mu_0}{2}(-\vec{\mathbf{j}} \times \vec{\mathbf{y}})$$

$$= \frac{\mu_0}{2}\vec{\mathbf{j}} \times (\vec{\mathbf{x}} - \vec{\mathbf{y}})$$

$$\vec{\mathbf{B}} = \frac{\mu_0}{2}(\vec{\mathbf{j}} \times \vec{\mathbf{a}})$$

Hence the magnetic field inside the cavity is uniform.

SOLENOID

Consider a solenoid with the individual loops of the wire somewhat separated from each other. For points very close to each turn, the magnetic field will be approximately that of a long wire and will circle the wire as shown. In the space between loops the field due to neighbouring loops is in opposite directions. For a closely packed solenoid, the close in field between the loops essentially cancel out. The field inside a solenoid is nearly uniform and parallel to the solenoid axis. The field outside the solenoid is very small compared to the field inside, except near the ends. *We can consider the field inside the solenoid to be uniform along the axis and essentially zero outside.*

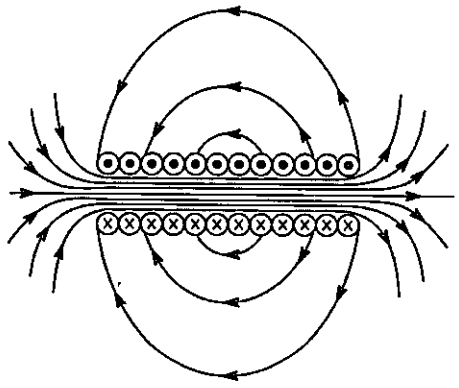
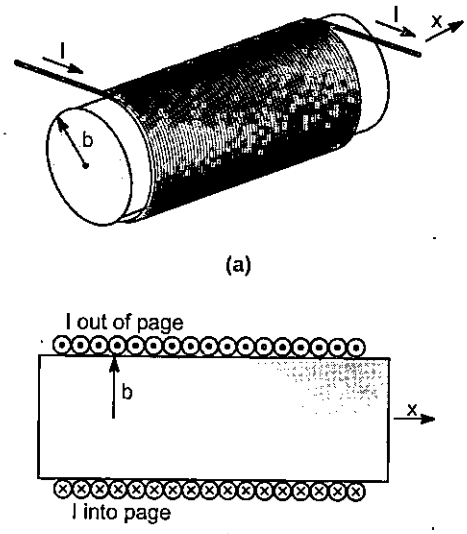


Fig. 4.67

Concepts:1. *Although the winding of the solenoid is helical, we make the approximation that it consists of a stack of circular loop.*



(a)

(b)

Fig. 4.68

The magnetic field of the solenoid is the sum of the contributions of the loops. Because the magnetic field on the axis of each loop is directed along the axis, the magnetic field on the axis of the solenoid is likewise directed along the axis.

If the solenoid is long—that is, if its length X is much greater than its radius b—the magnetic field is parallel to the axis at all locations within the solenoid that are far from its ends. In Fig. 4.69, the magnetic field at point P is the sum of the field contributions of all the turns. Consider a pair of such turns, 1 and 2, located equidistant from P in opposite directions. At an off-axis point such as P, the field contributions B_1 and B_2 will not be parallel to the axis. You can see this from the schematic sketch of the flux lines that would pass through P if the electric current were run through each turn separately. The magnetic field contributions B_1 and B_2 lie along the tangents to these flux lines. By symmetry, they make equal and opposite angles with the x (axial) direction. The vector sum $B_1 + B_2$ is evidently parallel to the axis.

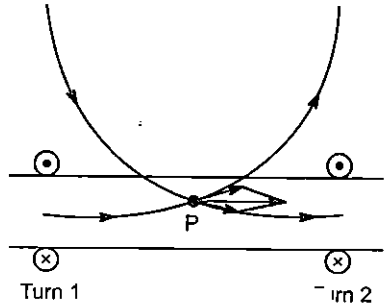


Fig. 4.69: Diagram for the proof that the magnetic field inside a solenoid, far from either end, is everywhere parallel to the solenoid axis.

What about the magnetic field outside the solenoid ? The requirement that flux lines close on themselves means that the lines that pass through the solenoid must return through the space outside. But there is infinite space outside the solenoid, and the returning flux lines are well spread out (except at or near the ends, where most of the lines emerge from the solenoid). Consequently, the magnetic field outside the solenoid, far from its ends, is very weak. Such magnetic field as exists is essentially antiparallel to the field inside the solenoid.

2. *What is the direction of B? Inside an ideal solenoid could it have a radial component ? No. For suppose B were radially outward; if we reversed the direction of the current, B would then be radially inward.*

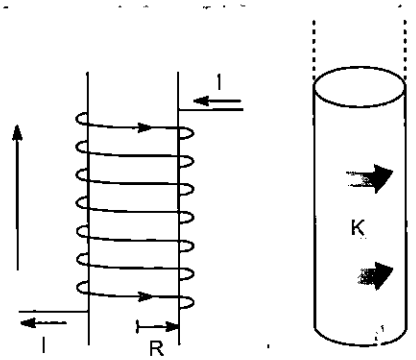


Fig. 4.70

Fig. 4.71

But switching I equivalent to turning the solenoid upside down, and that certainly should not change the radial field. Can it have a "circumferential" component? No. Because B_ϕ would be constant around an amperian loop concentric with the solenoid (Fig. 4.72), and hence

$$\oint B \cdot dl = B_\phi (2\pi s) = \mu_0 I_{enc} = 0$$

since the loop enclose no current.

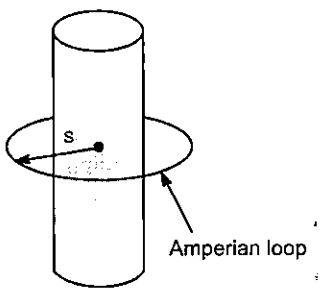


Fig. 4.72

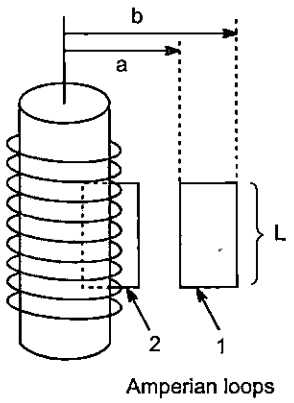


Fig. 4.73

In implies that so the magnetic field of an infinite, closely wound solenoid is parallel to the axis.

From the right-hand rule, we expect that it points upward inside the solenoid and downward outside. Now we apply Ampere's law to the two rectangular loops in Fig. 4.73. Loop 1 lies entirely outside the solenoid, with its sides at distances a and b from the axis:

$$\oint B \cdot dl = [B(a) - B(b)]L = \mu_0 I_{enc} = 0,$$

so $B(a) = B(b)$

Evidently the field outside does not depend on the distance from the axis. But we know that it is zero for large s. It must therefore be zero everywhere.

As for loop 2, which is half inside and half outside, Ampere's law gives

$$\oint B \cdot dl = BL = \mu_0 I_{enc} = \mu_0 nIL,$$

where B is the field inside the solenoid. (The right side of the loop contributes nothing, since B = 0 out there.) Conclusion:

$$B = \begin{cases} \mu_0 nI\hat{\mathbf{z}}, & \text{inside the solenoid,} \\ 0, & \text{outside the solenoid} \end{cases}$$

Notice that the field inside is uniform; in this sense the solenoid is to magnetostatics what the parallel-plate capacitor is to electrostatics: a simple device for producing strong uniform fields.

3. *A solenoid is a coil in which the wire is wound in a helix (Fig. 4.74).*

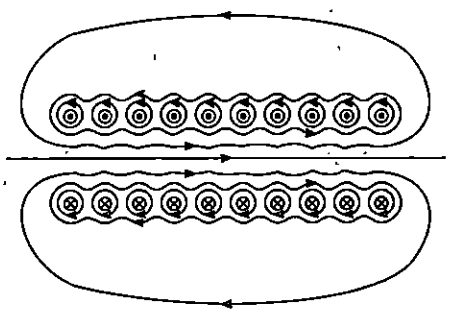


Fig. 4.74: The field inside the solenoid is almost uniform and parallel to the axis; outside, the field is almost zero.

Very close to the wire, the field lines form circles around the wire in accordance with Oersted's rule, but at distances comparable to the separation between loops, the lines merge into curves surrounding the entire coil. At great distances, the fields lines evolve into the familiar dipole pattern. For long, tightly wound coils, the pattern simplifies: within the coil the field is nearly uniform, while just outside the coil it is nearly zero. (Fig. 4.75).

The magnetic field produced by a solenoid at points P and Q just inside and just outside the coil is the sum of contributions from nearby coils ($\vec{B}_{local}$) and from pairs of coils at equal distance on either side of P and Q. One such pair is shown here, along with its contributions $\vec{B}_1$ and $\vec{B}_2$ whose sum is a vector in the +z-direction.

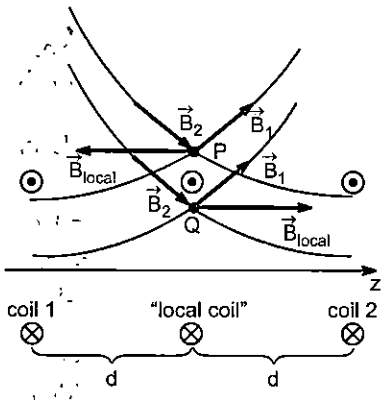


Fig. 4.75

The field $\vec{B}_{local}$ is in the +z-direction inside the solenoid at Q but in the –z-direction at P outside. Thus at P the total field opposes the field due to distant pairs of coils, and the sum is almost zero. Inside, at Q, the contributions reinforce each other. (The lengths of vectors $\vec{B}_1$ and $\vec{B}_2$ are shown exaggerated for clarity. Many such pairs sum to give $\vec{B}$ approximately equal to $-\vec{B}_{local}$ at P).

We now use Ampere's law to determine the magnetic field inside a long solenoid (ideally infinite). We choose a rectangular path as our curve, see figure. Ampere's law becomes

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = \int_a^b \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} + \int_b^c \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} + \int_c^d \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} + \int_d^a \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}}$$

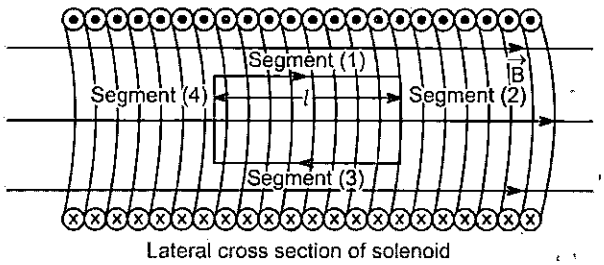


Fig. 4.76

The field outside the solenoid is small and negligible; thus the first term is zero. As $\vec{B}$ is perpendicular to the segments *bc* and *da* inside the solenoid and is nearly zero outside, so these terms are also zero. Therefore

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = \int_c^d \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = Bl$$

If a current *I* flows in the wire of the solenoid the total current enclosed in our closed curve is *NI* where *N* is the number of turns enclosed. Thus

$$\vec{\mathbf{B}} l = \mu_0 NI$$

$$\vec{\mathbf{B}} = \mu_0 \frac{N}{l} I = \mu_0 nI$$

where *n* is the number of turns per unit length.

This is the magnitude of magnetic field inside the solenoid. The field does not depend on the position of a point inside the solenoid.

Example 61

Use Ampere's law to determine the magnetic field (a) inside (b) outside a toroid. A toroid is like a solenoid bent into the shape of the circle as shown in the Fig. 4E.61.

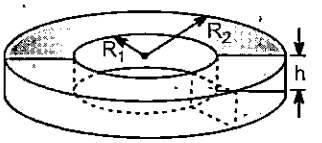


Fig. 4E.61

Solution: (a) The magnetic field lines inside the toroid are circles concentric with the toroid. We choose a circle of radius *r* inside the solenoid to perform integration of Ampere's law. We make this choice of the symmetry of the situation. *B* is same at all points along the path and tangential. So from Ampere's law,

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = \mu_0 I_{encl}$$

$$\vec{\mathbf{B}} (2\pi r) = \mu_0 NI$$

where *N* is the total number of coils and *I* is the current in each of the coils. Thus

$$\vec{\mathbf{B}} = \frac{\mu_0 NI}{2\pi r}$$

The magnitude of field is not constant over the cross-sectional area: it is largest along the inner edge (where *r* is smallest) and smallest at the outer edge. For a large and thin toroid, this difference can be negligible; the field will be uniform.

(b) Outside the toroid we choose as our closed curve a circle concentric with the toroid. This closed curve enclose *N* loops carrying current *I* in one direction and *N* loops carrying same current in opposite direction. Thus the net current enclosed is zero.

From Ampere's law,

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = \mu_0 I_{encl}$$

$$B(2\pi r) = 0$$

$$B = 0$$

The same is true for a circle with radius smaller than that of the toroid.

Example 62

A solenoid of length l and radius a has N turns of wire and carries a current I. Find the field strength at a point along the axis.

Solution: Since the solenoid is a series of closely packed loops, we use the expression for the field on the axis of a current-carrying loop. The solenoid is divided into current loops of width dz, as in Fig. 4E.62 (a), each of which contains $n\,dz$ turns, where $n = N/l$ is the number of turns per unit length. The current within such a loop is $(n\,dz)I$. From Fig. 4E.62(a) we see that

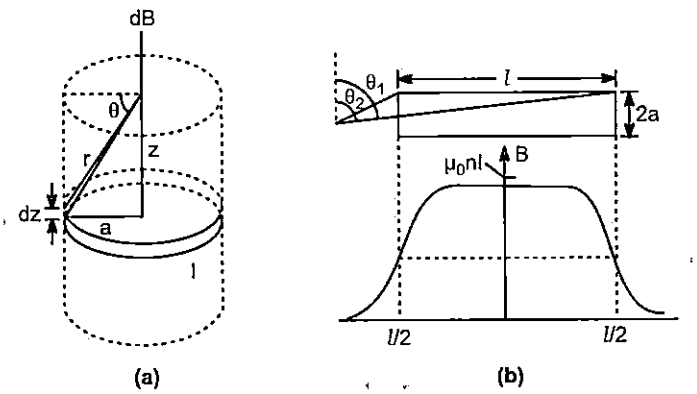


Fig. 4E.62

$$z = a\tan\theta$$

from which $$dz = a\sec^2\theta\,d\theta$$

Thus, current in loop,

$$nI\,dz = nIa\sec^2\theta\,d\theta$$

After substituting for I and z in the expression for field on the axis of a circular coil and simplifying, we find

$$d\vec{\mathbf{B}} = \frac{\mu_0 NIa^3\sec^2\theta\,d\theta}{2(a^2 + a^2\tan^2\theta)^{3/2}}$$

$$= \frac{1}{2}\mu_0 nI\cos\theta\,d\theta$$

For a finite solenoid, the total field strength is

$$\vec{\mathbf{B}} = \frac{1}{2}\mu_0 nI\int_{\theta_1}^{\theta_2}\cos\theta\,d\theta$$

$$= \frac{1}{2}\mu_0 nI(\sin\theta_2 - \sin\theta_1)$$

Fig. 4E.62 (b) shows the variation in $\vec{\mathbf{B}}$ along the axis of a solenoid for which $l = 10a$.

If the solenoid is infinite, $\theta_1 = -90°$ and $\theta_2 = 90°$, therefore, the field at the axis of the solenoid is

$$\vec{\mathbf{B}} = \mu_0 nI$$

This expression is also valid for points off the central axis; that is, the field is uniform. The field strength at the end of a very long solenoid is close to $\mu_0 nI/2$.

Example 63

A solenoid is a wire tightly wound into a helix of closely spaced turns as shown in Fig. 4E.63. A solenoid has n turns per unit length and radius R and carries a current I; its axis is along the x-axis with one end at $x = -\frac{1}{2}l$ and the other at $x = +\frac{1}{2}l$, where l is the total length of solenoid. Show that the magnetic field B on the axis at a point on the axis outside the solenoid is given by

$$B = \frac{1}{2}\mu_0 nI(\cos\theta_1 - \cos\theta_2)$$

where

$$\cos\theta_1 = \frac{x + \frac{1}{2}l}{\left[R^2 + \left(x + \frac{1}{2}l\right)^2\right]^{1/2}},$$

$$\cos\theta_2 = \frac{x - \frac{1}{2}l}{\left[R^2 + \left(x + \frac{1}{2}l\right)^2\right]^{1/2}}$$

Solution: To a good approximation, the current distribution can be considered as a set of closely spaced current loops. We can think of the field at the centre of the solenoid as the superposition of single coils extending along the entire solenoid. If n is the number of loops per unit length, there are $n\,dx$ turns of wire in a differential element dx of solenoid, with current $dI = nI\,dx$. The magnetic field at a point in the x-axis due to a loop at the origin carrying a current $nI\,dx$ is given by

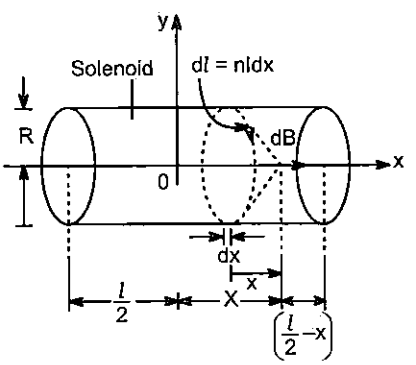


Fig. 4E.63

$$dB_x = \frac{\mu_0 2\pi R^2 nI\,dx}{4\pi(x^2 + R^2)^{3/2}}$$

Note that x is measured from the loop centre to the point B is to be computed. We place origin at the centre of the solenoid. The solenoid loops extend from $x = -(x + l/2)$ to $x = +(l/2 - x)$. Hence at point x the magnetic field is given by

$$B = \frac{\mu_0 nIR^2}{2}\int_{-(x+l/2)}^{+(l/2-x)}\frac{dx}{(x^2 + R^2)^{3/2}}$$

$$= \frac{\mu_0 nI}{2}\left(\frac{1}{\sqrt{x^2 + R^2}}\right)_{-(x+l/2)}^{+(l/2-x)}$$

$$= \frac{\mu_0 nI}{2}\left[\frac{l/2-x}{\sqrt{(l/2-x)^2+R^2}} + \frac{x+l/2}{\sqrt{(x+l/2)^2+R^2}}\right]$$

$$= \frac{\mu_0 nI}{2}[\cos\theta_1 - \cos\theta_2]$$

Example 64

Consider a sheet of current confined in the xz-plane, with a direction parallel to z-axis. The current per unit width on the x-axis is $\mathbf{j}$. *Find the magnetic field at a height H above the sheet, using Ampere's law.*

Solution: Imagine the sheet as being made-up of a large number of parallel wires all carrying current in the same direction. Field due to this wire circulates around it according to right-hand thumb rule. At a point above the sheet (say along *y*-axis) the magnetic field *B* from each wire element has an *x*-component and a *y*-component. The B_y components due to wires to the right of origin point toward

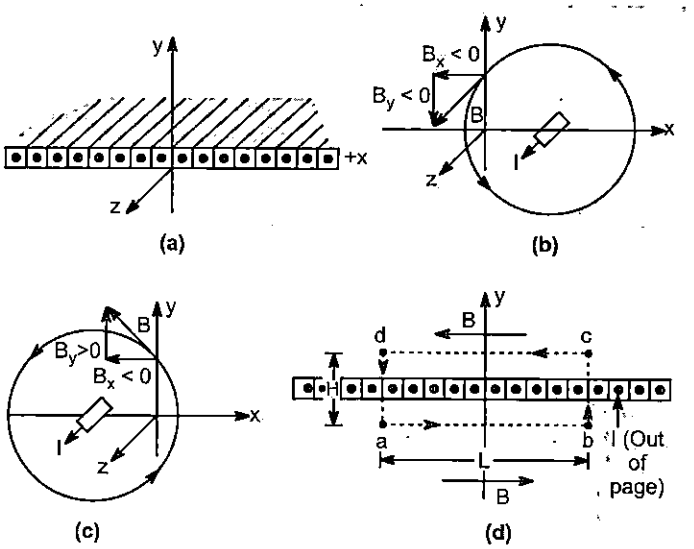


Fig. 4E.64

the sheet, whereas due to wires to the left point away from the sheet as shown in Fig. 4E.64 (a) ; they cancel out. We can take *y*-axis at any point on the sheet. So magnetic field everywhere has a negative *x*-component only. Similarly at any point below the sheet, the field has positive *x*-component only. Hence for $y > 0, B = -B\hat{\mathbf{i}}$ and for $y < 0, B = +B\hat{\mathbf{i}}$. To apply Ampere's law we consider a rectangular path of length *L* and height 2*H* as shown in figure. From Ampere's law we have

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = \mu_0 I_{encl}$$

$$\int_a^b \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} + \int_b^c \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} + \int_c^d \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} + \int_d^a \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}}$$

$$= \mu_0 (j \times L) \qquad \ldots(1)$$

Note that the total current enclosed in the closed curve is $j \times L$.

The path directions $b \to c$ and $d \to a$ are both perpendicular to the direction of *B*, so that for each of these paths

$$\vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = 0$$

Along paths $a \to b$ and $c \to d$ the magnetic field is parallel to the path direction and constant in magnitude; so eqn. (1) becomes

$$\int_a^b \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} + \int_c^d \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = \mu_0 jL$$

$$B\int_a^b dl + B\int_c^d dl = \mu_0 jL$$

$$BL + BL = \mu_0 jL$$

$$B = \frac{\mu_0 j}{2}$$

Magnitude of magnetic field is independent of the distance from the current sheet and is uniform above and below the infinite sheet.

Concept:

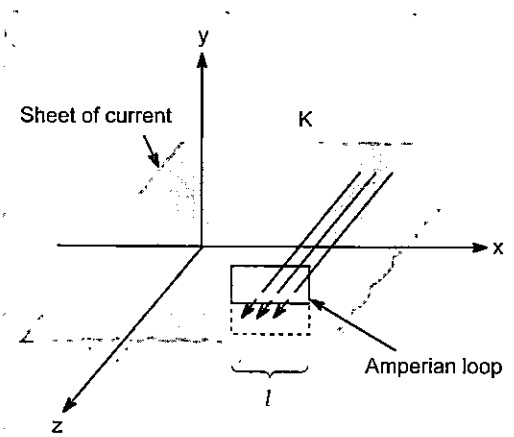


Fig. 4.77

What is the direction of B? Could it have any y-component ? No. At the Biot-Savart law shows that B is perpendicular to K. Could it have a y-component ? No again.

Any vertical contribution from a filament at +x is canceled by the corresponding filament at –x. Suppose the field pointed away from the plane. By reversing the direction of the current. It should point toward the plane (because in the Biot-Savart law, changing the sign of the current changes the sign of the field). But the x-component of B cannot possibly depend on the direction of the current in the xz-plane. So B can only have a x-component, and right-hand thumb rule shows that it points to the left above the plane and to the right below it.

We may determine the direction of the magnetic field by modeling the sheet as a set of parallel wires (Fig. 4.78). We choose the z-axis perpendicular sheet. The field directly above a single "wire" points in the –y-direction. "Wires" to the right produce magnetic field vectors with negative y- and z-components, while wires to the left produce magnetic field vectors with negative y-and positive z-components.

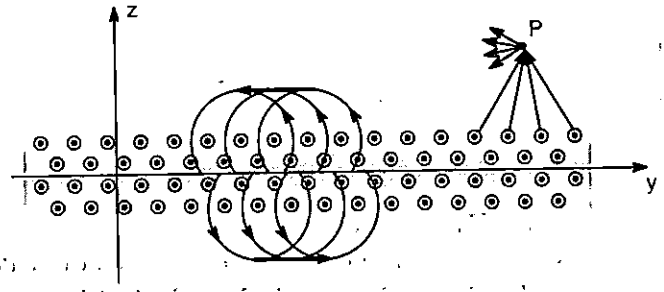


Fig. 4.78

We model the sheet as an infinite set of parallel wires. Each wire produces circular field lines. The superposition of fields due to different wires is parallel to the sheet and perpendicular to I. To be more precise, at any point P, we choose pairs of wires equidistant from P. The magnetic field due to each pair is in the –y-direction. The direction of $\vec{\mathbf{B}}$ *reverses across the sheet. In an infinite sheet, every wire on the right has a corresponding wire an equal distance away on the left. The sum of the vectors due to the two wires is in the –y-direction. Thus, the total magnetic field is in the –y-direction. The same argument shows that the magnetic field is in the +y-direction below the sheet. At equal distance from the sheet, the field strength is the same both above and below the sheet:* B_y *(above) =* $-B_y$ *(below). The same symmetry exists within the sheet.*

Example 65

A coaxial cable consists of a single wire surrounded by a cylindrical metallic braid as shown in Fig. 4E.65. The inner wire carries current to the other end of the cable and the outer braid carries the return current and is generally considered ground. Describe the magnetic field in the space between conductors B (a < r < b) and outside cable B (r > b).

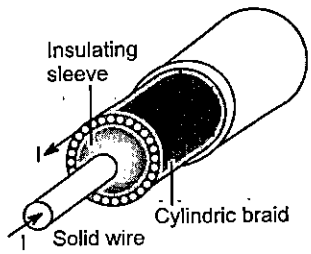


Fig. 4E.65

Solution: We take a circular closed path in the space between conductors, to apply Ampere's law.

$$\oint B \cdot dl = \mu_0 I_{\text{incl}}$$

$$B\oint dl = \mu_0 I$$

$$B2\pi r = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$

The result is same as in case of a long wire. The current in the outer conductor has no bearing on it. In Ampere's law the current that lies within the closed curve is included. As long as the currents outside the closed curve do not affect the symmetry of the field; they do not contribute to the field along the curve at all.

Similarly we can draw a circular closed curve, because from symmetry the expected field is circular. The net current enclosed in the closed curve is zero as there are two equal currents in opposite directions. Therefore Ampere's law becomes

$$B2\pi r = 0$$

or $$B = 0$$

The field outside the cable is zero.

Example 66

Show that in any region of space where, there are no currents, the magnetic field cannot be both unidirectional and non-uniform as shown in Fig. 4E.66(a).

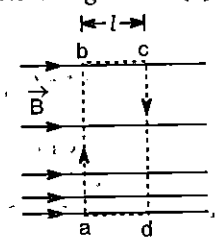


Fig. 4E.66 (a)

Solution: The number of lines per unit area in the upper region is smaller; it indicates the fields has a smaller magnitude at the top than in the lower region. We consider a rectangular closed curve *abcd* as shown in the Fig. 4E.66(a) to apply Ampere's law. Since no current is enclosed by this closed curve,

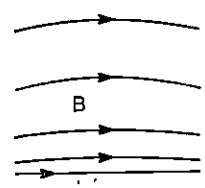


Fig. 4E.66 (b)

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = 0$$

$$\oint \vec{\mathbf{B}}\cdot d\vec{\mathbf{l}} = \int_a^b \vec{\mathbf{B}}\cdot d\vec{\mathbf{l}} + \int_b^c \vec{\mathbf{B}}\cdot d\vec{\mathbf{l}} + \int_c^d \vec{\mathbf{B}}\cdot d\vec{\mathbf{l}} + \int_d^a \vec{\mathbf{B}}\cdot d\vec{\mathbf{l}}$$

$$= B_{bc}l - B_{da}l = (B_{bc} - B_{da})l \qquad \ldots(1)$$

As integral along sections *ab* and *cd* is zero, since $B \perp dl$.

As $B_{bc} < B_{da}$, hence $\oint \vec{\mathbf{B}}\cdot d\vec{\mathbf{l}} \neq 0$.

There is a contradiction; $\oint \vec{\mathbf{B}}\cdot d\vec{\mathbf{l}}$ cannot be both zero (as $I_{encl} = 0$) and non-zero. So we can say that a non-uniform unidirectional field is not in accordance with Ampere's law. A non-uniform field whose direction changes as shown in Fig. 4E.66 (b), is in conformity with Ampere's law. Verify this statement yourself.

Example 67

A long hollow cylindrical conductor with inner radius a and outer radius b carries a current I uniformly distributed over the cross-sectional area of the conductor (see Fig. 4E.67). Use Ampere's law to show that the magnitude of the magnetic field at a radius r from the axis of the conductor is zero if $r < a$ and $\frac{\mu_0}{4\pi}\frac{2I}{r}\frac{r^2 - a^2}{b^2 - a^2}$ if $b < r < a$ and equal to $\frac{\mu_0 I}{2\pi r}$ if $r > b$.

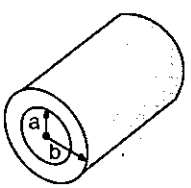


Fig. 4E.67

Solution: (a) Due to symmetry of the conductor we expect magnetic field lines to be circular. Therefore we choose a circular closed curve of radius $r < a$. From Ampere's law,

$$\oint \vec{\mathbf{B}}\cdot d\vec{\mathbf{l}} = B2\pi r = \mu_0 I_{encl}$$

as $I_{encl} = 0, \quad B(r < a) = 0$

(b) Current density $j = \frac{I}{\pi(b^2 - a^2)}$ inside the conductor. Now we take a circular closed curve in the region $a < r < b$. The current enclosed by curve,

$$I_{encl} = (j)[\pi(r^2 - a^2)]$$

$$= \frac{I(r^2 - a^2)}{(b^2 - a^2)}$$

From Ampere's law,

$$\oint \vec{\mathbf{B}}\cdot d\vec{\mathbf{l}} = B2\pi r = \mu_0 \frac{I(r^2 - a^2)}{(b^2 - a^2)}$$

$$B = \frac{\mu_0 I(r^2 - a^2)}{2\pi r(b^2 - a^2)}$$

(c) For a circular closed curve outside the conductor ($r > b$),

$$\oint \vec{\mathbf{B}}\cdot d\vec{\mathbf{l}} = B2\pi r = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$

As total current is enclosed in our circular loop.

Example 68

A long conductor of circular cross-section with radius R has current density

$$J(r) = \rho_0 \frac{r^2}{R^2}\left(for \frac{R}{2} \le r \le R\right)$$

$$= 0\left(for\ r < \frac{R}{2}\right)$$

(for $r < R/2$) into the plane of paper.
There is a point P at distance 'a' from the axis of the conductor [$a > R$]. Two infinitely long thin conducting wires carrying current I_0 in the same direction are placed at distance a from O perpendicular to OP and parallel to conductor at either side such that the magnetic field at P is zero. Find the current I_0 in the wires and the direction of current as compared with the direction of current in the conductor.

Solution: Current through conductor

$$I = \int_0^R 2\pi r\, dr\, J(r)$$

$$= \int_0^{R/2} 2\pi r\, J(r)\, dr + \int_{R/2}^R 2\pi r\, J(r)\, dr$$

$$= \int_0^{R/2} 2\pi r \cdot 0 \cdot dr + \int_{R/2}^R 2\pi r \rho_0 \frac{r^2}{R^2} dr$$

$$= 0 + \frac{2\pi\rho_0}{R^2}\int_{R/2}^R r^3 dr$$

$$= 0 + \frac{2\pi\rho_0}{R^2}\left[\frac{r^4}{4}\right]_{R/2}^R$$

$$= \frac{2\pi\rho_0}{R^2 4}\left(R^4 - \frac{R^4}{16}\right) = \frac{15\pi}{32}\rho_0 R^2$$

Let us consider a circle with centre O and radius OP in a plane $\perp$ to the conductor. For all points on the circle, due the symmetry, B is same due to the conductor. Applying Ampere's law

$$\int_0^{2\pi a} B \cdot dl = \mu_0 I$$

$$\Rightarrow \quad B \cdot 2\pi a = \mu_0 \frac{15}{32}\pi\rho_0 R^2$$

$$\Rightarrow \quad B = \frac{15}{64}\frac{\mu_0 \rho_0 R^2}{a}$$

As current is into the plane B is downward to P. Now field due to wires A_1 and A_2 must cancel B. That is possible when the current in the wires is out of the plane.

Field due to A_1, $\quad |\vec{\mathbf{B}}_1| = \frac{\mu_0}{4\pi}\frac{2I}{a\sqrt{2}}$

Due to A_2, $\quad |\vec{\mathbf{B}}_2| = \frac{\mu_0}{4\pi}\frac{2I}{a\sqrt{2}}$

Resultant of B_1 and B_2,

$$\vec{B}' = \vec{B}_1 + \vec{B}_2 = 2\frac{\mu_0}{4\pi}\frac{2I_0}{a\sqrt{2}}\cdot\frac{1}{\sqrt{2}}$$

$$= \frac{2\mu_0 I_0}{4\pi a} \text{ opposite to } \vec{B}.$$

Net field at P, $\vec{B} + \vec{B}' = 0$

$$\frac{\mu_0}{4\pi}\frac{2I_0}{a} - \frac{15}{64}\mu_0\frac{\rho_0 R^2}{a} = 0$$

$$I_0 = \frac{15\pi\rho_0 R^2}{32}.$$

CLASSICAL MAGNETISM

The line joining the two poles of a magnet is called **magnetic axis.** The plane in which a freely suspended magnet lies is called **magnetic meridian.**

The poles of a magnet do not exist exactly at the ends, the separation between two poles along the magnetic axis is called magnetic length.

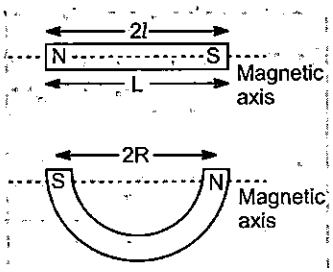


Fig. 4.79

The **pole strengths** of the north and south poles are expressed as $(+m)$ and $(-m)$ respectively. The pole strength depends on the nature of material of magnet, area of cross-section, the state of magnetisation. If a given magnet is divided into two equal parts such that the cross-section of each part is half that of the initial, each part will be a magnet with half the initial pole-strength [Fig. 4.80 (b)], if the magnet is cut into two equal parts such that its cross-section remains unchanged, each part will be a magnet with same polestrength [Fig 4.80 (c)].

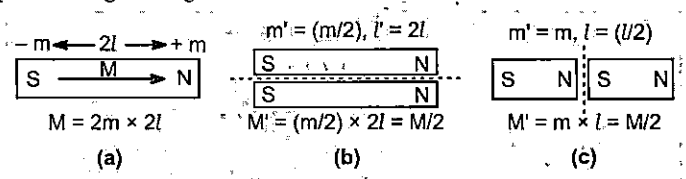


Fig. 4.80

The **magnetic moment** of a magnet is a vector quantity having magnitude equal to the product of pole-strength with effective length, and direction along the axis of the magnet from south to north pole, *i.e.*,

$$\vec{M} = m \times (2l) \times \vec{n} \quad \text{with} \quad M = 2ml$$

Here $\vec{n}$ is a unit vector along the axis of magnet directed from south pole to north pole.

If m is the pole-strength and L is the length of the wire, its magnetic moment

$$M = mL$$

If it is bent in a semicircular arc of radius r, $\pi r = L$, *i.e.*, $r = (L/\pi)$, the new effective length will become $2r = (2L/\pi)$ while pole-strength will remain unchanged, *i.e.*, m.

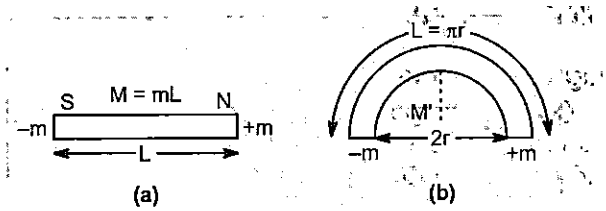


Fig. 4.81

So, its new magnetic moment

$$M' = m(2r) = m\frac{2L}{\pi} = \frac{2}{\pi}M \quad [\text{as } M = mL]$$

As magnetic moment is a vector, in case of two magnets having magnetic moment M_1 and M_2 with angle θ between them, the resulting magnetic moment,

$$M = [M_1^2 + M_2^2 + 2M_1M_2\cos\theta]^{1/2}$$

with $$\tan\phi = \left[\frac{M_2\sin\theta}{M_1 + M_2\cos\theta}\right]$$

Inverse-square law: The force between two isolated poles of strengths m_1 and m_2 separated by a distance r in vacuum is given by

$$\vec{F} = A\frac{m_1m_2}{(r)^3}\vec{r},$$

i.e., $$F = A\frac{m_1m_2}{(r)^2}$$

where A is a constant of proportionality and depends on the system of units.

In SI (Sommerfeld) units,

$$A = \frac{\mu_0}{4\pi} = 10^{-7}$$

$$F = \frac{\mu_0}{4\pi}\frac{m_1m_2}{(r)^2}$$

r in m, F in N and m in A-m.

Here μ_0 is a constant called **permeability** of *free space.*

Magnetic Field Vector $\vec{B}$

If a test pole of strength m_0 at a point P in a magnetic field experiences a force $\vec{F}$, the field vector at that point will be

$$\vec{B} = \frac{\vec{F}}{m_0}, \quad \textit{i.e.,} \quad \vec{F} = m_0\vec{B} \qquad ...(1)$$

Now if the field is produced by a pole of strength m from which the field point is at a distance r, by inverse square law,

$$\vec{F} = \frac{\mu_0}{4\pi}\frac{m_0m}{r^2}\vec{n}$$

so, $$\vec{B} = \frac{\vec{F}}{m_0} = \frac{\mu_0}{4\pi}\frac{m}{r^2}\vec{n}$$

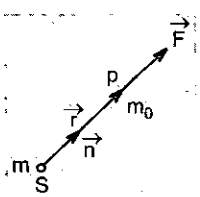


Fig. 4.82

Magnetic Dipole

(a) End-on position: A position on the magnetic axis of a bar magnet is called an *end-on position.* The magnetic field at P due to the north pole is

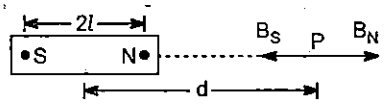


Fig. 4.83

$$B_N = \frac{\mu_0}{4\pi}\frac{m}{(d-l)^2}$$

directed away from the magnet. The field due to the south pole is

$$B_S = \frac{\mu_0}{4\pi}\frac{m}{(d+l)^2}$$

Directed towards the magnet. The resultant field is

$$B = B_N - B_S$$
$$= \frac{\mu_0 m}{4\pi}\left[\frac{1}{(d-l)^2} - \frac{1}{(d+l)^2}\right]$$
$$= \frac{\mu_0}{4\pi}\frac{2Md}{(d^2-l^2)^2}$$

where $M = 2ml$ is the magnetic moment of the magnet. If $d \gg l$, the magnet may be called a magnetic dipole and the field at an end-on position is

$$B = \frac{\mu_0}{4\pi}\frac{2M}{d^3}$$

(b) Broadside-on position: A position on a perpendicular bisector of the bar magnet is called *broadside-on position.* The field at P due to the north pole may be written as

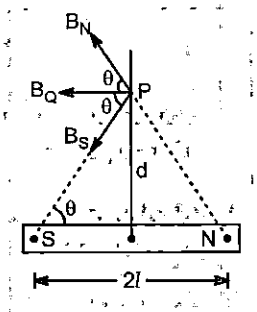


Fig. 4.84

$$|\vec{\mathbf{B}}_N| = \frac{\mu_0}{4\pi}\frac{m}{(d^2+l^2)}$$

This gives the magnitude as well as the direction of the field due to the north pole. The field due to the south pole is

$$|\vec{\mathbf{B}}_S| = \frac{\mu_0}{4\pi}\frac{m}{(d^2+l^2)},$$

Let $|\vec{\mathbf{B}}_N| = |\vec{\mathbf{B}}_S| = B$

The resultant field at P is

$$|\vec{\mathbf{B}}_R| = \vec{\mathbf{B}}_N + \vec{\mathbf{B}}_S = 2B\cos\theta$$
$$= \frac{\mu_0}{4\pi}\frac{2ml}{(d^2+l^2)^{3/2}}$$

The magnitude of the field is

$$|\vec{\mathbf{B}}_R| = \frac{\mu_0}{4\pi}\frac{m\cdot 2l}{(d^2+l^2)^{3/2}}$$

$$= \frac{\mu_0}{4\pi}\frac{M}{(d^2+l^2)^{3/2}}$$

where $M = 2ml$ is the magnetic moment of the magnet. The direction of the field is antiparallel to magnetic moment vector $\vec{\mathbf{M}}$.

If $d \gg l$, the magnetic field at a point in broadside-on position is

$$B = \frac{\mu_0}{4\pi}\frac{M}{d^3}$$

Analogous to an electric dipole, the field due to a dipole of moment $\vec{\mathbf{M}}$ at point (r, θ) is given by

$$B_r = \frac{\mu_0}{4\pi}\frac{2M\cos\theta}{r^3}$$

and $$B_\theta = \frac{\mu_0}{4\pi}\frac{M\sin\theta}{r^3} \quad ...(1)$$

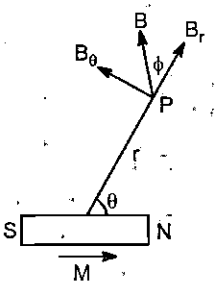


Fig. 4.85

so that, $B = \sqrt{(B_r^2 + B_\theta^2)}$

$$= \frac{\mu_0}{4\pi}\frac{M}{r^3}\sqrt{(1+3\cos^2\theta)} \quad ...(2)$$

with, $$\tan\phi = \frac{B_\theta}{B_r} = \frac{1}{2}(\tan\theta) \quad ...(3)$$

Magnetic Dipole in a Uniform Field

(a) Couple: When a dipole is placed in a uniform field as shown in Fig. 4.86, the net force on it,

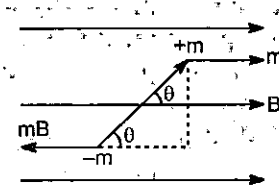


Fig. 4.86

$$F_R = mB + (-m)B = 0$$

while couple

$$C = mB \times 2l\sin\theta$$

i.e., $$\vec{\mathbf{C}} = MB\sin\theta \qquad [\text{as } M = m \times 2l]$$

i.e., $$\vec{\mathbf{C}} = \vec{\mathbf{M}} \times \vec{\mathbf{B}}$$

The forces tend to align the dipole with the magnetic field.

Potential Energy

Work done in rotating a dipole in uniform magnetic field through a small angle $d\theta$ will be

$$dW = C\,d\theta = MB\sin\theta\,d\theta$$

So work done in rotating a dipole from angular position θ_i to θ_j w.r.t. field,

$$W = \int_{\theta_i}^{\theta_j} MB\sin\theta\,d\theta = MB[\cos\theta_i - \cos\theta_j]$$

From work-energy theorem,

$$W = \Delta U = U_f - U_i$$
$$= (-MB\cos\theta_f) - (-MB\cos\theta_i)$$

Thus potential energy of dipole at an angle θ with dipole is

$$U = -MB\cos\theta = -\vec{\mathbf{M}}\cdot\vec{\mathbf{B}}$$

Consider a dipole suspended in a uniform field. Now if it is given a small angular displacement θ about its equilibrium position, the restoring couple will be

$$C = -MB\sin\theta = -MB\theta \qquad [\text{as } \sin\theta = \theta]$$

or $$I\frac{d^2\theta}{dt^2} = -MB\theta \qquad \left[\text{as } C = I\alpha = I\frac{d^2\theta}{dt^2}\right]$$

or $$\frac{d^2\theta}{dt^2} = -\omega^2\theta \quad \text{with } \omega^2 = \frac{MB}{I}$$

This is the standard equation of angular SHM with time period $T = (2\pi/\omega)$; so the dipole will execute angular SHM with time period

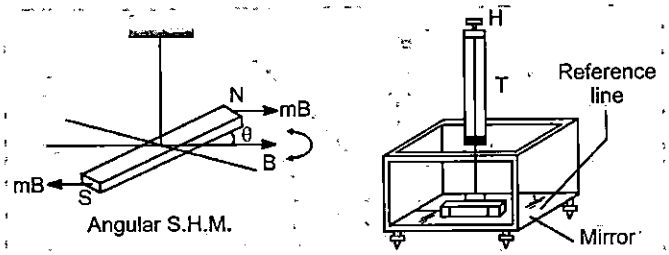


Fig. 4.87

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{MB}}$$

Fig. 4.87 (b) shows an oscillation magnetometer; its basic use is to measure MB_H for a bar magnet.

$$MB_H = \frac{4\pi^2 I}{T^2}$$

For a bar magnet of rectangular cross-section,

$$I = \frac{m(a^2+b^2)}{12}$$

where a is the geometric length, b is the breadth and m is the mass of the magnet.

Tangent Law

Fig. 4.88 shows a magnetic dipole of magnetic moment $\vec{\mathbf{M}}$ in two mutually perpendicular uniform magnetic fields $\vec{\mathbf{B}}_H$ and $\vec{\mathbf{B}}_V$. For the equilibrium of the magnet,

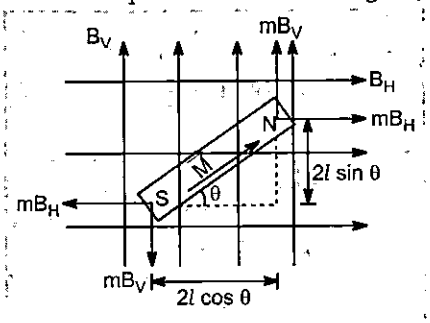


Fig. 4.88

$$\vec{\mathbf{M}}\times\vec{\mathbf{B}}_H = \vec{\mathbf{M}}\times\vec{\mathbf{B}}_V$$

$$MB_H\sin\theta = MB_V\sin(90-\theta)$$

or $$\tan\theta = \frac{B_V}{B_H}$$

Alternatively,

$$mB_H(2l\sin\theta) = mB_V\cdot 2l\cos\theta$$

or $$\tan\theta = \frac{B_V}{B_H}$$

The magnet aligns itself along the resultant magnetic field. This principle is employed in deflection magnetometer and tangent galvanometer.

Terrestrial Magnetism

Earth behaves as a magnetic dipole inclined at an angle of 11.5° to Earth's axis of rotation as shown in the figure. The north pole of Earth's dipole is towards the geographic south and the south pole towards the geographic north.

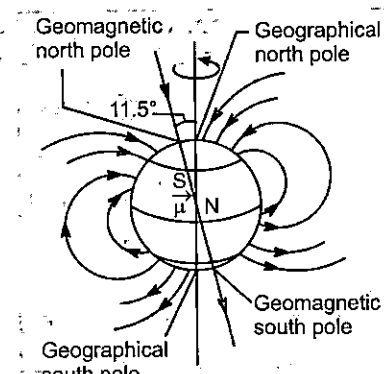


Fig. 4.89

Declination

A plane that includes the geographical poles, the axis of rotation of the Earth and a given point P on the Earth's surface, is called the *geographical meridian* at the point P. Similarly, the plane that includes the geomagnetic poles, the dipole-axis of the Earth and the point P is called the *magnetic meridian* at the point P.

The angle made by the magnetic meridian at a point with the geographical meridian is called the *declination* at that point.

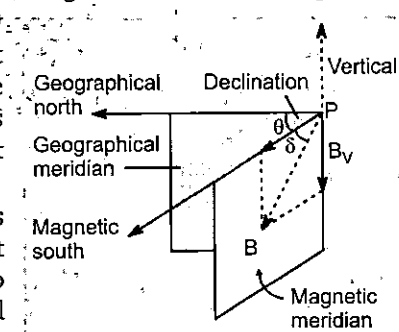


Fig. 4.90

A compass needle is a short and light magnetic needle, free to rotate about a vertical axis. The needle stays in equilibrium when it is in the magnetic meridian. The north direction shown by the needle makes an angle equal to the declination with the true north.

Inclination or Dip

A freely suspended magnet aligns itself in the direction of Earth's magnetic field. The magnet lies in the magnetic meridian and is inclined to the horizontal line.

The angle made by the Earth's magnetic field with the horizontal direction in the magnetic meridian, is called the *inclination* or *dip* at that point.

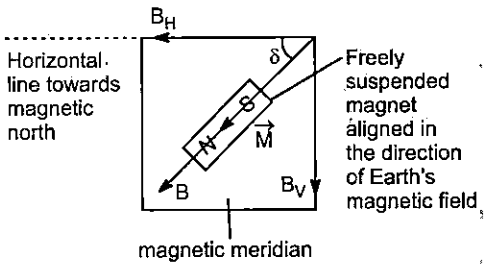


Fig. 4.91

The vertical component of the Earth's magnetic field points downwards in the northern hemisphere, the north pole of a freely suspended magnet, therefore, *dips*.

In Fig. 4.91 shown, the resultant magnetic field is in the magnetic meridian, δ is angle of dip, B_H and B_V denote horizontal and vertical components of Earth's magnetic field respectively.

$$B_H = B\cos\delta$$
$$B_V = B\sin\delta$$
$$|\vec{\mathbf{B}}| = \sqrt{B_H^2 + B_V^2}$$
$$\tan\delta = \frac{B_V}{B_H}$$

Determination of Dip at a Place

The dip at a place can be determined by an apparatus known as *dip circle*.

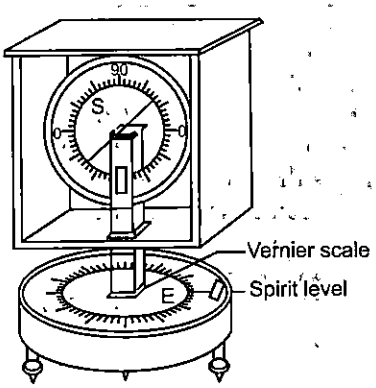


Fig. 4.92

During the experiment, the base of the dip circle is made horizontal, the vertical scale S and the needle is rotated about the vertical axis till the needle becomes vertical and reads 90°–90° on the vertical scale. In this condition, the plane of the circular scale S is perpendicular to the magnetic meridian. The horizontal component B_H is perpendicular to this plane and hence cannot rotate the needle. The needle is aligned with the vertical component B_V and hence reads 90°–90°. The reading of the vernier is noted and the glass cover is rotated exactly through 90° from this position. The plane of the circular scale S is now the same as the magnetic meridian, the Earth's magnetic field B is in this same plane, the needle rests in the direction of $\vec{\mathbf{B}}$. The inclination of the needle on the vertical scale now directly gives the value of the dip.

At the equator the Earth's magnetic field is parallel to the Earth's surface, angle of dip and vertical component of Earth's magnetic field are zero. Therefore a freely suspended magnet will be horizontal at the equator.

At the poles the Earth's magnetic field is perpendicular to the surface of the Earth, angle of dip is 90° and the horizontal component of the Earth's magnetic field is zero. Therefore a freely suspended magnet will become vertical at the poles.

Example 69

Considering the Earth as a short magnet with its centre coinciding with the centre of the Earth, show that the angle of dip ϕ is related to magnetic latitude λ through the relation

$$\tan\phi = 2\tan\lambda.$$

Solution: Considering the situation is shown in Fig. 4E.69, for a magnetic dipole, at position (r, θ) the radial (B_r) and tangential components of field are

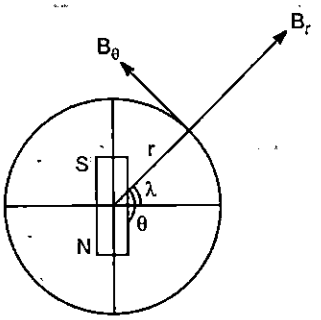


Fig. 4E.69

$$B_r = \frac{\mu_0}{2\pi}\frac{2M\cos\theta}{r^3}$$

and $$B_\theta = \frac{\mu_0}{4\pi}\frac{M\sin\theta}{r^3} \quad \ldots(1)$$

and as $$\tan\phi = \frac{B_V}{B_H} = -\frac{B_r}{B_\theta},$$

So from eqn. (1),

$$\tan\phi = -2\cot\theta$$

But from Fig. 4E.79,

$$\theta = 90° + \lambda$$

So, $$\tan\phi = -2\cot(90° + \lambda),$$

i.e., $$\tan\phi = 2\tan\lambda$$

Example 70

When the upper tip of a magnetic dip needle is loaded with a small mass, the dip recorded is found to drop from 45° to 30°. If the total magnetic intensity at the place is 0.42 oersted and the pole strength of the magnetic needle is 200 CGS units, what is the mass attached ?

Solution: As shown in Fig. 4E.70, on loading the upper end of the needle for its rotational equilibrium,

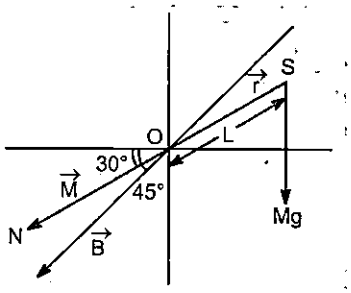


Fig. 4E.70

$$\vec{r}\times\vec{F}+\vec{M}\times\vec{B}=0$$

i.e., $Mg\times L\cos 30°=m\times 2L\times B\sin(45°-30°)$

i.e., $$M=\frac{2mB\sin 15°}{g\cos 30°}$$

Substituting the given data,

$$M=\frac{2\times 200\times 0.42\times 0.2588}{980\times(\sqrt{3}/2)}$$

$$=0.051\,g$$

Apparent Dip

If the dip circle is not kept in the magnetic meridian, angle made by the needle with the horizontal is called the *apparent dip* for this plane. If the dip circle is at an angle α to the meridian, the effective horizontal component in this plane is $B_H' = B_H\cos\alpha$. The vertical component is still B_V. If δ' is the apparent dip and δ is the true dip, we have

$$\tan\delta'=\frac{B_V}{B_H'}$$

or $$\cot\delta'=\frac{B_H'}{B_V}=\frac{B_H\cos\alpha}{B_V}$$

$$\cot\delta'=\cot\delta\cos\alpha \qquad ...(1)$$

Now the dip circle is rotated through an angle of 90° from this position. It will now make an angle $(90°-\alpha)$ with the meridian. The effective horizontal component in this plane is $B_H''=B_H\sin\alpha$. if δ'' be the apparent dip, we shall have

$$\cot\delta''=\cot\delta\sin\alpha \qquad ...(2)$$

Squaring and adding (1) and (2)

or $$\cot^2\delta'+\cot^2\delta''=\cot^2\delta$$

Thus, one can get the true dip δ without locating the magnetic meridian.

TANGENT GALVANOMETER

Tangent galvanometer measures electric current. As shown in Fig. 4.93 an aluminium pointer P is rigidly attached with the compass needle and perpendicular to it. The compass needle together with the pointer can rotate freely about the vertical axis.

When there is no current through the galvanometer, the compass needle is in the magnetic north-south direction. Initially, the base is rotated in such a way that the plane of the coil is parallel to the compass needle. The plane then coincides with the magnetic meridian.

When the current to be measured is passed through the coil, it produces a magnetic field at the centre and the compass needle deflects under its action, the pointer deflects through the same angle.

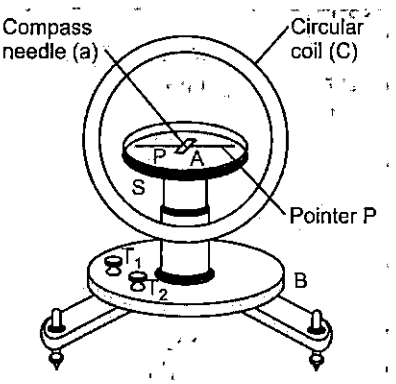


Fig. 4.93

Let the current through the coil be I, radius of the coil be r and the number of turns in it be n. The magnetic field produced at the centre is

$$B=\frac{\mu_0 In}{2r} \qquad ...(1)$$

This field is perpendicular to the plane of the coil (in accordance with the right hand thumb rule) and perpendicular to the magnetic meridian and hence to the horizontal component B_H of the Earth's magnetic field. The resultant horizontal magnetic field is

$$B_r=\sqrt{B^2+B_H^2}$$

in a direction making an angle θ with B_H (Fig. 4.94) where

$$\tan\theta=B/B_H \quad \text{[tangent law]} \qquad ...(2)$$

The deflection of the needle from its original position is θ as given by (2). Using (1) and (2),

$$B_H\tan\theta=\frac{\mu_0 In}{2r}$$

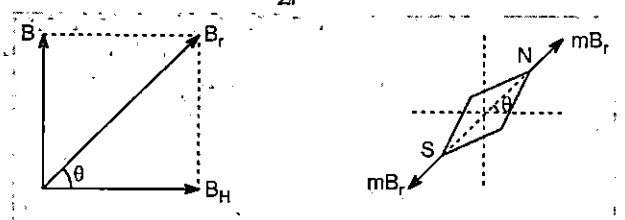


Fig. 4.94

or $$I=\frac{2rB_H}{\mu_0 n}\tan\theta$$

or $$I=K\tan\theta$$

where $K=\frac{2rB_H}{\mu_0 n}$ is a constant for the given galvanometer at a given place. This constant is called the *reduction factor* of the galvanometer. The reduction factor may be obtained by passing a known current I through the galvanometer, measuring θ.

Sensitivity

For good sensitivity the change in deflection is large for a given fractional change in current.

We have $I = K \tan\theta$

or $dI = K \sec^2\theta\, d\theta$

or $\frac{dI}{I} = \frac{K \sec^2\theta\, d\theta}{K \tan\theta} = \frac{2\, d\theta}{\sin 2\theta}$

or $d\theta = \frac{1}{2}\sin 2\theta\left(\frac{dI}{I}\right)$

Thus, for good sensitivity, $\sin 2\theta$ should be as large as possible. This is the case when $\theta = 45°$. So, the tangent galvanometer is most sensitive when the deflection is around 45°.

DEFLECTION MAGNETOMETER

Fig. 4.95 shows a deflection magnetometer which consists of a small compass needle A pivoted at the centre of a graduated circular scale. The graduations are marked from 0° to 90° in each quadrant.

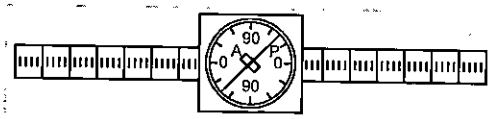


Fig. 4.95

The basic use of a deflection magnetometer is to determine M/B_H for a permanent bar magnet. Here M is the magnetic moment of the magnet and B_H is the horizontal component of the Earth's magnetic field. The quantity M/B_H can be measured in two standard positions of the magnetometer, Tan-A position of Gauss and Tan-B position of Gauss.

Tan-A Position

In this, the arms of the magnetometer are kept along the magnetic east-west direction. The magnet is kept on one of the arms, parallel to its length (Fig. 4.96). The needle

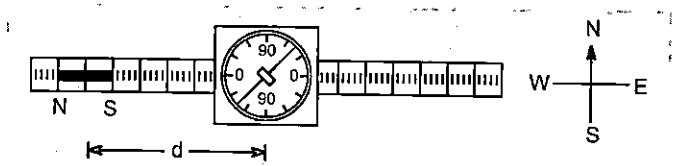


Fig. 4.96

deflects and the deflection θ in its equilibrium position is read from the circular scale. Let the distance of the centre of the magnet from the centre of the compass be d and $2l$ be the magnetic length of the magnet. In Tan-A position of the magnetometer, the compass needle is in end-on position of the bar magnet, its magnetic field at the site of the needle is

$$B = \frac{\mu_0}{4\pi}\frac{2Md}{(d^2 - l^2)^2}.$$

This field is along the length of the magnet, that is, towards east or towards west, perpendicular to the Earth's field B_H. From the tangent law, we have

$$B = B_H \tan\theta$$

or $\frac{\mu_0}{4\pi}\frac{2Md}{(d^2 - l^2)^2} = B_H \tan\theta$

or $\frac{M}{B_H} = \frac{4\pi}{\mu_0}\frac{(d^2 - l^2)^2}{2d}\tan\theta.$

knowing all the quantities on the right-hand side, we gets M/B_H.

Tan-B Position

In this position, the arms of the magnetometer are kept in the magnetic north-south direction.

In Tan-B position, the compass is in broadside-on position of the magnet, its magnetic field is

$$B = \frac{\mu_0}{4\pi}\frac{M}{(d^2 + l^2)^{3/2}}.$$

The field is parallel to the axis of the magnet and hence it is towards east or towards west. The Earth's magnetic field is from south to north. Using tangent law,

$$B = B_H \tan\theta$$

or $\frac{\mu_0}{4\pi}\frac{M}{(d^2 + l^2)^{3/2}} = B_H \tan\theta$

or $\frac{M}{B_H} = \frac{4\pi}{\mu_0}(d^2 + l^2)^{3/2}\tan\theta$

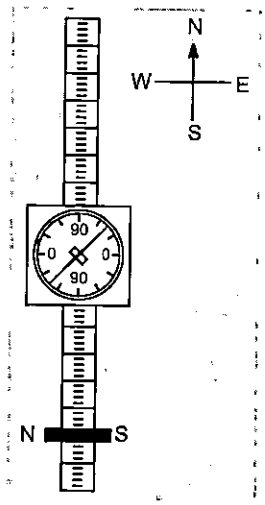


Fig. 4.97

* Comparison of the magnetic moments M_1 and M_2 of two magnets

It can be determined by calculating M_1/B_H and M_2/B_H separately for the two magnets and then get the ratio M_1/M_2. Another simple method known as *null method*. Experiments can be done either in Tan-A position or in Tan-B position. The two magnets are placed on the two arms of the magnetometer. The distances of the magnets from the center of the magnetometer are so adjusted that the deflection of the needle is zero. This will happen if the magnetic field at the needle due to the first magnet is equal in magnitude to the field due to the other magnet. If the magnetometer is used in Tan-A position.

$$\frac{\mu_0}{4\pi}\frac{2M_1 d_1}{(d_1^2 - l_1^2)^2} = \frac{\mu_0}{4\pi}\frac{2M_2 d_2}{(d_2^2 - l_2^2)^2}$$

or $\frac{M_1}{M_2} = \frac{d_2(d_1^2 - l_1^2)^2}{d_1(d_2^2 - l_2^2)^2}.$

If the magnetometer is used in Tan-B position,

$$\frac{\mu_0}{4\pi}\frac{M_1}{(d_1^2 + l_1^2)^{3/2}} = \frac{\mu_0}{4\pi}\frac{M_2}{(d_2^2 + l_2^2)^{3/2}}$$

or $\frac{M_1}{M_2} = \frac{(d_1^2 + l_1^2)^{3/2}}{(d_2^2 + l_2^2)^{3/2}}.$

Null method is easier and better because, here the deflection remains zero and the possible errors in the measurement of θ do not occur.

* Comparison of the horizontal components of the Earth's magnetic field at two places

Suppose the horizontal component of the Earth's magnetic field is B_{H_1} at the first place and B_{H_2} at the second. A deflection magnetometer is taken and a bar magnet is kept at a distance d in Tan-A position. The deflection θ_1 of the needle is noted. The magnetometer is now taken to the second place and the same magnet is kept at the same distance d in Tan-A position. The deflection θ_2 is noted. We have

$$\frac{M}{B_{H_1}} = \frac{4\pi}{\mu_0}\frac{(d^2 - l^2)^2}{2d}\tan\theta_1$$

and $$\frac{M}{B_{H_2}} = \frac{4\pi}{\mu_0}\frac{(d^2 - l^2)^2}{2d}\tan\theta_2.$$

Thus, $$\frac{B_{H_1}}{B_{H_2}} = \frac{\tan\theta_2}{\tan\theta_1}$$

The experiment can also be done in Tan-B position. We get the same relation for $\frac{B_{H_1}}{B_{H_2}}$.

Example 71

Two identical dipoles each of magnetic moment 1.0 A-m^2 are placed at a separation of 2 m with their axes perpendicular to each other. What is the magnetic field at a point midway between the dipoles ?

Solution: The point P will be in end-on position w.r.t. one dipole and in broadside on position w.r.t. the other.

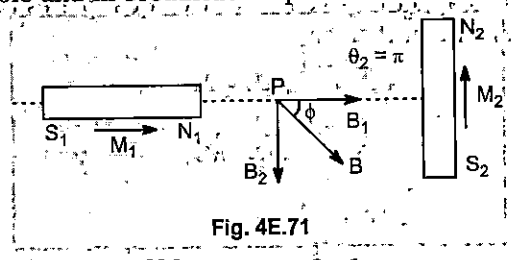


Fig. 4E.71

so, $$B_1 = \frac{\mu_0}{4\pi}\frac{2M_1}{r_1^3} = 10^{-7} \times \frac{2\times 1}{(1)^3} = 2\times 10^{-7}\,\text{T}$$

and $$B_2 = \frac{\mu_0}{4\pi}\frac{M_2}{r_2^3} = 10^{-7} \times \frac{1}{(1)^3} = 1\times 10^{-7}\,\text{T}$$

B_1 and B_2 are perpendicular to each other. The resultant magnetic field is

$$B = \sqrt{B_1^2 + B_2^2} = 10^{-7}\sqrt{2^2 + 1^2} = \sqrt{5}\times 10^{-7}\,\text{T}$$

and $$\tan\phi = \frac{B_2}{B_1} = \frac{1}{2},$$

i.e., $$\phi = \tan^{-1}(0.5) \approx 26.6°$$

Example 72

Two short magnets of magnetic moment M_1 and M_2 are fixed on a table as shown in Fig. 4E.72. What will be the direction and magnitude of the magnetic field produced by these magnets at the point P? $M_1 = 2700$ CGS units, $M_2 = 3200$ CGS units, $d_1 = 30$ cm and $d_2 = 40$ cm.

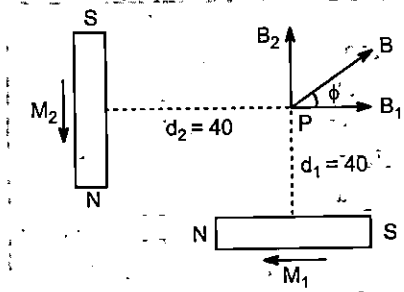


Fig. 4E.72

Solution: As the point P lies in broadside on position w.r.t. both the magnets, and in broadside on position in CGS system,

$$B = \frac{M}{d^3}$$

So, $$B_1 = \frac{2700}{(30)^3} = 0.1\,\text{G}$$

and $$B_2 = \frac{3200}{(40)^3} = 0.05\,\text{G}$$

But as here B_1 and B_2 are perpendicular to each other.

$$B = \sqrt{(B_1^2 + B_2^2)} = \sqrt{(0.1)^2 + (0.05)^2}$$

$$= \frac{\sqrt{5}}{20} = 0.112\,\text{G}$$

and $$\phi = \tan^{-1}\left(\frac{B_2}{B_1}\right) = \tan^{-1}\left(\frac{0.05}{0.10}\right)$$

$$= \tan^{-1}(0.5) \approx 26°$$

Example 73

A 6 cm long bar magnet possessing magnetic dipole moment 0.3 A-m^2 is placed vertically on a horizontal wooden table. The north pole of the magnet touches the table. A neutral point is found on the table at a distance of 8 cm south of the magnet. Find the horizontal component of Earth's magnetic field.

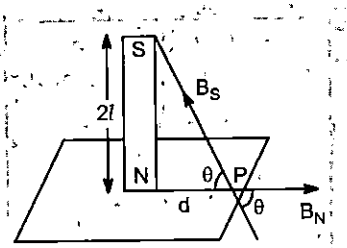


Fig. 4E.73

Solution: Magnetic field due to single pole of a magnet is given by

$$B = \frac{\mu_0}{4\pi}\frac{m}{d^2}$$ where m is pole strength

Field due to north pole, $B_N = \frac{\mu_0}{4\pi}\frac{m}{d^2}$ along NP

Field due to south pole, $B_S = \frac{\mu_0}{4\pi}\frac{m}{(d^2+4l^2)}$ along PS

Horizontal component of $B_S = B_S\cos\theta$

$$= \frac{\mu_0}{4\pi}\frac{m}{(d^2+4l^2)}\cdot\frac{d}{(d^2+4l^2)^{1/2}}$$

Resultant horizontal field

$$= \frac{\mu_0 m}{4\pi}\left[\frac{1}{d^2} - \frac{d}{(d^2+4l^2)^{3/2}}\right]$$

or $$B_H = \frac{\mu_0 M}{4\pi(2l)}\left[\frac{1}{d^2} - \frac{d}{(d^2+4l^2)^{3/2}}\right]$$

(where magnetic moment $M = m\times 2l$.)

On substituting numerical values, we get

$$B_H = 3.81\times 10^{-5} \text{ tesla}$$

Example 74

Two magnets of equal mass are joined at right angles to each other as shown in Fig. 4E.74. The magnet N_1S_1 has a magnetic moment 3 times that of N_2S_2. This arrangement is pivoted so that it is free to rotate in the horizontal plane. When in equilibrium, what angle will the magnet N_1S_1 subtend with the magnetic meridian?

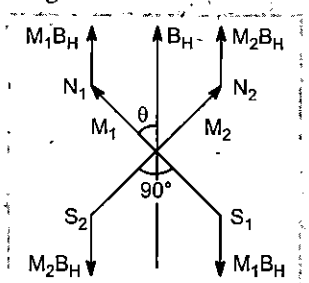


Fig. 4E.74

Solution: For equilibrium of the system, torque of M_1 and M_2 due to B_H must counter balance each other, *i.e.*,

$$\vec{\mathbf{M}}_1 \times \vec{\mathbf{B}}_H = \vec{\mathbf{M}}_2 \times \vec{\mathbf{B}}_H$$

If θ is the angle between M_1 and B_H then the angle between M_2 and B_H will be $(90°-\theta)$; so

$$M_1B_H\sin\theta = M_2B_H\sin(90°-\theta)$$

i.e., $$\tan\theta = \frac{M_2}{M_1} = \frac{M}{3M} = \frac{1}{3},$$

i.e., $$\theta = \tan^{-1}\left(\frac{1}{3}\right)$$

Example 75

A thin rectangular bar magnet suspended freely has a period of oscillation of 4 sec. What will be the period of oscillation if the magnet is broken into two halves (each having half the original length) and one of the pieces is made to oscillate in the same field ?

Solution: Time period of torsional vibrations of a magnet is given by

$$T = 2\pi\sqrt{\frac{I}{MB}} \qquad ...(1)$$

When the magnet is broken into two equal halves, perpendicular to its length,

$$M' = m\times l = (M/2) \qquad [\text{as } M = m\times 2l]$$

If A is the cross-section area and ρ the density of the material, the moment of inertia of the magnet is

$$I = \frac{1}{12}[(2\rho Al)\times(2l)^2]$$

After breaking into two halves,

and $$I' = \frac{1}{12}(\rho Al)\times(l)^2 = \left(\frac{I}{8}\right)$$

So, $$T' = 2\pi\sqrt{\frac{I'}{M'B}} = 2\pi\sqrt{\frac{(I/8)}{(M/2)B}} = \pi\sqrt{\frac{I}{MB}} \qquad ...(2)$$

and hence, dividing eqn. (2) by (1),

$$\frac{T'}{T} = \frac{1}{2},$$

i.e., $$T' = \frac{T}{2} = \frac{4}{2} = 2\,\text{sec}$$

* If the magnet is broken into two equal pieces parallel to its length, then as

$$M' = \left(\frac{m}{2}\right)(2l) = M/2$$

and $$I' = \frac{1}{12}\left(\frac{\text{mass}}{2}\right)(2l)^2 = \left(\frac{I}{2}\right)$$

So the new time period,

$$T'' = 2\pi\sqrt{\frac{(I/2)}{(M/2)B}}$$

$$= 2\pi\sqrt{\frac{I}{MB}} = T = 4\,\text{sec}$$

Example 76

Two bar magnets of the same length and breadth but having magnetic moment M and 2 M are joined together pole for pole and suspended by a string. The time of oscillation of this assembly in a magnetic field of strength B is 3 sec. What will be the period of oscillation if the polarity of one of the magnets is changed and the combination is again made to oscillate in the same field ?

Solution: As magnetic moment is a vector, so when magnets are joined together pole for pole, the resultant magnetic moment is $M + 2M = 3M$; so

$$T = 2\pi\sqrt{\frac{(I_1 + I_2)}{3MB}} \quad ...(1)$$

When the polarity of one of the magnets is reversed, $M' = M - 2M = M$; so

$$T' = 2\pi\sqrt{\frac{(I_1 + I_2)}{MB}} \quad ...(2)$$

Dividing eqn. (2) by (1),

$$\frac{T'}{T} = \sqrt{3},$$

i.e., $T' = (\sqrt{3})T = 3\sqrt{3}$ sec

Example 77

A vibration magnetometer consists of two identical bar magnets placed one over the other such that they are mutually perpendicular and bisect each other. The time period of oscillation in a horizontal magnetic field is 4 sec. If one of the magnets is taken away, find the period of oscillation of the other in the same field.

Solution: Time period of angular SHM of a dipole in a magnetic field,

$$T = 2\pi\sqrt{\frac{I}{MB}} \quad ...(1)$$

Initially there are two identical dipoles at right angles to each other, so resultant magnetic moment,

$$M_0 = \sqrt{(M^2 + M^2)} = (\sqrt{2})M$$

and $I_0 = I + I = 2I$

So, $$T_0 = 2\pi\sqrt{\frac{2I}{(\sqrt{2})MB}} = 2\pi\sqrt{\frac{(\sqrt{2})I}{MB}} \quad ...(2)$$

And when one magnet is taken away, the other will oscillate with period given by eqn. (1), So dividing Eqn. (1) by (2),

$$\frac{T}{T_0} = \frac{1}{(2)^{1/4}}$$

or $$T = \frac{T_0}{(2)^{1/4}} = \frac{4}{(2)^{1/4}} = 3.36\,s$$

Example 78

A magnet makes 10 oscillations per minute at a place where the angle of dip is 45° and the total intensity is 0.4 gauss. Calculate the number of oscillations made per sec by the same magnet at another place where the angle of dip is 60° and the intensity of the field is equal to 0.5 gauss.

Solution: Time period of angular SHM of a magnet in a magnetic field,

$$T = 2\pi\sqrt{\frac{I}{MB_H}} = 2\pi\sqrt{\frac{I}{MB\cos\delta}}$$

So for a given magnet at two different places,

$$\frac{T_2}{T_1} = \sqrt{\frac{B_1\cos\delta_1}{B_2\cos\delta_2}}$$

$$= \sqrt{\frac{0.4\cos 45}{0.5\cos 60}} = \sqrt{(0.8\times(2)^{1/2}} = 1.06$$

But as here $T_1 = (60/10) = 6$ s

So, $$T_2 = \frac{1}{f_2} = 1.06\times 6,$$

i.e., $$f_2 = \frac{1}{1.06\times 6} = 0.157\,\text{Hz}$$

Example 79

Discuss the magnetic potential energy, force and torque of interaction between two magnetic dipoles in the following cases:
(a) Magnetic dipoles lie along the line joining their centres,
(b) Dipoles are perpendicular to the line joining their centres,
(c) When one dipole is along and the other perpendicular to the line joining their centres.

Solution: (a) In the figure, the opposite poles of two dipoles face each other; the field due to $\vec{\mathbf{M}}_1$ at the position of $\vec{\mathbf{M}}_2$ (end-on position) is given by

$$\vec{\mathbf{B}}_1 = \frac{\mu_0}{4\pi}\frac{2M_1}{r^3}$$

So couple on $\vec{\mathbf{M}}_2$ due to $\vec{\mathbf{B}}_1$ magnetic field of $\vec{\mathbf{M}}_1$,

$$\vec{\mathbf{C}}_2 = \vec{\mathbf{M}}_2 \times \vec{\mathbf{B}}_1 = 0 \quad ...(1)$$

Because vector $\vec{\mathbf{M}}_2$ is parallel to vector $\vec{\mathbf{B}}_1$,

Also $\vec{\mathbf{C}}_1 = \vec{\mathbf{M}}_1 \times \vec{\mathbf{B}}_2 = 0$ [as $\vec{\mathbf{M}}_1$ is parallel to $\vec{\mathbf{B}}_2$] ...(2)

i.e., the dipoles will not exert any couple on each other.

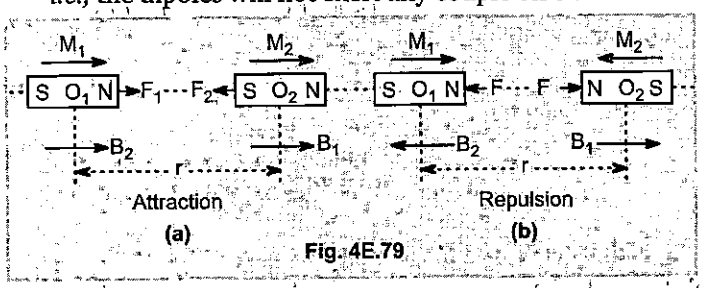


Fig. 4E.79

P. E. of $\vec{\mathbf{M}}_2$ in the field of $\vec{\mathbf{M}}_1$ or of $\vec{\mathbf{M}}_1$ in the field of $\vec{\mathbf{M}}_2$ can be expressed as

$$U = -\vec{\mathbf{M}}_2\cdot\vec{\mathbf{B}}_1 = -\vec{\mathbf{M}}_1\cdot\vec{\mathbf{B}}_2 = -\frac{\mu_0}{4\pi}\frac{2M_1M_2}{r^3} \quad ...(3)$$

[as $\vec{\mathbf{M}}_2$ is parallel to $\vec{\mathbf{B}}_1$]

For a conservative force, $F = -(dU/dr)$, so force on $\vec{\mathbf{M}}_1$ due to $\vec{\mathbf{M}}_2$ or on $\vec{\mathbf{M}}_2$ due to $\vec{\mathbf{M}}_1$ will be

$$F_1 = F_2 = -\frac{d}{dr}\left[-\frac{\mu_0}{4\pi}\cdot\frac{2M_1M_2}{r^3}\right] = -\frac{\mu_0}{4\pi}\frac{6M_1M_2}{r^4} \quad ...(4)$$

Interaction force is with negative sign (attractive) and varies as $(1/r^4)$.

If the similar poles of two dipoles face each other, θ will be 180° and so U and hence F will be positive (*i.e.,* repulsive) as shown in Fig. 4E.79 (b)

If the similar poles of two dipoles face each other as shown in Fig. 4E.79 (c), the field due to $\vec{\mathbf{M}}_1$ at the position of $\vec{\mathbf{M}}_2$ (broadside on position),

$$\vec{\mathbf{B}}_1 = \frac{\mu_0}{4\pi}\frac{M_1}{r^3}$$

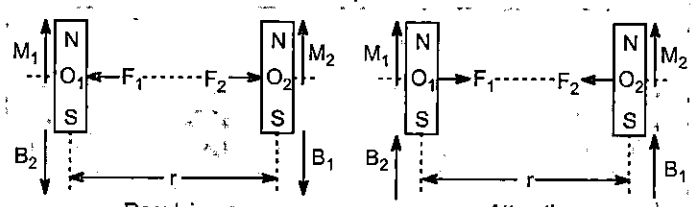


Fig. 4E.79

$\vec{\mathbf{B}}_1$ is antiparallel to $\vec{\mathbf{M}}_2$ and $\vec{\mathbf{B}}_2$ to $\vec{\mathbf{M}}_1$, so

$$\vec{\mathbf{C}}_2 = \vec{\mathbf{M}}_2 \times \vec{\mathbf{B}}_1 = 0 \text{ and } \vec{\mathbf{C}}_1 = \vec{\mathbf{M}}_1 \times \vec{\mathbf{B}}_2 = 0 \quad ...(5)$$

i.e., the dipoles will not exert any couple on each other the interaction energy of $\vec{\mathbf{M}}_2$ in the field of $\vec{\mathbf{M}}_1$ (or of $\vec{\mathbf{M}}_1$ in the field of $\vec{\mathbf{M}}_2$) will be

$$U = -\vec{\mathbf{M}}_2 \cdot \vec{\mathbf{B}}_1 = -\vec{\mathbf{M}}_1 \cdot \vec{\mathbf{B}}_2 = \frac{\mu_0}{4\pi}\frac{M_1 M_2}{r^3} \quad ...(6)$$

[as $\vec{\mathbf{M}}_2$ is antiparallel to $\vec{\mathbf{B}}_1$, *i.e.,* $\theta = 180°$]

As $F = -(dU/dr)$, so force on $\vec{\mathbf{M}}_1$ due to $\vec{\mathbf{M}}_2$ or on $\vec{\mathbf{M}}_2$ due to $\vec{\mathbf{M}}_1$ will be

$$F_1 = F_2 = -\frac{d}{dr}\left(\frac{\mu_0}{4\pi}\frac{M_1 M_2}{r^3}\right) = \frac{\mu_0}{4\pi}\frac{3M_1 M_2}{r^4} \quad ...(7)$$

Interaction force is with positive sign (repulsive) and varies as $(1/r^4)$.

If the opposite poles of the dipoles face each other, θ will be 0° and so U and hence F will be negative (*i.e.,* attractive) as shown in Fig. 4E.79(c).

(c) In the figure shown, $\vec{\mathbf{M}}_2$ is in broadside on position relative to $\vec{\mathbf{M}}_1$ while $\vec{\mathbf{M}}_1$ is in the end on position relative to $\vec{\mathbf{M}}_2$; so

$$\vec{\mathbf{B}}_1 = \frac{\mu_0}{4\pi}\frac{M_1}{r^3}$$

and $$\vec{\mathbf{B}}_2 = \frac{\mu_0}{4\pi}\frac{2M_2}{r^3}$$

So couple on $\vec{\mathbf{M}}_2$ due to $\vec{\mathbf{M}}_1$, *i.e.,* $\vec{\mathbf{B}}_1$,

$$\vec{\mathbf{C}}_2 = \vec{\mathbf{M}}_2 \times \vec{\mathbf{B}}_1 = \frac{\mu_0}{4\pi}\frac{M_1 M_2}{r^3} \text{ (clockwise)} \quad ...(8)$$

[as $\vec{\mathbf{M}}_2$ is perpendicular to $\vec{\mathbf{B}}_1$]

and couple on $\vec{\mathbf{M}}_1$ due to $\vec{\mathbf{M}}_2$, *i.e.,* $\vec{\mathbf{B}}_2$,

$$\vec{\mathbf{C}}_1 = \vec{\mathbf{M}}_1 \times \vec{\mathbf{B}}_2 = \frac{\mu_0}{4\pi}\frac{2M_1 M_2}{r^3} \text{ (clockwise)} \quad ...(9)$$

[as $\vec{\mathbf{M}}_1$ is perpendicular to $\vec{\mathbf{B}}_2$]

$\vec{\mathbf{B}}_1$ is perpendicular to $\vec{\mathbf{M}}_2$ and $\vec{\mathbf{B}}_2$ to $\vec{\mathbf{M}}_1$, so interaction energy is

$$U = -\vec{\mathbf{M}}_2 \cdot \vec{\mathbf{B}}_1 = -\vec{\mathbf{M}}_1 \cdot \vec{\mathbf{B}}_2 = 0 \quad ...(10)$$

We cannot use $F = -\frac{dU}{dr}$ for calculating force. To evaluate the force on $\vec{\mathbf{M}}_2$ due to $\vec{\mathbf{M}}_1$, let the length of dipole M_2 be $2l$ so that the field due to $\vec{\mathbf{M}}_1$ as S and N poles of M_2 will be respectively:

$$B_S = \frac{\mu_0}{4\pi}\frac{M_1}{(r-l)^3} \text{ opposite to } \vec{\mathbf{B}}_1$$

and $$B_N = \frac{\mu_0}{4\pi}\frac{M_1}{(r+l)^3} \text{ in the direction of } \vec{\mathbf{B}}_1$$

and as $\vec{\mathbf{F}} = m\vec{\mathbf{B}}$, forces on S and N poles of $\vec{\mathbf{M}}_2$ due to field of $\vec{\mathbf{M}}_1$ will be

$$\vec{\mathbf{F}}_S = \frac{\mu_0}{4\pi}\frac{M_1 m}{(r-l)^3} \text{ opposite to } \vec{\mathbf{B}}_1.$$

and $$\vec{\mathbf{F}}_N = \frac{\mu_0}{4\pi}\frac{M_1 m}{(r+l)^3} \text{ in the direction of } \vec{\mathbf{B}}_1.$$

So net force on $\vec{\mathbf{M}}_2$ due to $\vec{\mathbf{M}}_1$,

$$\vec{\mathbf{F}}_2 = \vec{\mathbf{F}}_S - \vec{\mathbf{F}}_N = \frac{\mu_0}{4\pi}\left[\frac{M_1 m}{(r-l)^3} - \frac{M_1 m}{(r+l)^3}\right] \text{ opposite to } \vec{\mathbf{B}}_1$$

i.e., $$\vec{\mathbf{F}}_2 = \frac{\mu_0}{4\pi}\frac{M_1 M_2 (3r^2 + l^2)}{(r^2 - l^2)^3} \text{ opposite to } \vec{\mathbf{B}}_1$$

[as $m \times 2l = M_2$]

or $$\vec{\mathbf{F}}_2 = \frac{\mu_0}{4\pi}\frac{3M_1 M_2}{r^4} \text{ opposite to } \vec{\mathbf{B}}_1$$

[as $r \gg l$] ...(11)

And as for every action there is an equal and opposite reaction, force $\vec{\mathbf{F}}_1$ on $\vec{\mathbf{M}}_1$ will be equal and opposite to $\vec{\mathbf{F}}_2$ as shown in Fig. 4E.79 (e).

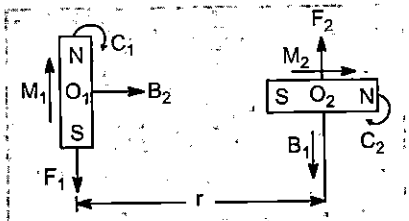


Fig. 4E.79 (e)

So if these magnets are free to rotate, they will rotate continuously clockwise due to the resultant couple

$$C = C_1 + C_2 = \frac{\mu_0}{4\pi}\frac{3M_1M_2}{r^3}.$$

It involves continuous work without any expenditure of energy. Infact the translatory forces F_1 and F_2 of value $\frac{\mu_0}{4\pi}\left(\frac{3M_1M_2}{r^4}\right)$ acting on M_1 and M_2 constitute an ACW couple of value

$$C_T = \frac{\mu_0}{4\pi}\left(\frac{3M_1M_2}{r^4}\right)\times r = \frac{\mu_0}{4\pi}\frac{3M_1M_2}{r^3}$$

which counterbalances the resultant couple C.

MAGNETIC MATERIALS-FERROMAGNETISM

Magnetic fields can be produced (*a*) by magnetic materials (magnets) and (*b*) by electric currents. Iron (and a few other materials) can be made into strong magnets. These materials are said to be **ferromagnetic.**

A bar magnet, with its two opposite poles near either end, resembles an electric dipole (equal-magnitude positive and negative charges separated by a distance). A bar magnet is sometimes referred to as a "magnetic dipole." There are opposite "poles" separated by a distance. And the magnetic field lines of a bar magnet form a pattern much like that for the electric field of an electric dipole:

Microscopic examination reveals that a magnet is actually made-up of tiny regions known as **domains,** which are at most about 1 mm in length or width, Each domain behaves like a tiny magnet with a north and a south pole. In an unmagnetized piece of iron, these domains are arranged randomly, as shown in Fig. 4.98 (a). The magnetic effects of the domains cancel each other out, so this piece of iron is not a magnet. In a magnet, the domains are preferentially aligned in one direction as shown in Fig. 4.98 (b) (downward in this case). A magnet can be made from an unmagnetized piece of iron by placing it in a strong magnetic field.

Careful observations show in this case that the magnetization of domains may actually rotate slightly so as to be more nearly parallel to the external field. Or, more commonly, the borders of domains move so that those domains whose magnetic orientation is parallel to the external field grow in size at the expense of other domains, as can be seen by comparing Fig. 4.98 (a) and (b). This explains how a magnet can pick up unmagnetized pieces of iron like paper clips. The magnet's field causes a slight alignment of the domains in the unmagnetized object so that the object becomes a temporary magnet with its north pole facing the south pole of the permanent magnet, and *vice versa*; thus, attraction results.

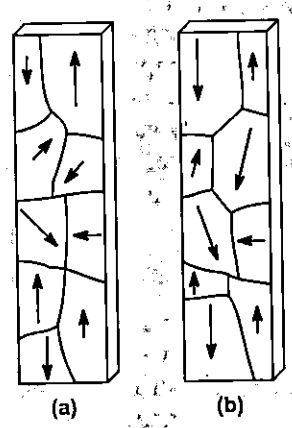


Fig. 4.98: (a) an unmagnetizd piece of iron is made-up of domains that are randomly arranged. Each domain is like a tiny magnet; the arrows represent the magnetization direction, with the arrowhead being the N pole. (b) In a magnet, the domains are preferentially aligned in one direction, and may be altered in size by the magnetization process.

An iron magnet can remains magnetized for a long time, and thus it is referred to as a "permanent magnet." However, if you drop a magnet on the floor or strike it with a hammer, you may jar the domains into randomness. The magnet can thus lose some or all of its magnetism. Heating a magnet too can cause a loss of magnetism, for raising the temperature increases the random thermal motion of the atoms which tends to randomize the domains. Above a certain temperature known as the **Curie temperature** (1043 K for iron), a magnet cannot be made at all.*

The striking similarity between the fields produced by a bar magnet and by a loop of electric current (Fig. 4.99)

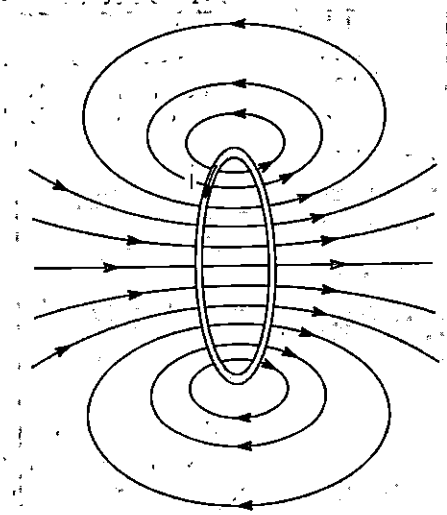


Fig. 4.99: Magnetic field due to a circular loop of wire

* Iron, nickel, cobalt, gadolinium, and certain alloys are ferromagnetic at room temperature; several other elements and alloys have low Curie temperature and thus are ferromagnetic only at low temperatures.

suggests that the magnetic field produced by a current may have something to do with ferromagnetism, an idea proposed by Ampere in the nineteenth century. According to modern atomic theory, the atoms that make-up any material can be roughly visualized as containing electrons that orbit around a central nucleus. Since the electrons are charged, they constitute an electric current and therefore produce a magnetic field.

But if there is no external field, the electron orbit in different atoms are arranged randomly, so the magnetic effects due to the many orbits in all the atoms in a material cancel out. However, electrons by themselves produce an additional intrinsic magnetic field—they have an intrinsic magnetic moment referred to as their "spin" magnetic moment. It is the magnetic field due to electron spin that is now believed to produce ferromagnetism. In most materials, the magnetic fields due to electron spin cancel out because they are oriented at random. But in iron and other ferromagnetic materials, a complicated cooperative mechanism, known as "exchange coupling," operates. The result is that the spin of the electrons contributing to the ferromagnetism in a domain point in the same direction. Thus the tiny magnetic fields due to each of these electrons add up to give the magnetic field of a domain. And when the domains are aligned, as we have seen, a strong magnet results.

ELECTROMAGNETS AND SOLENOIDS

The magnetic field within a solenoid can be fairly large since it will be the sum of the field due to the current in each loop (See Fig. 4.100). The solenoid acts like a magnet; one end can be considered the north pole and the other the south pole, depending on the direction of the current in the loops (use the right-hand rule). Since the magnetic field lines leave the north pole of a magnet, the north pole of the solenoid in Fig. 4.100 is on the right.

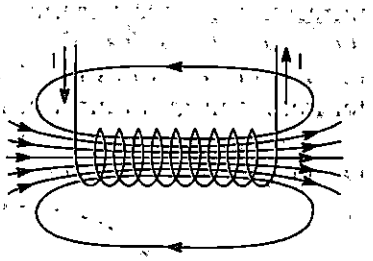


Fig. 4.100: Magnetic field of a solenoid. The north pole of this solenoid, through of as a magnet, is on the right, and the south pole is on the left.

If a piece of iron is placed inside a solenoid, the magnetic field is increased greatly because the domains of the iron are aligned by the magnetic field produced by the current. The resulting magnetic field is the sum of that due to the current and that due to the iron, and can be hundreds or thousands of times that due to the current alone. This arrangement is called an **electromagnet**. The iron used in electromagnets acquires and loses its magnetism quite readily when the current is turned on or off, and so is referred to as "so iron." (It is "soft" only in a magnetic sense.) Iron that holds its magnetism even when there is no externally applied field is called "hard iron." Hard iron is used in permanent magnets. Soft iron is usually used in electromagnets so that the field can be turned on and off readily.

When the current flows continuously in a normal electromagnet, a great deal of waste heat (I^2R power) can be produced. Cooling coils, which are tubes carrying water, must be used to absorb the heat.

Another useful device consists of a solenoid into which a rod of iron is partially inserted. This combination is also referred to as a solenoid. One simple use is as a doorbell (Fig. 4.101). When the circuit is closed by pushing the button the coil effectively becomes a magnet and exerts a force on the iron rod. The rod is pulled into the coil and strikes the bell, A larger solenoid is used in the starters of cars; when you engage the starter, you are closing a circuit that not only turns the starter motor, but activates a solenoid that first moves the starter into direct contact with the gears on the engine's flywheel.

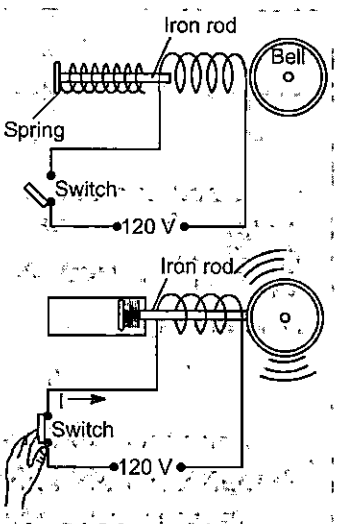


Fig. 4.101: Solenoid used

MAGNETIC FIELDS IN MAGNETIC MATERIALS; HYSTERESIS

The field of a long solenoid is directly proportional to the current. The field B_0 inside a solenoid is given by

$$B_0 = \mu_0 nI$$

If we put a piece of iron or other ferromagnetic material inside the solenoid, the field will be greatly increased, often by hundreds or thousands of times. This occurs because the domains in the iron become preferentially aligned by the external field. The resulting magnetic field is the sum of that due to the current and that due to the iron. It is sometimes convenient to write the total field in this case as a sum of two terms:

$$B = B_0 + B_M$$

Here, B_0 refers to the field due only to the current in the wire (the "external field"). It is the field that would be present in the absence of a ferromagnetic material. Then B_M represents the additional field due to the ferromagnetic material itself; often $B_M >> B_0$.

The total field inside a solenoid in such a case can also be written replacing the constant μ_0 by another constant μ, characteristic of the material inside the coil;

$$B = \mu nI;$$

μ is called the **magnetic permeability** of the material. For ferromagnetic materials, μ is much greater than μ_0. For all other materials, its value is very close to μ_0. The value of μ, however, is not constant for ferromagnetic materials; it depends on the value of the external field B_0, as the following experiment shows.

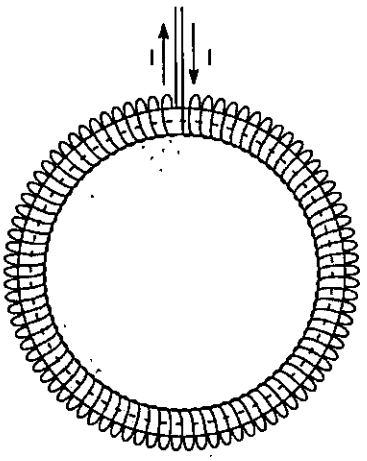


Fig. 4.102: Iron-core toroid

Measurements on magnetic materials are generally done using a toroid, (Fig. 4.102). So that practically all the lines of B remain within the toroid. Suppose the toroid has an iron core that is initially unmagnetized and there is no current in the windings of the toroid. Then the current I is slowly increased, and B_0 increases linearly with I. The total field B also increases, but follows the curved line shown in the graph of Fig. 4.103. (Note the different scales: $B \gg B_0$.). Initially, point a, the domains are randomly oriented.

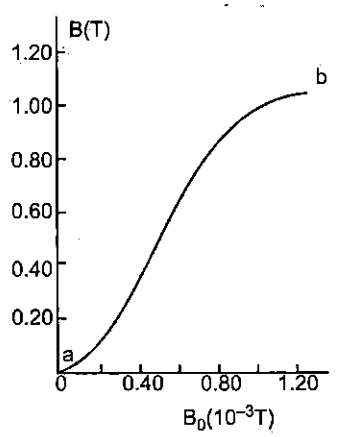


Fig. 4.103: Total magnetic field B in an iron-core toroid as a function of the external field B_0 (B_0 is caused by the current I in the coil).

As B_0 increases, the domains become more and more aligned until at point b, nearly all are aligned. The iron is said to be approaching **saturation.** (If B_0 is increased further, the curve continues to rise very slowly, and reaches 98 per cent saturation only when B_0 reaches a value about a thousandfold above that at point b; the last few domains are very difficult to align). Next, suppose the external field B_0 is reduced by decreasing the current in the coils. As the current is reduced to zero, shown as point c in Fig. 4.104, the domains do not become completely random. Some permanent magnetism remains. If the current is then reversed in direction, enough domains can be turned around so $B = 0$ (point d). As the reverse current is increased further, the iron approaches saturation in the opposite direction (point e). Finally, if the current is again reduced to zero and then increased in the original direction, the total field follows the path $efgb$, again approaching saturation at point b.

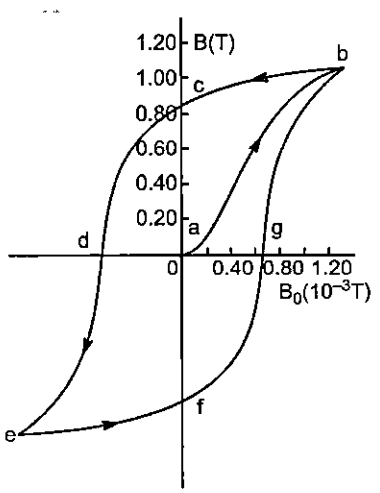


Fig. 4.104: Hysteresis curve

Notice that the field did not pass through the origin (point a) in this cycle. The fact that the curves do not retrace themselves on the same path is called **hysteresis.** The curve $bcdefgb$ is called a **hysteresis loop.** In such a cycle, much energy is transformed to thermal energy (friction) due to realigning of the domains. It can be shown that the energy dissipated in this way is proportional to the area of the hysteresis loop.

At points c and f, the iron core is magnetized even though there is no current in the coils. These points correspond to a permanent magnet. For a permanent magnet, it is desired that ac and af be as large as possible. Materials for which this is true are said to have high **retentivity.**

Materials with a broad hysteresis curve as in Fig. 4.106 are said to be magnetically "hard." On the other hand, a hysteresis curve such as that in Fig. 4.105 occurs for "soft" iron. This is preferred for electromagnets and transformers since the field can be more readily switched off, and the field can be reversed with less loss of energy.

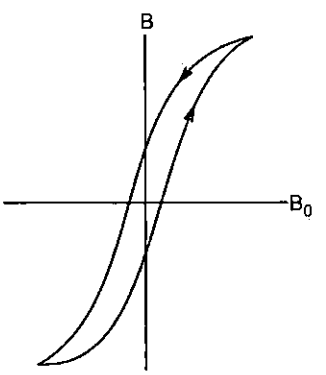


Fig. 4.105: Hysteresis curve for soft iron

PARAMAGNETISM AND DIAMAGNETISM

All materials are magnetic to some extent. Non-ferromagnetic materials fall into two principal classes: paramagnetic, in which the magnetic permeability μ is slightly greater than μ_0; and diamagnetic, in which, μ is slightly less than μ_0. The ratio of μ to μ_0 for any material is called the **relative permeability** K_m:

$$K_m = \frac{\mu}{\mu_0}$$

Another useful parameter is the **magnetic susceptibility** χ_m defined as

$$\chi_m = K_m - 1$$

Paramagnetic substances have $K_m > 1$ and $\chi_m > 0$, whereas diamagnetic substances have $K_m < 1$ and $\chi_m < 0$.

Paramagnetism occurs in materials whose molecules (or ions) have a permanent magnetic dipole moment. In the absence of an external field, the molecules are randomly oriented and no magnetic effects are observed. However, when an external magnetic field is applied, say, by putting the material in a solenoid, the applied field exerts a torque on the magnetic dipoles, tending to align them parallel to the field. The total magnetic field (external plus that due to aligned magnetic dipoles) will be slightly greater than B_0. The thermal motion of the molecules reduces the alignment, however. A useful quantity is the **magnetization vector, M,** defined as the magnetic dipole moment per unit volume,

$$M = \frac{\mu}{V}.$$

Where μ is the magnetic dipole moment of the sample and V its volume. It is found experimentally that M is directly proportional to the external magnetic field (tending to align the dipoles) and inversely proportional to the Kelvin temperature T (tending to randomize dipole directions). This is called Curie's law,

$$M = C\frac{B}{T},$$

where C is a constant. If the ratio B/T is very large (B very large or T very small) Curie's law is no longer accurate; as B is increased (or T decreased), the magnetization approaches some maximum value, M_{max}. This makes sense, of course, since M_{max} corresponds to complete alignment of all the permanent magnetic dipoles.

Ferromagnetic materials; are no longer ferromagnetic above a characteristic temperature called the Curie temperature (1043 K for iron). Above this Curie temperature, they generally are paramagnetic.

Diamagnetic materials (for which μ_m is slightly less than μ_0) are made-up of molecules that have no permanent magnetic dipole moment. When an external magnetic field is applied, magnetic dipoles are induced, but the induced magnetic dipole moment is in the direction opposite to that of the field. Hence the total field will be slightly less than the external field.

Electric dipoles in a dielectric align with an applied electric field and reduce the applied field. Magnetic dipoles in a paramagnetic material align with an applied magnetic field and increase the applied field (Fig. 4.106). Electric dipoles aligned with an externally applied electric field produce an electric field opposite $\vec{\mathbf{E}}_{external}$ within the material. Magnetic dipoles aligned with an externally applied magnetic field produce a magnetic field within the material that is in the same direction as $\vec{\mathbf{B}}_{external}$.

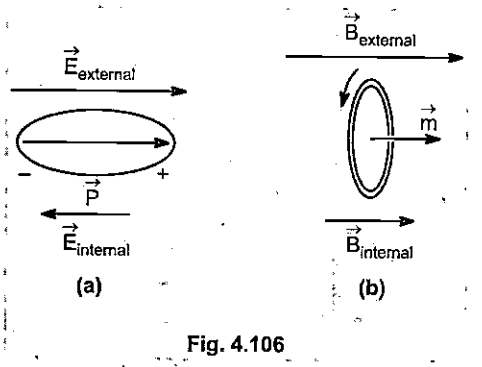


Fig. 4.106

This reversal of behavior occurs because electric field is produced by charge, and magnetic field is produced by current. Fig. 4.107 illustrates what happens when the atoms, each modeled as a small current loop, are aligned. The currents between atoms are in opposite direction and cancel. Each atom behaves like a miniature current loop. Between atoms the currents are in opposite directions, and cancel. At the edge of the material, there is no cancellation. The net current produces magnetic field in the material with direction into the page.

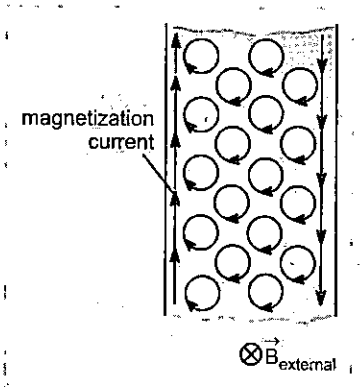


Fig. 4.107

On the surface of the material the atoms produce a net current called magnetization current that enhances the external field.

An external magnetic field changes the motion of the electrons and induces a net magnetic moment in each atom.

An electron in orbit about a nucleus (Fig. 4.108) forms a

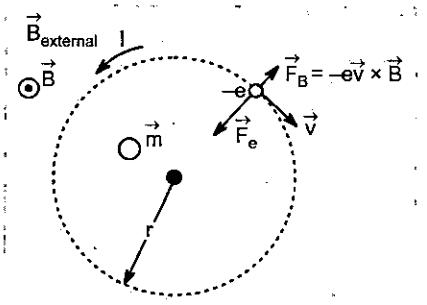


Fig. 4.108

current loop with magnetic moment $m = I(\pi r^2)$, where r is the radius of the electron orbit. The current is

$I = (e/T) = (ev/2\pi r)$, where T is the period of the orbit and v is the electron's speed. Thus:

$$m = \frac{ev}{2\pi r}\pi r^2 = \frac{evr}{2}$$

The electric force F_e exerted on the electron of mass M by the atomic nucleus causes the centripetal acceleration:

$$F_e = Mv^2/r$$

An external magnetic field applied parallel to $\vec{\mathbf{m}}$ cause a magnetic force $\vec{\mathbf{F}}_B = -e\vec{\mathbf{v}} \times \vec{\mathbf{B}}$, which is radially outward. Because of the reduced inward force, the centripetal acceleration of the electron decreases. Thus either the orbit radius r increases or the speed decreases. In fact it is the speed that changes. With the new speed $v + dv$, Newton's second law becomes:

$$\frac{M(v+dv)^2}{r} = F_e - evB$$

Expanding the square and ignoring the term in $(dv)^2$, we have:

$$\frac{Mv^2}{r} + \frac{2Mvdv}{r} = F_e - evB$$

The first term on the left-hand side equals the first term on the right, so:

$$dv = -\frac{eBr}{2M}$$

The change in speed is proportional to the applied field B. The magnetic moment changes by an amount:

$$dm = d\left(\frac{evr}{2}\right) = \frac{er}{2}dv$$

$$= \frac{er}{2}\left(-\frac{eBr}{2M}\right) = -\frac{e^2r^2B}{4M}$$

The magnetic moments of the electron orbit is reduced by an amount proportional to the applied field B.

If $\vec{\mathbf{B}}$ is applied antiparallel to $\vec{\mathbf{m}}$, then m is increased, so that $d\vec{\mathbf{m}}$ is always antiparallel to $\vec{\mathbf{B}}$.

The magnetic moments of the electrons in each atom sum to zero when there is no applied field. But since $d\vec{\mathbf{m}}$ is in the same direction for each electron, the net result of an applied $\vec{\mathbf{B}}$ is to create an induced moment opposite $\vec{\mathbf{B}}$.

Example 80

The region between $x = 0$ and $x = L$ is filled with uniform magnetic field $B_0\hat{\mathbf{k}}$. A particle of mass m, positive charge q and velocity $v_0\hat{\mathbf{i}}$ travels along x-axis and enters the region of the magnetic field. Neglecting gravity throughout the question, find the

(a) value of L if the particle emerges from the region of magnetic field with its final velocity at an angle 30° to its initial velocity.

(b) final velocity of the particle and the time spent by it in the magnetic field, if the magnetic field now expands upto 2.1 L.

Solution: (a) $\theta = 30°$

Since, we know that $\sin\theta = \frac{L}{R}$

where, $R = \frac{mv_0}{qB_0}$

$$\Rightarrow \quad \sin 30° = \frac{L}{\frac{mv_0}{qB_0}}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{qB_0L}{mv_0}$$

$$\Rightarrow \quad L = \frac{mv_0}{2qB_0}$$

(b) Since, we know that $\sin\theta = \frac{L}{R}$

$$\Rightarrow \quad \sin 30° = \frac{L}{R}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{L}{R}$$

$$\Rightarrow \quad L = \frac{R}{2}$$

Now when $L' = 2.1\ L = \frac{2.1R}{2} = 1.05R$

$$\Rightarrow \quad L' > R$$

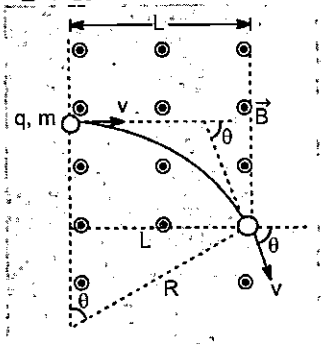


Fig. 4E.80

Therefore, deviation of the particle is $\theta = 180°$ is as shown.

$$\vec{v}_f = -v_0\hat{\mathbf{i}} = \vec{v}_B$$

and $$t_{AB} = \frac{T}{2} = \frac{\pi m}{qB_0}$$

Example 81

A particle of mass m and charge q is projected into a region having a perpendicular uniform magnetic field B. Find the angle of deviation θ of the particle as it comes out of the magnetic field if width d of the region is

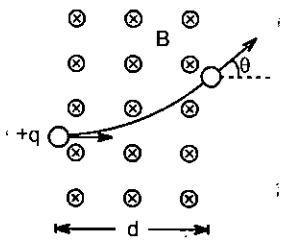


Fig. 4E.81 (a)

(a) $\frac{mv}{2qB}$ (b) $\frac{mv}{qB}$ (c) $\frac{2mv}{qB}$

Solution: (a) The radius of the circular orbit is given by $r = \frac{mv}{qB}$. So the angle of deviation for

$$d = \frac{mv}{2qB} = \frac{r}{2} \text{ is } \sin\theta = \frac{d}{r} = \frac{1}{2}$$

$$\Rightarrow \quad \theta = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6} = 30°$$

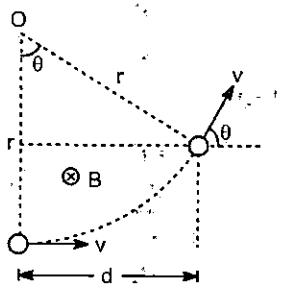


Fig. 4E.81 (b)

(b) when $d = \frac{mv}{qB} = r$, the charged particle deviates through an angle of $\frac{\pi}{2}$ as shown in Fig. 4E.81 (c), hence $\theta = \frac{\pi}{2}$

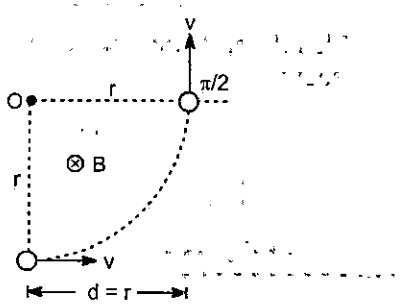


Fig. 4E.81 (c)

(c) When $d = 2\left(\frac{mv}{qB}\right) = 2r$, the charged particle completes one semi-circle and deviates through π, as shown in Fig. 4E.81 (d), hence $\theta = \pi$

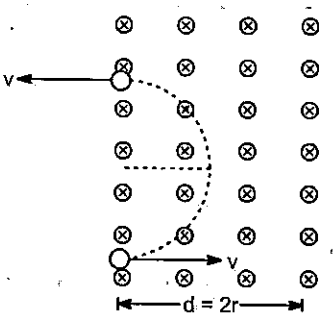


Fig. 4E.81 (d)

Example 82

Two circular coils of radius R, each with N turns, are perpendicular to a common axis. The centres of the coils are a distance R apart. Each coil carries a steady current I in the same direction, as shown in Fig. 4E.82.

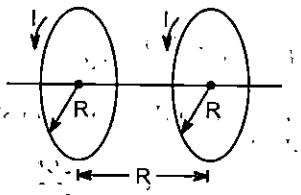


Fig. 4E.82 (a)

(a) Show that the magnetic field on the axis at a distance x from the center of one coil is

$$B = \frac{N\mu_0 IR^2}{2}\left[\frac{1}{(R^2+x^2)^{3/2}} + \frac{1}{(2R^2-x^2-2Rx)^{3/2}}\right]$$

(b) Show that $\frac{dB}{dx}$ *and* $\frac{d^2B}{dx^2}$ *are both zero at the point midway between the coils.*

Solution: (a) Let us use $B = \frac{\mu_0 NIR^2}{2(x^2+R^2)^{3/2}}$ twice, for both the coils. So, we get

$$B = B_{x_1} + B_{x_2}$$

$$= \frac{N\mu_0 IR^2}{2}\left[\frac{1}{(x^2+R^2)^{3/2}} + \frac{1}{[(R-x)^2+R^2]^{3/2}}\right]$$

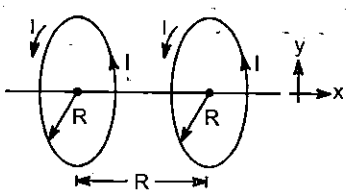


Fig. 4E.82 (b)

$$\Rightarrow \quad B = \frac{N\mu_0 IR^2}{2}\left[\frac{1}{(x^2+R^2)^{3/2}} + \frac{1}{[2R^2+x^2-2xR)^{3/2}}\right]$$

(b) $$\frac{dB}{dx}=\frac{N\mu_0 IR^2}{2}\left[-\frac{3}{2}(2x)(x^2+R^2)^{-5/2}-\frac{3}{2}(2R^2+x^2-2xR)^{-5/2}(2x-2R)\right]$$

Substituting $x=\frac{R}{2}$, we get

$$\frac{dB}{dx}=0$$

Further

$$\frac{d^2B}{dx^2}=\frac{d}{dx}\left(\frac{dB}{dx}\right)=-\frac{3N\mu_0 IR^2}{2}[(x^2+R^2)^{-5/2}-5x^2(x^2+R^2)^{-7/2}+(2R^2+x^2-2xR)^{-5/2}-5(x-R)^2(2R^2+x^2-2xR)^{-7/2}]$$

Again substituting $x=\frac{R}{2}$, we get

$$\frac{d^2B}{dx^2}=0$$

This means magnetic field in the region midway between the coils is uniform. Coils in this configuration are called Helmholtz coils.

Example 83

A rod of mass m and radius R rests on two parallel rails that are a distance l apart and have a length L. The rod carries a current I (in the direction shown) and rolls along the rails without slipping. A uniform magnetic field B is directed perpendicular to the rod and the rails. If it starts from rest, what is the speed of the rod as it leaves the rails ?

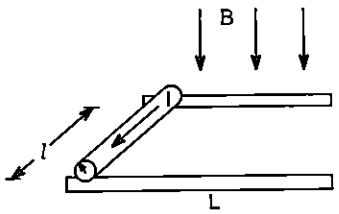


Fig. 4E.83 (a)

Solution: The magnetic force acting on the rod is given by

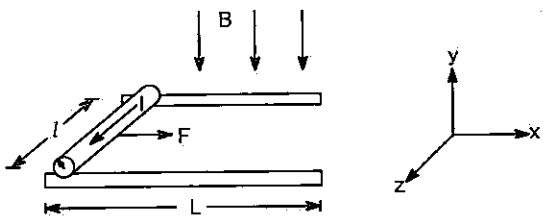


Fig. 4E.83 (b)

$$\vec{F}=I(\vec{l}\times\vec{B})=Il(\hat{k})\times B(\hat{j})=(BIl)\hat{i}$$

Form Work Energy Theorem, we have

$$(K_{trans}+K_{rot})_{initial}+\Delta E=(K_{trans}+K_{rot})_{final}$$

$$\Rightarrow\quad 0+0+Ps\cos\theta=\frac{1}{2}mv^2+\frac{1}{2}I\omega^2,$$

where $$I=\frac{1}{2}mR^2$$

$$\Rightarrow\quad (BIl)L\cos 0^\circ=\frac{1}{2}mv^2+\frac{1}{2}\left(\frac{1}{2}mR^2\right)\left(\frac{v}{R}\right)^2$$

$$\Rightarrow\quad BIlL=\frac{3}{4}mv^2$$

$$\Rightarrow\quad v=\sqrt{\frac{4BIlL}{3m}}$$

Example 84

A conducting wire of length l is placed on a rough horizontal surface, where a uniform horizontal magnetic field B perpendicular to the length of the wire exists. Least values of the forces required to move the rod when a current I is established in the rod are observed to be F_1 and F_2 ($<F_1$) for the two possible directions of the current through the rod respectively. Find the weight of the rod and the coefficient of friction between the rod and the surface.

Solution: Changing the direction of current in the wire, we can change the normal reaction on the wire by the surface. In one case magnetic force on the wire will be in upward direction while in the other case it will be in the downward direction. Hence normal reaction is given by

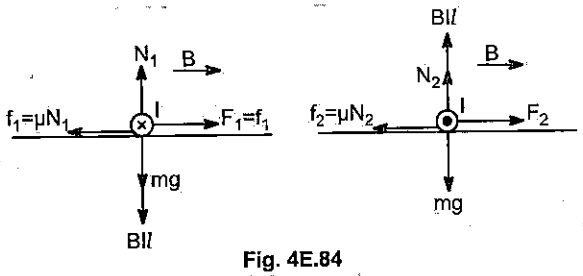


Fig. 4E.84

$$N=mg\pm BIl$$

$$\Rightarrow\quad f_{(\text{friction limiting})}=\mu(mg\pm BIl) \text{ as } F_1>F_2$$

$$\Rightarrow\quad F_1=f_1=\mu(mg+BIl) \qquad ...(1)$$

and $$F_2=f_2=\mu(mg-BIl) \qquad ...(2)$$

From eqns. (1) and (2),

$$\frac{F_1}{F_2}=\frac{mg+BIl}{mg-BIl}$$

$$\Rightarrow\quad mg=BIl\left(\frac{F_1+F_2}{F_1-F_2}\right) \qquad ...(3)$$

From eqns. (1) and (3), we get

$$\mu=\frac{F_1-F_2}{2BIl}$$

Example 85

A semi-circular conductor of radius R carrying a current I_0 is coplanar with a very long straight conductor carrying a current I. The distance of the centre of the straight conductor is 'a' from the straight conductor. Find the magnetic interaction force between the conductors for the position shown in Fig. 4E.85 (a).

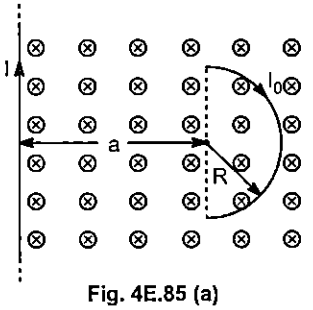


Fig. 4E.85 (a)

Solution: According to Biot-Savart's law the magnetic induction due to very long straight current I is directed normally into the plane of the Fig. 4E.85 (b) The magnitude of the magnetic induction due to straight current depends upon the perpendicular distance from it. Here the elements of semi-circular current are at different perpendicular distances from the straight current. Let us take an element of the circular current at an angle θ from horizontal as shown in the Fig. 4E.85 (b) Taken elemental wire experiences the Ampere force in accordance with the formula.

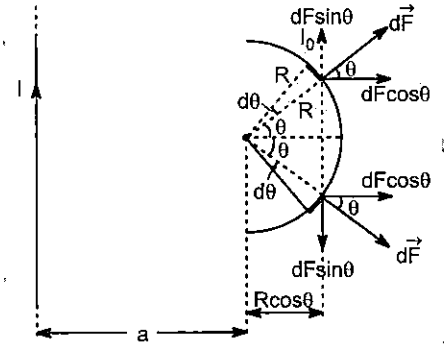


Fig. 4E.85 (b)

$d\vec{F} = I(d\vec{l} \times \vec{B})$, which gives

$d\vec{F} = I_0 R d\theta \frac{\mu_0}{2\pi} \frac{I}{(a + r\cos\theta)}$, directed away from the centre

From the symmetry of the problem the net Ampere force $\vec{F}$ is directed towards right and

$$F_{net} = \int dF \cos\theta \qquad \{\because \text{all sine components cancel}\}$$

$$\Rightarrow \quad F_{net} = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\mu_0}{2\pi} I I_0 R \frac{d\theta}{(a + R\cos\theta)}$$

$$= \frac{\mu_0}{2\pi} I I_0 R \left[2\int_0^{\frac{\pi}{2}} \frac{\cos\theta d\theta}{(a + r\cos\theta)} \right]$$

$$\Rightarrow \quad F_{net} = \frac{\mu_0}{\pi} I I_0 R \int_0^{\frac{\pi}{2}} \frac{\cos\theta d\theta}{(a + R\cos\theta)} = \frac{\mu_0 I I_0}{\pi}$$

$$\left(\frac{\pi}{2} - \frac{2a}{\sqrt{a^2 - R^2}} \tan^{-1} \sqrt{\frac{a-R}{a+R}} \right)$$

Please note that if the current I_0 were in a circular loop having its centre at O, then the two halves experience forces in the opposite directions. The left half experiences the Ampere force $F_{\text{Left half}}$ (Say) towards left and the right half experiences a net force $F_{\text{Right half}}$ towards right and so the net Ampere force on the circular current loop becomes $(F_{\text{Left half}} - F_{\text{Right half}})$, directed towards left.

Example 86

A coil carrying a current I is placed in a uniform magnetic field so that its axis coincides with the field direction. The single-layer winding of the coil is made of wire with diameter d, radius of turns is equal to R. At what value of the induction of the external magnetic field can the coil winding be ruptured? Assume the breaking stress of the wire to be σ_0.

Solution: Each element of length dl experiences a force $BIdl$. This causes a tension T in the wire. Consider an infinitesimal element of length dl, subtending an angle $d\theta$ at the centre, then for equilibrium,

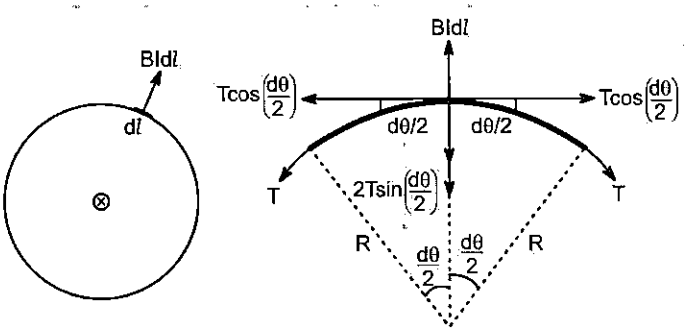


Fig. 4E.86

$$2T\sin\left(\frac{d\theta}{2}\right) = BIdl$$

Since $d\theta$ is very small, so we get

$$\sin\left(\frac{d\theta}{2}\right) \simeq \frac{d\theta}{2}$$

$$\Rightarrow \quad Td\theta = BIdl$$

$$\Rightarrow \quad Td\theta = BI(Rd\theta)$$

$$\Rightarrow \quad T = BIR$$

The wire experiences a stress σ, which is

$$\sigma = \frac{\text{Force}}{\text{Area}} = \frac{BIR}{\frac{\pi d^2}{4}}$$

This must equals the breaking stress σ_0 for the winding to rupture. Thus,

$$B_{max} = \frac{\pi d^2 \sigma_0}{4IR}$$

Example 87

A charged particle +q of mass m is placed at a distance d from another charged particle –2q of mass 2m in a uniform magnetic field B as shown in Fig. 4E.87. If the particles are projected towards each other with equal speeds v.

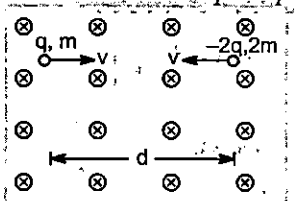


Fig. 4E.87 (a)

(a) Find the maximum value of projection speed v_m so that the two particles do not collide.
(b) Find the time after which collision occurs between the particles if projection speed equals $2v_m$.
(c) Assuming the collision to be perfectly inelastic find the radius of particle in subsequent motion.
(Neglect the electric force between the charges).

Solution: (a) The particles will move in circular paths, as velocity vector is perpendicular to magnetic field. Time period of both the particles is same $\left(T = \frac{2\pi m}{qB}\right)$. So, for collision not to take place, we must have

$$r_1 + r_2 < d$$

$$\Rightarrow \quad \frac{mv}{qB} + \frac{2mv}{2qB} < d$$

$$\Rightarrow \quad v < \frac{qBd}{2m}$$

Therefore, maximum speed should be $\frac{qBd}{2m}$

$$\Rightarrow \quad v_{max} = v_m = \frac{qBd}{2m}$$

(b) From symmetry, we observe that they collide at $\frac{d}{2}$.

$$\Rightarrow \quad \theta = \omega t = \left(\frac{qB}{m}\right)t$$

x-component of velocity at time t is,

$$v_x = 2v_m \cos\theta = 2v_m \cos\left(\frac{qB}{m}\right)t$$

$$\Rightarrow \quad \frac{d}{2} = \int_0^t v_x dt = 2v_m \int_0^t \cos\left(\frac{qB}{m}t\right)dt$$

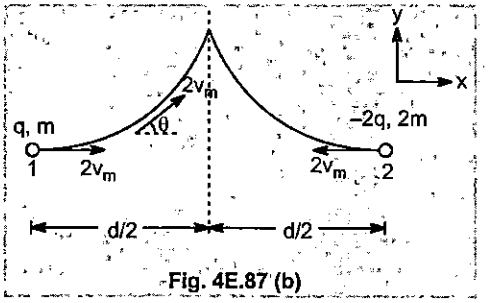


Fig. 4E.87 (b)

$$\Rightarrow \quad \frac{d}{2} = \frac{2mv_m}{qB}\left[\sin\left(\frac{qB}{m}t\right)\right]_0^t$$

$$\Rightarrow \quad \frac{d}{2} = \frac{2mv_m}{qB}\sin\left(\frac{qB}{m}t\right)$$

Substituting, $v_{max} = v_m = \frac{qBd}{2m}$, we get

$$\sin\left(\frac{qB}{m}t\right) = \frac{1}{2}$$

$$\Rightarrow \quad \frac{qBt}{m} = \frac{\pi}{6} \quad \Rightarrow \quad t = \frac{m\pi}{6qB}$$

(c) At the time of collision, $\theta = \omega t = \left(\frac{qB}{m}\right)\left(\frac{m\pi}{6qB}\right)$

$$= \frac{\pi}{6} = 30°$$

After the collision :

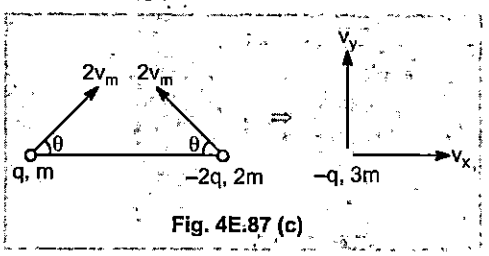


Fig. 4E.87 (c)

$$v_x = \frac{2mv_m \cos 30° - 4mv_m \cos 30°}{3m} = -\frac{v_m}{\sqrt{3}}$$

$$v_y = \frac{2mv_m \sin 30° + 4mv_m \sin 30°}{3m} = v_m$$

So, speed of combined mass is $v = \sqrt{v_x^2 + v_y^2} = \frac{2}{\sqrt{3}}v_m$

$$\Rightarrow \quad R = \frac{(3m)(v)}{qB} = \frac{(3m)\left(\frac{2}{\sqrt{3}}v_m\right)}{qB} = \frac{2\sqrt{3}mv_m}{qB}$$

Substituting, $v_m = \frac{qBd}{2m}$, we get $R = \sqrt{3}\,d$.

Example 88

A charged particle having charge 10^{-6}C and mass 10^{-10} kg is fired from the middle of the plate making an angle 30° with plane of the plate. Length of the plate is 0.17m and it is separated by 0.1m. An electric field $E = 10^{-3}$ NC^{-1} is present between the plates and just outside the plates a magnetic field is present. Find the velocity of projection of charged particle and magnitude of the magnetic field perpendicular to the plane of the Fig. 4E.88 (a), if it has to graze the plate at Q and P parallel to the surface of the plate. (Neglect gravity).

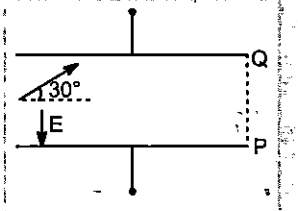


Fig. 4E.88 (a)

Solution: Since, $a = \frac{qE}{m}$

$$= \frac{(10^{-6})(10^{-3})}{(10^{-10})} = 10\,\text{ms}^{-2} \quad \text{(downwards)}$$

Path of the particle is a projectile. The particle will graze at Q if,

length of the plates $= \frac{\text{Range}}{2} = \frac{u^2 \sin 2\theta}{2g}$

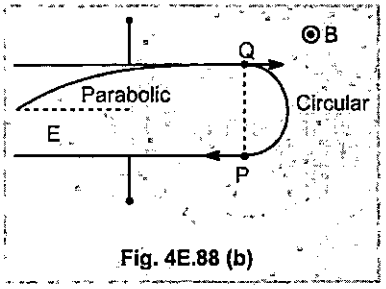


Fig. 4E.88 (b)

$$\Rightarrow \quad 0.17 = \frac{u^2 \sin(2\theta)}{2g} = \frac{u^2 \sin 60°}{20}$$

$$\Rightarrow \quad u = 1.98\,\text{ms}^{-1}$$

Speed of particle at Q is,

$$v = u\cos\theta = 1.98\cos 30° = 1.7\,\text{ms}^{-1}$$

It will graze at P if,

$PQ = 2$ (radius of circular path in magnetic field)

$$\Rightarrow \quad 0.1 = 2\left(\frac{mv}{qB}\right)$$

$$\Rightarrow \quad B = \frac{2\times 10^{-10}\times 1.7}{0.1\times 10^{-6}}$$

$$= 3.4\times 10^{-3}\,\text{T} = 3.4\,\text{mT}$$

Example 89

A ring of radius R having uniformly distributed charge Q is mounted on a rod suspended by two identical strings. The tension in strings in equilibrium is T_0. Now a vertical magnetic field is switched on the ring is rotated at constant angular velocity ω. Find the maximum ω with which the ring can be rotated if the strings can withstand a maximum tension of $\frac{3T_0}{2}$.

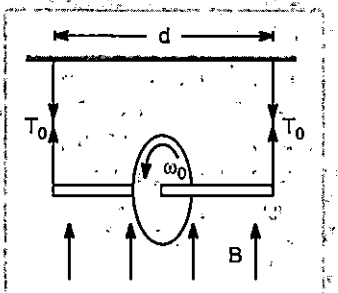


Fig. 4E.89

Solution: In equilibrium, $2T_0 = mg$

$$\Rightarrow \quad T_0 = \frac{mg}{2} \qquad \ldots(1)$$

Magnetic moment, $M = IA = \left(\frac{\omega}{2\pi}Q\right)(\pi R^2)$

$$\Rightarrow \quad \tau = MB\sin 90° = \frac{\omega BQR^2}{2}$$

Let T_1 and T_2 be the tensions in the two strings when magnetic field is switched on ($T_1 > T_2$).

For translational equilibrium of ring in vertical direction.

$$T_1 + T_2 = mg \qquad \ldots(2)$$

For rotational equilibrium,

$$(T_1 - T_2)\frac{D}{2} = \tau = \frac{\omega BQR^2}{2}$$

$$\Rightarrow \quad T_1 - T_2 = \frac{\omega BQR^2}{2} \qquad \ldots(3)$$

Solving equations (2) and (3), we have

$$T_1 = \frac{mg}{2} + \frac{\omega BQR^2}{2D}$$

As $T_1 > T_2$ and maximum values of T_1 can be $\frac{3T_0}{2}$, so we get

$$\frac{3T_0}{2} = T_0 + \frac{\omega_{\max} BQR^2}{2D} \quad \left(\frac{mg}{2} = T_0\right)$$

$$\Rightarrow \quad \omega_{\max} = \frac{DT_0}{BQR^2}$$

Example 90

A finite conductor AB carrying current I is placed near a fixed very long current carrying wire I_0 as shown in the Fig. 4E.90. Find the point of application and magnitude of the net ampere force on the conductor AB. What happens to the conductor AB if it is free to move. (Neglect gravitational field).

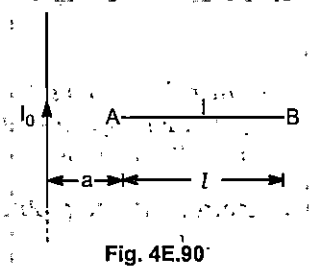


Fig. 4E.90

Solution: Consider an element of rod *AB* of length *dx* at a distance *x* from the wire. The field at this element is

$$B = \frac{\mu_0 I_0}{2\pi x}$$

$$|d\vec{F}| = BIdx = \frac{\mu_0 I_0 I}{2\pi}\frac{dx}{x}$$

$$\Rightarrow \quad F = \int dF = \frac{\mu_0 I I_0}{2\pi}\int_a^{a+l}\frac{dx}{x}$$

$$\Rightarrow \quad F = \frac{\mu_0 I I_0}{2\pi}\log_e\left(\frac{a+l}{a}\right)$$

[In the direction(I_c)]

For finding the point of application of force we shall make use of the following mathematical formula, according to which

$$r = \frac{\int x\,dF}{F}$$

where $$F = \frac{\mu_0 I I_0}{2\pi}\log_e\left(\frac{a+l}{a}\right)$$

Consider an element of length *dx* at a distance *x* from wire I_0.

Then $dF = B(x)Idx$ where $B(x) = \frac{\mu_0 I_0}{2\pi x}$

$$\Rightarrow \quad r = \frac{\int_a^{a+l} x\left(\frac{\mu_0 I_0}{2\pi x}\right)Idx}{F}$$

$$\Rightarrow \quad r = \frac{\left(\frac{\mu_0 I I_0}{2\pi}\right)l}{\left(\frac{\mu_0 I I_0}{2\pi}\right)\log_e\left(\frac{a+l}{a}\right)}$$

$$\Rightarrow \quad r = \frac{l}{\log_e\left(\frac{a+l}{a}\right)}$$

Example 91

A particle of mass m and charge q enters a region of electric field $\vec{E}$ as shown in the Fig. 4E.91 with some velocity at point P.

At the moment the particle collides elastically with smooth surface at N, the electric field $\vec{E}$ is switched off and a magnetic field $\vec{B}$ perpendicular to the plane of paper automatically switched on. If the particle hits the surface at point O, then prove that $B = 2\sqrt{\frac{mE}{qd}}$.

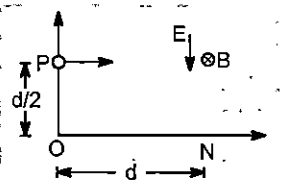


Fig. 4E.91 (a)

Solution: If *t* is the time taken to reach point *N*, then

$$\frac{d}{2} = \frac{1}{2}\left(\frac{qE}{m}\right)\times t^2$$

$$\Rightarrow \quad t = \sqrt{\left(\frac{m}{qE}\right)d}$$

If *u* is the velocity while entering the field, then

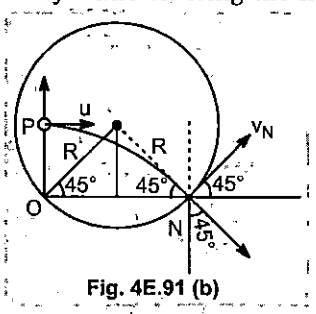


Fig. 4E.91 (b)

$$d = u \times t$$

$$\Rightarrow \quad u = \frac{d}{t} = \sqrt{\frac{qEd}{m}}$$

also if $v_{N\perp}$ is the velocity of the particle in the direction of $\vec{E}$ at *N*, then

$$v_{N\perp} = \sqrt{2\times\left(\frac{qE}{m}\right)\times\frac{d}{2}} = \sqrt{\frac{qEd}{m}}$$

So, net velocity at *N* is

$$v_N = \sqrt{v_{N\perp}^2 + u^2} = \sqrt{\frac{2qEd}{m}}$$

Since $$\tan\theta = \frac{v_{N\perp}}{u} = 1$$

$$\Rightarrow \quad \theta = 45°$$

Hence, the collision is elastic and the particle is reflected back with the same speed v_N.

The charge q now moves with a speed v_N in magnetic field $\vec{B}$. Radius of its path

$$R = \frac{mv_N}{qB} = \left(\sqrt{\frac{2mEd}{q}}\right) \times \frac{1}{B} \quad \ldots (1)$$

From Fig. 4E.91

$$R = \frac{d}{2} \times \sqrt{2} = \frac{d}{\sqrt{2}}$$

Putting in eqn. (1),

$$\frac{d}{\sqrt{2}} = \frac{1}{B}\sqrt{\frac{2mEd}{q}}$$

or $$B = 2\sqrt{\frac{mE}{qd}}$$

Example 92

A circular coil of radius r having n turns carries a current I. It is kept on a smooth inclined plane. It is prevented from slipping down by means of rigid support fixed along line perpendicular to the line of the greatest slope. The total mass of the coil is M. In which direction and of what minimum magnitude should a magnetic field be applied so that the starts tippingover about the line of rigid support.

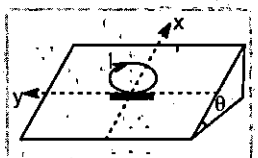


Fig. 4E.92

Solution: Direction of magnetic held should be such so as to produce a torque on the loop which opposes the torque of its weight and favors tippingover.

Now, it $\vec{B}$ be the magnetic field applied, the torque due to it on the loop would be

$$\vec{\tau}_B = \vec{M} \times \vec{B}$$

$= (NI\pi r^2)\hat{n}_1 \times B\hat{n}_2$, where $\hat{n}_1$ and $\hat{n}_2$ are the unit vector along $\vec{M}$ and $\vec{B}$.

$= (NI\pi r^2 B)\sin\alpha(-\hat{k}) \times \hat{n}_2$, respectively '$\alpha$' is angle between $\hat{n}_1$ and $\hat{n}_2$.

torque due to weight of loop

$$\vec{\tau}_w = Mgr\cos\theta\,\hat{j}$$

When the coil just starts tipping over

$$\vec{\tau}_w + \vec{\tau}_B = \vec{0}$$

$$\Rightarrow \quad Mgr\sin\theta\,\hat{j} = (NI\pi r^2 B)\sin\alpha(\hat{k} \times \hat{n})$$

$$\Rightarrow \quad \hat{n}_2 = \hat{i} \text{ (unit vector along } x\text{-axis)}$$

and $$B = \frac{Mgr\sin\theta}{NI\pi r^2\sin\alpha}$$

$$\Rightarrow \quad B_{min} = \frac{Mg\cos\theta}{NI\pi r}$$

Example 93

In a certain region of space there exists a uniform and constant electric field of magnitude E along the positive y-axis of a co-ordinate system. A charged particle of mass m and charge $-q(q > 0)$ is projected with speed $2v_0$ at an angle of 60° with the positive x-axis in xy-plane from the origin. When the x-co-ordinate of the particle becomes $\frac{\sqrt{3}mv_0^2}{qE}$, a uniform and constant magnetic field of strength B is also switched on along the positive y-axis. Find the co-ordinate of the particle as a function of time t after its projection.

Solution: According to the problem, we have

$$v = E\,\hat{j}$$

$$\vec{v} = 2v_0\cos 60°\,\hat{i} + 2v_0\sin 60°\,\hat{j} = v_0\hat{i} + v_0\sqrt{3}\hat{j}$$

$$\vec{v} = -\frac{qE}{m}\hat{j}$$

$$\vec{R}(t) = v_0 t\hat{i} + \left(v_0\sqrt{3}t - \frac{1}{2}\frac{qE}{m}t^2\right)\hat{j}$$

Given $$x = v_0 t = \frac{\sqrt{3}mv_0^2}{qE}$$

$$\Rightarrow \quad t = \frac{\sqrt{3}mv_0}{qE}$$

For $$t \le \frac{\sqrt{3}mv_0}{qE}$$

$$x(t) = v_0 t$$

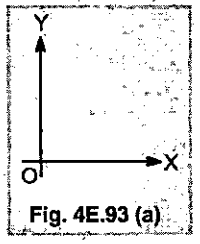


Fig. 4E.93 (a)

and $$v_y(t) = v_0\sqrt{3} - \frac{qE}{m}t$$

$$\Rightarrow \quad \left(t = \frac{\sqrt{3}mv_0}{2E}\right) = v_0\sqrt{3} - \left(\frac{qE}{m}\right)\frac{\sqrt{3}mv_0}{qE} = 0$$

For $t \ge \frac{\sqrt{3}mv_0}{qE}$, magnetic field is also present.

The particle will start moving on helical path. The cross-section of the helix will be in xz-plane

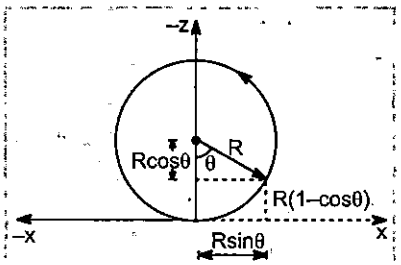


Fig. 4E.93 (b)

$$R = \frac{mv_0}{qB}$$

$$\omega = \frac{qB}{m}$$

$$\theta = \omega t = \frac{qBt}{m}$$

For $t \geq \frac{\sqrt{3}mv_0}{qE}$,

$$x(t) = \frac{\sqrt{3}mv_0^2}{qE} + R\sin\left(\frac{qB}{m}t\right)$$

$$y(t) = \frac{3mv_0^2}{2qE} - \frac{1}{2}\left(\frac{qE}{m}\right)t^2$$

$$z(t) = -R\left[1 - \cos\left(\frac{qB}{m}t\right)\right]$$

Example 94

An infinite wire 1, placed along z-axis, has current I_1 in positive z-direction. Another wire 2 is placed parallel to y-axis and carries a current I_2 in positive y-direction. The ends of the wire 2 subtend +30° and −60° at the origin with positive x-direction. The wire 2 is at a distance 'a' from the origin. Find net force on the wire 2.

Solution: The arrangement is shown here.

Please note that the direction of I_1 and the +z-axis, both are outwards towards the reader, denoted by ⊙.

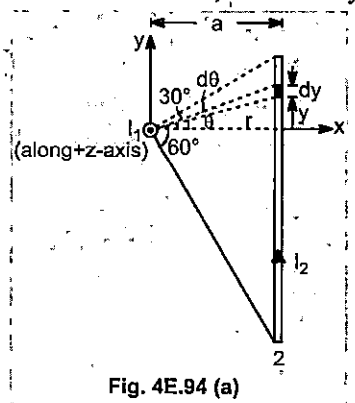


Fig. 4E.94 (a)

Let us consider an infinitesimal element of length dy at a distance y from x-axis. If $\vec{\mathbf{B}}$ be the field due to the wire 1 at the element is

$$B_1 = \frac{\mu_0 I_1}{2\pi r} = \frac{\mu_0 I_1}{2\pi\sqrt{a^2+y^2}}$$

The force on this element due to the field of wire 1 is

$$d\vec{\mathbf{F}} = I(d\vec{\mathbf{l}} \times \vec{\mathbf{B}})$$

$$\Rightarrow \quad d\vec{\mathbf{F}} = I_2(d\vec{\mathbf{y}} \times \vec{\mathbf{B}}_1)$$

$$\Rightarrow \quad |d\vec{\mathbf{F}}| = dF = (I_2 dy)\left(\frac{\mu_0 I_1}{2\pi\sqrt{a^2+y^2}}\right)\sin\theta$$

$$\Rightarrow \quad dF = \frac{\mu_0 I_1 I_2}{2\pi}\left(\frac{\sin\theta}{\sqrt{a^2+y^2}}\right)dy$$

[along positive z-direction, using right-hand thumb rule]

But $\sin\theta = \dfrac{y}{\sqrt{a^2+y^2}}$

$$\Rightarrow \quad dF = \frac{\mu_0 I_1 I_2}{2\pi}\left(\frac{ydy}{a^2+y^2}\right)$$

$$F = \frac{\mu_0 I_1 I_2}{2\pi}\int_{-\sqrt{3}a}^{\frac{a}{\sqrt{3}}} \frac{ydy}{a^2+y^2}$$

Since, $\int \frac{ydy}{a^2+y^2} = \frac{1}{2}\log_e(a^2+y^2)$

$$\Rightarrow \quad F = \frac{\mu_0 I_1 I_2}{4\pi}\left[\log_e(a^2+y^2)\Big|_{-\sqrt{3}a}^{\frac{a}{\sqrt{3}}}\right]$$

$$\Rightarrow \quad F = \frac{\mu_0 I_1 I_2}{4\pi}\log_e\left[\frac{a^2+\frac{a^2}{3}}{a^2+3a^2}\right]$$

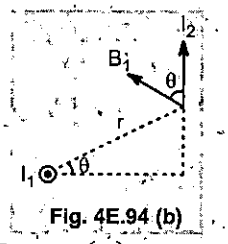


Fig. 4E.94 (b)

$$\Rightarrow \quad F = \frac{\mu_0 I_1 I_2}{4\pi}\log_e\left(\frac{1}{3}\right)$$

$$\Rightarrow \quad F = -\frac{\mu_0 I_1 I_2}{4\pi}\log_e(3)$$

Example 95

A long cylindrical conductor of radius 'a' has two cylindrical cavities of diameter 'a' through its entire length, as shown in Fig. 4E.95. A current I is directed out of the page and is uniform through a cross-section of the conductor. Find the magnitude and direction of the magnetic field in terms of μ_0, I, r and 'a' at
(a) point P_1 and
(b) point P_2.

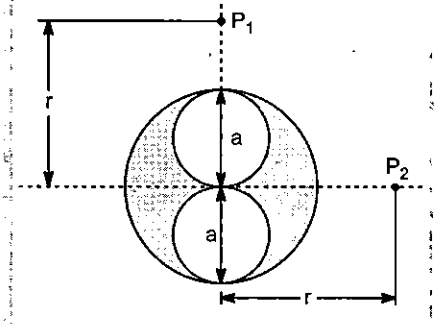


Fig. 4E.95 (a)

Solution: Note that the current I exists in the conductor with a current density $J = \frac{I}{A}$, where

$$A = \pi\left(a^2 - \frac{a^2}{4} - \frac{a^2}{4}\right) = \frac{\pi a^2}{2}$$

$$\Rightarrow \quad J = \frac{2I}{\pi a^2}$$

To find the field at either point P_1 or P_2 we follow the following steps.

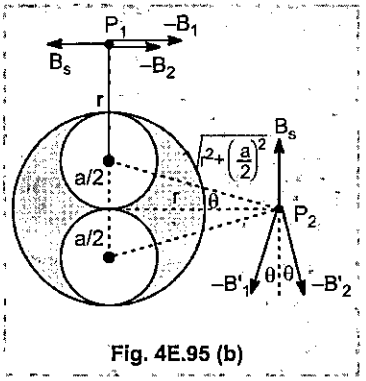


Fig. 4E.95 (b)

STEP I: Find B_s which would exist if the conductor were solid, using Ampere's law.

STEP II: Find B_1 and B_2 that would be due to the conductors of radius $\frac{a}{2}$ that could occupy the void where the holes exist.

STEP III: Then use the superposition principle and subtract the field that would be due to the part of the conductor where the holes exist from the field of the solid conductor.

(a) At point P_1, $B_s = \frac{\mu_0 J(\pi a^2)}{2\pi r}$,

$$B_1 = \frac{\mu_0 J\pi\left(\frac{a}{2}\right)^2}{2\pi\left(r - \frac{a}{2}\right)}$$

and

$$B_2 = \frac{\mu_0 J\pi\left(\frac{a}{2}\right)^2}{2\pi\left(r + \frac{a}{2}\right)}$$

$$\Rightarrow \quad B = B_s - B_1 - B_2$$

$$= \frac{\mu J\pi a^2}{2\pi}\left[\frac{1}{r} - \frac{1}{4\left(r - \frac{a}{2}\right)} - \frac{1}{4\left(r + \frac{a}{2}\right)}\right]$$

$$\Rightarrow \quad B = \frac{\mu_0(2I)}{2\pi}\left[\frac{4r^2 - a^2 - 2r^2}{4r\left(r^2 - \frac{a^2}{4}\right)}\right]$$

$$= \frac{\mu_0 I}{\pi r}\left[\frac{2r^2 - a^2}{4r^2 - a^2}\right]$$

directed to the left

(b) At point P_2, $B_s = \frac{\mu_0 J(\pi a^2)}{2\pi r}$ and

$$B_1' = B_2' = \frac{\mu_0 J\pi\left(\frac{a}{2}\right)^2}{2\pi\sqrt{r^2 + \left(\frac{a}{2}\right)^2}}$$

The horizontal components of B_1' and B_2' cancel while their vertical components add.

$$\Rightarrow \quad B = B_s - B_1'\cos\theta - B'_2\cos\theta$$

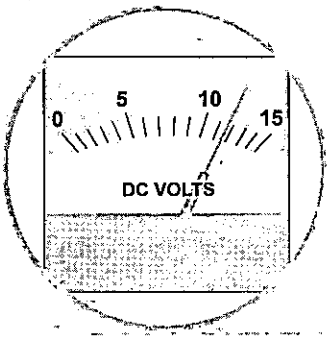
$$= \frac{\mu_0 J(\pi a^2)}{2\pi r} - 2\left(\frac{\frac{\mu_0 J\pi a^2}{4}}{2\pi\sqrt{r^2 + \left(\frac{a^2}{4}\right)}}\right)\frac{r}{\sqrt{r^2 + \left(\frac{a^2}{4}\right)}}$$

$$\Rightarrow \quad B = \frac{\mu_0 J(\pi a^2)}{2\pi r}\left[1 - \frac{r^2}{2\left(r^2 + \left(\frac{a^2}{4}\right)\right)}\right]$$

$$= \frac{\mu_0(2I)}{2\pi r}\left[1 - \frac{2r^2}{4r^2 + a^2}\right]$$

$$\Rightarrow \quad B = \frac{\mu_0 I}{\pi r}\left[\frac{2r^2 + a^2}{4r^2 + a^2}\right]$$

directed toward the top of the page.

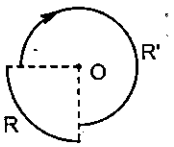


PROBLEMS

Level 1 Only One Alternative is Correct

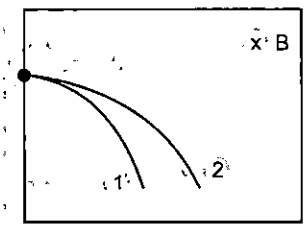
1. The dimensions of magnetic flux is :
(a) ML^2T^2I (b) $ML^2T^2I^{-1}$
(c) MT^2I^{-1} (d) $ML^2T^{-2}I^{-1}$

2. A current of I ampere is flowing through each of the bent wires as shown the magnitude and direction of magnetic field at 0 is:



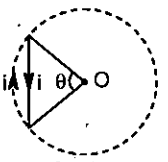
(a) $\frac{\mu_0 i}{4}\left(\frac{1}{R}+\frac{2}{R'}\right)$
(b) $\frac{\mu_0 i}{4}\left(\frac{1}{R}+\frac{3}{R'}\right)$
(c) $\frac{\mu_0 i}{8}\left(\frac{1}{R}+\frac{3}{2R'}\right)$
(d) $\frac{\mu_0 i}{8}\left(\frac{1}{R}+\frac{3}{R'}\right)$

3. Two charged particles having charges q_1 and q_2 and masses m_1 and m_2 are projected with same velocity in a region of uniform magnetic field. They follow the trajectory as shown in figure :

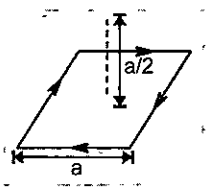


From this, we can conclude that :
(a) $q_1 > q_2$ (b) $q_1 < q_2$
(c) $m_1 < m_2$ (d) none of these

4. Net magnetic field at the centre of the circle O due to a current carrying loop as shown in figure is: $(\theta < 180^\circ)$
(a) zero
(b) perpendicular to paper inwards
(c) perpendicular to paper outwards
(d) is perpendicular to paper inwards if $\theta \le 90^\circ$ and perpendicular to paper outwards if $90^\circ \le \theta < 180^\circ$



5. The magnetic field due to a current carrying square loop of side a at a point located symmetrically at a distance of $a/2$ from its centre as shown in figure is:

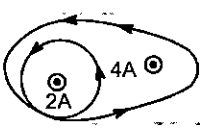


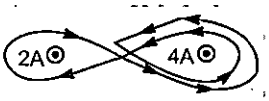
(a) $\frac{\sqrt{2}\mu_0 i}{\sqrt{3}\pi a}$
(b) $\frac{\mu_0 i}{\sqrt{6}\pi a}$
(c) $\frac{2\mu_0 i}{\sqrt{3}\pi a}$
(d) zero

6. A current carrying wire carries a current of $2A$, lies along the z-axis which is out of the page, y another wire carrying current of $4A$ in same direction lies parallel to the first, as shown in Fig.

Then around which loop linking both the wires $\int \vec{B} \cdot d\vec{l}$ will be zero :

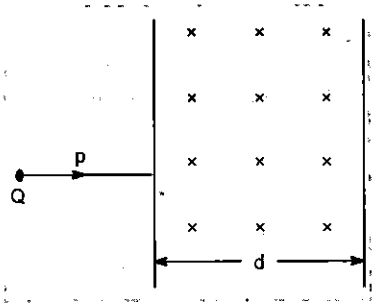
(a)

(b)


(c)


(d) None of these

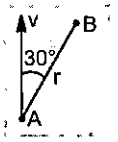
7. A particle with charge Q, moving with a momentum p, enters a uniform magnetic field normally. The magnetic field has magnitude B and is confined to a region of width d, where $d < \frac{p}{BQ}$. The particle is deflected by an angle θ in crossing the field. Then :



(a) $\sin\theta = \frac{BQd}{p}$ (b) $\sin\theta = \frac{p}{BQd}$

(c) $\sin\theta = \frac{Bp}{Qd}$ (d) $\sin\theta = \frac{pd}{BQ}$

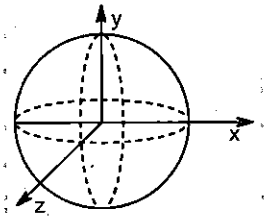
8. A charge particle A of charge $q = .2$ C has velocity $v = 100$ m/s. When it passes through point A and has velocity in the direction shown. The strength of magnetic field at point B due to this moving charge is ($r = 2$m):



(a) 2.5μT (b) 5.0μT

(c) 2.0μT (d) None of these

9. Three rings, each having equal radius R, are placed mutually perpendicular to each other and each having its centre at the origin of co-ordinate system. If current I is flowing through each ring then the magnitude of the magnetic field at the common centre is:



(a) $\sqrt{3}\frac{\mu_0 I}{2R}$ (b) zero

(c) $(\sqrt{2}-1)\frac{\mu_0 I}{2R}$ (d) $(\sqrt{3}-\sqrt{2})\frac{\mu_0 I}{2R}$

10. Which of the following pair of electric and magnetic fields represents the source of light?

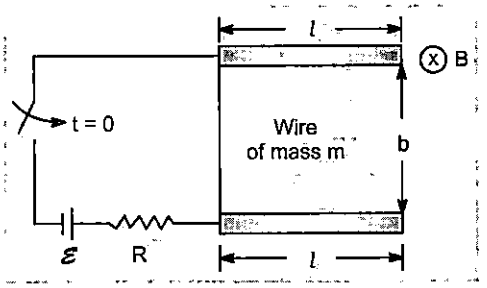
(a) $\vec{\mathbf{B}} = B_0 \sin(\omega t - kx)\hat{\mathbf{j}}$ and $\vec{\mathbf{E}} = E_0 \sin(\omega t - kz)\hat{\mathbf{k}}$

(b) $\vec{\mathbf{B}} = B_0\hat{\mathbf{i}} + B_0\hat{\mathbf{j}}$ and $\vec{\mathbf{E}} = E_0\hat{\mathbf{i}} - E_0\hat{\mathbf{j}}$

(c) $\vec{\mathbf{B}} = B_0 \sin(\omega t - kx)\hat{\mathbf{j}} + B_0 \sin(\omega t - kx)\hat{\mathbf{k}}$ and $\vec{\mathbf{E}} = E_0 \sin(\omega t - kx)\hat{\mathbf{j}} - E_0 \sin(\omega t - kx)\hat{\mathbf{k}}$

(d) none of the above

11. Two metal strips of length l each are placed parallel to each other in contact with an electric circuit as shown in figure. The entire structure is placed on a smooth horizontal floor in a region in which uniform magnetic field is existing in vertical direction.



The distance travelled by the wire as a function of time is best given by :

(a) $\frac{1}{2}\times\frac{\mathcal{E}Bb}{Rm}\times t^2$ for $t < t_0$ and $l + \frac{\mathcal{E}Bb}{Rm}(t - t_0)$ for $t \geq t_0$

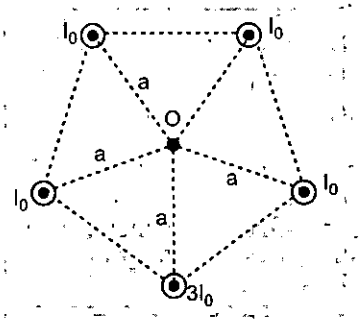
(b) $\frac{1}{2}\times\frac{\mathcal{E}Bb}{Rm}\times t^2$ for all t

(c) $\frac{1}{2}\times\frac{\mathcal{E}Bb}{Rm}\times t^2$ for $t \leq t_0$ and then the wire stops

(d) $\frac{1}{2}\times\frac{\mathcal{E}Bb}{2m}\times t^2$ for $t \leq t_0$ and $l + \frac{\mathcal{E}Bb}{Rm}\times t$ for $t > t_0$

where $t_0 = \sqrt{\frac{2lmR}{\mathcal{E}Bb}}$

12. Five parallel infinite wires are placed at the vertices of a regular pentagon. Four wires carry current I_0 each while the fifth wire carries current $3I_0$ as shown. The resultant magnetic field at the centre O is :



(a) zero (b) $\frac{\mu_0 I_0}{2\pi a}$

(c) $\frac{\mu_0 I_0}{\pi a}$ (d) infinite

13. Two concentric coils X and Y of radii 16 cm and 10 cm lie in the same vertical plane containing N-S direction. X has 20 turns and carries 16A. Y has 25 turns & carries 18A. X has current in anticlockwise direction and Y has current in clockwise direction for an observer, looking at the coils facing the west. The magnitude of net magnetic field at their common centre is:

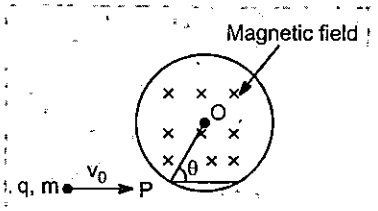
(a) $5\pi\times10^{-4}$ T towards west

(c) $13\pi\times10^{-4}$ T towards east

(b) $13\pi \times 10^{-4}$ T towards west
(d) $5\pi \times 10^{-4}$ T towards east

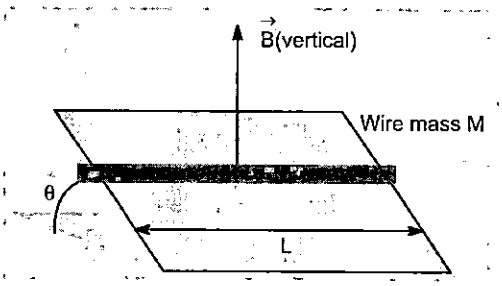
14. The dimension of $\sqrt{\frac{\mu}{\varepsilon}}$ where μ is permeability & ε is permittivity is same as:
(a) Resistance (b) Inductance
(c) Capacitance (d) None of these

15. A particle of charge q and mass m is projected with a velocity v_0 towards a circular region having uniform magnetic field B perpendicular and into the plane of paper form point P as shown in figure. R is the radius and O is the centre of the circular region. If the line OP makes an angle θ with the direction of v_0 then the value of v_0 so that particle passes through O is :



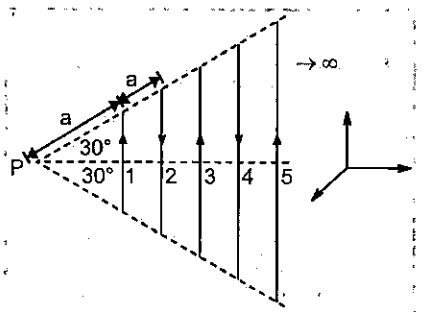
(a) $\frac{qBR}{m\sin\theta}$ (b) $\frac{qBR}{2m\sin\theta}$
(c) $\frac{2qBR}{m\sin\theta}$ (d) $\frac{3qBR}{2m\sin\theta}$

16. A straight piece of conducting wire with mass M and length L is placed on a frictionless incline tilted at an angle θ from the horizontal (as shown in figure). There is a uniform, vertical magnetic field at all points (produced by an arrangement of magnets not shown in figure):



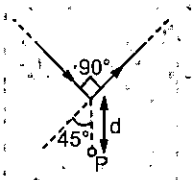
To keep the wire from sliding down the incline, a voltage source is attached to the ends of the wire. When just the right amount of current flows through the wire, the wire remains at rest. Determine the magnitude and direction of the current in the wire that will cause the wire to remain at rest.
(a) $\frac{Mg\tan\theta}{2LB}$ to the left
(b) $\frac{Mg\tan\theta}{LB}$ to the right
(c) $\frac{Mg\tan\theta}{LB}$ to the left
(d) $\frac{3Mg\tan\theta}{2LB}$ to the left

17. Infinite number of straight wires each carrying current I are equally placed as shown in the figure. Adjacent wires have current in opposite direction. Net magnetic field at point P is:



(a) $\frac{\mu_0 I}{4\pi}\frac{\ln 2}{\sqrt{3}a}\hat{\mathbf{k}}$ (b) $\frac{\mu_0 I}{4\pi}\frac{\ln 4}{\sqrt{3}a}\hat{\mathbf{k}}$
(c) $\frac{\mu_0 I}{4\pi}\frac{\ln 4}{\sqrt{3}a}(-\hat{\mathbf{k}})$ (d) Zero

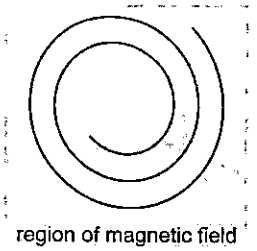
18. Find the magnetic filed at P due to the arrangement shown:



(a) $\frac{\mu_0 i}{\sqrt{2}\pi d}\left(1-\frac{1}{\sqrt{2}}\right)\otimes$ (b) $\frac{2\mu_0 i}{\sqrt{2}\pi d}\otimes$
(c) $\frac{\mu_0 i}{\sqrt{2}\pi d}\otimes$ (d) $\frac{\mu_0 i}{\sqrt{2}\pi d}\left(1+\frac{1}{\sqrt{2}}\right)\otimes$

19. A direct current is passing through a wire. It is bent to form a coil of one turn. Now it is further bent to form a coil of two turns but at smaller radius. The ratio of the magnetic induction at the centre of this coil and at the centre of the coil of one turn is:
(a) 1 : 4 (b) 4 : 1
(c) 2 : 1 (d) 1 : 1

20. An electron is moving in air at right angles to a uniform magnetic field. The diagram below shows the path of the electron. The electron is slowing down.



Which **one** of the following correctly gives the direction of motion of the electron and the direction of the magnetic field ?

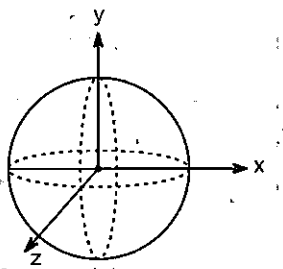
	Direction of motion	Direction of magnetic field
(a)	clockwise	into plane of paper
(b)	clockwise	out of plane of paper
(c)	anti – clockwise	into plane of paper
(d)	anti – clockwise	out of plane of paper

21. An equilateral triangular loop of wire of side l carries a current i. The magnetic filed produced at the circumcentre of the loop is :

(a) $\frac{\mu_0}{4\pi}\frac{3\sqrt{3}i}{l}$ (b) $\frac{\mu_0}{4\pi}\frac{9i}{l}$

(c) $\frac{\mu_0}{4\pi}\frac{18i}{l}$ (d) $\frac{\mu_0}{4\pi}\frac{6i}{l}$

22. Three rings, each having equal radius R, are placed mutually perpendicular to each other and each having its center at the origin of coordinate system. If current I is flowing through each ring, then the magnitude of the magnetic field at the common center is :

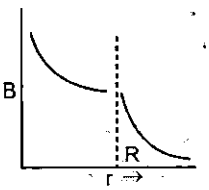


(a) $\sqrt{3}\frac{\mu_0 I}{2R}$ (b) zero

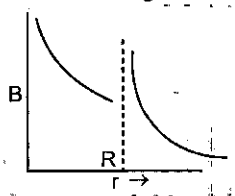
(c) $(\sqrt{2}-1)\frac{\mu_0 I}{2R}$ (d) $(\sqrt{3}-\sqrt{2})\frac{\mu_0 I}{2R}$

23. A thin, straight conductor lies along the axis of a hollow conductor of radius R. The two carry equal currents in the same direction. The magnetic field B is plotted against the distance r from the axis. Which of the following best represents the resulting curve?

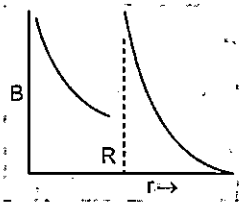
(a)



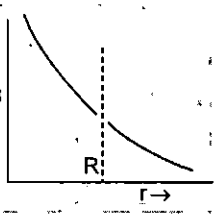
(b)



(c)



(d)



24. Equal current i is flowing in the three infinitely long wires along positive x, y and z directions. The magnitude field at a point $(0, 0, -a)$ would be:

(a) $\frac{\mu_0 i}{2\pi a}(\hat{\mathbf{j}}-\hat{\mathbf{i}})$ (b) $\frac{\mu_0 i}{2\pi a}(\hat{\mathbf{i}}+\hat{\mathbf{j}})$

(c) $\frac{\mu_0 i}{2\pi a}(\hat{\mathbf{i}}-\hat{\mathbf{j}})$ (d) $\frac{\mu_0 i}{2\pi a}(\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}})$

25. Two mutually perpendicular conductors carrying currents I_1 and I_2 lie in one plane. Locus of the point at which the magnetic induction is zero, is a:

(a) circle with centre as the point of intersection of the conductor.

(b) parabola with vertex as the point of intersection of the conductors

(c) straight line passing through the point of intersection of the conductors.

(d) rectangular hyperbola

26. The working principle of the mass spectrograph is that for a given combination of accelerating potential and magnetic field. The radius of curvature of the ion beam of charge q and mass m collected at different positions on the collector will depend on the value of :

(a) $\sqrt{\frac{m}{2q}}$ (b) $\frac{m}{q}$

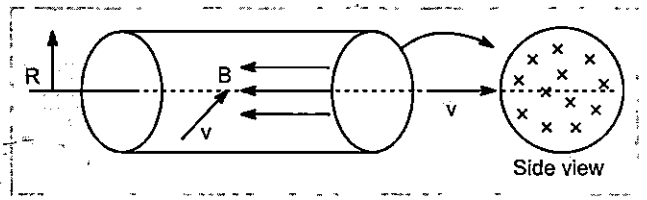
(c) $\sqrt{\frac{m}{q}}$ (d) $\frac{m}{2q}$

27. An electron moving in a circular orbit of radius R makes n turns per second. The magnetic field at the centre has magnitude :

(a) $\frac{2\mu_0 ne}{R}$ (b) $\frac{\mu_0 ne}{2R}$

(c) $\frac{\mu_0 ne}{\pi R}$ (d) zero

28. In a cylindrical region, uniform magnetic field is present as shown in figure. The cylinder is kept on a horizontal plane and its axis is horizontal. A charge particle of mass m and charge q is projected horizontally with velocity v through a hole normal to the axis of the cylinder as shown in the diagram. An observer states the particle moves first undeviated and subsequently:



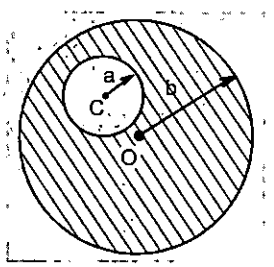
(a) oscillates inside the cylinder along a horizontal diameter passing through axis of the cylinder with time period $\frac{4RqB}{mg}$

(b) oscillates inside the cylinder along a horizontal diameter passing through axis of the cylinder with time period $\frac{2qRB}{mg}$

(c) oscillates inside the cylinder along a horizontal diameter passing through axis of the cylinder with time period $\frac{mg}{qBR}$

(d) it is not possible for the particle to oscillate in this given situation

29. A long straight metal rod has a very long hole of radius 'a' drilled parallel to the rod axis as shown in the figure. If the rod carries a current 'i' find the value of magnetic induction on the axis of the hole, where $OC = c$



(a) $\frac{\mu_0 ic}{\pi(b^2-a^2)}$ (b) $\frac{\mu_0 ic}{2\pi(b^2-a^2)}$

(c) $\frac{\mu_0(b^2-a^2)}{2\pi c}$ (d) $\frac{\mu_0 ic}{2\pi a^2 b^2}$

30. A hollow cylinder having infinite length and carrying uniform current per unit length λ along the circumference as shown. Magnetic field inside the cylinder is:

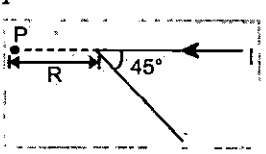
(a) $\frac{\mu_0\lambda}{2}$

(b) $\mu_0\lambda$

(c) $2\mu_0\lambda$

(d) none of these

31. A long straight wire, carrying current I, is bent at its midpoint to form an angle of 45°. Induction of magnetic field at point P, distant R from point of bending is equal to:



(a) $\frac{(\sqrt{2}-1)\mu_0 I}{4\pi a}$ (b) $\frac{(\sqrt{2}+1)\mu_0 I}{4\pi R}$

(c) $\frac{(\sqrt{2}-1)\mu_0 I}{4\sqrt{2}\pi R}$ (d) $\frac{(\sqrt{2}+1)\mu_0 I}{4\sqrt{2}\pi R}$

32. Two parallel conductors carrying current in the same direction attract each other. while two parallel beams of electrons moving in the same direction repel each other. Which of the following statements cannot be the reason for this ?

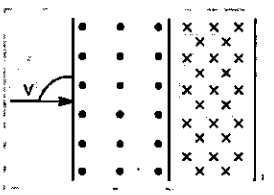
(a) The conductors are electrically neutral

(b) The conductors produce magnetic fields on each other

(c) The electron beams do not produce magnetic fields on each other

(d) The magnetic forces caused by the electron beams on each other are weaker than the electrostatic forces between them

33. The magnetic field shown in the Fig. consists of two uniform regions. The width of the first part is 5 cm and the magnetic induction here is 0.001 T. The width of the other part is also 5 cm, with the direction of the induction being opposite in direction and 0.002 T in magnitude. What should be the minimum speed of the electron arriving form the direction indicated in the Fig. so that it can pass through the magnetic field ? Mass of electron $= 9\times10^{-31}$ kg :

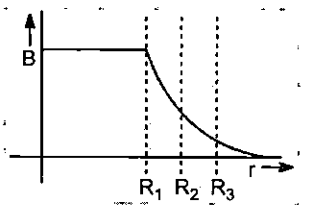


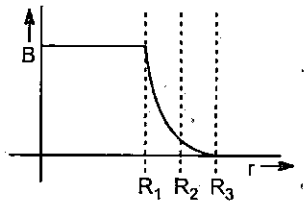
(a) $\frac{8}{9}\times10^7$ m/s (b) $\frac{4}{9}\times10^7$ m/s

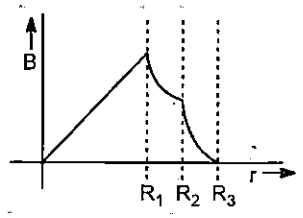
(c) $\frac{16}{9}\times10^7$ m/s (d) none of these

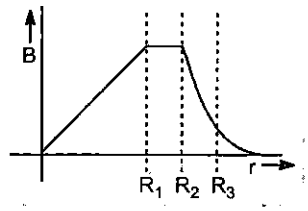
34. A coaxial cable is made-up of two conductors. The inner conductor is solid and is of radius R_1 and the outer conductor is hollow of inner radius R_2 and outer radius R_3. The space between the conductors is filled with air. The inner and outer conductors are carrying currents of equal magnitudes and in opposite directions. Then, the variation of magnetic field with distance from the axis is best plotted as :

(a)

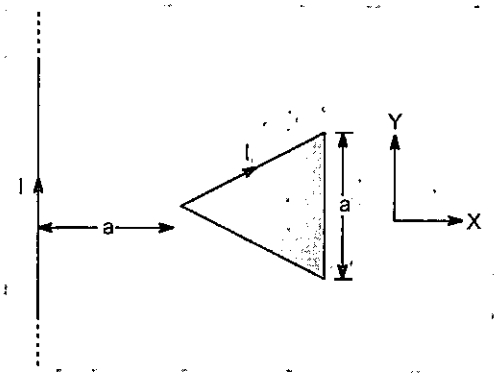


(b)


(c)


(d)


35. An equilateral triangular loop is kept near to a current carrying long wire as shown in figure. Under the action of magnetic force alone, the loop :



(a) must move along positive or negative X-axis.
(b) must move in XY plane and not along X- or Y-axis
(c) does not move
(d) moves but which way we cannot predict

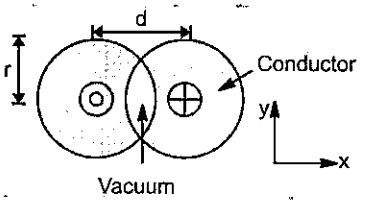
36. A uniform magnetic field $\vec{\mathbf{B}} = B_0\hat{\mathbf{j}}$ exists in a space. A particle of mass m and charge q is projected towards negative x-axis with speed v from the a point $(d, 0, 0)$. The maximum value v for which the particle does not hit y-z plane is:

(a) $\frac{2Bq}{dm}$ (b) $\frac{Bqd}{m}$
(c) $\frac{Bq}{2dm}$ (d) $\frac{Bqd}{2m}$

37. An electron is projected with velocity v_0 in a uniform electric field E perpendicular to the field. Again it is projected with velocity v_0 perpendicular to a uniform magnetic field . If r_1 is initial radius of curvature just after entering in the electric field and r_2 is initial radius of curvature just after entering in magnetic field then the ratio r_1/r_2 is equal to:

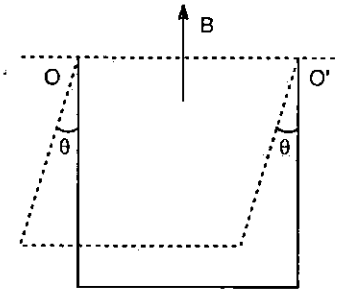
(a) $\frac{Bv_0^2}{E}$ (b) $\frac{B}{E}$
(c) $\frac{Ev_0}{B}$ (d) $\frac{Bv_0}{E}$

38. Two long conductors are arranged as shown above to form overlapping cylinders, each of radius r, whose centers are separated by a distance d. Current of density J flows into the plane of the page along the shaded part of one conductor and an equal current flows out of the plane of the page along the shaded portion of the other, as shown. What are the magnitude and direction of the magnetic field at point A?



(a) $(\mu_0 / 2\pi)\pi dJ$, in the $+y$-direction
(b) $(\mu_0 / 2\pi)d^2/r$, in the $+y$-direction
(c) $(\mu_0 / 2\pi)4d^2J/r$, in the $-y$-direction
(d) $(\mu_0 / 2\pi)Jr^2/d$ in the $-y$-direction

39. A wire of cross-sectional area A forms three sides of a square and is free to rotate about axis OO'. If the structure is deflected by an angle θ from the vertical when current I is passed through it in a magnetic field B acting vertically upward and density of the wire is ρ, then the value of θ is given by :



(a) $\frac{2A\rho g}{IB} = \cot\theta$ (b) $\frac{2A\rho g}{IB} = \tan\theta$
(c) $\frac{A\rho g}{IB} = \sin\theta$ (d) $\frac{A\rho g}{2IB} = \cos\theta$

40. A particle of charge q and mass m starts moving from the origin under the action of an electric field $\vec{\mathbf{E}} = E_0\hat{\mathbf{i}}$ and $\vec{\mathbf{E}} = B_0\hat{\mathbf{i}}$ with velocity $v = v_0\hat{\mathbf{j}}$. The speed of the particle will become $2v_0$ after a time

(a) $t = \frac{2mv_0}{qE}$ (b) $t = \frac{2Bq}{mv_0}$

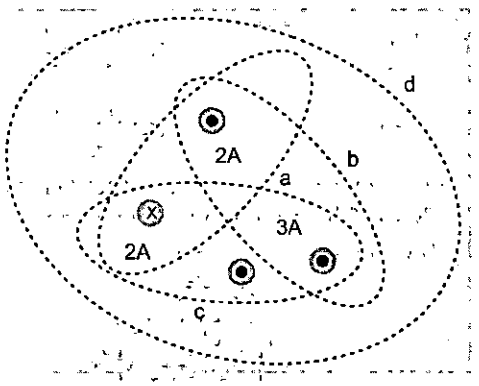
(c) $t = \frac{\sqrt{3}Bq}{mv_0}$ (d) $t = \frac{\sqrt{3}mv_0}{qE}$

41. An electron is moving along positive x-axis. A uniform electric field exists towards negative y-axis. What should be the direction of magnetic field of suitable magnitude so that net force of electron is zero ?

(a) positive z-axis (b) negative z-axis

(c) positive y-axis (d) negative y-axis

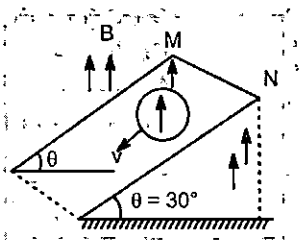
42. Rank the value of $\oint \vec{B} \cdot d\vec{I}$ for the closed paths shown in figure from the smallest to largest:



(a) a, b, c, d (b) a, c, d, b

(c) a, d, c, b (d) a, c, b, d

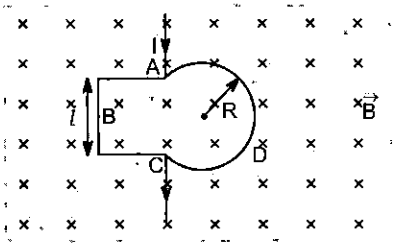
43. Two parallel frictionless rails having negligible resistance are inclined at an angle θ with horizontal. The rails are joined at the top. A ring of radius r and total resistance R, with its plane in the plane of the rails, slides down smoothly while maintaining contact with the rails. A vertical magnetic field $\vec{B}$ exists in the region. The current in the section MN when the ring attains a speed v will be :



(a) $\frac{rvB}{R}$ (b) $\frac{rvB}{2R}$

(c) $\frac{4\sqrt{3}rvB}{R}$ (d) zero

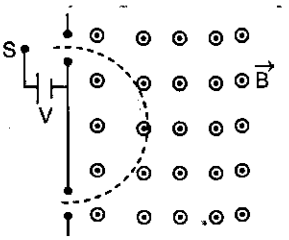
44. The Fig. shows a conducting loop $ABCD\,A$ placed in a uniform magnetic field perpendicular to its plane. The part ABC is the $(3/4)^{th}$ portion of the square of side length l. The part ADC is a circular arc of radius R. The points A and C are connected to a battery which supply a current I to the circuit. The magnetic force on the loop due to the field B is :



(a) zero (b) BIl

(c) $2BIR$ (d) $\frac{BIlR}{l+R}$

45. A mass spectrometer is a device which select particle of equal mass. An iron with electric charge $q > 0$ and mass m starts at rest from a source S and is accelerated through a potential difference V. It passes through a hole into a region of constant magnetic field $\vec{B}$ perpendicular to the plane of the paper as shown in the figure. The particle is deflected by the magnetic field and emerges through the bottom hole at a distance d from the top hole. The mass of the particle is:



(a) $\frac{qBd}{mV}$ (b) $\frac{qB^2d^2}{4V}$

(c) $\frac{qB^2d^2}{8V}$ (d) $\frac{qBd}{2mV}$

46. A electron experiences a force $(4.0\hat{i} + 3.0\hat{j}) \times 10^{13}$N in a uniform magnetic field when its velocity is $2.5\,\hat{k} \times 10^7\ ms^{-1}$. When the velocity is redirected and becomes $(1.5\hat{i} - 2.0\hat{j}) \times 10^7\ ms^{-1}$, the magnetic force of the electron is zero. The magnetic field vector $\vec{B}$ is:

(a) $0.075\hat{i} + 0.1\hat{j}$ (b) $0.1\hat{i} + 0.075\hat{j}$

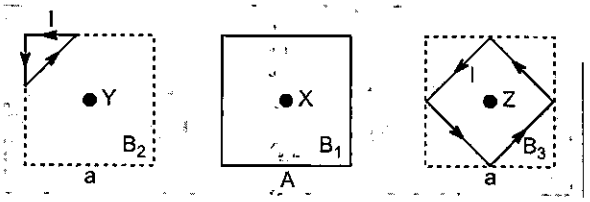
(c) $0.075\hat{i} - 0.1\hat{j} + \hat{k}$ (d) $0.075\hat{i} - 0.1\hat{j}$

47. An electron (mass $= 9.1 \times 10^{-31}$, charge $= -1.6 \times 10^{-19}$C) experiences no deflection if subjected to an electric field of 3.2×10^5 V/m and a magnetic field of 2.0×10^{-3} Wb/m^2. Both the fields are normal to the path of electron and to each other. If the electric field is removed, then the electron will revolve in an orbit of radius:

(a) 45 m (b) 4.5 m

(c) 0.45 m (d) 0.045 m

48. The magnetic flux density in vaccum at the centre of any square coil (or one turn) of side 'a' and carrying a current I is kI/a where k is independent. The magnitude of the induction at X, Y, Z are B_1, B_2 and B_3 respectively. Then :



(a) $B_3 > B_1 > B_2$ (b) $B_2 > B_3 > B_1$
(c) $B_2 > B_1 > B_3$ (d) $B_1 > B_2 > B_3$

49. An insulating rod of length l carries a charge q distributed uniformly on it. The rod is pivoted at its mid point and is rotated at a frequency f about a fixed axis perpendicular to the rod and passing through the pivot. The magnetic moment of the rod system is :

(a) $\frac{1}{12}\pi q f l^2$ (b) $\pi q f l^2$
(c) $\frac{1}{6}\pi q f l^2$ (d) $\frac{1}{3}\pi q f l^2$

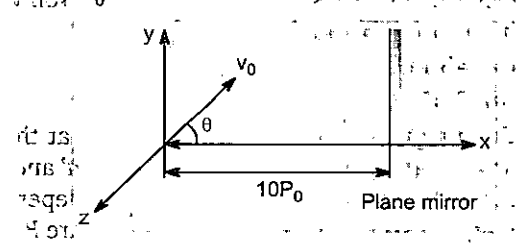
50. A electron having kinetic energy T is moving in a circular orbit of radius R perpendicular to a uniform magnetic induction $\vec{\mathbf{B}}$. If kinetic energy is doubled and magnetic induction tripled, the radius will become:

(a) $\frac{3R}{2}$ (b) $\sqrt{\frac{3}{2}}R$
(c) $\sqrt{\frac{2}{9}}R$ (d) $\sqrt{\frac{4}{3}}R$

51. In order to measure the speed v of blood flowing through an artery, a uniform magnetic field B is applied in a direction perpendicular to the flow and a voltmeter measures the voltage across the diameter D of the artery, at right angles to B. If positive and negative ions in the blood are longitudinally at rest with respect to the flow, the speed of the flow is closet to :

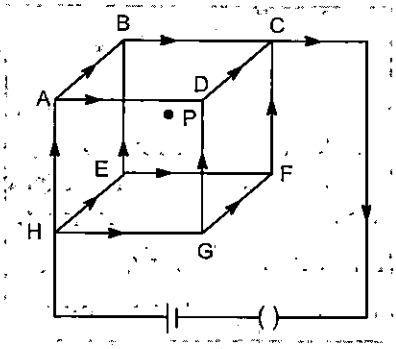
(a) $v = V/BD$ (b) $v = BD/V$
(c) $v = VD/B$ (d) $v = B/VD$

52. In the plane mirror, the coordinates of image of a charged particle (initially at origin as shown in figure) after two and a half time periods are (Initial velocity of charge particle is v_0 in the xy-plane and the plane mirror is perpendicular to the x-axis. A uniform magnetic field $B\hat{\mathbf{i}}$ exists in the space. P_0 is pitch of helix, R_0 is radius of helix):



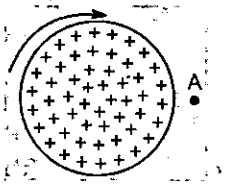
(a) $17P_0, 0, -2R_0$ (b) $3P_0, 0, -2R_0$
(c) $17.5P_0, 0, -2R_0$ (d) $3P_0, 0, 2R_0$

53. A steady current is set up in a cubic network composed of wires of equal resistance and length d as shown in figure. What is the magnetic field at the centre P due to the cubic network ?



(a) $\frac{\mu_0}{4\pi}\frac{2I}{d}$ (b) $\frac{\mu_0}{4\pi}\frac{3I}{\sqrt{2}d}$
(c) zero (d) $\frac{\mu_0}{4\pi}\frac{\theta\pi I}{d}$

54. A positively charged disk is rotated clockwise as shown in figure. The direction of the magnetic field at point A in the plane of the disk is :



(a) ⊗ (into the page)
(b) 3/4 →
(c) ← 3/4
(d) ⊙ (out of the page)

55. A charged particle moves in a magnetic field $\vec{\mathbf{B}} = 10\hat{\mathbf{i}}$ with initial velocity $\vec{\mathbf{u}} = 5\hat{\mathbf{i}} + 4\hat{\mathbf{j}}$. The path of the particle will be:

(a) straight line (b) circle
(c) helical (d) none of these

56. Three ions H^+, He^+ and O^{2+} having same kinetic energy pass through a region in which there is a uniform magnetic field perpendicular to their velocity, then:

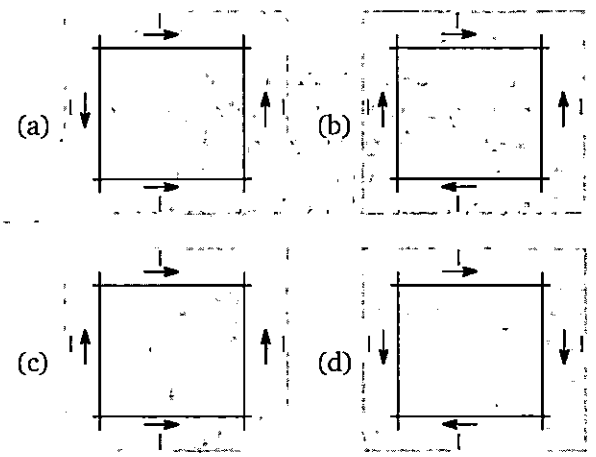
(a) H^+ will be least deflected
(b) He^+ and O^{2+} will be deflected equally
(c) O^{2+} will be deflected most
(d) all will be deflected equally

57. A charged particle of specific charge α is released from origin at time $t = 0$ with velocity $\vec{\mathbf{V}} = V_0\hat{\mathbf{i}} + V_0\hat{\mathbf{j}}$ in magnetic field $\vec{\mathbf{B}} = B_0\hat{\mathbf{i}}$. The coordinates of the particle at time $t = \frac{\pi}{B_0\alpha}$ are (specific charge $\alpha = q/m$)

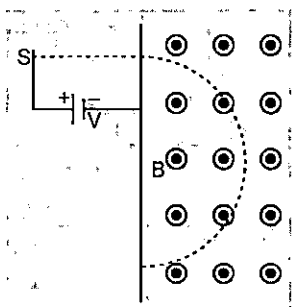
(a) $\left(\frac{V_0}{2B_0\alpha}, \frac{\sqrt{2}V_0}{aB_0}, \frac{-V_0}{B_0\alpha}\right)$ (b) $\left(\frac{-V_0}{2B_0\alpha}, 0, 0\right)$

(c) $\left(0, \frac{2V_0}{B_0\alpha}, \frac{V_0 p}{2B_0\alpha}\right)$ (d) $\left(\frac{V_0\pi}{B_0\alpha}, 0, -\frac{2V_0}{B_0\alpha}\right)$

58. Figure shows four different sets of wires that cross each other without actually touching. The magnitude of the current is the same in all four case and the directions of current flow are as indicated. For which configuration will the magnetic field at the center of the square formed by the wires be equal to zero ?



59. A mass spectrometer is a device that selects particles of a fixed mass. An ion with electric charge $q > 0$ and mass m starts at rest at a source S and is accelerated through a potential difference V. It passes through a hole into a region with a constant magnetic field $\vec{\mathbf{B}}$ perpendicular to the plane as in the figure. The particle is deflected by the magnetic field and emerges through the hole in the bottom of the figure that is a distance d from the hole at the top. What is the mass of the particle ?



(a) $m = \frac{8qV}{B^2}$ (b) $m = \frac{qBd}{8V}$

(c) $m = \frac{q^2B}{3Vd^3}$ (d) $m = \frac{qB^2d^2}{8V}$

60. Two protons move parallel to each other, keeping distance r between them, both moving with same velocity $\vec{\mathbf{V}}$. Then the ratio of the electric and magnetic force of interaction between them is:

(a) c^2/V^2 (b) $2c^2/V^2$

(c) $c^2/2V^2$ (d) None of these

61. Electrons moving with different speeds enter a uniform magnetic field in a direction perpendicular to the field. They will move along circular paths:

(a) of same radius

(b) with larger radii for the faster electrons

(c) with smaller radii for the faster electrons

(d) either (b) or (c) depending on the magnitude of the magnetic field.

62. In the previous question, time periods of rotation will be:

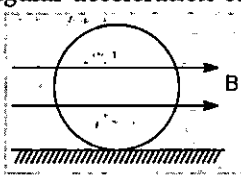
(a) same for all electrons

(b) greater for the faster electrons

(c) smaller for the faster electrons

(d) either (b) or (c) depending on the magnitude of the magnetic field

63. A conducting ring of mass 2 kg and radius 0.5 m is placed on a smooth horizontal plane. The ring carries a current of $I = 4$A. A horizontal magnetic field $B = 10$T is switched on at time $t = 0$ as shown in figure. The initial angular acceleration of the ring will be :



(a) 40π rad s^{-2} (b) 20π rad s^{-2}

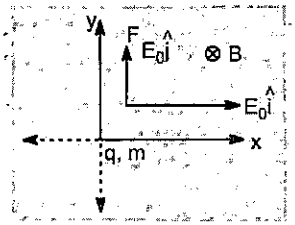
(c) 5π rad s^{-2} (d) 15π rad s^{-2}

64. A particle of specific charge (charge/mass) α starts moving from the origin under the action of an electric field $\vec{\mathbf{E}} = E_0\hat{\mathbf{i}}$ and magnetic field $\vec{\mathbf{B}} = B_0\hat{\mathbf{k}}$. Its velocity at $(x_0, y_0, 0)$ is $(4\hat{\mathbf{i}} + 3\hat{\mathbf{j}})$. The value of x_0 is:

(a) $\frac{13}{2}\frac{\alpha E_0}{B_0}$ (b) $\frac{16\alpha B_0}{E_0}$

(c) $\frac{25}{2\alpha E_0}$ (d) $\frac{5\alpha}{2B_0}$

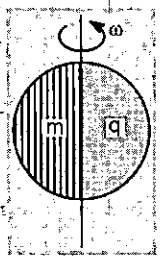
65. A free point charge q and mass m is at rest at the origin as shown in the figure. A constant electric field $E_0\hat{\mathbf{i}} - E_0\hat{\mathbf{j}}$ and constant magnetic filed $B(-\hat{\mathbf{k}})$ is present in whole region. When this charge reaches at position $(2, 20)$, its speed will be :



(a) $\sqrt{\frac{8qE_0}{m}}$ (b) $e\sqrt{\frac{4qE_0}{m}}$

(c) $\sqrt{\frac{qE_0}{Bm}}$ (d) $\sqrt{\frac{2qB}{m}}$

66. Consider a hypothetic spherical body. The body is cut into two parts about the diameter. One of hemispherical portion has mass distribution m while the other portion has identical charge distribution q. The body is rotated about the axis with constant speed ω. Then, the ratio of magnetic moment to angular momentum is :



(a) $\frac{q}{2m}$ (b) $> \frac{q}{2m}$

(c) $< \frac{q}{2m}$

(d) cannot be calculated

67. A charged particle is released from rest in a region of uniform electric and magnetic fields, which are parallel to each other. The locus of the particle will be:

(a) helix of constant pitch

(b) straight line

(c) helix of varying pitch

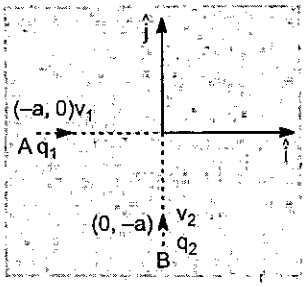
(d) cycloid

68. A particle having charge of 1C, mass 1 kg and speed 1 m/s enters a uniform magnetic field, having magnetic induction of 1T, at an angle $\theta = 30^\circ$ between velocity vector and magnetic induction. The pitch of its helical path is (in meters):

(a) $\frac{\sqrt{3}\pi}{2}$ (b) $\sqrt{3}\pi$

(c) $\frac{\pi}{2}$ (d) π

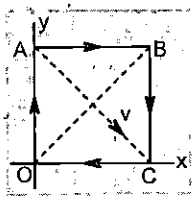
69. Two positive charges q_1 and q_2 are moving with velocities v_1 and v_2 when they are at points A and B, respectively, as shown in figure. The magnetic force experienced by charge q_1 due to the other charge q_2 is:



(a) $\frac{\mu_0 q_1 q_2 v_1 v_2}{8\sqrt{2}\pi a^2}$ (b) $\frac{\mu_0 q_1 q_2 v_1 v_2}{4\sqrt{2}\pi a^2}$

(c) $\frac{\mu_0 q_1 q_2 v_1 v_2}{2\sqrt{2}\pi a^2}$ (d) $\frac{\mu_0 q_1 q_2 v_1 v_2}{\sqrt{2}\pi a^2}$

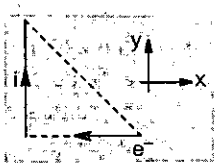
70. $OABC$ is a current carrying square loop an electron is projected from the centre of loop along its diagonal AC as shown. Unit vector in the direction of initial acceleration will be:



(a) $\hat{k}$ (b) $-\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$

(c) $-\hat{k}$ (d) $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$

71. The direction of magnetic force on the electron as shown in the diagram is along:



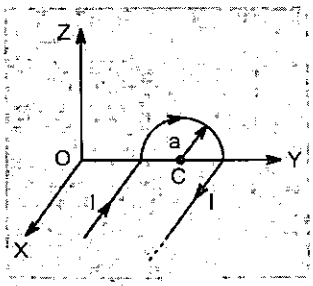
(a) y-axis

(b) $-y$-axis

(c) z-axis

(d) $-z$-axis

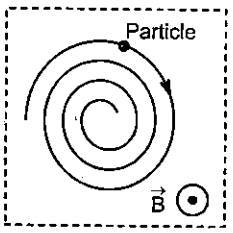
72. A long wire bent as shown in figure carries current I. If the radius of the semicircular portion is a, the magnetic field at the center C is :



(a) $\frac{\mu_0 I}{4a}$ (b) $\frac{\mu_0 I}{4\pi a}\sqrt{\pi^2+4}$

(c) $\frac{\mu_0 I}{4a} + \frac{\mu_0 I}{4\pi a}$ (d) $\frac{\mu_0 I}{4\pi a}\sqrt{(\pi^2-4)}$

73. A uniform magnetic filed is directed out of the page. A charged particle, moving in the plane of the page, follows a clockwise spiral of decreasing radius as shown. A reasonable explanation is :



(a) the charge is positive and slowing down
(b) the charge is negative and slowing down
(c) the charge is positive and speeding up
(d) the charge is negative and speeding up.

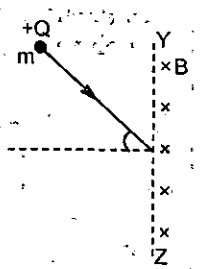
74. A charged particle is projected perpendicular to a uniform magnetic field. It's path is as shown number eight. The frequency of reversal of direction of magnetic field if mass of particle is m, charge is q and the applied magnetic field has magnitude B.
Consider only magnetic force on the particle.

(a) $\frac{qB}{4\pi m}$ (b) $\frac{qB}{2\pi m}$
(c) $\frac{qB}{\pi m}$ (d) none of these

75. In the previous question, if the particle has $-Q$ charge, the time spend by the particle in the field will be:

(a) $T\theta$ (b) $2T\theta$
(c) $T\left(\frac{\pi+2\theta}{2\pi}\right)$ (d) $T\left(\frac{\pi-2\theta}{2\pi}\right)$

76. A particle with charge $+Q$ and mass m enters a magnetic field of magnitude B, existing only to the right of the boundary YZ. The direction of the motion of the particle is perpendicular to the direction of B. Let $T = 2\pi\frac{m}{QB}$. The time spent by the particle in the field will be:



(a) $T\theta$ (b) $2T\theta$
(c) $T\left(\frac{\pi+2\theta}{2\pi}\right)$ (d) $T\left(\frac{\pi-2\theta}{2\pi}\right)$

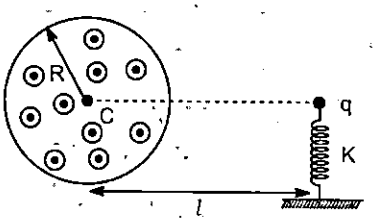
77. A particle of charge Q and mass M moves in a circular path of radius R in a uniform magnetic field of magnitude B. The same particle now moves with the same speed in a circular path of same radius R in the space between the cylindrical electrodes of the cylindrical capacitor. The radius of the inner electrode is $R/2$ while that of the outer electrode is $3R/2$. Then the potential difference between the capacitor electrodes must be:

(a) $QBR(\ln 3)/M$ (b) $QB^2R^2(\ln 3)/2M$
(c) $QB^2R^2(\ln 3)/M$ (d) None of these

78. An electron moving with a velocity $\vec{V}_1 = 2\hat{i}$ m/s at a point in a magnetic field experiences a force $\vec{F}_1 = -2\hat{j}$N. If the electron is moving with a velocity $\vec{V}_2 = 2\hat{j}$ m/s at the same point, it experiences a force $\vec{F}_2 = +2\hat{i}$N. The force on the electron would experience if it were moving with a velocity $\vec{V}_3 = 2\hat{k}$ m/s at the same point is:

(a) zero
(b) $2\hat{k}$N
(c) $-2\hat{k}$N
(d) information is insufficient

79. There is a horizontal cylindrical uniform but time-varying magnetic field increasing at a constant rate $\frac{dB}{dt}$ as shown in figure. A charged particle having charge q and mass m is kept in equilibrium, at the top of a spring of spring constant K, in such a way that it is on the horizontal line passing through the center of the magnetic field as shown in the figure. The compression in the spring will be :



(a) $\frac{1}{K}\left[mg - \frac{qR^2}{2l}\frac{dB}{dt}\right]$ (b) $\frac{1}{K}\left[mg + \frac{qR^2}{l}\frac{dB}{dt}\right]$
(c) $\frac{1}{K}\left[mg + \frac{2qR^2}{l}\frac{dB}{dt}\right]$ (d) $\frac{1}{K}\left[mg + \frac{qR^2}{2l}\frac{dB}{dt}\right]$

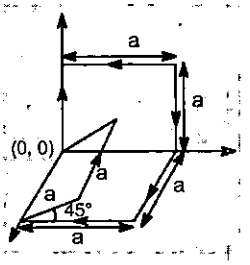
80. A particle of specific charge (q/m) is projected from the origin of coordinates with initial velocity $[ui - vj]$. Uniform electric magnetic fields exist in the region along the $+y$-direction, of magnitude E and B. The particle will definitely return to the origin once if:

(a) $[vB/2\pi E]$ is an integer
(b) $(u^2+v^2)^{1/2}$ $[B/\pi E]$ is an integer
(c) $[vB/\pi E]$ in an integer
(d) $[uB/\pi E]$ is an integer

81. Two particles of charges $+Q$ and $-Q$ are projected from the same point with a velocity v in a region of uniform magnetic field B such that the velocity vector makes an angle q with the magnetic field. Their masses are M and $2M$, respectively. Then, they will meet again for the first time at a point whose distance from the point of projection is:

(a) $2\pi Mv\cos\theta/QB$ (b) $8\pi Mv\cos\theta/QB$
(c) $\pi Mv\cos\theta/QB$ (d) $4\pi Mv\cos\theta/QB$

82. In the given loop length of each side is a. Current flowing through the loop is $I = 1$ amp. Find its magnetic dipole moment.



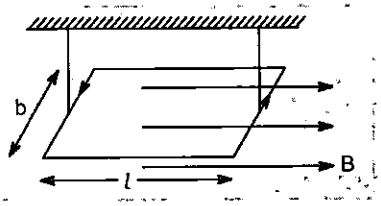
(a) $\frac{a^2}{\sqrt{2}}(\hat{j}) + \frac{a^2(1-\sqrt{2})}{\sqrt{2}}(\hat{i}) - a^2(\hat{k})$

(b) $\frac{a^2}{\sqrt{2}}(-\hat{i}) + \frac{a^2(1-\sqrt{2})}{\sqrt{2}}(\hat{j}) - a^2(-\hat{k})$

(c) $-a^2(\hat{i}) - \frac{a^2(1-\sqrt{2})}{\sqrt{2}}(\hat{j}) - \frac{a^2}{\sqrt{2}}(\hat{k})$

(d) none of these

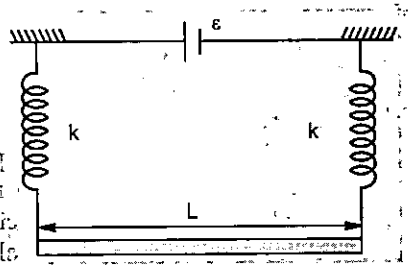
83. A uniform conducting rectangular loop of sides l, b and mass m carrying current I is hanging horizontally with the help of two vertical strings. There exists a uniform horizontal magnetic field B which is parallel to the longer side of loop. The value of tension which is least is :



(a) $\frac{mg - Bb}{2}$ (b) $\frac{mg + Bb}{2}$

(c) $\frac{mg - 2IBb}{2}$ (d) $\frac{mg + 2Bb}{2}$

84. A straight rod of mass m and length L is suspended from the identical spring as shown in the figure. The spring stretched by a distance of x_0 due to the weight of the wire. The circuit has total resistance $R\Omega$. When the magnetic field perpendicular to the plane of the paper is switched on, springs are observed to extend further by the same distance. The magnetic field strength is:



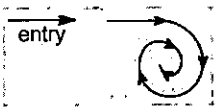
(a) $\frac{mgR}{\varepsilon L}$; directed outward from the plane of the paper

(b) $\frac{mgR}{2\varepsilon x_0}$; directed outward from the plane of the paper

(c) $\frac{mgR}{\varepsilon L}$; directed into the plane of the paper

(d) $\frac{mgR}{\varepsilon x_0}$; directed into the plane of the paper

85. A charged particle enters a uniform magnetic field perpendicular to its initial direction travelling in air. The path of the particle is seen to follow the path in figure. Which of statements 1–3 is/are correct?



[1] The magnetic field strength may have been increased while the particle was travelling in air

[2] The particle lost energy by ionising the air

[3] The particle lost charge by ionising the air

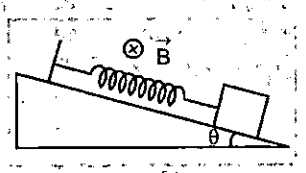
(a) 1, 2, 3 are correct

(b) 1, 2 only are correct

(c) 2, 3 only are correct

(d) 1 only

86. A small block of mass m, having charge q, is placed on a frictionless inclined plane making an angle θ with the horizontal. There exists a uniform magnetic field B parallel to the inclined plane but perpendicular to the length of spring. If m is slightly pulled on the incline in downward direction, the time period of oscillation will be : (assume that the block does not leave contact with the plane)



(a) $2\pi\sqrt{\frac{m}{K}}$ (b) $2\pi\sqrt{\frac{2m}{K}}$

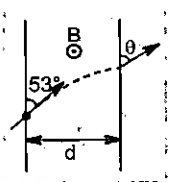
(c) $2\pi\sqrt{\frac{qB}{K}}$ (d) $2\pi\sqrt{\frac{qB}{2K}}$

87. Constant current I is flowing through a circular coil of radius R. At what distance from the centre will the magnetic field (on the axis) be maximum :

(a) $\frac{R}{\sqrt{2}}$ (b) $\frac{R}{2}$

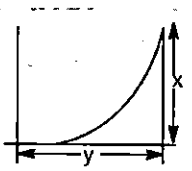
(c) R (d) zero

88. A particle moving with velocity v having specific charge (q/m) enters a region of magnetic field B having width $d = \frac{3mv}{5qB}$ at angle 53° to the boundary of magnetic field. Find the angle θ in the diagram.



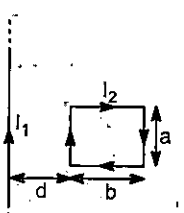
(a) 37° (b) 60°
(c) 90° (d) none

89. A particle having charge q enters a region of uniform magnetic field $\vec{B}$ (directed inwards) and is deflected a distance x after travelling a distance y. The magnitude of the momentum of the particle is:



(a) $\frac{qBy}{2}$ (b) $\frac{qBy}{x}$
(c) $\frac{qB}{2}\left(\frac{y^2}{x}+x\right)$ (d) $\frac{qBy^2}{2x}$

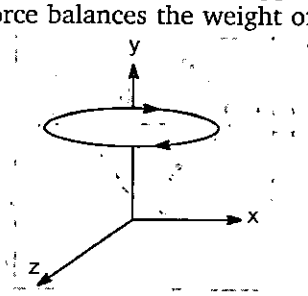
90. An infinitely long straight wire carrying current I_1, is situated in the plane of and parallel to one side of a rectangular loop of wire carrying current I_2 as shown in the Fig. The force experienced by the infinitely long wire due to the magnetic field of the rectangular loop of the wire is:



(a) $\frac{\mu_0 I_1 I_2 a}{2\pi}\left[\frac{1}{b}-\frac{1}{d+b}\right]$ (b) $\frac{2\mu_0 I_1 I_2 a}{\pi}\left[\frac{1}{b}-\frac{1}{d+b}\right]$
(c) $\frac{\mu_0 I_1 I_2 a}{\pi}\left[\frac{1}{b}-\frac{1}{d+b}\right]$ (d) $\frac{4\mu_0 I_1 I_2 a}{\pi}\left[\frac{1}{b}-\frac{1}{d+b}\right]$

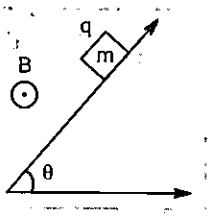
91. Antiparallel currents are directed so that one is out of the page and the other is into the page Compare the magnitude of the magnetic field B_2 at any arbitrary point equidistant from the wires to he magnitude of the field B_1 at that point form one wire alone.
(a) $B_2 > B_1$ for all equidistant points
(b) $B_2 < B_1$ for all equidistant points
(c) $B_2 > B_1$ for closer equidistant points only
(d) $B_2 < B_1$ for closer equidistant points only

92. A circular coil having mass m is kept above the ground ($x-z$ plane) at some height. The coil carries a current i in the direction shown in figure. In which direction a uniform magnetic field $\vec{B}$ be applied so that the magnetic force balances the weight of the coil ?



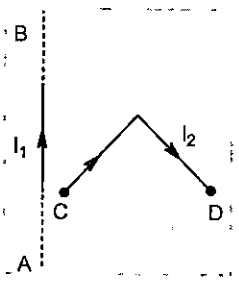
(a) Positive x-direction
(b) Negative x-direction
(c) Positive z-direction
(d) None of these

93. A block of mass m & charge q is released on a long smooth inclined plane magnetic field B is constant, uniform, horizontal and parallel to surface as shown. Find the time from start when block loses contact with the surface:



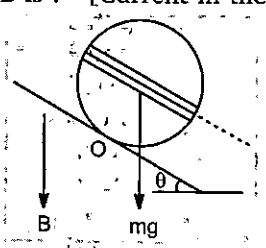
(a) $\frac{m\cos\theta}{qB}$ (b) $\frac{m\,\mathrm{cosec}\,\theta}{qB}$
(c) $\frac{m\cot\theta}{qB}$ (d) none

94. In figure shown a current I_1 is established in the long straight wire AB. Another wire CD carrying current I_2 is placed in the plane of the paper. The line joining the ends of this wire is perpendicular to the wire AB. The force on the wire CD is:



(a) zero
(b) towards left
(c) directed upwards (d) none of these

95. In figure, a coil of single turn is wound on a sphere of radius r and mass m. The plane of the coil is parallel to the inclined plane and lies in the equatorial plane of the sphere. If the sphere is in rotational equilibrium, the value of B is : [Current in the coil is I]

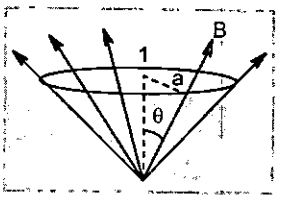


(a) $\frac{mg}{\pi Ir}$ (b) $\frac{mg\sin\theta}{\pi I}$
(c) $\frac{mg\sin\theta}{\pi I}$ (d) none of these

96. A conductor of length l and mass m is placed along the east-west line on a table. Suddenly a certain amount of charge is passed through it and it is found to jump to a height h. The earth's magnetic induction is B. The charge passed through the conductor is:

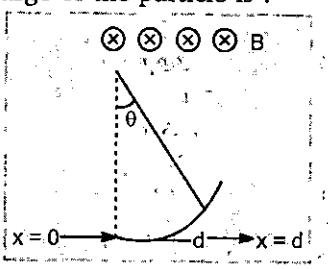
(a) $\frac{1}{Bmgh}$ (b) $\frac{\sqrt{2gh}}{Blm}$
(c) $\frac{gh}{Blm}$ (d) $\frac{m\sqrt{2gh}}{Bl}$

97. A circular current loop of radius a is placed in a radial field B as shown. The net force acting on the loop is:



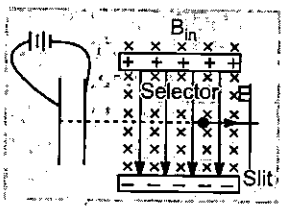
(a) zero
(b) $2\pi BaI\cos\theta$
(c) $2\pi BaI\sin q$
(d) None of these

98. A charge particle moving along + ve x-direction with a velocity v enters a region where there is a uniform magnetic field $B_0(-\hat{k})$, from $x = 0$ to $x = d$. The particle gets deflected at an angle θ from its initial path. The specific charge of the particle is :



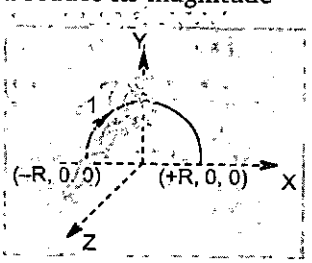
(a) $\frac{v\cos\theta}{Bd}$ (b) $\frac{v\tan\theta}{Bd}$
(c) $\frac{v}{Bd}$ (d) $\frac{v\sin\theta}{Bd}$

99. The given figure. shows a set-up for accelerating protons from rest, through a potential (fixed) by the battery, and then through a velocity selector. The E and B fields are carefully adjusted so that the accelerated protons go straight through the slit after exiting the selector. You want to modify this so that electrons instead of protons are used. You reverse the leads on the accelerating battery. What do you need to do to the potential difference between the plates that create the electric field in the selector ?



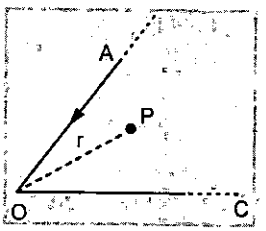
(a) Nothing
(b) Reduce its magnitude
(c) Increase its magnitude
(d) Reverse the polarity, and reduce its magnitude

100. A semi-circular current carrying wire having radius R is placed in x-y plane with its centre at origin 'O'. There is non-uniform magnetic field $\vec{B} = \frac{B_0 x}{2R}\hat{k}$ (here B_0 is +ve constant) is existing in the region. The magnetic force acting on semi circular wire will be along:



(a) $-x$-axis (b) $+y$-axis
(c) $-y$-axis (d) $+x$-axis

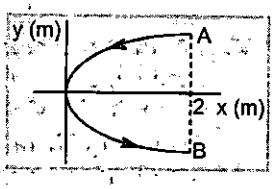
101. Two wires AO and OC carry equal currents I as shown in figure. One end of both the wires extends to infinity. Angle AOC is α.



The magnitude of magnetic field at a point P on the bisector of these two wires at a distance r from point O is :

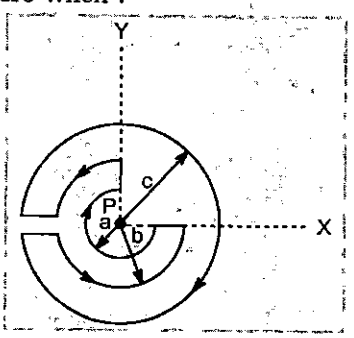
(a) $\frac{\mu_0 I}{2\pi r}\cot\left(\frac{\alpha}{2}\right)$ (b) $\frac{\mu_0 I}{4\pi r}\cot\left(\frac{\alpha}{2}\right)$
(c) $\frac{\mu_0}{2\pi}\frac{I}{r}\frac{\left(1+\cos\frac{\alpha}{2}\right)}{\sin\left(\frac{\alpha}{2}\right)}$ (d) $\frac{\mu_0 I}{4\pi r}\left(\frac{\alpha}{2}\right)$

102. A conducting wire bent in the form of a parabola $y^2 = 2x$ carries a current $I = 2A$ as shown in figure. This wire is placed in a uniform magnetic field $\vec{B} = -4\hat{k}$ Tesla. The magnetic force on the wire is (in newton)



(a) $-16\hat{i}$ (b) $32\hat{i}$
(c) $-32\hat{i}$ (d) $16\hat{i}$

103. For $c = 2a$ and $a < b < c$, the magnetic field at point P will be zero when :



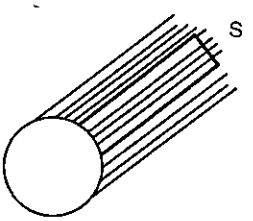
(a) $a = b$ (b) $a = \frac{3}{5}b$
(c) $a = \frac{5}{3}b$ (d) $a = \frac{1}{3}b$

104. Two long parallel straight conductors carry current i_1 and i_2 $(i_1 > i_2)$. When the currents are in the same direction, the magnetic field at a point midway

between the wires is 20μT. If the direction of i_2 is reversed, the field becomes 50μT. The ratio of the currents i_1/i_2 is :

(a) 5/2 (b) 7/3
(c) 4/3 (d) 5/3

105. A long copper wire of circular cross-section carries a current of 10A. S is a plane surface parallel to the axis of the copper wire wholly inside the wire. The magnetic flux per meter of wire, for the plane surface is, in Weber/m.



(a) 1×10^{-5} (b) 1.5×10^{-6}
(c) 1×10^{-6} (d) 1.5×10^{-5}

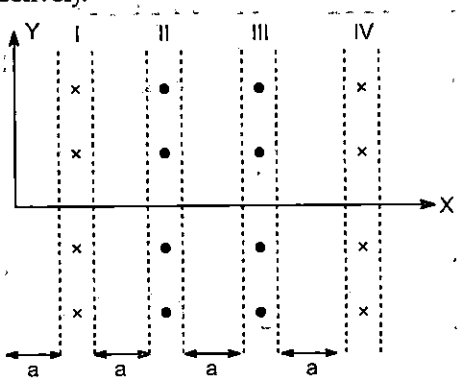
106. The dimensional formula for the physical quantity $\frac{E^2\mu_0\varepsilon_0}{B^2}$ is:
(E = electric field and B = magnetic field)

(a) $L^0M^0T^0$ (b) $L^1M^0T^{-1}$
(c) $L^{-1}M^0T^1$ (d) $L^{1/2}M^0T^{-1/2}$

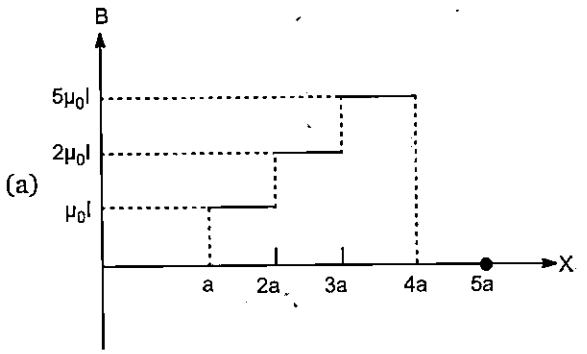
107. The magnetic moment of a circular orbit of radius 'r' carrying a charge 'q' and rotating with velocity v is given by:

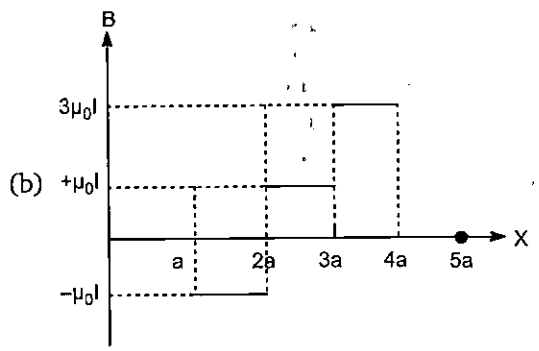
(a) $\frac{qvr}{2\pi}$ (b) $\frac{qvr}{2}$
(c) $qv\pi r$ (d) $qv\pi r^2$

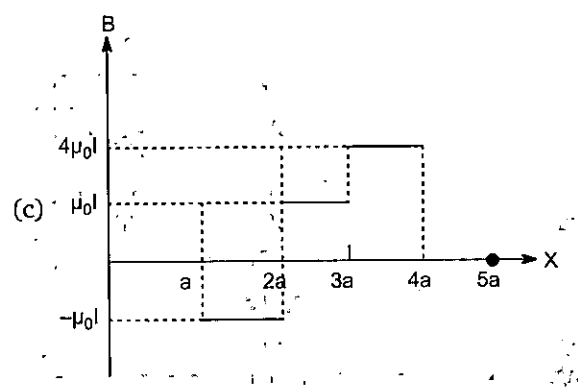
108. For infinite thin current carrying sheets are placed in YZ plane. The $2D$ view of the arrangement is as shown in figure. Direction of current has also been shown in the figure. The linear current density *i.e.*, current per unit width in the four sheets are $I, 2I, 3I$ and $4I$, respectively.

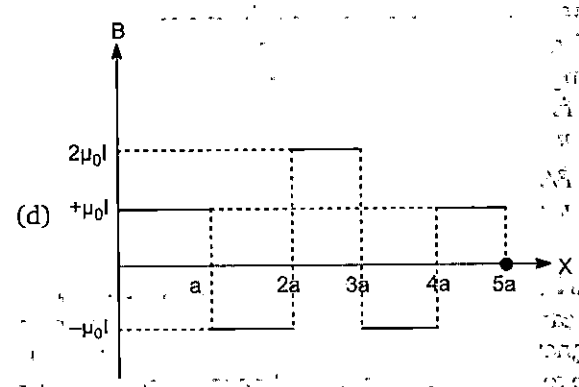


The magnetic field as a function of x is best represented by :

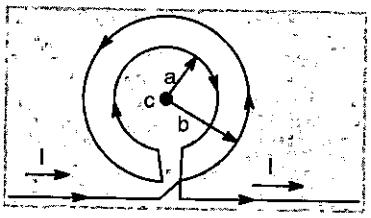








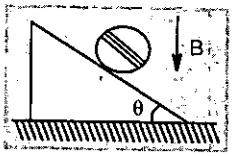
109. An otherwise infinite, straight wire has two concentric loops of radii a and b carrying equal currents in opposite directions as shown in figure. The magnetic field at the common center is zero for :



(a) $\frac{a}{b}=\frac{\pi-1}{\pi}$ (b) $\frac{a}{b}=\frac{\pi}{\pi+1}$

(c) $\frac{a}{b}=\frac{\pi-1}{\pi+1}$ (d) $\frac{a}{b}=\frac{\pi+1}{\pi-1}$

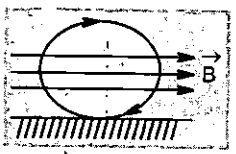
110. In the figure shown a coil of single turn is wound on a sphere of radius R and mass m. The plane of the coil is parallel to the plane and lies in the equatorial plane of the sphere. Current in the coil is i. The value of B if the sphere is in equilibrium is:



(a) $\frac{mg\cos\theta}{\pi iR}$ (b) $\frac{mg}{\pi iR}$

(c) $\frac{mg\tan\theta}{\pi iR}$ (d) $\frac{mg\sin\theta}{\pi iR}$

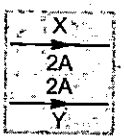
111. A conducting ring of mass 2 kg and radius 0.5 m is placed on a smooth horizontal plane. The ring carries a current i = 4A. A horizontal magnetic field B = 10 T is switched on at time t = 0 as shown in figure. The initial angular acceleration of the ring will be:



(a) 40 π rad/s^2 (b) 20 π rad/s^2

(c) 50 π rad/s^2 (d) 15 π rad/s^2

112. In given figure, X and Y are two long straight parallel conductors each carrying a current of 2A. The force on each conductor is F newton. When the current in each is changed to 1 A and reversed in direction, the force on each is now:



(a) $F/4$ and unchanged in direction

(b) $F/2$ and reversed in direction

(c) $F/2$ and unchanged in direction

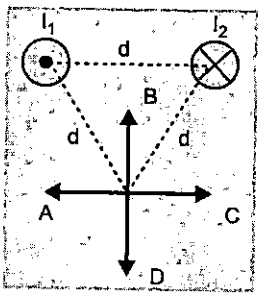
(d) $F/4$ and reversed in direction

113. A circular coil of 100 turns and effective diameter 20 cm carries a current of 0.5A It is to be turned in a magnetic field $B = 2T$ from a position in which 0 equal zero to θ equals 180°. The work required in this process is :

(a) $1\pi J$ (b) $2\pi J$

(c) $4\pi J$ (d) $8\pi J$

114. The figure shows two long wires carrying equal currents I_1 and I_2 flowing in opposite directions. Which of the arrows labeled A to D correctly represents the direction of the magnetic field due to the wires at a point located at equal distance d from each wire ?



(a) A (b) B

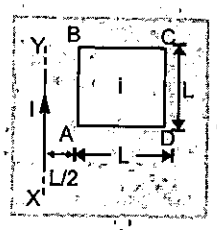
(c) C (d) D

115. A metal ring of radius r = 0.5 m with its plane normal to a uniform magnetic field B of induction 0.2 T carries a current I = 100 A. The tension in Newtons developed in the ring is:

(a) 100 (b) 50

(c) 25 (d) 10

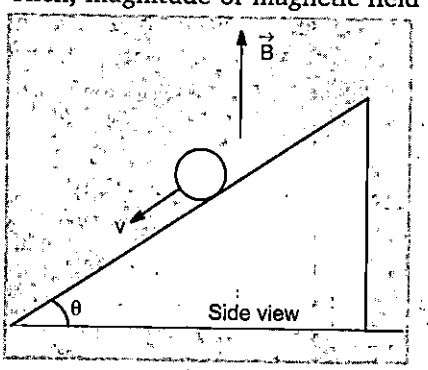
116. A square loop $ABCD$, carrying a current i, is placed near and coplanar with a long straight conductor XY carrying a current I, the net force on the loop will be:



(a) $\frac{2\mu_0 Ii}{3\pi}$ (b) $\frac{\mu_0 Ii}{2\pi}$

(c) $\frac{2\mu_0 Iil}{3\pi}$ (d) $\frac{\mu_0 Iil}{2\pi}$

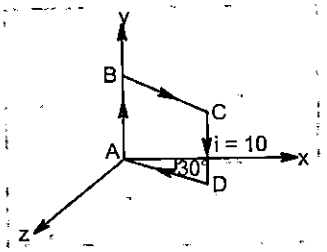
117. A conducting rod of length l and mass m is moving down a smooth inclined plane of inclination θ with constant velocity v. A current I is flowing in the conductor in a direction perpendicular to paper inward. A vertically upward magnetic field $\vec{B}$ exists in space. Then, magnitude of magnetic field $\vec{B}$ is :



(a) $\frac{mg}{Il}\sin\theta$ (b) $\frac{mg}{Il}\tan\theta$

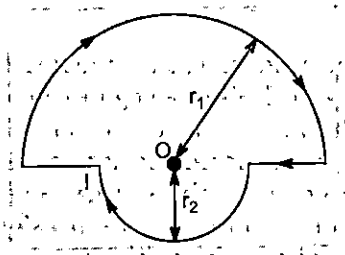
(c) $\frac{mg\cos\theta}{Il}$ (d) $\frac{mg}{Il\sin\theta}$

118. Fig. shows a square current carrying loop *ABCD* of side 10 cm and current $I = 10$A. The magnetic moment $\vec{M}$ of the loop is:



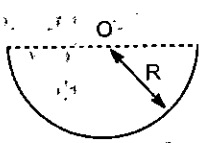
(a) $(0.05)\,(\hat{i}-\sqrt{3}\hat{k})$ A - m^2
(b) $(0.05)\,(\hat{j}+\hat{k})$ A - m^2
(c) $(0.05)\,(\sqrt{3}\hat{i}+\hat{k})$ A - m^2
(d) $(\hat{i}+\hat{k})$ A - m^2

119. In figure there are two semicircles of radii r_1 and r_2 in which a current I is flowing. The magnetic induction a the center O will be :



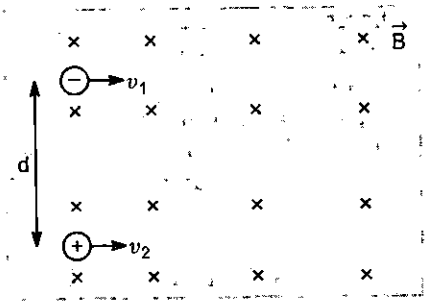
(a) $\frac{\mu_0}{r}(r_1+r_2)$ (b) $\frac{\mu_0 I}{4}(r_1-r_2)$
(c) $\frac{\mu_0 I}{4}\left(\frac{r_1+r_2}{r_1 r_2}\right)$ (d) $\frac{\mu_0 I}{4}\left(\frac{r_2-r_1}{r_1 r_2}\right)$

120. A current I flows in a long straight wire with its cross-section having the form of a this half ring of radius R (see Fig.). O is the centre of its mid-section perpendicular to its length. Assume the wire extends to infinite length on either side of the mid-section. Then the magnetic induction field B at the centre O will be :



(a) $B = \frac{\mu_0 I}{\pi R}$ (b) $B = \frac{\mu_0 I}{2\pi R}$
(c) $B = \frac{\mu_0 I}{\pi^2 R}$ (d) $B = \frac{\mu_0 I}{2R}$

121. Two identical particles having the same mass m and charges $+q$ and $-q$ separated by a distance d enter in a uniform magnetic field B directed perpendicular to paper inward with speeds v_1 and v_2 as shown in figure. The particles will not collide if :

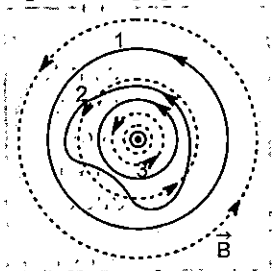


(a) $d > \frac{m}{Bq}(v_1+v_2)$ (b) $d < \frac{m}{Bq}(v_1+v_2)$
(c) $d > \frac{2m}{Bq}(v_1+v_2)$ (d) $v_1 = v_2$

122. A straight current carrying conductor is placed in such a way that the current inn the conductor flows in the direction out of the plane of the paper. The conductor is placed between two poles of two magnets, as shown. The conductor will experience a force in the direction towards:

(a) P (b) Q
(c) R (d) S

123. Consider the three closed loops drawn using solid line in the magnetic field (magnetic field lines are drawn using dotted line) of an infinite curent-carrying wire normal to the plane of paper as shown.

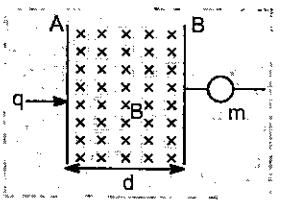


Rank the line integral of the magnetic field along each path in order of increasing magnitude :
(a) $1 > 2 > 3$ (b) $1 = 3 > 2$
(c) $1 = 2 = 3$ (d) $3 > 2 > 1$

124. A charged particle of specific charge (charge/mass) α is released from origin at time $t = 0$ with velocity $\vec{v} = v_0(\hat{i}+\hat{j})$ in uniform magnetic field $\vec{B} = B_0\hat{i}$. Coordinates of the particle at time $t = \frac{\pi}{B_0\alpha}$ are :

(a) $\left(\frac{v_0}{2B_0\alpha}, \frac{\sqrt{2}v_0}{\alpha B_0}, \frac{-v_0}{B_0\alpha}\right)$ (b) $\left(\frac{-v_0}{2B_0\alpha}, 0, 0\right)$
(c) $\left(0, \frac{2v_0}{B_0\alpha}, \frac{v_0\pi}{2B_0\alpha}\right)$ (d) $\left(\frac{v_0\pi}{B_0\alpha}, 0, \frac{-2v_0}{B_0\alpha}\right)$

125. Two uncharged plates A and B are at the same electrical potential, and parallel and separated by a distance d. A uniform magnetic induction field B acts perpendicular to plane of paper (and parallel to plates) between the plates. Particles, each with charge q, mass m and kinetic energy K are shot through a hole in A directly towards B, each particle entering the hole perpendicular to A. An ammeter m connected to an electrical network is connected to B and measures the current collected by the plate B, as the particles hit it. The limiting value of the filed B for which the ammeter will stop showing a current is :



(a) $\frac{\sqrt{2mK}}{qd}$ (b) $\frac{\sqrt{3mK}}{qd}$

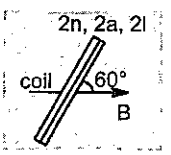
(c) $\frac{\sqrt{mK}}{2qd}$ (d) $\frac{2\sqrt{mK}}{qd}$

126. A particle of charge per unit mass α is released from origin with a velocity $\vec{\mathbf{v}} = v_0\hat{\mathbf{i}}$ in a uniform magnetic field $\vec{\mathbf{B}} = -B_0\hat{\mathbf{k}}$. If the particle passes through $(0, y, 0)$, then y is equal to :

(a) $-\frac{2v_0}{B_0\alpha}$ (b) $\frac{v_0}{B_0\alpha}$

(c) $\frac{2v_0}{B_0\alpha}$ (d) $-\frac{v_0}{B_0\alpha}$

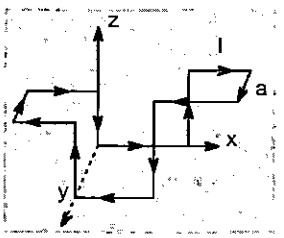
127. A rectangular coil PQ has $2n$ turns, an area $2a$ and carries a current $2I$, (see figure). The plane of the coil is at 60° to a horizontal uniform magnetic field of flux density B. The torque on the coil due to magnetic force is:



(a) $BnaI$ sin 60° (b) 8 $BnaI$ cos 60°

(c) 4na$I$ B sin 60° (d) none of these

128. The magnetic moment of the loop shown in the adjacent figure is :



(a) $-3a^2I\hat{\mathbf{k}}$ (b) $-2a^2I\hat{\mathbf{k}}$

(c) $-2a^2I\hat{\mathbf{j}}$ (d) $-3a^2I\hat{\mathbf{j}}$

129. A particle of charge per unit mass α is released from origin with velocity $\vec{\mathbf{v}} = v_0\hat{\mathbf{i}}$ in a magnetic field

$\vec{\mathbf{B}} = -B_0\hat{\mathbf{k}}$ for $x \le \frac{\sqrt{3}}{2}\frac{v_0}{B_0\alpha}$

and $\vec{\mathbf{B}} = 0$ for $x > \frac{\sqrt{3}}{2}\frac{v_0}{B_0\alpha}$

The x-coordinate of the particle at time $t\left(> \frac{\pi}{3B_0\alpha}\right)$ would be :

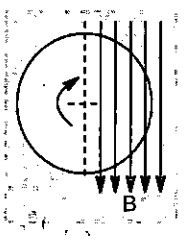
(a) $\frac{\sqrt{3}}{2}\frac{v_0}{B_0\alpha} + \frac{\sqrt{3}}{2}v_0\left(t - \frac{\pi}{B_0\alpha}\right)$

(b) $\frac{\sqrt{3}}{2}\frac{v_0}{B_0\alpha} + v_0\left(t - \frac{\pi}{3B_0\alpha}\right)$

(c) $\frac{\sqrt{3}}{2}\frac{v_0}{B_0\alpha} + \frac{v_0}{2}\left(t - \frac{\pi}{3B_0\alpha}\right)$

(d) $\frac{\sqrt{3}}{2}\frac{v_0}{B_0\alpha} + \frac{v_0 t}{2}$

130. A thin non conducting disc of radius R is rotating clockwise (see figure) with an angular velocity ω about its central axis, which is perpendicular to its plane. Both its surfaces carry +ve charges of uniform surface density. Half the disc is in a region of a uniform, unidirectional magnetic field B parallel to the plane of the disc, as shown. Then:



(a) the net torque of the disc is zero.

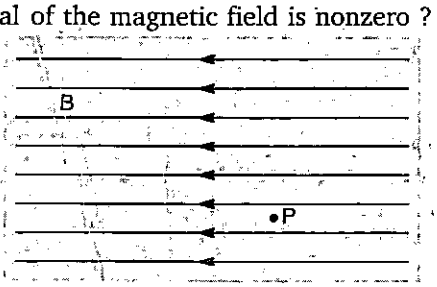
(b) the net torque vector on the disc is directed leftwards.

(c) the net torque vector on the disc is directed rightwards.

(d) the net torque vector on the disc is parallel to B.

131. Consider the uniform magnetic field shown :

Starting from point P and without leaving the region of magnetic field, is it possible to choose a closed path(that is, a path that returns to P) for which the line integral of the magnetic field is nonzero ?



(a) Yes, but only positive

(b) Yes, but only negative

(c) Yes, both positive and negative

(d) No.

132. A particle of charge q and mass m starts moving from the origin under the action of an electric field $\vec{\mathbf{E}} = E_0\,\hat{\mathbf{i}}$ and $\vec{\mathbf{B}} = B_0\,\hat{\mathbf{i}}$ with a velocity $\vec{\mathbf{v}} = v_0\hat{\mathbf{j}}$. The speed of the particle will become $2v_0$ after a time :

(a) $t = \dfrac{2mv_0}{qE}$ (b) $t = \dfrac{2Bq}{mv_0}$

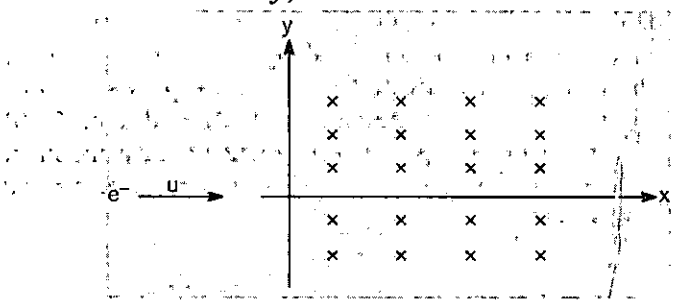
(c) $t = \dfrac{\sqrt{3}\,Bq}{mv_0}$ (d) $t = \dfrac{\sqrt{3}\,mv_0}{qE}$

133. A given length L of wire carries a current I. The wire can be formed into a circular coil of any number of turns. The maximum value of the torque τ that can be developed on the coil when placed in a given magnetic field is :

(a) $\tau = \dfrac{L^2IB}{2\pi}$ (b) $\tau = \dfrac{L^2IB}{4\pi}$

(c) $\tau = \dfrac{L^2IB}{2}$ (d) $\tau = L^2IB$

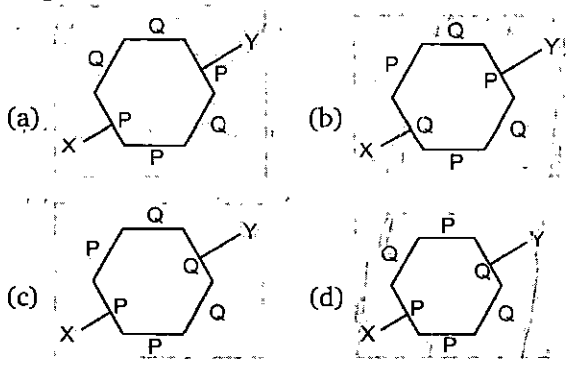
134. An electron moving with a speed u along the positive x-axis at $y = 0$ enters a region of uniform magnetic field which exists to the right of y-axis. The electron exits from the region after some time with the speed v at coordinate y, then :



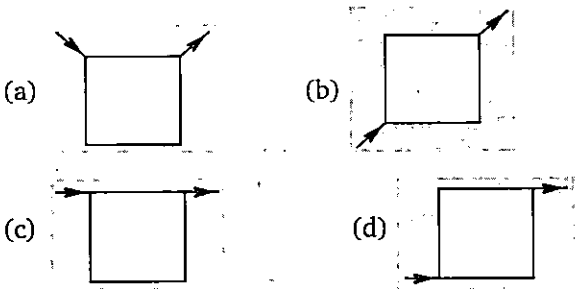
(a) $v > u,\ y < 0$ (b) $v = u,\ y > 0$

(c) $v > u,\ y > 0$ (d) $v = u,\ y < 0$

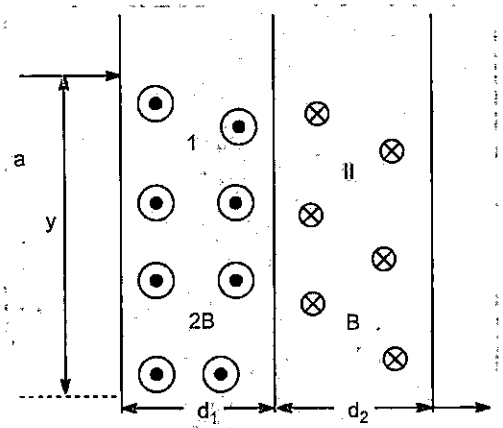
135. In the following hexagons, made up of two different material P and Q, current enters and leaves from points X and Y respectively. In which case the magnetic field at its centre is not zero:



136. Current flows through uniform, square frames as shown. In which case is the magnetic field at the centre of the frame not zero?



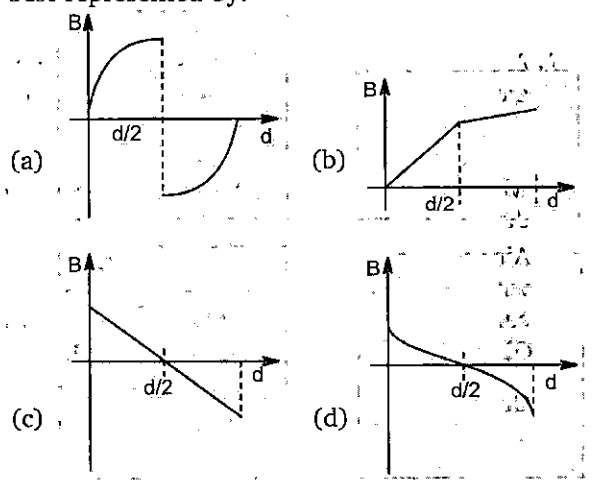
137. In the situation in the diagram, the given separation (y) between the lines along which the positive charged particle entres into and leaves the rigion of magnetic field is (Given $mv/qB = 2.00\,\text{m}$, $d_1 = 0.5\,\text{m}$ and $d_2 = 1\,\text{m}$)



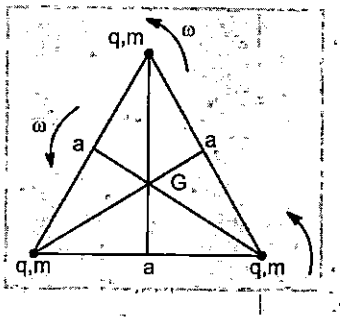
(a) $\dfrac{3}{2}(2-\sqrt{3})\,\text{m}$ (b) $\dfrac{3}{2}(2+\sqrt{3})\,\text{m}$

(c) $\dfrac{3}{2}\,\text{m}$ (d) $\dfrac{1}{2}(2+\sqrt{3})\,\text{m}$

138. A uniform beam of positively charged particles is moving with a constant velocity parallel to another beam of negatively charged particles moving with the same velocity in opposite direction separated by a distance d. The variation of magnetic field B along a perpendicular line draw between the two beams is best represented by:



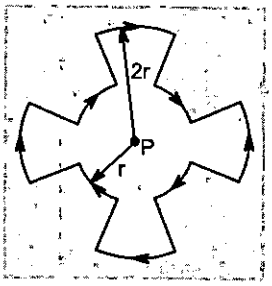
139. Three point charges, each of mass m and charge q, are placed at the vertices of an equilateral triangle of side 'a'. G is the centroid. The system is rotated with a constant angular frequency ω about an axis passing through the centroid and perpendicular to the plane of triangle. The ratio of magnetic dipole moment to angular momentum of the system will be :



(a) $\frac{q}{2m}$ (b) $\frac{3q}{2m}$

(c) $\frac{q}{3m}$ (d) $\frac{2q}{3m}$

140. A current I flows around a closed path in the horizontal plane of the circle as shown in the figure. The path consists of eight arcs with alternating radii r and $2r$. Each segment of arc subtends equal angle at the common centre P. The magnetic field produced by current path at point P is:



(a) $\frac{3}{8}\frac{\mu_0 I}{r}$; perpendicular to the plane of the paper and directed inward.

(b) $\frac{3}{8}\frac{\mu_0 I}{r}$; perpendicular to the plane of the paper and directed outward.

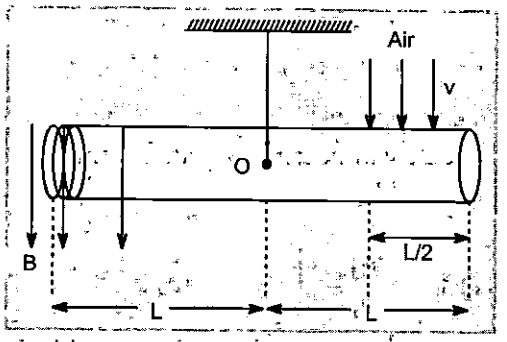
(c) $\frac{1}{8}\frac{\mu_0 I}{r}$; perpendicular to the plane of the paper and directed inward.

(d) $\frac{1}{8}\frac{\mu_0 I}{r}$; perpendicular to the plane of the paper and directed outward.

141. A non-conducting non-magnetic rod having circular cross-section of radius R is suspended from a rigid support as shown in the Fig. A light and small coil of 300 turns is warped tightly at the left end of the rod where magnetic field B exists in vertically downward direction.

Air of density ρ hits the half of the right part of the rod with velocity v (see the Fig.). What should be the current in ampere in clockwise direction (as seen from O) in the coil so that the rod remains horizontal ?

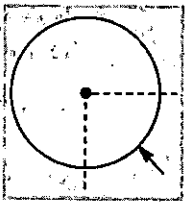
Given: $v = \frac{2}{L}\sqrt{\frac{\pi RB}{\rho}}$ m/s



(a) $0.01\,A$ (b) $0.1\,A$

(c) $0.02\,A$ (d) $0.2\,A$

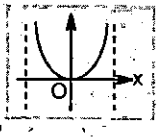
142. A long thin walled pipe of radius R, carries a current I along its length. The current density is uniform over the circumference of the pipe. The magnetic field at the center of the pipe due to quarter portion of the pipe shown, is:

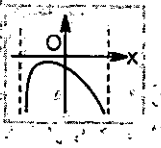


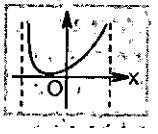
(a) $\frac{\mu_0 I\sqrt{2}}{4\pi^2 R}$ (b) $\frac{\mu_0 I}{\pi^2 R}$

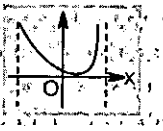
(c) $\frac{2\mu_0 I\sqrt{2}}{\pi^2 R}$ (d) None of these

143. Two very long straight parallel wires, parallel to y-axis, carry currents $4I$ and I, along $+y$ direction and $-y$ direction, respectively. The wires are passes through the x-axis at the points $(d, 0, 0)$ and $(-d, 0, 0)$ respectively. The graph of magnetic field z-component as one moves along the x-axis from $x = -d$ to $x = +d$, is best given by:

(a) 

(b) 

(c) 

(d) 

144. Taking a radius of the earth as 6400 km and the horizontal component of earth's magnetic field at the equator is 0.4×10^{-4} wb/m^2, with what minimum momentum could a proton circle the earth at the equator ?

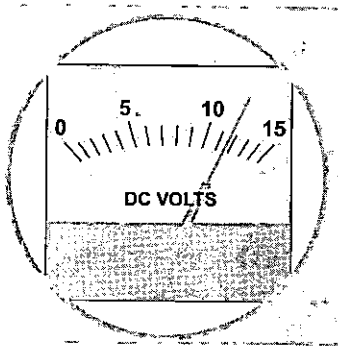
(a) 6.4×10^{-20} kg m/s (b) 4.1×10^{-20} kg m/s

(c) 7.1×10^{-19} kg m/s (d) 4.1×10^{-19} kg m/s

ANSWERS

Level-1: Only One Alternative is Correct

1.	(d)	**2.**	(d)	**3.**	(d)	**4.**	(c)	**5.**	(c)	**6.**	(a)	**7.**	(a)	**8.**	(a)
9.	(a)	**10.**	(c)	**11.**	(a)	**12.**	(c)	**13.**	(a)	**14.**	(a)	**15.**	(b)	**16.**	(c)
17.	(b)	**18.**	(a)	**19.**	(b)	**20.**	(a)	**21.**	(c)	**22.**	(a)	**23.**	(b)	**24**	(a)
25.	(c)	**26.**	(c)	**27.**	(b)	**28.**	(d)	**29.**	(b)	**30.**	(b)	**31.**	(a)	**32.**	(c)
33.	(a)	**34.**	(c)	**35.**	(a)	**36.**	(b)	**37.**	(d)	**38.**	(a)	**39.**	(a)	**40.**	(d)
41.	(b)	**42.**	(b)	**43.**	(c)	**44.**	(b)	**45.**	(c)	**46.**	(a)	**47.**	(c)	**48.**	(a)
49.	(a)	**50.**	(c)	**51.**	(a)	**52.**	(c)	**53.**	(c)	**54.**	(d)	**55.**	(c)	**56.**	(b)
57.	(d)	**58.**	(c)	**59.**	(d)	**60.**	(a)	**61.**	(b)	**62.**	(a)	**63.**	(a)	**64.**	(c)
65.	(a)	**66.**	(a)	**67.**	(b)	**68.**	(b)	**69.**	(a)	**70**	(b)	**71.**	(a)	**72.**	(b)
73.	(a)	**74.**	(b)	**75.**	(d)	**76.**	(c)	**77.**	(c)	**78.**	(a)	**79.**	(d)	**80.**	(c)
81.	(d)	**82.**	(b)	**83.**	(c)	**84.**	(a)	**85.**	(b)	**86.**	(b)	**87.**	(d)	**88.**	(c)
89.	(c)	**90.**	(a)	**91.**	(c)	**92.**	(d)	**93.**	(c)	**94.**	(d)	**95.**	(a)	**96.**	(d)
97.	(c)	**98.**	(d)	**99.**	(c)	**100.**	(a)	**101.**	(c)	**102.**	(b)	**103.**	(c)	**104.**	(b)
105.	(c)	**106.**	(a)	**107.**	(b)	**108.**	(c)	**109.**	(b)	**110.**	(b)	**111.**	(a)	**112.**	(a)
113.	(b)	**114.**	(b)	**115.**	(d)	**116.**	(a)	**117.**	(b)	**118.**	(a)	**119.**	(c)	**120.**	(c)
121.	(c)	**122.**	(b)	**123.**	(c)	**124.**	(d)	**125.**	(a)	**126.**	(c)	**127.**	(b)	**128.**	(a)
129.	(c)	**130.**	(b)	**131.**	(d)	**132.**	(d)	**133.**	(b)	**134.**	(d)	**135.**	(a)	**136.**	(c)
137.	(a)	**138.**	(d)	**139.**	(a)	**140.**	(a)	**141.**	(a)	**142.**	(a)	**143.**	(c)	**144.**	(b)



SOLUTION

Level-1: Only One Alternative is Correct

1. (d) $ILB = F$

$$B = \frac{F}{IL} = \frac{MLT^{-2}}{IL} = MT^{-2}I^{-1}$$

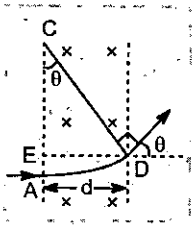
Magnetic flux

$$\phi = BA = (MT^{-2}I^{-1})\,(L^2)$$

3. (d) $R_1 < R_2$

and $$R = \frac{mv}{qB} \text{ of } \left(\frac{m}{q}\right)_1 < \left(\frac{m}{q}\right)_2$$

7. (a)



A to D is part of a circle with center C.

$$CD = r$$

$$mv = p = BQr$$

or $$r = p/BQ$$

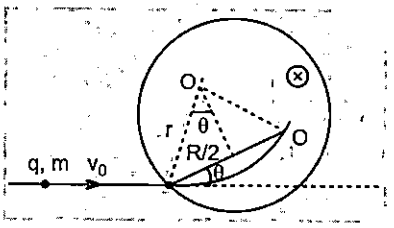
$$\sin\theta = \frac{ED}{CD} = \frac{d}{r} = \frac{BQd}{p}$$

11. (a) Use $F = IBl\sin\theta$ and the kinematics concepts.

12. (c) By symmetry, magnetic field at the centre is only due to wire having current $3I_0$, but effective current is only $2I_0$

$$\therefore \quad |\vec{\mathbf{B}}| = \frac{\mu_0 2I_0}{2\pi a} = \frac{\mu_0 I_0}{\pi a}$$

15. (b) If the particle passes through O, the situation can be shown in the figure. Let r the radius of circular path then from the given figure.



$$\sin\theta = \frac{R/2}{r} = \frac{R}{2r}$$

$$r = \frac{mv}{qB}$$

$$\sin\theta = \frac{RqB}{2mv};\ v_0 = \frac{qBR}{2m\sin\theta}$$

16. (c) The current is to the left, so the force is into the plane.

$$\Sigma F_y = N\cos\theta - Mg = 0$$

and $$\Sigma F_x = N\sin\theta - F_B = 0$$

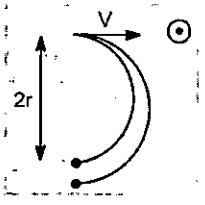
$$F_B = Mg\tan\theta = ILB$$

$$\Rightarrow \quad I = \frac{Mg\tan\theta}{LB}$$

22. (a) $$\vec{\mathbf{B}} = \frac{\mu_0 I}{2R}(\pm\hat{\mathbf{i}}) + \frac{\mu_0 I}{2R}(\pm\hat{\mathbf{j}}) + \frac{\mu_0 I}{2R}(\pm\hat{\mathbf{k}})$$

$$|\vec{\mathbf{B}}| = \frac{\mu_0 I}{2R}\sqrt{3}$$

26. (c) Let velocity with which charge enters magnetic field is v



$$\frac{1}{2}mv^2 = qv_0$$

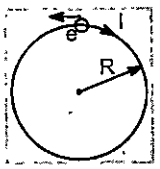
$$\Rightarrow \quad v = \sqrt{\frac{2qv_0}{m}}$$

Charge is collected at $2r$

$$2r = 2\frac{mv}{qB} = \frac{2m}{qB}\sqrt{\frac{2qv_0}{m}}$$

$$= \sqrt{\frac{m}{q}}\left(\frac{2\sqrt{2}v_0}{B}\right)$$

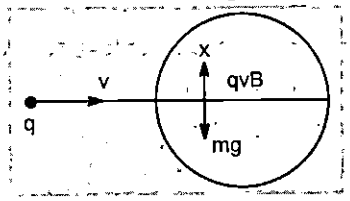
27. (b) This is equivalent to a current loop of radius R



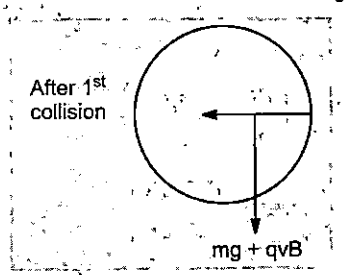
$$I = ne$$

$$\begin{pmatrix}\text{Magnetic}\\ \text{field}\end{pmatrix} B = \frac{\mu_0}{4\pi}\left(\frac{2\pi IR}{R^2}\right) = \frac{\mu_0 ne}{2R}$$

28. (d) Consider the 2D view of the situation. Before any collision with the walls of cylinder, the particle will move in horizontal plane if $qvB = mg$.



But after 1st collision, qvB and mg start acting in the same direction along vertical and the particle is not able to move in horizontal plane.

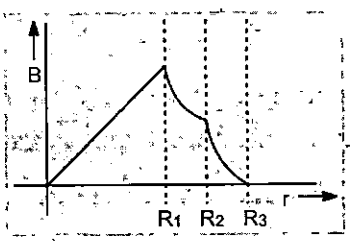


33. (a) Suppose in 1st region radius of circular path is r_1 & in region 2 this is r_2.

$\therefore \quad r_1 > 5 \;\&\; r_2 > 5$

$r = mv/(qB)$

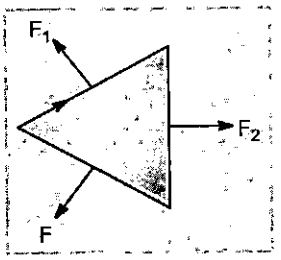
34. (c)



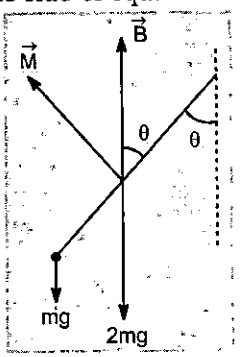
From Ampere's law, the field at the axis is zero. From $x = 0$ to R_1, the field increases linearly as the charge enclosed increases.

From $x = R_1$ to R_2 and from $x = R_2$ to R_3, the field decreases hyperbolically but with different slopes as the media are different.

35. (a) The net force acting on the loop would be along X-axis (to determine whether it is along +ve or –ve X-axis, calculation has to be carried out) as shown in figure.



39. (a) Let a is the side of square.



Torque of current $= MB\sin(90°-\theta)$

$= MB\cos\theta = I a^2 B\cos\theta$

Mass of each side $= m = \rho Aa$

Torque of gravity

$= 2mg\frac{a}{2}\sin\theta + mg\,a\sin\theta$

$= 2mg\,a\sin\theta = 2\rho Aga^2\sin\theta$

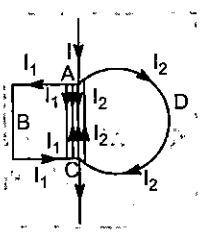
Now, both torques should be same

i.e., $Ia^2B\cos\theta = 2\rho Aga^2\sin\theta$

$\Rightarrow \quad \cot\theta = \frac{2\rho Ag}{IB}$

42. (b) Use $\oint \vec{B}\cdot d\vec{l} = \mu_0 I$, where I is the current enclosed by the loop.

44. (b) Introducing two equal and opposite current I_1 and also I_2 between A&C.



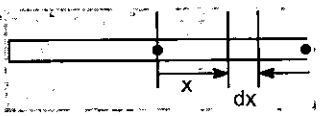
Force on $ABCDA$ closed loop zero

Force on $ADCA$ closed loop zero

Force on extra I_1 & I_2

$F = (I_1 + I_2)lB = IlB$

49. (a) At a distance x consider a small element of width dx. Magnetic moment of the small elements is :



$$dM = IA = (dq)f\,\pi x^2 = \frac{q}{l}dx f\,\pi x^2$$

$$M = \frac{qf\,\pi}{l}\int_{-l/2}^{l/2} x^2 dx = \frac{q\pi f\,l^2}{12}$$

51. (a) $qUB = qE$

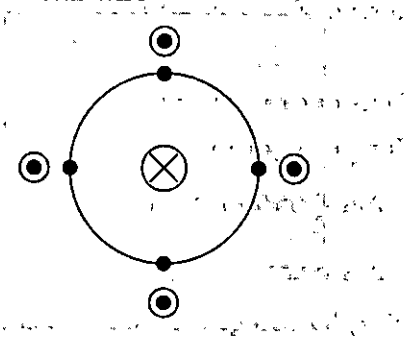
$\Rightarrow \quad UB = \dfrac{V}{D}$

$\Rightarrow \quad U = \dfrac{V}{BD}$

52. (c) After two and a half time periods, its x-coordinate will be $2.5P_0$ (so x-coordinate of image will be $17.5P_0$). It will be at distance $2R_0$ (equal to diameter) on the negative z-axis and y-coordinate will be zero.

53. (c) Net magnetic field at P will be zero, because the magnetic field at center P is cancelled by opposite pairs like $AD - EF$, etc.

54. (d) For a circular wire



58. (c) The field due to clockwise current cancels the field due to anticlockwise current.

63. (a) $\tau = I\alpha \;\Rightarrow\; MB = I\alpha$

$\Rightarrow \quad I\pi r^2 B = \dfrac{1}{2}mr^2\alpha$

$\Rightarrow \quad \alpha = \dfrac{2IB\pi}{m}$

$= \dfrac{2\times 4\times 10\pi}{2} = 40\pi\ \text{rad s}^{-2}$

65. (a) $2qE_0 - 2qE_0 = \dfrac{1}{2}mv_0^2$

$4qE_0 = \dfrac{1}{2}mv_0^2$

$v_0 = \sqrt{\dfrac{8qE_0}{m}}$

66. (a) The ratio M/L is always $\dfrac{q}{2m}$

69. (a) $|\vec{\mathbf{F}}_{B/A}| = \left|-\dfrac{\mu_0}{4\pi}\dfrac{q_1q_2v_1v_2}{2\sqrt{2}a^2}\hat{\mathbf{i}}\right|$

$|\vec{\mathbf{F}}_{12}| = \left|-\dfrac{\mu_0}{4\pi}\dfrac{q_1q_2v_1v_2}{2\sqrt{2}a^2}\hat{\mathbf{j}}\right|$

72. (b) $B_1 = \dfrac{\mu_0}{4\pi}\times\dfrac{2\pi I}{a}\times\dfrac{1}{2}$ (due to semicircular part)

$B_2 = \dfrac{\mu_0}{4\pi}\times\dfrac{2I}{a}$ (due to parallel parts of currents)

These two fields are at right angles to each other.

Hence, resultant field

$$B = \sqrt{B_1^2 + B_2^2} = \frac{\mu_0 I}{4\pi a}\sqrt{\pi^2 + 4}$$

74. (b) Frequency of reversal of magnetic field is same as frequency of revolution of charged particle.

79. (d) $E2\pi l = \pi R^2\left(\dfrac{dB}{dt}\right);\quad E = \dfrac{R^2}{2l}\left(\dfrac{dB}{dt}\right)$

$qE + mg = Kx$

$x = \dfrac{qR^2}{K2l}\left(\dfrac{dB}{dt}\right) + \dfrac{mg}{K};$

$x = \dfrac{1}{K}\left[mg + \dfrac{qR^2}{2l}\dfrac{dB}{dt}\right]$

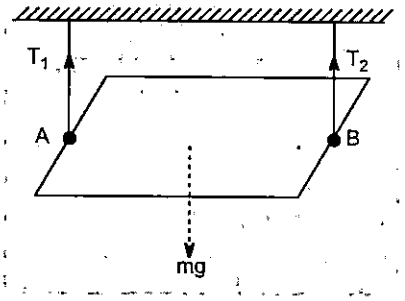
82. (b)

$\tau = Ia^2(-\hat{\mathbf{k}}) + Ia^2(-\hat{\mathbf{j}}) + Ia^2\cos 45°(-\hat{\mathbf{j}}) + Ia^2\sin 45°(\hat{\mathbf{j}})$

$= \dfrac{Ia^2}{\sqrt{2}}(-\hat{\mathbf{i}}) + \left(\dfrac{Ia^2}{\sqrt{2}} - Ia^2\right)\hat{\mathbf{j}} + Ia^2(-\hat{\mathbf{k}})$

$= \dfrac{a^2}{\sqrt{2}}(-\hat{\mathbf{i}}) + \dfrac{a^2(1-\sqrt{2})}{\sqrt{2}}(\hat{\mathbf{j}}) + a^2(-\hat{\mathbf{k}})$

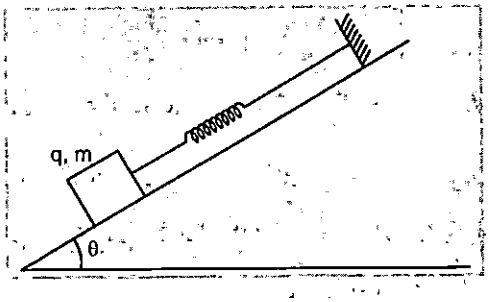
83. (c) Taking moments about point B to be zero.



$T_1 l + IblB = mg\dfrac{l}{2};$

$T_1 = \dfrac{mg - 2IbB}{2}$

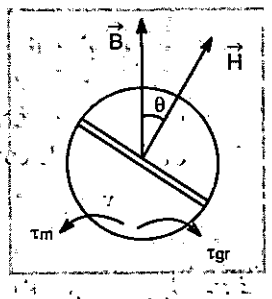
86. (b) There will be no effect of magnetic force on time period because the magnetic force will be perpendicular to the inclined plane.



90. (a) $F/1 = \frac{\mu_0 I_1 I_2}{2\pi r}$

92. (d) Net force on a current carrying loop in a uniform magnetic field is zero. So, magnetic force cannot balance its weight.

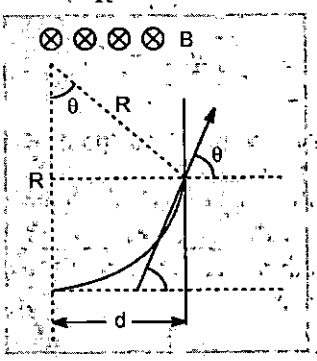
95. (a) The gravitational torque must be counter balanced by the magnetic torque about O, for equilibrium of the sphere. The gravitational torque $\tau_m = \pi I r^2 B \sin\theta$



$\therefore \quad \pi I r^2 B \sin\theta = mgr \sin\theta$

$\Rightarrow \quad B = \frac{mg}{\pi I r}$

98. (d) $\sin\theta = \frac{d}{R}$

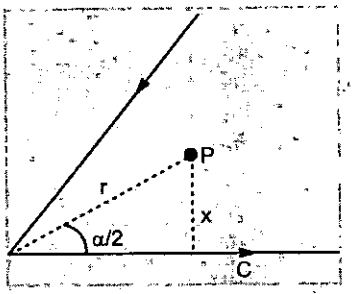


$d = R\sin\theta = \frac{mv}{Bd}\sin\theta$

$\Rightarrow \quad \frac{q}{m} = \frac{v\sin\theta}{Bd}$

99. (c) Same $q\Delta V$ so same KE but $m_e < m_p$ So $V_e > V_p$ this in velocity selection potential different must be increased.

101. (c) $x = r\sin\frac{\alpha}{2}$



$\therefore \quad B_P = 2\left(\frac{\mu_0}{4\pi}\right)\left(\frac{I}{x}\right)\left[\sin\left(90° - \frac{\alpha}{2}\right) + \sin 90°\right]$

$= \frac{\mu_0}{2\pi}\frac{I}{r}\frac{\left(1+\cos\frac{\alpha}{2}\right)}{\sin\frac{\alpha}{2}}$

103. (c) Magnetic field at the center

$$B = \frac{\mu_0 I}{2c}\otimes + \frac{\mu_0 I}{2b}\left(\frac{3}{4}\right)\odot + \frac{\mu_0 I}{2a}\left(\frac{3}{4}\right)\otimes$$

As per problem:

$$0 = \frac{\mu_0 I}{2}\left[\frac{1}{c} - \frac{3}{4b} + \frac{3}{4a}\right]$$

$$\Rightarrow \quad \frac{3}{4}\left[\frac{1}{b} - \frac{1}{a}\right] = \frac{1}{c}$$

If $c = 2a$,

$$\frac{3}{4}\left[\frac{1}{b} - \frac{1}{a}\right] = \frac{1}{2a} \Rightarrow a = \frac{5}{3}b$$

105. (c) By Ampere's circuital law, the magnetic induction field B inside a wire of circular cross section of radius R carrying a current I at a distance $r < R$ from the axis is given by $B = \frac{\mu_0 I r}{2\pi R^2}$. This field acts tangentially to the circle of radius r.

Consider a small strip of area ds at distance r from centre $(r < R)$

The flux through this area $= d\phi = \left(d\vec{\mathbf{s}} \times \vec{\mathbf{B}}\right) = Bds$ normal to the elementary area

Putting $ds = l.\,dr$ where l is the length of the planar strip, $df = B.\,l.\,dr$

But B (at distance r) is $B = \frac{\mu_0 I r}{2\pi R^2}$

108. (c) Magnetic field due to infinite current carrying sheet is given by $B = \frac{\mu_0 J}{2}$, where J is linear current density.

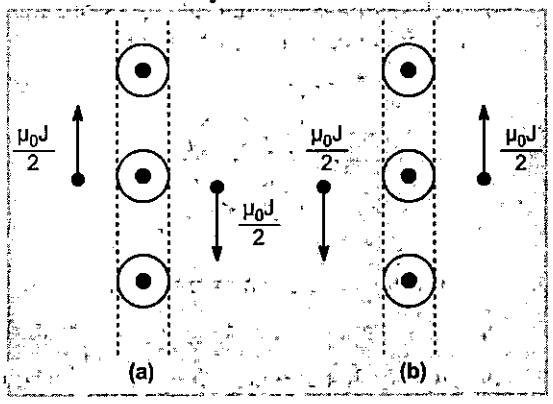


Fig. (a) and (b) represent the direction of magnetic field due to current carrying sheets.

For $x < a$, $B_{\text{resultant}} = \frac{\mu_0 J}{2} - \frac{\mu_0(2J)}{2} - \frac{\mu_0(3J)}{2} + \frac{\mu_0(4J)}{2}$

For $a < x < 2a$,

$$B_{\text{resultant}} = -\frac{\mu_0 J}{2} - \frac{\mu_0(2J)}{2} - \frac{\mu_0(3J)}{2} + \frac{\mu_0(4J)}{2} = -\mu_0 J$$

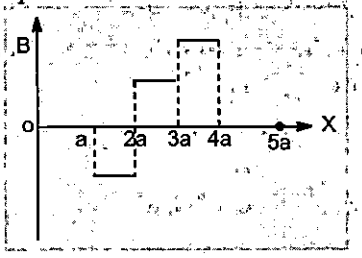
For $2a < x < 3a$,

$$B_{\text{resultant}} = -\frac{\mu_0 J}{2} + \frac{\mu_0(2J)}{2} - \frac{\mu_0(3J)}{2} + \frac{\mu_0(4J)}{2} = \mu_0 J$$

For $4a < x$,

$$B_{\text{resultant}} = \frac{-\mu_0 J}{2} + \frac{\mu_0(2J)}{2} + \frac{\mu_0(3J)}{2} - \frac{\mu_0(4J)}{2} = 0$$

So, the required curve is



109. (b) $B_{\text{center}} = 0$

$$\frac{\mu_0 I}{4\pi b}\odot + \frac{\mu_0 I}{2b}\odot + \frac{\mu_0 I}{2b}\odot + \frac{\mu_0 I}{2a}\otimes = 0$$

$$\frac{\mu_0 I}{2\pi b} + \frac{\mu_0 I}{2b} - \frac{\mu_0 I}{2a} = 0$$

$$\frac{1}{2\pi b} + \frac{1}{2b} = \frac{1}{2a}$$

$$\frac{a}{b} = \frac{\pi}{\pi + 1}$$

113. (b) Magnetic moment of loop

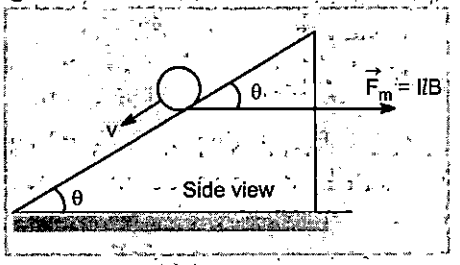
$$\mu = NIA \qquad 100 = \frac{1}{2} \cdot \pi r^2$$

(Potential Energy) $U = -\vec{\mu} \cdot \vec{B}$

$$\Delta U = U_t - U_i = -(-\mu B) - (-\mu B) = 2\mu B$$

$$= 2 \times 100 \times \frac{1}{2} \times \pi \left(\frac{1}{10}\right)^2 \times 2 = 2\pi J$$

117. (b) Magnetic force acts in the direction shown in figure



Road will move downward with constant velocity if net force on it is zero.

or $F_m \cos\theta = mg \sin\theta$

or $IlB\cos\theta = mg\sin\theta$

$$B = \left(\frac{mg}{Il}\right)\tan\theta$$

119. (c) $B = \frac{\mu_0 I}{4r_1} + \frac{\mu_0 I}{4r_2}$

or $B = \frac{\mu_0 I}{4}\left[\frac{1}{r_1} + \frac{1}{r_2}\right]$

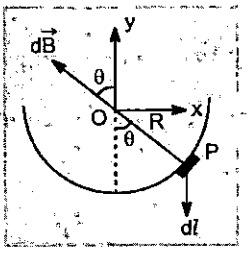
$$= \frac{\mu_0 I}{4}\left[\frac{r_2 + r_1}{r_1 r_2}\right]$$

120. (c) Consider a longitudinal section of the given wire parallel to its axis, through the point P. It will be a straight line of infinite length perpendicular to plane with the point P as at its centre. Consider an element dl of arc at P and the infinitely long strip of width dl.

The magnetic induction field $d\vec{B}$ due to this infinitely long straight strip, at O will be

$$|d\vec{B}| = \frac{\mu_0 (dI)}{2\pi R}$$

where the elemental current is dI.



Now the total current flowing along the arc is I.

Hence current per unit length of arc

$$= \frac{1}{\text{length of arc}} = \frac{1}{\pi R}$$

Hence, $|dI| = \left(\frac{1}{\pi R}\right) dl$ for elemental length (dl)

Choosing angle θ, as in the diagram, we see that $dl = R\,d\theta$, the $d\vec{\mathbf{B}}$ acts along $\overrightarrow{PO}$. Resolving $d\vec{\mathbf{B}}$, horizontally and vertically, we see that the horizontal components will cancel each other out between the two halves of the arc while the vertical components will add up. Hence, total magnetic field $\vec{\mathbf{B}}$ due to the total current in the wire will be :

$$\vec{B} = \left|\overrightarrow{B_y}\right| = \int_{-\pi/2}^{+\pi/2} \left(\frac{\mu_0}{2\pi R}\right)\left(\frac{1}{\pi R}\right)\cos\theta (R\,d\theta)$$

$$= \frac{\mu_0 I}{2\pi^2 R}\left(\int_{-\pi/2}^{+\pi/2} \cos\theta\, d\theta\right) = \frac{\mu_0 I}{\pi^2 R}$$

121. (c) The particles will not collide if

$$d > 2(r_1 + r_2)$$

or $$d > 2\left(\frac{mv_1}{Bq} + \frac{mv_2}{Bq}\right)$$

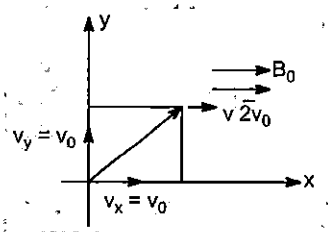
or $$d > \frac{2m}{Bq}(v_1 + v_2)$$

123. (c) $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = \mu_0 I_{\text{enclosed}}$

since I_{enclosed} is same

124. (d) $\alpha = \frac{q}{m}$, path of the particle will be a helix of time period,

$$T = \frac{2\pi m}{B_0 q} = \frac{2\pi}{B_0 \alpha}$$



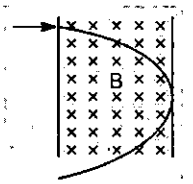
The given time $t = \frac{\pi}{B_0\alpha} = \frac{T}{2}$

$\therefore$ Coordinates of particle at time $t = T/2$ would be $(v_x\, T/2, 0, -2r)$

Here, $r = \frac{mv_r}{B_0 q} = \frac{v_0}{B_0\alpha}$

$\therefore$ The coordinate are $\left(\frac{v_0\pi}{B_0\alpha}, 0, \frac{-2v_0}{B_0\alpha}\right)$

125. (a) Obviously for the limiting field, the path of the particles will be semi-circular arcs of radius equal to the distance between the plates.

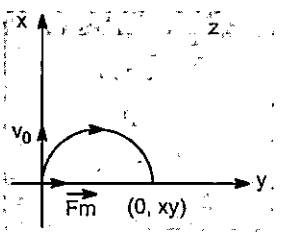


(*i.e.*) radius r of circular path is $r = d$

Now for a particle of charge q, mass m and kinetic energy K, where velocity $v = v = \sqrt{\frac{2K}{m}}$ the radius r is $r = d = \frac{mv}{qB} = \frac{m}{qB}\sqrt{\frac{2K}{m}}$

Hence, $B = \frac{m}{qd}\sqrt{\frac{2K}{m}} = \frac{\sqrt{2K}}{qd}$

126. (c) $y = 2r = \frac{2mv_0}{B_0 q} = \frac{2v_0}{B_0\alpha}$



Here, $\frac{q}{m} = \alpha$

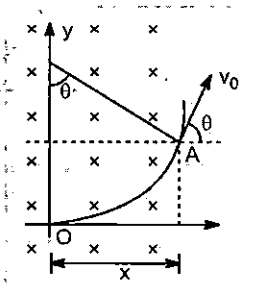
128. (a) $\hat{\mathbf{u}} = -3a^2\hat{\mathbf{i}}$ (see figure)

129. (c) $r = \frac{mv_0}{B_0 q} = \frac{v_0}{B_0\alpha}$

$$\frac{x}{r} = \frac{\sqrt{3}}{2} = \sin\theta$$

$\therefore$ $\theta = 60°$

$$t_{OA} = \frac{T}{6} = \frac{\pi}{3B_0\alpha}$$



Therefore, x-coordinate of particle at any time $t > \frac{\pi}{3B_0\alpha}$ will be

$$x = \frac{\sqrt{3}}{2}\frac{v_0}{B_0\alpha} + v_0\left(t - \frac{\pi}{3B_0\alpha}\right)\cos 60°$$

$$= \frac{\sqrt{3}}{2}\frac{v_0}{B_0\alpha} + \frac{v_0}{2}\left(t - \frac{\pi}{3B_0\alpha}\right)$$

131. (d) Any path can be broken into small horizontal and vertical segments. Because the magnetic field is horizontal only the horizontal segments contribute to the line integral. Moving away form P and then returning to it, involves as many horizontal segments to the left as horizontal segments to the right and so the net contribution is always zero.

132. (d) $\vec{\mathbf{E}}$ is parallel to $\vec{\mathbf{B}}$ and $\vec{\mathbf{v}}$ is perpendicular to both. Therefore, path of the particle is a helix with increasing pitch. Speed of particle at any time t is

$$v = \sqrt{v_x^2 + v_y^2 + v_z^2} \quad ...(1)$$

Here, $v_y^2 + v_z^2 = v_0^2$

and $v = 2v_0$

Substituting the values in eqn. (1), we get

$$t = \frac{\sqrt{3}\,mv_0}{qE}$$

133. (b) If n be the number of turns and r, the radius of each turn then $n(2\pi r) = L$ = constant

Also, for n turns, each turn of radius r, the torque on the total coil, when carrying the current I and placed in a magnetic field B is

$$\tau = nI\left(\vec{\mathbf{A}} \times \vec{\mathbf{B}}\right) = nI\,(\pi r^2)B = (\pi IB)\,nr^2$$

We have to maximise τ with respect to n and r subject to condition (1), treating n and r as variables substituting for nr from (1),

$$\tau = (\pi IB)\left(\frac{L}{2\pi}\right)r = \frac{IBL}{2}r$$

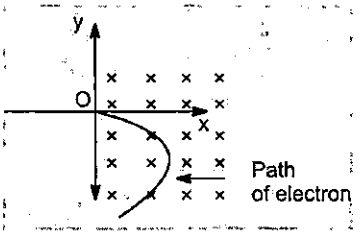
Hence, τ will be maximum when r will be maximum and the r maximumn $n = 1$ (*i. e.*) there is only one single turn of coil.

Then $r = \frac{L}{2\pi}$

$$\tau_{max} = \frac{IBL}{2}\cdot\frac{L}{2\pi} = \frac{L^2IB}{4\pi}$$

134. (d) y will be less than zero.

The trajectory will be as shown in figure



137. (a) $R_1 = \frac{mv}{2qB} = \frac{2.0}{2} = 1.0\,\text{m}$

$$R_2 = \frac{mv}{qB} = 2.0\,\text{m}$$

$$y_1 + y_2 = R_1(1 - \cos 30°) + R_2(1 - \cos 30°)$$

$$= 3\left(1 - \frac{\sqrt{3}}{2}\right) = \frac{3}{2}(2 - \sqrt{3})$$

139. (a) Magnetic dipole moment,

$$\mu = IA = \frac{3q\omega}{2\pi} \times \pi \times \left(\frac{a}{\sqrt{3}}\right)^2 = \frac{q\omega a^2}{2}$$

Angular momentum,

$$L = 3m\left(\frac{a}{\sqrt{3}}\right)^2\omega = m\omega a^2$$

$$\therefore \quad \frac{\mu}{L} = \frac{q\omega a^2}{2m\omega a^2} = \frac{q}{2m}$$

141. (a) Force exerted by air on the rod

$$= \rho(L/2 \times 2R)v^2 = \rho LRv^2$$

Balancing torque about O,

$$NI(\pi R^2)B = (\rho LRv^2) \times \frac{3L}{4}$$

$$\Rightarrow \quad 300\pi IBR = \frac{3\rho v^2L^2}{4}$$

$$\Rightarrow \quad I = \frac{1}{400} \times \frac{\rho L^2v^2}{\pi BR} = 0.01\,\text{A}$$

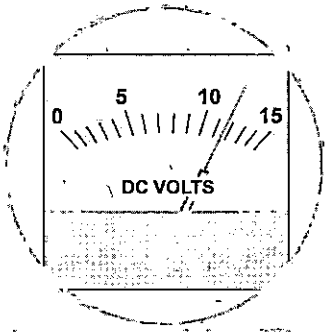
144. (b) $qVB = \frac{mv^2}{R}$

$$mV = qBR$$

$$\Rightarrow \quad P_{min} = qBR$$

putting the values, we get

$$P_{min} = 4.1 \times 10^{-20}\,\text{kg. m/s}$$

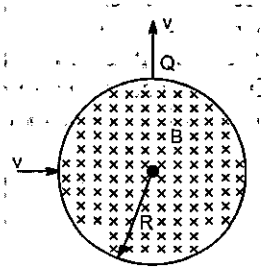


PROBLEMS

Level 2 More than One Alternative is/are Correct

1. Consider the magnetic field produced by a finitely long current carrying wire.
(a) the lines of field will be concentric circles with centres on the wire.
(b) There can be two points in the same plane where magnetic fields are same.
(c) There can be large number of points where the magnetic field is same.
(d) The magnetic field at a point is inversally proportional to the distance of the point from the wire.

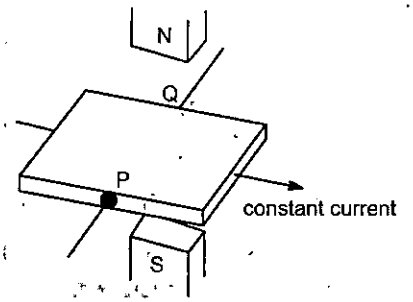
2. A particle of charge 'q' and mass 'm' enters normally (at point P) in a region of magnetic field with speed v. It comes out normally from Q after time T as shown in figure. The magnetic field B is present only in the region of radius R and is uniform. Initial and final velocities are along radial direction and they are perpendicular to each other. For this to happen, which of the following expression(s) is/are correct ?



(a) $B = \frac{mv}{qR}$ (b) $T = \frac{\pi R}{2v}$

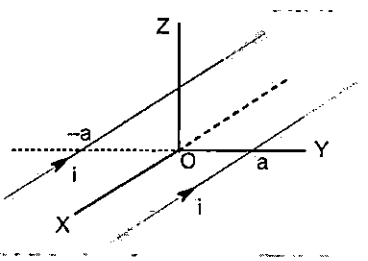
(c) $T = \frac{\pi m}{2qB}$ (d) none of these

3. Fig. shows the essential parts of an apparatus to demonstrate the Hall effect. Which of the following statements is/are correct ?



(a) In the arrangement above, the Hall potential difference is developed across PQ
(b) The magnitude of the Hall potential is greater if the applied magnetic flux density is increased
(c) The magnitude of the Hall potential is less if the width PQ of the specimen is decreased
(d) Hall potential is independent of material used for conductor

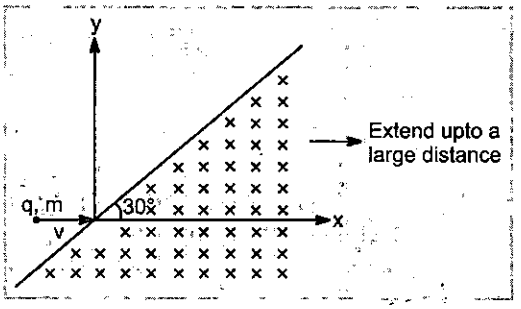
4. Two long thin, parallel conductors carrying equal currents in the same direction are fixed parallel to the x-axis, one passing through $y = a$ and the other through $y = -a$. the resultant magnetic field due to the two conductors at any point is B. Which of the following are correct?



(a) $B = 0$ for all points on the x-axis
(b) At all points on the y-axis, excluding the origin, B has only a z-component.
(c) At all points on the z-axis, excluding the origin, B has only a y-component.
(d) B cannot have an x-component.

5. Consider three quantities $x = E/B, y = \sqrt{1/\mu_0\varepsilon_0}$ and $z = \frac{l}{CR}$. Here, l is length of a wire, C is a capacitance and R is a resistance. All other symbols have standard meanings.
(a) x, y have the same dimensions
(b) y, z have the same dimensions
(c) z, x have the same dimensions
(d) none of the three pairs have the same dimensions

6. A charge particle of charge q and mass m is moving with velocity v as shown in figure in a uniform magnetic field B along $-$ve z-direction. Select the correct alternative(s):



(a) Velocity of the particle when it comes out from the magnetic field is $\vec{\mathbf{v}} = v\cos 60°\,\hat{\mathbf{i}} + v\sin 60°\,\hat{\mathbf{j}}$
(b) Time for which the particle was in magnetic field is $\frac{\pi m}{3qB}$
(c) Distance travelled in magnetic field is $\frac{\pi m v}{2qB}$.
(d) None of these

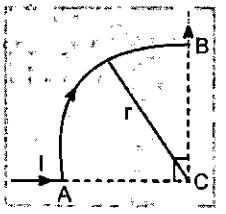
7. A particle of mass m and charge q moving with velocity $\vec{\mathbf{v}}$ enters a region of uniform magnetic field of induction $\vec{\mathbf{B}}$. Then :
(a) its path in the region of the field is always circular
(b) its path in the region of the field is circular if $\vec{\mathbf{v}} \cdot \vec{\mathbf{B}} = 0$
(c) its path in the region of the field is a straight line if $\vec{\mathbf{v}} \times \vec{\mathbf{B}} = 0$
(d) distance travelled by the particle in time T does not depend on the angle between $\vec{\mathbf{v}}$ and $\vec{\mathbf{B}}$.

8. A long straight wire carries a current along the x-axis. Consider the points $A(0, 1)$, $B(0, 1, 1)$, $C(1, 0, 1)$ and $D(1, 1, 1)$. Which of the following pairs of points will have magnetic fields of the same magnitude?
(a) A and B (b) A and C
(c) B and C (d) B and D

9. In the previous question 8, if the current is i and the magnetic field at D has magnitude B,
(a) $B = \frac{\mu_0 i}{2\sqrt{2}\pi}$
(b) $B = \frac{\mu_0 i}{2\sqrt{3}\pi}$
(c) B is parallel to the x-axis
(d) B makes an angle of 45° with the xy plane

10. A charged particle of specific charge α moves with a velocity $\vec{\mathbf{v}} = v_0\hat{\mathbf{i}}$ in a magnetic field $\vec{\mathbf{B}} = \frac{B_0}{\sqrt{2}}(\hat{\mathbf{j}} + \hat{\mathbf{k}})$. Then (specific charge = charge per unit mass)
(a) path of the particle is a helix
(b) path of the particle is circle
(c) distance moved by the particle in time $t = \frac{\pi}{B_0\alpha}$ is $\frac{\pi v_0}{B_0\alpha}$
(d) velocity of the particle after time $t = \frac{\pi}{B_0\alpha}$ is $\left(\frac{v_0}{2}\hat{\mathbf{i}} + \frac{v_0}{2}\hat{\mathbf{j}}\right)$

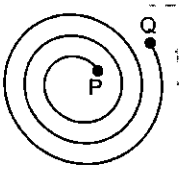
11. A wire carrying I is shaped as shown. Section AB is a quarter circle of radius r. The magnetic field at C is directed:



(a) along the bisector of the angle ACB, away from AB
(b) along the bisector of the angle ACB, towards AB
(c) perpendicular to the plane of the paper, directed into the paper
(d) at an angle $\pi/4$ to the plane of the paper

12. A proton is moving in gravity free space having a uniform electric field, E, of unknown magnitude and direction. There is a uniform magnetic field of $0.01T$ pointing in the y-direction. The proton moves in the x-direction at a constant velocity of 10 km/s. All directions are with respect to the usual right handed x-y-z coordinate system. Which of the following is/are correct ?
(a) $E_z = 100$ V/m (b) $E_z = -100$ V/m
(c) $E_x = 100$ V/m (d) $E_x = 0$

13. A charged particle enters into a region which offers a resistance against its motion and a uniform magnetic field exists in the region. The particle traces a spiral path as shown in figure. Which of the following statement(s) is/are correct ?



(a) Component of magnetic field in the plane of spiral is zero
(b) The particle enters the region at Q
(c) If magnetic field is outward, then the particle is positively charged
(d) If magnetic field is outward, then the particle is negatively charged

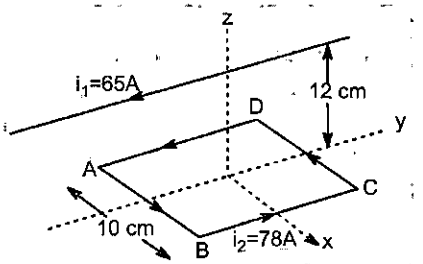
14. Which of the following statement is correct:
(a) A charged particle enters a region of uniform magnetic field at an angle 85° to magnetic lines of force. The path of the particle is a circle.
(b) An electron and proton are moving with the same kinetic energy along the same direction.When they pass through uniform magnetic field perpendicular to their direction of motion, they describe circular path.
(c) There is no change in the energy of a charged particle moving in a magnetic field although magnetic force acts on it.
(d) Two electrons enter with the same speed but in opposite direction in a uniform transverse magnetic field. Then the two describe circle of the same radius and these move in the same direction.

15. A charged particle of unit mass and unit charge moves with velocity $\vec{\mathbf{v}} = (8\hat{\mathbf{i}} + 6\hat{\mathbf{j}})$ m/s in a magnetic field of $\vec{\mathbf{B}} = 2\hat{\mathbf{k}}$ T. Choose the correct alternative(s).
(a) The path of the particle may be $x^2 + y^2 - 4x - 21 = 0$
(b) The path of the particle may be $x^2 + y^2 = 25$
(c) The path of the particle may be $y^2 + z^2 = 25$
(d) The time period of the particle will be $3.14s$

16. Consider the following statements regarding a charged particle in a magnetic field. Which of the statements are true:
(a) Starting with zero velocity, it accelerates in a direction perpendicular to the magnetic field.
(b) While deflecting in magnetic field its energy gradually increases
(c) Only the component of magnetic field perpendicular to the direction of motion of the charged particle is effective in deflecting it.
(d) Direction of deflecting force on the moving charged particle is perpendicular to its velocity.

17. Two identical charged particles enter a uniform magnetic field with same speed but at angles 30° and 60° with field. Let a, b and c be the ratio of their time periods, radii and pitches of the helical paths than:
(a) $abc = 1$ (b) $abc > 1$
(c) $abc < 1$ (d) $a = bc$

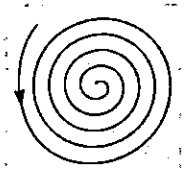
18. Fig. shows a square loop 10 cm on each side in the x-y plane with its centre at the origin. An infinite wire is at $z = 12$ cm above y-axis



Select correct options :
(a) Resultant magnetic force on loop due to infinite wire is 6×10^{-4} N
(b) Magnetic force on wire segment AB due to infinite wire in positive x-direction
(c) Magnetic force on wire segment CD due to infinite wire is in negative x-direction
(d) Torque on loop due to magnetic force is 7.2×10^{-5} N-m

19. A particle of charge q and velocity v passes undeflected through a space with non-zero electric field E and magnetic field B. The underflecting conditions will hold if.
(a) signs of both q and E are reversed.
(b) signs of both q and B are reversed.
(c) both B and E are changed in magnitude, but keeping the product of $|B|$ and $|E|$ fixed.
(d) both B and E are doubled in magnitude.

20. An electron moves in a uniform magnetic field and follows a spiral path as shown in figure. Which of the following statement(s) is/are correct ?



(a) Angular velocity of the electron remains constant
(b) Magnitude of velocity of the electron decreases continuously
(c) Net force on the particle is always perpendicular to its direction of motion
(d) Magnitude of net force on the electron decreases continuously

21. Let $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ denote the electric and magnetic fields in a certain region of space. A proton moving with a velocity along a straight line enters the region and is

found to pass through it undeflected. Indicate which of the following statements are consistent with the observations:

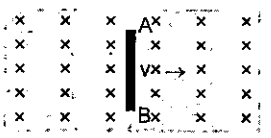
(a) $\vec{E} = 0$ and $\vec{B} = 0$

(b) $\vec{E} \neq 0$ and $\vec{B} = 0$

(c) $\vec{E} \neq 0$ and $\vec{B} \neq 0$ and both $\vec{E}$ and $\vec{B}$ are parallel to $\vec{v}$

(d) $\vec{E}$ is parallel to $\vec{v}$ but $\vec{B}$ is perpendicular to $\vec{v}$

22. A rod AB moves with a uniform velocity v in a uniform magnetic field as shown in figure.



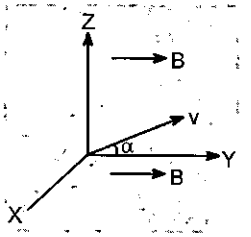
(a) The rod becomes electrically charged.

(b) The end A becomes positively charged.

(c) The end B becomes positively charged.

(d) The rod becomes hot because of Joule heating.

23. In a region of space, a uniform magnetic field B exists in the y-direction. A proton is fired from the origin, with its initial velocity v making a small angle α with the y-direction in the yz plane. In the subsequent motion of the proton,



(a) its x-coordinate can never be positive

(b) its x- and y-coordinates cannot both be zero at the same time

(c) it z-coordinate can never be negative

(d) its y-coordinate will be proportional to the square of its time of flight

24. An electron (mass $= m_e$) and a proton (mass $= m_p$) initially at rest move through a certain distance in a uniform electric field in times t_1 and t_2. Neglect the effect of gravity. Then:

(a) the acceleration of electron is much greater than that of proton

(b) the acceleration of proton is much greater than that of electron

(c) $\dfrac{t_1}{t_2} = \left(\dfrac{m_e}{m_p}\right)^{1/2}$

(d) $\dfrac{t_1}{t_2} = \left(\dfrac{m_p}{m_e}\right)^{1/2}$

25. Force experienced by a moving charge in a magnetic field is conservative as :

(a) Magnetic lines of force form closed loops

(b) Magnetic lines of force do not form closed loops

(c) Work done by magnetic force in a closed path is zero

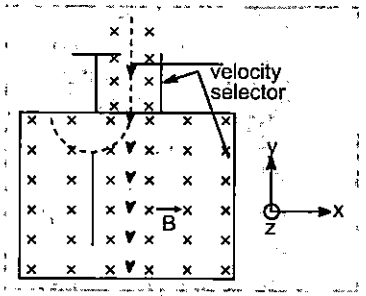
(d) No work is done by magnetic force in any path.

26. An electron is moving along the positive X-axis. You want to apply a magnetic field for a short time so that the electron may reverse its direction and move parallel to the negative X-axis. This can be done by applying the magnetic field along:

(a) Y-axis (b) Z-axis

(c) Y-axis only (d) Z-axis only

27. Two charged particle A and B each of charge $+e$ and masses 12 amu and 12 amu respectively follow a circular trajectory in chamber X after the velocity selector as shown in the figure. Both particles enter the velocity selector with speed $1.5 \times 10^6 \text{ms}^{-1}$. A uniform magnetic field of strength 1.0 T is maintained within the chamber X and in the velocity selector.



(a) Electric field across the conducting plate of the velocity selector is $-10^6 \text{NC}^{-1}\hat{\mathbf{i}}$.

(b) Electric field across the conducting plate of the velocity selector is $10^6 \text{NC}^{-1}\hat{\mathbf{i}}$.

(c) The ratio r_A / r_B of the radii of the circular paths for the two particles is 12/13

(d) The ratio r_A / r_B of the radii of the circular paths for the two particles is 13/12.

28. A particle of charge q and mass m moves rectilinearly under the action of an electric field $E = \alpha - \beta x$. Here, α and β are positive constants and x is the distance from the point where the particle was initially at rest. Then,

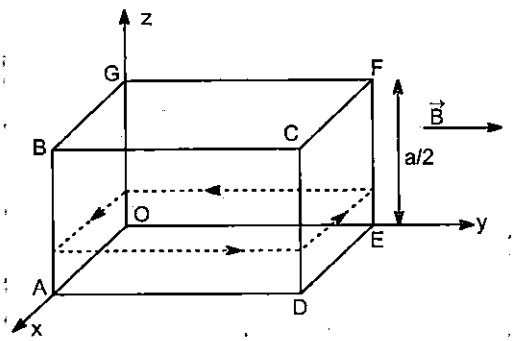
(a) the motion of the particle is oscillatory

(b) the amplitude of the particle is (α/β)

(c) the mean position of the particle is at $x = (\alpha/\beta)$

(d) the maximum acceleration of the particle is $\dfrac{q\alpha}{m}$

29. A wooden cubical block $ABCDEFG$ of mass m and side a is wrapped by a square wire loop of perimeter $4a$, carrying current I. The whole system is placed at frictionless horizontal surface in a uniform magnetic field $\vec{B} = B_0\hat{\mathbf{j}}$ as shown in figure. In this situation, normal force between horizontal surface and block passes through a point at a distance x from centre. Select the correct statement(s).



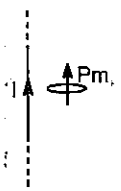
(a) The block must not topple if $I < \dfrac{mg}{aB_0}$

(b) The block must not topple if $I < \dfrac{mg}{2aB_0}$

(c) $x = \dfrac{a}{4}$ if $I = \dfrac{mg}{2aB_0}$

(d) $x = \dfrac{a}{4}$ if $I = \dfrac{mg}{4aB_0}$

30. The magnetic dipole $\vec{\mathbf{p}}_m$ is placed parallel to an infinitely long straight wire as shown in figure:



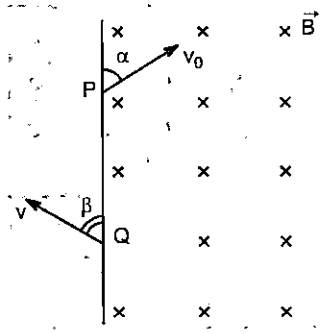
(a) the potential energy of the dipole is minimum

(b) the torque acting on the dipole is zero

(c) the force acting on the dipole is zero

(d) none of these

31. A particle of charge $-q$ and mass m enters a uniform magnetic field $\vec{\mathbf{B}}$ (perpendicular to paper inward) at P with a velocity v_0 at an angle α and leaves the field at Q with velocity v at angle β as shown in figure:



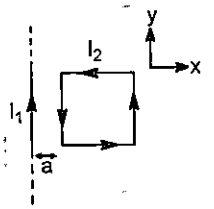
(a) $\alpha = \beta$

(b) $v = v_0$

(c) $PQ = \dfrac{2mv_0 \sin\alpha}{Bq}$

(d) The particle remains in the field for time $t = \dfrac{2m(\pi - \alpha)}{Bq}$

32. A square loop of side l is placed in the neighbourhood of an infinitely long straight wire carrying a current I_1. The loop carries a current I_2 as shown in figure.



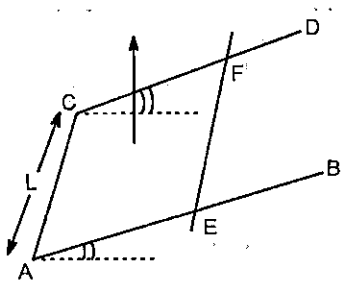
(a) The magnetic moment of the loop is $\vec{\mathbf{p}}_m = l^2 I_2 \hat{\mathbf{k}}$

(b) The magnetic moment of the loop is $\vec{\mathbf{p}}_m = -l^2 I_2 \hat{\mathbf{k}}$

(c) The potential energy of the loop is minimum

(d) The torque experienced by the loop is maximum

33. AB and CD are smooth parallel rails, separated by a distance L and inclined to the horizontal at an angle θ. A uniform magnetic field of magnitude B, directed vertically upwards, exists in the region. EF is a conductor of mass m, carrying a current I. For EF to be in equilibrium:



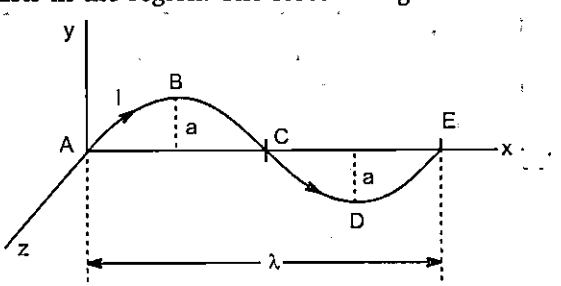
(a) I must flow form E to F

(b) $BIL = mg \tan\theta$

(c) $BIL = mg \sin\theta$

(d) $BIL = mg$

34. A conductor $ABCDE$, shaped as shown, carries current I. It is placed in the x - y plane with the ends A and E on the x-axis. A uniform magnetic field of magnitude B exists in the region. The force acting on it will be :



(a) zero, if B is in the x-direction

(b) λBI in the z-direction, if B is in the y-direction

(c) λBI in the negative y-direction, if B is in the z-direction

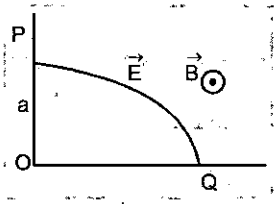
(d) λaBI, if B is in the x-direction

35. A charged particle with velocity $\hat{\mathbf{v}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$ moves in a magnetic field $\vec{\mathbf{B}} = y\hat{\mathbf{i}} + x\hat{\mathbf{j}}$. The force acting on the particle has magnitude F. Which one of the following statements is/are correct ?

(a) No force will act on charged particle if $x = y$
(b) If $x > y$, $f \propto (x^2 - y^2)$
(c) If $x > y$, the force will act along y-axis
(d) If $y > x$, the force will act along y-axis

36. A charged particle goes undeflected in a region containing electric and magnetic fields. It is possible that :
(a) $\vec{E} || \vec{B}, \vec{v} || \vec{E}$
(b) $\vec{E}$ is not parallel to $\vec{B}$
(c) $\vec{v} || \vec{B}$ but $\vec{E}$ is not parallel to $\vec{B}$
(d) $\vec{E} || \vec{B}$ but $\vec{v}$ is not parallel to $\vec{E}$

37. A particle with charge $+q$ and mass m, moving under the influence of a uniform electric field $E\hat{i}$ and a uniform magnetic field $B\hat{k}$, follows a trajectory from P to Q as shown in figure. The velocities at P and Q $v\hat{i}$ and $-2v\hat{j}$. Which of the following is correct ?

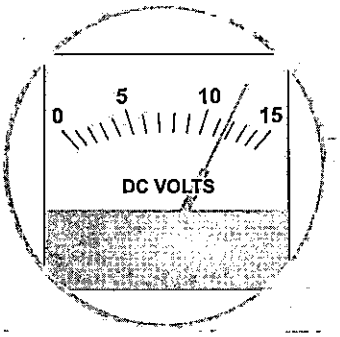


(a) $E = \frac{3}{4}\left(\frac{mv^2}{qa}\right)$
(b) The rate of work done by the electric field at P is $\frac{3}{4}\left(\frac{mv^3}{a}\right)$
(c) The rate of work done by the electric field at P is 0
(d) The rate of work done by both the fields at Q is 0

ANSWERS

Level-2: More than One Alternative is/are Correct

1.	(a, b, c)	**2.**	(a, b, c)	**3.**	(a, b)	**4.**	(a, b, c, d)	**5.**	(a, b, c)	**6.**	(a, b, c)
7.	(b, c, d)	**8.**	(b, d)	**9.**	(a, d)	**10.**	(b, c)	**11.**	(c)	**12.**	(b, d)
13.	(a, b, c)	**14.**	(b, c)	**15.**	(a, c, d)	**16.**	(c, d)	**17.**	(a, d)	**18.**	(a, d)
19.	(d)	**20.**	(a, b, d)	**21.**	(a, b, c)	**22.**	(b)	**23.**	(a)	**24.**	(a, c)
25.	(c, d)	**26.**	(a, b)	**27.**	(c)	**28.**	(a, b, c, d)	**29.**	(b, d)	**30.**	(c)
31.	(a, b, c, d)	**32.**	(a)	**33.**	(a, b)	**34.**	(a, b, c)	**35.**	(a, b, c)	**36.**	(a, b)
37.	(a, b, d)										

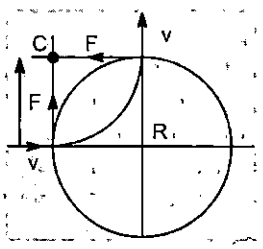


SOLUTION

Level-2: More than One Alternative is/are Correct

2. (a, b, c)

The particle will move along an arc which is part of a circle of radius: $r = \frac{mv}{Bq}$



From figure, we can see that $r = R$

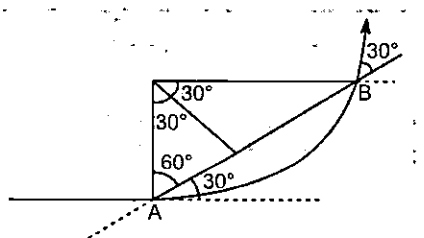
$$\therefore \quad R = \frac{mv}{Bq}$$

$$T = \frac{\pi r/2}{v} = \frac{\pi r}{2v} = \frac{\pi R}{2v} \quad \left(\because r = R = \frac{mv}{Bq}\right)$$

$$\therefore \quad T = \frac{\pi m}{2Bq}$$

6. (a, b, c)

Arc length: $AB = \frac{\pi}{3} r = \frac{\pi mv}{3qB}$



$$\text{Time: } t = \frac{\theta}{\omega} = \frac{\pi/3}{2\pi/T} = \frac{T}{6} = \frac{\pi m}{3qB}$$

$$\text{Distance} = vt = \frac{\pi mv}{3qB}$$

7. (b, c, d)

If $\vec{\mathbf{v}} \cdot \vec{\mathbf{B}} = 0 \Rightarrow$ force is $\perp$ to velocity as $\vec{\mathbf{v}} \perp \vec{\mathbf{B}}$

if $\vec{\mathbf{v}} \times \vec{\mathbf{B}} = 0 \Rightarrow F = 0$ so it is straight line.

speed is constant.

10. (b, c)

$$\vec{\mathbf{v}} \perp \vec{\mathbf{B}}$$

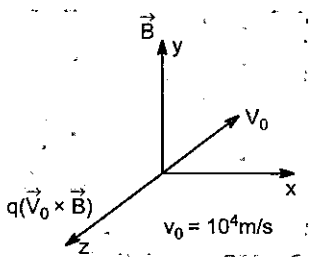
Therefore, path of the particle is a circle. In magnetic field, speed of the particle remains constant. Therefore, distance moved by the particle in time

$$t = \frac{\pi}{B_0 \alpha} \text{ is } v_0 t \quad \text{or} \quad \frac{\pi v_0}{B_0 \alpha}.$$

Magnitude of velocity is always v_0.

12. (b, d)

$$q\vec{\mathbf{v}}_0 \times \vec{\mathbf{B}} = q\vec{\mathbf{E}}$$

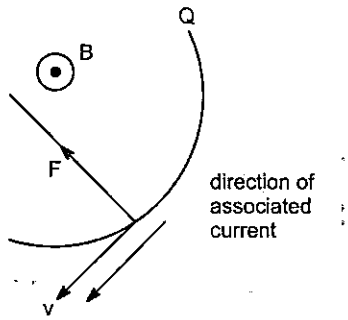


$E = VB \sin 90° = 10^4 \times 10^{-2} = 100 \text{V/m}$

$\vec{\mathbf{E}}$ along $(-z)$ direction

13. (a, b, c)

(a) Whenever a particle moves along a spiral path, its velocity always remains in the plane of spiral or the component of the velocity normal to the plane of spiral is always equal to zero. Force exerted by magnetic field is always normal to the direction of motion of the particle.



Force exerted by the magnetic field is in the plane of spiral. Direction of magnetic field is always normal to both the velocity vector and the direction of force. It means, magnetic field is normal to the plane of spiral.

(b) Since the particle experiences a resistance against its motion, therefore its speed decreases continuously. Radius of the circular path followed by a charged particle moving in a magnetic field is given by $R = \frac{mv}{qB}$. Since v is continuously decreasing, therefore radius R also decreases continuously. So, a charged particle should follow a spiral path of decreasing radius. Hence, the particle enters the magnetic field at Q.

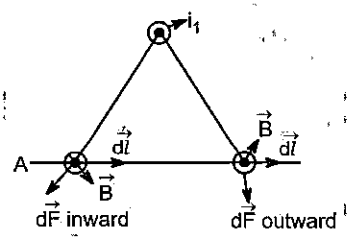
(c) Force experienced by the particle is towards the center and direction of the force is found out by Fleming's left hand rule. Since magnetic field is outward, therefore according to Fleming's left hand rule, the direction of current associated with its motion should be as shown in figure. Since the particle is moving in the same direction, therefore the particle is positively charged.

option (d) is not correct

15. (a, c, d)

As magnetic field due to a long wire $B = \frac{\mu_0 I}{2r}$. The magnetic field lines are concentric with wire. The direction may be either clockwise or anticlockwise.

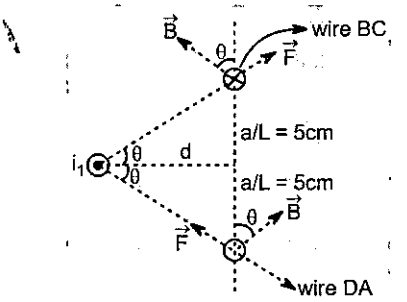
18. (a, d)



force on AB and CD is zero.

$\cos\theta = d/r$

$r = \sqrt{d^2 + \frac{a^2}{4}} = 13\,\text{cm}$



$\text{Torque} = \left(\frac{\mu_0 i_1 i_2}{2\pi r} a\cos\theta\right) a$

$\text{Resultant force} = \frac{\mu_0 i_1 i_2}{\pi r}\sin\theta$

20. (a, b, d)

(a) Period of revolution of a charged particle moving in a uniform magnetic field is given by $T = \frac{2\pi m}{qB}$. This period T does not depend upon speed of the particle. In this particular question, the moving particle is an electron. Hence, its mass and charge (q) both are constant. Magnetic field is also uniform. Hence, its period of revolution remains constant. It means, the electron moves with constant angular velocity.

(b) In previous question, we have already discussed that if a charged particle experiences a resisting force against its motion, then it follows a decreasing radius spiral path. In this question, the electron is moving along a spiral path of decreasing radius. It means, its speed is decreasing continuously.

(c) Since speed of the electron is countinuously decreasing, therefore it is experiencing a tangential retardation. It is possible only when the component of resultant force opposite to the direction of motion of electron has non-zero value. It means, net force on the electron cannot be perpendicular to its direction of motion.

(d) Since speed of the electron is decreasing continuously, therefore the force exerted by the magnetic field ($F = qvB$) is also decreasing continuously. Hence, magnitude of net force acting on the electron is decreasing continuously.

21. (a, b, c)

If a charge particle experiences no electromagnetic force, then electric field must be zero (as $\vec{F} = q\vec{E}$). The magnetic field may or may not be zero as a charge

particle at rest or moving in the direction of magnetic field experiences no force.

(a) When both $\vec{E}$ and $\vec{B}$ are 0, it is undeflected.

(b) If $\vec{E}$ is non-zero, it will move in the direction of E and remains undeflected.

(c) $\vec{E}$ is non-zero, force will be in direction of E. B is non-zero and V is parallel to $\vec{B}$.

$\vec{F} = q\vec{v} \times \vec{B}$ will become zero due to $\vec{B}$ and proton will move in a direction of $\vec{E}$ only.

24. (a, c)

$$a_e = \frac{F}{m_e}, a_p = \frac{F}{m_p}$$

Since $m_e << m_p$, so

$a_e >> a_p$

Now, $d = \frac{1}{2}a_e t_1^2$ and $d = \frac{1}{2}a_p t_2^2$

$$\frac{t_1}{t_2} = \left(\frac{m_e}{m_p}\right)^{1/2}$$

28. (a, b, c, d)

$$a = \frac{F}{m} = \frac{qE}{m} = \frac{q}{m}(\alpha - \beta x) \quad ...(1)$$

$a = 0$ at $x = \frac{\alpha}{\beta}$

i.e., force on the particle is zero at $x = \frac{\alpha}{\beta}$

So, mean position of the particle is at $x = \frac{\alpha}{\beta}$

Equation (1) can be written as $v\frac{dv}{dx} = \frac{q}{m}(\alpha - \beta x)$

$$\therefore \int_0^v v\,dv = \frac{q}{m}\int_0^x \left(\alpha - \frac{\beta}{2}x\right)dx$$

$$\therefore v = \sqrt{\frac{2qx}{m}\left(\alpha - \frac{\beta}{2}x\right)}$$

$v = 0$ at $x = 0$ and $x = \frac{2\alpha}{\beta}$

So, the particle oscillates between $x = 0$ and $x = \frac{2\alpha}{\beta}$ with mean position at $x = \frac{\alpha}{\beta}$.

Maximum acceleration of the particle is at extreme positions (at $x = 0$ or $x = 2\alpha/\beta$) and $a_{max} = q\alpha/m$ [from equation (1)].

29. (b, d)

For rotation equilibrium, taking torque about the point from where normal force passes.

$$IaB_0\frac{a}{2} + IaB_0\frac{a}{2} - mgx = 0$$

$$x = \frac{Ia^2B_0}{mg}$$

For the block not to topple:

$$x < \frac{a}{2} \Rightarrow I < \frac{mg}{2aB_0}$$

31. (a, b, c, d)

$$r = \frac{mv_0}{qB}$$

$$PQ = 2r\sin\alpha = 2\frac{mv_0}{qB}\sin\alpha$$

$$\alpha = \beta$$

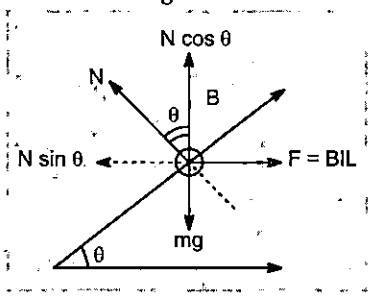
Time taken $= T = \frac{2\pi r}{v}$

$$T = \frac{2\pi m}{qB}$$

For t time, $t = \frac{T}{2\pi}(2\pi - 2\alpha) = \frac{2m}{qB}(\pi - \alpha)$

33. (a, b)

$N\cos\theta = mg$



[⊗ indicates current I is flowing into the paper]

$N\sin\theta = BIL$

$$\tan\theta = \frac{BIL}{mg}$$

34. (a, b, c)

To find the Ampere's force on a conductor of any shape, replace the conductor by an imaginary straight conductor joining the two ends of the given conductor. So, if B is in x-direction, then the imaginary straight conductor will be along the field and the force acting on it will be zero. If B is in y-direction, then the force will be λBI acting along the z-direction. Similarly, if B is in the z-direction, then the force will be λBI acting along the negative y-direction.

35. (a, b, c)

If $x = y$, then $\vec{v} \| \vec{B}$, *i.e.*, $\vec{F}_{mag} = 0$.

Hence, (a) is correct.

$\vec{F}_{mag} = q(\vec{v} \times \vec{B}) = q[(x\hat{i} + y\hat{j}) \times (y\hat{i} + x\hat{j})]$

$\vec{\mathbf{F}}_{mag} = q(x^2 + y^2)\hat{\mathbf{k}}$

If $x > y$, the the force is along z-axis.

36. (a, b)

If electric field is parallel to magnetic field, the charged particle moving parallel to $\vec{\mathbf{E}}$ experiences a force in the direction of $\vec{\mathbf{E}}$; due to $\vec{\mathbf{B}}$ there will not be any force.

Hence, no deflection.

The particle may go undeflected in the case when forces due to electric field and magnetic field balance each other.

37. (a, b, d)

In going from P to Q, increase in kinetic energy

$$\frac{1}{2}m(2v)^2 - \frac{1}{2}mv^2 = \frac{1}{2}m(3v^2) = \text{work done by electric field.}$$

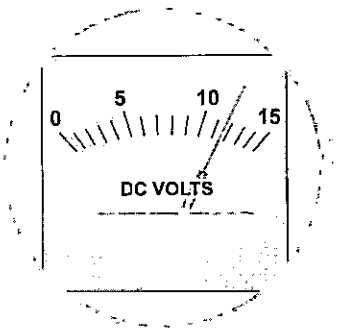
or $$3mv^2 = Eq \times 2a$$

or $$E = \frac{3}{2}\left(\frac{mv^2}{qa}\right)$$

The rate of work done by E at P = force due to $E \times$ velocity.

$$= (qE)v = qv\left[\frac{3}{2}\left(\frac{mv^2}{qa}\right)\right] = \frac{3}{4}\left(\frac{mv^3}{a}\right)$$

At q, is perpendicular to and, and no work is done by either field.

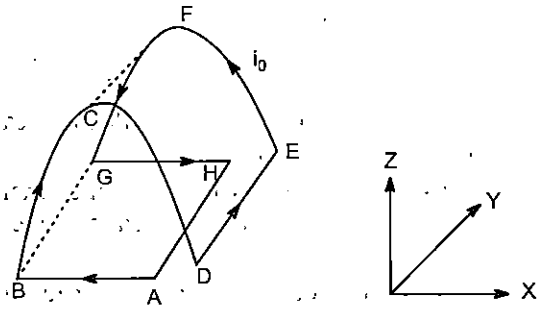


PROBLEMS

Level ③ Comprehension Based Problems

Passage 1

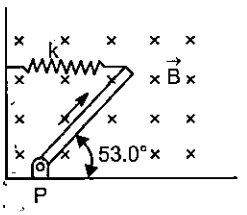
A coil *ABCDEFGH* is in the form of prism having each side equal to length l carries a current i_0 (assume gap between parallel sides *AH* and *DE* negligible)



1. The direction of magnetic field at the geometric centre of three dimensional prism shape coil will be :
   (a) along Y-axis (b) along X-axis
   (c) along Z-axis (d) none of these
2. If an external magnetic field B_0 is applied along $+ve$ Z-axis, then the force on the coil *ABCDEFGH* due to B_0 only will be equal to :
   (a) Zero (b) i_0B_0l
   (c) $6i_0B_0l$ (d) $2i_0B_0l$
3. If the mass of coil is m, in which direction an external magnetic field should be applied so that net normal reaction from the ground is on *AH* ?
   (a) along Y-axis (b) along X-axis
   (c) along Z-axis (d) not possible.

Passage 2

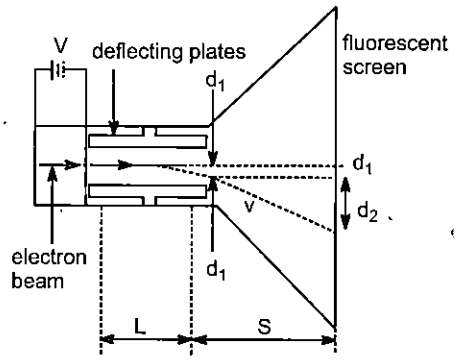
A thin, uniform rod with negligible mass and length 0.200 m is attached to the floor by a frictionless hinge at point *P* (as shown in figure). A horizontal spring with force constant $k = 4.80\,\text{Nm}^{-1}$ connects the other end of the rod to a vertical wall. The rod is in a uniform magnetic field $B = 0.340\,\text{T}$ directed into the plane of the figure. There is current $I = 6.50\,\text{A}$ in the rod, in the direction shown.



1. Calculate the torque due to the magnetic force on the rod, for an axis at *P*.
   (a) $0.0442\,\text{Nm}^{-1}$, clockwise
   (b) $0.0442\,\text{Nm}^{-1}$, anticlockwise
   (c) $0.0442\,\text{Nm}^{-1}$, clockwise
   (d) $0.0442\,\text{Nm}^{-1}$, anticlockwise
2. When the rod is in equilibrium and makes an angle of 53.0° with the floor, is the spring stretched or compressed ?
   (a) 0.05765 m, stretched
   (b) 0.05765 m, compressed
   (c) 0.0242 m, stretched
   (d) 0.0242 m, compressed

Passage 3

The following experiment was performed by J. J. Thomson in order to measure the ratio of the charge e to the mass m of an electron. Figure shows a modern version of Thomson's apparatus. Electrons emitted from a hot filament are accelerated by a potential difference V. As the electrons pass through the deflector plates, they encounter both electric and magnetic fields. When the electrons leave the plates they enter a field-free region that extends to the fluorescent screen. The beam of electrons can be observed as a spot of light on the screen. The entire region in which the electrons travel is evacuated with a vacuum pump.



Thomson's procedure was to first set both the electric and magnetic fields to zero, not the position of the undeflected electron beam on the screen, then turn on only the electric field and measure the resulting deflection. The deflection of an electron in an electric field of magnitude E is given by $d_1 = eEL^2/2mv^2$, where L is the length of the deflecting plates, and v is the speed of the electron. The deflection d_1 can also be calculated from the total deflection of the spot on the screen, $d_1 + d_2$ and the geometry of the apparatus. In the second part of the experiment. Thomson adjusted the magnetic field so as to exactly cancel the force applied by the electric field, leaving the electron beam undeflected. This gives $eV = evB$. By combing this relation with the expression of d_1, one can calculate the charge to mass ratio of the electron as a function of the known quantities. The result is:

$$\frac{e}{m} = \frac{2d_1E}{B^2L^2}$$

1. Why was it important for Thomson to evacuate the air from the apparatus?
- (a) Electrons travel faster in a vacuum, making the deflection d_1 smaller
- (b) Electromagnetic waves propagate in a vacuum.
- (c) The electron collisions with the air molecules cause them to be scattered, and a focused beam will not be produced.
- (d) It was not important and could have been avoided.

2. One might have considered a different experiment in which no magnetic field is needed. The ratio e/m can then be calculated directly from the expression for d_1. Why might Thomson have introduced the magnetic field B in his experiment?
- (a) To verify the correctness of the equation for the magnetic force.
- (b) To avoid having to measure the electron speed v.
- (c) To cancel unwanted effects of the electric field E.
- (d) To make sure that the electric field does not exert a force on the electron.

3. If the electron speed were doubled by increasing the potential difference V, which of the following would have to be true in order to correctly measure e/m?
- (a) The magnetic field would be to be cut in half in order to cancel the force applied by the electric field.
- (b) The magnetic field would have to be doubled in order to cancel the force applied by the electric field.
- (c) The length of the plates L , would have to be doubled to keep the deflection, d_1, from changing.
- (d) Nothing needs to be changed.

4. The potential difference V, which accelerates the electrons, also creates an electric field. Why did Thomson NOT consider the deflection caused this electric field in his experiment?
- (a) This electric field is much weaker than the one between the deflecting plates and can be neglected.
- (b) Only the deflection, $d_1 + d_2$ caused by the deflecting plates is measured in the experiment.
- (c) There is no deflection from this electric field
- (d) The magnetic field cancels the force caused by this electric field.

5. If the electron is deflected downward when only the electric field is turned on (as shown in figure) then in what directions do the electric and magnetic fields point in the second part of the experiment?
- (a) The electric field points to the bottom, while the magnetic field points into the page.
- (b) The electric field points to the bottom, while the magnetic field points out of the page.
- (c) The electric field points to the top, while the magnetic field points into the page.
- (d) The electric field points to the top, while the magnetic field points out of the page.

Passage 4

A particle of mass m and charge q is accelerated by a potential difference V volt and made to enter a magnetic field region at an angle θ with the field. At the same moment, another particle of same mass and charge is projected in the direction of the field from the same point. Magnetic field of induction is B.

1. What would be the speed of second particle so that both particles meet again and again after a regular interval of time, which should be minimum ?

(a) $\sqrt{\frac{qv}{m}}\cos\theta$ (b) $\sqrt{\frac{2qv}{m}}\cos\theta$

(c) $2\sqrt{\frac{qv}{m}}\sin\theta$ (d) $2\sqrt{\frac{qv}{m}}\cos\theta$

2. Find the time interval after which they meet.

(a) $\frac{2\pi m}{qB}$ (b) $\frac{\pi m}{2qB}$

(c) $\frac{\pi m}{qB}$ (d) $\frac{3\pi m}{2qB}$

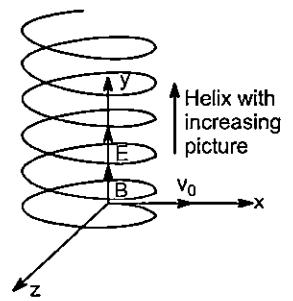
3. Find the distance travelled by the second particle during that interval mentioned in the above problem.

(a) $\sqrt{\frac{vm}{q}}\frac{2\pi}{B}\cos\theta$ (b) $\sqrt{\frac{2vm}{3q}}\frac{2\pi}{B}\cos\theta$

(c) $\sqrt{\frac{2vm}{q}}\frac{2\pi}{B}\cos\theta$ (d) $\frac{2}{3}\sqrt{\frac{vm}{q}}\frac{\pi}{m}\cos\theta$

Passage 5

Uniform electric and magnetic fields with strength E and induction B, respectively, are along y-axis as shown in figure. A particle with specific charge q/m leaves the origin O in the direction of x-axis with an initial non-relativistic velocity v_0.



1. The coordinate y_n of the particle when it crosses the y-axis for the n^{th} time is :

(a) $\frac{2\pi^2 mn^2E}{qB^2}$ (b) $\frac{\pi^2 mn^2E}{qB^2}$

(c) $\frac{2\pi^2 mn^2E}{3qB^2}$ (d) $\frac{\sqrt{3}\pi^2 mn^2E}{qB^2}$

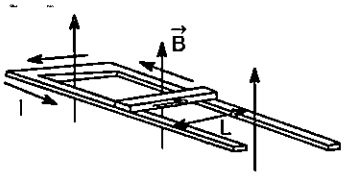
2. The angle α between the particle's velocity vector and y-axis at that moment is :

(a) $\tan^{-1}\left(\frac{3v_0B}{2\pi nE}\right)$ (b) $\tan^{-1}\left(\frac{v_0B}{\pi nE}\right)$

(c) $\tan^{-1}\left(\frac{v_0B}{\sqrt{2}\pi nE}\right)$ (d) $\tan^{-1}\left(\frac{v_0B}{2\pi nE}\right)$

Passage 6

A conducting bar with mass m and length L slides over horizontal rails that are connected to a voltage source. The voltage source maintains a constant current I in the rails and bar, and a constant, uniform, vertical magnetic field $\vec{B}$ fills the region between the rails (as shown in figure).



1. Find the magnitude and direction of the net force on the conducting bar. Ignore friction, air resistance and electrical resistance :

(a) ILB, to the right (b) ILB, to the left

(c) $2ILB$, to the right (d) $2ILB$, to the left

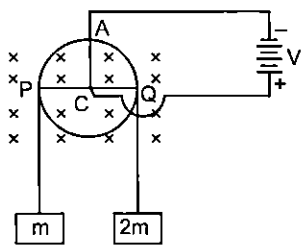
2. If the bar has mass m, find the distance d that the bar must move along the rails from rest to attain speed v.

(a) $\frac{3v^2m}{2ILB}$ (b) $\frac{5v^2m}{2ILB}$

(c) $\frac{v^2m}{ILB}$ (d) $\frac{v^2m}{2ILB}$

Passage 7

A conducting ring of mass m and radius r has a weightless conducting rod PQ of length $2r$ and resistance $2R$ attached to it along its diameter. It is pivoted at its center C with its plane vertical, and two blocks of mass m and $2m$ are suspended by means of a light in-extensible string passing over it as shown in figure. The ring is free to rotate about C and the system is placed in a magnetic field B (into the plane of the ring). A circuit is now completed by connecting the ring at A and C to a battery of e.m.f. V. It is found that for certain value of V, the system remains static.



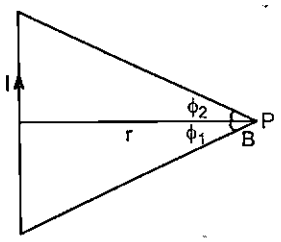
1. In static condition, find the current through rod *PC*.
   (a) V/R (b) $V/2R$
   (c) $4V/R$ (d) $2V/R$
2. Net torque applied by the tension in string can be related as :
   (a) $\frac{3BVr^2}{R}$ (b) $\frac{BVr^2}{R}$
   (c) $\frac{BVr^2}{3R}$ (d) $\frac{BVr^2}{2R}$
3. The value of V can be related with m, B and r as :
   (a) $2mgR/Br$ (b) mgR/Br
   (c) $mgR/2Br$ (d) $3mgR/Br$

Passage 8

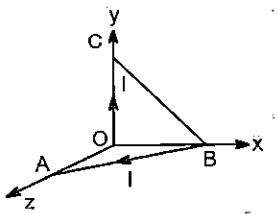
According to Biot-Savarat's Law, magnetic field due to a straight current carrying wire at a point at a distance r from it is given by

$$B = \frac{\mu_0 I}{4\pi r}(\sin\phi_1 + \sin\phi_2)$$

The direction of magnetic field being perpendicular to the plane containing the wire and that point.



1. Figure shows a closed loop *AOCB*. *A* in which current I is flowing as shown. Given $OA = OB = OC = a$. Find the magnetic field at point *B* due to this loop :



   (a) $-\frac{\mu_0 I}{4\pi\sqrt{2}a}(\hat{i}+\hat{k})$ (b) $-\frac{\mu_0 I}{4\pi\sqrt{2}a}(\hat{j}+\hat{k})$
   (c) $\frac{-\mu_0 I}{2\sqrt{2}\pi a}(\hat{j}+\hat{k})$ (d) none of these
2. Find magnetic field at point *O* in Fig. 9.314.
   (a) $-\frac{\mu_0 I}{4\pi a}(\hat{i}+\hat{k})$ (b) $-\frac{\mu_0 I}{4\pi a}(\hat{j}+\hat{k})$
   (c) $\frac{-\mu_0 I}{2\pi a}(\hat{j}+\hat{k})$ (d) $\frac{-\mu_0 I}{2\sqrt{2}\pi a}(\hat{j}+\hat{k})$

Passage 9

In a certain region of space, there exists a uniform and constant electric field of magnitude E along the positive y-axis of a coordinate system. A charged particle of mass 'm' and charge '$-q$' ($q > 0$) is projected from the origin with speed $2v$ at an angle of 60° with the positive x-axis in x-y plane. When the x-coordinate of particle becomes $\frac{\sqrt{3}mv^2}{qE}$, a uniform and constant magnetic field of strength B is also switched on along positive y-axis.

1. Velocity of the particle just before the magnetic field is switched on is :
   (a) $v\hat{i}$ (b) $v\hat{i}+\frac{\sqrt{3}v}{2}\hat{j}$
   (c) $v\hat{i}-\frac{\sqrt{3}v}{2}\hat{j}$ (d) $2v\hat{i}-\frac{\sqrt{3}v}{2}\hat{j}$
2. x-coordinate of the particle as a function of time after the magnetic field is switched on is :
   (a) $\frac{\sqrt{3}mv^2}{qE} - R\sin\left(\frac{qB}{m}t\right)$ (b) $\frac{\sqrt{3}mv^2}{qE} + R\sin\left(\frac{qB}{m}t\right)$
   (c) $\frac{\sqrt{3}mv^2}{qE} - R\cos\left(\frac{qB}{m}t\right)$ (d) $\frac{\sqrt{3}mv^2}{qE} + R\cos\left(\frac{qB}{m}t\right)$
3. z-coordinate of the particle as a function of time after the magnetic field is switched on is :
   (a) $R\left[1+\cos\left(\frac{qB}{m}t\right)\right]$ (b) $R\left[1+\sin\left(\frac{qB}{m}t\right)\right]$
   (c) $-R\left[1-\cos\left(\frac{qB}{m}t\right)\right]$ (d) $-R\left[1+\cos\left(\frac{qB}{m}t\right)\right]$

Passage 10

Ampere's law provides us an easy way to calculate the magnetic field due to a symmetrical distribution of current. Its mathematical expression is as $\oint \vec{B}\cdot dl = \mu_0 I_m$

The quantity on the left hand side is known as line integral of magnetic field over a closed Ampere's loop.

1. Only the current inside the Amperian loop contributes in :
   (a) finding magnetic field at any point on the Ampere's loop
   (b) line integral of magnetic field
   (c) in both of the above
   (d) in neither of them

2. If the current density in a linear conductor of radius a varies with r according to relation $J = kr^2$, where k is a constant and r is the distance of a point from the axis of conductor, find the magnetic field induction at a point distance r from the axis when $r < a$. Assume relative permeability of the conductor to be unity.
   (a) $\frac{\mu_0 k\pi a^4}{4r}$ (b) $\frac{\mu_0 kr^3}{2}$
   (c) $\frac{\mu_0 k\pi a^4}{2r}$ (d) $\frac{\mu_0 kr^3}{2}$

3. In the above question, find the magnetic field induction at a point distance r from the axis when $r > a$. Assume relative permeability of the medium surrounding the conductor to be unity :
   (a) $\frac{\mu_0 k\pi a^4}{4r}$ (b) $\frac{\mu_0 kr^3}{2}$
   (c) $\frac{\mu_0 k\pi a^4}{2r}$ (d) $\frac{\mu_0 kr^3}{4}$

Passage 11

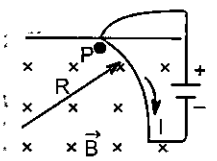
A charged particle of mass m and charge q is projected on a rough horizontal $X-Y$ plane, both electric and magnetic fields are acting in the region and given by $\vec{E} = -E_0\,\hat{k}$ and $\vec{B} = -B_0\,\hat{k}$, respectively. At $t = 0$, the particle enters into the field at $(A_0, 0, 0)$ with velocity $\vec{v} = v_0\hat{j}$. The particle starts moving into a circular path on the plane. If coefficient of friction between the particle and the plane is μ.

1. Time when the particle will come to rest :
   (a) $\frac{mv_0}{\mu(mg + qE)}$ (b) $\frac{2mv_0}{\mu(mg + 2qE)}$
   (c) $\frac{3mv_0}{\mu(2mg + qE)}$ (d) none of these

2. Length of the path travelled by the particle when it comes to rest :
   (a) $\frac{mv_0^2}{\mu(mg + qE)}$ (b) $\frac{3mv_0^2}{2\mu(mg + qE)}$
   (c) $\frac{mv_0^2}{2\mu(mg + qE)}$ (d) none of these

Passage 12

As shown in figure, a thin, flexible wire hangs from point P in a region where there is a uniform, horizontal, magnetic field of magnitude B directed into, and perpendicular to, the plane of the figure. A weight is attached to the bottom of the wire to provide a uniform tension T throughout the wire. (The weight of the wire itself is negligible.) When a current I flows from the top to bottom of wire, the wire curves into a circular are of radius R.



1. By considering the forces on a small segment of wire that subtends an angle θ, find the radius of curvature of the wire.
   (a) $R = 2T/IB$ (b) $R = T/3IB$
   (c) $R = \sqrt{2}\,T/B$ (d) $R = T/IB$

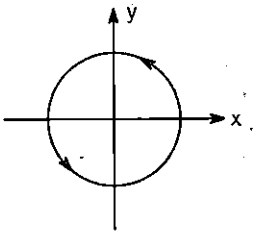
2. The wire is now removed. A negatively charged particle of charge $-q$ and mass m is launched from the same point P from which the wire was hung, in the same direction as that in which the wire extended from P. If the trajectory of the particle follows the same circular arc as in previous problem, the speed of the particle will be :
   (a) Tq/mI (b) $2Tq/mI$
   (c) $Tq/3mI$ (d) $\sqrt{2}\,Tq/mI$

MATCHING TYPE PROBLEMS

1. A charged particle having charge q and mass m is to be subjected to a combination of constant uniform magnetic field ($\vec{B}$) and a constant uniform gravitational field ($\vec{E}$). Apart from these field forces there exists no other force. Now match the column.

	Column-I		Column-II
(a)	The charged particle moves without change in direction	(p)	It is possible that both $\vec{B}$ and $\vec{E}$ are zero
(b)	The charged particle moves without change in its velocity	(q)	If the possible that both $\vec{B}$ and $\vec{E}$ are noon zero
(c)	The charged particle takes a circular path	(r)	It is possible that $\vec{B}$ is zero $\vec{E}$ is not zero
(d)	The charged particle takes a parabolic path	(s)	It is possible that $\vec{B}$ is non zero and $\vec{E}$ is zero.

2. A circular current carrying loop of 100 turns and radius 10 cm is placed in x-y plane as shown in figure. A uniform magnetic field $\vec{B} = (-\hat{i} + \hat{k})$ tesla is present in the region. If current in the loop is 5 A, then match the Column-I with Column-II.



	Column-I		Column-II
(a)	Magnitude and direction of moment (in A-m) of the loop are	(p)	Zero
(b)	Magnitude and direction of torque (in N-m) on the loop are	(q)	5π
(c)	Magnitude and direction of net force (in N) on the current loop are	(r)	along positive z-axis
(d)	Direction of magnetic field of loop at the center is	(s)	along negative y-axis

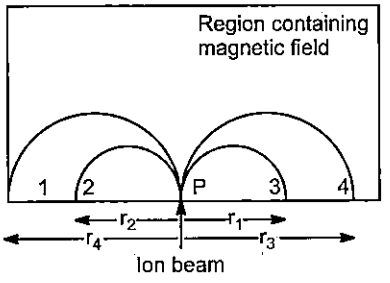
3. A charged particle with some initial velocity is projected in a region where non-zero electric and/or magnetic fields are present. In column I, information about the existence of electric and/or magnetic field and direction of initial velocity of charged particle are given, while in Column II the probable path of the charged particle is mentioned. Match the Column-I with the Column-II.

	Column-I		Column-II
(a)	$\vec{E} = 0, \vec{B} \neq 0$, and initial velocity is at any angle with $\vec{B}$	(p)	Straight line
(b)	$\vec{E} \neq 0, \vec{B} = 0$ and initial velocity is at any angle with $\vec{E}$	(q)	Parabola
(c)	$\vec{E} \neq 0, \vec{B} \neq 0, E \parallel \vec{B}$ and initial velocity is $\perp$ to both	(r)	Circular
(d)	$\vec{E} \neq 0, \vec{B} \neq 0, \vec{E}$ perpendicular $\vec{B}$ to and $\vec{v}$ perpendicular to both $\vec{E}$ and $\vec{B}$	(s)	Helical path with non-uniform pitch

4. Match the Column-I with Column-II.

	Column-I		Column-II
(a)	Electric field	(p)	Stationary charge
(b)	Magnetic field	(q)	Moving charge
(c)	Electric force	(r)	Changes the kinetic energy
(d)	Magnetic force	(s)	Does not change kinetic energy

5. A beam consisting of four types of ions A, B, C and D enters a region at P that contains a uniform magnetic field as shown in figure. The field is perpendicular to the plane of the paper, but its precise direction is not given. All ions in the beam travel with the same speed. The following table shows the masses and charges of the ions.



$r_4 > r_3 = r_2 > r_1$

Ion	Mass	Charge
A	$2m$	e
B	$4m$	$-e$
C	$2m$	$-e$
D	m	$+e$

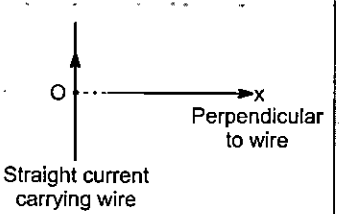
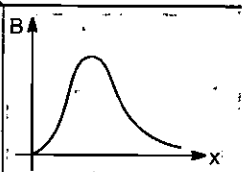
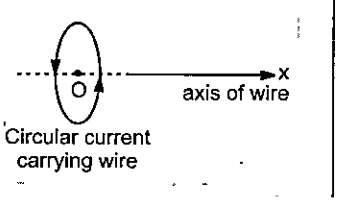
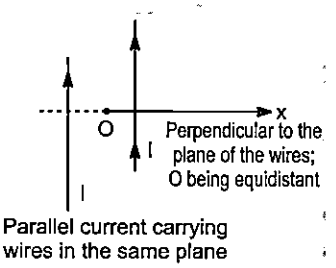
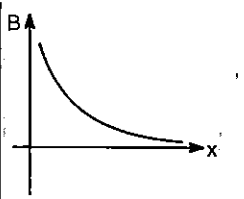
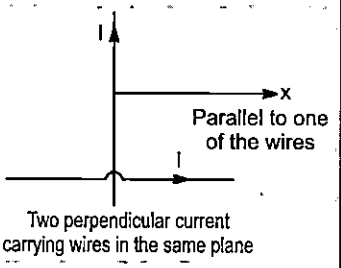
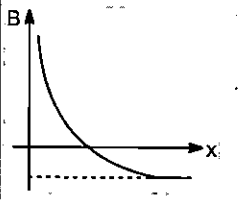
The ions fall at different positions 1, 2, 3 and 4 as shown. Correctly match the ions with their respective positions of fall.

	Column-I		Column-II
(a)	A	(p)	1
(b)	B	(q)	2
(c)	C	(r)	3
(d)	D	(s)	4

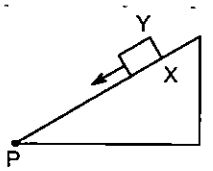
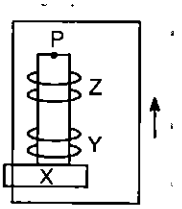
6. Match the Column-I with Column-II

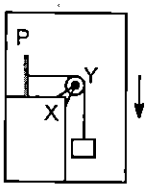
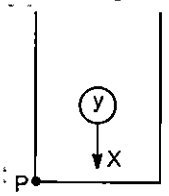
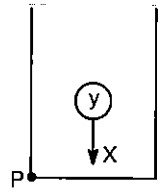
	Column-I		Column-II
(a)	A charge particle is moving in uniform electric and magnetic fields in gravity free space.	(p)	Velocity of the particle may be constant.
(b)	A charge particle is moving in uniform electric magnetic and gravitational fields.	(q)	Path of the particle may be straight line.
(c)	A charge particle is moving in uniform magnetic and gravitational fields (where electric field is zero).	(r)	Path of the particle may be circular.
(d)	A charge particle is moving in only uniform electric field.	(s)	Path of the particle may be helical.

7. The entries in Column-I depict certain current distributions, while the entries in Column-II depict the variation of the magnetic field (B) as one moves along the x-axis for each of these distributes, but in a different order. Match the entries in Column-I with the proper entries in Column-II.

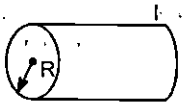
	Column-I		Column-II
(a)		(p)	
(b)		(q)	
(c)		(r)	
(d)		(s)	

8. Column-II shows five systems in which two objects are labelled as X and Y. Also in each case a point P is shown. Column-I gives some statements about X and/or Y. Match these statements to the appropriate system(s) from Column-II.

	Column-I		Column-II
(a)	The force exerted by X on Y has a magnitude Mg.	(p)	Block Y of mass M left on a fixed inclined plane X, slides on it with a constant velocity. 
(b)	The gravitational potential energy of X is continuously increasing.	(q)	Two ring magnets Y and Z, each of mass M are kept in frictionless vertical plastic stand so that they repel each other. Y rests on the base X and Z hangs in air in equilibrium. P is the topmost point of the stand on the common axis of the two rings. The whole system is in a lift that is going up with a constant velocity. 

(c)	Mechanical energy of the system $X+Y$ is continuously decreasing.	**(r)**	A pulley Y of mass m_0 is fixed to a table through a clamp X. A block of mass M hangs from a string that goes over the pulley and is fixed at point P of the table. The whole system is kept in a lift that is going down with a constant velocity 
(d)	The torque of the weight of Y about point P is zero.	**(s)**	A sphere Y of mass M is put in a non viscous liquid X kept in a container at rest. The sphere is released and it moves down in the liquid. 
		(t)	A sphere Y of mass M is falling with its terminal velocity in a viscous liquid X kept in a container. 

9. A long straight conductor of radius R carries a current I as shown. The current density j varies as a function of radius according to $j = br$ where b is a constant. Match the quantities in Column-I with those in Column-II.

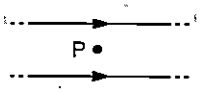
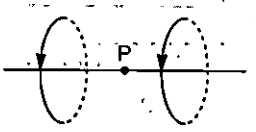
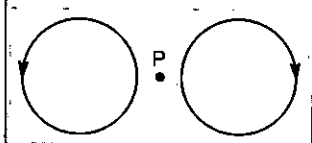
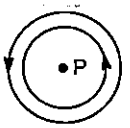


	Column-I		Column-II
(a)	B_{inside}	**(p)**	Maximum
(b)	$B_{surface}$	**(q)**	$\frac{\mu_0 br^2}{3}$
(c)	$B_{outside}$	**(r)**	$2\pi br^3$
(d)	I_{inside}	**(s)**	$\frac{\mu_0 bR^3}{3r}$
		(t)	Continuous

10. A current flows along the length of a long thin cylindrical shell of radius R. Match the fields in different regions given in Column-I to their respective matches in Column-II.

	Column-I		Column-II
(a)	$B\,(r<R)$	**(p)**	$B \propto \frac{1}{r}$
(b)	$B\,(r>R)$	**(q)**	Zero
(c)	$B\,(r=R)$	**(r)**	$B \propto r$
(d)	$B\,(r=0)$	**(s)**	Maximum
		(t)	Discontinuous

11. Two wires each carrying a steady current I are shown in four configurations in Column-I. Some of the resulting effects are described in Column-II. Match the statements in Column-I with the statements in Column-II.

	Column-I		Column-II
(a)	Point P is situated midway between the wires. 	**(p)**	The magnetic field (B) at P due to the currents in the wires are in the same direction.
(b)	Point P is situated at the mid-point of the line joining the centers of the circular wires, which have same radii. 	**(q)**	The magnetic fields (B) at P due to the currents in the wires are in opposite directions.
(c)	Point P is situated at the mid-point of the line joining the centers of the circular wires, which have same radii. 	**(r)**	There is no magnetic field at P.
(d)	Point P is situated at the common centre of the wires. 	**(s)**	The wires repel each other.

12. Match the physical quantities in Column-II with the respective SI units in Column-I

	Column-I		Column-II
(a)	$NA^{-1}m^{-1}$	**(p)**	Magnetic permeability (μ_0)
(b)	Am^2	**(q)**	Magnetic flux
(c)	NmA^{-1}	**(r)**	Magnetic potential energy
(d)	NA^{-2}	**(s)**	Magnetic flux density
		(t)	Magnetic dipole moment

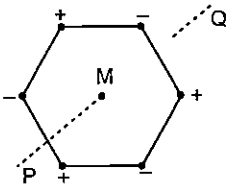
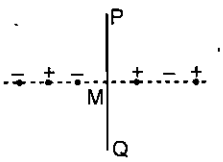
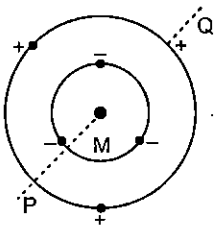
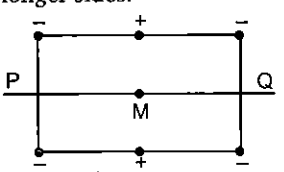
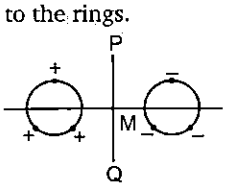
13. Match the Column-I with Column-II

	Column-I		Column-II
(a)	Magnetic field inside a long straight solenoid is	**(p)**	Not constant
(b)	Magnetic field inside a toroidal solenoid is	**(q)**	Zero
(c)	Magnetic field inside a conducting hollow pipe having current parallel to its axis	**(r)**	Constant
(d)	Magnetic field due to current carrying wire on its surface is	**(s)**	Maximum

14. For a charged particle of charge q, mass m that enters a region of space the field options are given in Column-I. Match those to the trajectories in Column-II.

	Column-I		Column-II
(a)	Electric field	**(p)**	Straight line path
(b)	Magnetic field	**(q)**	Circular path
(c)	In crossed field for $v = \frac{E}{B}$	**(r)**	Helical path
(d)	In mutually perpendicular electric and magnetic field, charge initially at rest	**(s)**	Cycloidal path
		(t)	Parabolic path

15. Six point charges, each of the same magnitude q, are arranged in different manners as shown in Column-II. In each case, a point M and a line PQ passing through M are shown. Let E be the electric field and V be the electric potential at M (potential at infinity is zero) due to the given charge distribution when it is at rest. Now, the whole system is set into rotation with a constant angular velocity about the line PQ. Let B be the magnetic field at M and μ be the magnetic moment of the system in this condition. Assume each rotating charge to be equivalent to a steady current.

	Column-I		Column-II (Effect)
(a)	$E = 0$	**(p)**	Charges are at the corners of a regular hexagon. M is at the centre of the hexagon. PQ is perpendicular to the plane of the hexagon. 
(b)	$V \neq 0$	**(q)**	Charges are on a line perpendicular to PQ at equal intervals. M is the midpoint between the two innermost charges. 
(c)	$B = 0$	**(r)**	Charges are placed on two coplanar insulating rings at equal intervals. M is the common centre of the rings. PQ is perpendicular to the plane of rings. 
(d)	$\mu \neq 0$	**(s)**	Charges are placed at the corners of a rectangle of sides a and $2a$ and at the mid points of the longer sides. M is at the centre of the rectangular. PQ is parallel to the longer sides. 
		(t)	Charges are placed on two coplanar, identical insulating rings at equal intervals. M is the mid-point between the centres of the rings. PQ is perpendicular to the line joining the centres and coplanar to the rings. 

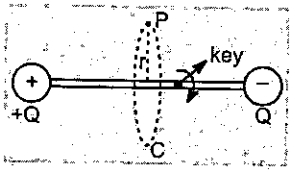
16. In a region of space, uniform electric and/or magnetic field(s) exist. The direction(s) of field(s) are indicated in column I and possible trajectories of a charged particle in the fields are mentioned in column II. In option $A, B \& C$ particle can have any initial velocity (zero or non-zero).

	Column-I		Column-II
(a)	$\longrightarrow E, B = 0$	(p)	Parabola
(b)	$\longrightarrow B, E = 0$	(q)	Straight line
(c)	E → , B	(r)	Helix (with constant or variable pitch)
(d)	E ↑, B →, particle starts from rest	(s)	Cycloid

ASSERTION AND REASON

(a) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
(b) Statement-1 is true, statement-2 is NOT the correct explanation for statement-1.
(c) Statement-1 is true, statement-2 is false.
(d) Statement-1 is false, statement-2 is true.

1. Two small spherical conductors with initial charge $+Q$ and $-Q$ are connected with a very long conducting straight wire with a switch. When switch is closed.



Statement-1: Ampere's law is applicable for the curve C encircling the wire.

Statement-2: From symmetry considerations magnetic field is tangent at each point of circle C.

2. Statement-1: Two long parallel conductors carrying currents in the same direction experience a force of attraction.

Statement-2: The magnetic produced in the space between the conductors are in the same direction.

3. Statement-1: Net force on a current carrying loop in a non-uniform magnetic field must be non-zero.

Statement-2: Force on a current carrying wire of length $d\vec{l}$ placed in magnetic field $\vec{B}$ is given by

$$d\vec{F} = i\, d\vec{l} \times \vec{B}$$

4. Statement-1: A charged particle can never move along a magnetic field line in absence of any other force.

Statement-2: Force due to magnetic field is given by $\vec{F} = q(\vec{v} \times \vec{B})$

ANSWERS

Level-3: Comprehension Based Problems

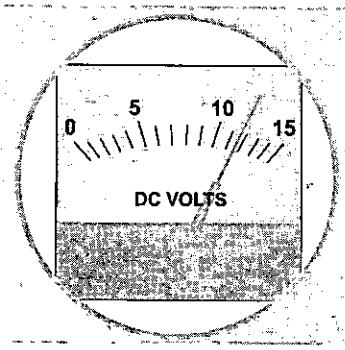
Passage-1:	**1.** (d)	**2.** (a)	**3.** (d)		**Passage-2:**	**1.** (a)	**2.** (a)	
Passage-3:	**1.** (c)	**2.** (b)	**3.** (a)		**Passage-4:**	**1.** (b)	**2.** (a)	**3.** (c)
	4. (c)	**5.** (d)						
Passage-5:	**1.** (a)	**2.** (d)			**Passage-6:**	**1.** (a)	**2.** (d)	
Passage-7:	**1.** (a)	**2.** (b)	**3.** (b)		**Passage-8:**	**1.** (b)	**2.** (d)	
Passage-9:	**1.** (a)	**2.** (b)	**3.** (c)		**Passage-10:**	**1.** (b)	**2.** (d)	**3.** (a)
Passage-11:	**1.** (a)	**2.** (c)			**Passage-12:**	**1.** (d)	**2.** (a)	

Matching Type Problems

1. (a)-p, q, r, s; (b)-p, q, s; (c)-s; (d)-r
2. (a)-q, r; (b)-q, s; (c)-p; (d)-r
3. (a)-p, r; (b)-p, q; (c)- s; (d)-p
4. (a)-p, q; (b)-q; (c)-p, q, r; (d)-q, s
5. (a)-s; (b)-p; (c)-q; (d)-r
6. (a)-p, q, s; (b)-p, q, r, s; (c)-p, q, r, s; (d)-q
7. (a)-r; (b)-q; (c)-p; (d)-s
8. (a)-p, t; (b)-q, s, t; (c)-p, r, t; (d)-p
9. (a)-q; (b)-p, t; (c)-s; (d)-r
10. (a)-q; (b)-p; (c)-s, t; (d)-q
11. (a)-q, r; (b)-p; (c)-q, r; (d)-q
12. (a)-s; (b)-t; (c)-q; (d)-p
13. (a)-r; (b)-p; (c)-q, r; (d)-s
14. (a)-p, t; (b)-p, q, r; (c)-p; (d)-s
15. (a)-p, r, s; (b)-r, s; (c)-p, q, t; (d)-r, s
16. (a)-p, q; (b)-q, r; (c)-q, r; (d)-s

Assertion and Reason

1. (d) **2.** (c) **3.** (d) **4.** (d)



SOLUTION

Level-3: Comprehension Based Problems

Passage-1

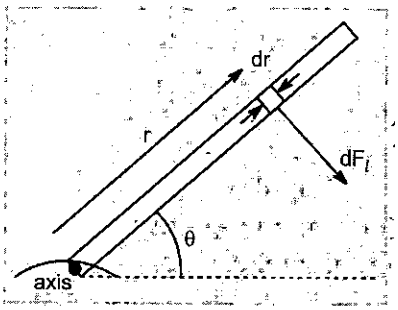
1. (d) Due to symmetry

2. (a) The net force acting on the close loop is zero.

3. (d) The net force acting on the closed loop is zero.

Passage -2

1. (a) By examining a small piece of the wire (figure), we find:



Divide the rod into infinitesimal sections of length dr.

The magnetic force on this section is $dF_1 = IB\,dr$ and is perpendicular to the rod. The torque dt due to force on this section is $d\tau = r\,dF_1 = IBr\,dr$

The total torque is

$$\int d\tau = IB\int_0^l r\,dr = \frac{1}{2}Il^2B = 0.0442\,\text{Nm}^{-1},$$

clockwise. This the same torque calculated from a force diagram in which the total magnetic force $F_1 = ILB$ acts at the center of the rod.

2. (b) F_1 produces a clockwise torque, so the spring force must produce a counterclockwise torque. The spring force must be to the left, the spring is stretched.

Find x, the amount the spring is stretched:

axis at hinge, counterclockwise torques positive

$$(kx)\,l\sin 53° - \frac{1}{2}Il^2B = 0$$

$$x = \frac{IlB}{2k\sin 53.0°} = \frac{(6.50)(0.200)(0.340)}{2\times 4.80\sin 53.0°}$$

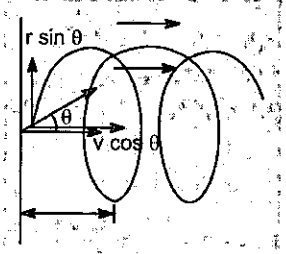
$$= 0.05765\,\text{m}$$

Passage -4

1. (b), 2. (a), 3. (c)

The first particle will have a helical path and the second particle will move rectilinearly along the field. For the two particles to meet again and again, $v_{\|}T = v'T$

where v' is the speed of the second particle.



$$\therefore \quad v' = v_{\|} = v\cos\theta$$

$$\frac{1}{2}mv^2 = qv \Rightarrow v = \sqrt{\frac{2qv}{m}}$$

$$\therefore \quad v' = \sqrt{\frac{2qv}{m}}\cos\theta$$

$$T = \frac{2\pi m}{qB}$$

Distance travelled = pitch

$$\text{Distance} = \sqrt{\frac{2qv}{m}}\cos\theta \times \frac{2\pi m}{qB}$$

$$\text{Distance} = \sqrt{\frac{2vm}{q}}\frac{2\pi}{B}\cos\theta$$

Passage -5

1. (a) Proceeding similar to example, we can see that the particle's acceleration is in y-direction,

$$\frac{dv_y}{dt} = \frac{qE}{m} = \text{constant}$$

The motion of the particle is equivalent to circular motion in xz plane with uniform acceleration in y-direction.

Hence, $v_0^2 = v_x^2 + v_z^2 = \text{constant}$

The magnetic force cannot change the magnitude of v_0

The y-component of velocity, $v_y = \frac{qE}{m}t$

The y-coordinate at time t, $y = \frac{1}{2}\frac{qE}{m}t^2$

The time period of circular motion in xz plane,

$$T = \frac{2\pi m}{Bq}$$

Let the particle cross y-axis after n rotations, then

$$t = nT = \frac{2\pi mn}{qB}$$

Thus, $y_n = \frac{qE}{2m} \times \left(\frac{2\pi mn}{qB}\right)^2 = \frac{2\pi^2 mn^2 E}{qB^2}$

2. (b) As $v_y = a_y t \left(\frac{qE}{m}\right)\left(\frac{2\pi mn}{qB}\right) = \frac{2\pi nE}{B}$

Thus, $\tan\alpha = \frac{v_0}{v_y} = \frac{v_0 B}{2\pi nE}$

$$\Rightarrow \quad \alpha = \tan^{-1}\left(\frac{v_0 B}{2\pi nE}\right)$$

Passage -6

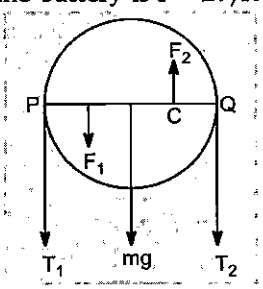
1. (a) $F = ILB$, to the right.

2. (b) $v^2 = 2ad \Rightarrow d = \frac{v^2}{2a} = \frac{v^2 m}{2ILB}$

Passage -7

1. (a) 2. (b) 3. (b)

PC and CQ are in parallel. So, equivalent resistance of the circuit is $R/2$. In static condition, current through the battery is $I = 2V/R$.



The current through CP and CQ is $i = I/2 = V/R$

Hence,

$$F_1 = F_2 = Bir$$

Consider the FBD of ring and the blocks

FBD of ring:

Consider torque about 'C',

$$T_1 r + + F_1 r/2 + F_2 r/2 = T_2 r$$

FBD of block: $T_1 = mg$ and $T_2 = 2mg$

So, $mgr + Bir^2/2 + Bir^2/2 = 2mgr$

or $$i = \frac{mg}{Br} \text{ or } \frac{V}{R} = \frac{mg}{Br}$$

or $$v = \frac{mgR}{Br}$$

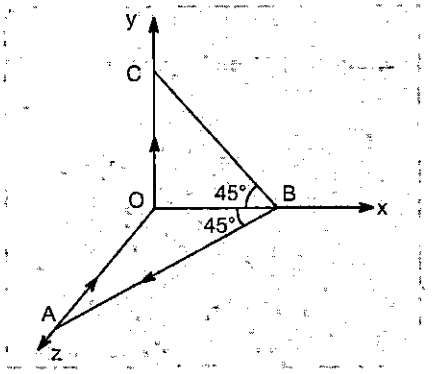
Net torque of tension

$$= (T_2 - T_1) r = mgr = \frac{Bvr^2}{R}$$

Passage -8

1. (b) 2. (d)

At B, magnetic field will be zero due to CB and AB parts.



$$\vec{\mathbf{B}}_{AO} = \frac{\mu_0}{4\pi}\frac{I}{a}[\sin 0° + \sin 45°]\,(-\hat{\mathbf{j}}) = \frac{-\mu_0 I}{4\pi\sqrt{2}a}\hat{\mathbf{j}}$$

$$\vec{\mathbf{B}}_{OC} = \frac{\mu_0}{4\pi}\frac{I}{a}[\sin 0° + \sin 45°]\,(-\hat{\mathbf{k}}) = \frac{-\mu_0 I}{4\pi\sqrt{2}a}\hat{\mathbf{k}}$$

Net magnetic field at B

$$= \vec{\mathbf{B}}_{AO} + \vec{\mathbf{B}}_{OC} = -\frac{\mu_0 I}{4\pi\sqrt{2}a}(\hat{\mathbf{j}} + \hat{\mathbf{k}})$$

At O, magnetic field due to AO and OC will be zero.

Due to BC: $r = a\cos 45° = \frac{a}{\sqrt{2}}$

$$\vec{\mathbf{B}}_{CB} = \frac{\mu_0}{4\pi}\frac{I}{r}[\sin 45° + \sin 45°](-\hat{\mathbf{k}}) = \frac{-\mu_0 I}{2\pi a}\hat{\mathbf{k}}$$

Similarly, due to BA: $\vec{\mathbf{B}}_{BA} = \frac{-\mu_0 I}{2\pi a}\hat{\mathbf{j}}$

Net magnetic field

$$= \vec{\mathbf{B}}_{CB} + \vec{\mathbf{B}}_{BA} = \frac{-\mu_0 I}{2\pi a}(\hat{\mathbf{j}} + \hat{\mathbf{k}})$$

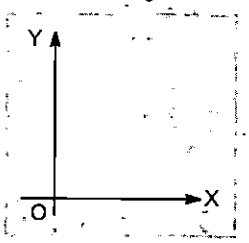
Passage -9

1. (a) 2. (b) 3. (c)

$$\vec{\mathbf{E}} = E\hat{\mathbf{j}}$$

$$\hat{\mathbf{v}}_0 = 2v\cos 60°\hat{\mathbf{i}} + 2v\sin 60°\hat{\mathbf{j}}$$

$$= v\hat{\mathbf{i}} + v\sqrt{3}\hat{\mathbf{j}}$$



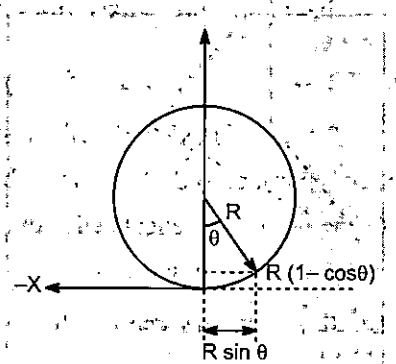
$$\vec{\mathbf{a}} = -\frac{qE}{m}\hat{\mathbf{j}}$$

$$\vec{\mathbf{R}}(t) = vt\hat{\mathbf{i}} + \left(v\sqrt{3}t - \frac{1}{2}\frac{qE}{m}t^2\right)\hat{\mathbf{j}}$$

$$\therefore \quad vt = \frac{\sqrt{3}mv^2}{qE} \Rightarrow t = \frac{\sqrt{3}mv}{qE}$$

For $t \le \frac{\sqrt{3}mv}{qE}$,

$$x(t) = vt \text{ and } v_y(t) = v\sqrt{3} - \frac{qE}{m}t$$

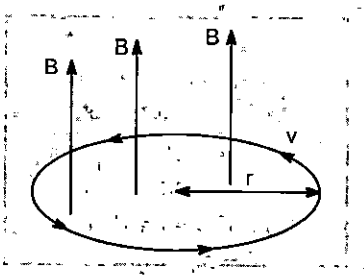


$$\Rightarrow \quad v_y = \left(t - \frac{\sqrt{3}mv}{2E}\right)$$

$$v\sqrt{3} = \frac{qE}{m} \times \frac{\sqrt{3}mv}{qE} = 0$$

for $t > \frac{\sqrt{3}mv}{qE}$, magnetic field is also present

The particle will start moving on helical path. The cross-section of the helix will be in x-z plane.



$$R = \frac{mv}{qB};\ \omega = \frac{qB}{m};\ \theta = \omega t = \frac{qBt}{m}$$

For $t \ge \frac{\sqrt{3}mv}{qE}$,

$$x(t) = \frac{\sqrt{3}mv^2}{qE} + R\sin\left(\frac{qB}{m}t\right),$$

$$y(t) = \frac{3v^2m}{2qE} - \frac{1}{2}\frac{qE}{m}t^2,$$

$$z(t) = -R\left[1 - \cos\left(\frac{qB}{m}t\right)\right]$$

Passage -10

1. (b) 2. (d) 3. (a)

Magnetic field at any point on Ampere's loop can be due to all currents passing through inside or outside the loop. But net contribution in the left hand side will come from inside current only.

For $r < a$, current passing through within the cylinder of radius r is given by

$$\int_0^r JdA = \int_0^r kr^2 2\pi r dr = 2\pi k\int_0^r r^3 dr$$

$$= k\pi r^4/2$$

Now using Ampere's law:

$$B \times 2\pi r = \mu_0 I = \mu_0 k\pi r^4/2$$

$$\Rightarrow \quad B = \frac{\mu_0 k r^3}{4}$$

For $r > a$, $I = k\pi a^4/2$ (by putting $r = a$)

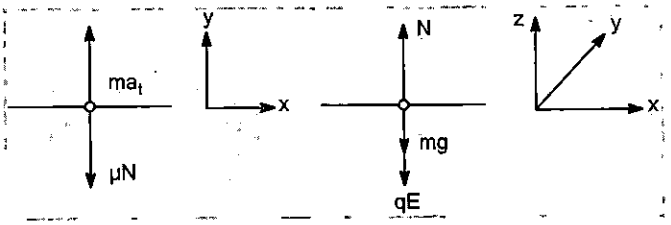
$$\therefore \quad B = \frac{\mu_0 k\pi a^4}{4r}$$

Passage -11

1. (a) 2. (c)

The particle enters into the magnetic field perpendicular to it. It will start moving into a circular path. Let the initial radius of the path is R.

Free body diagram



Hence, $N = mg + qE$ ...(1)

$$qBv = \frac{mv^2}{R} \quad ...(2)$$

$$-m\frac{dv}{dt} = \mu N \quad ...(3)$$

From eqn. (2), $R = \frac{mv}{qB}$ ...(4)

From (1) and (2),

$$-m\frac{dv}{dt} = \mu(mg + qE) \quad ...(5)$$

$$-m\int_{v_0}^{0}\frac{dv}{dt} = \mu(mg + qE)\int_0^t dt$$

which gives $t = \frac{mv_0}{\mu(mg + qE)}$.

From eqn. (4),

$$dR = \frac{m}{qB}dv = -\frac{\mu(mg + qE)\,dt}{qB}$$

$$\int_R^0 dR = -\frac{\mu(mg + qE)}{qB}t$$

$$-R = \frac{\mu(mg + qE)}{qB}t;$$

$$t = \frac{\mu(mg + qE)}{qB}$$

From eqn. (5),

$$-mv\frac{dv}{dl} = \mu(mg + qE)$$

$$-m\int_{v_0}^{0} v\,dv = \mu(mg + qE)\int_0^l dl$$

$$\frac{mv_0^2}{2} = \mu(mg + qE)l$$

$$l = \frac{mv_0^2}{2\mu(mg + qE)}$$

Passage -12

1. (d) By examining a small piece of the wire (figure), we find

$$F_B = ILB = 2T\sin(\theta/2)$$

$$ILB \approx \frac{2T\theta}{2} = \frac{2TL/R}{2}$$

$$\Rightarrow \quad \frac{T}{IB} = R$$

2. (a) For a particle:

$$qvB = \frac{mv^2}{R}$$

$$\Rightarrow \quad B = \frac{mv}{Rq} = \frac{mvIB}{Tq}$$

$$\Rightarrow \quad v = \frac{Tq}{mI}$$

Matching Type Problems

2. (a) → q, r ; (b) → q, s; (c) → p; (d) → r.

(a) $M = NIA = 100 \times 5 \times \pi(0.1)^2 = 5\pi$Am

As the current in xy plane is along anticlockwise direction, so moment will be along z-axis by right hand thumb rule.

$$\vec{\mathbf{M}} = 5\pi\hat{\mathbf{k}}$$

(b) $\vec{\tau} = \vec{\mathbf{M}} \times \vec{\mathbf{B}} = 5\pi\hat{\mathbf{k}} \times (-\hat{\mathbf{i}} + \hat{\mathbf{k}}) = -5\pi\hat{\mathbf{j}}$

(c) Net force on a closed current carrying loop in a uniform magnetic field is zero.

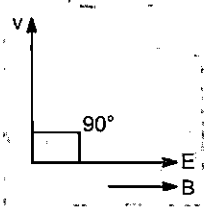
(d) Magnetic field at center of loop due to current in loop will be along +ve z-axis from right hand thumb rule.

3. (a) → p, r; (b) → p, q; (c) → s; (d) → p.

(a) Since $\vec{\mathbf{E}} = 0$ and $\vec{\mathbf{B}} \neq 0$, so path will be straight line if velocity is parallel to $\vec{\mathbf{B}}$. Or path will be circular if $\vec{\mathbf{v}} \perp \vec{\mathbf{B}}$. Or path will be helical (with uniform pitch) if $\vec{\mathbf{v}}$ is at some other angle to $\vec{\mathbf{B}}$.

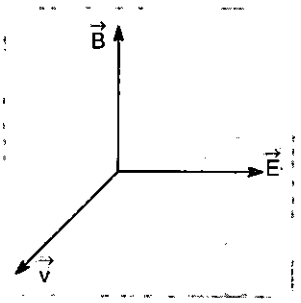
(b) Since $\vec{\mathbf{E}} \neq 0$ and $\vec{\mathbf{B}} \neq 0$, so path will be straight line if $\vec{\mathbf{v}} \perp \vec{\mathbf{B}}$ or parabola otherwise.

(c) $\vec{\mathbf{E}} \neq 0$, $\vec{B} \neq 0$, $\vec{\mathbf{E}} \| \vec{\mathbf{B}}$



Helical path with non-uniform pitch

(d) Straight line path if $\vec{\mathbf{v}} \times \vec{\mathbf{B}} = \vec{\mathbf{E}}$



5. (a) → s; (b) → p; (c) → q; (d) → r.

(a) We know that $r = mv/qB$. So, larger the mass, larger is the radius. Radii of paths '2' and '4' are same. If path '2' corresponds to C, then path '4' should correspond to ion A, because masses of A and C are same.

(b) Now radius of path '1' is largest, so path '1' should correspond to ion B.

(c) Now path '2' should correspond to ion C, because charge on both B and C is –ve, so their paths should be on the same side.

(d) Now, remaining path '3' should correspond to D.

6. (a) → p, q, s; (b) → p, q, r, s; (c) → p, q, r, s; (d) → q.

(a) Velocity of the particle may be constant, if forces of electric and magnetic fields balance each other. Then, path of particle will be straight line. Also, path of particle may be helical if magnetic and electric fields are in same direction. But path of particle cannot be circular. Path can be circular if only magnetic field is present, or if some other force is present which can cancel the effect of electric fields.

(b) Here, all the possibilities are possible depending upon the combinations of the three fields.

(c) The situation is similar to part (a).

(d) In a uniform electric field, path can be only straight line or parabolic.

16. (a) → p, q; (b) → q, r; (c) → q, r; (d) → s.

(a) It is contant acceleration motion so it can be either straight line or parabola.

(b) It can be zero acceleration motion if $\vec{u}$ & $\vec{B}$ are parallel or anti parallel other motion is helix with contant pitch

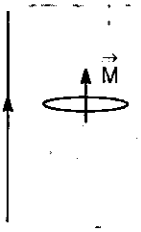
(c) Motion is helix with variable pitch

(d) Cycloid

Assertion and Reason

1. (d) Ampere's law is valid for steady current only.

3. (d) Statement-1 is false *eg.* consider the situations shown in diagram



4. (d) If v is parallel to B, charge can move along B in absence of any other force [$\because \vec{v} \times \vec{B} = 0$]

5 ELECTROMAGNETIC INDUCTION & A.C. CIRCUITS

MAGNETIC FLUX

The magnetic flux passing through a circuit of loop of area A is defined as

$$\phi_B = \vec{\mathbf{B}} \cdot \vec{\mathbf{A}} \text{ (for uniform } \vec{\mathbf{B}}\text{)}$$
$$= BA\cos\theta$$
$$= B_\perp A$$

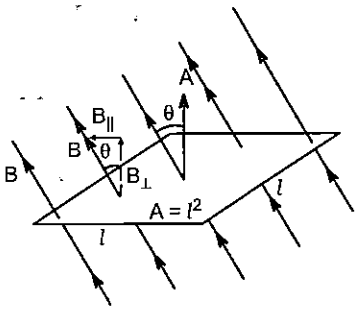


Fig. 5.1 (a)

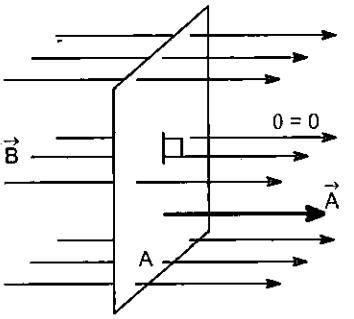
where $B_\perp$ is the component of the magnetic field B perpendicular to the face of the loop, and θ is the angle between $\vec{\mathbf{B}}$ and the vector $\vec{\mathbf{A}}$ (representing area). Direction of area vector is outward normal to the face of the loop.

☛ If the magnetic field is non-uniform or area is of arbitrary shape, the magnetic flux is

$$\phi_B = \int \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}$$

The units of magnetic flux are tesla-metre^2 which is called **weber** (Wb) in honour of Wilhelm Weber. 1 Wb $= 1\,\text{T m}^2$. Clearly B can be measured in Wb/m^2. 1 Wb/m^2 $= 1$ T. Sometimes $\vec{\mathbf{B}}$ is referred to as **flux density**.

- $\vec{B}$ and $\vec{A}$ are parallel (the angle between $\vec{B}$ and $\vec{A}$ is $\theta = 0$).
- The magnetic flux $\phi_B = \vec{B} \cdot \vec{A} = BA$



Surface is tilted by an angle θ:

- The angle between $\vec{B}$ and $\vec{A}$ is θ
- The magnetic flux $\phi_B = \vec{B} \cdot \vec{A} = BA\cos\theta$

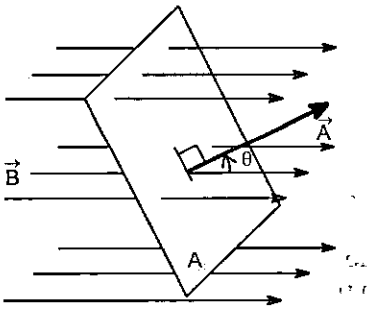


Fig. 5.1 (b)

- $\vec{B}$ and $\vec{A}$ are perpendicular (the angle between $\vec{B}$ and $\vec{A}$ is $\theta = 90°$).
- The magnetic flux $\phi_B = \vec{B} \cdot \vec{A} = BA \cos 90° = 0$

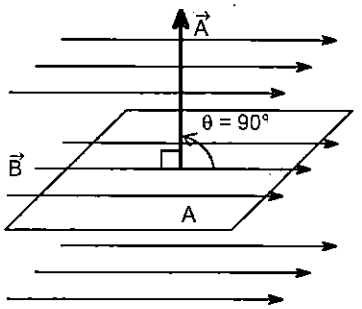


Fig. 5.1 (c)

Faraday's Experiments

Experiment 1 : A magnet and a loop of wire.

A closed coil of conducting wire has a very sensitive galvanometer in series with it. The galvanometer can detect a small current. A permanent magnet is held near the closed loop. The magnetic lines of the magnet thread the coil; there is flux of magnetic field through the coil. The following qualitative observations are made:

(1) When the magnet is moved toward the coil a current is detected in the loop. Bringing the magnet closer to the loop increases the magnetic flux through the loop. The current is detected in the wire as long as the magnetic flux through the loop is changing with time. When the magnet is moved rapidly, the time rate of change of the magnetic flux increases, and the current is increased. Now the magnet is stopped: there is no current. Even if the magnet were to be inside the ring, no current would be observed so long as the magnet remains at rest.

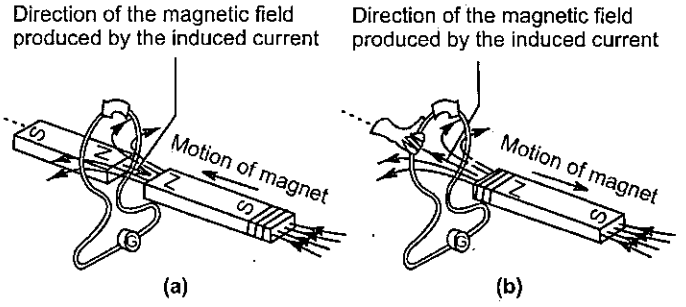


Fig. 5.2

(2) When the magnet is moved away from the loop, as shown in Fig. 5.2(b), the current observed in the wire is in the opposite direction. In this case, the magnetic flux through the loop is decreasing. Once again, the faster the magnet is moved, the greater is the change in the magnetic flux through the loop and the greater is the current observed in the wire.

Conclusions

(1) A current is observed in the loop as long as the magnitude of flux through the loop is changing with time.

(2) Greater the time rate of change of the magnetic flux, the greater the current in the loop.

(3) When magnetic flux is constant, there is no current.

(4) When the magnetic flux is increasing, the current is in one direction, and is in the opposite direction when the flux is decreasing.

Changing flux causes an emf to be induced around the loop and this induced emf causes a current to flow in the loop.

Experiment 2 : Two loops of wire, a battery and a switch.

Fig. 5.3 shows two conducting coils with a common axis. The coil of experiment 1 is placed near a coil with a battery and a switch. The following observations are made:

(1) When we close the switch in the circuit on the right, a current grows from zero to some steady state final value. When we close the switch and the current begins to flow, a magnetic field is generated by the current in the right loop, which grows with the current; therefore the magnitude of magnetic flux through the left loop increases during the interval current increases. This increasing flux induces an emf in the left loop and a current flows in this loop. The galvanometer detects a current in the left loop only during the brief interval during which magnetic flux is changing with time.

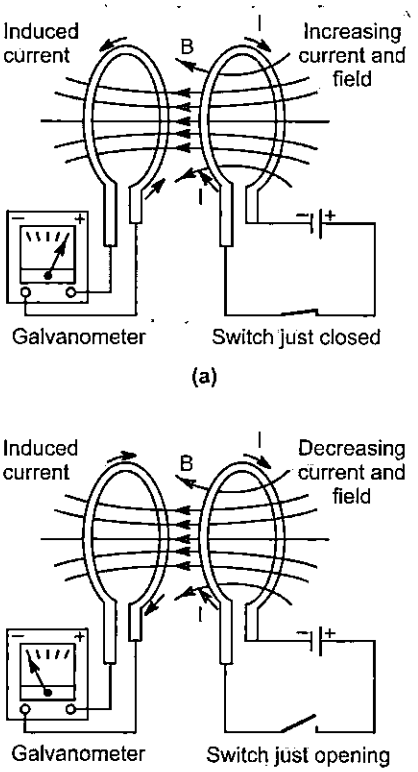


Fig. 5.3

(2) When the switch in the right circuit is opened, the galvanometer needle once again momentarily deflects, but this time deflection is in opposite direction from the deflection when we closed the switch. Just after the switch is

opened, the current in the right loop decreases rapidly. Consequently its magnetic field and the resulting flux through the left loop decreases. The galvanometer detects a current in the left loop during the interval the current in the right loop reduces to zero.

Conclusions

(1) A current is detected in the loop as long as magnetic flux through it changes with time.

(2) The current is in one direction when the magnetic flux through the loop is increasing and in the opposite direction when the flux is decreasing with time.

INDUCED EMF

- A static magnetic field produces no electrical effect.
- A changing magnetic field induces an emf in the coil.
- Induced emf results from change of magnetic flux.
- The total magnetic flux through a coil with N turns is the sum of the fluxes passing through each of its turns.

$$\phi_B = \sum_{i=1}^{N} \phi_{B,i} = N\phi_{B,\text{each loop}}$$

- The induced emf $\mathcal{E}$ is proportional to the rate at which flux, ϕ_B, changes.

In his experiments, Faraday demonstrated that $\mathcal{E}$ is always proportional to $d\phi_B/dt$: in SI, they are equal. Thus we may state Faraday's law.

The magnitude of the emf $\mathcal{E}$ induced in a circuit equals the rate at which magnetic flux through the circuit changes.

$$|\mathcal{E}| = \left|\frac{d\phi_B}{dt}\right|$$

The SI unit of magnetic flux is the weber, after Wilhelm Weber (1804-1891):

$$1 \text{ Wb} = 1 \text{ T.m}^2$$

As a corollary to Faraday's law, a coil of wire consisting of N turns in a changing magnetic field acts like N batteries in series. The induced emf of each of the coils adds, so the induced emf of the entire coil is

$$\mathcal{E}_{\text{coil}} = N\left|\frac{d\phi_{\text{per coil}}}{dt}\right|$$

(Faraday's law for an N-turn coil)

As a first example of using Faraday's law, Fig. 5.4, where a wire moves through a magnetic field by sliding on a U-shaped conducting rail. Fig. 5.4 shows the circuit. The magnetic field $\vec{B}$ is perpendicular to the plane of the conducting loop, so $\theta = 0°$ and the magnetic flux is $\phi = AB$, where A is the area of the loop. If the slide wire is distance x from the end, the area is $A = xl$ and the flux at that instant of time is

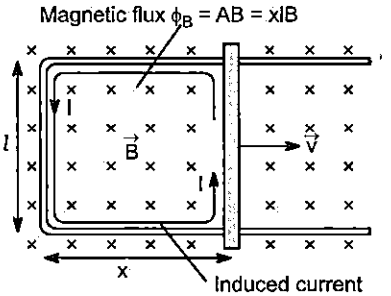


Fig. 5.4

$$\phi_B = AB = xlB$$

The flux through the loop increases as the wire moves. According to Faraday's law, the induced emf is

$$\mathcal{E} = \left|\frac{d\phi_B}{dt}\right| = \frac{d}{dt}(xlB)$$

$$= \frac{dx}{dt}lb = vlB$$

where the wire's velocity is $v = dx/dt$ and the induced current is

$$I = \frac{\mathcal{E}}{R} = \frac{vlB}{R}$$

The flux is increasing into the loop, so the induced magnetic field opposes this increase by pointing out of the loop. This requires a ccw induced current in the loop. Faraday's law leads us to the conclusion that the loop will have a ccw induced current $I = vlB/R$.

There are two fundamentally different ways to change the magnetic flux through a conducting loop:

1. The loop can move or expand or rotate, creating a motional emf.

2. The magnetic field can change.

We can see both of these if we write Faraday's law as

$$\mathcal{E} = \left|\frac{d\phi_B}{dt}\right| = \left|\vec{\mathbf{B}}\cdot\frac{d\vec{\mathbf{A}}}{dt} + \vec{\mathbf{A}}\cdot\frac{d\vec{\mathbf{B}}}{dt}\right|$$

The first term on the right side represents a motional emf.

The second term on the right side is the new physics in Faraday's law. It says that an emf can also be created simply by changing a magnetic field, even if nothing is moving.

Consider the loop shown in Fig. 5.5. A long, tightly wound solenoid of radius r_1 passes through the center of a conducting loop having a larger radius r_2. Even though the loop is completely outside the magnetic field of the solenoid, changing the current through the solenoid causes an induced current around the loop.

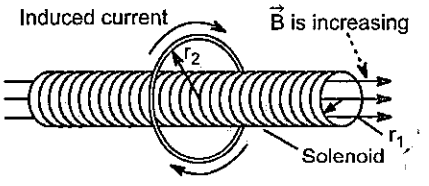


Fig. 5.5: A changing current in the solenoid induces a current in the loop.

How can the charge carriers in the conducting loop possibly move ? In the case of a motional emf, the mechanism that causes an induced current is the magnetic force on the moving charges. But here, where there's no motion, what is the mechanism that creates a current when the magnetic flux changes ?

Lenz's Law

The negative sign in the Faraday's law tells us the direction in which induced emf acts. The Lenz's law is an interpretation of this minus sign.

The Lenz's law states that :

The direction of the induced emf is such as to oppose the change in magnetic flux that causes the induced emf.

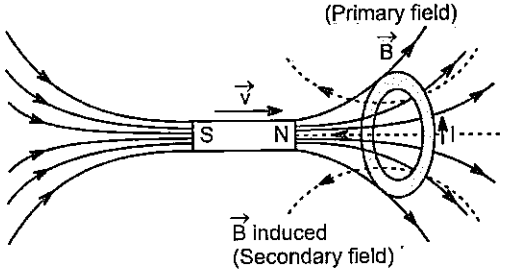
Concept: ***(1)*** *When the magnetic flux is changed at a fixed rate, the magnitude of induced emf is the same whether flux increases or decreases.*

(2) *The sense of induced emf depends on the sense of the flux change.*

The rule that gives the sense of induced emf is credited to Heinrich Lenz (1804-1865).

Lenz's Law: *Induced emf acts so as to oppose the change in flux that creates it.*

The changing magnetic flux through the coil induces an emf; if the coil is closed, the induced emf produces a current in the coil. This induced current produces a magnetic field. The externally applied magnetic field will be referred to as **primary field** and the magnetic field due to the induced current as **secondary field.**



When the bar magnet is moving toward the loop, the emf induced in the loop produces a current in the direction shown. The magnetic field due to the induced current in the loop (indicated by the dashed lines) produces a flux that opposes the increase in flux through the loop due to the motion of the magnet.

Fig. 5.6

(1) If the magnitude of the flux through a circuit is increasing, the induced magnetic field (secondary field) is in the opposite direction to the primary field.

(2) If the magnitude of the flux through a circuit is decreasing, the induced magnetic field (secondary field) is in the same direction as the primary field.

The Lenz's law is in consistency with the law of conservation of energy. Let us consider a situation in which reverse of Lenz's law is followed. The induced current will produce a flux in the same direction as the original change. This greater change in flux would produce an even larger current followed by a greater change in flux; this process will continue indefinitely. The current should grow to an unlimited value, producing power ($P = I^2R$) greater than that supplied by the source. This would violate the law of conservation of energy.

The Role of Magnetic Flux in Faraday's Law

According to Faraday's law *an emf is always induced when the magnetic flux through a conducting loop is changing, no matter how that flux change is effected.* Because the magnetic flux is the dot product of the magnetic field $\vec{B}$ and area vector $d\vec{A}$, we can list the following ways that the emf $\mathcal{E}$ can be induced in a circuit:

(1) The magnetic field B varies with time, decreases or increases with time.

(2) The area enclosed by the circuit can vary with the time.

(3) The orientation of the coil changes so that the angle between the magnetic field and area vector of the coil changes with time.

(4) Any combination of events listed in 1, 2, 3 can take place.

The magnetic flux $\phi_B = \vec{B} \cdot \vec{A}$

$= BA\cos\theta$

From Faraday's law,

$$\mathcal{E}_{\text{induced}} = -\frac{d\phi_B}{dt} = -\frac{d}{dt}(BA\cos\theta)$$

$$= -B\cos\theta\frac{dA}{dt} - A\cos\theta\frac{dB}{dt} + BA\sin\theta\frac{d\theta}{dt}$$

The first term denotes change in flux due to the area change.

The second term denotes change in flux due to magnitude of magnetic field.

The third term denotes the change in flux due to changing orientation (rotational motion).

Example 1

Use Lenz's law to determine the direction of induced current in each of the following cases :

(a) In the Faraday's first experiment permanent magnet is moved toward coil. [See Fig. 5.2 (a)].

(b) The permanent magnet is moved away from the coil. [See Fig 5.2(b)].

(c) In Fig. 5E.1(c) shown south magnetic pole is moving toward the coil, out of the plane of the paper.

(d) North magnetic pole is moving toward the coil in the plane of the page Fig. 5E.1(d).

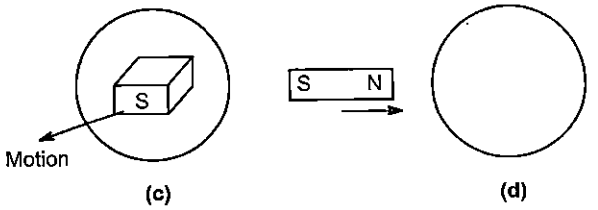
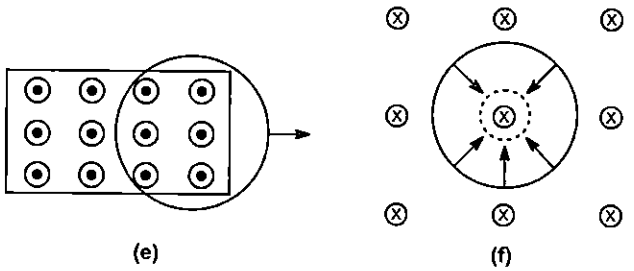


Fig. 5E.1

(e) A conducting coil is pulled to the right out of a magnetic field that points out of the page Fig. 5E.1(e).

(f) A flexible conducting coil is shrunk in a magnetic field pointing into the page Fig. 5E.1(f).



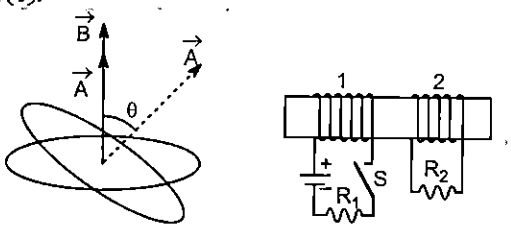
(e) (f)

Fig. 5E.1

(g) In the Faraday's second experiment, the switch is closed in the right coil. [See Fig. 5.3(a)].

(h) In the Faraday's second experiment, the switch is opened in the right coil. [See Fig. 5.3(b)].

(i) A coil is rotated about a horizontal diameter as shown in Fig. 5E.1(i).

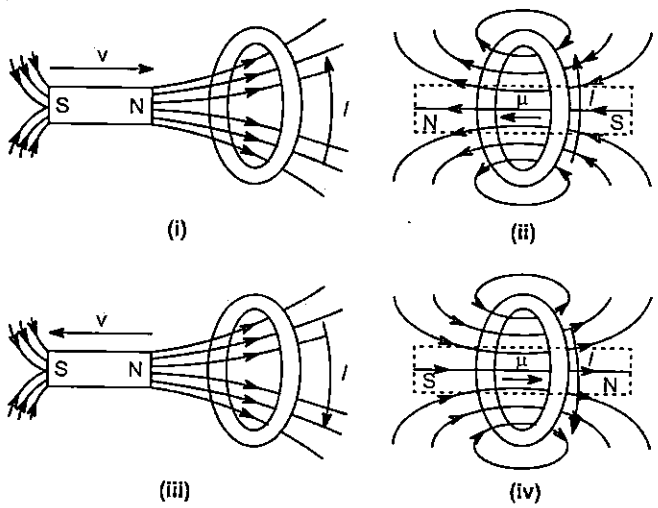


(i) (j)

Fig. 5E.1

(j) In the Fig. 5E.1(j) shown case (1) switch S is closed, and case (2) switch S is opened.

Solution : (a) When the magnet is moved toward the coil the magnetic flux through it increases because the primary magnetic field is increasing through the loop. The direction of the induced current will be such that the magnetic field created by it (secondary magnetic field) is opposite to primary magnetic field. The primary magnetic field through the coil is from right to left; the secondary magnetic field will be from left to right. Point your thumb in the direction of secondary magnetic field; the fingers will curl in the sense of current as shown in Fig. 5E.1(i), (ii). The magnetic moment of the loop $\vec{\mu}$ (shown in outlines as if it were a magnet) due to the induced current is such as to oppose the motion of the bar magnet. Here the bar magnet is moving toward the loop, so the induced magnetic moment repels the bar magnet.



(i) (ii)

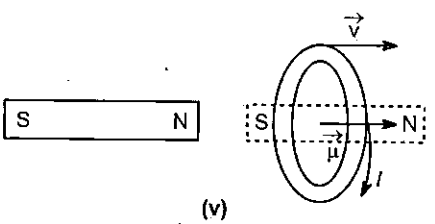
(iii) (iv)

Magnetic field lines

Fig. 5E.1

(b) When the motion of the magnet is away from the coil , the magnetic flux through it is decreasing because the primary magnetic field is decreasing. The direction of the induced current will be such that the secondary magnetic field created by it will try to oppose decrease of magnetic flux. That is, the field of the induced current will be in the same direction as the primary field of permanent magnet. Once again use right hand thumb rule to determine sense of current, as shown in Fig. 5E.1(iii), (iv). When the loop is moving away from the stationary bar magnet, the induced magnetic moment in the loop attracts the bar magnet, again opposing the relative motion.

(c) The magnetic field lines of magnet point toward south pole, inward normal to the plane of the page. As the magnet approaches near the coil the primary magnetic field is getting stronger. The magnetic flux through the coil is increasing. The secondary magnetic field due to induced current will oppose the change of flux, that is it will be outward. Point your thumb in the direction of magnetic field; your fingers will curl in the sense of current. It is anticlockwise.



(v)

Fig. 5E.1

(d) The magnetic field of the permanent magnet is in the plane of the coil, so the flux through the loop is zero throughout the process. Hence there is no change in magnetic flux with time and there will be no induced emf or current in the loop.

(e) When the coil is withdrawn from the magnetic field the magnetic flux decreases, since the area through which the magnetic lines pass has decreased. The secondary magnetic field due to induced current will be in same direction as primary magnetic field, that is outward normal to the plane of the page; the current will be anticlockwise in accordance with right hand thumb rule.

(f) When the coil is shrunk, the area inside the magnetic field decreases. There is decrease in magnetic flux, therefore secondary magnetic field will help primary magnetic field; the induced current will be clockwise.

(g) In the brief interval during which the current in right coil increases, the magnetic field created by this current also increases with time, so that the flux through the left coil increases. The secondary magnetic field will oppose the primary magnetic field; it will be left to right. Point your thumb in the direction of magnetic field; the fingers will be curl in the sense of current.

(h) The current decreases to zero in a brief interval, so does the magnetic field of this current. The magnetic flux of primary field decreases, hence the secondary magnetic field is parallel to the primary magnetic field. The current is in accordance with the right hand thumb rule.

(i) The initial flux through the coil is

$$\phi = \vec{\mathbf{B}} \cdot \vec{\mathbf{A}} = BA\cos\theta = BA$$

As the coil is turned the angle between area vector and primary magnetic field increases, so flux decreases (decreasing $\cos\theta$). The secondary magnetic field is in the direction of primary field. Use right hand thumb rule for direction of current (see figure).

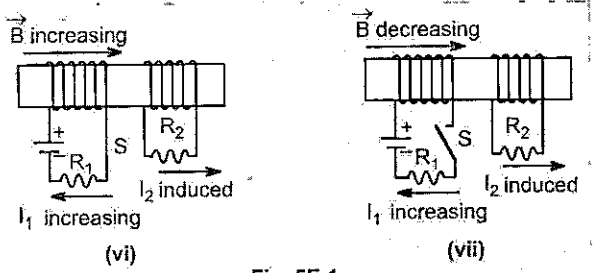


Fig. 5E.1

(j) (1) Just after the switch is closed, I_1 is increasing in the direction shown. The changing flux in circuit 2 induces the current I_2. The flux due to I_2 opposes the increase in flux due to I_1. (2) As the switch is opened, I_1 decreases and B decreases. The induced current I_2 then tends to maintain the flux in the circuit, opposing the change.

Example 2

A small loop of resistance R and radius a lies on the axis of a solenoid with n turns per meter (Fig. 5E.2). The current in the solenoid is increased uniformly such that $\frac{dI}{dT} = B$. *What is the current through an ammeter connected to the small loop ?*

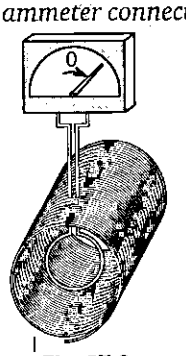


Fig. 5E.2

Solution: As the current in the solenoid increases, so does the magnetic field it produces. Consequently, magnetic flux through the loop increases, inducing an emf in the loop and producing current in it. We assume the magnetic field inside the solenoid is uniform at any one time.

At any instant, the magnetic field inside the solenoid has magnitude $|\vec{\mathbf{B}}| = \mu_0 nI$. Since $\vec{\mathbf{B}}$ is perpendicular to the loop, the flux through the loop at any time is:

$$\phi_B = \int \vec{\mathbf{B}} \cdot \hat{\mathbf{n}} dA = \pi a^2 |\vec{\mathbf{B}}| = \pi a^2 \mu_0 nI$$

From Faraday's law, the emf induced in the loop equals the rate of change of this flux. The current I is the only time-dependent variable in ϕ_B:

$$|\mathcal{E}| = \left|\frac{d\phi_B}{dt}\right| = \pi a^2 \mu_0 n \left|\frac{dI}{dt}\right|.$$

Finally, the current in the loop is found from the definition of resistance:

$$I_l = \frac{|\mathcal{E}|}{R} = \frac{\pi a^2 \mu_0 n}{R}\left|\frac{dI}{dt}\right|$$

The current in the loop exists only while the solenoid current is changing. It is all right to use the result of static field calculations to find $\vec{\mathbf{B}}$ when the current and field are varying as slowly as in this example. In rapidly varying systems, however, the fields propagate through the system as waves at the speed of light.

Example 3

The magnetic field inside a solenoid is increasing at a constant rate $|d\vec{\mathbf{B}}/dt| = \alpha$. *Find the induced electric field inside the solenoid.*

Solution: The changing magnetic field has cylindrical symmetry, and the electric field lines circulate around this source like the magnetic field lines around a cylindrical current distribution. The electric field lines from circles around the axis of the solenoid. To find the magnitude of the field at any point, we may apply Faraday's law to a circular path passing through that point (Fig. 5E.3). The direction of the field lines in the figure follows from Lenz's law.

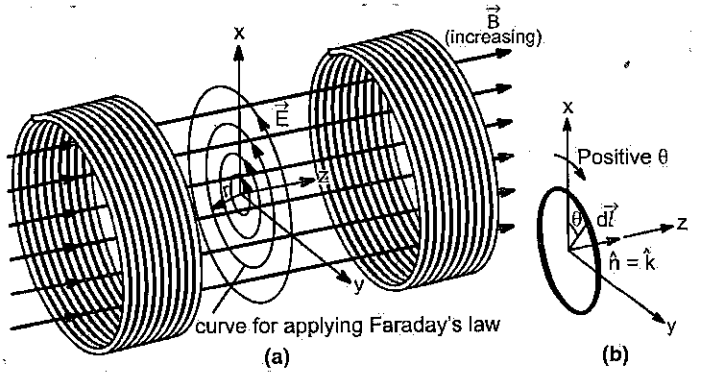


Fig. 5E.3: Finding the induced electric field inside a solenoid. We choose the z-axis to be along the axis of the solenoid, parallel to $\vec{\mathbf{B}}$, and apply Faraday's law to a circle of radius r centered on the axis and parallel to the xy-plane. (b) We choose to go around the circle in the same direction that the angle θ increases, as shown so that according to the right-hand rule, $\hat{n} = \hat{k}$.

We choose the z-axis to be along the axis of the solenoid, with the circle in the xy-plane. Using the right-hand rule and choosing $\hat{\mathbf{n}} = \hat{\mathbf{k}}$, we go around the circle as shown in the figure. Since B_z is spatially uniform, the flux passing through the area of the circle is $\phi_B = \pi r^2 B_z$. Then:

$$\frac{d\phi_B}{dt} = \pi r^2 \frac{dB_z}{dt} = \pi r^2 \alpha$$

Here the curve is stationary, so there is no motional contribution to the emf. The electric field has only a θ -component, and its magnitude is the same at each point on the chosen circle. As a result, the integral reduces to a multiplication:

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}} = \oint E_\theta dl = E_\theta \oint dl = 2\pi r E_\theta$$

Applying Faraday's law, we find

$$2\pi r E_\theta = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}} = \mathcal{E}$$

$$= -\frac{d\phi_B}{dt} = -\pi r^2 \alpha$$

or $$E_\theta = -\alpha r/2$$

The electric field exists whether or not there is a material circuit within the solenoid. If a conducting loop were placed there, the induced field would cause a current. The negative value of E_θ confirms the direction of $\vec{\mathbf{E}}$ shown in the figure.

Example 4

A circular coil of wire consists of exactly 200 turns with a total resistance 0.20Ω. The area of the coil is 100 cm^2. The coil is kept in a uniform magnetic field B. The magnetic field is increased at a constant rate of 1.5T/s. Find the current induced in the coil, state the sense of induced current.

Solution: The magnetic flux through single turn of the coil

$$\phi_B = \vec{\mathbf{B}} \cdot \vec{\mathbf{A}} = BA\cos\theta$$

The total flux through N turns of the coil,

$$\phi_{\text{total}} = NBA\cos\theta$$

According to Faraday's law of electromagnetic induction,

$$\mathcal{E}_{\text{induced}} = -\frac{d\phi}{dt} = -\frac{d}{dt}(NBA\cos\theta)$$

$$= -(NA\cos\theta)\frac{dB}{dt}$$

Note that only the magnitude of magnetic field is changing.

$$\mathcal{E}_{\text{induced}} = -(200)(100)(\cos 30°)(1.5)$$

$$= -2.59\text{V}$$

The current induced in the coil,

$$I_{\text{induced}} = \frac{\mathcal{E}_{\text{induced}}}{R}$$

$$= \frac{2.59}{0.20} = 12.95\text{A}$$

Since the magnetic flux through the coil is increasing with time, the magnetic field due to induced current will be opposite to the primary field. Apply right hand thumb rule, point your thumb in the direction of induced field, your fingers will curl in the directional sense of induced current.

Example 5

A metal rod slides to the right at constant velocity v while in contact with a stationary U-shaped conductor. The left side of the conductor has resistance R and there is negligible resistance elsewhere, including the rod, which has length l. A uniform magnetic field B is directed downward perpendicular to the plane of the loop. Calculate the current induced in the loop.

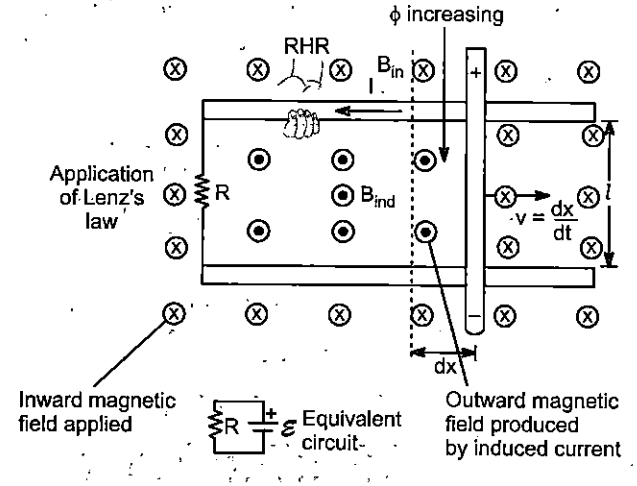


Fig. 5E.5

Solution: Here the flux increases because the area of the circuit increases with time. Let l be the length of the area enclosed by rod and U-shaped conductor at some instant. Then the flux through the circuit is

$$\phi_B = \int \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = \int B(\cos 0°)\, dA = B\int dA = Blx$$

Recognize that the magnetic field is constant in magnitude and parallel area vector. From Faraday's law,

$$\mathcal{E}_{\text{induced}} = -\frac{d\phi}{dt} = -\frac{d}{dt}(Blx)$$

$$= -Bl\frac{dx}{dt} = -Blv$$

Thus, $$I = \frac{Blv}{R}$$

MOTIONAL EMF

Consider a conductor of length l that moves with velocity $\vec{\mathbf{v}}$ through a uniform magnetic field $\vec{\mathbf{B}}$, as shown in Fig. 5.7.

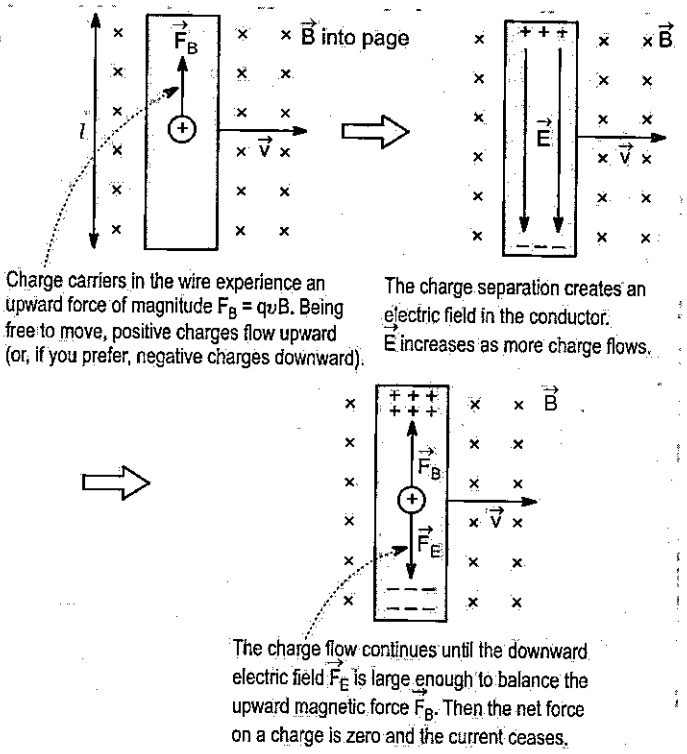


Fig. 5.7: The magnetic force on the charge carries in a moving conductor creates an electric field inside the conductor.

The charge carriers inside the wire also move with velocity $\vec{\mathbf{v}}$, so they each experience a magnetic force $\vec{\mathbf{F}}_B = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$. For simplicity, we will assume that $\vec{\mathbf{v}}$ is perpendicular to $\vec{\mathbf{B}}$, in which case the strength of the force is $F_B = qvB$. This force causes the charge carriers to move, separating the positive and negative charges. The separated charges then create an electric field inside the conductor.

The magnitude of electric field is calculated in following example.

Example 6

A conducting wire of length l moves with a velocity v in a direction transverse to its length. A uniform magnetic field B exists over the entire region through which the wire moves. The direction of B is perpendicular both to the length of the wire and to v.

(a) What is the induced electric field in the wire?

(b) What is the potential difference between the ends of the conductor? Which end is at higher potential?

Solution: (a) When a conductor moves with a speed v in the plane perpendicular to the magnetic field, an emf is induced in it; but in equilibrium there is no current through it. The free electrons in the conductor experience a magnetic force given by

$$\vec{F}_B = -e\vec{\mathbf{v}} \times \vec{\mathbf{B}}$$

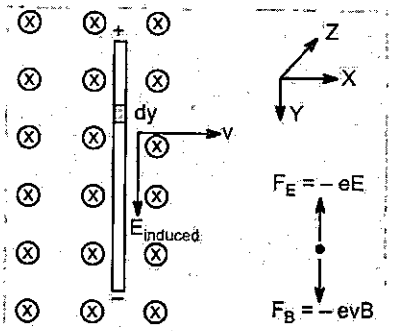


Fig. 5E.6

The free electrons move to the lower end of the bar. This charge displacement leaves the upper end positive. The positive charge at the upper end and a negative charge at the lower end create an electric field directed from the upper end to the lower end. The electric field increases until the resulting electric force $F_E = qE$ just balances the magnetic force. The Lorentz force is then zero, and we have

$$F = F_E + F_B$$

$$= -e[\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}}] = 0$$

$$E = -\vec{\mathbf{v}} \times \vec{\mathbf{B}},$$

This electric field is uniform throughout the wire, because v and B are uniform.

(b) The direction of the electric field is the positive y-direction. The potential difference is

$$V = -\int_0^l \vec{\mathbf{E}} \times d\vec{\mathbf{y}} = -Eyl$$

Since $v \perp B$, $\quad |\vec{\mathbf{v}} \times \vec{\mathbf{B}}| = vB$

and the direction of $\vec{\mathbf{v}} \times \vec{\mathbf{B}}$ is parallel to $d\vec{\mathbf{y}}$.

We have $\quad V = Blv$

Concept:

THE NATURE OF EMF

1. Potential difference and EMF: *The emf of an ideal battery is the energy it provides to the circuit per unit of charge that passes through it, and also equals the potential difference the battery maintains between its terminals.*

Fig. 5.8 shows three circuit, each with the same emf, resistance and current. Their differences are in the nature of the emf and in the role of static fields.

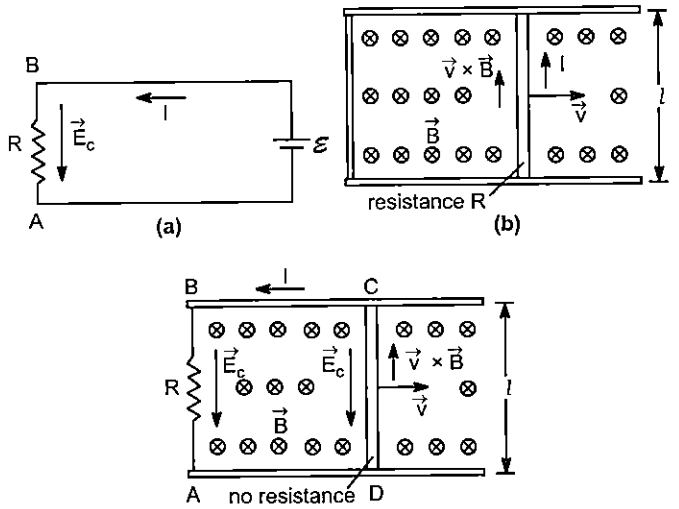


Fig. 5.8

Circuit (a) is a DC circuit. The battery provides energy that is used at the resistor. Potential rises across the battery and decreases again across the resistor.

Circuit (b) is the rod-on-rails system. Here the energy source is the external agent pulling the rod. The motional emf arising from magnetic force on electrons in the rod makes the energy available to the circuit. The energy is dissipated as heat by resistance in the rod. In the rest of the circuit, no energy is needed and no fields are required to drive current. Consequently, there is no electric field, or potential difference.

In circuit (c) the energy source and emf are the same as in circuit (b), but here the rod has negligible resistance and the rails are connected by a stationary resistor. The energy is used in the resistor.

Once the electric and magnetic forces on the electrons in the rod balance ($\vec{\mathbf{E}}_c = -\vec{\mathbf{v}} \times \vec{\mathbf{B}}$), *the charge buildup ceases. The coulomb electric force drives current through the resistor, and electrons flow at constant speed through the perfectly conducting rod, where the force on them is now zero. The potential rises by* $V_{BA} = vBl$ *across the rod.*

2. *The potential difference between any two points is given by the integral of the Coulomb electric field along any path between the two points:*

$$\Delta V \equiv V_B - V_A = -\int_A^B \vec{\mathbf{E}}_c \cdot d\vec{\mathbf{l}}$$

If the integral is taken around a closed path starting and ending at A, the potential difference is zero:

$$V_A - V_A = \oint \vec{\mathbf{E}}_c \cdot d\vec{\mathbf{l}} = 0.$$

This is Kirchhoff's rule–the change of potential around any closed path is zero.

Faraday's law tells us that the emf around a closed path is not zero. The total electromagnetic force acting on a charge is $\vec{\mathbf{F}} = q(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}})$.

So:

The induced emf acting around a closed path is the work done per unit charge as the charge is moved around the path:

$$\mathcal{E} \equiv \frac{W}{q} \equiv \oint (\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}}) \cdot d\vec{\mathbf{l}} \qquad ...(1)$$

Although the total electric field appears in eqn. (1), only the induced electric field contributes to the emf, since the integral of the Coulomb field around a closed curve is zero. On the other hand, potential is defined by the Coulomb field alone; the induced field does not contribute.

3. EMF and choice of reference frame: *If we push a magnet toward a wire loop at speed v [Fig. 5.9 (a)], the changing magnetic field induces electric field at the loop, and current flows in the loop. If instead, we push the loop toward the magnet at speed v [Fig. 5.9 (b)], magnetic forces produce a motional emf that drives current in the loop.*

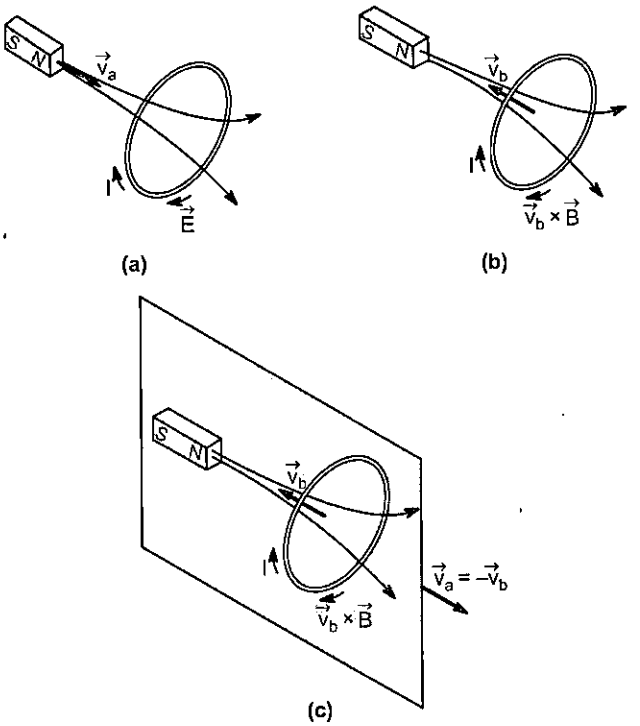


Fig. 5.9

(a) In the lab frame, as we push a magnet toward a wire loop, the increasing flux through the loop creates an induced electric field that drives current.

(b) If instead we push the loop toward the magnet, magnetic force $-e\vec{\mathbf{v}} \times \vec{\mathbf{B}}$ *drives the current in the loop.*

(c) Experiment (a) viewed in a reference frame moving with the magnet is indistinguishable from experiment (b). Thus the electric field $\vec{\mathbf{E}}$ *in experiment (a) equals* $\vec{\mathbf{v}} \times \vec{\mathbf{B}}$ *in experiment (b).*

4. Magnetic force and EMF: *Since magnetic force is always perpendicular to the velocity of the charge it acts on magnetic fields cannot themselves do work on charges.*

Yet we have used $dW = q(\vec{v} \times \vec{B}) \cdot d\vec{l}$

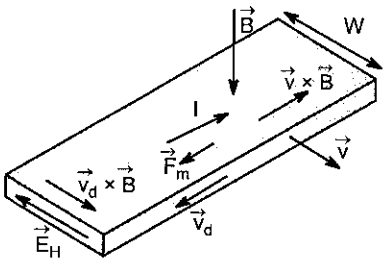


Fig. 5.10

In fact, it is the Hall electric field $\vec{E}_H = -\vec{v}_d \times \vec{B}$ that does the work. The power delivered to a single electron is:

$$P_e = -e\vec{E}_H \cdot \vec{v}$$

$$= eE_H v = evBv_d$$

The power delivered to all the electrons in a length dl of the rod is:

$$dP = ndVP_e = nAdlevBv_d$$

$$= neAv_d Bvdl = I\,d\mathcal{E},$$

with $d\mathcal{E} = Bvdl.$

5. Concept of work done by motional EMF: Motional emf arise when we move a wire through a magnetic field. Fig. 5.11 shows there is a uniform magnetic field $\vec{B}$, pointing into the page, and the resistor R. If the entire loop is pulled to the right with speed v, the charges in segment ab experience a magnetic force whose vertical component qvB drives current around the loop, in the clockwise direction. The emf is

$$\mathcal{E} = \oint \vec{F}_{mag} \cdot d\vec{l} = vBh$$

where h is the width of the loop. Displacement vector $d\vec{l}$, for the segment ab in Fig. 5.11, points s traight up, even though the loop is moving to the right.

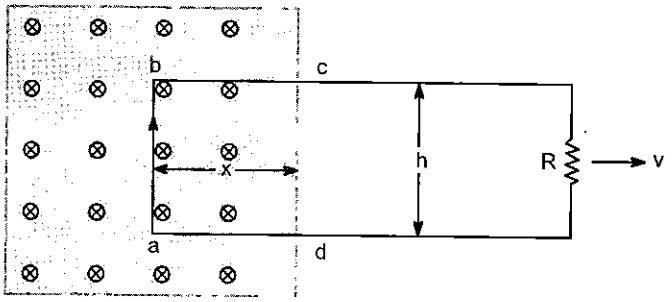


Fig. 5.11

The magnetic force is responsible for establishing the emf but it is certainly not doing any work—magnetic force never do work.

With the current flowing, charges in segment ab have a vertical velocity (call it $\vec{u}$) in addition to the horizontal velocity $\vec{v}$ they acquire from the motion of the loop. Accordingly, the magnetic force has a component quB to the left. To counteract this, the external agent pulling on the wire must exert a force per unit charge

$$F_{pull} = uB$$

to the right (Fig. 5.12). This force is transmitted to the charge by the structure of the wire. Meanwhile, the particle is actually moving in the direction of the resultant velocity $\vec{v}$, and the distance it goes is (h/cos θ). The work done per unit charge is therefore

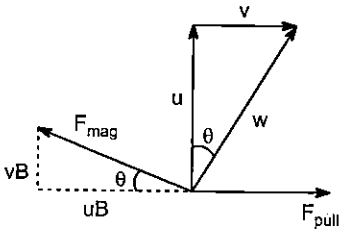


Fig. 5.12

$$\int \vec{F}_{pull} \cdot d\vec{l} = (uB)\left(\frac{h}{\cos\theta}\right)\sin\theta$$

$$= vBh = \mathcal{E}$$

(sin θ coming from the dot product). As it turns out, then, the work done per unit charge is exactly equal to the emf. Thus work is supplied by external agent.

Example 7

A conducting rod of length l is rotated at angular velocity $\vec{\omega}$ about an end in a uniform magnetic field $\vec{B}$. The magnetic field is normally inward to the plane of the page and parallel to $\vec{\omega}$ as shown in Fig. 5E.7. Find the induced emf and indicate which end of the rod is at higher electric potential.

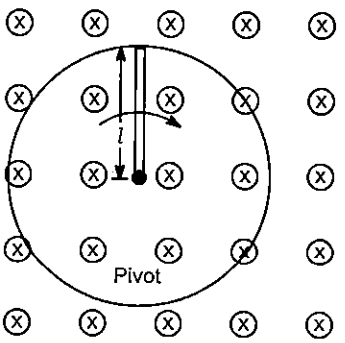


Fig. 5E.7

Solution: Method 1. The free electrons in the conductor experience a magnetic force

$$\vec{\mathbf{F}}_B = -e\vec{\mathbf{v}} \times \vec{\mathbf{B}}$$

This force is directed toward the pivot. Since $\vec{\mathbf{v}}$ is perpendicular to $\vec{\mathbf{B}}$ the magnitude of the force is

$$F_B = evB$$

Due to this magnetic force displacement of electron takes place, with the tip of the rod becoming positive and the pivoted end negative. The separation of charges results in an electric field, directed from the tip to the pivoted end. The charge separation does not continue indefinitely; it stops when magnetic forces on electron and electric force are balanced.

$$eE = evB \quad \Rightarrow \quad E = vB$$

At a point distant r from the pivot, the velocity is $v = r\omega$ We take a unit vector $\hat{\mathbf{r}}$ radially outward from the pivot; the electric field is

$$\vec{\mathbf{E}} = -\omega B r\,\hat{\mathbf{r}}$$

The potential difference between the ends of the rod is

$$V_f - V_i = -\int_i^f \vec{\mathbf{E}} \cdot d\vec{\mathbf{r}}$$

$$V_{tip} - V_{pivot} = -\int_0^l (-\omega B r\,\hat{\mathbf{r}}) \cdot (dr\,\hat{\mathbf{r}})$$

$$= \omega B \int_0^l r\,dr = \frac{1}{2}\omega B l^2$$

This is the potential difference between two ends.

Method 2. When the rod rotates through an angle θ, the area swept by the rod is $\frac{1}{2}l^2\theta$. The magnetic flux through the swept area is

$$\phi = \int \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \frac{1}{2}Bl^2\theta$$

The magnitude of induced emf is

$$|\mathcal{E}_{induced}| = \left|\frac{d\phi}{dt}\right| = \frac{Bl^2}{2}\frac{d\theta}{dt} = \frac{\omega B l^2}{2}$$

Example 8

A conducting rod of length l, mass m and resistance R slides on frictionless U-shaped metal rails that are inclined at an angle θ above the horizontal. The rails have negligible resistance. The rod is released from rest.

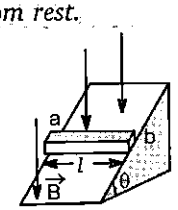


Fig. 5E.8 (a)

(a) Is the current in the rod from a to b or from b to a?
(b) What is the terminal speed of the bar?
(c) What is the induced current in the bar when the terminal speed has been reached?
(d) After the terminal speed has been reached, at what rate is electric energy being converted to thermal energy in the resistance of the rod?
(e) After the terminal speed has been reached, at what rate is work being done on the rod by gravity? Compare your answer to that in part (d).

Solution: (a) When the rod slides downward, the area of the circuit in the magnetic field decreases; therefore flux decreases. The induced current must flow in such a direction that its magnetic field is in the same direction as the primary magnetic field. In the top view the current is in clockwise direction, from a to b.

(b) Let the position of the rod be at x from the base.

Flux linked with circuit,

$$\phi_B = Blx\cos\theta$$

According to Faraday's law of electromagnetic induction.

$$\mathcal{E}_{induced} = -\frac{d\phi}{dt} = -Bl\cos\theta\frac{dx}{dt}$$

$$|\mathcal{E}_{induced}| = Blv\cos\theta$$

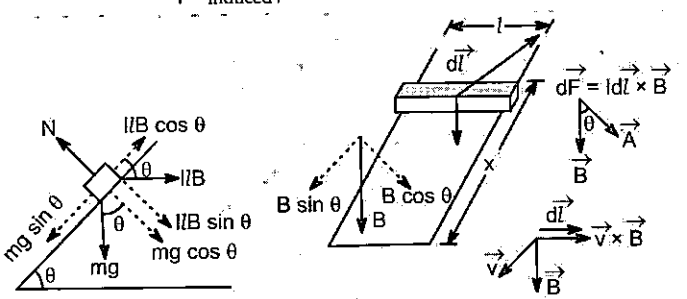


Fig. 5E.8 (b)

Note that $\frac{dx}{dt}$ is negative as x is decreasing; we can also use

$$\mathcal{E}_{induced} = (\vec{\mathbf{v}} \times \vec{\mathbf{B}}) \cdot d\vec{\mathbf{l}}$$

The current induced in the circuit is therefore

$$I = \frac{\mathcal{E}_{induced}}{R} = \frac{Blv\cos\theta}{R}$$

The magnetic force on the current carrying conductor is given by

$$F_B = Id\vec{\mathbf{l}} \times \vec{\mathbf{B}}$$

Magnetic force is normal to the plane of $d\vec{\mathbf{l}}$ and $\vec{\mathbf{B}}$ vectors. From right hand thumb rule for magnetic force (or Fleming's left hand rule), we see that magnetic force is horizontal as shown in the figure.

The conductor will slide with terminal velocity when component of weight along the incline is balanced by component of magnetic force along the incline.

$$mg\sin\theta = IlB\cos\theta = \left(\frac{Blv\cos\theta}{R}\right)lB\cos\theta$$

$$v_T = \frac{mgR\sin\theta}{B^2l^2\cos^2\theta}$$

(c) The current when terminal velocity is achieved,

$$I = \frac{Blv_T\cos\theta}{R} = \frac{mg}{Bl}\tan\theta$$

(d) The rate of conversion of electrical energy to thermal energy.

$$P = I^2R = \frac{B^2l^2v^2\cos^2\theta}{R}$$

(e) Rate of work done by gravity

$$= (mg\sin\theta)v = (IlB\cos\theta)v = \frac{B^2l^2v^2\cos^2\theta}{R}$$

We see that the rate at which gravitational potential is lost, is equal to thermal energy dissipated in resistor.

Example 9

Fig. 5E.9 shows a thin circular horizontal laminated aluminium ring, with square cross-section of area A, containing horizontal laminae. It is released from rest and falls vertically under gravity through a horizontal radial magnetic field B produced by north, N and south, S magnetic poles. It has mass m, radius r and resistivity ρ. The centre of the ring falls down the vertical line of symmetry of the system. Obtain an expression for terminal velocity of the ring.

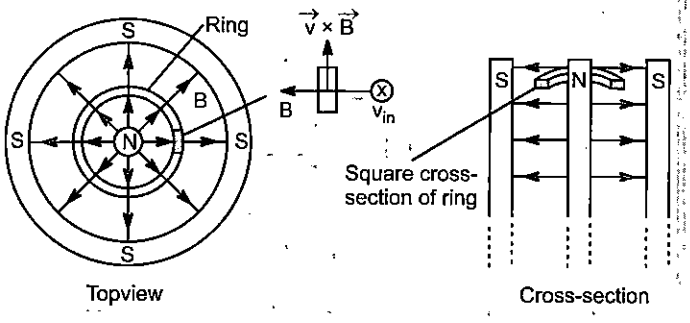


Fig. 5E.9

Solution: The emf induced in a differential element on the ring is

$$d\mathcal{E}_{\text{induced}} = (\vec{\mathbf{v}}\times\vec{\mathbf{B}})\cdot d\vec{\mathbf{l}}$$

Note that vector $\vec{\mathbf{v}}\times\vec{\mathbf{B}}$ is parallel to vector $d\vec{\mathbf{l}}$ and vectors v and B are perpendicular to each other. Hence

$$d\mathcal{E}_{\text{induced}} = vB\oint dl = 2\pi rBv$$

If A is the cross-sectional area of the ring and ρ the resistivity, the resistance of the ring is

$$R = \frac{\rho(2\pi r)}{A}$$

The current in the ring is $I = \frac{\mathcal{E}}{R} = \frac{2\pi rBvA}{\rho 2\pi r} = \frac{BvA}{\rho}$

The induced electric field in the ring is anticlockwise, so is the current in the ring. From Fleming's left hand rule, the magnetic force is upward.

$$F_B = IlB = B\left(\frac{ABv}{\rho}\right)2\pi r = \frac{2\pi AB^2vr}{\rho}$$

According to Newton's second law, the equation of motion of the ring is

$$mg - \frac{m\pi rAB^2}{\rho}v = ma$$

The terminal velocity is reached when the magnetic force balances the weight of the ring.

Terminal velocity is thus given by

$$\frac{2\pi AB^2rv_T}{\rho} = mg \quad \text{or} \quad v_T = \frac{mg\rho}{2\pi AB^2r}$$

Example 10

In Fig. 5E.10 below a wire is formed into an open semicircle of radius b. End O of the semicircle is attached to an axis about which the semicircle rotates with an angular speed ω The axis is normal to the plane of the semicircle. A uniform magnetic field exists parallel to the axis, extends over the entire region. Find the emf induced in the wire between points O and P.

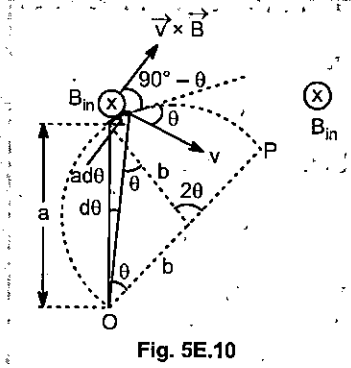


Fig. 5E.10

Solution: We consider a differential element dr on the semicircle, which subtends an angle $d\theta$ at O, as shown in the figure. From cosine law,

$$a^2 = b^2 + b^2 - 2b^2\cos(\pi - 2\theta)$$

$$= 2b^2[1 + \cos 2\theta] = \cdot\ ^2\cos^2\theta$$

$$a = 2b\cos\theta \qquad \ldots(1)$$

Magnitude of velocity of this element $v = \omega r = 2a\omega\cos\theta$

From Faraday's law of electromagnetic induction,

$$\mathcal{E}_{\text{induced}} = \int (\vec{v}\times\vec{B})\cdot d\vec{l}$$

Vector $\vec{v}\times\vec{B}$ is perpendicular to v and B, direction is determined from screw rule or right hand thumb rule.

$$|\vec{v}\times\vec{B}| = vB\sin 90° = (2b\omega\cos\theta)B$$

$d\vec{r}$ vector and $\vec{v}\times\vec{B}$ vector are at an angle $(90°-\theta)$

$$(\vec{v}\times\vec{B})\cdot d\vec{r} = (vB)\,dr\cos(90°-\theta)$$
$$= vBdr\sin\theta$$
$$= (2b\omega\cos\theta)\,B\,(dr\sin\theta)$$

From figure, $dr\cos\theta = a\,d\theta$

$$(\vec{v}\times\vec{B})\cdot d\vec{r} = 2b\omega Ba\sin\theta\,d\theta$$
$$= 2b\omega B(2b\cos\theta)\sin\theta\,d\theta$$
$$= 2b^2\omega B\sin 2\theta\,d\theta$$
$$\mathcal{E}_{\text{induced}} = \int_0^{\pi/2} 2b^2\omega B\sin 2\theta\,d\theta$$
$$= 2b^2\omega B\cdot\frac{1}{2} = b^2\omega B$$

Example 11

An L-shaped rod with a non-conducting part OA and a conducting part AB rotates with a constant angular velocity ω. Find the emf induced between ends A and B.

Solution: We take a differential element dx on the rod AB at a distance x from the end A. This element rotates with a velocity $R\omega$ about O. From Faraday's law,

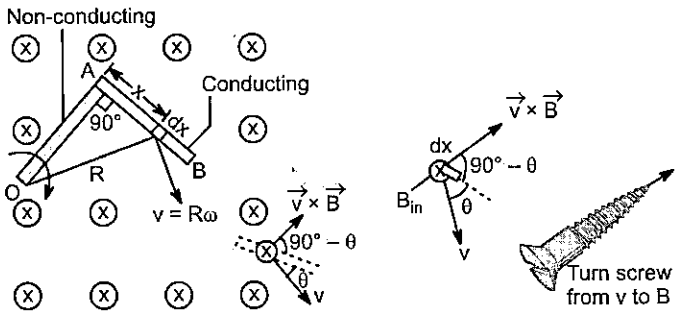


Fig. 5E.11

$$\mathcal{E}_{\text{induced}} = \int (\vec{v}\times\vec{B})\cdot d\vec{l}$$

$$|\vec{v}\times\vec{B}| = vB\sin 90° = vB = (R\omega)B$$

Vector $\vec{v}\times\vec{B}$ is normal to v as well as B vector, as shown in the figure. Vector $\vec{v}\times\vec{B}$ and $d\vec{x}$ are at an angle $(90°-\theta)$; so we have

$$\mathcal{E}_{\text{induced}} = \int_0^L (vB\sin 90°)\,dx\cos(90°-\theta)$$
$$= \int_0^L (R\omega)\,B\,dx\sin\theta$$
$$= \omega B\int_0^L x\,dx = \frac{1}{2}\omega BL^2$$

The induced emf is same as if the point A is fixed and the rod of length L is rotated about it.

Concepts: 1. *A rectangular loop, as shown in the figure moves away from an infinitely long wire carrying a current I. The emf induced in the rectangular loop is given by*

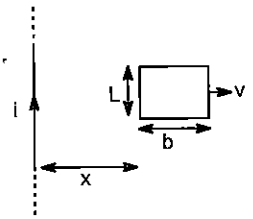


Fig. 5.13

$$\mathcal{E} = B_1Lv - B_2Lv$$
$$= \frac{\mu_0 I}{2\pi x}Lv - \frac{\mu_0 I}{2\pi(x+b)}Lv$$
$$= \frac{\mu_0 ILbv}{2\pi x(x+b)}$$

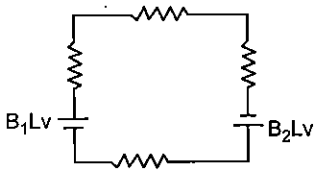


Fig. 5.14

or Consider a small segment of width dy at a distance y from the wire. Let flux through the segment be dϕ

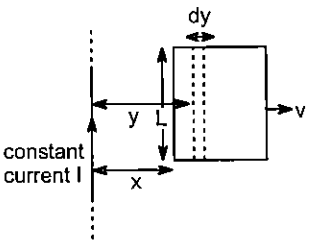


Fig. 5.15

$$\therefore \quad d\phi = \frac{\mu_0 I}{2\pi y}L\,dy$$

$$\therefore \quad \phi = \frac{\mu_0 IL}{2\pi}\int_x^{x+b}\frac{dy}{y}$$
$$= \frac{\mu_0 IL}{2\pi}(\ln(x+b) - \ln x)$$

Now
$$\frac{d\phi}{dt} = \frac{\mu_0 IL}{2\pi}\left[\frac{1}{x+b}\frac{dx}{dt} - \frac{1}{x}\frac{dx}{dt}\right]$$
$$= \frac{\mu_0 IL}{2\pi}\left[\frac{(-b)}{x(x+b)}\right]v = \frac{-\mu_0 IbLv}{2\pi x(x+b)}$$

$$\therefore \quad \text{Induced emf} = \frac{\mu_0 IbLv}{2\pi x(x+b)}$$

2. *A rod of length l is placed perpendicular to a long wire carrying current I. The rod is moved parallel to the wire with a velocity v. The emf induced in the rod, if its nearest end is at a distance 'a' from the wire can be calculated as*

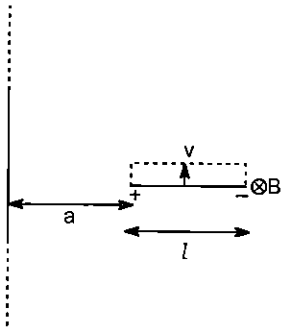


Fig. 5.16

Consider a segment of rod of length dx, at a distance x from the wire. Emf induced in the segment

$$d\mathcal{E} = \frac{\mu_0 I}{2\pi x} dx \cdot v$$

$$\therefore \quad \mathcal{E} = \int_a^{a+l} \frac{\mu_0 Iv\,dx}{2\pi x} = \frac{\mu_0 Iv}{2\pi} \ln\left(\frac{l+a}{a}\right)$$

3. *A rectangular loop is moving parallel to a long wire carrying current I with a velocity v. The emf induced in the loop, if its nearest end is at a distance 'a' from the wire can be calculated by drawing equivalent electrical diagram.*

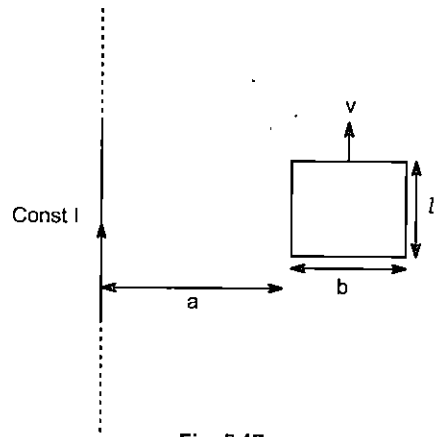


Fig. 5.17

emf = 0;

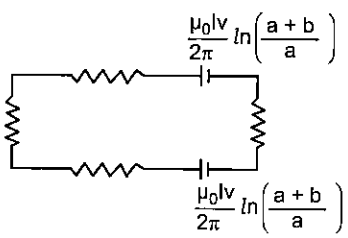


Fig. 5.18

$$I = \frac{\mathcal{E}}{R+r} = \frac{\mu_0 Iv}{2\pi(R+r)} \ln\left(\frac{x+l}{l}\right)$$

4. *Consider a conducting rod of length l rotating in a uniform magnetic field.*

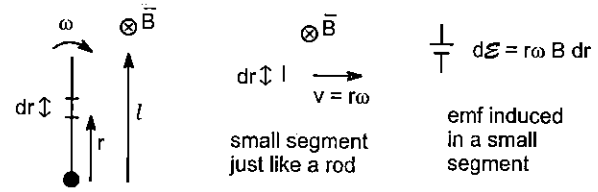


Fig. 5.19

Emf induced in a small segment of length dr, of the rod $= vBdr = r\omega Bdr$

$$\therefore \text{emf induced in the rod} = \omega B \int_0^l r\,dr = \frac{1}{2} B\omega l^2$$

equivalent of this rod is as following

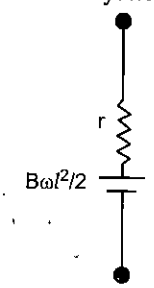


Fig. 5.20

$$\text{or} \quad \mathcal{E} = \frac{d\phi}{dt} = \frac{\text{flux through the area swept by the rod in time } dt}{dt}$$

$$= \frac{B\frac{1}{2} l^2 \omega dt}{dt} = \frac{1}{2} B\omega l^2$$

5. *A rod PQ of length 2l is rotating about one end P in a uniform magnetic field B which is perpendicular to the plane of rotation of the rod. Point M is the mid point of the rod. The induced emf between M and Q can be calculated as*

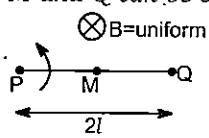


Fig. 5.21

$$E_{MQ} + E_{PM} = E_{PQ}$$

$$\text{Corner} \rightarrow \frac{B\omega l^2}{2} = 100$$

$$E_{MQ} + \frac{B\omega\left(\frac{l}{2}\right)^2}{2} = \frac{B\omega l^2}{2} \quad E_{MQ} = \frac{3}{8} B\omega l^2$$

6. *A rod PQ of length 2l is rotating about its mid point C, in a uniform magnetic field B which is perpendicular to the plane of rotation of the rod. The induced emf between PQ and PC. Can be calculated by drawing the circuit diagram of parts PC and CQ.*

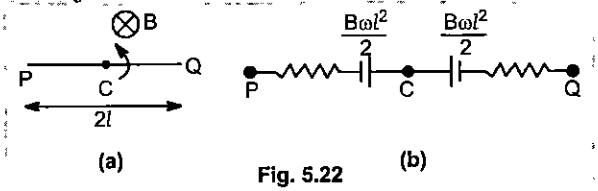


Fig. 5.22

$$emf_{PQ} = 0; \quad emf_{PC} = \frac{B\omega l^2}{2}$$

7. *A rod of length l and resistance r rotates about one end as shown in figure. Its other end touches a conducting ring a of negligible resistance. A resistance R is connected between centre and periphery. By drawing the electrical equivalence and find the current in the resistance R. There is a uniform magnetic field B directed as shown.*

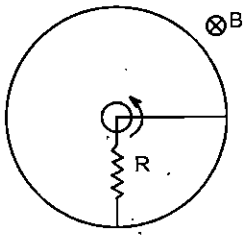


Fig. 5.23

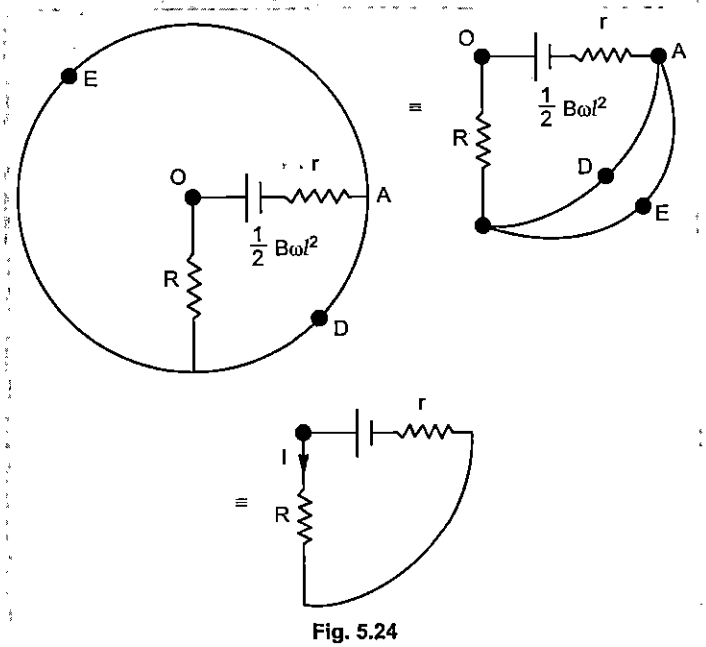


Fig. 5.24

Current $$I = \frac{\frac{1}{2}B\omega l^2}{R + r}$$

8. *In the above concept if the length of rod is 2l and resistance 2r and it is rotating about its centre. Both ends of the rod now touch the conducting ring.*

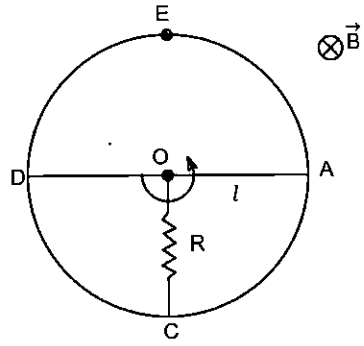


Fig. 5.25

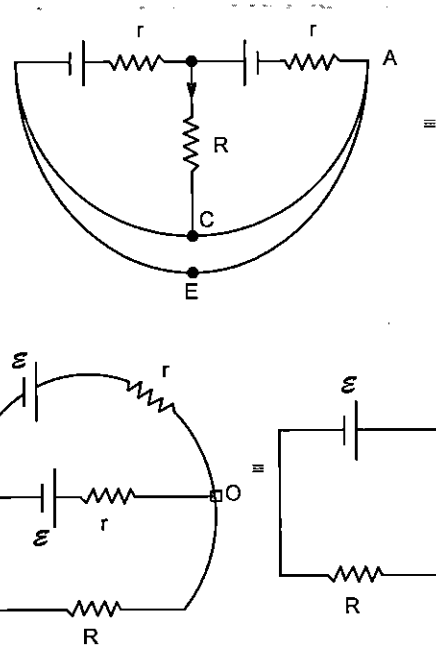


Fig. 5.26

$$I = \frac{\varepsilon}{R + \frac{r}{2}} = \frac{\frac{1}{2}B\omega l^2}{R + \frac{r}{2}}$$

9. *A rod of length l is rotating with an angular speed ω about its one end which is at a distance 'a' from an infinitely long wire carrying current I. The emf induced in the rod at the instant shown in the figure can be calculated as. Consider a small segment of rod of length dx, at a distance x from one end of the rod. Emf induced in the segment.*

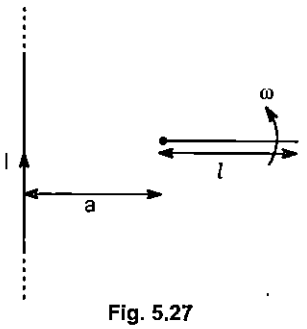


Fig. 5.27

$$d\mathcal{E} = \frac{\mu_0 I}{2\pi(x+a)}(x\omega)dx$$

$$\therefore \quad \mathcal{E} = \int_0^l \frac{\mu_0 I}{2\pi(x+a)}(x\omega)dx = \frac{\mu_0 I\omega}{2\pi}\left[l - a\cdot\ln\left(\frac{l+a}{a}\right)\right]$$

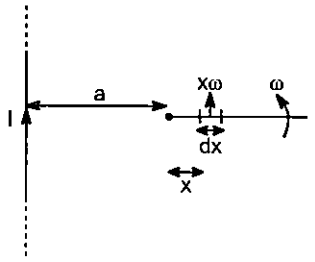


Fig. 5.28

10. *A rod PQ of mass m and resistance r is moving on two fixed, resistanceless, smooth conducting rails (closed on both sides by resistance R_1 and R_2). The current in the rod at the instant its velocity is v can be calculated as*

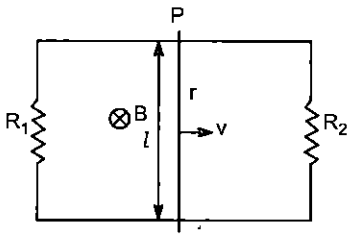


Fig. 5.29

$$I = \frac{Blv}{r + \dfrac{R_1R_2}{R_1+R_2}}$$

this circuit is equivalent to the following diagram.

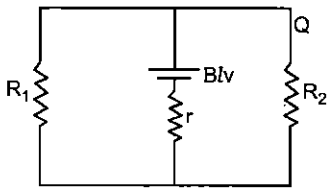


Fig. 5.30

11. *If one resistance is replaced by a capacitor of capacitance C as shown. Find the velocity of the moving rod at time t if the initial velocity of the rod is v and a constant force F is applied on the rod. Neglect the resistance of the rod.*

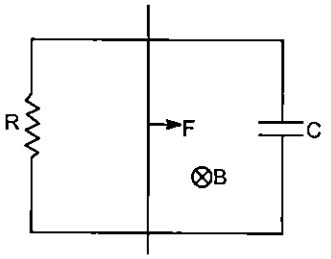


Fig. 5.31

At any time t, let the velocity of the rod be v

Applying Newtons law: $F - IlB = ma$ ...(1)

Also $Blv = I_1R = \dfrac{q}{c}$

Applying KCl,

$$I = I_1 + \frac{dq}{dt} = \frac{Blv}{R} + \frac{d}{dt}(BlvC)$$

or $$I = \frac{Blv}{R} + BlCa$$

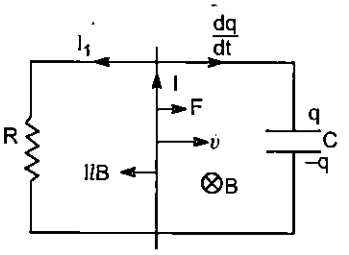


Fig. 5.32

Putting the value of I in eq (1),

$$F - \frac{B^2l^2v}{R} = (m + B^2l^2C)a$$

$$= (m + B^2l^2C)\frac{dv}{dt}$$

$$(m + B^2l^2C)\frac{dv}{F - \dfrac{B^2l^2v}{R}} = dt$$

Integrating both sides, and solving we get

$$v = \frac{FR}{B^2l^2}\left(1 - e^{-\frac{tB^2l^2}{R(m+CB^2l^2)}}\right)$$

Example 12

A circular loop of area A and resistance R rotates with an angular velocity ω about diameter. Initially the plane of the loop is perpendicular to the magnetic field B. Find the induced current in the loop.

Solution: Let the area vector make an angle θ with the magnetic field at some instant. The magnetic flux through the loop

$$\phi_B = \int \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}$$

$$= BA\cos\theta$$

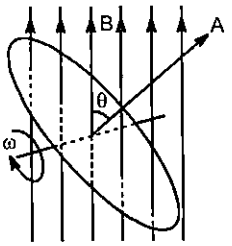


Fig. 5E.12

When the loop rotates at constant angular speed ω, the angle θ is given by $\theta = \omega t$, hence

$$\phi_B = BA\cos(\omega t)$$

From Faraday's law of electromagnetic induction,

$$\mathcal{E} = -\frac{d\phi_B}{dt}$$

$$= -\frac{d}{dt}[BA\cos\omega t]$$

$$\mathcal{E}(t) = BA\omega\sin(\omega t)$$

The current $I(t)$ is given by

$$I(t) = \frac{\mathcal{E}}{R} = \frac{BA\omega}{R}\sin(\omega t)$$

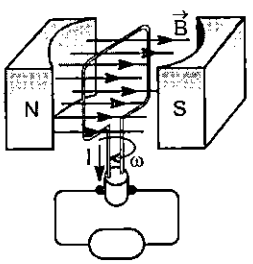
Note that the induced emf and the current vary sinusoidally with time.

Amplitude of induced emf $= BA\omega$

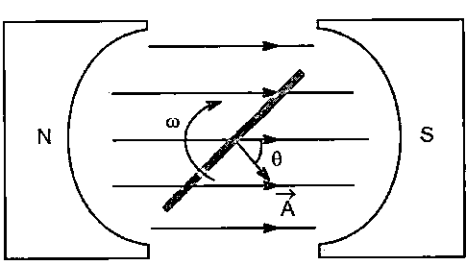
Amplitude of induced current $= \frac{BA\omega}{R}$.

GENERATORS

One of the most important applications of Faraday's law of induction is to generators and motors. A generator converts mechanical energy into electric energy, while a motor converts electrical energy into mechanical energy.



(a) A simple generator.



(b) The rotating loop as seen from above

Fig. 5.33

Fig. 5.33 (a) is a simple illustration of a generator. It consists of an N-turn loop rotating in a magnetic field which is assumed to be uniform. The magnetic flux varies with time, thereby inducing an emf. From Fig. 5.33 (b), we see that the magnetic flux through the loop may be written as

$$\phi_B = \vec{\mathbf{B}} \cdot \vec{\mathbf{A}} = BA\cos\theta = BA\cos\omega t$$

The rate of change of magnetic flux is

$$\frac{d\phi_B}{dt} = -BA\omega\sin\omega t$$

Since there are N turns in the loop, the total induced emf across the two ends of the loop is

$$\mathcal{E} = -N\frac{d\phi_B}{dt} = NBA\omega\sin\omega t$$

If we connect the generator to a circuit which has a resistor R, then the current generated in the circuit is given by

$$I = \frac{|\mathcal{E}|}{R} = \frac{NBA\omega}{R}\sin\omega t$$

The current is an alternating current which oscillates in sign and has amplitude $I_0 = NBA\omega/R$. The power delivered to this circuit is

$$P = I|\mathcal{E}| = \frac{(NBA\omega)^2}{R}\sin^2\omega t$$

On the other hand, the torque exerted on the loop is

$$\tau = \mu B\sin\theta = \mu B\sin\omega t$$

Thus, the mechanical power supplied to rotate the loop is

$$P_m = \tau\omega = \mu B\omega\sin\omega t$$

Since the dipole moment of the N-turn current loop is

$$\mu = NIA = \frac{N^2A^2B\omega}{R}\sin\omega t$$

the above expression becomes

$$P_m = \left(\frac{N^2A^2B\omega}{R}\sin\omega t\right)B\omega\sin\omega t$$

$$= \frac{(NAB\omega)^2}{R}\sin^2\omega t$$

As expected, the mechanical power put in is equal to the electric power output.

Example 13

A metal rod of mass m and length l slides on a frictionless rail of negligible resistance, with a resistor R at the termination point (see figure). A uniform magnetic field is directed perpendicular to the plane of the rails. The initial velocity of the rod is v_0. There is no external force on the rod. Find (a) the velocity of the rod as function of time, (b) the total distance travelled by the rod, (c) the total energy dissipated in the resistance and show that it is equal to $(1/2)mv_0^2$.

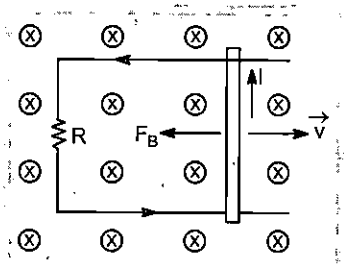


Fig. 5E.13

Solution: (a) The motion of the rod in a magnetic field induces an emf $\mathcal{E} = Blv$ and therefore a current $I = \frac{\mathcal{E}}{R} = \frac{Blv}{R}$. As the flux increases, in accordance with Lenz's law, the current in the circuit is anticlockwise. The magnetic force on a current carrying conductor is given by $d\vec{\mathbf{F}} = Id\vec{\mathbf{l}} \times \vec{\mathbf{B}}$; from Fleming's left hand rule it is directed to left.

Magnetic force on the conductor

$$= IBl = \left(\frac{Blv}{R}\right)Bl = \frac{B^2l^2v}{R}$$

From Newton's second law,

$$\frac{-B^2l^2v}{R} = m\frac{dv}{dt}$$

The magnetic force is opposite to direction of motion; hence it is negative.

On separating variables, then integrating the speed from v_0 to v and integrating the time from 0 to t.

$$\frac{dv}{v} = -\frac{B^2l^2}{mR}dt$$

$$\int_{v_0}^{v}\frac{dv}{v} = -\frac{B^2l^2}{mR}\int_0^t dt$$

$$\ln\frac{v}{v_0} = -\frac{B^2l^2}{mR}t$$

$$v = v_0e^{-(B^2l^2/mR)t} \quad \ldots(1)$$

or $$v = v_0e^{-t/\tau}$$

where $$\tau = \frac{mR}{B^2l^2}$$

The velocity of the rod long time after projection, *i.e.*, at $t \to \infty$, is zero as $e^{-\infty} = \frac{1}{e^{\infty}} = 0$.

(b) From eqn. (1), we have

$$\frac{dx}{dt} = v_0e^{-t/\tau}$$

$$dx = v_0e^{-t/\tau}dt$$

$$\int_0^x dx = v_0\int_0^t e^{-t/\tau}\,dt$$

$$= v_0\tau\left|-e^{-t/\tau}\right|_0^t$$

$$x = v_0\tau(1-e^{-t/\tau})$$

As we have seen, at $t \to \infty, v \to 0$, *i.e.*, the rod stops. Thus distance travelled till the rod can stop, can be obtained by taking limit $t \to \infty$ of the expression for x.

$$x = v_0\tau \qquad (\text{as } e^{-\infty} = 0)$$

(c) The energy dissipated per second in resistance is

$$P = I^2R$$

Energy dissipated in time dt is

$$dH = I^2R\,dt = \left(\frac{Blv}{R}\right)^2 R\,dt$$

$$= \frac{B^2l^2}{R}(v_0e^{-t/\tau})^2\,dt$$

Total energy dissipated in time t can be obtained by integrating the above expression from $t = 0$ to t.

$$H = \int_0^t \frac{B^2l^2}{R}(v_0e^{-t/\tau})^2dt$$

$$= \frac{B^2l^2v_0^2}{R}\int_0^t (e^{-t/\tau})^2dt$$

$$= \left(\frac{B^2l^2}{R}v_0^2\right)\tau[1-e^{-2t/\tau}]$$

$$= \frac{1}{2}mv_0^2(1-e^{-2t/\tau})$$

Total energy dissipated till the rod stops is given by taking limit $t \to \infty$ of expression for H.

$$H = \frac{1}{2}mv_0^2$$

Example 14

A rod slides on a U-shaped conductor, similar to previous problem: if the resistance of the circuit at any instant is R, find the force needed to pull the wire with speed v. What external power is needed to pull the rod ? Show that the power input equals that dissipated in the resistance at any moment.

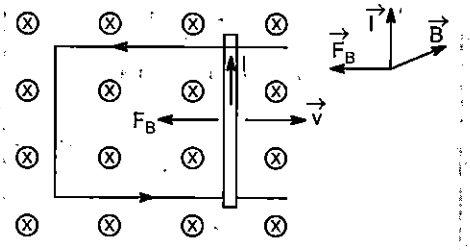


Fig. 5E.14

Solution: When the rod moves to the right at speed v, a clockwise current is set up in the circuit. From Lenz's law we can see that the induced current must oppose the increase of flux due to motion of the rod. Thus the induced current is anticlockwise so as to produce a magnetic field out of the page (right hand thumb rule). The force experienced by a current carrying wire in a magnetic field is $I\vec{\mathbf{l}} \times \vec{\mathbf{B}}$, for a

constant B. From Fleming's left hand rule this force resists our effort to move the rod to the right.

The induced current is $I = \frac{Blv}{R}$

The magnetic force is $F_B = IlB = \frac{B^2l^2v}{R}$

In order to move the rod at constant speed we must exert a force equal in magnitude to F_B and directed to right.

$$|F_{external}| = |F_B| = \frac{B^2l^2v}{R}$$

The external power needed to move the rod for constant R is

$$P = Fv = \frac{B^2l^2v^2}{R}$$

The power dissipated in resistor is, $P = I^2R$.

$$P = \left(\frac{Blv}{R}\right)^2 R = \frac{B^2l^2v^2}{R}$$

Thus the power input equals the power lost as heat in resistor.

Example 15

A flip coil consists of N turns of circular coils which lie in a uniform magnetic field. Plane of the coils is perpendicular to the magnetic field as shown in Fig. 5E.15. The coil is connected to a current integrator which measures the total charge passing through it. The coil is turned through 180° about its diameter. Find the charge passing through the coil.

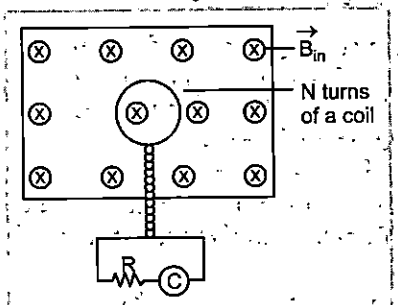


Fig. 5E.15

Solution: When the flip coil is rotated, the magnetic flux through it changes, resulting in an induced emf $\mathcal{E}$ and an induced current $I = \mathcal{E}/R$, where R is the total resistance of the circuit.

As $I = \frac{dQ}{dt}$

$$Q = \int dQ = \int I\,dt$$

From Faraday's law, induced emf

$$|\mathcal{E}| = \frac{d\phi_B}{dt}$$

Thus $Q = \int I\,dt = \int \frac{\mathcal{E}}{R}dt$

$$= \frac{1}{R}\int \frac{d\phi_B}{dt}dt$$

$$Q = \frac{1}{R}\int d\phi_B = \frac{\Delta\phi_B}{R}$$

This is a generalised result and can be used to determine charge through a circuit in which an emf and a current is induced.

Initial flux through the coil, $\phi_{Bi} = +NBA$

Final flux through the coil, $\phi_{Bf} = -NBA$.

When the coil is turned through 180° its flux reverses, angle between magnetic field and area vector is reversed.

$$\Delta\phi_B = \phi_{Bf} - \phi_{Bi} = NBA - (-NBA) = 2NBA$$

$$Q = \frac{2NBA}{R}$$

Example 16

An elasticized conducting band is around a spherical balloon. Its plane passes through the centre of the balloon. A uniform magnetic field of magnitude 0.04T is directed perpendicular to the plane of the band. Air is let out of the balloon at 100 cm^3/s at an instant when the radius of the balloon is 6 cm. What is the induced emf in the band?

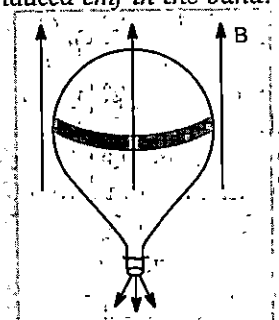


Fig. 5E.16

Solution: Volume of the balloon at any instant, when radius is r,

$$V = \frac{4}{3}\pi r^3$$

Time rate of change of volume,

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

Time rate of change of radius of balloon,

$$\frac{dr}{dt} = \frac{1}{4\pi r^2}\frac{dV}{dt}$$

Flux through rubber band at the given instant,

$$\phi = B(\pi r^2)$$

$$\text{Induced emf} = -\frac{d\phi}{dt} = -\frac{d}{dt}(B\pi r^2)$$

$$= -2\pi rB\frac{dr}{dt}$$

$$= -2\pi rB\left(\frac{1}{4\pi r^2}\frac{dV}{dt}\right)$$

$$= -\frac{B}{2r}\frac{dV}{dt}$$

As volume of the balloon is decreasing, $\frac{dV}{dt}$ is negative.

$$\mathcal{E}_{\text{induced}} = -\frac{(0.4)}{2\times 6\times 10^{-2}} \times (-100\times 10^{-6})$$

$$= 333\,\mu\text{V}$$

Example 17

In Fig. 5E.17 shown, the rod has a resistance R, the horizontal rails have negligible friction. A battery of emf $\mathcal{E}$ and negligible internal resistance is connected between points a and b. The rod is initially at rest. (a) Find the force on the rod as a function of the speed v. (b) Show that the rod will approach a terminal speed and find an expression for it. (c) What is the current when the rod attains its terminal speed?

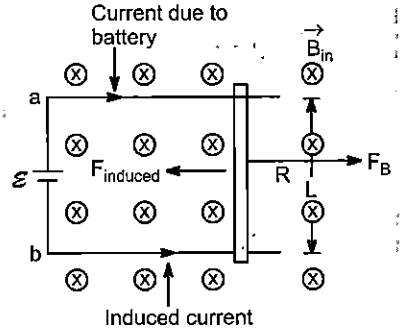


Fig. 5E.17

Solution: (a) The current of the battery at any instant $I = \mathcal{E}/R$.

The magnetic force due to this current $F_B = IBl = \frac{\mathcal{E}Bl}{R}$.

This magnetic force will accelerate the rod from its position of rest. The motional emf developed in the rod is Blv; the induced current $I_{\text{induced}} = \frac{Blv}{R}$.

The magnetic force due to induced current,

$$F_{\text{induced}} = I_{\text{induced}}Bl = \frac{B^2l^2v}{R}$$

From Fleming's left hand rule, force F_B is to the right and F_{induced} is to the left. Net force on the rod $= F_B - F_{\text{induced}}$.

From Newton's second law,

$$F_B - F_{\text{induced}} = m\frac{dv}{dt}$$

$$\frac{\mathcal{E}Bl}{R} - \frac{B^2l^2v}{R} = m\frac{dv}{dt}$$

On separating variables and integrating speed from v_0 and v and time from 0 to t, we have

$$\frac{dv}{\mathcal{E}-Bvl} = \frac{Bl}{mR}dt$$

$$\int_0^v \frac{dv}{(\mathcal{E}-Bvl)} = \frac{Bl}{mR}\int_0^t dt$$

$$\ln\left(\frac{\mathcal{E}-Bvl}{\mathcal{E}}\right) = -\frac{B^2l^2}{mR}t$$

$$\frac{\mathcal{E}-Bvl}{\mathcal{E}} = e - \frac{B^2l^2}{mR}t \quad \Rightarrow \quad v = \frac{\mathcal{E}}{Bl}(1-e^{-t/\tau})$$

where $\tau = \frac{mR}{(Bl)^2}$

(b) The rod will attain a terminal velocity at $t \to \infty$, *i.e.*, when $e^{-t/\tau} = 0$, the velocity is independent of time.

Thus $v_T = \frac{\mathcal{E}}{Bl}$

(c) The induced current $I_{\text{induced}} = \frac{Blv}{R}$. When the rod has attained terminal speed,

$$I_{\text{induced}} = \frac{Bl}{R} \times \left(\frac{\mathcal{E}}{Bl}\right) = \mathcal{E}/R$$

The current of battery and induced current are of same magnitude; hence net current through the circuit is zero.

Example 18

An infinitesimal bar magnet of dipole moment M is pointing and moving with speed v in the x-direction. A closed circular conducting loop of radius a and negligible self-inductance lies in the y-z plane with its centre at x = 0 and its axis coinciding with x-axis. Find the force opposing the motion of the magnet, if the resistance of the loop is R. Assume that the distance x of the magnet from the centre of the loop is much greater than a.

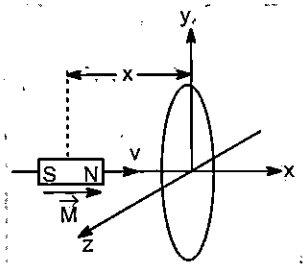


Fig. 5E.18

Solution: The magnetic field of a bar magnet at its axial point, distance x from its centre, is given by

$$B = \frac{\mu_0}{4\pi}\frac{2M}{x^3}$$

Magnetic flux linked with the loop

$$\phi = BA = \left(\frac{\mu_0}{4\pi}\frac{2M}{x^3}\right)\pi a^2$$

The induced emf in the loop due to motion of magnet,

$$\mathcal{E} = -\frac{d\phi}{dt} = \frac{\mu_0}{4\pi}\frac{6\pi Ma^2}{x^4}\frac{dx}{dt}$$

$$= \frac{\mu_0}{4\pi}\frac{6\pi Ma^2}{x^4}v$$

The induced current in the loop,

$$I = \frac{\mathcal{E}}{R} = \frac{\mu_0}{4\pi}\frac{6\pi Ma^2}{Rx^4}v$$

The induced magnetic moment in the coil

$$= I \times A = \frac{\mu_0}{4\pi}\frac{6\pi^2 a^4 M}{R \times x^4}v$$

The magnetic force between two magnetic dipoles is given by

$$F = \frac{\mu_0}{4\pi}\frac{6M_1M_2}{x^4}$$

Hence opposing force, $F = \frac{9}{4}\frac{\mu_0^2 M^2 a^4}{Rx^8}v$

Example 19

Fig. 5E.19 (a) shows a toroidal solenoid whose cross-section is rectangular. Find the magnetic flux through this cross-section if the current through the winding equals $I = 1.7$ A, the total number of turns is $N = 1000$, the ratio of the outside diameter to the inside one is $\eta = 1.6$ and the height is equal to $h = 5.0$ cm.

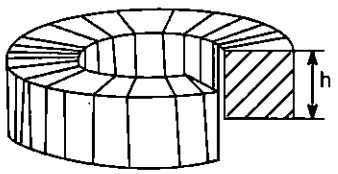


Fig. 5E.19 (a)

Solution: The magnetic field at distance r from centre of toroid is

$$B = \frac{\mu_0 NI}{2\pi r}$$

We consider a strip of thickness dr at distance r from centre O. The area of considered element is

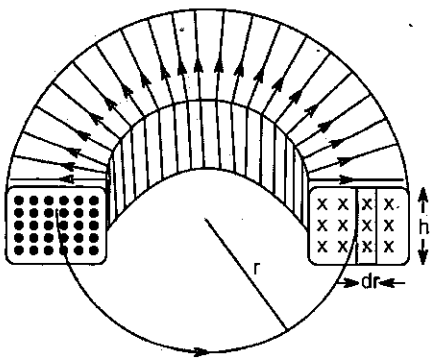


Fig. 5E.19 (b)

$$dA = h\,dr$$

$$d\phi = \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}$$

$$= Bh\,dr\cos\theta$$

$$\therefore \quad \phi = \int_a^b \frac{\mu_0 NI}{2\pi r} h\,dr$$

$$= \frac{\mu_0 NIh}{2\pi}\ln\frac{b}{a}$$

$$\therefore \quad \phi = \frac{\mu_0 NIh}{2\pi}\ln\eta$$

Example 20

The Earth's magnetic flux density is 5×10^{-5} T and the angle of dip is 66°, at a place, at north pole it is 6×10^{-5} T. The wing span of a jumbo jet is 80 m, its length 60 m, and its depth 8 m. Estimate the potential differences that could be detected over the surface of the jet when it flies horizontally at 200 m/s :

(i) over the North Pole,

(ii) northwards over the Equator,

(iii) eastwards along the Equator,

(iv) northwest over London.

Solution: (i) The magnetic field is vertically downwards, and the induced voltage is $80 \times 6 \times 10^{-5} \times 200 = 960$ mV, with the starboard (right) wing tip at the higher potential.

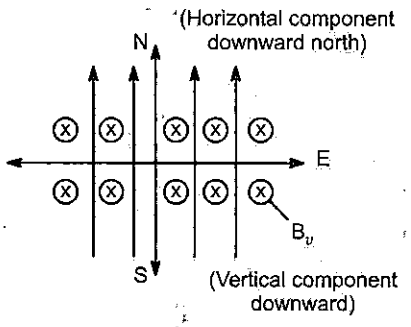


Fig. 5E.20

(ii) The motion is parallel to the field, and so no potential difference is developed.

(iii) Since the field is a dipole, its strength at the Equator is one-half that at the Pole. The field is horizontal, and the induced voltage is $8 \times 3 \times 10^{-5} \times 200 = 48$ mV, with the bottom of the jet at the higher potential.

(iv) The vertical field is $5 \times 10^{-5} \times \sin 66°$ T and the northward component of the velocity is $200/\sqrt{2}\ \mathrm{m\,s^{-1}}$, leading to a 520 mV potential difference across the wings with the starboard tip at the higher potential. In the same way, a P.D. of 23 mV appears between the top (higher potential) and the bottom of the jet's body.

Example 21

A wire ACBOA shaped as a semi-circle ACB with the bounding diameter BOA with OA = OB = OC = a has resistance R. It is placed in the same plane as an infinitely long wire XY with AB parallel to XY at a distance a and C further away from XY. A total charge Q flows through XY such that the current decreases linearly from an initial value to 0 in time T; find:

(a) Current in XY as a function of time.
(b) Magnetic flux through ACBOA when current in XY is I.
(c) Induced current in ACBOA as a function of time.
(d) Heat generated in ACBOA in time T.
(e) Force acting on ACB as a function of time.

Solution: (a) Variation of I with time is shown in the diagram.

Total charge = area of the triangle $= \frac{I_0 T}{2} = Q$

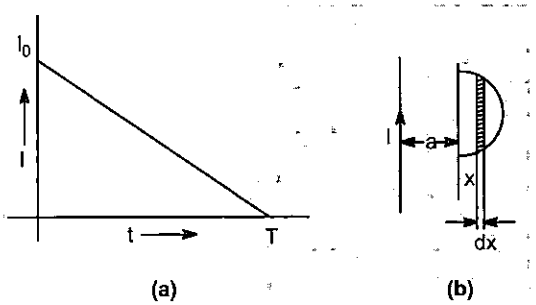


Fig. 5E.21

$$I_0 = \frac{2Q}{T}$$

$\therefore$ Now $$I = I_0 - \frac{I_0}{T}t$$

or $$I = \frac{2Q}{T}\left[1 - \frac{t}{T}\right]$$

(b) $$\phi = \int B \cdot dA$$

$$= \int_0^a \frac{\mu_0 I}{2\pi(a+x)}(2\sqrt{a^2 - x^2}\,dx)$$

$$= \frac{\mu_0 I}{\pi}\frac{a}{2}(\pi - 2)$$

(c) Induced emf,

$$\mathcal{E} = \frac{-\mu_0 a(\pi-2)}{2\pi}\frac{dI}{dt}$$

$$= \frac{\mu_0 a(\pi-2)}{2\pi}\left(\frac{2Q}{T^2}\right)$$

$\therefore$ Induced current

$$I_1 = \frac{\mu_0 a(\pi-2)}{\pi R}\frac{Q}{T^2}$$

(d) $H = \int_0^T I_1^2 R\,dt = I_1^2 RT$ (I_2 is constant)

$$= \frac{a^2 u_0^2 (\pi-2)^2 Q^2}{\pi^2 RT^3}$$

(e) Force on the straight part

$$F_1 = BI_1 2a = \frac{\mu_0}{2\pi a} I I_1 (2a)$$

$$= \frac{\mu_0}{2\pi a}\frac{2Q}{T}\left(1 - \frac{t}{T}\right)\frac{\mu_0 a(\pi-2)Q}{\pi RT^2}$$

$$= \frac{\mu_0^2 Q^2(\pi-2)}{\pi^2 RT^3}\left(1 - \frac{t}{T}\right)$$

The force is directed towards the wire as the currents are parallel.

Example 22

A thermocole vessel contains 0.5 kg of distilled water at 30° C. A metal coil of area $5 \times 10^{-3}\ m^2$, number of turns 100, mass 0.06 kg and resistance 1.6Ω is lying horizontally at the bottom of the vessel. A uniform, time varying magnetic field is set up to pass vertically through the coil at time t = 0. The field is first increased from zero to 0.8 T at a constant rate between 0 and 0.2 s and then decreased to zero at the same rate between 0.2 and 0.4 s. This cycle is repeated 12000 times. Make sketches of the current through the coil and the power dissipated in the coil as functions of time for the first two cycles. Clearly indicate the magnitudes of the quantities on the axes. Assume that no heat is lost to the vessel or the surroundings. Determine the final temperature of the water under thermal equilibrium. Specific heat of the metal = 500 J $kg^{-1}K^{-1}$ and the specific heat of water = 4200 J $kg^{-1}K^{-1}$. Neglect the inductance of the coil.

Solution: The variation of magnetic field is shown in Fig. 5E.22.

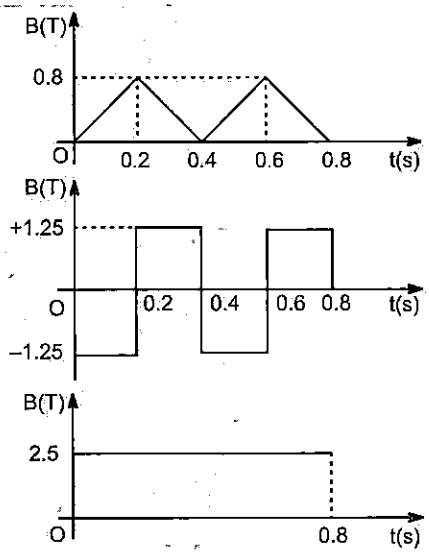


Fig. 5E.22

The rate of variation of B is

$$\left|\frac{dB}{dt}\right| = 4\,\text{T/s}$$

From Faraday's law of electromagnetic induction,

$$\mathcal{E} = \frac{d\phi}{dt}$$

$$\mathcal{E} = AN\left|\frac{dB}{dt}\right|$$

Here $A = 5\times10^{-3}\,m^2$; $N = 100$

Then $|\mathcal{E}| = (5\times10^{-3})(100)(4) = 2\,V$

$$I = \frac{\mathcal{E}}{R} = \frac{2}{1.6} = 1.25\,A$$

Power dissipated in the coil is

$$P = I^2R = (1.25)^2(1.6) = 2.5\,W$$

Let T be the final temperature of water, then using the law of calorimetry,

$$m_w s_w (T-30) + m_C s_C (T-30) = 2.5t$$

here $t = (12000)(0.4) = 4800\,s$

$m_w = 0.5\,kg$; $m_C = 0.06\,kg$. $s_w = 4200\,J\,kg^{-1}\,K^{-1}$

$s_C = 500\,J\,kg^{-1}\,K^{-1}$

$\because$ $(0.5)(4200)(T-30) + (0.06)(500)(T-30) = 2.5(4800)$

$(2100+30)(T-30) = 12000$

$T = 35.6°C$

Example 23

A metal disc of radius $R = 25$ cm rotates with a constant angular velocity $\omega = 130$ rad/s about its axis. Find the potential difference between the centre and rim of the disc if:
(a) the external magnetic field is absent,
(b) the external uniform magnetic field $B = 5.0$ mT is directed perpendicular to the disc.

Solution: (a) Centripetal force required for circular motion of electron is generated by a radial electric field caused by the redistribution of the electrons in the disc.

$$F = eE = mr\omega^2$$

or $$E = \frac{mr\omega^2}{e}$$

From $dV = -E\,dr$,

we have $$dV = -\frac{m\omega^2}{e} r\,dr$$

or $$\int_{V_1}^{V_2} dV = -\frac{m\omega^2}{e}\int_0^R r\,dr$$

or $$V_1 - V_2 = \frac{m\omega^2R^2}{2e}$$

$$= \frac{(9.1\times10^{-31})(130)^2(0.25)^2}{(2)(1.6\times10^{-19})}$$

$$= 3.0\times10^{-9}\text{ volt} = 3.0\text{ nV}$$

$V_1 > V_2$, *i.e.*, potential at centre is more than the potential at edge.

(b) A disc may be assumed to be made-up of a large number of radial, conducting, differential elements rotating with angular velocity ω about the centre of disc O.

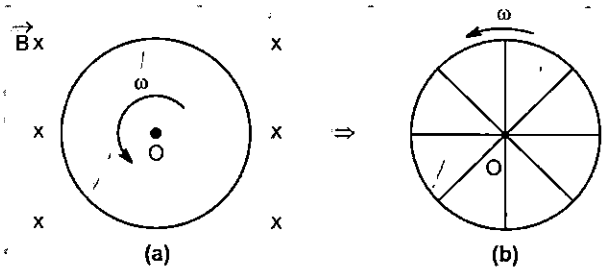


Fig. 5E.23

Thus, $V_{centre} \sim V_{edge} = \frac{1}{2}BR^2\omega$

$V_{centre} > V_{edge}$ [anticlockwise rotation]

$V_{edge} > V_{centre}$ [clockwise rotation]

$$V_{centre} - V_{edge} = \frac{1}{2}BR^2\omega$$

Substituting the values, we have

$$V_{centre} - V_{edge} = \frac{1}{2}\times5.0\times10^{-3}\times0.25\times0.25\times130$$

$$= 0.02\,\text{volt} = 20\,\text{mV}$$

Example 24

A wire loop enclosing a semicircle of radius R is located on the boundary of a uniform magnetic field B. At the moment $t = 0$, the loop is set into rotation with a constant angular acceleration α about an axis O coinciding with a line of vector $\vec{B}$ on the boundary. Find the emf induced in the loop as a function of time. Draw the approximate plot of this function. The arrow in Fig. 5E.24(a) shows the emf direction taken to be positive.

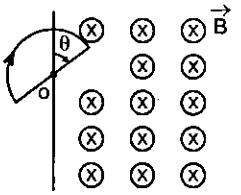


Fig. 5E.24 (a)

Solution: Emf is induced in the loop because area inside the magnetic field is continually changing.

From $\theta = 0$ to π, 2π to 3π, 4π to 5π, the loop begins to enter the magnetic field. Thus the magnetic field passing through the loop is increasing. Hence, current in the loop is anticlockwise, and for $\theta = \pi$ to 2π, 3π to 4π, 5π to 6π etc. magnetic field passing through the loop is decreasing. Hence current in the loop is clockwise.

Angular acceleration of the loop is constant, therefore angle turned in time t is $\theta = 1/2\,\alpha t^2$ and time taken, $t = \sqrt{2\alpha/\theta}$.

$$t_1 = \text{time taken to rotate angle } \pi = \sqrt{\frac{2\pi}{\alpha}}$$

From $t = 0$ to $t = t_1$, emf is negative,

t_2 = time taken to rotate angle $2\pi = \sqrt{\frac{4\pi}{\alpha}}$

From $t = t_1$ to $t = t_2$, emf is positive

... ...

t_n = time taken to rotate angle $n\pi = \sqrt{\frac{2n\pi}{\alpha}}$

Area inside the field is $A = \frac{1}{2}R^2\theta$

or $A = \frac{1}{4}R^2\alpha t^2$

So flux passing through the loop,

$$\phi = BA = \frac{1}{4}BR^2\alpha t^2$$

$$\mathcal{E} = \left|\frac{d\phi}{dt}\right| = \frac{1}{2}BR^2\alpha t$$

$$\mathcal{E} \propto t$$

i.e., $\mathcal{E}$- t graph is a straight line passing through origin. $\mathcal{E}$- t equation with sign can be written as

$$\mathcal{E} = (-1)^n\left(\frac{1}{2}BR^2\alpha t\right)$$

Here $n = 1, 2, 3, \ldots$ is the number of half revolutions that the loop performs at the given moment t.

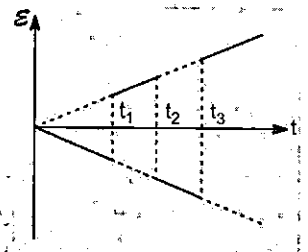


Fig. 5E.24 (b)

The $\mathcal{E}$- t graph is as shown in Fig. 5E.24(b).

Example 25

An equilateral triangular frame PQR of mass M and side a is at rest is under the influence of horizontal magnetic field B and gravitational field as shown in Fig. 5E.25(a).

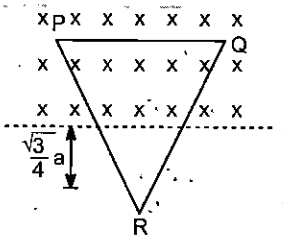


Fig. 5E.25 (a)

(a) Find the magnitude and direction of current in the frame so that the frame remains at rest.

(b) If the frame is slightly displaced in its plane perpendicular to PQ, show that its motion is simple harmonic and find its period of oscillation.

[Assuming small displacement neglect emf induced due to motion of the loop.]

Solution: (a) The current 'I' in the loop is clockwise in accordance with Lenz's law.

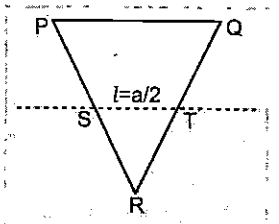


Fig. 5E.25 (b)

Now for the equilibrium of the loop,

$$Mg = I \times \frac{a}{2} \times B$$

or $$I = \frac{2Mg}{aB}$$

(b) Now if the loop is displaced downward by distance 'x',

Restoring force, $F = -[I(S'T')B - Mg]$

$$= -I\left[\frac{2}{\sqrt{3}}\left(\frac{\sqrt{3}}{4}a + x\right)\right]B + Mg$$

$$= -\frac{IaB}{2} - \frac{2IB}{\sqrt{3}}x + Mg$$

$$= -\frac{2IB}{\sqrt{3}}x$$

As the force restoring and proportional to x, motion is simple harmonic.

$$a = \frac{-2IB}{\sqrt{3}M}x$$

or $$\omega = \sqrt{\frac{2IB}{\sqrt{3}M}}$$

or $$T = 2\pi\left[\frac{\sqrt{3}M}{2IB}\right]^{1/2} = \pi\sqrt{\frac{\sqrt{3}a}{g}}$$

Example 26

A circular metal ring of radius of r rotates about a vertical diameter with constant angular velocity. As shown in the Fig. 5E.26, a small magnetic needle that can turn freely about a vertical axis sits in the middle of the ring.

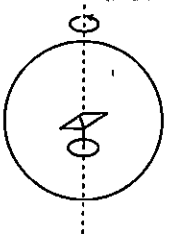


Fig. 5E.26

When the ring is stationary, the needle points in the direction of the horizontal component of the Earth's magnetic field. However, when it rotates at the rate of ten turns per second, the magnet deviates by an average of α from this position. What is the electrical resistance R of the ring?

Solution: We resolve the magnetic field of the Earth into its horizontal and vertical components. The vertical component induces no current in the ring, since its flux through the ring is always zero (angle between area vector and vertical magnetic field is 90°). Let the horizontal component of the magnetic field be $\vec{B}$ and the angular velocity of the ring be ω. The magnetic flux linked with the ring is

$$\phi = \pi r^2 B \cos\omega t,$$

and the induced voltage is $V = -\frac{d\phi}{dt} = \pi r^2 B\omega \sin\omega t$. The current,

$$I = \frac{V}{R} = \frac{r^2 \pi B\omega}{R}\sin\omega t,$$

flowing in the ring, induces a magnetic field at the centre of the ring of magnitude

$$B_I = \mu_0 \frac{I}{2r} = \mu_0 \frac{\pi r B\omega}{2R}\sin\omega t.$$

The direction of the magnetic field $\vec{B}_I$ is perpendicular to the plane of the ring and rotates with it. Resolve the vector $\vec{B}_I$ into a component parallel to $\vec{B}$ and a component perpendicular to it. The parallel component is proportional to $\cos\omega t \times \sin\omega t = \frac{1}{2}\sin 2\omega t$, which averages to zero over time. The perpendicular component can be written as

$$B_\perp = \mu_0 \frac{\pi r B\omega}{2R}\sin^2\omega t$$

$$= \mu_0 \frac{\pi r B\omega}{4R}(1 - \cos 2\omega t).$$

This expression consists of a term varying (relatively rapidly) with time and which, on average, is zero, and a constant term that causes the magnetic needle to deviate by α from its original (north-south) direction. Since the needle aligns itself with the direction of the (average) resultant field,

$$\tan\alpha = \frac{B_\perp}{B} = \mu_0 \frac{\pi r\omega}{4R}.$$

The needle will make small oscillations about the above position, with an amplitude determined by the mechanical and magnetic characteristics of the needle and by the damping forces.

Note that the angle of deviation of the magnetic needle does not depend on the magnitude of the Earth's magnetic field, although its horizontal component is *non-zero.*

Example 27

Two infinitely long parallel wires carrying currents $I = I_0 \sin\omega t$ in opposite directions are placed a distance 3a apart. A square loop of side a of negligible resistance with a capacitor of capacitance C is placed in the plane of wires as shown. Find:

(i) the maximum current in the square loop.

(ii) Also sketch the graph showing the variation of charge on the upper plate of the capacitor as a function of time for one complete cycle taking anticlockwise direction for the current in the loop as positive.

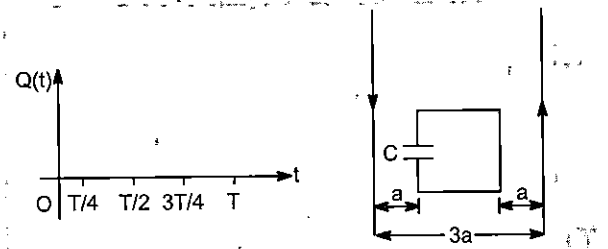


Fig. 5E.27 (a)

Solution: (i) For an elemental strip of thickness dx at a distance x from left wire:

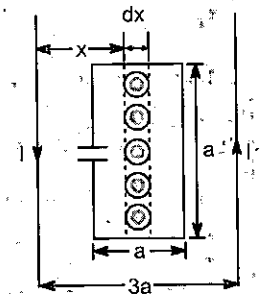


Fig. 5E.27 (b)

Net magnetic field (due to both wires),

$$B = \frac{\mu_0}{2\pi}\frac{I}{x} + \frac{\mu_0}{2\pi}\frac{I}{3a-x} \quad \text{(outwards)}$$

$$= \frac{\mu_0 I}{2\pi}\left(\frac{1}{x} + \frac{1}{3a-x}\right)$$

Magnetic flux in this strip,

$$d\phi = BdS = \frac{\mu_0 I}{2\pi}\left(\frac{1}{x} + \frac{1}{3a-x}\right)a\,dx$$

$\therefore$ Total flux,

$$\phi = \int_a^{2a} d\phi = \frac{\mu_0 Ia}{2\pi}\int_a^{2a}\left(\frac{1}{x} + \frac{1}{3a-x}\right)dx$$

or $$\phi = \frac{\mu_0 Ia}{\pi}\ln(2)$$

$$\phi = \frac{\mu_0 a \ln(2)}{\pi}(I_0 \sin\omega t) \quad \ldots(1)$$

Magnitude of induced emf,

$$\mathcal{E} = \left|-\frac{d\phi}{dt}\right| = \frac{\mu_0 a I_0 \omega \ln(2)}{\pi}\cos\omega t = \mathcal{E}_0 \cos\omega t$$

where $$\mathcal{E}_0 = \frac{\mu_0 a I_0 \omega \ln(2)}{\pi}$$

Charge stored in the capacitor,

$$q = C\mathcal{E} = C\mathcal{E}_0 \cos\omega t \quad \ldots(2)$$

and current in the loop,

$$I = \frac{dq}{dt} = C\omega\mathcal{E}_0 \sin\omega t \quad \ldots(3)$$

$$I_{max} = C\omega\mathcal{E}_0 = \frac{\mu_0 a I_0 \omega^2 C \ln(2)}{\pi}$$

(ii) Magnetic flux passing through the square loop,

$$\phi \propto \sin\omega t$$

[from equation (1)]

i.e., magnetic field passing through the loop is increasing at $t = 0$. Hence, the induced current will produce

⊗ magnetic field (from Lenz's law). Or the current in the circuit at $t = 0$ will be clockwise (or negative as per the given convention).

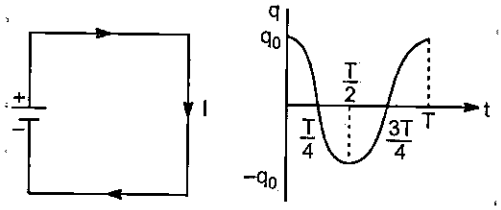


Fig. 5E.27 (c)

Therefore, charge on upper plate could be written as

$$q = +q_0 \cos\omega t$$

[from equation (2)]

Here $$q_0 = C\mathcal{E}_0 = \frac{\mu_0 aCI_0\omega \ln(2)}{\pi}$$

The corresponding q-t graph is shown in Fig. 5E.27 (c)

Example 28

One end of a horizontal track, of gauge l and negligible resistance, is connected to a capacitor of capacitance C charged to voltage V_0. The inductance of the assembly is negligible. The system is placed in a homogeneous, vertical magnetic field of induction B, as shown in Fig. 5E.28.

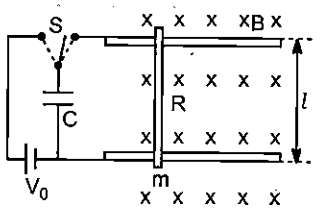


Fig. 5E.28

A frictionless conducting rod of mass m and resistance R is placed perpendicularly onto the track. The polarity of the capacitor is such that the rod is repelled from the capacitor when the switch is turned over.

(i) What is the maximum velocity of the rod?

(ii) Under what conditions is the efficiency of this 'electromagnetic gun' maximal?

Solution: (i) At the instant when the capacitor is connected, a current $I = V_0/R$ starts flowing in the rod, which experiences a force $F = BIl$ and an initial acceleration $a = BlV_0/mR$. In accordance with Lenz's law, the voltage induced in the moving rod causes the current flowing in the rod to decrease. The charge Q on the capacitor decreases and consequentyl so does the voltage across it. Meanwhile the voltage induced in the rod increases, until the two voltages cancel each other out. The rod then continues with its maximum velocity given by

$$Blv_{max} = \frac{Q_{min}}{C}. \qquad \ldots(1)$$

The equation of motion of the rod is

$$m\frac{dv}{dt} = ma = BlI = -Bl\frac{dQ}{dt} \qquad \ldots(2)$$

where the acceleration and the current have been expressed as the rates of change in velocity and charge, respectively. The proportionality between the two rates of change holds throughout. The speed of the rod increases from zero to v_{max}, whilst the charge on the capacitor decreases from $Q_0 = CV_0$ to Q_{min}. Equation (2) can therefore be rewritten as

$$mv_{max} = Bl(Q_0 - Q_{min}).$$

The final velocity and the residual charge on the capacitor can be calculated using equations (1) and (2),

$$v_{max} = \frac{BlCV_0}{m + B^2l^2C}$$

and $$Q_{min} = \frac{B^2l^2C^2V_0}{m + B^2l^2C}$$

(ii) The above relations show that the maximum velocity of the rod is proportional to the initial voltage V_0 across the capacitor. Thus, the final kinetic energy of the rod is proportional to V_0^2 (for given values of C and m), *i.e.*, proportional to the initial energy of the system. The coefficient of proportionality can be regarded as the efficiency η of the apparatus (considered as an electromagnetic gun), and can be written in the form

$$\eta = \frac{\frac{1}{2}mv_{max}^2}{\frac{1}{2}CV_0^2} = \frac{1}{\left(\frac{\sqrt{m}}{Bl\sqrt{C}} + \frac{Bl\sqrt{C}}{\sqrt{m}}\right)^2}$$

The product of the two terms in the brackets is 1, and from the inequality between arithmetic and geometric means, it follows that their sum is at least 2. This means that the efficiency of the electromagnetic gun cannot be more than 25 per cent.

Remark: *If the condition for maximum efficiency $\eta = CB^2l^2$ is substituted into the expression for the final charge on the capacitor we find that $Q_{min} = V_0C/2$ i.e., only half of the initial charge on the capacitor is left. Thus, only one-quarter of the initial energy of the capacitor is left; one-quarter of it is transformed into the kinetic energy of the rod, and the other half is dissipated in the rod as Joule heat.*

Example 29

A wire loop of radius r and resistance R is kept in a horizontal plane. A uniform magnetic field B exists in the vertical direction. A rod AB of length $L > 2r$ and negligible resistance slides on the loop with a uniform velocity v_0 directed perpendicular to the length, as shown in Fig. 5E.29 (a). Find the variation of the electromagnetic force F acting on the rod with the angle θ in the range $0 \le \theta \le 2\pi$

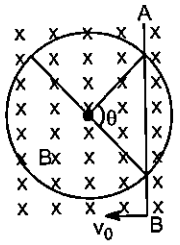


Fig. 5E.29 (a)

Solution: At an instant t the length of conductor between contact points is $2r\sin(\theta/2)$.

The induced emf across the contact points $\mathcal{E} = B(2r\sin\theta/2)v_0$

The rod divides the circular loop in two parallel resistors R_1 and R_2.

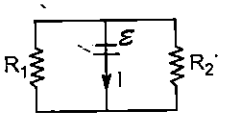


Fig. 5E.29 (b)

The resistance,

$$R_1 = R\left(1-\frac{\theta}{2\pi}\right) \quad \text{and} \quad R_2 = \left(\frac{R\theta}{2\pi}\right)$$

∴ The induced current

$$I = \frac{\mathcal{E}}{R_{eq}} = \frac{\mathcal{E}}{\frac{\theta R}{2\pi}\left(1-\frac{\theta}{2\pi}\right)}$$

Magnetic force F is

$$F = I\left[2r\sin\left(\frac{\theta}{2}\right)\right]$$

$$B = \frac{8\pi^2 B^2 v_0 r^2 \sin^2(\theta/2)}{R\theta(2\pi-\theta)}$$

Example 30

Fig. 5E.30 shows a wire bent into the shape of a right angle that is fixed on a horizontal plane. A very long rod of mass m initially starts with a velocity v_0 from the apex A of the bent wire. The resistance of the wire and the rod is $\lambda\Omega/m$. The whole arrangement is put in a region of uniform magnetic induction B. Find the distance travelled by the rod before it becomes stationary.

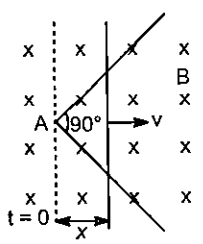


Fig. 5E.30

Solution: Let x be the distance travelled by the rod in time t, then the resistance of the circuit is

$$R = (2\sqrt{2}x + 2x)\lambda$$

The instantaneous induced emf is

$$\mathcal{E} = B(2x)v$$

Induced current $I = \frac{\mathcal{E}}{R} = \frac{Bv}{\lambda(1+\sqrt{2})}$

Magnetic force on the rod is

$$F = BI(2x) = \frac{2B^2vx}{\lambda(1+\sqrt{2})}$$

From Newton's second law, we have

$$F = -m\frac{dv}{dt} = \frac{2B^2vx}{\lambda(1+\sqrt{2})} \quad \ldots(1)$$

Eliminating t from the equation (1) and then integrating, we get

$$-\int_{v_0}^{0} dv = \left(\frac{2B^2}{m\lambda(1+\sqrt{2})}\right)\int_0^x x\,dx$$

or $$x = \frac{\sqrt{mv_0\lambda(1+\sqrt{2})}}{B}$$

Example 31

A plane loop is shaped in the form as shown in Fig. 5E.31 with radii $a = 20$ cm and $b = 10$ cm and is placed in a uniform time varying magnetic field $B = B_0 \sin\omega t$ where $B_0 = 10$ mT and $\omega = 100$ rad/s. Find the amplitude of the current induced in the loop if its resistance per unit length is equal to 50×10^{-3} Ω/m. The inductance of the loop is negligible.

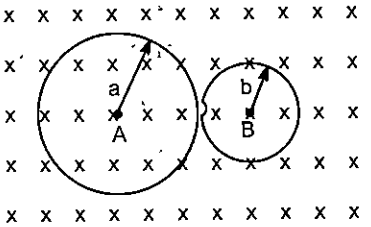


Fig. 5E.31

Solution: Instantaneous flux through loop is

$$\phi = \pi a^2 B\cos 0° + \pi b^2 B\cos 180°$$

$$= \pi(a^2-b^2)B = \pi(a^2-b^2)B_0\sin\omega t$$

Differentiating w.r.t. time,

$$\frac{d\phi}{dt} = \pi(a^2-b^2)B_0\omega\cos\omega t$$

$$I = \frac{\pi(a^2-b^2)B_0\omega\cos\omega t}{R}$$

(only the modulus)

$$I_{max} = \frac{\pi(a^2-b^2)B_0\omega}{2\pi(a+b)\rho},$$

ρ = resistance per unit length

$$= \frac{1}{2\rho}(a-b)B_0\omega$$

$$= \frac{1}{2}(20-10)\times10^{-2}\times10\times10^{-3}\times\frac{100}{50\times10^{-3}}$$

$$I_{max} = 1\text{ A}$$

Example 32

Fig. 5E.32 shows two parallel and co-axial loops. The smaller loop (radius r) is above the larger loop (radius R), by a distance $x >> R$. The magnetic field due to current I in the larger loop is nearly constant throughout the smaller loop. Suppose that x is increasing at the constant rate $\frac{dx}{dt} = v$.

(a) Determine the magnetic flux through the smaller loop as a function of x.

(b) Find the induced emf and the direction of induced current in the smaller loop.

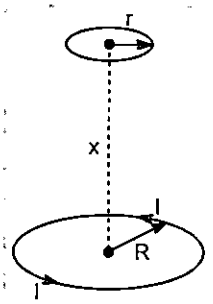


Fig. 5E.32

Solution: (a) Magnetic field on the axis of a circular coil is given by

$$B = \frac{\mu_0 I R^2}{2(R^2 + x^2)^{3/2}}$$

Since $x >> R$, therefore, magnetic field at the centre of the smaller loop is

$$B \approx \frac{\mu_0 I R^2}{2x^3}$$

Flux linked with coil is

$$\phi = B(\pi r^2) = \frac{\mu_0 \pi I R^2 r^2}{2x^3}$$

(b) From Faraday's law, we have

$$\mathcal{E} = -\frac{d\phi}{dt} = \frac{3}{2}\frac{\mu_0 \pi I R^2 r^2}{x^4} v$$

As the external magnetic field at the position of smaller coil is decreasing, the induced magnetic field is parallel to external field.

Hence the direction of current is anti-clockwise when seen from the top.

Example 33

A metal rod OA of mass m can rotate about a horizontal axis passing through O, sliding along a circular-conducting ring of radius r as shown in Fig. 5E.33 (a). This arrangement is located in a uniform magnetic field of induction B, directed perpendicular to the plane of the ring and directed inwards. The axis and the rim of the ring are connected to a source of emf to form a non-inductive circuit of resistance R.

The kinetic frictional force between the rod and the ring is constant and is equal to f. Neglecting ring resistance, find an expression for the emf in terms of time t, so that the rod rotates with constant angular velocity.

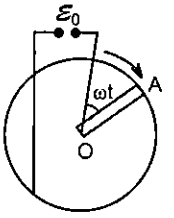


Fig. 5E.33 (a)

Solution: F_m = magnetic force acting on rod

f = frictional force

$$\mathcal{E} = -\frac{d\phi}{dt} = \frac{1}{2}Br^2\omega \quad \ldots(1)$$

Net emf of the rod-ring circuit = $\mathcal{E}_0 - \mathcal{E}$.

$$\therefore \quad I = \frac{\mathcal{E}_0 - \mathcal{E}}{R} = \frac{\mathcal{E}_0 - \frac{1}{2}r^2\omega B}{R} \quad \ldots(2)$$

$F_m = BIr \sin 90°$

$F_g = mg$, taking moments about O,

$$\therefore \quad F_m \times \frac{r}{2} + f \times r = mg \sin \omega t \times \frac{r}{2}$$

$$\therefore \quad \left\{\frac{\mathcal{E}_0 - \frac{1}{2}r^2\omega B}{R}\right\} rB = mg \sin \omega t - 2f$$

$$\therefore \quad \mathcal{E}_0 = \frac{1}{2rB}\{r^3B^2\omega + 2mgR \sin \omega t - 4fR\}$$

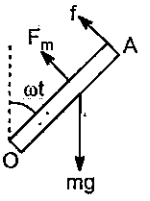


Fig. 5E.33 (b)

Example 34

In Fig. 5E.34 shown, AB is a conducting rod of length 'l', mass 'm' and resistance R. The rod is free to rotate in a horizontal plane about a vertical axis through the end A. The end B of the rod is always in contact with a conducting ring of negligible resistance as shown in the Fig. 5E.34. The end A of the rod and point C on the circumference of the ring are connected to a cell of emf '$\mathcal{E}$'. Find:

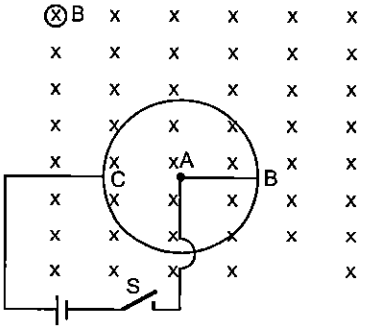


Fig. 5E.34

(a) the angular speed of the rod as a function of time,
(b) the heat produced in the rod in the first t seconds.

Solution: (a) Let at any moment the angular speed of the rod be ω. Hence emf induced across the end of the rod is $\frac{B\omega l^2}{2}$.

Therefore current in the rod is

$$I = \frac{\left(\mathcal{E} - \frac{B\omega l^2}{2}\right)}{R} = \frac{2\mathcal{E} - B\omega l^2}{2R}$$

Consider the differential element of the rod of length dx at distance x from the point A. Torque on this element about A is

$$d\tau = B\left(\frac{2\mathcal{E} - B\omega l^2}{2R}\right) x\,dx$$

$$\Rightarrow \quad \tau = \int d\tau = B\left(\frac{2\mathcal{E} - B\omega l^2}{2R}\right)\int_0^l x\,dx$$

$$\Rightarrow \quad \frac{ml^2}{3}\frac{d\omega}{dt} = \frac{B(2\mathcal{E} - B\omega l^2)l^2}{4R}$$

$$\Rightarrow \quad \frac{d\omega}{dt} = \frac{3B(2\mathcal{E} - B\omega l^2)}{4Rm}$$

$$\Rightarrow \quad \int_0^{\omega}\frac{d\omega}{(2\mathcal{E} - B\omega l^2)} = \frac{3B}{4Rm}\int_0^t dt$$

$$\Rightarrow \quad \omega = \frac{2\mathcal{E}}{Bl^2}(1 - e^{-3B^2l^2t/4Rm})$$

(b) Instantaneous current in the rod is

$$I = \frac{2\mathcal{E} - Bl^2\omega}{2R} = \frac{\mathcal{E}}{R}e^{-3B^2l^2t/4Rm}$$

Hence heat produced $Q = \int_0^t I^2R\,dt$

or $$Q = \frac{2\mathcal{E}^2m}{3B^2l^2}(1 - e^{-3B^2l^2t/2Rm})$$

Example 35

A conducting rod PQ of mass M rotates without friction on a horizontal plane about O as circular rails of diameter 'l'. The centre O and the periphery are connected by resistance R. The system is located in a uniform magnetic field perpendicular to the plane of the loop. At t = 0, PQ starts rotating clockwise with angular velocity ω_0 see Fig. 5E.35 (a). Neglecting the resistance of the rails and rod, as well as self-inductance, find:
(a) magnitude of current as a function of time
(b) total charge flown through the resistance
(c) the heat generated in the circuit by t = ∞.

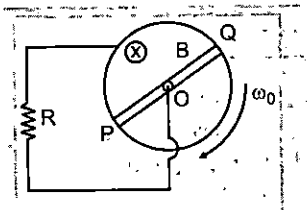


Fig. 5E.35 (a)

Solution: (a) Let ω be the angular velocity at time t. The induced emf across OP (or OQ)

$$= \frac{1}{2}B\omega\left(\frac{l}{2}\right)^2 = \frac{B\omega l^2}{8}$$

$$\text{Current } I = \frac{B\omega l^2}{8R} \qquad \ldots(1)$$

Torque on the rod $= 2\int_0^{l/2}(BIx\,dx) = \frac{Bll^2}{4}$

Substituting the value of I from (1),

$$\frac{Ml^2}{12}\left(\frac{d\omega}{dt}\right) = -\frac{B^2\omega l^4}{32R}$$

or $$-\int_{\omega_0}^{\omega}\frac{d\omega}{\omega} = \frac{3B^2l^2}{8RM}\int_0^t dt$$

Solving this equation, we get

$$\omega = \omega_0 e^{-\alpha t}$$

where $$\alpha = \frac{3B^2l^2}{8RM}$$

Substituting in Eqn. (1), we get

$$I = \frac{B\omega_0 l^2}{8R}e^{-\alpha t}$$

(b) $$Q = \int_0^{\infty} I\,dt = \frac{B\omega_0 l^2}{8R\alpha}$$

$$= \left(\frac{B\omega_0 l^2}{8R}\right)\left(\frac{8RM}{3B^2l^2}\right) = \frac{\omega_0 M}{3B}$$

(c) Heat generated $= \frac{1}{2}I\omega_0^2 = \frac{Ml^2\omega_0^2}{24}$

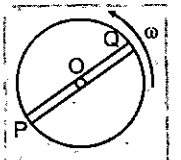


Fig. 5E.35 (b)

Example 36

Two long parallel conducting rods with negligible resistance are connected by capacitor C and semicircular conducting wire in the same plane as shown. This system lies in a perpendicular magnetic field.
Now loop ACB is moved on rails with its displacement given by $x = a\sin\omega t$ from the initial position. Find the 'maximum' current in the capacitor.

Solution: Let in initial position $BM = AN = l$. At any time area of closed loop

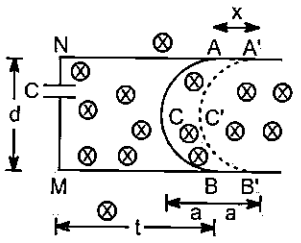


Fig. 5E.36

$$A'NMB' = A$$

$$A = (l + x)\,d - \pi\frac{d^2}{8}$$

Flux through the loop,

$$\phi = A.B = B\left[ld - \frac{\pi d^2}{8} + xd\right]$$

Emf induced, $$\mathcal{E} = \left|\frac{d\phi}{dt}\right| = B\frac{d}{dt}\left(ld - \frac{\pi d^2}{8}\right) + \frac{d}{dt}(Bxd)$$

$$= Bd\frac{dx}{dt} = Bd\frac{d}{dt}(a\sin\omega t) = Bda\omega\cos\omega t$$

As net potential drop in the loop is zero,

$$|\mathcal{E}| = \frac{q}{C}$$

$$\Rightarrow \quad q = CBa\omega d\cos\omega t$$

$$I = \frac{dq}{dt} = -CBad\omega^2\sin\omega t$$

$$I_{max} = CBad\omega^2$$

Example 37

In the arrangement shown in Fig. 5E.37, there is a uniform magnetic field B_0 normal to the plane of paper. The connector is smooth and conducting and it has a mass m and length l. The connector is pushed against the spring so that the spring has compression x_0. The connector is released at t = 0. Find the time it will take to come to its original position again. The spring is non-conducting. The resistance of the rails is zero and neglect its self-inductance.

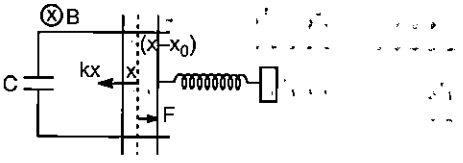


Fig. 5E.37

Solution: Let us say at any time t the rod is at a distance x from its original position.

$$v = \frac{dx}{dt}$$

$$\mathcal{E}\ (\text{induced emf}) = Blv = Bl\frac{dx}{dt}$$

Charge on the capacitor $q = C\mathcal{E} = BlC\dfrac{dx}{dt}$

$$\text{Current } \mathcal{E} = \frac{dq}{dt} = BlC\frac{d^2x}{dt^2}$$

The force on the connector due to this current,

$$F = IlB = B^2l^2C\frac{d^2x}{dt^2}$$

Net force on the connector to the right,

$$m\frac{d^2x}{dt^2} = -B^2l^2C\frac{d^2x}{dt^2} + kx$$

$$\Rightarrow \quad \frac{d^2x}{dt^2} = \frac{k}{m + B^2l^2C}x$$

The acceleration is proportional to x and opposite to it and hence we can say the motion will be SHM.

With $$T = 2\pi\sqrt{\frac{m + B^2l^2C}{k}}$$

and the time taken will be

$$\frac{T}{4} = \frac{\pi}{2}\sqrt{\frac{m + B^2l^2C}{k}}$$

INDUCED EMF AND ELECTRIC FIELDS

The induced electric currents are changes in motion which move in response to electric forces caused by electric fields, according to the equation

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}}$$

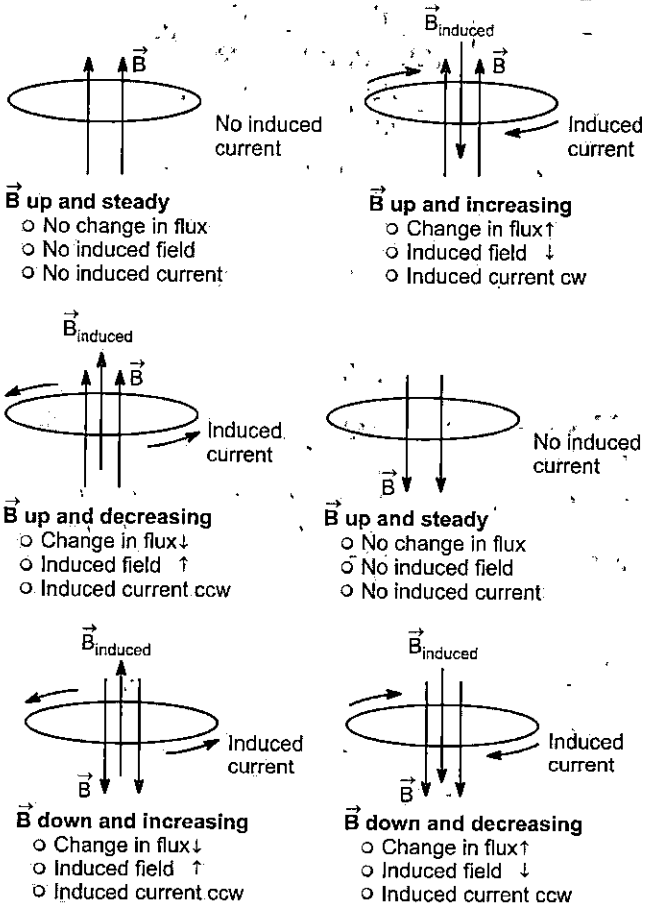


Fig. 5.34

The changing magnetic flux through a loop induces an electric field called an **induced electric field,** which causes the changes to move.

We consider a circular loop of area A, lying in a magnetic field $\vec{B}$ parallel to area vector $\vec{A}$. When the magnetic field increases in magnitude, the magnetic flux through the loop increases, the direction of induced current is observed to be in clockwise sense. When the magnetic field decreases in magnitude, the magnetic flux through the loop decreases and the direction of induced current is observed to be in anticlockwise sense. *The induced electric field is directed around the loop in the same sense as the induced current.* **The electric field lines representing the induced electric field form closed curves.**

Even if the conductor is absent the induced electric field is still present in space. The work done by induced electric field in taking a charge q around a circular closed path when the magnetic flux is changing,

$$w_{\text{elect}} = \oint \vec{F} \cdot d\vec{l}$$

[$\oint$ sign denotes integration along closed path]

$$= q\oint \vec{E}_{\text{induced}} \cdot d\vec{l}$$

The induced electric field is parallel to $d\vec{l}$ around the entire path; so

$$w_{\text{elect}} = qE_{\text{induced}} \oint dl$$

$$= qE_{\text{induced}} (2\pi r)$$

The induced emf by definition is

$$\frac{w_{\text{elect}}}{q} = \oint \vec{E} \cdot d\vec{l}\ .$$

Note that *net work done by induced electric field is not zero; hence it is a* ***non-conservative force.***

The electrostatic electric field arising from stationary charges is conservative; it is not along a closed path. Static electric field lines begin on positive charge and end on negative charge. The induced electric field lines from closed path so that their flux through any closed path is zero; the Gauss's law for electric field still holds.

All the electric fields we have examined until now have been created by charges. Electric field vectors pointed away from positive charges and toward negative charges. An electric field created by charges is called a **Coulomb electric field.** The induced electric field of Fig. 5.35 is caused not by charges but by a changing magnetic field. It is called **non-Coulomb electric field.** There are two different ways to create an electric field:

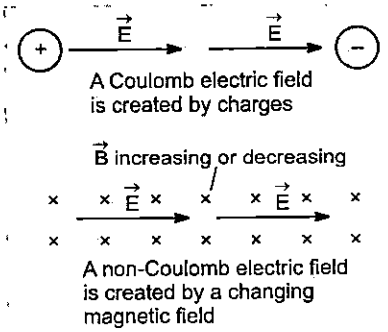


Fig. 5.35 : Two ways to create an electric field

1. A Coulomb electric field is created by positive and negative charges.

2. A non-Coulomb electric field is created by a changing magnetic field.

Both exert force $\vec{F} = q\vec{E}$ on a charge, and both create a current in a conductor. However, the origins of the fields are very different.

The electric field is not just a pictorial representation; it is real.

$$\oint \vec{E} \cdot d\vec{s} = A\left|\frac{dB}{dt}\right|$$

Above equation is an alternative statement of Faraday's law that relates the induced electric field to the changing magnetic field.

The solenoid if Fig. 5E.36 (a) provides a good example of the connection between $\vec{E}$ and $\vec{B}$. If there were a conducting loop inside the solenoid, we could used Lenz's law to determine that the direction of the induced current would be clockwise. But Faraday's law, tells us that **an induced electric field is present whether there's a conducting loop or not.** The electric field is induced simply due to the fact that $\vec{B}$ is changing.

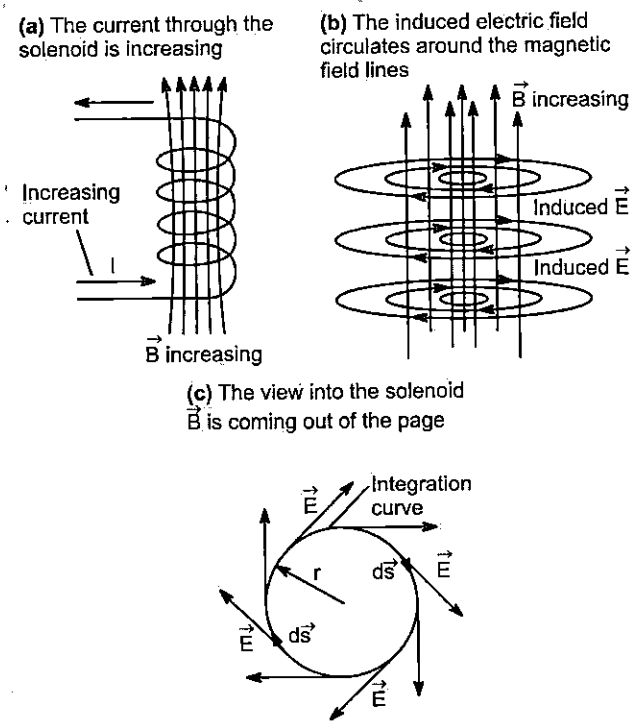


Fig. 5.36

The shape and direction of the induced electric field have to be such that it *could* drive a current around a conducting loop, if one were present, and it has to be consistent with the cylindrical symmetry of the solenoid.

The only possible choice, shown in Fig. 5.36 (b) is an electric field that circulates clockwise around the magnetic field lines.

Circular electric field lines violate the rule that electric field lines have to start and stop on charges. However, that rule applied only to Coulomb field created by source charges. An induced electric field is a non-Coulomb field created not by source charges but by a changing magnetic field. Without source charges, induced electric field lines *must* from closed loops.

Because of cylindrical symmetry, the electric field $\vec{E}$ has the same magnitude at every point on the circle and is tangent to it at each point. Symmetry would also permit the field to be *radial,* but then Gauss's law would require the presence of a net charge inside the circle, and there is none.

The Faraday's law of electromagnetic induction states that **the induced emf** (the work done per unit charge by induced electric field around a closed path is equal to the negative of the time rate of change of the magnetic flux through the same path.

$$\text{Induced emf} = \oint \vec{E} \cdot d\vec{l} = -\frac{d\phi}{dt}$$

Now we apply the Faraday's law to circular loop under consideration.

$$\phi_B = \int \vec{B} \cdot d\vec{A} = BA$$

$$\frac{d\phi_B}{dt} = A\frac{dB}{dt}$$

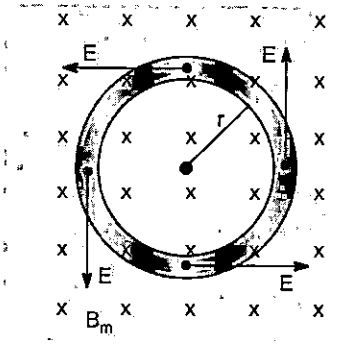
$$E_{\text{induced}}\, 2\pi r = -A\frac{dB}{dt} = -\pi r^2 \frac{dB}{dt}$$

$$E_{\text{induced}} = -\frac{r}{2}\frac{dB}{dt}$$

Thus **a changing magnetic field gives rise to an induced electric field.** These electric and magnetic fields are mutually perpendicular.

An electric field is created in the conductor as a result of the changing flux. This induced electric field has two important properties that distinguish it form the electrostatic field produced by stationary charges: the induced field is non-conservative and time varying.

Consider a conducting loop of radius r situated in a uniform magnetic field that is perpendicular to the plane of the loop, as shown in Fig. 5.37. If the magnetic field changes with time, then, according to Faraday's law, an emf $\mathcal{E} = -d\phi_B/dt$ is induced in the loop. The induction of a current in the loop implies the presence of an induced electric field $\vec{E}$, which must be tangent to the loop because all points on the loop are equivalent. The work done in moving a test charge q once around the loop is equal to $q\mathcal{E}$. Because the electric force acting on the charge is $q\vec{E}$, the work done by this force in moving the charge once around the loop is $qE(2\pi r)$, where $2\pi r$ is the circumference of the loop. These two expressions for the work must be equal; therefore, we see that



A conducting loop of radius r in a uniform magnetic field perpendicular to the plane of the loop. If $\vec{B}$ changes in time, an electric field is induced in a direction tangent to the circumference of the loop.

Fig. 5.37

$$q\mathcal{E} = q\vec{E}(2\pi r)$$

$$\vec{E} = \frac{\mathcal{E}}{2\pi r}$$

Using this result and the fact that $\phi_B = BA = \pi r^2 B$ for a circular loop, we find that the induced electric field can be expressed as

$$\vec{E} = -\frac{1}{2\pi r}\frac{d\phi_B}{dt} = -\frac{r}{2}\frac{dB}{dt} \qquad \ldots(1)$$

The negative sign indicates that the induced electric field opposes the change in the magnetic field.

The emf for any closed path can be expressed as the line integral of $\vec{E} \cdot d\vec{s}$ over that path: $\mathcal{E} = \oint \vec{E} \cdot d\vec{s}$. In more general cases, $\vec{E}$ may not be constant, and the path may not be a circle. Hence, Faraday's law of induction, $\mathcal{E} = -d\phi_B/dt$, can be written in the general form

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\phi_B}{dt} \qquad \ldots(2)$$

The induced electric field $\vec{E}$ is a non-conservative, time-varying field that is generated by a changing magnetic field. The field $\vec{E}$ cannot possibly be an electrostatic field for the following reason: If the field were electrostatic, and hence conservative, the line integral of $\vec{E} \cdot d\vec{s}$ over a closed loop would be zero: this would be in contradiction to Eqn. (2).

Concept: *Conservative and Non-conservative Electric Fields:* *Static electric fields, those beginning and ending on stationary charge distribution, are conservative, meaning that the work required to move a charge between two points is path independent. A consequence is that it takes no work to move around a closed path in an electrostatic field; mathematically,*

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{r}} = 0 \qquad \text{(electrostatic field)}$$

Induced electric field generally form closed loops, and here Faraday's law shows that the line integral of the electric field around a closed path is decidedly not zero. That means the induced electric field does work on a charge moved around a closed path and that the work done in moving between two points cannot be independent of the path taken (Fig. 5.38). The induced electric field is therefore not conservative.

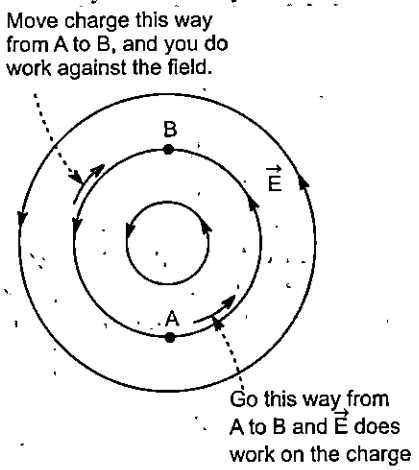


Fig. 5.38

The Fig. 5.39 shows three resistors in series surrounding an infinitely long solenoid with a changing magnetic field; the resulting induced electric field drives a current counterclockwise, as shown. Two identical voltmeters are shown connected to the same points A and B. What does each read ?

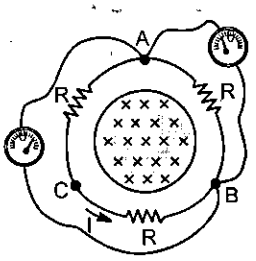


Fig. 5.39

Left-hand meter reads 2IR, right-hand meter reads IR—ever though they're electrically connected to the same points. There's no contradiction because the field isn't conservative, and electric potential therefore can't be defined unambiguously.

Example 38

A long solenoid of radius R has n turns of wire per unit length and carries a time-varying current that varies sinusoidally as $I = I_{max} \cos \omega t$, where I_{max} is the maximum current and ω is the angular frequency of the alternating current source (Fig. 5E.38). Determine the magnitude of the induced electric field outside the solenoid, a distance $r > R$ from its long central axis.

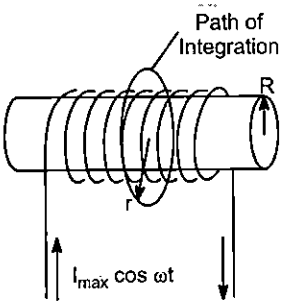


Fig. 5E.38

Solution: First let us consider an external point and take the path for our line integral to be a circle of radius r centered on the solenoid, as illustrated in Fig. 5E.38. By symmetry we see that the magnitude of $\vec{\mathbf{E}}$ is constant on this path and that $\vec{\mathbf{E}}$ is tangent to it. The magnetic flux through the area enclosed by this path is $BA = B\pi R^2$; hence Equation gives

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\frac{d}{dt}(B\pi R^2) = -\pi R^2 \frac{dB}{dt}$$

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = E(2\pi r) = -\pi R^2 \frac{dB}{dt} \qquad \ldots(1)$$

The magnetic field inside a long solenoid is given by $B = \mu_0 nI$. When we substitute $I = I_{max} \cos \omega t$ into this equation and then substitute the result into Eqn. (1), we find that

$$E(2\pi r) = -\pi R^2 \mu_0 n I_{max} \frac{d}{dt}(\cos \omega t)$$

$$= \pi R^2 \mu_0 n I_{max} \omega \sin \omega t$$

$$E = \frac{\mu_0 n I_{max} \omega R^2}{2r} \sin \omega t \quad (\text{for } r > R) \qquad \ldots(2)$$

Hence, the electric field varies sinusoidally with time and its amplitude falls off as $1/r$ outside the solenoid.

Proceeding as above, we find that

$$E(2\pi r) = -\pi r^2 \frac{dB}{dt} = \pi r^2 \mu_0 n I_{max} \omega \sin \omega t$$

$$E = \frac{\mu_0 n I_{max} \omega}{2} r \sin \omega t \quad (\text{for } r < R) \qquad \ldots(3)$$

This shows that the amplitude of the electric field induced inside the solenoid by the changing magnetic flux through the solenoid increases linearly with r and varies sinusoidally with time.

Example 39

A line charge λ is wound around an insulating disc of mass M and radius R, which is then suspended horizontally as shown in Fig. 5E.39, so that it is free to rotate. In the central region, out to radius a, there is a uniform magnetic field B_0, pointing up. Now the magnetic field is switched off, which causes the disc to rotate. Find the angular speed ω with which the disc starts rotating.

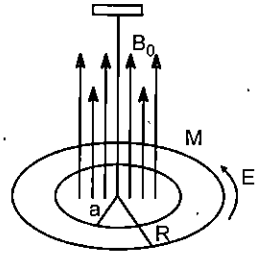


Fig. 5E.39

Solution: The induced electric field E due to changing magnetic field is given by (from Faraday's law)

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}} = \frac{-d\phi_B}{dt} = -\pi a^2 \frac{dB}{dt}$$

or $$E.2\pi R = -\pi a^2 \frac{dB}{dt} \quad \Rightarrow \quad E = \frac{-a^2}{2R}\frac{dB}{dt}$$

Hence induced electric field is tangential to the disc as shown in the figure and its magnitude is

$$E = \frac{a^2}{2R}\frac{dB}{dt}$$

This electric field causes the disc to rotate. Now torque on the disc is

$$\tau = (\lambda 2\pi R)\cdot E \cdot R = \pi\lambda a^2 R\frac{dB}{dt}$$

If ω is the angular speed of the disc, then

$$\tau = \frac{dL}{dt} \quad \text{(where } L \text{ is the angular momentum)}$$

or $$\pi\lambda a^2 R\frac{dB}{dt} = \frac{dL}{dt}$$

or $$\pi\lambda a^2 R(B_0 - 0) = \left(\frac{MR^2}{2}\omega - 0\right)$$

or $$\omega = \frac{2\pi\lambda a^2 B_0}{MR}$$

Example 40

A uniform magnetic field of induction B fills a cylindrical volume of radius R. A rod AB of length 2l is placed as shown in Fig. 5E.40 (a). If B is changing at the rate $\frac{dB}{dt}$, find the emf that is produced by the changing magnetic field and that acts between the ends of the rod.

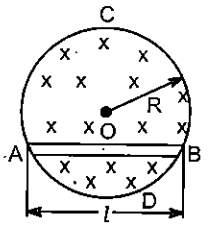


Fig. 5E.40 (a)

Solution: Consider a point on the circumference of a circle of radius r ($r < R$). Let E be the electric field along the tangents to the circle. Then $\mathcal{E}$ (electromotive force) $= \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}} = E \cdot 2\pi r$.

Also $$\mathcal{E} = -\frac{d\phi_B}{dt} = -\frac{d}{dt}(\pi r^2 B)$$

$$= -\pi r^2 \frac{dB}{dt}.$$

$$\therefore \quad E \times 2\pi r = -\pi r^2 \frac{dB}{dt}$$

$$\Rightarrow \quad E = -\frac{1}{2} r \frac{dB}{dt}.$$

The minus sign suggests that the induced electric field acts to oppose the change of the magnetic field.

Now consider a point P in the rod and a small distance $dl = PQ$ along AB. The electric field at P is $E = \frac{1}{2} r \frac{dB}{dt}$ along tangent to the circle as shown in the Fig. 5E.40 (b).

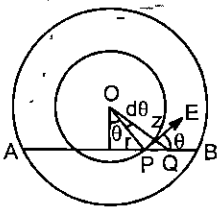


Fig. 5E.40 (b)

Elementary work done $= (E\cos\theta)\,dl = E\,dl\cos\theta$

Draw perpendicular PN from P on OQ.

Then $PN = r\,d\theta = dl\cos\theta$

$\therefore$ Elementary work done $= Erd\theta = \left(\frac{1}{2} r\frac{dB}{dt}\right) rd\theta$

$$= \left(\frac{1}{2} r^2 d\theta\right)\frac{dB}{dt}.$$

But $\frac{1}{2} r^2 d\theta$ = area of the triangle OPQ

$\therefore$ Total work done in taking unit charge from A to B.

$= \frac{dB}{dt} \times$ summation of the areas of elementary triangle

$$= \frac{dB}{dt} \times \text{area } OAB.$$

= P. D. between the ends of AB.

$$\therefore \quad V = \frac{dB}{dt}\cdot\frac{1}{2}\cdot 2\cdot l\sqrt{R^2 - l^2}$$

$$= \frac{dB}{dt} l\sqrt{R^2 - l^2}.$$

Example 41

A long solenoid contains another coaxial solenoid (whose radius R is half of its own). Their coils have the same number of turns per unit length and initially both carry no current. At the same instant currents start increasing linearly with time in both solenoids. At any moment the current flowing in the inner coil is twice as large as that in the outer one and their directions are the same. As a result of the increasing currents a charged particle, initially at rest between the solenoids, starts moving along a circular trajectory (see Fig. 5E.41). What is the radius r of the circle?

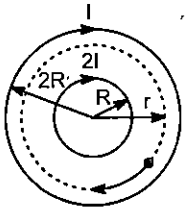


Fig. 5E.41

Solution: The current at time t is $I = kt$ in the outer coil, and $2I = 2kt$ in the inner one, where k is a constant. The magnetic field is

$$B(R < r < 2R) = \mu_0 nkt = B_0$$

$$B(r < R) = 3\mu_0 nkt = 3B_0$$

where n is the number of turns per unit length. The magnetic flux enclosed by the particle's trajectory of radius r is

$$\phi = \pi R^2 \times 2B + \pi r^2 \times B = (2R^2 + r^2)\pi\mu_0 nkt.$$

The (constant) magnitude of the induced electric field E is given by

$$E \times 2\pi r = \frac{d\phi}{dt} = (2R^2 + r^2)\pi\mu_0 nk,$$

and so $$E = \frac{(2R^2 + r^2)}{r}\frac{\mu_0 nk}{2}.$$

The charged particle is held in its circular orbit by the magnetic field, so we have

$$\frac{mv^2}{r} = qvB. \qquad \ldots(1)$$

The particle is accelerated along its circular orbit by the tangential component of the net force according to $ma_t = qE$, where m is the mass and q the electric charge of the particle.

As the magnitude of the electric field is constant, the speed of the particle increases uniformly with time,

$$v = a_t t = \frac{qE}{m}t$$

$$= \frac{(2R^2 + r^2)}{r}\frac{\mu_0 nk}{2}\frac{q}{m}t$$

Inserting this and the value of B into equation (1), we get

$$\frac{m}{r}\frac{(2R^2 + r^2)}{r}\frac{\mu_0 nk}{2}\frac{q}{m}t = q\mu_0 nkt,$$

which is satisfied if

$$\frac{(2R^2 + r^2)}{2r^2} = 1, \quad i.e., \quad r = \sqrt{2}R.$$

Example 42

Charge Q is uniformly distributed on a thin insulating ring of mass m which is initially at rest. To what angular velocity will the ring be accelerated when a magnetic field B, perpendicular to the plane of the ring, is switched on?

Solution: In accordance with Faraday's law of electromagnetic induction the changing magnetic field induces an electric field in the ring. Let us imagine the ring to be divided into differential elements of length ds and denote the tangential component of the induced electric field by E_t. The charge on element ds of the ring is

$$dQ = Q\frac{ds}{2\pi r}$$

where r is the radius of the ring. The force exerted on it is

$$dF_t = dQE_t$$

and the resultant torque is

$$d\tau = r\,dF_t.$$

The total torque experienced by the ring is thus

$$\tau = \int d\tau = \int rQ\frac{ds}{2\pi r}E_t = \frac{Q}{2\pi}\int E_1\,ds.$$

The induced electromotive force along the ring is directly proportional to the rate of change in the magnetic flux, we have

$$\int E_t ds = -\frac{d\phi}{dt} = -\pi r^2\frac{dB}{dt}.$$

As a result of the torque, the ring, which has a moment of inertia $I = mr^2$, starts to spin with angular acceleration α. During a time interval dt its angular velocity changes by

$$d\omega = \alpha\,dt = \frac{\tau}{I}dt$$

$$= \frac{Q}{2\pi}\left(-\pi r^2\frac{dB}{dt}\right)\frac{1}{mr^2}dt = -\frac{Q}{2m}dB.$$

Since the magnetic field strength increases from zero to B, the final angular velocity of the ring will be

$$\omega = -\frac{QB}{2m}.$$

- The negative sign shows that the direction of the angular velocity vector is opposite to the magnetic induction if Q is positive.
- The final angular velocity does not depend on the radius of the ring, the time over which the magnetic flux changes, or even on how the magnetic flux increases with time.
- In our calculation we ignored the magnetic field produced by the rotating ring.

- Except in the case of a *cylindrically symmetric* uniform field, it is not possible to find the actual value of the induced electric field within the ring because the geometrical structure of the magnetic field is unknown and we do not know the position of the ring in the magnetic field. We can determine the *total* induced electromotive force, but not the electric field itself.

EDDY CURRENT

Fig. 5.40 show a rigid square loop of wire between the poles of a magnet. The magnetic field in Fig. 5.40 (a) passes through the loop, but the wires are not in the field. None of the charge carriers in the wire experience a magnetic force, so there is no induced current and it takes no force to pull the loop to the right.

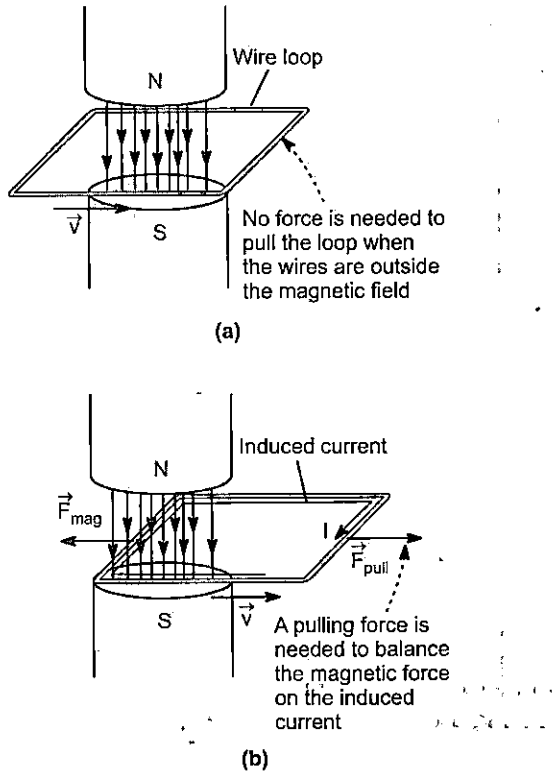


Fig. 5.40: Pulling a loop of wire out of a magnetic field

But when the left edge of the loop enters the field, as shown in Fig. 5.40 (b), the magnetic force on the charge carriers induces a current in the loop. The magnetic field then exerts a retarding magnetic force on this current, so **a pulling force must be exerted to pull the loop out of the magnetic field.** Note that the wire, typically copper, is not a magnetic material. It is the induced current in the complete loop that causes the wire to experience a retarding force.

Consider pulling a sheet of metal through a magnetic field, as shown in Fig. 5.41 (a). The metal is not a magnetic material, so it experience no magnetic force if it is at rest. The charge carriers in the metal experience a magnetic force as the sheet is dragged between the pole tips of the magnet. A current is induced, just as in the loop of wire, but here the currents do not have wires to define their path. As a consequence, localized loops of current begin to circulate in the metal. These spread-out current localized loops in a solid metal are called **eddy currents.**

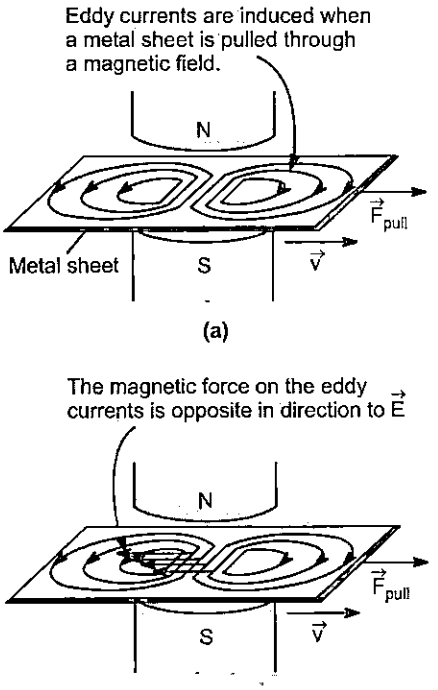


Fig. 5.41: Eddy currents

Fig. 5.41 (b) shows the magnetic force on the eddy current as it passes between the pole tips. This force is to the left, acting as a retarding force. Thus **an external force is required to pull a metal through a magnetic field.** If the pulling force ceases, the retarding magnetic force quickly causes the metal to decelerate until it stops. Similarly, a force is required to push a sheet of metal into a magnetic field.

The power dissipation of eddy currents can cause unwanted heating, and the magnetic forces on eddy currents implies that extra energy must be expended to move metals in magnetic fields.

The moving train car has an electromagnet that straddles the rail, as shown in Fig. 5.42. During normal travel, there is no current through the electromagnet and no magnetic field. To stop the car, a current is switched into the electromagnet. The magnetic force between the electromagnet and the eddy currents acts as a braking force on the magnet and, thus, on the car.

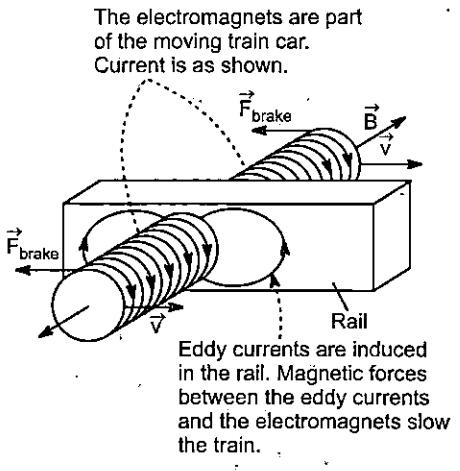
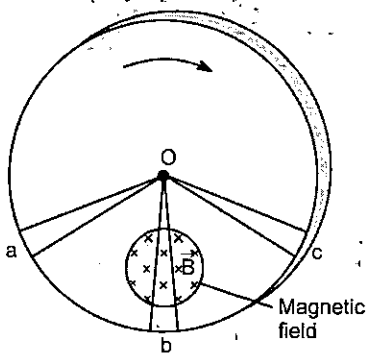
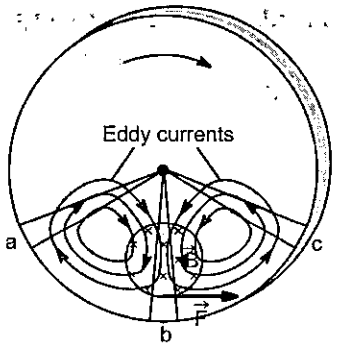


Fig. 5.42: Magnetic braking systems are an application of eddy currents.

Consider a metallic disk rotating in a magnetic field perpendicular to the plane of the disk but confined to a limited portion of the disk's area, as shown in Fig. 5.43 (a) Sector *Ob* is moving across the field and has an emf induced in it. Sectors *Oa* and *Oc* are not in the field, but they provide return conducting paths for charges displaced along *Ob* to return from *b* to *O*. The result is a circulation of eddy currents in the disk, somewhat as sketched in Fig. 5.43 (b).



(a) Metal disk rotating through a magnetic field



(b) Resulting eddy currents and braking force

Fig. 5.43

This current experiences a magnetic force $\vec{\mathbf{F}} = I\vec{\mathbf{L}} \times \vec{\mathbf{B}}$ that opposes the rotation of the disk, and so this force must be to the right in Fig. 5.43 (b). The return currents lie outside the field, so they do not experience magnetic forces. The interaction between the eddy currents and the field causes a braking action on the disk. Such effects can be used to stop the rotation of a circular saw quickly when the power is turned off.

Metal Detectors

Metal detectors work because of induced currents.

A metal detector, shown in Fig. 5.44, consist of two coils: a transmitter coil and a receiver coil. A high-frequency alternating current in the transmitter coil generates an alternating magnetic field along the axis. This magnetic field creates a changing flux through the receiver coil and causes an alternating induced current.

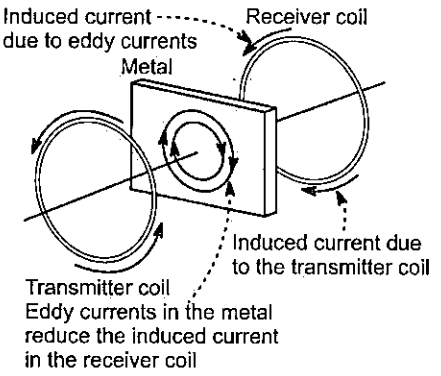


Fig. 5.44: A metal detector

Suppose a piece of metal is placed between the transmitter and the receiver. The alternating magnetic field through the metal induces eddy currents in a plane parallel to the transmitter and receiver coils. The receiver coil then responds to the superposition of the transmitter's magnetic field and the magnetic field of the eddy currents. The eddy currents attempt to prevent the flux from changing, the net field at the receiver decreases when a piece of metal is inserted between the coils.

INDUCTANCE

Self Induction

Consider a solenoid as shown in the figure. The current I in the solenoid creates a magnetic field directed along axis of the solenoid. Let the current I be increasing with time. Hence magnetic flux also increases with time. According to Lenz's law, a current is induced in the coil in a direction that opposes the change in flux taking place. Since the flux is increasing with time, the direction of induced current (due to induced emf) tends to decrease the flux through the solenoid by decreasing the current I. Thus the induced current direction opposes that of I and the polarity of the induced flux is as shown in the figure. If the current is

decreasing in the solenoid, the decreasing flux induces an emf in the same direction as the current, thus tending to maintain the applied current.

The magnitude of magnetic field inside a long solenoid, having n turns per unit length, is

$$B = \mu_0 nI$$

The flux through a single coil of the solenoid is BA, where A is the cross-sectional area. The number of turns of wire in length l of the solenoid is nl. Thus the total flux

$$\phi_B = nlBA = nl(\mu_0 nI)A$$
$$= n^2 lA\mu_0 I$$

Thus we can see that

$$\phi_B \propto I, \qquad \phi_B = LI$$

The constant of proportionality ($L = \mu_0 n^2 Al$) is called **self-inductance** of the solenoid. The solenoid is a circuit element called **inductor**. The self-inductance depends on geometric factors associated with the coil; L is independent of the magnetic flux through the coil and the current in it. In a circuit diagram the inductor is shown by the symbol.

—WWW— (inductor symbol)

The magnetic flux ϕ_B is proportional to the current I in the coil; so we define self-inductance as

$$L = \frac{N\phi_B}{I}$$

From Faraday's law,

$$\mathcal{E}_{\text{induced}} = -N\frac{d\phi_B}{dt} = -L\frac{dI}{dt}$$

Suppose a current passes through an inductor from left to right and is increasing with time. From Lenz's law the induced emf must oppose the change in magnetic flux. The induced emf opposes the increasing flux by acting like a source of emf that opposes the external source of emf driving the current (see figure). So we treat an inductor as a common circuit element labelled with polarity (+) and (–) marked according to Lenz's law.

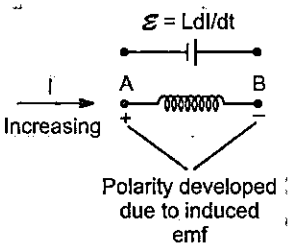


Fig. 5.45

If the current is decreasing, then by Lenz's law the induced emf acts to help the decreasing flux. The inductor acts like a source of emf reinforcing the external emf driving the current. The induced emf acts to increase I (see figure).

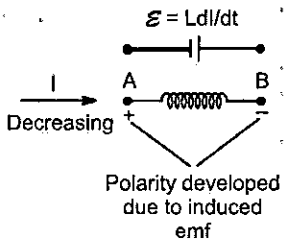


Fig. 5.46

Thus we can consider an inductor as a battery whose polarity is decided according to Lenz's law.

Concept:1-

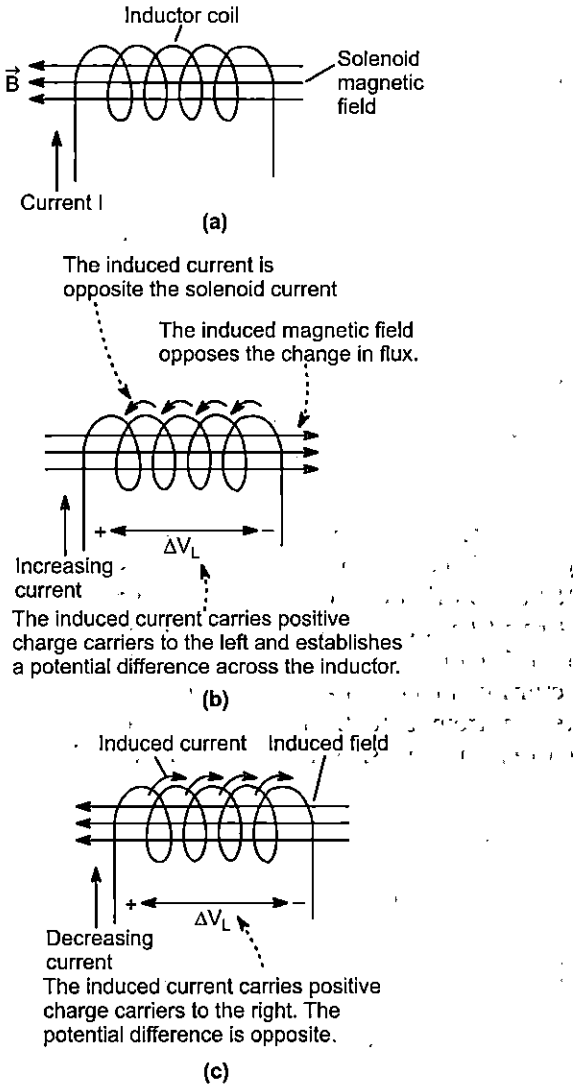


Fig. 5.47

2. Convention for an Inductor: *The potential difference across an inductor, measured along the direction of the current, is*

$$\Delta V_L = -L\frac{dI}{dt}$$

If the current is increasing ($dI/dt > 0$), the input side of the inductor is more positive than the output side and the potential decreases in the direction of the current ($\Delta V_L < 0$). If the current is decreasing ($dI/dt > 0$), the input side is more negative and the potential increases in the direction of the current ($\Delta V_L > 0$).

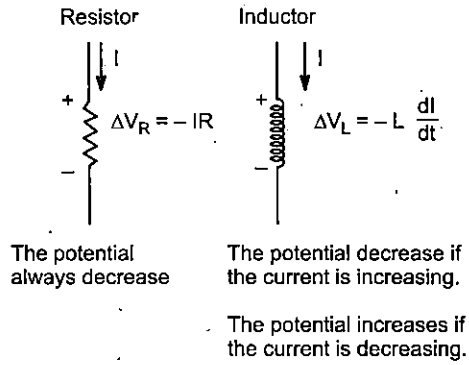


Fig. 5.48: The potential difference across a resistor and an inductor

SERIES AND PARALLEL COMBINATION OF INDUCTORS

Series Combination

Figure shows a collection of inductors in series, all of them will have same current through them. We have for each inductor

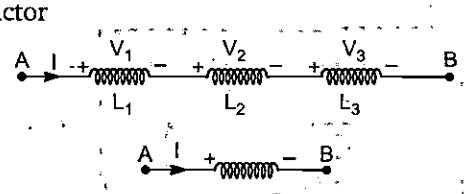


Fig. 5.49

$$V_1 = L_1\frac{dI}{dt}, \qquad V_2 = L_2\frac{dI}{dt}, \qquad V_3 = L_3\frac{dI}{dt}$$

We replace the series with a single equivalent inductance $L_{eq.}$ with the same potential difference V between the terminals A and B, as shown in Fig. 5.49.

$$V = L_{eq.}\frac{dI}{dt}$$

The potential difference V is the sum of individual potential differences.

$$V = V_1 + V_2 + V_3$$

$$L_{eq.}\frac{dI}{dt} = L_1\frac{dI}{dt} + L_2\frac{dI}{dt} + L_3\frac{dI}{dt}$$

$$\boldsymbol{L_{eq.} = L_1 + L_2 + L_3}$$

Parallel Combination

When the circuit elements are in parallel, the potential difference across each circuit element is same; thus for each inductor we have

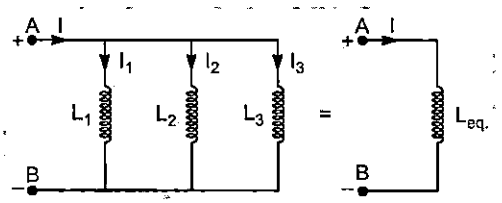


Fig. 5.50

$$V = L_1\frac{dI_1}{dt},\ V = L_2\frac{dI_2}{dt},\quad V = L_3\frac{dI_3}{dt} \qquad ...(1)$$

If we replace the combination with a single equivalent inductor $L_{eq.}$, we have

$$V = L_{eq.}\frac{dI}{dt} \qquad ...(2)$$

From KCL, we get $I = I_1 + I_2 + I_3$

Differentiating this equation with respect to t, we get

$$\frac{dI}{dt} = \frac{dI_1}{dt} + \frac{dI_2}{dt} + \frac{dI_3}{dt} \qquad ...(3)$$

On substituting derivatives from eqns. (1) and (2) in eqn. (3), we have

$$\frac{V}{L_{eq.}} = \frac{V}{L_1} + \frac{V}{L_2} + \frac{V}{L_3}$$

$$\frac{1}{L_{eq.}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$$

(inductors in parallel)

SELF-INDUCTANCE AND THE MODIFIED KIRCHHOFF'S LOOP RULE

The addition of time-changing magnetic fields to simple circuits means that the closed line integral of the electric field around a circuit is no longer zero. Instead, we have, for any open surface

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = \frac{d}{dt}\int \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$$

Any circuit where the current changes with time will have time-changing magnetic fields, and therefore induced electric fields. How do we solved simple circuits taking such effects into account ? We discuss here a consistent way to understand the consequences of introducing time-changing magnetic fields into circuit theory-that is, inductance.

As soon as we introduce time-changing magnetic fields, the electric potential difference between two points in our circuit is no longer well-defined, because when the line integral of the electric field around a closed loop is nonzero, the potential difference between two points, say a and b, is no longer independent of the path taken to get from a to b. That is, the electric field is no longer a conservative field, and the electric potential is no longer an appropriate concept, since we can no longer write $\vec{\mathbf{E}}$ as the negative gradient of a scalar potential. However, we can still write down in a straightforward fashion the equation that determines the behavior of a circuit.

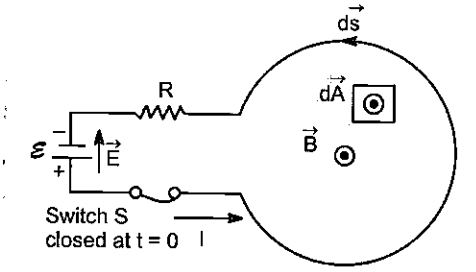


Fig. 5.51: One-loop inductor circuit

To show how to do this, consider the circuit shown in Fig. 5.51. We have a battery, a resistor, a switch S that is closed at $t = 0$, and a "one-loop inductor". It will become clear what the consequences of this "inductance" are as we proceed. For $t > 0$, current will flow in the direction shown (from the positive terminal of the battery to the negative, as usual). What is the equation that governs the behavior of our current $I(t)$ for $t > 0$?

To investigate this, apply Faraday's law to the open surface bounded by our circuit, where we take $d\vec{\mathbf{A}}$ to be out of the page and $d\vec{\mathbf{s}}$ right-handed with respect to that choice (counter-clockwise). First, what is the integral of the electric field around this circuit ? There is an electric field in the battery, directed from the positive terminal to the negative terminal, and when we go through the battery in the direction of $d\vec{\mathbf{s}}$ that we have chosen, we are moving against that electric field, so that $\vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} < 0$. Thus the contribution of the battery to our integral is $-\mathcal{E}$. Then, there is an electric field in the resistor, in the direction of the current, so when we move through the resistor in that direction, $\vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$ is greater than zero, and that contribution to our integral is $+IR$. What about when we move through our one-loop inductor ? There is no electric field in this loop if the resistance of the wire making up the loop is zero. Thus, going around the closed loop clockwise against the current, we have

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\mathcal{E} + IR$$

Now, what is the magnetic flux ϕ_B through our open surface ? First of all, we arrange the geometry so that the part of the circuit which includes the battery, the switch, and the resistor makes only a small contribution to ϕ_B as compared to the (much larger in area) part of the open surface which includes our one-loop inductor. Second, we know that ϕ_B is positive in that part of the open surface, because current flowing counterclockwise will produce a magnetic field $\vec{\mathbf{B}}$ pointing out of the page, which is the same direction we have assumed for $d\vec{\mathbf{A}}$. Thus, the dot product $\vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} > 0$. Note that $\vec{\mathbf{B}}$ is the self magnetic field-that is, the magnetic field produced by the current flowing in the circuit, and not by any external currents.

We also see that the magnetic flux ϕ_B is proportional to I, and may be written as $\phi_B = LI$, where L is the self-inductance which depends on the geometry of the circuit. The time rate of change of ϕ_B is just $L(dI / dt)$ so that we have from Faraday's law

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\mathcal{E} + IR$$

$$= -\frac{d\phi_B}{dt} = -L\frac{dI}{dt}$$

We can write the governing equation for $I(t)$ from above as

$$\Delta V = \mathcal{E} - IR - L\frac{dI}{dt} = 0$$

where the expression has been cast in a form that resembles Kirchhoff's loop rule, namely that the sum of the potential drops around a circuit is zero. To preserve the loop rule, we must specify the "potential drop" across an inductor.

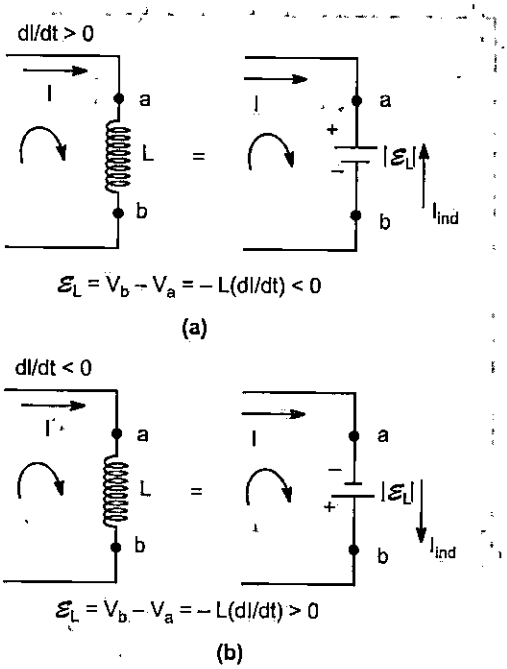


Fig. 5.52: Modified Kirchhoff's rule for inductors (a) with increasing current, and (b) with decreasing current

The modified rule for inductors may be obtained as follows: The polarity of the self-induced emf is such as to oppose the change in current, in accord with Lenz's law. If the rate of change of current is positive, as shown in Fig. 5.52 (a), the self-induced emf $\mathcal{E}_L$ sets up an induced current I_{ind} moving in the opposite *ba* direction of the current I to oppose such an increase. The inductor could be replaced by an emf $|\mathcal{E}_L| = L\,|dI / dt| = +L\,(dI / dt)$ with the polarity shown in Fig. 5.52 (a). On the other hand, if $dI / dt < 0$, as shown in Fig. 5.52 (b), the induced current I_{ind} set up by the self-induced emf $\mathcal{E}_L$ flows in the same direction as I to oppose such a decrease.

We see that whether the rate of change of current in increasing ($dI/dt > 0$) or decreasing ($dI/dt < 0$), in both cases, the change in potential when moving from a to b along the direction of the current I is $V_b - V_a = -L(dI/dt)$. Thus, we have

Kirchhoff's Loop Rule Modified for Inductors

If an inductor is traversed in the direction of the current, the "potential change" is $-L(dI/dt)$. On the other hand, if the inductor is traversed in the direction opposite of the current, the "potential change" is $+L(dI/dt)$.

Use of this modified Kirchhoff's rule will give the correct equations for circuit problems that contain inductors. However, keep in mind that it is **misleading** at best, and at some level **wrong**, in terms of the physics. Again we emphasize that kirchhoff's loop rule was originally based on the fact that the line integral of $\vec{\mathbf{E}}$ around a closed loop was zero.

With time-changing magnetic fields, this is no longer so, and thus the sum of the "potential drops" around the circuit, if we take that to mean the negative of the closed loop integral of $\vec{\mathbf{E}}$, is **no longer zero**—in fact it is $+L(dI/dt)$.

A SERIES *LR* CIRCUIT

In this section, we analyse the transient behaviour of a **series *LR* circuit,** consisting of an emf $\mathcal{E}$, an inductor L and a resistor R in series as shown in Fig. 5.53. This current is analogous to the sereis RC circuit, we studied earlier.

Case I: Current after closing the switch

We wish to determine the variation of current with time after switch (1) is closed at time $t = 0$. Switch (2) is left open. The potential difference across the resistor is IR, and inductor is $L\,dI/dt$. After closing the switch current begin to flow clockwise, the current is increasing. So the inductor acts as a source of emf V whose sense opposes the battery voltage. In order to oppose this current, the direction of the induced emf, must be as indicated by plus and minus sign in the figure. Recall that emf drives the current from their positive terminal around the external circuit toward their negative terminal. Hence the emf provided by the inductor L has shown polarity to oppose the current I.

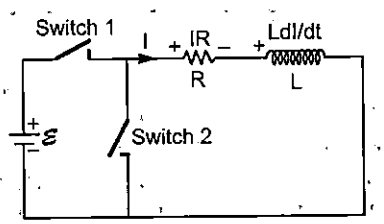


Fig. 5.53

Immediately after closing switch (1), the IR drop across resistor is negligibly small and KVL becomes

$$\mathcal{E} - V_L - 0 \cdot R = 0$$

i.e., the entire battery voltage is initially across the inductor

$$\mathcal{E} - L\left|\frac{dI}{dt}\right|_{t=0} = 0$$

The initial value of $\frac{dI}{dt}$ is thus

$$\left|\frac{dI}{dt}\right|_{t=0} = \frac{\mathcal{E}}{L} \qquad \ldots(1)$$

As I increasing, the IR drop across the resistor increases and the voltage drop across the inductor decreases.

We apply Kirchhoff's voltage law (KVL), traversing the circuit clockwise.

$$-\mathcal{E} + IR + L\frac{dI}{dt} = 0 \qquad \ldots(2)$$

Rearranging the terms, we have

$$\frac{dI}{dt} + \frac{R}{L}I = \frac{\mathcal{E}}{L}$$

This is a first order differential equation.

$$\frac{dI}{dt} = \frac{\mathcal{E}}{L} - I\frac{R}{L} = \frac{R}{L}\left(\frac{\mathcal{E}}{R} - I\right)$$

$$\frac{dI}{\mathcal{E}/R - I} = \frac{R}{L}dt \qquad \ldots(3)$$

To find the current I at any time $t > 0$, we integrate both sides of above equation from time $t = 0$, when the current has the initial value 0, to any particular time t when current is I.

$$\int_0^I \frac{dI}{\mathcal{E}/R - I} = \frac{R}{L}\int_0^t dt$$

$$\left[-\ln\left(\frac{\mathcal{E}}{R} - I\right)\right]_0^I = \frac{R}{L}t$$

$$-\ln\left(\frac{\mathcal{E}/R - I}{\mathcal{E}/R}\right) = \frac{R}{L}t$$

$$\ln\left[1 - \frac{R}{\mathcal{E}}I\right] = \frac{-R}{L}t$$

Taking antilogarithm of both sides of this equation, we have

$$1 - \frac{R}{\mathcal{E}}I = e^{-(R/L)t}$$

$$I = \frac{\mathcal{E}}{R}(1 - e^{-(R/L)t})$$

$$= \frac{\mathcal{E}}{R}(1 - e^{-t/\tau_L}) \qquad \ldots(4)$$

where τ_L is time constant of the circuit.

Note that immediately after closing the switch, when $t = 0$ s, the current is $I = 0$. A long time after closing the switch, as $t \to \infty$, the exponential term tends to zero and the current tends to $\mathcal{E}/R$. *After a long time, I becomes constant; the inductor has no effect. An inductor has effect only while the current is changing with time.*

As $V_L = L\frac{dI}{dt}$, when I is constant, its derivative is zero making $V_L = 0$.

The exponent for e in equation (4) must be dimensionless; hence the quantity L/R must have SI units of

second. This characteristic time is called the time constant of the circuit.

$$\tau = L/R$$

For example, after a time t equal to five time constants ($t = 5\tau$), the exponential term is then $e^{-5} = 0.00674$, and the current I is within 1% of its final value of $\mathcal{E}/R$. After one time constant the current reaches $1/e \simeq .37$ or 37 per cent of maximum value of $\mathcal{E}/R$. A graph of current *versus* time is shown in Fig. 5.54.

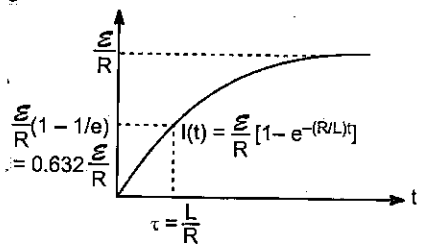


Fig. 5.54

Case II. Disconnecting the battery from circuit

After a sufficiently long time the current reaches its steady state value, the switch (2) is closed just and switch (1) is opened.

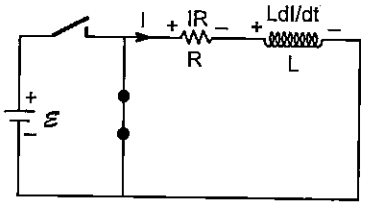


Fig. 5.55

At the moment when the switching occurs (call it $t = 0$) and the current is I_0, the steady state current $\mathcal{E}/R$, prior to closing switch (2), cannot vanish instantaneously. The current continues for some time in the same sense as before. In accordance with the Lenz's law the inductor tends to oppose the change in current. The energy required to drive the current through resistor comes from the inductor, whose magnetic flux is decreasing as the energy is transferred to resistor, where it is dissipated as heat.

We apply KVL to the circuit, traverse the circuit in clockwise sense.

$$-IR - L\frac{dI}{dt} = 0$$

On rearranging the terms and integrating, we have

$$\int_{I_0}^{I} \frac{dI}{I} = -\int_0^t \frac{R}{L}dt$$

where $I = I_0$ at $t = 0$

and $I = I$ at time t.

$$\ln\frac{I}{I_0} = -\frac{R}{L}t$$

or $$I = I_0 e^{-(R/L)t} = \frac{\mathcal{E}}{R}e^{-(R/L)t}$$

This indicates that the current in the circuit decreases exponentially. The exponent R/L has dimensions of time; we have the same time constant as before. After a time t equal to five time constants, the exponential term is $e^{-5} = 0.00674$ and the current I is less than 1% of its initial value.

Concepts:1- *The voltage across a capacitor can't change instantaneously. We can make an analogous statement for inductors. Because the inductor emf depends on the rate of change of current and because an infinite emf is impossible,* ***the current through an inductor can't change instantaneously.***

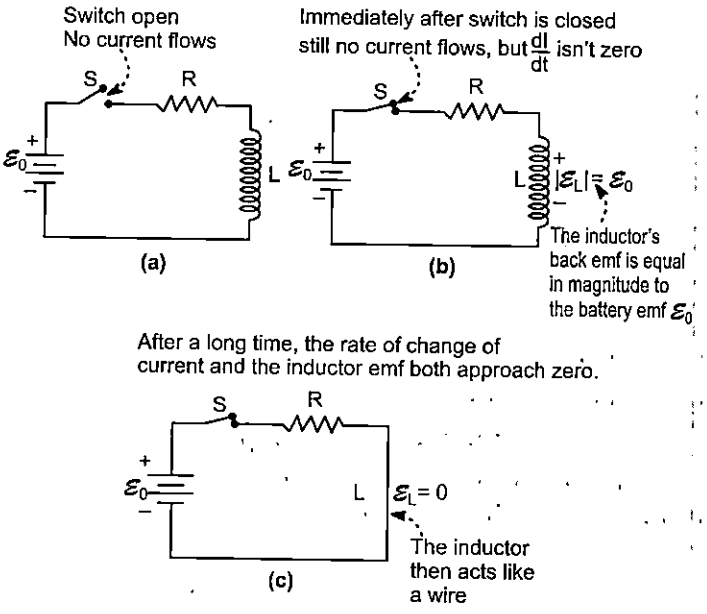


Fig. 5.56: An RL circuit at three different times

Fig. 5.56 shows a circuit with a battery, switch, resistor, and inductor. With the switch open there's no current [Fig. 5.56 (a)]. Close the switch, and the current at that instant is still zero because the inductor current can't change instantaneously. With no current, there's no voltage across the resistor, so the inductor must be producing a back emf equal in magnitude to the battery emf [Fig. 5.56 (b)]

Although at this instant there's zero current in the inductor, the nonzero emf $\mathcal{E}_L = -L(dI/dt)$ shows that the rate of change of current, dI/dt, isn't zero.

So the inductor current rises from zero, and with it the resistor current and therefore the resistor voltage IR. The battery emf $\mathcal{E}_0$ is constant, so as IR increases, the magnitude of the inductor emf drops. Eqn. $\mathcal{E}_L = -L\frac{dI}{dt}$ shows that the rate of change of current drops as well. Eventually the whole circuit reaches a steady state in which dI/dt and therefore the inductor emf are both zero [Fig. 5.56 (c)]. At this point the inductor acts like a wire, and the resistor determines the current: $I = \mathcal{E}_0/R$. Fig. 5.57 summarizes this analysis of the RL circuit.

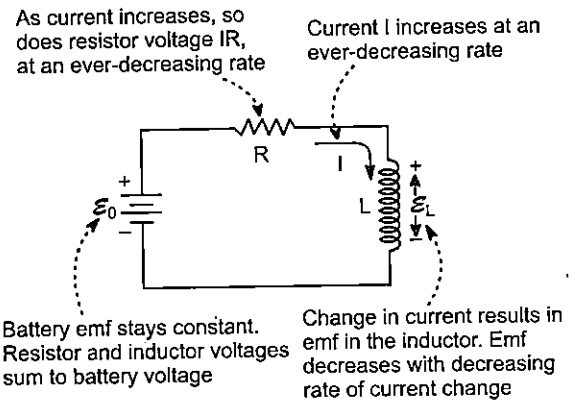


Fig. 5.57

2. *Figure 5.58 shows a circuit with a two-way switch. Throw the switch to A, and current builds up as we just described. Throw it to B, and current continues to flow through the inductor and resistor because the inductor current can't change instantaneously. The current decays exponentially with the same time constant L/R:*

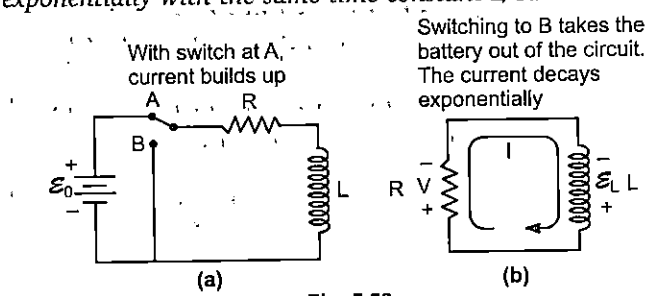


Fig. 5.58

$$I = I_0 e^{-Rt/L}$$

This is analogous to our result for the discharging capacitor. For short times inductor current can't change instantaneously, and for long times inductors have no emfs and therefore act like wires.

3. *The switch in Fig. 5.59 (a) is initially open. Find the current in R_2 immediately and a long time after the switch is closed. Long after the switch is closed, it's reopened. Find the current in R_2 immediately after this second switch opening.*

There's no current in the inductor before the switch closes, so there's no current right afterward; the inductor might as well be an open circuit. That's why we drew Fig. 5.59 (b) with only the two resistors. After a long time the inductor becomes a wire, as we show in Fig. 5.59 (c). Finally, whatever current was flowing in the inductor continues to flow after we reopen the switch. Opening the switch takes out R_1 and the battery, so in Fig. 5.59 (d) we show only the inductor and R_2 with current continuing to flow.

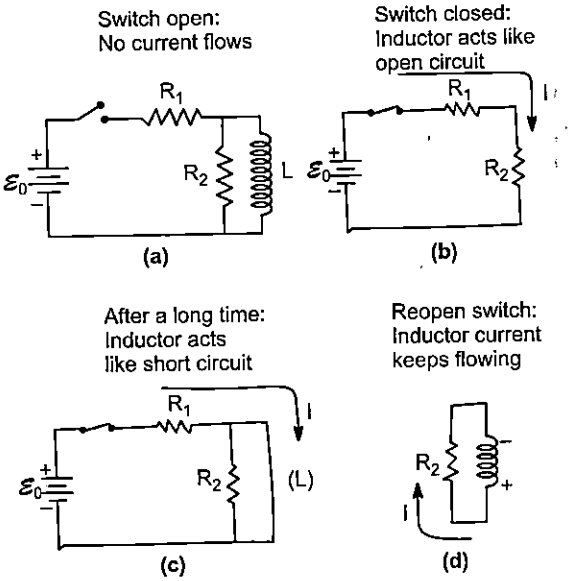


Fig. 5.59

Fig. 5.59 (b) is a simple series circuit, with the current in both resistors given by $I = \mathcal{E}_0 / (R_1 + R_2)$. In Fig. 5.59 (c) there's no inductor emf, which is why we represent the inductor by a wire. So there is no voltage across R_2, and the current in R_2 is therefore zero. But there is current in the inductor, given by $I = \mathcal{E}_0 / R_1$. In Fig. 5.59 (d) that same current $\mathcal{E}_0 / R_1$ continues to flow right after the switch opens, but now it goes upward through R_2. This current then decays exponentially to zero. Fig. 5.60 summarizes the time dependence of the currents in R_2 and L.

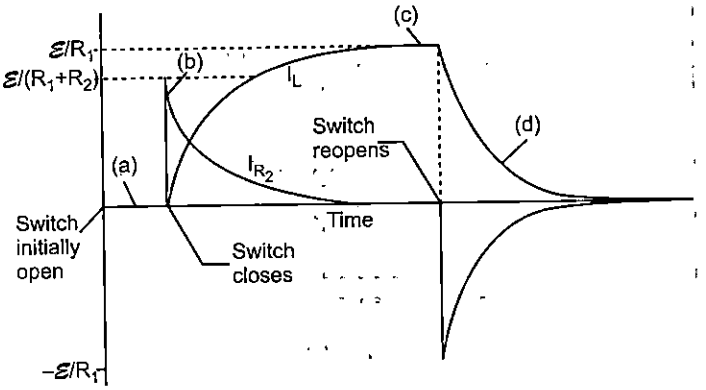


Fig. 5.60

The current in R_2 determined entirely by the battery voltage and the resistance R_1. Doesn't R_2 have any role to play ? No: right after the switch opens, the inductor current must be whatever it was before, and R_1 had determined that current. With current $I = \mathcal{E}_0/R_1$, the voltage across R_2 is $IR_2 = \mathcal{E}_0(R_2/R_1)$ and is equal in magnitude to the inductor emf. The inductor will produce whatever emf is needed to keep the current from changing instantaneously. If R_2 is large compared with R_1, that implies a voltage much higher than the battery voltage. If R_2 isn't in the circuit, then the voltage will rise dangerously high as the inductor tries at all cost to keep the current flowing. Resistors are often placed in parallel with large inductors to alleviate this danger.

4. *The potential difference across an inductor can be very large if the current changes very abruptly (large dI/dt). Suppose the switch is suddenly opened. A very large induced voltage is created across the inductor as the current rapidly drops to zero. This potential difference (plus ΔV_{bat}) appears across the gap of the switch as it is opened. A large potential difference across a small gap often creates a spark.*

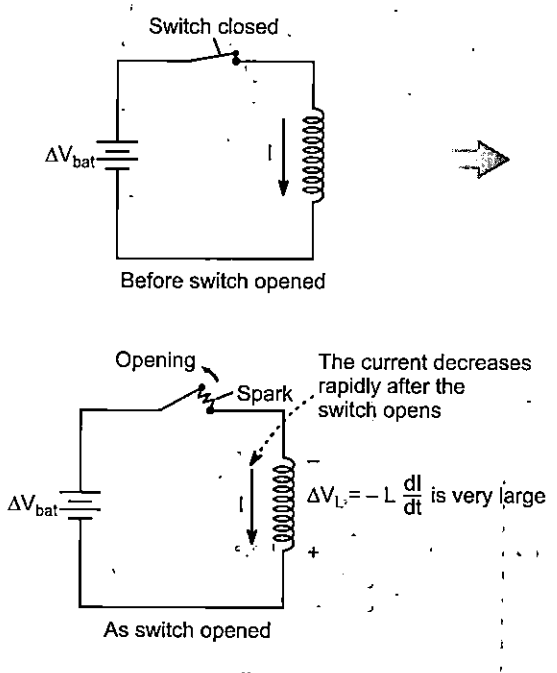


Fig. 5.61

The spark plugs in car work on similar principle. The car's generator sends a large current through the coil, which is a big inductor. A switch in the distributor is suddenly opened, breaking the current. The induced voltage, typically a few thousand volts, appears across the terminals of the spark plug, creating the spark that ignites the petrol. A similar phenomenon happens if we unplug appliances such as toaster ovens or hair dryers while they are running. The heating coils in these devices have inductance. Suddenly pulling the plug is like opening a switch. The large induced voltage often causes a spark between the plug and the electric outlet.

Example 43

Fig. 5E.43 (a) shows an LR circuit.

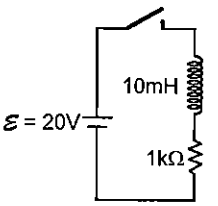


Fig. 5E.43 (a)

(a) Immediately after closing the switch determine.
(i) the current in the resistor
(ii) the current in inductor
(iii) the potential difference across the resistor
(iv) the potential difference across the inductor
(v) the rate at which the current through the inductor is changing with time.
(b) Determine same quantities a long time after closing the switch, that is after many time constants (for $t \to \infty$).
(c) Show explicitly that the KVL is satisfied in the circuit at any time t.
(d) Plot the graphs of the potential difference across the resistor and inductor as functions of time.

Solution: (a) The currents through resistor and inductor are same as they are in series. For the current through inductor the equation is

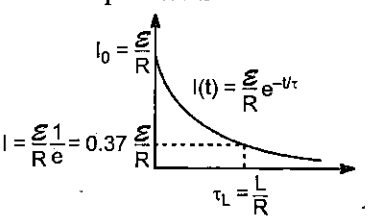


Fig. 5E.43 (b)

$$I(t) = \frac{\mathcal{E}}{R}[1 - e^{-(R/L)t}] \quad \ldots(1)$$

When $t = 0$, $\dfrac{1}{e^{(R/L)t}} = 1$

$$I(0) = 0$$

Current through inductor and resistor is zero.

The graph of this function shows that the current decreases with time. This result is expected because the source of emf has been removed.

The emf across inductor $\mathcal{E}_L = -L\left(\dfrac{dI}{dt}\right)$ is positive because the slope $\dfrac{dI}{dt}$ of the graph is always negative. The positive emf is generated in accordance with the Lenz's law; the flux is decreasing; the emf must oppose this decrease of flux.

(b) As $$\frac{dI}{dt} = \frac{\mathcal{E}}{L} e^{-(R/L)t} \quad \ldots(2)$$

at $t = \infty$, $e^{-(R/L)t} = 0$

$$\frac{dI}{dt} = 0$$

The current is no longer changing with time; hence the potential difference across the inductor is zero.

The current through inductor is constant, from eqn. (1), at $t = \infty$.

$$e^{-(R/L)t} = 0$$

$$I = \frac{\mathcal{E}}{R} = \frac{20}{1.0\times 10^3} = 20\,\text{mA}$$

$$= 0$$

The same current flows through the resistor.

The potential difference across the resistor is

$$V_R = IR = (20\times 10^{-3})(1.0\times 10^3) = 20\,\text{V}$$

The inductor behaves as short circuit (a conducting wire) after a long time interval.

(c) The potential difference across resistor at any time $V_R(t)$ is

$$V_R(t) = I(t)R$$

$$= \frac{\mathcal{E}}{R}[1 - e^{-(R/L)t}]\times R$$

$$= \mathcal{E}(1 - e^{-(R/L)t})$$

The potential difference across inductor $V_L(t)$ at any time t is

$$V_L(t) = L\frac{dI}{dt}$$

From eqn. (2), $\frac{dI}{dt} = \frac{\mathcal{E}}{L}e^{-(R/L)t}$

The current through resistor is zero at $t = 0$; the potential difference across it is zero; hence from KVL applied on the loop the potential difference on the inductor must be equal to that of the emf source, *i.e.*, 20 V

The potential difference on an inductor is

$$\mathcal{E} = L\frac{dI}{dt}$$

$$20\text{V} = (10\times 10^{-3}\,\text{H})\frac{dI}{dt}$$

Solving for $\frac{dI}{dt}$, we have

$$\frac{dI}{dt} = \frac{20\text{V}}{10\times 10^{-3}\,\text{H}} = 2.0\times 10^3\ \text{A/s}$$

The potential difference on inductor depends on the rate of change of current.

Note that $\frac{dI}{dt}$ is slope of a graph of I *versus* t. Initial slope of graph is $dI/dt = 2.0\times 10^3$ A/s. The current through resistor changes at the same rate as it is in series. Let us see another way to determine dI/dt. The current through the inductor is given by

$$I(t) = \frac{\mathcal{E}}{R}[1 - e^{-(R/L)t}] \qquad \ldots(1)$$

Differentiating w.r.t. t, we get

$$\frac{dI}{dt} = \frac{\mathcal{E}}{R}(-1)e^{-(R/L)t}\left(-\frac{R}{L}\right) = \frac{\mathcal{E}}{L}e^{-(R/L)t} \qquad \ldots(2)$$

At $t = 0$, $e^{-(R/L)t} = 1$, we have

$$\frac{dI}{dt} = \frac{\mathcal{E}}{L}, \text{ same result as obtained before}$$

$$V_L = L\frac{dI}{dt} = \mathcal{E}e^{-(R/L)t}$$

The polarities are shown in the figure. Now we apply KVL around the closed loop, traversing the circuit in clockwise sense. Beginning from left corner, we get

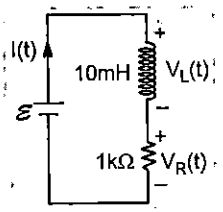


Fig. 5E.43 (c)

$$\mathcal{E} - V_L(t) - V_R(t) = 0$$

$$\mathcal{E} - \mathcal{E}e^{-(R/L)t} - \mathcal{E}[1 - e^{-(R/L)t}] = 0$$

$$0\ \text{V} = 0\ \text{V}$$

The KVL is satisfied at any instant.

(d) The time constant of circuit is

$$\tau = \frac{L}{R} = \frac{10\times 10^{-3}\,\text{H}}{1.0\times 10^3\,\Omega} = 1.0\times 10^{-5}\,\text{s}$$

The potential difference across the resistor is

$$V_R(t) = \mathcal{E}[1 - e^{-(t/\tau)}]$$

$$= 20\,\text{V}[1 - e^{-(1.0\times 10^{-5}\,\text{s}^{-1})t}]$$

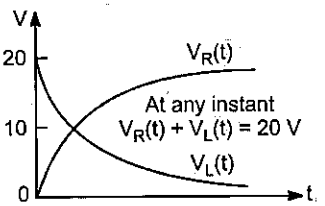


Fig. 5E.43 (d)

The potential difference across the inductor is

$$V_L(t) = \mathcal{E}e^{-(t/\tau)}$$

$$= (20\,\text{V})e^{-(1.0\times 10^{-5}\,\text{s}^{-1})t}$$

Example 44

What is the total heat produced in the resistor R in an RL circuit when the current in the inductor decreases from I_0 to 0.

Solution: The power dissipated in the resistor,

$$P = \frac{dW}{dt} = I^2R$$

Since the current through resistor varies with time we must integrate.

The total energy produced as heat in the resistor

$$W = \int_0^\infty I^2R\,dt$$

The current in an *RL* circuit is

$$I = I_0e^{-(R/L)t}$$

$$W = \int_0^\infty I_0^2e^{-2Rt/L}\,R\,dt$$

We can integrate by substituting

$$x = 2Rt/L,\ dt = (L/2R)\,dx$$

$$W = I_0^2 R \frac{L}{2R}\int_0^\infty e^{-x}\,dx$$

$$= \frac{1}{2}LI_0^2\left[-e^{-x}\right]_0^\infty = \frac{1}{2}LI_0^2$$

Note that the total heat produced equals the energy $(1/2)LI_0^2$ originally stored in the inductor.

Example 45

Consider the RL circuit of figure. When the switch is closed in position 1 and open in position 2, electrical work must be performed on the inductor and on the resistor. The energy stored in the inductor is for the magnetic field inside it which increases as I increases. In the resistor energy appears as heat. (a) What is the ratio P_L/P_R of the rate at which energy is stored in the inductor to the rate at which work is done in the resistor? (b) Express the ratio P_L/P_R as a function of time. (c) If the time constant of circuit is τ, what is the time at which $P_L = P_R$?

Solution: (a) The energy stored in the inductor is

$$U_L = \frac{1}{2}LI^2$$

The power

$$P_L = \frac{dU}{dt} = \frac{d}{dt}\left(\frac{1}{2}LI^2\right) = LI\frac{dI}{dt} \qquad ...(1)$$

The power of a resistor

$$P_R = I^2R \qquad ...(2)$$

So the ratio is

$$\frac{P_L}{P_R} = \frac{L}{R}\frac{1}{I}\frac{dI}{dt} = \tau\frac{1}{I}\frac{dI}{dt} \qquad ...(3)$$

(b) As $$I = \frac{\mathcal{E}}{R}[1 - e^{-(t/\tau)}] \qquad ...(4)$$

$$\frac{dI}{dt} = \frac{d}{dt}\left[\frac{\mathcal{E}}{R}(1 - e^{-(t/\tau)})\right] = \frac{\mathcal{E}}{R\tau}e^{-(t/\tau)} \qquad ...(5)$$

Inserting the value of dI/dt and I in eqn. (3), we have

$$\frac{P_L}{P_R} = \tau\frac{\frac{\mathcal{E}}{R\tau}e^{-t/\tau}}{\frac{\mathcal{E}}{R}(1 - e^{-t/\tau})} = \frac{e^{-(t/\tau)}}{1 - e^{-(t/\tau)}}$$

$$= \frac{1}{e^{(t/\tau)} - 1} \qquad ...(6)$$

When $t = 0$, this ratio tends to infinity, due to large initial value of dI/dt and small initial value of I. When t tends to infinity, after many time constants, the current tends to zero. As a result P_L vanishes and the ratio goes to zero.

(c) From eqn. (6), $P_L = P_R$ when

$$\frac{1}{e^{(t/\tau)} - 1} = 1$$

which on simplification yields

$$e^{(t/\tau)} = 2$$

or $$\frac{t}{\tau} = \ln 2$$

$$t = 0.693\tau$$

ENERGY STORED IN AN INDUCTOR

As we have seen earlier in the *RC* circuit that the part energy supplied by source of emf $\mathcal{E}$ is dissipated in the resistor and the remainder is stored in the uniform electric field between the parallel plates of a capacitor, with an energy density $u_E = (1/2)\varepsilon_0 E^2$; a similar situation exists with *RL* circuit.

Applying KVL to *RL* circuit of Fig. 5.53, we have

$$\mathcal{E} = IR + L\frac{dI}{dt}$$

On multiplying this equation by I, we have

$$\mathcal{E}I = I^2R + LI\frac{dI}{dt}$$

The term on the left hand side ($\mathcal{E}I$) is the power at which the source supplies energy to the circuit.

The first term on the right hand side (I^2R) is the rate at which energy is dissipated in the resistor. From conservation of energy on this closed system the remaining term is the rate at which the energy is stored in the inductor *L*. i.e.,

$$\frac{dU}{dt} = LI\frac{dI}{dt}$$

On integrating the above expression w.r.t. time with initial condition $U = 0$ and $I = 0$, we have

$$\int_0^t \frac{dU}{dt}dt = \int_0^I LI\frac{dI}{dt}dt$$

$$\int_0^U dU = L\int_0^I I\,dI$$

$$U = \frac{1}{2}LI^2$$

- Note that, just as the energy stored in a capacitor can be considered to reside in the electric field between its plates, so the energy in an inductor can be considered to be stored in the magnetic field of inductor.
- The energy stored in the inductor is independent of dI/dt; it depends on I only. The inductor stores absorbed as long as $I \neq 0$, even when the current is not changing with time.
- The power observed by an inductor

$$P = \frac{dU}{dt} = IL\frac{dI}{dt}$$

depends on the rate at which the current is changing with time.

- When the current is increasing with time, $dI/dt > 0$, the power absorbed by the inductor is positive. If the current is decreasing with time, $dI/dt < 0$, the power absorbed is negative, which implies that the inductor is actually supplying power. When I is constant, $dI/dt = 0$, the inductor absorbs zero power.

- When we increase the current through the inductor, it results in the induction of a back emf $V = -L\,dI/dt$. In order to increase the current this back emf must be overcome, so we must derive the current with forward emf $V = L\,dI/dt$. In this process power must be expended. From Joule's law, $P = VI$, so

$$P = \frac{dW}{dt} = LI\frac{dI}{dt}$$

This is the work done by external source to overcome the resistance of inductor to increasing current.

- The self-inductance at a solenoid is given by $L = \mu_0 n^2 Al$. The magnitude of the magnetic field inside a long solenoid is given by $B = \mu_0 nI$. The energy stored in the magnetic field of inductor is

$$U = \frac{1}{2}LI^2 = \frac{1}{2}\mu_0 n^2 Al\left(\frac{B}{\mu_0 n}\right)^2$$

$$= \frac{1}{2}\frac{AlB^2}{\mu_0}$$

$$\frac{U}{Al} = \frac{1}{2}\frac{B^2}{\mu_0}$$

as Al is the volume of the solenoid; the energy stored per unit volume in the solenoid is

$$U_B = \frac{1}{2}\frac{B^2}{\mu_0}$$

The quantity $\frac{1}{2}\frac{B^2}{\mu_0}$ is called **magnetic energy density.** This result is analogous to energy density of an electric field $(1/2)\varepsilon_0 E^2$. The result for energy density of magnetic field is although derived for a solenoid, but it is general. Note that at any location both energy densities $u_B = B^2/2\mu_0$ and $u_E = \varepsilon_0 E^2/2$ are proportional to the square of the respective field strengths.

Example 46

Consider a toroid of circular cross-section of radius b, major radius R much greater than minor radius b, see Fig. 5E.46. Derive (a) an expression for the self-inductance of this toroid, (b) an expression for total energy in the inductor and show that it is $u_B = B^2/2\mu_0$.

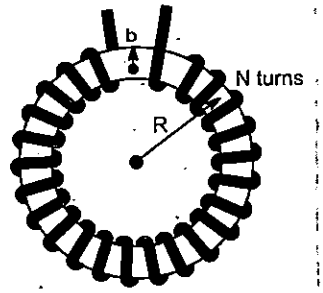


Fig. 5E.46

Solution: (a) The magnitude of magnetic field inside the cavity of a toroid is

$$B = \frac{\mu_0}{2\pi}\frac{NI}{r}$$

where N is the number of turns and r is the distance from the toroidal axis. As $b << R$, so B does not vary appreciably within the cavity. So the magnetic field can be assumed to be uniform.

$$B = \frac{\mu_0}{2\pi}\frac{NI}{R} \quad \ldots(1)$$

The magnetic flux through each turn of the toroid is

$$\phi = \pi b^2 B = \frac{\mu_0 NIb^2}{2R} \quad \ldots(2)$$

The flux with N turns is

$$\phi_{\text{total}} = \frac{\mu_0 N^2 b^2}{2R} I$$

as $\phi = LI$

The self-inductance of the toroid is

$$L = \frac{\mu_0 N^2 b^2}{2R} \quad \text{with} \quad b << R \quad \ldots(3)$$

The energy in the magnetic field of toroid is

$$U = \frac{\mu_0 N^2 l^2}{4R} b^2 \quad \ldots(4)$$

Let us express U in terms of magnetic field B.

As $$B = \frac{\mu_0 NI}{2\pi R}$$

$$B^2 = \frac{\mu_0^2 N^2 I^2}{4\pi^2 R^2}$$

or $$\frac{4\pi^2 R^2 B^2}{\mu_0^2 N^2 I^2} = 1 \quad \ldots(5)$$

We can multiply eqn. (5) to (4); we get

$$U = \frac{B^2}{\mu_0}\pi^2 b^2 R$$

The volume of the cavity is

$$\pi b^2 \times 2\pi R = 2\pi^2 b^2 R$$

So we have $$u = \frac{U}{2\pi^2 b^2 R} = \frac{B^2}{2\mu_0}$$

Example 47

Fig. 5E.47 (a) shows a coaxial cable. The cable consists of two conductors in the form of thin cylindrical shells having a common axis. The inner conductor has radius a and the outer conductor has radius b. Coaxial cables are used to connect electrical devices such as stereo system and a loud speaker. The two conductors carry the current in the opposite directions. You can imagine that the inner conductor carries current to a device and the outer one acts as a return path carrying the current back to the source.

(a) Find the magnetic energy stored in the cable per unit length when the current in it is I. (b) Use the result of part (a) to determine L/l, the self-inductance per unit length. (c) Determine the total flux through the entire cross-section.

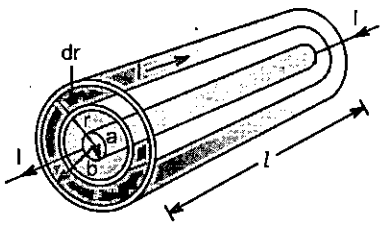


Fig. 5E.47 (a)

Solution: (a) The magnetic lines are circles surrounding the inner conductor. From Ampere's law $\oint B \cdot dl = \mu_0 I$, the magnitude of the magnetic field at a distance r from the centre when the inner conductor carries a current is

$$B = \frac{\mu_0 I}{2\pi r}$$

The magnetic field is zero outside the outer shell ($r > b$) because net current in an Ampere's loop outside is zero. The magnetic field is zero inside the inner shell because the shell is hollow and no current is present within a radius $r < a$.

The energy density at radius r is, therefore

$$u_B = \frac{B^2}{2\mu_0}$$

$$= \frac{1}{2\mu_0}\left(\frac{\mu_0 I}{2\pi r}\right)^2 = \frac{\mu_0 I^2}{8\pi^2 r^2}$$

So, the energy density per unit volume can be written as

$$u_B = \frac{dU_B}{dV}$$

where dV is a differential volume element between the cylinders.

$$dU_B = u_B dV$$

$$U_B = \int \frac{\mu_0 I^2}{8\pi^2 r^2} dV$$

As energy density is cylindrically symmetric, depending on radius r, we choose a cylindrical shell of length l and radius r, thickness dr, as a volume element dV.

$$dV = (2\pi r) l\, dr$$

The total energy between the cylinder is, therefore,

$$U_B = \int_a^b \frac{\mu_0 I^2}{8\pi^2 r^2}(2\pi r) l\, dr$$

$$= \frac{\mu_0 I^2 l}{4\pi}\int_a^b \frac{dr}{r}$$

$$= \frac{\mu_0 I^2 l}{4\pi}\ln\left(\frac{b}{a}\right)$$

The energy per unit length is

$$\frac{U_B}{l} = \frac{\mu_0 I^2}{4\pi}\ln\left(\frac{b}{a}\right)$$

(b) As energy of an inductor is

$$U = \frac{1}{2}LI^2$$

energy per unit length of inductor is

$$\frac{U}{l} = \frac{1}{2}\frac{L}{l}I^2$$

On comparing the result of part (a) with the above expression, we have

$$\frac{L}{l} = \frac{\mu_0}{2\pi}\ln\left(\frac{b}{a}\right)$$

(c) **Method 1 :** We take a rectangle of width dr and length l (along the cable), a distance r from the centre; the magnetic flux through it,

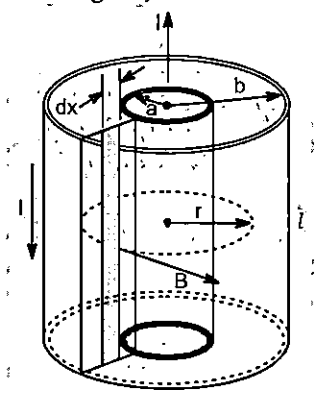


Fig. 5E.47 (b)

$$d\phi_B = B(l\, dr) = \frac{\mu_0 I}{2\pi r}(l\, dr)$$

The total flux in a length of the cable is

$$\phi_B = \int d\phi_B = \frac{\mu_0 I l}{2\pi}\int_a^b \frac{dr}{r}$$

$$= \frac{\mu_0 I l}{2\pi}\ln\left(\frac{b}{a}\right)$$

Method 2 : We take a rectangle of length l and width $b - a$, the cross-section under consideration. The magnetic field varies with r across the rectangle. We take a rectangular strip of thickness dr; the flux through each strip is

$$B\, dA = B\, l dr$$

$$\phi_B = \int B\, dA = \int_a^b \frac{\mu_0 I}{2\pi r} l\, dr$$

$$= \frac{\mu_0 I l}{2\pi}\int_a^b \frac{dr}{r}$$

$$= \frac{\mu_0 I l}{2\pi}\ln\left(\frac{b}{a}\right)$$

Example 48

Fig. 5E.48 shows a section of a coaxial cable. The cable consists of two conductors in the form of thin cylindrical shells having a common axis. The inner conductor has radius r_1, and the outer conductor has radius r_2. The two conductors carry the current I in opposite directions. (a) Find U/l, the magnetic energy stored in the cable per unit length, when the cable carries a current I. (b) Use the result of part a to determine L/l, the self-inductance per unit length.

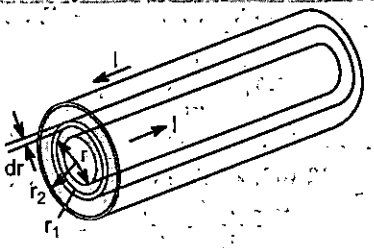


Fig. 5E.48

Solution: (a) An Amperean circle of radius $r < r_1$ is threaded by no current. The net current threading an Amperean circle of radius $r > r_2$ is likewise zero. Thus it follows from Ampere's law that the magnetic field is zero at all points $r > r_1$ and $r > r_2$. Between the two conductors, the field is due entirely to the current in the inner conductor and is given by

$$B = \frac{\mu_0}{2\pi}\frac{I}{r}$$

Because B depends on r, the energy density is not uniform throughout the space between the conductors. The energy density at any point as

$$u = \left(\frac{\mu_0}{2\pi}\frac{I}{r}\right)^2 \frac{1}{2\mu_0} = \frac{\mu_0 I^2}{8\pi^2}\frac{1}{r^2}$$

for $r_1 < r < r_2$.

Now, consider a thin cylindrical shell of radius r and thickness dr. The volume of a length l of this shell is $2\pi lrdr$. So the energy dU of the magnetic field contained in the shell is the product of the energy density and volume:

$$du = \frac{\mu_0 I^2}{4\pi}\frac{dr}{r}$$

To find the total magnetic energy within a length l of the cable, you integrate:

$$U = \frac{\mu_0 I^2}{4\pi}\int_{r_1}^{r_2}\frac{dr}{r}$$

$$= \frac{\mu_0 I^2}{4\pi}\ln\frac{r_2}{r_1}$$

The energy per unit length is

$$\frac{U}{l} = \frac{\mu_0 I^2}{4\pi}\ln\frac{r_2}{r_1}$$

(b) The energy per unit length of cable as

$$\frac{U}{l} = \frac{1}{2}\frac{LI^2}{l}$$

Thus,

$$\frac{L}{l} = \frac{\mu_0}{2\pi}\ln\frac{r_2}{r_1}$$

for the inductance per unit length.

Example 49

An LR circuit with emf $\mathcal{E}$ is switched on at t = 0. Find (a) the charge that flows through the battery during t = 0 to t; (b) voltage across resistor; (c) voltage across inductor, (d) power from battery; (e) energy in the inductor; (f) the heat developed during time 0 to t; (g) the time taken for the magnetic energy stored in the circuit to charge from one-fourth of the steady state value to half of the steady state value.

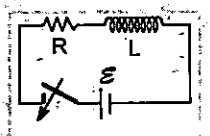


Fig. 5E.49

Solution: The current in RL circuit at time t is

$$I = \frac{\mathcal{E}}{R}(1 - e^{-Rt/L})$$

(a) Charge:

$$q = \int_0^t I\,dt = \frac{\mathcal{E}}{R}t - \frac{\mathcal{E}L}{R^2}(1 - e^{-Rt/L})$$

(b) Voltage across resistor:

$$V_R = IR = \mathcal{E}(1 - e^{-Rt/L})$$

(c) Voltage across inductor:

$$\mathcal{E} - V_R = \mathcal{E}e^{-Rt/L}$$

(d) Power from battery :

$$P = \mathcal{E}I = \frac{\mathcal{E}^2}{R}(1 - e^{-Rt/L})$$

(e) Energy in inductor:

$$U_L = \frac{1}{2}LI^2 = \frac{L\mathcal{E}^2}{2R^2}(1 - e^{-Rt/L})^2$$

(f) Heat developed in the resistor:

$$H = \int_0^t I^2 R\,dt$$

$$= \frac{\mathcal{E}^2}{R}\int_0^t (1 - e^{-Rt/L})^2\,dt$$

$$= \frac{\mathcal{E}^2}{R}\int_0^t (1 + e^{2Rt/L} - 2e^{-Rt/L})\,dt$$

$$= \frac{\mathcal{E}^2}{R}\left(t - \frac{L}{2R}e^{-2Rt/L} + \frac{2L}{R}e^{-Rt/L}\right)_0^t$$

$$= \frac{\mathcal{E}^2}{R}\left[t + \frac{L}{2R} - \frac{L}{2R}e^{-2Rt/L} - \frac{2L}{R} + \frac{2L}{R}e^{-Rt/L}\right]$$

$$= \frac{\mathcal{E}^2}{R}\left[t + \frac{L}{2R}(1 - e^{-2Rt/L}) - \frac{2L}{R}(1 - e^{-Rt/L})\right]$$

(g) The steady state current $= \frac{\mathcal{E}}{R}$

Magnetic field energy stored in the inductor in steady state

$$= \frac{1}{2}LI^2 = \frac{1}{2}\frac{\mathcal{E}^2}{R^2}L$$

Energy at any time, $t = \frac{1}{2}L\frac{\mathcal{E}^2}{R^2}(1-e^{-Rt/L})^2$

Let time taken for energy to be halved be t_1

$$\frac{1}{4}L\frac{\mathcal{E}^2}{R^2} = \frac{1}{2}L\frac{\mathcal{E}^2}{R^2}(1-e^{-Rt_1/L})^2$$

$$e^{-Rt_1/L} = \frac{\sqrt{2}-1}{\sqrt{2}} = \frac{2-\sqrt{2}}{2}$$

$$\frac{-Rt_1}{L} = \ln(2-\sqrt{2}) - \ln 2$$

$$t_1 = \frac{L}{R}\left[\ln 2 - \ln\left(\frac{1}{2-\sqrt{2}}\right)\right]$$

Let time taken for energy to be one-fourth be t_2.

$$\frac{1}{8}L\frac{\mathcal{E}^2}{R^2} = \frac{1}{2}L\frac{\mathcal{E}^2}{R^2}(1-e^{-Rt_2/L})^2$$

$$1-e^{-Rt_2/L} = \frac{1}{2}$$

$$e^{-Rt_2/L} = \frac{1}{2} \quad \text{or} \quad t_2 = \frac{L}{R}\ln 2$$

Hence $t_2 - t_1 = \frac{L}{R}\ln\left(\frac{1}{2-\sqrt{2}}\right)$

Example 50

An inductor of inductance $L = 400\ mH$ and resistors of resistances $R_1 = 2\Omega$ and $R_2 = 2\Omega$ are connected to a battery of emf $\mathcal{E} = 12\ V$ as shown in Fig. 5E.50. The internal resistance of the battery is negligible. The switch S is closed at time $t = 0$. What is the potential drop across L as a function of time? After the steady state is reached the switch is opened. What is the direction and the magnitude of current through R_1 as a function of time?

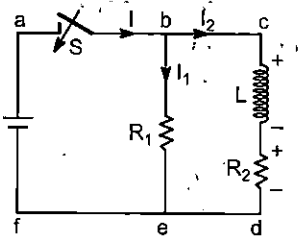


Fig. 5E.50

Solution: Let the current distribution in the circuit be as shown in the figure.

From KCL, $I = I_1 + I_2$ ...(1)

Applying KVL in loop *acdfa*, traversing the circuit clockwise, beginning at left corner,

$$\mathcal{E} - L\frac{dI_2}{dt} - I_2R_2 = 0 \qquad ...(2)$$

which on rearranging gives

$$\frac{dI_2}{\mathcal{E} - I_2R_2} = \frac{1}{2}dt \qquad ...(3)$$

On integrating the eqn. (3), we get

$$I_2 = \frac{\mathcal{E}}{R_2}[1-e^{-R_2t/L}]$$

Note that loop *acdfa* is a simple *RL* circuit with time constant L/R_2. In the steady state, the inductor is short circuited, resistors R_1 and R_2 are in parallel arrangement. So $I_1 = \mathcal{E}/R_1$, $I_2 = \mathcal{E}/R_2$.

Potential drop across inductor

$$|V_L| = L\frac{dI_2}{dt} = \mathcal{E}e^{-R_2t/L}$$

On substituting numerical values, we have

$$\tau_L = \frac{L}{R_2} = \frac{0.4}{2} = 0.2\text{s}$$

Steady state current,

$$I_2 = \frac{\mathcal{E}}{R_2} = \frac{12}{2} = 6\text{ A}$$

$$I_2 = 6(1-e^{-5t})$$

$$|V_L| = \mathcal{E}e^{-R_2t/2} = 12e^{-5t}$$

When the switch is opened the loop *abefa* is an open circuit, current in R_1 is zero instantly while in loop *bcdeb* the current decays to zero. Time constant of this loop is

$$\tau'_L = \frac{L}{R_1+R_2} = \frac{0.4}{2+2} = 0.1\text{s}$$

Current through R_1 at any time t is

$$I = I_0e^{-t/\tau'_L} = 6e^{-10t}$$

Example 51

In terms of inductive time constant τ_L, how long must we wait, for current in an RL circuit to build up to within 0.100 % of its equilibrium value?

Solution: The current in an *RL* circuit at time t, with $I = 0$ at $t = 0$, is given by

$$I = \frac{\mathcal{E}}{R}(1-e^{-t/\tau_L})$$

We wish to calculate the time t for which

$$I = 0.9990\frac{\mathcal{E}}{R}$$

$$0.9990\frac{E}{R} = \frac{E}{R}(1-e^{-t/\tau_L})$$

or $0.9990 = 1-e^{-t/\tau_L}$

or $e^{-t/\tau_L} = 0.0010$

Take the natural logarithm of both sides to obtain

$$-t/\tau = \ln(0.0010) = -6.91$$

Thus, $t = 6.91\,\tau_L$

The time elapsed is 6.91 inductive time constants.

Example 52

In the circuit shown in Fig. 5E.52, $\mathcal{E} = 100\,V$, $R_1 = 10.0\,\Omega$, $R_2 = 20.0\,\Omega$, $R_3 = 30.0\,\Omega$ and $L = 2.00\,H$. Find the values of I_1 and I_2 (a) immediately after closing of switch S; (b) a long time after closing the switch, (c) immediately after reopening the switch (d) a long time after reopening the switch.

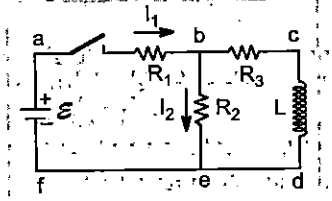


Fig. 5E.52

Solution: (a) The current through a simple *RL* circuit is

$$I = \frac{\mathcal{E}}{R}(1 - e^{-t/RL})$$

At $t = 0$, $I = 0$, as exponential term is unity.

At $t = \infty$, $I = \frac{\mathcal{E}}{R}$, as exponential term is zero.

The rate of change of current in *RL* circuit is

$$\frac{dI}{dt} = \frac{\mathcal{E}}{L} e^{-t/RL}$$

At $t = 0$, $\frac{dI}{dt} = \frac{\mathcal{E}}{L}$ (maximum value)

At $t = \infty$, $\frac{dI}{dt} = 0$

Immediately after closing the switch, $I = 0$ and $\frac{dI}{dt}\left(= \frac{\mathcal{E}}{L}\right)$ is maximum. The inductor opposes the fast build up of the current through it; current in the inductor is zero. This means

$$I_1 = I_2 = \frac{\mathcal{E}}{R_1 + R_2}$$

$$= \frac{100}{10 + 20} = 3.33\,A$$

(b) A long time after the current reaches its steady state value, the emf across the inductor is zero, the inductor behaves as if it were replaced by a wire (*i.e.*, short-circuited). The current in R_3 is $(I_1 - I_2)$.

From KVL,

loop *abefa* $\quad \mathcal{E} - I_1R_1 - I_2R_2 = 0$

loop *acdfa* $\quad \mathcal{E} - I_1R_1 - (I_1 - I_2)R_3 = 0$

On solving these equations simultaneously we have

$$I_1 = \frac{\mathcal{E}(R_2 + R_3)}{R_1R_2 + R_1R_3 + R_2R_3} = \frac{(100)(20 + 30)}{(10)(20) + (10)(30) + (20)(30)}$$

$$= 4.55\,A$$

and

$$I_2 = \frac{\mathcal{E}R_3}{R_1R_2 + R_1R_3 + R_2R_3}$$

$$= \frac{(100)(30)}{(10)(20) + (10)(30) + (20)(30)}$$

$$= 2.73\,A$$

(c) When the switch is reopened, the left loop is an open circuit; if its inductance is zero the current immediately drops to zero when the switch is opened; hence $I_1 = 0$. The current in R_3 changes only slowly because there is an inductor in its branch. Immediately after the switch is opened it has the same value as it had just before the switch was opened, *i.e.*, $4.55\,A - 2.73\,A = 1.82\,A$. The current in R_2 is the same as that in R_3, 1.82 A.

(d) In the absence of any source of emf all the currents drop to zero.

Example 53

In the circuit shown in Fig. 5E.53, switch S is closed at $t = 0$. Thereafter the constant current source by varying its emf maintains a constant current I out of its upper terminal. (a) Derive an expression for the current through the inductor as a function of time. (b) Show that the current through the inductor equals the current through the resistor at time $t = (L/R)\ln 2$.

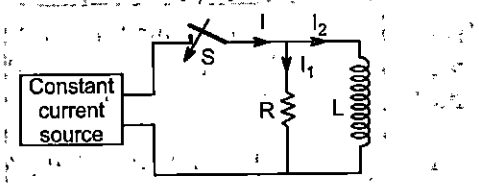


Fig. 5E.53

Solution: Let I_1 and I_2 be the currents through inductor and resistor respectively.

From KCL, $\quad I = I_1 + I_2 \quad \ldots(1)$

From KVL, $\quad I_1R - L\frac{dI_2}{dt} = 0 \quad \ldots(2)$

As I = constant, from eqn. (1) we have

$$\frac{dI_1}{dt} = -\frac{dI_2}{dt} \quad \ldots(3)$$

Substituting eqn. (3) in eqn. (2), we obtain

$$L\frac{dI_1}{dt} + I_1R = 0$$

This is a familiar differential equation with the solution of the type

$$I_1 = I_0 e^{-Rt/L}$$

where I_0 is the current through the resistor at $t = 0$, just after the switch is closed. At $t = 0$, the current through inductor is zero, $I_2 = 0$, as inductor opposes fast build up of current.

So at $t = 0$, $I_1 = I$, thus $I_0 = I$; so

$$I_1 = Ie^{-Rt/L}$$

and $\quad I_2 = I - I_1 = I[1 - e^{-Rt/L}]$

(b) When $I_2 = I_1$,

$$e^{-Rt/L} = 1 - e^{-Rt/L}$$

$$e^{-Rt/L} = 1/2$$

Take natural logarithm of both sides to obtain

$$\frac{Rt}{L} = \ln 2$$

or $$t = \frac{L}{R}\ln 2$$

Example 54

In the circuit shown in Fig. 5E.54 the battery has negligible internal resistance. Show that the current in the circuit through the battery rises instantly to its steady state value $\mathcal{E}/R$ when the switch is closed, provided that the resistance R is $\sqrt{L/C}$.

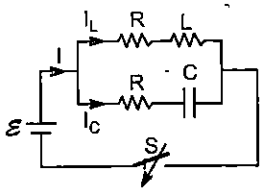


Fig. 5E.54

Solution: Let the currents through inductive branch and capacitive branch be I_L and I_C respectively. Then the current through the battery, from KCL, is

$$I = I_L + I_C$$

Since the battery is connected in parallel to the *RL* and *RC* branches of the circuit, the current in the *RL* branch is unaffected by the presence of the *RC* branch; so

$$I_L = \frac{\mathcal{E}}{R}[1 - e^{-(R/L)t}]$$

and $$I_C = \frac{\mathcal{E}}{R}e^{-(t/RC)}$$

Hence the current through battery is

$$I = \frac{\mathcal{E}}{R}[1 - e^{-(R/L)t} + e^{-t/RC}]$$

If the current has to reach its final value $\mathcal{E}/R$ instantaneously, the exponential terms must cancel out, *i.e.*,

$$e^{-t/\tau_L} = e^{-t/\tau_C}$$

which is possible if $$\tau_L = \tau_C$$

$$L/R = RC$$

or $$R = \sqrt{(L/C)}$$

For all $t > 0$, this is the desired result.

MUTUAL INDUCTANCE

Consider two circuits close to each other (Fig. 5.62). When the switch S_1 is closed in upper circuit a current I_1, begins to flow around the circuit. This current does not reach the steady state value instantaneously. The increasing current creates a magnetic field and thereby a changing magnetic flux through same loop. In accordance with Lenz's law an emf is induced that opposes the current I in the loop. This is the phenomenon of self-induction. The changing magnetic field generated by the changing current I_1 can produce a changing magnetic flux through circuit 2. Hence an emf will be induced in the circuit 2 by changing current in circuit 1. Similarly if the current I_2 in circuit 2 changes in time, an emf will be induced in circuit 1 due to the variation of the current I_2.

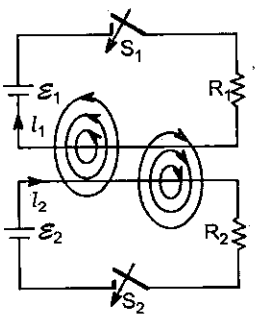


Fig. 5.62

The magnetic flux through circuit 1 is proportional to I_1 and I_2, because change in both the currents induces emf in circuit 1. Total magnetic flux through circuit 1 is the sum of the flux from both circuits.

Total magnetic flux in circuit 1 is

$$\phi_{B_1} = L_1 I_1 + M_{12} I_2$$

The constant L_1 is **self-inductance** of circuit 1 and the constant M_{12} is the **mutual inductance** on circuit 1 by circuit 2. The total flux through circuit 2 depends on the size, shape, relative locations and relative orientations of the two coils.

Similarly the flux through circuit 2 can be written as

$$\phi_{B_2} = L_2 I_2 + M_{21} I_1$$

We will not prove the theorem, but it is always true that

$$M_{21} = M_{12} = M$$

From Faraday's law of electromagnetic induction,

$$\mathcal{E}_1 = -\frac{d}{dt}(\phi_{B_1})$$

$$= -L_1 \frac{dI_1}{dt} - M\frac{dI_2}{dt}$$

$$\mathcal{E}_2 = -\frac{d}{dt}(\phi_{B_2})$$

$$= -L_2 \frac{dI_2}{dt} - M\frac{dI_1}{dt}$$

Example 55

Consider two tightly wound concentric solenoids as shown in Fig. 5E.55. Let l be the length of both the solenoids and let the inner solenoid have N_1 turns and radius r_1 and the outer solenoid have N_2 turns and radius r_2. Calculate the mutual inductance of M_{21} by assuming that all the flux from coil 2 passes through coil 1.

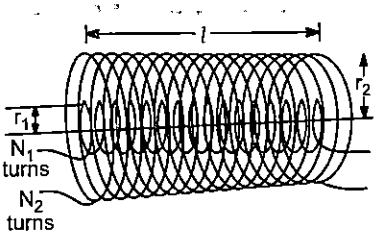


Fig. 5E.55

Solution: The magnetic field due to the current in the inner solenoid is constant in the space within the solenoid and has magnitude

$$B_1 = \mu_0\left(\frac{N_1}{l}\right)I_1 = \mu_0 n_1 I_1$$

The magnetic flux through the outer solenoid due to this magnetic field is therefore

$$\phi_{B_2} = N_2 B_1(\pi r_1^2) = n_1 l B_1(\pi r_1^2)$$
$$= \mu_0 n_1 n_2 l(\pi r_1^2) I_1$$

The mutual inductance M_{21} is thus

$$M_{21} = \frac{\phi_{B_2}}{I_1} = \mu_0 n_2 n_1 l \pi r_1^2$$

We have calculated the mutual inductance M_{21} by assuming that the inner solenoid carries a current I_1. Note that the area used to calculate the flux through the outer solenoid is not πr_2^2 but πr_1^2 the area of inner solenoid. As the magnetic field due to inner solenoid is zero outside it.

If we assume a current I_2 in the outer solenoid, its magnetic field is

$$B_2 = \mu_0 \left(\frac{N_2}{l}\right) I_2 = \mu_0 n_2 I_2$$

The magnetic flux through the inner solenoid due to this magnetic flux is

$$\phi_{B_1} = N_1 B_2(\pi r_1^2) = n_1 l B_2(\pi r_1^2)$$
$$= \mu_0 n_1 n_2 l(\pi r_1^2) I_2$$

The mutual inductance M_{12} is thus

$$M_{12} = \frac{\phi_{B_1}}{I_2} = \mu_0 n_1 n_2 l \pi r_1^2$$

Note that $M_{12} = M_{21}$ and it is a generalised result; therefore we usually drop the subscripts and write mutual inductance as M.

Example 56

In Fig. 5E.56 (a) coil 1 and coil 2 are wound on a long cylindrical insulator. The ends A' and B are joined together and a current I is passed. The self-inductances of the two coils are L_1 and L_2, their mutual inductance is M. (a) Show that this combination can be replaced by a single coil of equivalent inductance given by

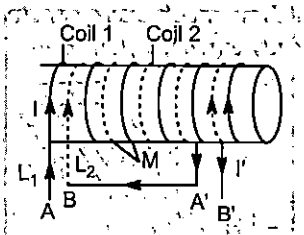


Fig. 5E.56 (a)

$$L_{eq.} = L_1 + L_2 + 2M$$

(b) How could the coils be reconnected to yield an equivalent inductance of $L_{eq.} = L_1 + L_2 - 2M$?

Solution: (a) When the terminals A' and B are connected, the sense of current in both the coils is same. Let the current be changing at the rate dI/dt. The magnetic fields of coil 1 and coil 2 both point to left. When the current increases both fields increase and both changes in flux contribute emf's in the same direction.

Thus the induced emf in coil 1 is

$$\mathcal{E}_1 = -(L_1 + M)\frac{dI}{dt} \quad \ldots(1)$$

Similarly, the magnetic field in coil 2 due to it and field due to coil 1 point to left. The two induced emf's are again in the same direction.

$$\mathcal{E}_2 = -(L_2 + M)\frac{dI}{dt} \quad \ldots(2)$$

Hence the total emf across both coils is

$$\mathcal{E} = \mathcal{E}_1 + \mathcal{E}_2 = -(L_1 + L_2 + 2M)\frac{dI}{dt} \quad \ldots(3)$$

If these coils were replaced by single coil

$$\mathcal{E} = -L_{eq.}\frac{dI}{dt} \quad \ldots(4)$$

On comparing eqns. (3) and (4), we have

$$L_{eq.} = L_1 + L_2 + 2M$$

(b) When the terminals A and B are connected the sense of the current flow in coil 1 and coil 2 are opposite. Then at the site of coil 1 the field produced by coil 2 is opposite the field produced by coil 1 itself. An increasing current in coil 1 tends to increase the flux in that coil but an increasing current in coil 2 tends to decrease it. The emf across coil 1 is

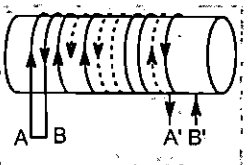


Fig. 5E.56 (b)

$$\mathcal{E}_1 = -(L_1 - M)\frac{dI}{dt}$$

Similarly emf across coil 2 is

$$\mathcal{E}_2 = -(L_2 - M)\frac{dI}{dt}$$

The total emf

$$\mathcal{E} = \mathcal{E}_1 + \mathcal{E}_2$$
$$= -(L_1 + L_2 - 2M)\frac{dI}{dt}$$

Thus $L_{eq.} = (L_1 + L_2 - 2M)$

Example 57

Derive an expression for the total magnetic energy stored in two coils with inductances L_1 and L_2 and mutual inductance M, when the currents in the coils are I_1 and I_2, respectively.

Solution: When the currents are increasing in the circuit, we have for emf's

$$\mathcal{E}_1 = -L_1\frac{dI_1}{dt} \pm M\frac{dI_2}{dt}$$
$$\mathcal{E}_2 = -L_2\frac{dI_2}{dt} \pm M\frac{dI_1}{dt}$$

where the ± sign appears consistently in both equations and depends on the geometry of the coils and the sense of current. Work done in pushing charges dq_1 and dq_2 through each circuit, respectively, is

$$dW = -\mathcal{E}_1 dq_1 - \mathcal{E}_2 dq_2$$
$$= L_1\frac{dI_1}{dt}dq_1 \mp M\frac{dI_2}{dt}dq_1 + L_2\frac{dI_2}{dt}dq_2 \mp M\frac{dI_1}{dt}dq_2$$

As $I_1 = \frac{dq_1}{dt}, \quad dq_1 = I_1 dt$

$I_2 = \frac{dq_2}{dt}, \quad dq_2 = I_2 dt$

we have $dW = L_1 I_1 dI_1 + L_2 I_2 dI_2 \mp (MI_1 dI_2 + MI_2 dI_1)$

$= L_1 I_1 dI_1 + L_2 I_2 dI_2 \mp Md(I_1 I_2)$

On integrating the above expression from 0 to final current, we have

$$U = \int dW = L_1 \int_0^{I_1} I_1 dI_1 + L_2 \int_0^{I_2} I_2 dI_2 \mp M \int_0^{I_1 I_2} d(I_1 I_2)$$

$$= \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 + MI_1 I_2$$

Example 58

Fig. 5E.58 shows a rectangular coil near a long wire. Calculate the mutual inductance of the combination.

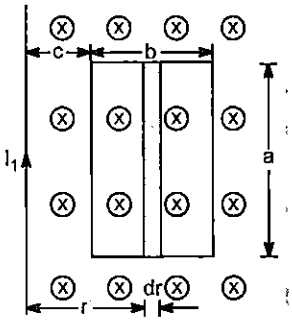


Fig. 5E.58

Solution: Let the current I_1 in the straight wire be upward. Then the magnetic field due to the straight wire has magnitude $B_1 = \mu_0 I_1 / 2\pi r$ at a distance r. In accordance with right hand rule B_1 points inward to the plane of page. We consider a differential strip of thickness dr, area $dA_2 = a\,dr$. Magnetic flux through area dA, $d\phi_B = B_1 (a\,dr)$.

Total flux through the loop,

$$\phi_{B_{12}} = \int B_1 \, dA_2 = \int_c^{c+b} \frac{\mu_0 I_1}{2\pi r} a \, dr$$

$$= \frac{\mu_0 I_1 a}{2\pi} \int_c^{c+b} \frac{dr}{r} = \frac{\mu_0 I_1 a}{2\pi} \ln\left(\frac{c+b}{c}\right)$$

Therefore the mutual inductance,

$$M = M_{12} = \frac{\phi_{12}}{I_1} = \frac{\mu_0 a}{2\pi} \ln\left(1 + \frac{b}{c}\right)$$

Example 59

A toroidal tube has a rectangular cross-section with sides Δa and b. The inner radius of the toroid is a and the outer radius $a + \Delta a$. The height of the rectangular cross-section is b. The toroid is overwound with a coil that has a total of n turns. Show that the self-inductance L of the coil is given approximately by

$$L = \frac{\mu_0 b n^2 \Delta a}{2\pi a},$$

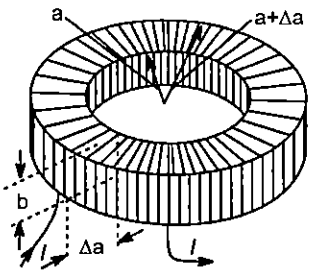


Fig. 5E.59

if a is much greater than Δa, which in turn is greater than b.

Solution: Fig. 5E.59 shows the toroid overwound with the coil. The magnetic field B at all points on the perimeter of a circle of radius x is $B(x) = \mu_0 nI / 2\pi x$. Since $\Delta a > b$, and $a >> \Delta a$, we make the approximation that the field is independent of height within the rectangular tube and varies only over the longer side of the tube. The magnetic flux through a small area $b\,dx$ of the n turns is then $d\phi = b\,dx \mu_0 nI / 2\pi x n$. The flux through the whole coil is

$$\phi = \frac{b\mu_0 n^2 I}{2\pi} \int_a^{a+\Delta a} \frac{dx}{x},$$

and the self-inductance is

$$L = \frac{b\mu_0 n^2}{2\pi} \ln\left(\frac{a+\Delta a}{a}\right)$$

This reduces to $L = \frac{b\mu_0 n^2}{2\pi} \frac{\Delta a}{a}$

when $a >> \Delta a$.

Example 60

Two parallel rectangular superconducting plates of length l, width b and separation a ($l >> b >> a$) are joined at each end to form a one-turn coil of negligible resistance. What is its self-inductance? How much energy is stored in the magnetic field when a steady current I flows?

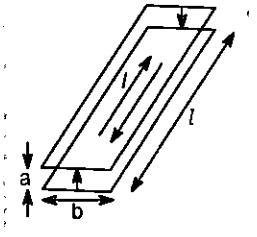


Fig. 5E.60 (a)

The separation a increases by a small amount δa. Calculate the various energy changes (a) when the coil includes in its circuit a lossless battery which maintains a constant current I, and (b) when the circuit consists of the coil alone. Deduce the magnitude and direction of the force per unit area between the two plates.

Solution: The magnetic field is perpendicular to the current in the plates. The cross-section of the system is as shown in Fig. 5E.60 (b).

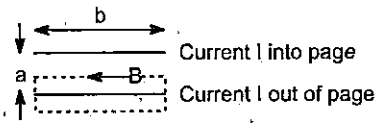


Fig. 5E.60 (b)

Assuming that the magnetic field strength B is constant between the plates, and zero outside them, application of Ampere's theorem round the dashed curve gives

$$B = \frac{\mu_0 I}{b}.$$

To calculate the flux linked by the coil we must multiply this magnetic field by the area of the coil measured perpendicularly to the direction of $\vec{\mathbf{B}}$. So the flux linked by the circuit is

$$\phi = \frac{\mu_0 Ila}{b},$$

and the self-inductance of the coil is given by ϕ/I, so $L = \mu_0 la/b$.

The energy U stored by the magnetic field is given by $U = LI^2/2$. Substituting our expression for the self-inductance L gives

$$U = \frac{\mu_0 laI^2}{2b}.$$

(a) If the self-inductance is changed without changing the current, the change in the energy stored by the magnetic field is clearly

$$\delta U = \frac{\mu_0 I^2 l}{2b}\delta a.$$

(b) If the circuit contains only the coil, no emf is supplied, so there can be no change in the flux linked by the coil, and so the current will change as the inductance changes. We can rewrite the energy stored in terms of the flux linked as

$$U = \frac{\phi^2 b}{2\mu_0 la},$$

so the change in U when a is changed by δa is given by

$$\delta U = -\frac{\phi^2 b}{2\mu_0 la^2}\delta a.$$

Rewriting this in terms of the current I gives

$$\delta U = -\frac{\mu_0 I^2 l}{2b}\delta a.$$

Since this is negative, the change is energetically favourable, so the forces on the plates must be outwards (repulsive). The force F is $-dU/da$, so the force per unit area on each plate is

$$\frac{\mu_0 I^2 l}{2b}\cdot\frac{1}{lb} = \frac{\mu_0 I^2}{2b^2}.$$

Example 61

The circuit shown in Fig. 5E.61 (a) consisting of three identical lamps and two coils, is connected to a direct current source. The ohmic resistance of the coils is negligible.

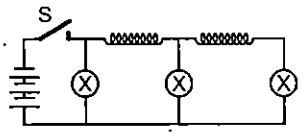


Fig. 5E.61 (a)

After some time, switch S is opened. What are the relative brightnesses of three lamps immediately afterwards?

Solution: When the switch is closed, currents, as shown in Fig. 5E.61 (b), flow round the circuit.

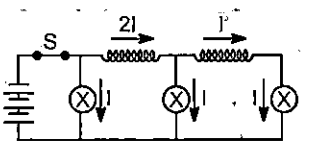


Fig. 5E.61 (b)

In the period immediately after opening the switch, the current flowing in each coil is practically unchanged; if this were not the case, there would be a rapid change in its magnetic flux which would induce a very high voltage in the coil. Currents of $2I$ and I therefore continue to flow in the coils, and these determine the currents flowing through the lamps [see Fig. 5E.61 (c)].

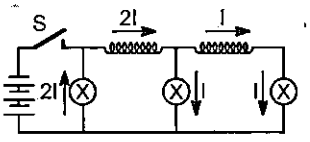


Fig. 5E.61 (c)

This means that the lamp closest to the switch suddenly flashes, but the brightness of the two other lamps does not change. This all takes place in a very brief period and later all three lamps fade and go out.

Example 62

A resistor and an inductor in series are connected to a battery through a switch.

After the switch has been closed:

(i) What is the magnitude of the current flowing when the rate of the increase of magnetic energy stored in the coil is at a maximum?

(ii) When will the Joule heat dissipated in the resistor change at the fastest rate?

Solution: (i) The rate of increase of magnetic energy ($E_{\text{magn.}} = LI^2/2$) is the difference between the power output of the battery and the power dissipated in the resistor,

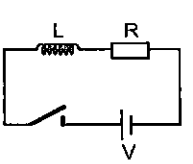


Fig. 5E.62 (a)

$$\frac{dE_{\text{magn.}}}{dt} = VI - RI^2$$

$$= -R\left(I - \frac{V}{2R}\right)^2 + \frac{V^2}{4R} \le \frac{V^2}{4R}.$$

It is clear that the rate of increase is maximal when $I = V/(2R)$.

(ii) After the switch has been closed, we can write Kirchhoff's law for the circuit as

$$V = IR + L\frac{dI}{dt},$$

which gives the current-time relationship [see Fig. 5E.84(b)]

$$I = \frac{V}{R}(1 - e^{-(R/L)t})$$

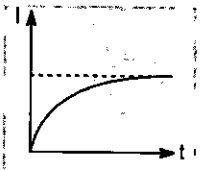


Fig. 5E.62 (b)

The power dissipated in the resistor by Joule heating is

$$P = RI^2 = \frac{V^2}{R}(1 - e^{-(R/L)t})^2.$$

Fig. 5E.62(c) is a sketch of the power-time graph, and shows that the power increases monotonically. Whilst the rate of change of power initially increases, it later reaches a maximum, and beyond this point decreases monotonically to zero as shown in Fig. 5E.62(d).

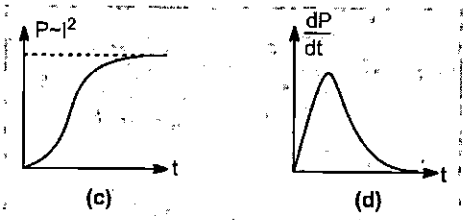


Fig. 5E.62

The fastest rate of change of energy occurs, for both the inductor and resistor, when I^2 is changing most rapidly. In part (i) we found that this happens when $I = V/2R$. Substituting this value for I into the above expression for the power, shows that it occurs when $e^{-(R/L)t} = 1/2$, and hence that $t = (L/R)\ln 2 \approx 0.69(L/R)$.

Remark: *(i) The rate of energy loss from the battery is VI, and so is proportional to the current, which increases monotonically.*
(ii) The fastest increase in the current takes place at $t = 0$, but that in I^2 (which is proportional to the magnetic energy of the coil) occurs later.

Example 63

In Fig. 5E.63 shown, $I_1 = 10e^{-2t}$ A, $I_2 = 4$ A and $V_C = 3e^{-2t}$ V. Determine:
(a) I_L and V_L, (b) V_{AC}, V_{AB} and V_{CD}.

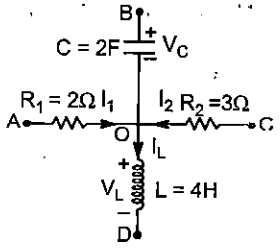


Fig. 5E.63

Solution: (a) Instantaneous in the capacitor,

$$q = CV_C = (2)(3e^{-2t}) = 6e^{-2t}$$

Current, $$I_C = \frac{dq}{dt} = -12e^{-2t} \text{ A}$$

Current flows from B to O.

From KCL, we have

$$I_L = I_1 + I_2 + I_C = 10e^{-2t} + 4 - 12e^{-2t}$$
$$= (4 - 2e^{-2t})\text{A} = [2 + 2(1 - e^{-2t})]\text{A}$$

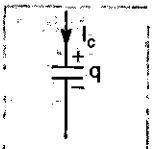


Fig. 5E.63 (a)

I_L *vs.* time graph is as shown in Fig. 5E.63 (b).

I_L increases from 2 A to 4 A exponentially.

$$V_L = L\frac{dI_L}{dt}$$
$$= (4)\frac{d}{dt}(4 - 2e^{-2t})$$
$$= 16e^{-2t} \text{ V}$$

V_L decreases exponentially from 16 V to 0 as shown in Fig. 5E.63 (c).

(b) To determine V_{AC}, we begin from A and end at C. From KVL, we have

$$V_A - I_1R_1 + I_2R_2 = V_C$$
or $$V_A - V_C = I_1R_1 - I_2R_2$$

Substituting the values we have,

$$V_{AC} = (10e^{-2t})(2) - (4)(3)$$
$$V_{AC} = (20e^{-2t} - 12) \text{ V}$$

At $t = 0$, $V_{AC} = 8$ V

At $t = \infty$, $V_{AC} = -12$ V

Therefore, V_{AC} decreases exponentially from 8 V to −12 V.

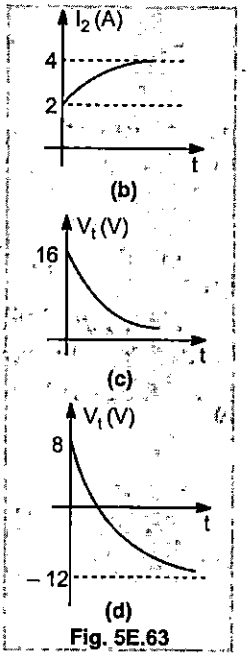


Fig. 5E.63

Similarly, we have from A to B

$$V_A - I_1R_1 + V_C = V_B,$$
or $$V_{AB} = V_A - V_B = I_1R_1 - V_C$$

Substituting the values we have,

$$V_{AB} = (10e^{-2t})(2) - 3e^{-2t}$$
or $$V_{AB} = 17e^{-2t} \text{ V}$$

Thus, V_{AB} decreases exponentially from17 V to 0.

As we move from C to D,

$$V_C - I_2R_2 - I_L = V_D$$
$$V_{CD} = V_C - V_D = I_2R_2 + I_L$$

Substituting the values we have,

$$V_{CD} = (4)(3) + 16e^{-2t}$$
or $$V_{CD} = (12 + 16e^{-2t}) \text{ V}$$

At $t = 0$, $V_{CD} = 28$ V

and at $t = \infty$, $V_{CD} = 12$ V

i.e., V_{CD} decreases exponentially from 28 V to 12 V.

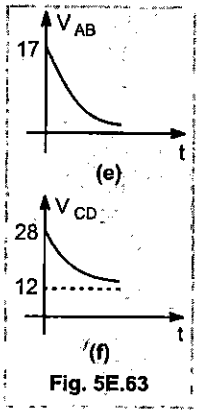


Fig. 5E.63

Example 64

In the circuit shown, the switch S is shifted to position 2 from position 1 at $t = 0$, having been in position 1 for a long time. Find the current in the circuit as a function of time.

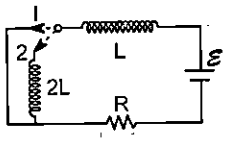


Fig. 5E.64

Solution: Let I_0 be the initial current in the steady state condition before shifting the switch.

$$\therefore \quad I_0 = \frac{\mathcal{E}}{R}$$

Let I'_0 be the current in the circuit at time $t = 0$, when the switch is shifted to position 2.

Since the flux associated with inductors will be same just before and just after the shifting of switch, otherwise the induced emf would be infinite.

$$\phi = I_0 L = I'_0 (L + 2L)$$

or $$I'_0 = \frac{I_0}{3} = \frac{\mathcal{E}}{3R}$$

If I is the instantaneous current in the circuit at time t, then

$$\mathcal{E} - 3L\frac{dI}{dt} - IR = 0 \quad \text{or} \quad 3L\frac{dI}{dt} = \mathcal{E} - IR$$

or $$\int_{I'_0}^{I} \frac{dI}{\mathcal{E} - IR} = \frac{2}{3L}\int_0^t dt$$

or $$\ln\left(\frac{\mathcal{E} - IR}{\mathcal{E} - I'_0 R}\right) = \frac{-2Rt}{3L}$$

or $$\frac{\mathcal{E} - IR}{\mathcal{E} - \frac{\mathcal{E}}{3R}R} = e^{-2Rt/3L}$$

or $$\mathcal{E} - IR = \frac{2\mathcal{E}}{3}e^{-2Rt/3L}$$

or $$I = \frac{\mathcal{E}}{R}\left(1 - \frac{2}{3}e^{-2Rt/3L}\right)$$

Example 65

In the Fig. 5E.65 (a) shown electrical network at $t < 0$, key was placed on (1) till the capacitor get fully charged. Key is placed on (2) at $t = 0$. Determine the time when the energy in both capacitor and inductor will be same for the first time.

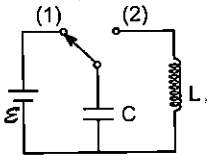


Fig. 5E.65 (a)

Solution: When switch 2 is closed, applying KCL in loop 2, we get

$$\frac{q}{C} + L\frac{dI}{dt} = 0$$

or $$L\frac{d^2q}{dt^2} + \frac{q}{C} = 0$$

$$\frac{d^2q}{dt^2} = \frac{-q}{CL}$$

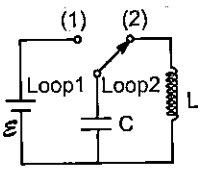


Fig. 5E.65 (b)

$$\Rightarrow \quad \frac{d^2q}{dt} + \omega^2 q = 0$$

Hence $$q = q_0 \cos\omega t, \ \omega = \sqrt{\frac{1}{LC}}$$

and $$I = -q_0\omega \sin\omega t$$

U_c = Energy stored in capacitor $= \frac{q^2}{2C} = \frac{q_0^2 \cos^2 \omega t}{2C}$

U_L = Energy stored in inductor

$$= \frac{1}{2}LI^2 = \frac{1}{2}Lq_0^2\omega^2 \sin^2 \omega t$$

When $U_C = U_L$, $$\frac{q_0^2 \cos^2 \omega t}{2C} = \frac{q_0^2\omega^2 L \sin^2 \omega t}{2}$$

$$\cot^2 \omega t = \omega^2 LC = 1$$

For minimum value of t, $\omega t = \frac{\pi}{4}$

Hence $$t = \frac{\pi}{4}\sqrt{LC}$$

Example 66

A U-frame ABCD and a sliding rod PQ of resistance R, start moving with the velocities v and $2v$ respectively, parallel to a long wire carrying current I when the distance $AP = l$ at $t = 0$. Determine the current through the inductor of inductance L if just before connecting, the rod PQ looses contact with the U-frame.

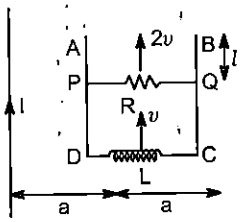
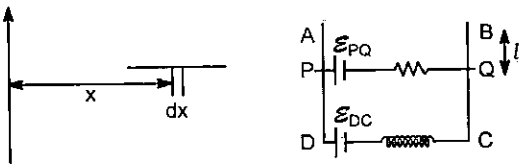


Fig. 5E.66 (a)

Solution: The motional emf will be induced across both the rods PQ and DC.

Integrating, P.D. across differential dx is



(b) (c)

Fig. 5E.66

$$\mathcal{E}_{DC} = \int dE = \int_a^{2a} v\left(\frac{\mu_0 I}{2\pi x}\right)dx$$

$$\mathcal{E}_{DC} = \frac{v\mu_0 I}{2\pi}\ln 2$$

(with D at higher potential)

Similarly, PD. across PQ is

$$\mathcal{E}_{PQ} = \frac{2v\mu_0 I}{2\pi}\ln 2$$

(with P at higher potential)

The relative velocity of the rod PQ w.r.t.U-frame,

$$v_{rel} = 2v - v = v$$

Now time taken by it to loose the contact, $t = l/v$

From equivalent electrical network,net emf in the closed loop $QPDC$,

$$\mathcal{E} = \mathcal{E}_{PQ} - \mathcal{E}_{DC} = \frac{v\mu_0 I}{2\pi}\ln 2$$

Growth of current in the LR circuit is given by

$$I = I_0(1-e^{-tR/L}) = \left(\frac{\mathcal{E}}{R}\right)(1-e^{-tR/L})$$

At $t = l/v$, $\quad I = \left(\frac{\mathcal{E}}{R}\right)(1-e^{-lR/Lv})$

Example 67

An iron core solenoid of length l and cross-sectional area A having N turns on it is connected to a battery through a resistance as shown in Fig. 5E.67(a). At instant $t = 0$, the iron rod of permeability μ from the core is abruptly removed. (a) Find the current as a function of time, (b) The work done to pull out the rod.

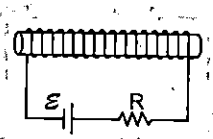


Fig. 5E.67 (a)

Solution: Self-inductance of the solenoid

$$L = \frac{\mu_0\mu N^2 A}{l}$$

and initial current $I_0 = \frac{\mathcal{E}}{R}$ because circuit is in the steady state.

When iron rod is abruptly removed from the core, the number of flux linkages abruptly do not change.

$$N\phi = LI_0 = L'I'_0$$

$$\left(\frac{\mu_0\mu N^2 A}{l}\right)I_0 = \left(\frac{\mu_0 N^2 A}{l}\right)I'_0$$

$\therefore$ Just after $t = 0$, initial current $I'_0 = \mu I_0 = \mu\left(\frac{\mathcal{E}}{R}\right)$

(a) At this instant, if the current in the circuit is I, then applying KVL in the loop, we get

$$L\frac{dI}{dt} + IR = \mathcal{E}$$

$$-L\frac{dI}{(\mathcal{E}-IR)} = dt$$

On integration, we get

$$\frac{L}{-R}\ln(\mathcal{E}-IR) = t + c_1$$

At $t = 0$, $\quad I = I'_0 = \left(\mu\frac{\mathcal{E}}{R}\right)$

$$\Rightarrow \quad \left(\frac{L}{-R}\right)\ln(\mathcal{E}-I'_0R) = c_1$$

$$\therefore \quad \ln\left(\frac{\mathcal{E}-IR}{\mathcal{E}-I'_0R}\right) = -\frac{tR}{L}$$

$$\left(\frac{\mathcal{E}-IR}{\mathcal{E}-I'_0R}\right) = e^{-tR/L}$$

$$\Rightarrow \quad \left(\frac{\mathcal{E}-IR}{\mathcal{E}-\left(\mu\frac{\mathcal{E}}{R}\right)R}\right) = e^{-tR/L}$$

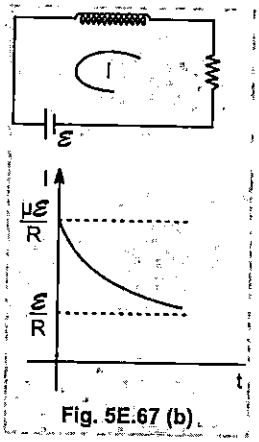


Fig. 5E.67 (b)

On rearranging the equation, we get

$$I = \frac{\mathcal{E}}{R}[1 + (\mu - 1)e^{-tR/L}]$$

At $t \to 0$, $\quad I = \frac{\mu\mathcal{E}}{R}$

and at $t \to \infty$, $\quad I = \frac{\mathcal{E}}{R}$

(b) From work-energy theorem,

$$W = \Delta U = U_f - U_i$$

$$= \frac{1}{2}L'I'^2 - \frac{1}{2}LI^2$$

$$= \frac{1}{2}\left(\frac{\mu_0 N^2 A}{l}\right)\left(\frac{\mathcal{E}}{R}\right)^2 - \frac{1}{2}\left(\frac{\mu\mu_0 N^2 A}{l}\right)\left(\frac{\mu\mathcal{E}}{R}\right)^2$$

$$= \frac{1}{2}\left(\frac{\mu_0 N^2 A}{l}\right)\left(\frac{\mathcal{E}}{R}\right)^2[1-\mu^3]$$

Example 68

A metal bar AB can slide on two parallel thick metallic rails separated by a distance l. A resistance R and an inductance L are connected to the rails as shown in Fig. 5E.68. A long straight wire carrying a constant current I_0 is placed in the plane of the rails and perpendicular to them as shown. The bar AB is held at rest at a distance x_0 from the long wire. At $t = 0$, it is made to slide on the rails away from the wire. Answer the following questions:

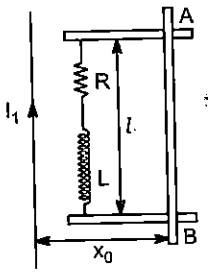


Fig. 5E.68

(a) Find a relation among I, $\frac{dI}{dt}$ and $\frac{d\phi}{dt}$, where I is the current in the circuit and ϕ is the flux of the magnetic field due to the long wire through the circuit.

(b) It is observed that at time $t = T$, the metal bar AB is at a distance $2x_0$ from the long wire and the resistance R carries a current I_1. Obtain an expression for the net charge that has flown through resistance R from $t = 0$ to $t = T$.

(c) The bar is suddenly stopped at time T. The current through resistance R is found to be $I_1/4$ at time 2T. Find the value of L/R in terms of the other given quantities.

Solution: (a) From KVL,

$$\frac{d\phi}{dt} - L\frac{dI}{dt} - IR = 0 \qquad \ldots(1)$$

(b) $$\frac{d\phi}{dt} = L\frac{dI}{dt} + \frac{dq}{dt}R$$

$\Rightarrow$ $$d\phi = L\,dI + R\,dq$$

$\Rightarrow$ $$\Delta\phi = L\Delta I + R\,\Delta q$$

or $$\Delta q = \frac{1}{R}(\Delta\phi - L\,\Delta I)$$

or $$\Delta q = \frac{1}{R}(\Delta\phi - LI_1) \quad (\text{since } \Delta I = I_1 - 0 = I_1) \qquad \ldots(2)$$

Now $$\Delta\phi = \int_{x_0}^{2x_0} B\,dA$$

$$= \int_{x_0}^{2x_0} \frac{\mu_0 I_0}{2\pi x}\,l\,dx = \frac{\mu_0 I_0 l}{2\pi}\ln 2$$

Putting this value of $\Delta\phi$ in equation (2),

$$\Delta q = \frac{1}{R}\left(\frac{\mu_0 I_0 l}{2\pi}\ln 2 - LI_1\right)$$

(c) When the rod is stopped,

Induced emf, $\frac{d\phi}{dt} = 0$

Therefore from equation (1),

$$-L\frac{dI}{dt} - IR = 0$$

or $$\frac{dI}{I} = \frac{-R}{L}dt$$

Integrating, $$\int_{I_1}^{I_1/4}\frac{dI}{I} = \frac{-R}{L}\int_T^{2T} dt$$

or $$\ln\frac{1}{4} = \frac{-R}{L}T$$

or $$\frac{L}{R} = \frac{T}{\ln 4}$$

Example 69

A thin superconducting (zero resistance) ring is held above a vertical, cylindrical magnetic rod, as shown in Fig. 5E.69. The axis of symmetry of the ring is the same as that of the rod. The cylindrically symmetrical magnetic field around the ring can be described approximately in terms of the vertical and radial components of the magnetic field vector as $B_z = B_0(1 - \alpha z)$ and $B_r = B_0\beta r$, where B_0, α and β are constants, and z and r are the vertical and radial position coordinates, respectively.

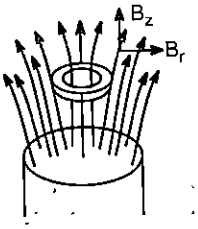


Fig. 5E.69

Initially, the ring has no current flowing in it. When released, it starts to move downwards with its axis still vertical. From the data below, determine how the ring moves subsequently? What current flows in the ring?

Data :

Properties of the ring :

Mass	$m = 50\,mg$
Radius	$r_0 = 0.5\,cm$
Inductance	$L = 1.3 \times 10^{-8} H$

Initial coordinates of the centre of the ring : $z = 0$, $r = 0$

Magnetic field constants:

$$B_0 = 0.01\,T,\ \alpha = 2\ m^{-1},\ \beta = 32\ m^{-1}$$

Solution: The total magnetic flux at the position of the ring is made up of that due to the external magnetic field and the effects of self-induction,

$$\phi = B_z\pi r_0^2 + LI$$

Any change in magnetic flux induces a current in the ring in accordance with

$$RI = \frac{\Delta\phi}{\Delta t}.$$

However, this has to be zero since the ohmic resistance of a superconducting ring is zero. Accordingly, the magnetic flux through the ring has to be constant, *i.e.*,

$$\phi = B_0(1 - \alpha z)\pi r_0^2 + LI = \text{constant}.$$

From the initial conditions ($z = 0, I = 0$), the value of the constant is $\phi = B_0\pi r_0^2$.

The current in the ring can be determined using the above equations which give

$$I = \frac{1}{L}B_0\alpha\pi r_0^2 z.$$

The Lorentz force acting on the ring (which can only be vertical, because of the symmetry of the assembly) can be expressed as

$$F_z = -B_r I(z)\,2\pi r_0$$

$$= -B_0\beta r_0 \frac{1}{L} B_0 \alpha \pi r_0^2 2\pi r_0 z = -kz.$$

The Lorentz force is thus directly proportional to the vertical displacement of the ring, with the coefficient of proportionality calculable from the given data. (This result is only valid for small displacements, since the magnetic induction is not adequately described by the given formulae for large ones.)

The equation of motion of the ring is

$$ma_z = F_z - mg = -kz - mg.$$

This means that the ring makes harmonic oscillations about the equilibrium position $z_0 = -mg/k$ with

$$z(t) - z_0 = A\cos\omega t,$$

where $\omega = \sqrt{k/m}$. From the initial conditions it follows that $A = -z_0$, and so

$$z(t) = \frac{g}{\omega^2}(\cos\omega t - 1).$$

The vertical z-coordinate is never positive, and it follows that the Lorentz force always points upwards, being zero at the topmost point of the oscillation. The current always flows in the same direction around the ring.

Substituting the numerical data gives $\omega = 31.2\,s^{-1}$ and $A = 1$ cm. The time dependence of the current flowing in the ring can be expressed in terms of $z(t)$ as

$$I = \frac{1}{L} B_0 \alpha \pi r_0^2 z(t)$$

$$= \frac{1}{L} B_0 \alpha \pi r_0^2 A(\cos\omega t - 1).$$

The maximum value of the current, which flows at the bottom of the oscillation, is $I_{max} = 39$ A.

Example 70

A coil of inductance L connects the upper ends of two vertical copper bars separated by a distance l. A horizontal conducting connector of mass m starts falling with zero initial velocity along the bars without losing contact with them. The whole system is located in a uniform magnetic field B perpendicular to the plane of the bars. Find the law of motion x(t) of the connector.

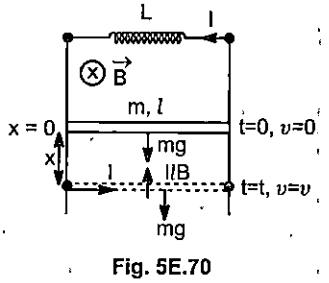


Fig. 5E.70

Solution: Assuming that the connector is released from rest at $t = 0$ from the position $x = 0$, and at time t, it is at position x and its velocity is v (downwards). Let the current in the connector is I as shown in Fig. 5E.70.

From KVL, $L\frac{dI}{dt} = Bvl = Bl\frac{dx}{dt}$

$\therefore$ $L\,dI = Bl\,dx$

Integrating, we get

$$LI = Blx$$

or $$I = \left(\frac{Bl}{L}\right)x \quad \ldots(1)$$

Magnetic force on conductor,

$$F_B = IlB = \left(\frac{B^2l^2}{L}\right)x$$

Resulting force $= mg - \frac{B^2l^2}{L}x.$ $\ldots(2)$

From Newton's second law, we have

$$m\left(\frac{dv}{dt}\right) = mg - \left(\frac{B^2l^2}{L}\right)x$$

or $$\frac{dv}{dt} = g - \left(\frac{B^2l^2}{mL}\right)x$$

Differentiating this equation w.r.t. time, we have

$$\frac{d^2v}{dt^2} = -\left(\frac{B^2l^2}{mL}\right)\frac{dx}{dt} = -\left(\frac{B^2l^2}{mL}\right)v$$

or $$\frac{d^2v}{dt^2} = -\omega^2 v$$

where $$\omega = \frac{Bl}{\sqrt{mL}} \quad \ldots(3)$$

Eq. (3) shows that, v oscillates simple harmonically with angular frequency $\omega = \frac{Bl}{\sqrt{mL}}$. At $t = 0$, $v = 0$, so v_0 can be calculated by noting that

$$v = v_0 \sin\omega t \quad \ldots(4)$$

At $t = 0$, $\frac{dv}{dt} = g$

or $v_0\omega = g$

or $v_0 = \frac{g\sqrt{mL}}{Bl}$ $\left[\text{as } \omega = \frac{Bl}{\sqrt{mL}}\right]$

As $\frac{dx}{dt} = v_0 \sin\omega t$

or $\int_0^x dx = \int_0^t v_0 \sin\omega t\, dt$

or $x = \frac{v_0}{\omega}(1 - \cos\omega t)$

Example 71

A loop is formed by two parallel conductors connected by a solenoid with inductance L and a conducting rod of mass m which can freely (without friction) slide over the conductors. The conductors are located in a horizontal plane in a uniform vertical magnetic field B. The distance between the conductors is l. At the moment $t = 0$, the rod is imparted an initial velocity v_0 directed to the right. Find the law of its motion, $x(t)$ if the electric resistance of the loop is negligible.

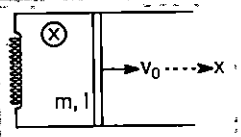


Fig. 5E.71 (a)

Solution: Let the instantaneous velocity of the rod be v towards right, and the current in the circuit be I.

From KVL, we have

$$L\frac{dI}{dt} = Bvl = Bl\frac{dx}{dt}$$

i.e., $$L\,dI = Bl\,dx$$

Integrating, we get

$$LI = Blx$$

or $$I = \frac{Bl}{L}x \quad \ldots(1)$$

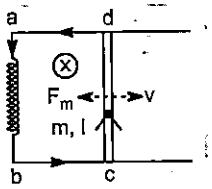


Fig. 5E.71 (b)

Instantaneous magnetic force on the rod is

$$F = IlB = \frac{B^2l^2}{L}x \quad \ldots(2)$$

When the rod moves, flux linked with the loop increases, hence the induced current is in such a direction so as to oppose the forward motion of the rod.

From Newton's second law, we have

$$m\left(\frac{d^2x}{dt^2}\right) = -\frac{B^2l^2}{L}x$$

or $$\left(\frac{d^2x}{dt^2}\right) = -\frac{B^2l^2}{mL}x$$

Comparing this with the equation of SHM, *i.e.*,

$$\frac{d^2x}{dt^2} = -\omega^2x$$

we have, $$\omega = \frac{Bl}{\sqrt{mL}}$$

or $$T = \frac{2\pi}{Bl}\sqrt{LM}$$

The rod oscillates simple harmonically with angular frequency $\omega = Bl/\sqrt{mL}$. Because the rod moves from mean position $x = 0$, x-t equation on of rod is

$$x = A\sin\omega t \quad \ldots(3)$$

At $t = 0$, $\frac{dx}{dt}$ has a value v_0.

Hence $$\frac{dx}{dt} = v = A\omega\cos\omega t$$

or $$A\omega = v_0 \qquad (\text{at } t = 0)$$

or $$A = \frac{v_0}{\omega}$$

Substituting in Eq. (3), we have

$$x = \frac{v_0}{\omega}\sin\omega t,$$

where $$\omega = \frac{Bl}{\sqrt{mL}}$$

Alternate method of finding *A*:

At $x = A$, $v = 0$, *i.e.*, whole of its kinetic energy is converted into magnetic energy. Thus,

$$\frac{1}{2}LI^2 = \frac{1}{2}mv_0^2$$

Substituting the value of I from eqn. (1), with $x = A$, we have

$$L\left(\frac{Bl}{L}A\right)^2 = mv_0^2$$

or $$A = \frac{\sqrt{mL}}{Bl}v_0 = \frac{v_0}{\omega}$$

as $$\omega = \frac{Bl}{\sqrt{mL}}$$

Example 72

A homogeneous field of magnetic induction $\vec{\mathbf{B}}$ *is perpendicular to a track of gauge l which is inclined at an angle α to the horizontal. A frictionless conducting rod of mass m straddles the two rails of the track as shown in Fig. 5E.72.*

How does the rod move, after being released from rest, if the circuit formed by the rod and the track is closed by:

(i) a resistor of resistance R,

(ii) a capacitor of capacitance C, or

(iii) a coil of inductance L ?

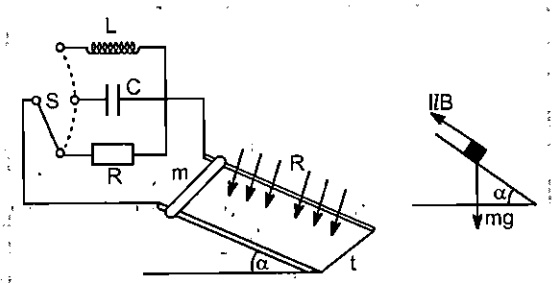


Fig. 5E.72

Solution: Let the rod move with speed v and acceleration a along the inclined plane, while current I flows in it. The magnetic field exerts a force that retards the rod in accordance with Lenz's law, and its equation of motion is

$$ma = mg\sin\alpha - BlI.$$

This equation is the same in all three cases but there is a different relationships between the induced voltage and the current flowing in the rod.

(i) If the circuit is closed by a resistor R, the current $I = V/R = Blv/R$. The braking force $[IlB = B^2l^2v/R]$ increases in proportion to the speed, with the result that the rod experiences decreasing acceleration and ultimately travels with uniform speed. This final maximum speed v_{max} can be found from the equation of motion by setting $a = 0$,

$$v_{max} = \frac{mgR\sin\alpha}{B^2l^2}$$

(ii) If the circuit is closed by a capacitor of capacitance C, the charge on the capacitor is given by

$$Q = CV = CBlv$$

the current flowing through the rod is

$$I = \frac{dQ}{dt} = C\frac{dV}{dt} = CBI\frac{dv}{dt} = CBla.$$

If the above expression for the current is substituted into the equation of motion, the rod is found to move on the rails with uniform acceleration

$$a = \frac{mg\sin\alpha}{m + B^2l^2C}$$

Induction decreases the acceleration caused by gravity by, in effect, increasing the inertial mass of the rod . The speed of the rod and the charge on the capacitor are both directly proportional to the time elapsed.

(iii) If the circuit is closed by a coil of inductance L, the relationship between the induced voltage and the current is

$$L\frac{dI}{dt} = Blv = Bl\frac{dx}{dt}$$

We note that, since $I = 0$ and $x = 0$ at the start of the motion, the above formula implies that the current is proportional to the x-coordinate, $LI = Blx$. Substituting for the current, from this relationship into the equation of motion, gives

$$ma = mg\sin\alpha - \frac{B^2l^2}{L}x.$$

The force acting on the rod is therefore the sum of a constant term and a negative term proportional to the displacement. This is the same as the equation of motion of a body hung on a spring and then released . Thus the rod makes harmonic oscillations about an equilibrium position

$$x_0 = \frac{mgL\sin\alpha}{B^2l^2}$$

The amplitude of the oscillation is $A = x_0$, and the dependence of the displacement of the rod on time is

$$x(t) = A(1 - \cos\omega t),$$

Where $\omega^2 = \dfrac{B^2l^2}{mL}$.

A PARALLEL *LC* CIRCUIT

Fig. 5.63 shows a LC circuit consisting of an ideal capacitor of capacitance C and an ideal inductor of inductance L.

We assume that the capacitor is initially charged with charge Q_0 and the switch is closed at $t = 0$. The following process takes place after closing the switch:

(1) When the switch is closed at $t = 0$, the capacitor begins to discharge and a current I passes through inductor. Initially current increases and dI/dt is positive . In accordance with Lenz's law the induced emf in inductor opposes the increase in current.

(2) As I increases with time, dI/dt is positive , the potential difference across the capacitor decreases because charge on capacitor's plates is decreasing. Consequently the rate of change dI/dt decreases and reduces to zero when capacitor is fully discharged. At that instant the current reaches its maximum value.

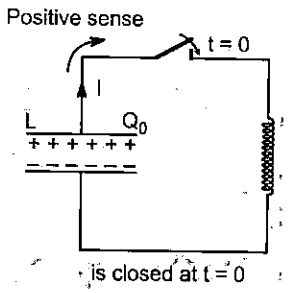


Fig. 5.63

(3) The potential difference across capacitor plates has dropped to zero, it cannot drive further charges. But inductor produces an emf whose sense opposes a decrease in I, in accordance with Lenz's law. This induced emf propels charges in the positive sense as the current decreases . At this instant I is still positive , but dI/dt is now negative.

(4) According to step (3), the current persists after capacitor has discharged; the capacitor now charges with polarity opposite its initial polarity. The growing potential difference opposes the current, which falls to zero when the capacitor is fully charged. At this instant the value dI/dt reaches a minimum (negative) value.

What we have described in steps 1 to 4 is first half of the oscillation cycle. In the second half cycle the capacitor discharges with current in negative sense. The second half is exactly reverse of the first half . Thus the system behaves in an oscillatory manner.

At any instant the total energy U_{tot} stored in the circuit is the sum of that stored in the capacitor and that stored on the inductor .

$$U_{tot} = \frac{1}{2}LI^2 + \frac{Q^2}{2C} \quad \ldots(1)$$

In the absence of losses in the circuit, the electrical energy U_{tot} must remain constant and

$$\frac{dU_{tot}}{dt} = 0$$

i.e., $$\frac{d}{dt}\left(\frac{1}{2}LI^2 + \frac{Q^2}{2C}\right) = 0$$

[C and L are constant, I and Q are variable]

$$LI\frac{dI}{dt} + \frac{1}{C}Q\frac{dQ}{dt} = 0 \quad \ldots(2)$$

But $$\frac{dQ}{dt} = I \text{ and } \frac{dI}{dt} = \frac{d^2Q}{dt^2}$$

So our equation becomes

$$\frac{d^2Q}{dt^2} + \frac{1}{LC}Q = 0 \quad \ldots(3)$$

This linear differential equation has the same form as that for a simple harmonic oscillator composed of mass m attached to a spring of spring constant k and vibrating with frequency

$$\omega = \sqrt{k/m}$$

$$\frac{d^2x}{dt^2} + \omega^2 x = 0 \qquad ...(4)$$

The solution of this equation is

$$x(t) = A\cos(\omega t + \delta)$$

where A is amplitude and δ is phase constant determined from initial conditions of oscillator. Therefore we are led to the solution of equation (3)as

$$Q(t) = A\cos(\omega t + \delta) \qquad ...(5)$$

By comparing eqns. (3) and (4), we recognize angular frequency to be

$$\omega = \sqrt{\frac{1}{LC}} \qquad ...(6)$$

The current is given by

$$I(t) = \frac{dQ}{dt} = -\omega A\sin(\omega t + \delta)$$

As current is zero at $t = 0$,

$$I(0) = -\omega_0 A\sin\delta = 0 \text{ ; thus } \delta = 0$$

As charge is Q_0 at $t = 0$,

$$Q(0) = Q_0 = A\cos 0 \text{ , thus } A = Q_0$$

- Thus the final solution is

$$Q(t) = Q_0\cos\omega t \qquad ...(7)$$

$$I(t) = -\omega Q_0\sin\omega t \qquad ...(8)$$

- **The passage of charge through the *LC* circuit is oscillatory.**

The quantity ω is natural angular frequency of the system.

- The potential difference across the capacitor is

$$V_C = \frac{Q(t)}{C} = \frac{Q_0}{C}\cos\omega t = V_0\cos\omega t \qquad ...(9)$$

- The potential difference across the inductor is

$$V_L = L\frac{dI(t)}{dt} \qquad ...(10)$$

$$\frac{dI(t)}{dt} = -\omega^2 Q_0\cos\omega t$$

Thus $$V_L = -\omega^2 L Q_0\cos\omega t \qquad ...(11)$$

- The energy stored in the inductor at time t,

$$U_B = \frac{1}{2}LI^2 = \frac{1}{2}L\omega^2 Q^2\sin^2\omega t$$

$$= \frac{Q^2\sin^2\omega t}{2C} \quad \text{as} \quad \omega^2 = \frac{1}{LC}$$

- The energy stored in the capacitor at time t,

$$U_C = \frac{Q^2}{2C} = \frac{Q_0^2\cos^2\omega t}{2C}$$

- The total energy U_{tot} of the LC circuit is the sum of $U_B + U_C$.

$$U_{tot} = \frac{Q_0^2\sin^2\omega t}{2C} + \frac{Q_0^2\cos^2\omega t}{2C}$$

$$= \frac{Q_0^2}{2C}[\sin^2\omega t + \cos^2\omega t] = \frac{Q_0^2}{2C}$$

This is the expected result;the total energy of the system is constant and equal to initial energy

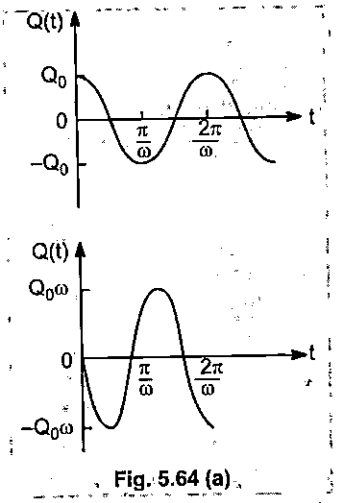


Fig. 5.64 (a)

- Fig 5.64 (a) shows the graphs of charge and current oscillations as functions of time . Fig 5.64 (b) shows the energy stored by the capacitor and inductor as functions of time.

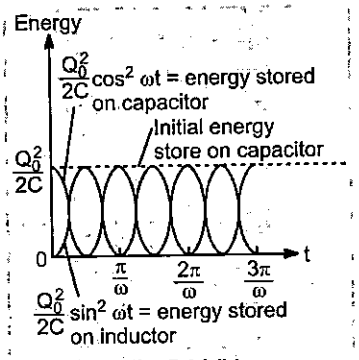


Fig. 5.64 (b)

- The table shows the analogies between the LC oscillator and mass-spring oscillator:

Table: Analogies between the *LC* Oscillator and the Mass Spring Oscillator

LC Oscillator	Mass-Spring Oscillator
$\frac{d^2q}{dt} = -\frac{1}{LC}q$	$\frac{d^2x}{dt} = -\frac{k}{m}x$
$q = q_0\cos(\omega_0 t + \delta)$	$x = A\cos(\omega t + \delta)$
$I = \frac{dq}{dt} = -\omega_0 q_0\sin(\omega_0 t + \delta)$	$v = \frac{dx}{dt} = -\omega A\sin(\omega t + \delta)$
$\frac{V_L}{L} = \frac{d^2q}{dt^2} = -\omega_0^2 q_0\cos(\omega t + \delta)$	$a = \frac{d^2x}{dt^2} = -\omega^2 A\cos(\omega t + \delta)$
$1/C$	k
L	m
$\omega_0 = \frac{1}{\sqrt{LC}}$	$\omega = \sqrt{(k/m)}$
$U_C = \frac{1}{2}\frac{1}{C}q^2$	$U = \frac{1}{2}kx^2$
$U_L = 1/2LI^2$	$K = 1/2mv^2$

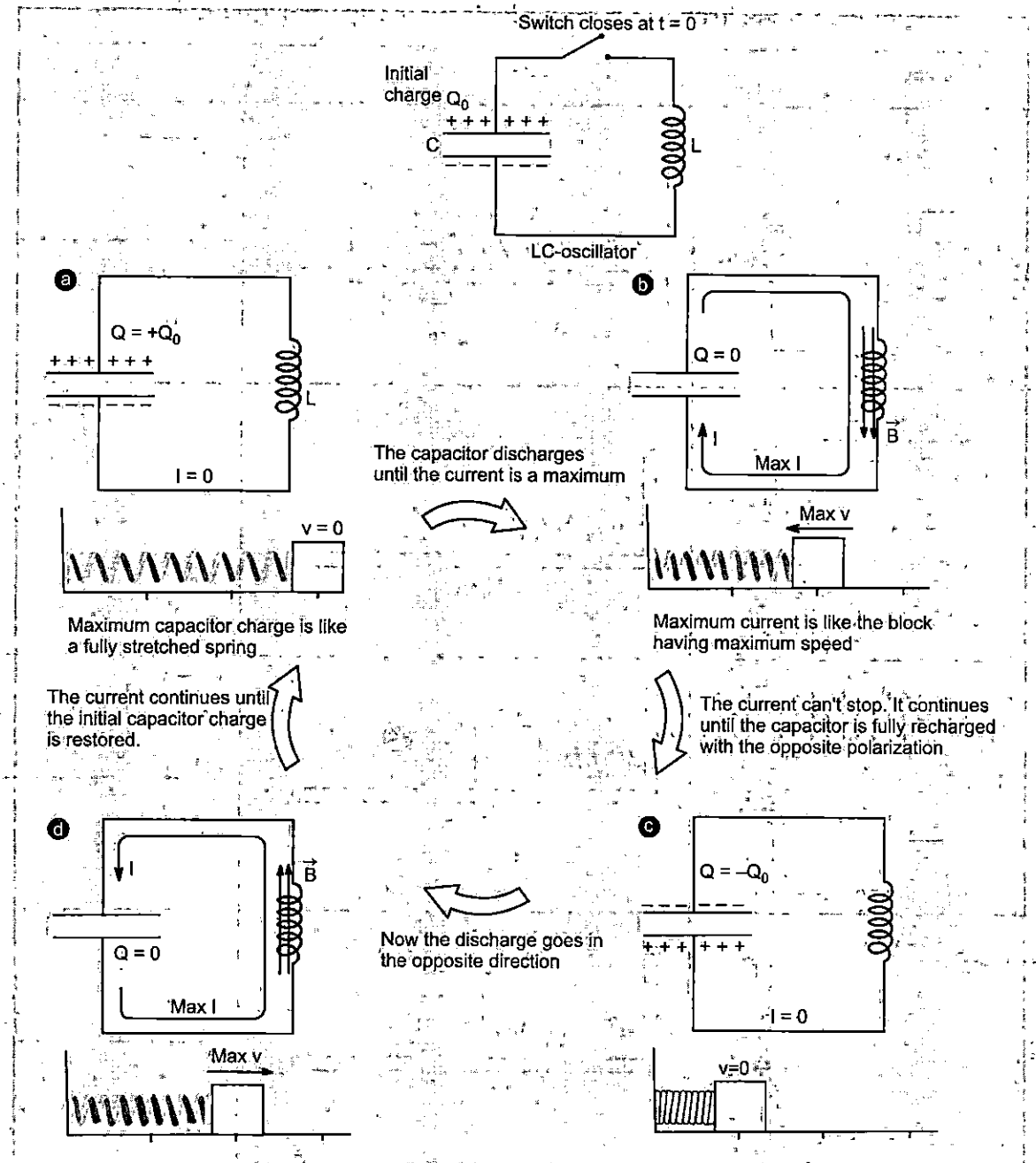
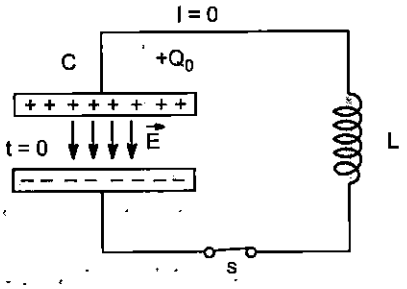
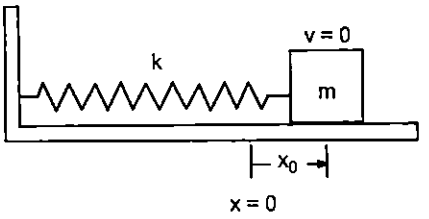
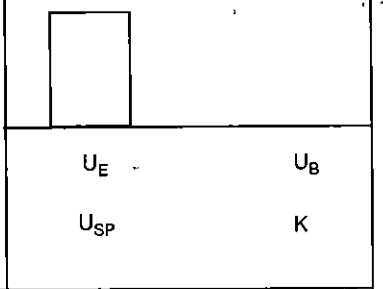
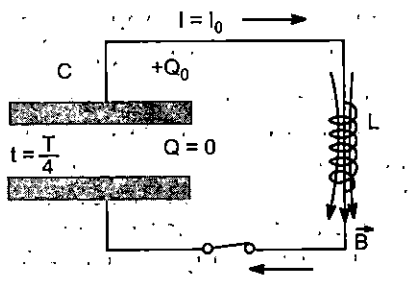
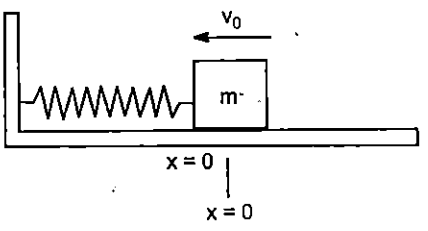
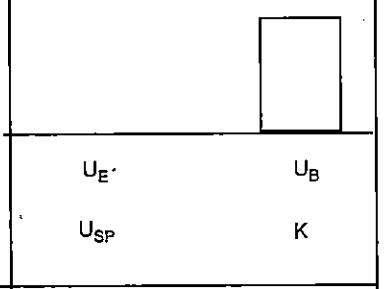
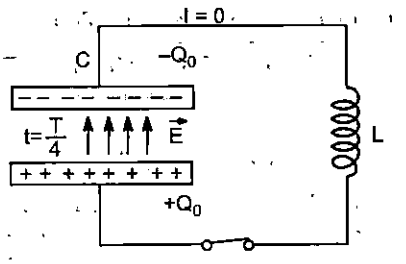
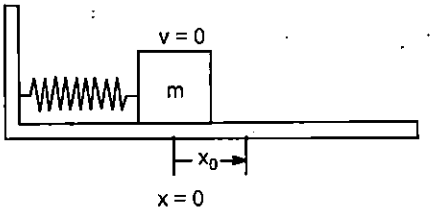
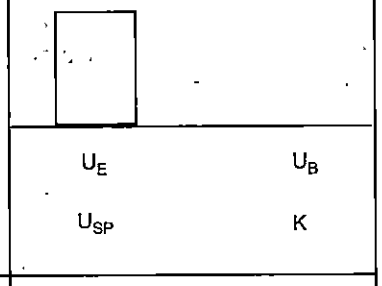
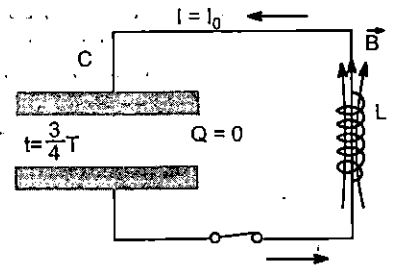
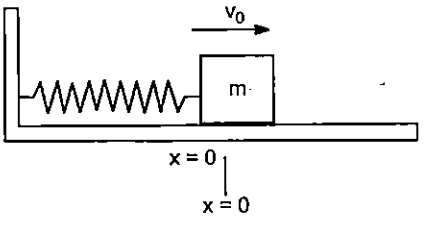
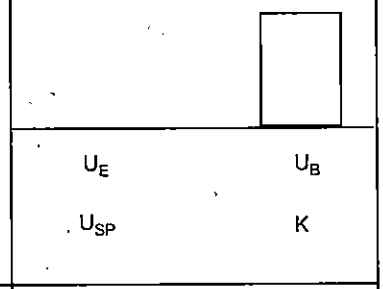
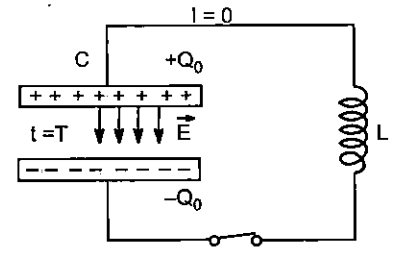
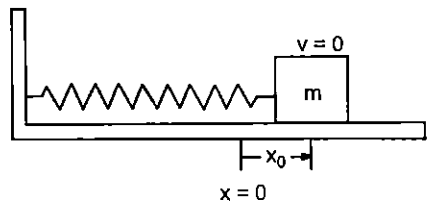
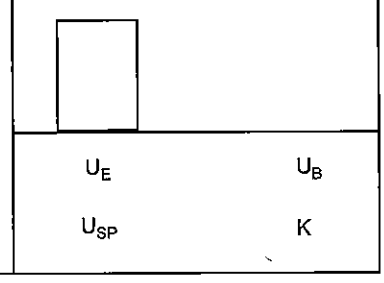


Fig. 5.65: The capacitor charge oscillates like a block attached to a spring

Table: Illustrates Transformation Electric Energy to Magnetic Energy

LC circuit	Mass-spring system	Energy
		
		
		
		
		

Example 73

Consider the circuit shown in Fig. 5E.73. Suppose the switch which has been connected to point a for a long time is suddenly thrown to b at $t = 0$.

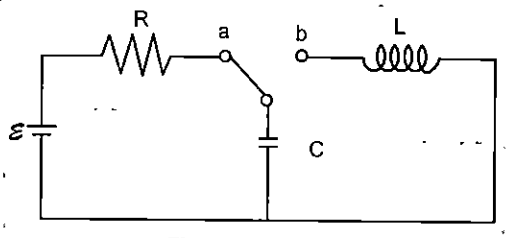


Fig. 5E.73: LC circuit

Find the following quantities:

(a) the frequency of oscillation of the LC circuit.

(b) the maximum charge that appears on the capacitor.

(c) the maximum current in the inductor.

(d) the total energy in the circuit possesses at any time t.

Solution: (a) The (angular) frequency of oscillation of the LC circuit is given by $\omega = 2\pi f = 1/\sqrt{LC}$. Therefore, the frequency is:

$$f = \frac{1}{2\pi\sqrt{LC}}$$

(b) The maximum charge stored in the capacitor before the switch is thrown to b is:

$$Q = C\mathcal{E}$$

(c) The energy stored in the capacitor before the switch is thrown is:

$$U_E = \frac{1}{2}C\mathcal{E}^2$$

On the other hand, the magnetic energy stored in the inductor is:

$$U_B = \frac{1}{2}LI^2$$

Thus, when the current is at its maximum, all the energy originally stored in the capacitor is now in the inductor:

$$\frac{1}{2}C\mathcal{E}^2 = \frac{1}{2}LI_0^2$$

This implies a maximum current

$$I_0 = \mathcal{E}\sqrt{\frac{C}{L}}$$

(d) At any time, the total energy in the circuit would be equal to the initial energy that the capacitance stored, that is

$$U = U_E + U_B = \frac{1}{2}C\mathcal{E}^2$$

Example 74

Initially the 900 μF capacitor is charged to 100 V and the 100 μF capacitor is uncharged in Fig. 5E.74 shown. Then the switch S_2 is closed for a time t_1, after which it is opened and at the same instant switch S_1 is closed for a time t_2 and then opened.

It is now found that the 100 μF capacitor is charged to 300 V. Find the minimum possible values of the time intervals t_1 and t_2.

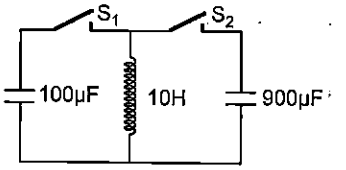


Fig. 5E.74

Solution: Initial energy stored in the 900 μF capacitor is

$$U_1 = (1/2)\times 900\times 10^{-6}\times (100)^2 = 4.5\,\text{J}$$

Finally, energy stored in the 100 μF capacitor is

$$U_2 = (1/2)\times 100\times 10^{-6}\times (300)^2 = 4.5\,\text{J}$$

The entire energy of the 900 μF capacitor has been transferred to the 100 μF capacitor. First electrical energy of the 900 μF capacitor is converted into magnetic energy in the inductor and then this energy is converted into electrical energy once again using S_2 and S_1 appropriately. In a LC circuit the transfer of electrical energy into magnetic energy and *vice-versa* takes place in a time $T/4$ where $T = 2\pi\sqrt{LC}$ is the time period of the electrical oscillations. Thus

$$T_1 = 2\pi\sqrt{10\times 900\times 10^{-6}} = 0.6\,\text{s}$$

and

$$T_2 = 2\pi\sqrt{10\times 100\times 10^{-6}} = 0.2\,\text{s}$$

Therefore, switch S_2 is first closed for time $0.6/4 = 0.15$ s, during which time the 900 μF capacitor gets fully discharged and the current in the inductor is fully established. Next, the switch S_2 is opened and simultaneously switch S_1 is closed for time $0.2/4 = 0.05$ s during which time the current in the inductor disappears and the 100 μF capacitor gets fully charged. After this time, the switch S_1 is also opened. The 100 μF capacitor is now charged to 300 V.

Thus $t_1 = 0.15\,\text{s}$

and $t_2 = 0.05\,\text{s}$

Example 75

The circuit shown in Fig. 5E.75 (a) is in the steady state with switch S_1 closed. At $t = 0$, S_1 is opened and switch S_2 is closed.

(a) Derive an expression for the charge on the capacitor C_2 as a function of time.

(b) Determine the first instant t, when the energy in the inductor becomes one-third of that in the capacitor.

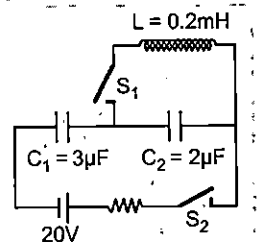


Fig. 5E.75 (a)

Solution: (a) In the steady state C_1 and C_2 are in series arrangement, their equivalent is.

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = 1.2\,\mu F$$

Charge on the capacitor C_2, $Q_0 = C_{eq} V = 1.2 \times 20 = 24$ μC

When S_1 is open and S_2 is closed. Capacitor C_2 starts discharging through the inductor and let at any time 't' charge on the capacitor be Q, then.

Applying Kirchhoff's voltage law,

$$\frac{Q}{C_2} - L\frac{dI}{dt} = 0$$

$$\frac{Q}{C_2} + L\frac{d^2Q}{dt^2} = 0 \qquad \left[I = \frac{-dQ}{dt}\right]$$

The solution of this equation is

$$Q = Q_0 \sin(\omega t + \phi)$$

where $\omega = \frac{1}{\sqrt{LC_2}} = 5 \times 10^4$ rad/s

At $t = 0$, $Q = Q_0$

Hence $\phi = \frac{\pi}{2}$

Thus the charge on the capacitor at any time t is

$$Q = Q_0 \cos \omega t$$

(b) In a LC circuit, total energy remains always conserved.

∴ Electrical energy stored in the capacitor + magnetic energy stored in the inductor $= \frac{1}{2}\frac{Q_0^2}{C_2}$.

$$U_E + U_B = \frac{Q_0^2}{2C_2}$$

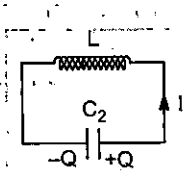


Fig. 5E.75 (b)

At the time $t = t_1$,

$$U_B = \frac{1}{3}U_E$$

Hence $U_E = \frac{3}{4}\left(\frac{1}{2}\frac{Q_0^2}{C_2}\right)$

and $\frac{1}{2}\frac{Q^2}{C_2} = \frac{3}{4}\left(\frac{1}{2}\frac{Q_0^2}{C_2}\right)$

or $Q = \frac{\sqrt{3}}{2}Q_0$

or $Q_0 \cos \omega t_1 = \frac{\sqrt{3}}{2}Q_0$

or $\omega t_1 = \frac{\pi}{6}$

or $t_1 = \frac{\pi}{6\omega} = 1.05 \times 10^{-5} = 10.5\,\mu s$

Example 76

A copper ring is placed co-axially in a time varying cylindrically symmetric magnetic field given by $\vec{B} = dt\,\vec{k}$. *Radius of the copper ring is r. Resistance of one half of the copper ring is double that of the remaining half. Obtain the magnitude of the electric field (conservative) inside the copper ring.*

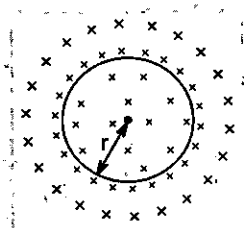


Fig. 5E.76

Solution: The time varying magnetic field will produce induced electric field (non-conservative) which will have same magnitude everywhere inside the tude and directed tangentially to the tube at any point. If E_n be its magnitude

$$E_n(2\pi r) = (\pi r^2)\frac{dB}{dt}$$

$$\Rightarrow \quad E_n = \frac{r}{2}\frac{dB}{dt}$$

$$\Rightarrow \quad E_n = \frac{rb}{2}$$

Let $\vec{E}_T$ be the total field at any point then

This induced field will set the free electrons in motion. Eventually the current will be the same everywhere in the ring. Let I be the current, then $-\int \vec{E}_T \cdot d\vec{l} = IR$ [over one half of ring with resistance R]

$= 2IR$ [over another half of the ring resistance $2R$]

The difference in the two integrals is on account of redistribution of free charges creating another field (conservative) such as to oppose E_n in one half having lesser resistance, and support E_n on the half having more resistance. Thus if E be the magnitude of the field (conservative),

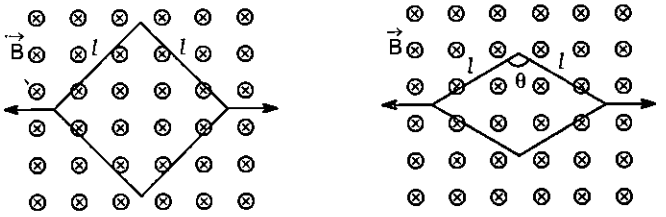
$$(E_n + E)\pi r = 2IR$$

and $(E_n - E)\pi r = IR$

Solving we get, $E = \frac{br}{6}$

Example 77

A square loop with length l on each side is placed in a uniform magnetic field pointing into the page. During a time interval, Δt, the loop is pulled from its two edges and turned into a rhombus, as shown in Fig. 5E.77 Assuming that the total resistance of the loop is R, find the average induced current in the loop and its direction.



Conducting loop changing area

Fig. 5E.77

Solution: Using Faraday's law, have

$$\mathcal{E} = -\frac{\Delta\phi_B}{\Delta t} = -B\left(\frac{\Delta A}{\Delta t}\right)$$

Since the initial and the final areas of the loop are $A_i = l^2$ and $A_f = l^2 \sin\theta$, respectively (recall that the area of a parallelogram defined by two vectors $\vec{l}_1$ and $\vec{l}_2$ is $A = \left|\vec{l}_1 \times \vec{l}_2\right| = l_1 l_2 \sin\theta$), the average rate of change of area is

$$\frac{\Delta A}{\Delta t} = \frac{A_f - A_i}{\Delta t} = -\frac{l^2(1-\sin\theta)}{\Delta t} < 0$$

$$\Rightarrow \qquad \mathcal{E} = \frac{Bl^2(1-\sin\theta)}{\Delta t} > 0$$

Thus, the average induced current is

$$I = \frac{\mathcal{E}}{R} = \frac{Bl^2(1-\sin\theta)}{\Delta t R}$$

Since $\left(\frac{\Delta A}{\Delta t}\right) < 0$, the magnetic flux into the page decreases. Hence, the current flows in the clockwise direction to compensate the loss of flux.

Example 78

A current I flowing a cylindrical wire of radius R is uniformly distributed over its cross-section. Obtain an expression for the energy stored in a co-axil cylindrical volume near its centre as shown in Fig. 5E.78.

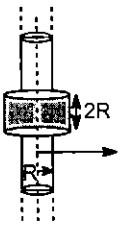


Fig. 5E.78

Solution: Let us divide the total volume in two parts, one within the cylindrical wire and another outside it. For the lst part, energy density u_1 is given by

$$u_1 = \frac{1}{2\mu_0}\left[\frac{\mu_0}{2\pi}\left(\frac{I}{\pi R^2}\right)\frac{\mu r^2}{r}\right]^2$$

$$= \frac{1}{8}\frac{\mu_0 I^2 r^2}{\pi^2 R^4} \qquad 0 \le r \le R$$

For the 2nd part, energy density, u_2 is given by.

$$u = \frac{I}{2\mu_0}\left[\frac{\mu_0 I}{2\pi r}\right]^2 \qquad R \le r \le 2R$$

Total energy

$$u = \int_{\substack{\text{voln of}\\ \text{part 1}}} u_1 dv_1 + \int_{\substack{\text{voln of}\\ \text{part 2}}} u_2 dv_2$$

$$= \int_0^R \frac{1}{8}\left(\frac{\mu_0 I^2 r^2}{\pi^2 R^4}\right) 2\pi r l\, dr + \int_R^{2R} \frac{1}{2\mu_0}\left(\frac{\mu_0 I}{2\pi r}\right) l\, 2\pi r dr$$

$$= \frac{m_0 I^2 l}{16\pi}[1 + 4\ln 2]$$

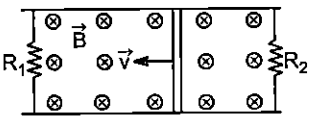
Example 79

A conducting rod of length l is free to side on two parallel conducting bars as in Fig. 5E.79. In addition, two resistors R_1 and R_2 are connected across the ends of the bars. There is a uniform magnetic field pointing into the page. Suppose an external agent pulls the bar to the left at a constant speed v. Find the :

(a) current through both resistors.

(b) total power delivered to the resistors.

(c) applied force required to maintain a constant velocity of the rod.



Sliding rod

Fig. 5E.79

Solution: (a) The emf induced between the ends of the moving rod is

$$\mathcal{E} = \frac{d\phi_B}{dt} = -Blv$$

The currents through the resistors are

$$I_1 = \frac{|\mathcal{E}|}{R_1} \quad \text{and} \quad I_2 = \frac{|\mathcal{E}|}{R_2}$$

Since the flux into the page for the left loop is decreasing, I_1 flows clockwise to produce a magnetic field pointing into the page. On the other hand, the flux into the page for the right loop is increasing. To compensate the change, according to Lenz's law, I_2 must flow counterclockwise to produce a magnetic field pointing out of the page.

(b) The total power dissipated in the two resistors is

$$P_R = I_1|\mathcal{E}| + I_2|\mathcal{E}| = (I_1 + I_2)|\mathcal{E}| = \mathcal{E}^2\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$= B^2l^2v^2\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

(c) The total current flowing through the rod is $I = I_1 + I_2$. Thus, the magnetic force acting on the rod is

$$F_B = BIl = |\mathcal{E}|\mathcal{E}^2\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$= B^2l^2v^2\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

and the direction is to the right. Thus, an external agent must apply an equal but opposite force $\vec{F}_{ext} = -\vec{F}_B$ to the left in order to maintain a constant speed.

Alternatively, we note that since the power dissipated in the resistors must be equal to P_{ext}, the mechanical power supplied by the external agent. The same result is obtained since

$$P_{ext} = \vec{F}_{ext} \cdot \vec{v} = F_{ext}v$$

Example 80

Find the current through section PQ of length a in Fig. 5E.80(a). The circuit is located in a time varying magnetic field $B = B_0t$. Assume the resistance per length of the wire is λ.

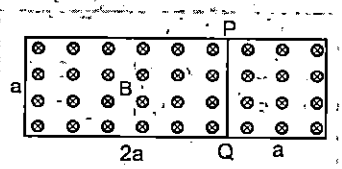


Fig. 5E.80 (a)

Solution: First of all we must understand how to get the current direction in the left loop *PRSQP* and the right loop *PTUQP*. Since B is increasing with t, so the induced current in both the loops must not allow B to increase, as a result of which the branches must have induced current which opposes the increase in the value of B inwards. So, an outward B must be set up due to currents in both the loops. The directions taken satisfy the argument supplied. Now, applying Kirchhoff's loop law for loops *PRSQP* and *PTUQP*, we get

$$\mathcal{E}_1 - (5\lambda a)I_1 - (\lambda a)I_{PQ} = 0 \text{ and} \quad ...(1)$$

$$-\mathcal{E}_2 + (3\lambda a)I_2 - (\lambda a)I_{PQ} = 0$$

$$\Rightarrow \quad \mathcal{E}_2 - (3\lambda a)I_2 + (\lambda a)I_{PQ} = 0 \quad ...(2)$$

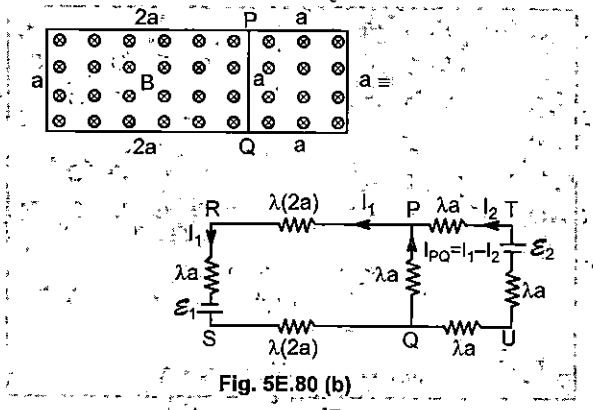


Fig. 5E.80 (b)

where, $\mathcal{E}_1 = (A_{\text{left loop}})\frac{dB}{dt} = (2a^2)B_0$,

$$\mathcal{E}_2 = (A_{\text{right loop}})\frac{dB}{dt}a^2B_0$$

and $I_2 + I_{PQ} = I_1$

i.e., $I_{PQ} = I_1 - I_2$

So, equations (1) and (2) are rewritten as

$$2a^2B_0 - (5\lambda a)I_1 - (\lambda a)(I_1 - I_2) = 0$$

$$\Rightarrow \quad 2a^2B_0 - (6\lambda a)I_1 + (\lambda a)I_2 = 0 \quad ...(3)$$

$$a^2B_0 - (3\lambda a)I_2 + (\lambda a)(I_1 - I_2) = 0$$

$$\Rightarrow \quad a^2B_0 + (\lambda a)I_1 - (4\lambda a)I_2 = 0 \quad ...(4)$$

6(4) + (3), gives

$$8a^2B_0 - (23\lambda a)I_2 = 0$$

$$\Rightarrow \quad I_2 = \frac{8aB_0}{23\lambda}$$

Similarly, doing 4(3)+(4), we get

$$9a^2B_0 - (23\lambda a)I_1 = 0$$

$$\Rightarrow \quad I_1 = \frac{9aB_0}{23\lambda}$$

$$\Rightarrow \quad I_{PQ} = I_1 - I_2 = \frac{aB_0}{23\lambda}$$

Example 81

One end of a spring of force constant k is connected to the mid-point of a conducting rod of mass m and l, which can slide freely on a pair of parallel, smooth, horizontal conducting rails. The other end of the spring is connected to a rigid wall (see Fig. 5E.81).

The spring is non-conducting. There exists a uniform magnetic field $\vec{B}$ normal to the plane of paper and into it. Electrical resistance of rails and rod is negligible. The rails are connected to a capacitor of capacitance C. If the wire is pushed towards left and released, show that the subsequent motion is oscillatory. Find its time period.

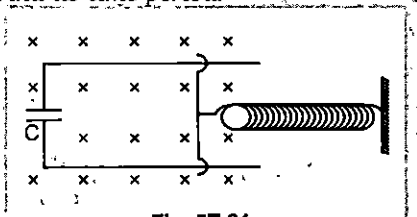


Fig. 5E.81

Solution: Let $\vec{a}$ and x be the acceleration of the rod and change in length of spring respectively.

F.B.D. of rod when it is moving towards right and x is +ve.

$$\Rightarrow \quad CBl\left|\frac{dv}{dt}\right| = \left|\frac{dq}{dt}\right|$$

$$\Rightarrow \quad CBla = I$$

$$\Rightarrow \quad |F_B| = CB^2l^2a$$

From the shown free body diagram

$$kx - CB^2l^2a = ma$$

$$\Rightarrow \quad a = kx/(m + CB^2l^2)$$

and $$\vec{a} = -\left(\frac{kx}{m + CB^2l^2}\right)$$

F.B.D. of rod when it is moving towards right and x is –ve.

From the shown free body diagram

$$ma = CB^2l^2a + k|x|$$

$$\Rightarrow \quad a = \frac{k|x|}{m - CB^2l^2}$$

$$\Rightarrow \quad \vec{a} = -kx/(m - CB^2l^2)$$

Time period of the oscillatory motion = T. say

$$= 2T_1 + 2T_2$$

Where T_1 = time taken by the rod to move from left extreme to its equilibrium position and T_2 = time taken by the rod to move from equilibrium position to the right extreme.

$$T_1 = \frac{2\pi}{4}\sqrt{\frac{m + CB^2l^2}{k}}$$

$$T_2 = \frac{2\pi}{4}\sqrt{\frac{m - CB^2l^2}{k}}$$

$$\Rightarrow \quad T = \pi\left(\sqrt{\frac{m + CB^2l^2}{k}} + \sqrt{\frac{m - CB^2l^2}{k}}\right)$$

Example 82

An inductor of inductance 2 mH is connected across a charged capacitor of capacitance 5 μF and the resulting LC circuit is set oscillating at its natural frequency. Let Q denote the instantaneous charge on the capacitor and I the current in the circuit. It is found that the maximum value of Q is 200 μC.

(a) When Q = 100 μC, what is the value of $\left|\frac{dI}{dt}\right|$?

(b) When Q = 200 μC, what is the value of I ?

(c) Find the maximum value of I.

(d) When I is equal to one-half its maximum value, what is the value of |Q| ?

Solution: This problem is dealing with *LC* oscillations. The charge stored in the capacitor oscillates simple harmonically as,

$$Q = Q_0 \sin(\omega t \pm \phi)$$

Here Q_0 = maximum value of $Q = 200\mu C = 2 \times 10^{-4}$ C

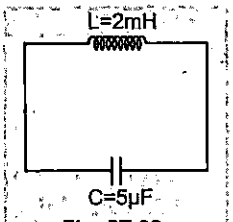


Fig. 5E.82

$$\omega = \frac{1}{\sqrt{LC}}$$

$$= \frac{1}{\sqrt{(2\times10^{-3}\text{H})(5\times10^{-6}\text{F})}} = 10^4\ \text{s}^{-1}$$

Let at $t = 0, Q = Q_0$ then,

$$Q(t) = Q_0\cos(\omega t) \quad \ldots(1)$$

$$\Rightarrow \quad I(t) = \frac{dQ}{dt} = -Q_0\omega\sin(\omega t) \text{ and} \quad \ldots(2)$$

$$\Rightarrow \quad \frac{dI(t)}{dt} = -Q_0\omega^2\cos(\omega t) \quad \ldots(3)$$

(a) $$Q = 100\mu C = \frac{Q_0}{2}$$

At $\cos(\omega t) = \frac{1}{2}$, from eqn. (3), we get

$$\left|\frac{dI}{dt}\right| = (2\times10^{-4}\text{C})(10^4\ \text{s}^{-1})^2\left(\frac{1}{2}\right)$$

$$\Rightarrow \quad \left|\frac{dI}{dt}\right| = 10^4\ \text{As}^{-1}$$

(b) $Q = 200\mu C = Q_0$ when $\cos(\omega t) = 1$, *i.e.*, $\omega t = 0, 2\pi \ldots$

At this time $I(t) = -Q_0\omega\sin(\omega t)$ $\{\sin 0° = \sin 2\pi = 0\}$

$$\Rightarrow \quad I(t) = 0$$

(c) $$I(t) = -Q_0\omega\sin(\omega t)$$

The maximum value of I is $Q_0\omega$

$$\Rightarrow \quad I_{max} = Q_0\omega = (2\times10^{-4}\text{C})(10^4\ \text{s}^{-1}) = 2\text{A}$$

(d) From energy conservation,

$$\frac{1}{2}LI_{max}^2 = \frac{1}{2}LI^2 + \frac{1}{2}\frac{Q^2}{C}$$

$$\Rightarrow \quad Q = \sqrt{LC(I_{max}^2 - I^2)}$$

For, $I = \frac{I_{max}}{2} = 1\text{A}$, we get

$$Q = \sqrt{(2\times10^{-3})(5\times10^{-6})(2^2-1^2)}$$

$$\Rightarrow \quad Q = \sqrt{3}\times10^{-4}\text{C}$$

$$\Rightarrow \quad Q = 1.732\times10^{-4}\text{C}$$

Example 83

Two parallel long smooth conducting rails separated by a distance l are connected by a movable conducting connector of mass m. Terminals of the rails are connected by the resistor R and the capacitor C as shown in Fig. 5E.83 (a). A uniform magnetic field B perpendicular to the plane of the rails is switched on. The connector is dragged by a constant force F. Find the speed of the connector as a function of time if the force F is applied at t = 0. Also find the terminal velocity of the connector.

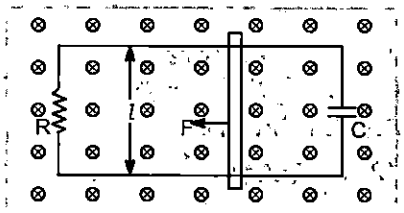


Fig. 5E.83 (a)

Solution: Due to the external force F the connector begins to accelerate such that an emf is induced in the connector due to which an induced current flows through it. If the induced current is I (downwards), then due to this current a magnetic force BIl (acting opposite to F) acts on the connector. Let at some instant t, the connector has a velocity v. Then the current passing through the resistor is

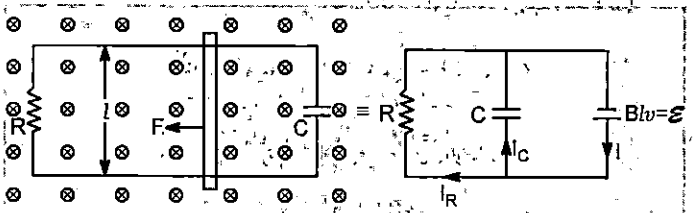


Fig. 5E.83 (b)

$$I_R = \frac{\mathcal{E}}{R} = \frac{Blv}{R} \qquad ...(1)$$

Since the source of emf (which is the moving connector) is connected in parallel to both R and C, hence the potential across both is Blv. So, at that instant if Q be the charge on the capacitor, then

$$Q = C\mathcal{E} = C(Blv)$$

$$\Rightarrow \quad I_C = \frac{dQ}{dt} = BlC\left(\frac{dv}{dt}\right) \qquad ...(2)$$

Total induced current is then given by

$$I = I_B + I_C$$

$$\Rightarrow \quad I = \frac{Blv}{R} + BlC\left(\frac{dv}{dt}\right) \qquad ...(3)$$

Let us now draw a free body diagram of the connector. Then, from Newton's second law, we get

$$F - BIl = ma$$

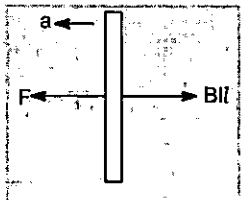


Fig. 5E.83 (c)

$$\Rightarrow \quad F - B\left(\frac{Blv}{R} + BlC\frac{dv}{dt}\right)l = m\left(\frac{dv}{dt}\right)$$

$$\Rightarrow \quad F - \frac{B^2l^2v}{R} = (B^2l^2C + m)\frac{dv}{dt}$$

$$\Rightarrow \quad \frac{dv}{F - \left(\frac{B^2l^2}{R}\right)v} = \frac{dt}{(B^2l^2C+m)}$$

$$\Rightarrow \quad \int_0^t \frac{dv}{F - \left(\frac{B^2l^2}{R}\right)v} = \frac{1}{(B^2l^2C+m)}\int_0^t dt$$

$$\Rightarrow \quad \frac{R}{B^2l^2}\log_e\left(F - \frac{B^2l^2v}{R}\right)\Bigg|_0^v = \frac{t}{(B^2l^2C+m)}$$

$$\Rightarrow \quad \log_e\left(\frac{F - \frac{B^2l^2v}{R}}{F}\right) = -\left[\frac{B^2l^2}{R(B^2l^2C+m)}\right]t$$

$$\Rightarrow \quad F - \frac{B^2l^2v}{R} = Fe^{-\left(\frac{B^2l^2}{R(B^2l^2C+m)}\right)t}$$

$$\Rightarrow \quad v = \frac{FR}{B^2l^2}\left(1 - e^{-\left(\frac{B^2l^2}{R(B^2l^2C+m)}\right)t}\right)$$

If v_T is the terminal velocity, then at $v = v_T$, $\frac{dv}{dt} = 0$. Then from eqn. (1), we get

$$v_T = \frac{FR}{B^2l^2}$$

Example 84

Switch S in Fig. 5E.84 (a) is closed for $t < 0$ and is opened at $t = 0$. When current I_1 through L_1 and current I_2 through L_2 are first equal to each other, what is their common value? (The resistors have the same resistance R.) Also calculate the value of I_2 at time t (> 0).

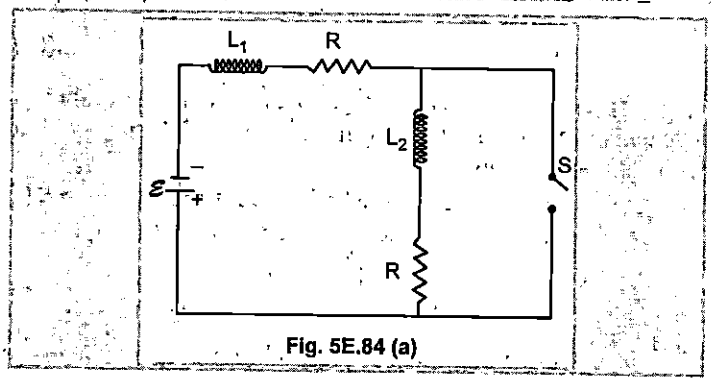


Fig. 5E.84 (a)

Solution: At $t < 0$, $I_1 = \mathcal{E}/R$, $I_2 = 0$

When switch is open, at that time total flux will be constant. Let after opening the switch the common current in the circuit is I_2.

$$\therefore \quad L_1 I_1 = (L_1 + L_2) I_2$$

$$\Rightarrow \quad I_2 = \left(\frac{L_1}{L_1 + L_2}\right) I_1 = \frac{\mathcal{E} L_1}{R(L_1 + L_2)}$$

Applying KVL after time t,

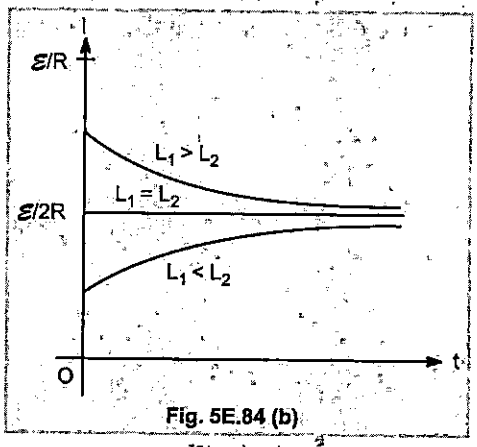


Fig. 5E.84 (b)

$$(L_1 + L_2)\frac{dI}{dt} + 2IR - \mathcal{E} = 0$$

$$\Rightarrow \quad \int \frac{dI}{\mathcal{E} - 2IR} = \frac{1}{(L_1 + L_2)} \int_0^1 dt$$

$$\Rightarrow \quad \ln \frac{\mathcal{E} - 2IR}{\mathcal{E} - 2I_2 R} = \frac{-2tR}{L_1 + L_2}$$

$$\Rightarrow \quad \mathcal{E} - 2IR = (\mathcal{E} - 2I_2 R)\, e^{\frac{2tR}{L_1 + L_2}}$$

$$= \left(\frac{L_2 - L_1}{L_1 + L_2}\right) \mathcal{E} e^{\frac{2tR}{L_1 + L_2}}$$

$$\Rightarrow \quad I = \frac{\mathcal{E}}{2R}\left[1 - \frac{L_2 - L_1}{L_1 + L_2} e^{\frac{-2tR}{L_1 + L_2}}\right]$$

Example 85

A thin ring of mass 0.1 kg and diameter 0.5 m carrying charge 'q' is free to rotate about an axis as shown in the Fig. 5E.85. At $t = 0$, time dependent magnetic field of 8t Tesla is applied vertically down on the ring. If the ring is at rest before applying magnetic field and induced magnetic effects due to rotation of the ring are neglected, the power developed by the force acting on the ring after 10sec is found to be 6.4 mw. Find the charge 'q' that the ring is carrying?

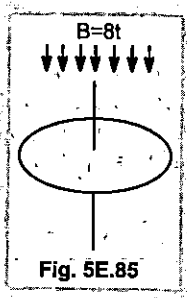


Fig. 5E.85

Solution: We know that,

$$E(2\pi R) = (\pi R^2) \cdot \frac{dB}{dt}$$

$$\Rightarrow \quad E = \frac{R}{2} \cdot B_0$$

$$F = qE = \frac{qB_0 R}{2}$$

$$\Rightarrow \quad \tau = R(F) = \frac{qB_0 R^2}{2}$$

$$\therefore \quad \alpha = \frac{\tau}{I} = \frac{qB_0 R^2}{2MR^2} = \frac{qB_0}{2M} \qquad ...(1)$$

$$\therefore \quad \text{Power} = \frac{d(K.E.)}{dt}$$

$$= \frac{d}{dt}\left(\frac{1}{2} I\omega^2\right) = I\omega \frac{d\omega}{dt} \qquad ...(2)$$

$$p = I\omega\alpha = |(\alpha t)\alpha| = |\alpha^2 t|$$

From eqn. (1) $\quad P = MR^2 \left(\frac{qB_0}{2M}\right)^2 t$

(as $B = 8t$ given, $B_0 = \frac{dB}{dt} = 8$)

$$q = \sqrt{\frac{(4M)P}{B_0^2 R^2 t}} = \sqrt{\frac{4 \times \frac{1}{10} \times 64}{(8)^2 \left(\frac{1}{4}\right)^2 \times 10 \times 10^4}}\,\text{c}$$

$$= 8 \times 10^{-3} \text{ coulomb.}$$

AC CIRCUIT

Alternating Current Circuits

The emf produced by an AC generator can be represented by a sine function of angular frequency $\omega = 2\pi\nu = \frac{2\pi}{T}$. The time dependent potential drop V across terminals of a particular circuit element has the form

$$V(t) = V_M \sin \omega t \quad ...(1)$$

where $V(t)$ is the **instantaneous voltage.** A plot of $V(t)$ varies between two peak or **maximum voltage** values $+V_M$ and $-V_M$.

With a purely resistive load R across the terminals of an AC source, the **instantaneous** current has the form

$$I(t) = \frac{V(t)}{R} = \frac{V_M}{R} \sin \omega t$$

and with **maximum current** given by $I_M = V_M/R$, it becomes

$$I(t) = I_M \sin \omega t \quad ...(2)$$

Eqns. (1) and (2) show that in an AC circuit the current through a resistor is exactly in phase with the potential drop across its terminals.

The Root-Mean-Square Potential and Current

The voltage and current *versus* time graph shows that the current and voltage oscillate and have corresponding positive and negative values during each cycle. Thus, the sum of all the instantaneous current values (positive and negative) over one complete cycle is zero, so average current and voltage are zero. Mathematically as we know that the time averaged value of the sine function over one or more complete (360°) cycles is zero.

The average value of $V(t)$ is zero and the average value of $I(t)$ is zero does not imply that there is no joule heating. Since both AC and DC produce joule heating, the effective value of the alternating current is defined as the quantity of current that produces the same amount of heating in a resistor as does a direct current during the same interval of time.

The instantaneous power dissipated in a resistor is

$$P = I^2R$$

The actual power measured is average over many cycles, as R is constant.

$$P_{av.} = [I^2]_{av.} R$$

If effective value of current is $I_{eff.}$,

$$P_{av.} = I_{eff.}^2 R$$

Thus on comparing expressions for average power. We have

$$I_{eff.} = \sqrt{[I^2]_{av.}}$$

This effective current is called **root-mean-square current.**

Thus

$$I_{rms} = \sqrt{\langle I^2 \rangle} \quad ...(1)$$

Similarly,

$$V_{rms} = \sqrt{\langle V^2 \rangle} \quad ...(2)$$

where $\langle\rangle$ means average over a whole number of cycles, I and V are instantaneous values of current and voltage respectively.

If T is the period over which a periodic function $f(t)$ repeats itself, then the time average of $f(t)$ is defined as

$$\langle f(t) \rangle = \frac{\int_0^T f(t)\,dt}{T} \quad ...(3)$$

Here we will use a simple technique to get average value.

$$V^2(t) = V_M^2 \sin^2 \omega t$$

$$\langle V^2(t) \rangle = \langle V_M^2 \sin^2 \omega t \rangle$$

$$= V_M^2 \langle \sin^2 \omega t \rangle \quad ...(4)$$

As $\sin^2 \omega t + \cos^2 \omega t = 1$

$$\langle \sin^2 \omega t \rangle + \langle \cos^2 \omega t \rangle = 1$$

Fig. 5.66 shows that $\sin^2 \omega t$ and $\cos^2 \omega t$ functions have the same average over a whole number of periods.

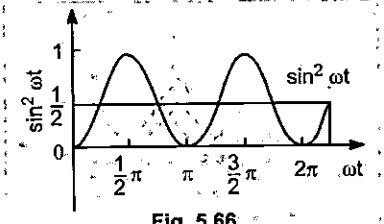


Fig. 5.66

The area under the curve $\sin^2 \omega t$ divided by the extent of the curve (ωt) equals the average value: $[\sin^2 \omega t]_{av.} = 1/2$

$$\langle \cos^2 \omega t \rangle = \langle \sin^2 \omega t \rangle \quad ...(5)$$

$$2\langle \sin^2 \omega t \rangle = 1$$

$$\langle \sin^2 \omega t \rangle = 1/2 \quad ...(6)$$

So from eqn. (4), we get

$$\langle V^2(t) \rangle = \frac{1}{2}(V_M)^2$$

or

$$V_{rms} = \frac{V_M}{\sqrt{2}} = 0.707\, V_M$$

Similarly,

$$I_{rms} = \frac{I_M}{\sqrt{2}} = 0.707 I_M$$

The effective or average power is

$$\langle P \rangle = \frac{1}{2} I_M^2 R = I_{rms}^2 R$$

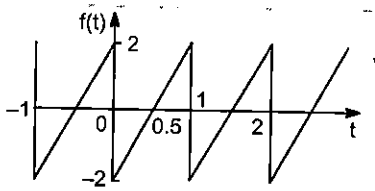
$$= V_{rms} I_{rms} = \frac{V_{rms}^2}{R}$$

It is customary to measure and specify rms values for AC quantities, for example 220 V AC is an rms value with a peak voltage of

$$V_M = \sqrt{2}V_{rms}$$
$$= \sqrt{2} \times 220\text{V} = 311\text{V}$$

Example 86

Find the rms and average values of the saw tooth wave-form shown in Fig. 5E.86 (a).

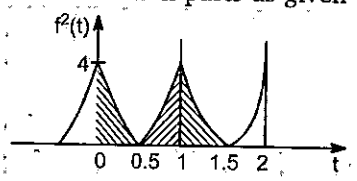


5E.86 (a)

Solution: The required values can be found by using either graphical method or analytical method.

Graphical Method

The average value can be found by averaging the function from $t = 0$ to $t = 1$ in parts as given below.



5E.86 (b)

Average value of $f(t)$

$$= \frac{1}{T}\int_0^T f(t)\,dt$$

$$= \frac{1}{T} \times \text{(net area over one cycle)}$$

Now, area of a right-angled triangle

$$= (1/2) \times \text{(base)} \times \text{(altitude)}.$$

Hence, area of the triangle during $t = 0$ to $t = 0.5$ second is

$$A_1 = \frac{1}{2} \times (\Delta t) \times (-2)$$
$$= \frac{1}{2} \times \frac{1}{2} \times -2 = -\frac{1}{2}$$

Similarly, area of the triangle from $t = 0.5$ to $t = 1$ second is

$$A_2 = \frac{1}{2} \times (\Delta t) \times (+2)$$
$$= \frac{1}{2} \times \frac{1}{2} \times 2 = \frac{1}{2}$$

Net area from $t = 0$ to $t = 1.0$ second is

$$A_1 + A_2 = -\frac{1}{2} + \frac{1}{2} = 0$$

Hence, average value of $f(t)$ over one cycle is zero.

For finding the rms value, we will first square the ordinates of the given function and draw a new plot for $f^2(t)$ as shown in Fig. 5E.86 (b). It would be seen that the squared ordinates form a parabola.

Area under a parabolic curve $= \frac{1}{3} \times$ base $\times$ altitude. The area under the curve from $t = 0$ to $t = 0.5$ second is:

$$A_1 = \frac{1}{3}(\Delta t) \times 2^2$$
$$= \frac{1}{3} \times \frac{1}{2} \times 4 = \frac{2}{3}$$

Similarly, for $t = 0.5$ to $t = 1.0$ second,

$$A_2 = \frac{1}{3}(\Delta t) \times 4$$
$$= \frac{1}{3} \times \frac{1}{2} \times 4 = \frac{2}{3}$$

$$\text{Total area} = A_1 + A_2 = \frac{2}{3} + \frac{2}{3} = \frac{4}{3}$$

$$\text{rms value} = \sqrt{\frac{1}{T}\int_0^T f^2(t)\,dt} = \sqrt{\text{average of } f^2(t)}$$

$$\text{rms value} = \sqrt{4/3} = 1.15$$

Analytical Method

The equation of the straight line from $t = 0$ to $t = 1$ in Fig. 5E.86 (a) is

$$f(t) = 4t - 2;$$
$$f^2(t) = 16t^2 - 16t + 4$$

$$\text{Average value} = \frac{1}{T}\int_0^T (4t - 2)\,dt$$
$$= \frac{1}{T}\left|\frac{4t^2}{2} - 2t\right|_0^T = 0$$

$$\text{rms value} = \sqrt{\frac{1}{T}\int_0^T (16t^2 - 16t + 4)\,dt}$$
$$= \sqrt{\frac{1}{T}\left|\frac{16t^3}{3} - \frac{16t^2}{2} + 4t\right|_0^T} = 1.15$$

A Capacitor in an AC Circuit

Since $V = q/C$ and the source voltage is $V = V_M \sin \omega t$, it follows that $q = CV_M \sin \omega t$.

charge and the voltage are in phase. Now the instantaneous current $I(t) = dq/dt$.

$$I = \frac{d}{dt}(CV_M \sin \omega t)$$
$$= \omega CV_M \cos \omega t$$
$$= \omega CV_M \sin(\omega t + \pi/2)$$
$$= \frac{V_M \sin(\omega t + \pi/2)}{(1/\omega C)}$$

Similar to DC circuit, a capacitor provides opposition to the charging process in an AC circuit. The quantitative measure of the opposition of a capacitor to current is referred as **capacitive reactance** (X_C), which is defined as

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C}.$$

SI unit of capacitive reactance is ohm (Ω).

So voltage across a capacitor is

$$V = IX_C$$

Now we take a look at the changing conditions of voltage current and charge. We arbitrarily choose initial time ($t = 0$) when applied voltage is at its maximum positive value.

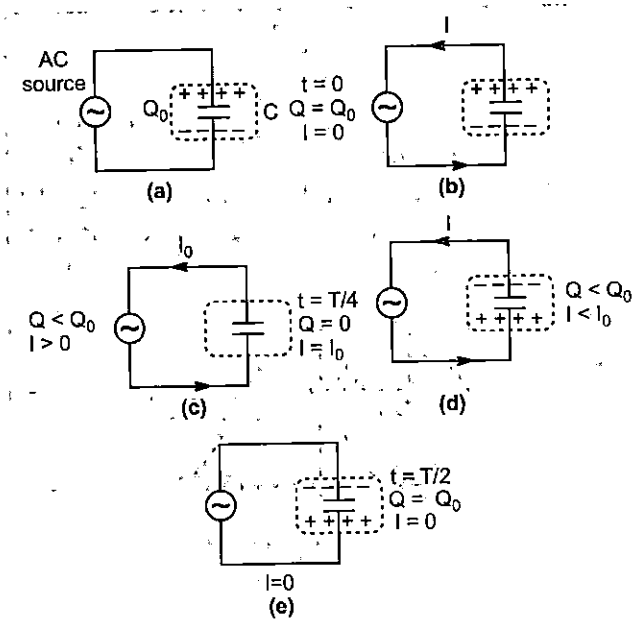


Fig. 5.67 A capacitor under AC conditions: This sequence shows the voltage, charge, and current in a circuit containing only a capacitor and an AC voltage source. All five circuit diagrams taken together represent physically what is plotted in the first half of the cycle (from t=0 to t = T/2) in the graph shown here.

- At $t = 0$ capacitor is fully charged ($Q = CV_M$). As the capacitor cannot absorb further charge, so current is zero.
- When the voltage decreases, the capacitor begins to discharge, an anticlockwise current arises.
- When voltage drops to zero, capacitor is discharged, current is maximum, This happens at $t = T/4$.
- The voltage source reverses its polarity and starts to increase in magnitude. The capacitor begins to charge reversed polarity. Initially the current is maximum, it decreases as the capacitor plates are charged.
- At $t = T/2$ the capacitor is fully charged with reverse polarity. All the five Fig. 5.67 (a) to (e), represent the first half cycle (from $t = 0$ to $t = T/2$).]

An Inductor in an AC Circuit

Fig. 5.68 shows an inductor L, having negligible resistance, connected with an AC source.

instantaneous current in the inductor is given by Faraday's law of electromagnetic induction

$$V_L = -L\frac{dI}{dt}$$

The AC source voltage is $V(t) = V_M \sin\omega t$.

From Kirchhoff's law around closed loop

$$V(t) + V_L = 0$$

Hence $$V(t) - L\frac{dI}{dt} = 0$$

Solving for $\frac{dI}{dt}$ and integrating w.r.t. time,

$$\int \frac{V_M}{L}\sin\omega t\, dt = \int dI$$

$$I(t) = -\frac{V_M}{\omega L}\cos\omega t$$

As $$-\cos\omega t = \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$= \frac{V_M}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right)$$

- Comparing the current $I(t)$ with potential drop across inductor $V_L(t) = V(t) = V_M \sin\omega t$ we see that the current lags the voltage by one quarter cycle ($90° = \pi/2$ rad).
- The instantaneous voltage depends on the inductance and rate of change of current.
- The voltage is zero when the slope of the current curve is zero; current is at its maximum value.
- Voltage is maximum when the slope of the current is maximum, the current curve crosses the time axis.

A real inductor inhibits current because of its resistance and inductance; the net effect is referred as **impedance,** a part of which, the **reactance,** is purely due to inductive behaviour.

We can write $I(t) = \frac{V_M}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right)$

as $$I(t) = I_M \sin\left(\omega t - \frac{\pi}{2}\right),$$

where $$I_M = \frac{V_M}{\omega L} = \frac{V_0}{X_L}.$$

This result is almost like Ohm's law, with the term X_L called **inductive reactance,** that inhibits current, whereupon

$$V = IX_L$$

SI unit of inductive reactance is ohm.

Now we take a look at the varying current and voltage situation in a purely inductive circuit.

- At the instant $t = 0$ (arbitrarily chosen initial time) an increasing clockwise current exists, creating an increasing flux in the coil.
- In response an induced current I_1 will appear in the windings and in accordance with Lenz's law it will oppose the change in flux. The induced current at that moment emerges from the inductor at point A. A potential difference is established across terminals of an inductor, that is, + at point A and – at point B, see Fig. 5.68 (b). The inductor now behaves as a battery being charged.

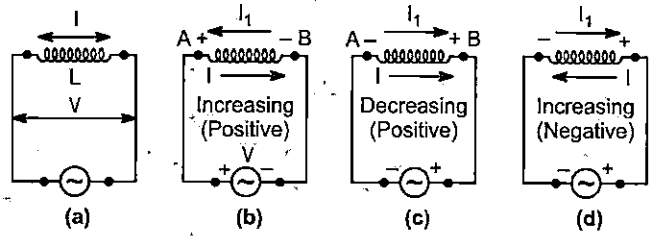


Fig. 5.68 (a) An inductor placed across the terminals of an AC source. (b) Note that when I is positive (clockwise) and increasing, the terminal voltage V is positive. (c) When I is positive and decreasing, V is negative, and (d) when I is negative and increasing, V is negative.

- As the source is sinusoidal, there will be a continuously changing flux and a sustained simple harmonic induced emf.
- As shown in Fig. 5.68 (c), when I is positive and decreasing, the terminal voltage across an inductor is negative.
- As shown in Fig. 5.68 (d) when I is negative and increasing, the terminal voltage across the inductor is negative.

Phasor Diagram

A phasor diagram graphically represents phase relationships between the current $I(t)$ flowing through a circuit and the potential changes $V(t)$ throughout a circuit. A **phasor** is vector like line segment that rotates anticlockwise in a cartesian plane with an angular velocity ω. The tail of the phasor is fixed to origin, the length represents amplitude of either the current of the potential drop.

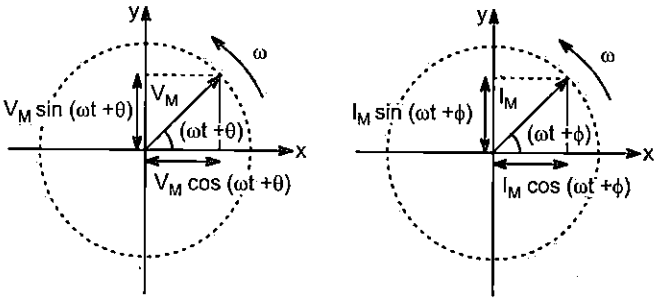


Fig. 5.69

Fig. 5.69 shows the phasor diagram for a purely resistive circuit. As $I(t)$ and $V_R(t)$ are in phase, both the phasors I_M and V_M are parallel and at an angle $(\omega t + \theta)$ with positive x-axis. If the applied voltage is $V_M \sin(\omega t + \theta)$, the projection of phasors V_M and I_M on the y-axis gives the functions $V(t)$ and $I(t)$ respectively.

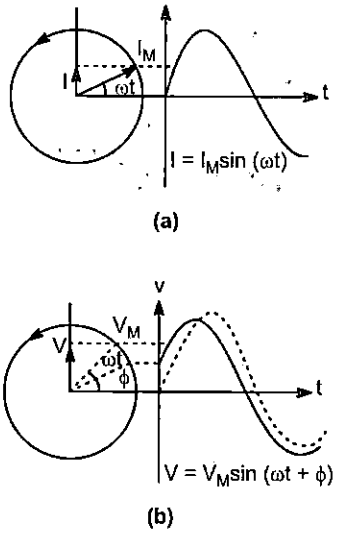


Fig. 5.70 (a) As the current phasor I_m rotates, its vertical component is the instantaneous current I. (b) The potential difference phasor V_m is shifted by a phase angle ϕ relative to the current phasor.

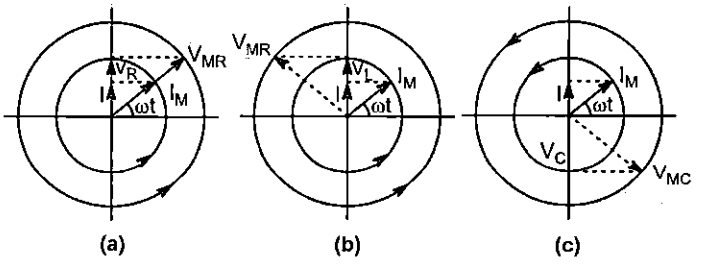
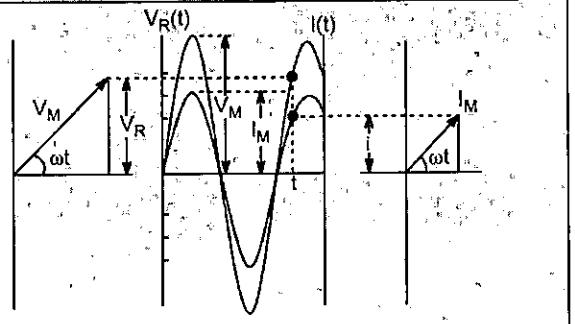
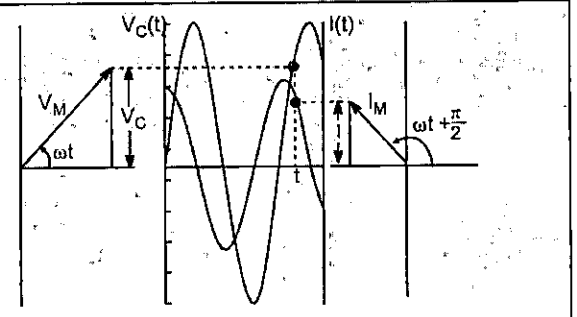
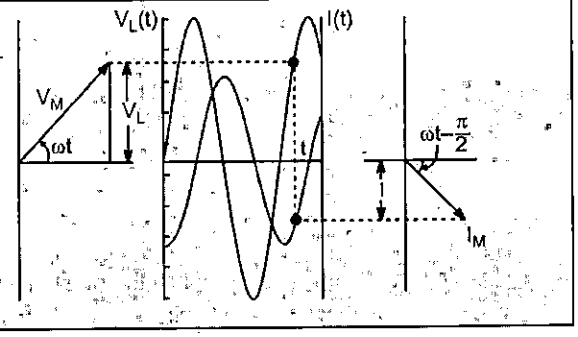


Fig. 5.71 The phase relationship between the instantaneous current and the potential difference for (a) a resistor, (b) an inductor, and (c) a capacitor

RELATIONSHIP BETWEEN $I(t)$ AND $V_x(t)$ FOR AN AC CIRCUIT

Circuit element	Impedance (Z)	Phase relation	At any instant	Amplitude relation	Phasor diagram, $V_x(t)$ and $I_x(t)$ curve
R — ∿∿∿ —	R	$V_R(t)$ and $I(t)$ are in phase	$V_R(t) = I(t)R$	$V_{RM} = I_M R$	
C — ⊣⊢ —	$X_C = \frac{1}{\omega C}$	$V_C(t)$ lags $I(t)$ by $\pi/2$ rad	$V_C(t) = I(t)X_C$	$V_{CM} = I_M X_C$	
L — ∿∿∿ —	$X_L = \omega L$	$V_L(t)$ leads $I(t)$ by $\pi/2$ rad	$V_L(t) = I(t)X_L$	$V_{LM} = I_M X_L$	

Note that the curves $I(t) = I_M \sin \omega t$ and $V(t) = V_M \sin \omega t$ are traced out by projections of phasors on the y-axis.
If applied voltage is $V_M \cos \omega t$, the x-projections will be taken

Analysis of Series in AC Circuits

Fig. 5.72 shows an *LCR* circuit consisting of a series combination of a resistance R, an inductance L and a capacitance C driven by a current source.

$$I(t) = I_M \sin \omega t$$

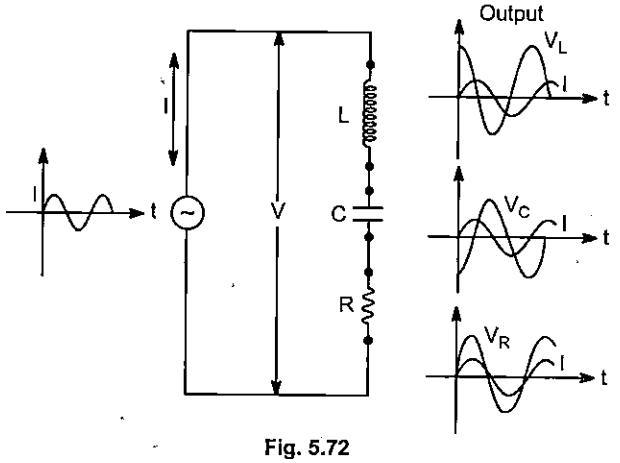


Fig. 5.72

Current through each element is same, the voltage are not in phase

We assume that the terminal potential of current source is

$$V(t) = V_M \sin(\omega t + \phi)$$

where V_M is maximum emf applied by current source and ϕ is the **phase angle** between $I(t)$ and $V(t)$. We wish to determine V_M and ϕ in terms of R, L, C, I_M and ω

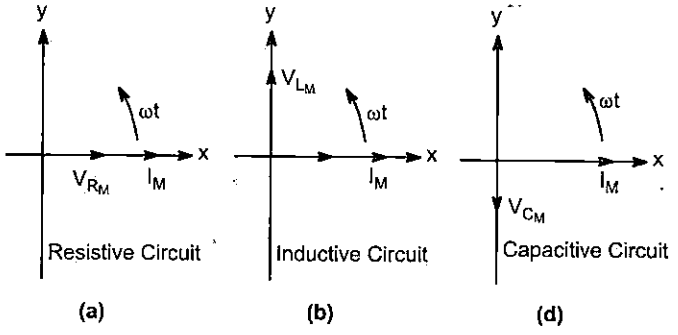


Fig. 5.73

Fig. 5.73 shows the phasor diagram for each element in a series *LCR* circuit. In each diagram I_M is aligned in the same direction because the current must be same in each circuit element. At an instant when phasor I_M is aligned along the x-axis, Fig. 5.73 (a), (b) represent resistor, inductor and capacitor phasor diagrams respectively.

Fig. 5.74 (a) represents combined phasor diagram for an *LCR* circuit. At any instant the potential drop across the capacitor and inductor are of opposite sign; these two phasors are always π radians out of phase. The combination of phasors (similar to vector representation) V_{L_M} and V_{C_M} is written as $V_X = V_{L_M} - V_{C_M}$.

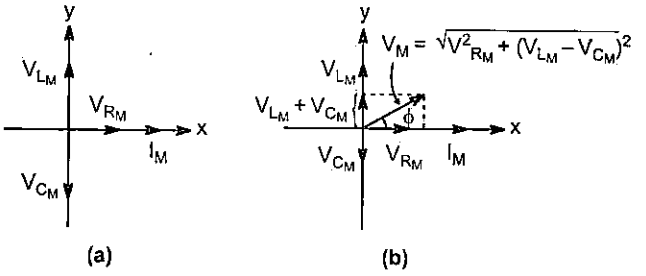


Fig. 5.74

The magnitude of V_X is $|V_{L_M} - V_{C_M}|$; it represents the net potential drop across the two reactive elements.

Condition	Contribution to total potential drops across *LC* combination	Phase relation
(i) $V_{L_M} > V_{C_M}$	Inductive	V_X leads the current I_M by $\pi/2$ radians
(ii) $V_{L_M} < V_{C_M}$	Capacitive	V_X lags the current I_M by $\pi/2$ radians

In either condition (i) or (ii) the potential drop across resistor is $\pi/2$ radian out of phase with the phasor V_X, which represents the potential drop across inductor and capacitor.

The magnitude of the phasor V_M is

$$V_M = \sqrt{V_{R_M}^2 + V_X^2}$$
$$= \sqrt{V_{R_M}^2 + (V_{L_M} - V_{C_M})^2}$$

The phase angle between the phasor V_{max} and the current phasor I_{max} is

$$\tan \phi = \frac{V_{L_M} - V_{C_M}}{V_{R_M}} \quad ...(1)$$

Now substituting

$$V_{R_M} = I_M R$$
$$V_{L_M} = I_M X_L$$
$$V_{C_M} = I_M X_C$$

We have

$$V_M = \sqrt{(I_M R)^2 + (I_M X_L - I_M X_C)^2}$$
$$V_M = I_M \sqrt{R^2 + (X_L - X_C)^2} \quad ...(2)$$

Comparing the equation with

$$V_M = I_M Z$$

where Z is impedance of circuit, given by

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \quad ...(3)$$

The phase angle ϕ can be written as

$$\phi = \tan^{-1}\left(\frac{I_M X_L - I_M X_C}{I_M R}\right)$$
$$= \tan^{-1}\left(\frac{X_L - X_C}{R}\right) \quad ...(4)$$

☛ **Impedance diagram:** A simplification of the circuit analysis can be obtained by drawing the impedance diagram. The eqns. (3) and (4) just require impedances, R, X_L and X_C. Maintaining correct phase relations, a diagram of R, X_L and X_C is sufficient to compute Z and ϕ such a diagram is called impedance diagram (see Fig. 5.75).

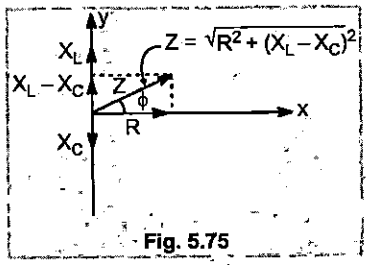


Fig. 5.75

☛ Following steps should be adopted in solving AC circuit problems.

Step 1. Draw a phasors diagram and impedance diagram.

Step 2. If applied current is

$$I(t) = I_M \sin(\omega t)$$

assume the voltage as

$$V(t) = V_M \sin(\omega t + \phi)$$

where $V_M = I_M Z$ and ϕ is the phase angle between current and voltage.

Step 3. Impedance Z and phase angle ϕ can be calculated from

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

voltage can lead, lag, or be in phase with the current as shown in Table.

Phase relations between $V(t)$ and $I(t)$ for an LCR circuit

When	$X_L > X_C$	$\phi > 0$ and	$V(t)$ leads $I(t)$ by ϕ
	$X_L < X_C$	$\phi < 0$ and	$V(t)$ lags $I(t)$ by ϕ
	$X_L = X_C$	$\phi = 0$ and	$V(t)$ and $I(t)$ are in phase

☛ If emf source is given as

$$V(t) = V_M \sin(\omega t)$$

then current can be written as

$$I(t) = I_M \sin(\omega t - \phi)$$

The phase angle ϕ and impedance Z can be obtained as before.

Summary of LCR circuit equations

Case (i)

Given: $I(t) = I_M \sin(\omega t)$, then $V(t) = V_M \sin(\omega t + \phi)$

$V_R(t) = V_{R_M} \sin(\omega t)$	$V_R(t)$ is in phase with $I(t)$
$V_L(t) = V_{L_M} \sin(\omega t + \pi/2)$	$V_L(t)$ leads $I(t)$ by $\pi/2$ rad
$V_C(t) = V_{C_M} \sin(\omega t - \pi/2)$	$V_C(t)$ lags $I(t)$ by $\pi/2$ rad

Case (ii)

Given: $V(t) = V_M \sin(\omega t)$, then $I(t) = I_M \sin(\omega t - \phi)$

$V_R(t) = V_{R_M} \sin(\omega t - \phi)$	$V_R(t)$ is in phase with $I(t)$
$V_L(t) = V_{L_M} \sin(\omega t - \phi + \pi/2)$	$V_L(t)$ leads $I(t)$ by $\pi/2$ rad
$V_C(t) = V_{C_M} \sin(\omega t - \phi - \pi/2)$	$V_C(t)$ lags $I(t)$ by $\pi/2$ rad

In both cases, we may use

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

and $V_M = I_M Z \Rightarrow V_{R_M} = I_M R$

$V_{L_M} = I_M X_L \Rightarrow V_{C_M} = I_M X_C$

Series AC *RL* Circuit

Let applied voltage

$$V = V_M \sin \omega t$$

The reactance, $X = X_L = \omega L$

Impedance, $Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + \omega^2 L^2}$

Maximum current,

$$I_M = \frac{V_M}{Z} = \frac{V_M}{\sqrt{R^2 + (\omega L)^2}}$$

Phase angle, $\phi = \tan^{-1}\left(\frac{X}{R}\right) = \tan^{-1}\left(\frac{\omega L}{R}\right)$

The current in the circuit at any instant is given by

$$I = I_M \sin(\omega t - \phi)$$

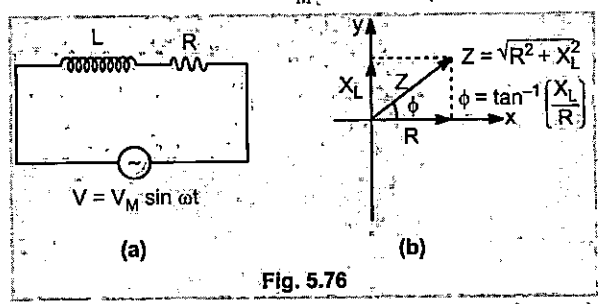


Fig. 5.76

In a series LR circuit, current lags voltage by phase angle

$$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

Voltage across resistor R and inductor L is given by

$$V_R = IR$$

and $V_X = IX_L = I\omega L$

and $V^2 = V_R^2 + V_X^2$

Series AC *CR* Circuit

Let applied voltage,

$$V = V_M \sin \omega t$$

The reactance, $X = X_C = \frac{1}{\omega C}$

Impedance, $Z = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$

Maximum current, $I_M = \frac{V_M}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}}$

Phase angle, $\phi = \tan^{-1}\frac{X}{R} = -\tan^{-1}\left(\frac{1}{\omega CR}\right)$

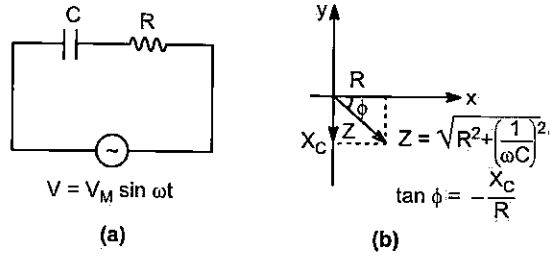


Fig. 5.77

Phase angle ϕ is negative because in a capacitive circuit voltage lags by $\pi/2$ with the current.

Current in the circuit, $I = I_M \sin(\omega t - \phi)$

Voltages across resistor and capacitor are

$$X_R = IR, \quad V_C = IX_C = \frac{I}{\omega C}$$

and $$V^2 = V_R^2 + V_C^2$$

Series AC *LC* Circuit

In this case, $R = 0$

Net reactance,

$$X = X_L - X_C = \left(\omega L - \frac{1}{\omega C}\right)$$

$$Z = \sqrt{R^2 + X^2} = X$$

$$I_M = \frac{V_M}{Z} = \frac{V_M}{\left(\omega L - \frac{1}{\omega C}\right)}$$

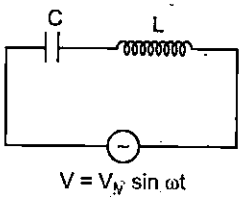


Fig. 5.78

and $$\phi = \tan^{-1}\frac{X}{R} = \tan^{-1}\frac{\left(\omega L - \frac{1}{\omega C}\right)}{0}$$

$$= \tan^{-1}(\infty) = \pi/2$$

Hence current at any instant is given by

$$I = I_M \sin(\omega t - \pi/2)$$

Current in the circuit lags the applied voltage by $\pi/2$

☛ If $X_L = X_C$

or $$\omega L = \frac{1}{\omega C} \quad \text{or} \quad \omega = \frac{1}{\sqrt{LC}}$$

then $I_M = \infty$ and $Z = 0$

☛ As $\frac{1}{\sqrt{LC}}$ is natural frequency of circuit the amplitude of current (I_M) becomes infinite due to zero impedance. This is called a resonance condition.

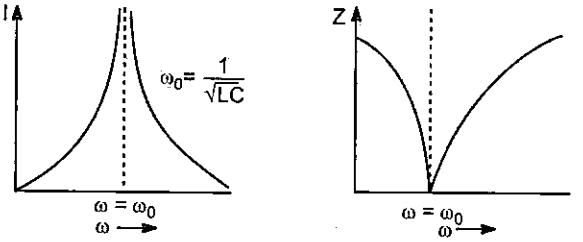


Fig. 5.79

Power in AC Circuits

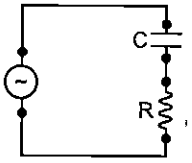
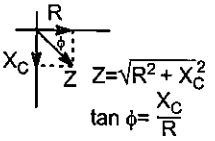
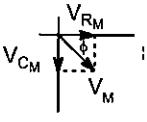
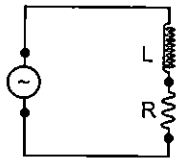
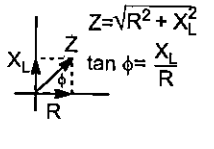
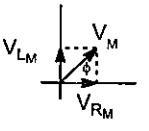
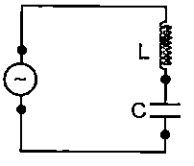
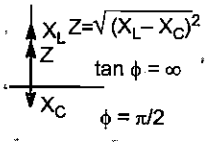
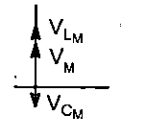
Power provided to a circuit by a source of emf V that delivers current I is

$$P = VI$$

In the case of AC circuits, V and I are sinusoidal functions expressed as

$$V(t) = V_M \sin(\omega t)$$
$$I(t) = I_M \sin(\omega t - \phi)$$

A SUMMARY OF THE BEHAVIOUR OF AC SERIES CIRCUITS

Circuit diagram	Impedance diagram	Phasor diagram	$I(t), V(t)$
			$I(t) = I_M \sin(\omega t)$ $V(t) = V_M \sin(\omega t - \phi)$
			$I(t) = I_M \sin \omega t$ $V(t) = V_M \sin(\omega t + \phi)$
			$I(t) = I_M \sin \omega t$ $V(t) = V_M \sin(\omega t + \pi/2)$

Instantaneous power,

$$P = V(t)\, I(t)$$
$$= V_M I_M \sin(\omega t)\sin(\omega t - \phi)$$

Average power is given by

$$P_{av.} = \langle V_M I_M \sin(\omega t)\sin(\omega t - \phi)\rangle_{av.}$$
$$= V_M I_M \langle [\sin(\omega t)\cos\phi - \sin\phi\cos(\omega t)]\sin(\omega t)\rangle_{av.}$$
$$= V_M I_M [\langle \sin^2(\omega t)_{av.}\cos\phi - \sin(\phi)\rangle \langle \sin(\omega t)\cos(\omega t)\rangle_{av.}]$$

As $\langle \sin^2(\omega t)\rangle_{av.} = \frac{1}{2}$.

and $\langle \sin(\omega t)\cos(\omega t)\rangle_{av.} = 0$

Hence, $P_{av.} = V_M I_M \left[\frac{1}{2}\cos(\phi) + 0\right]$

$$= \frac{V_M I_M}{2}\cos\phi$$
$$= \frac{V_M}{\sqrt{2}}\frac{I_M}{\sqrt{2}}\cos\phi$$
$$= V_{rms} I_{rms}\cos\phi$$

the term $\cos(\phi)$ is called the **power factor** and can be determined from the impedance diagram shown in figure

$$\cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}}$$

Substituting expression for $\cos\phi$ in P_{av}. we get

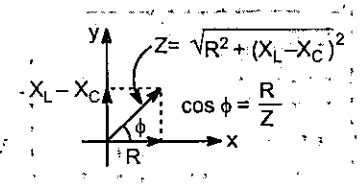


Fig. 5.80

$$P_{av.} = V_{rms} I_{rms}\frac{R}{Z}$$
$$= \left(\frac{V_{rms}}{Z}\right) I_{rms} R$$
$$= I_{rms}^2 R$$

- When sinusoidal current or emf source is applied to a capacitor, the charge on the capacitor starts increasing, the potential drop $V(t)$ on capacitor increases and so does electric field between plates. Instantaneous energy in the capacitor is $U(t) = \frac{1}{2} C V_C^2(t)$, it reaches its maximum value when capacitor is fully charged. At this instant maximum energy is stored in the electric field. In the next half cycle, the capacitor begins to discharge and does work on the charges flowing in the circuit. In the second half cycle, the capacitor is charged but with opposite polarity. So twice during each cycle $\frac{1}{2} C V_{C_M}^2$ amount of energy is stored on the capacitor and then returned to the other part of the circuit.
- Similar to charging of capacitor, an inductor also store energy in its magnetic field B, as the current varies sinusoidally. When the current reaches its amplitude I_M, the energy in the inductor is $U_L = \frac{1}{2} L I_M^2$. Twice during each cycle the energy of the inductor reaches to maximum and then returned to the remaining circuit.
- Thus no energy is dissipated in an ideal capacitor (no leakage current) and an ideal inductor (no resistance). In an *LCR* circuit energy is dissipated as heat only in a resistor.
- The power factor $\cos\phi$ expresses the relative influence of the resistor in dissipating power. The expression $P_{av.} = VI\cos\theta$ gives **real** or **dissipated power.**

For the purely resistive circuit, $\cos\phi = 1$ and the power dissipated is VI.

For either a purely inductive and capacitive circuit the phase angle is ±90° and no real power is transferred from the source.

The product VI is called **apparent power** and is measured in volt amps (VA). Practically an amount of power VI must be supplied even if a portion of that $VI(1 - \cos\theta)$ is stored in the electric field of capacitor or magnetic field of inductor and returned to the source.

If in an AC circuit, phase difference between V and I is $\pi/2$, *i.e.*, if voltage and current are given by

$$V = V_0 \sin\omega t.$$
$$I = I_0 \cos\omega t,$$

the power consumed by the circuit is zero. The current corresponding to such a circuit is called wattless current.

Resonance in Series *LCR* Circuit

From the expression for power factor and impedance of an *LCR* circuit, we see that

$$\cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}}$$

and $$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

If $X_C = X_L$, $\cos\phi = 1$ and $Z = R$, the average power delivered to the resistor in circuit is a maximum, and impedance of *LCR* circuit is minimum. In this condition the circuit is said to be in **resonance.** The condition for resonance exists when

$$X_L = X_C$$
$$\omega L = \frac{1}{\omega C} \quad \text{or} \quad \omega_{res.} = \frac{1}{\sqrt{LC}}$$

At resonance amplitude of current is

$$I_M = \frac{V_M}{Z} = \frac{V_M}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V_M}{R}$$

and average power,

$$P_{av.} = I_{rms}^2 R = \frac{V_M^2}{[R^2 + (X_L - X_C)^2]} R$$

$$X_L - X_C = \omega L - \frac{1}{\omega C} = \frac{L}{\omega}(\omega^2 - \omega_{res.})^2$$

After a bit of algebraic manipulations, we get

$$P_{av.} = \frac{V^2 R\omega^2}{R^2\omega^2 + L^2(\omega^2 - \omega_{res.}^2)}$$

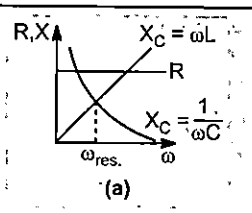


Fig. 5.81 (a) at the resonance frequency, the inductive and capacitive reactances are equal.

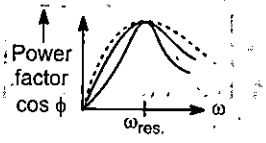


Fig. 5.81 (b) shows the behaviour of power factor cos ϕ with angular frequency ω

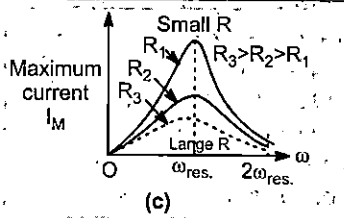


Fig. 5.81 (c) shows the current amplitude *vs.* angular frequency curve.

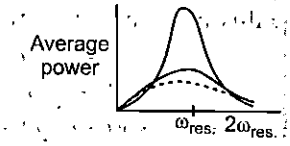


Fig. 5.81 (d) shows the average power $P_{av.}$ as a function of angular frequency. The peak occurs at $\omega_{res.} = 1/\sqrt{LC}$.

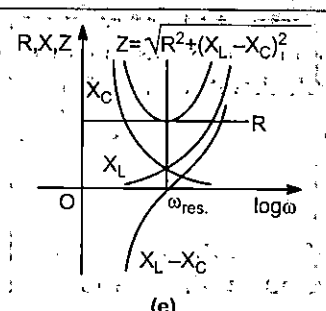


Fig. 5.81 (e) shows the reactance, resistance and impedance as functions of angular frequency (on a logarithmic angular frequency scale).

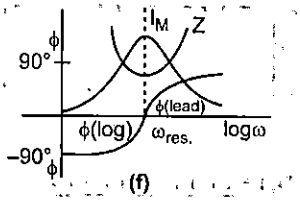


Fig. 5.83 (f) shows impedance current amplitude and phase angle as functions of angular frequency (logarithmic angular frequency scale).

☛ A parameter called quality factor Q is used to describe the sharpness of resonance peak of electrical as well as mechanical oscillating systems. The "Q" of a system is defined as

$$Q = \frac{2\pi(\text{maximum energy stored in the system})}{\text{energy dissipated in each cycle}} = \frac{2\pi U_M}{\Delta U}$$

In each cycle, energy is alternately stored in capacitor and inductor. Maximum energy stored in the inductor is $U = \frac{1}{2}LI_M^2$. Energy is dissipated only in resistor, at the average rate

$$P_{av.} = I_{rms}^2 R = I_M^2 R/2$$

In each cycle the electrical energy dissipated is $\Delta U = \frac{I_M^2 RT}{2}$, where T is the period of oscillation.

Thus
$$Q = 2\pi\frac{U}{\Delta U} = 2\pi\frac{(LI_M^2/2)}{(I_M^2 RT/2)} = \frac{\omega_{res.}L}{R}$$
$$= \frac{\omega_{res.}}{2\beta}\left(\text{where } \beta = \frac{R}{2L}\right)$$

as $\omega_{res.} = 2\pi/T$.

At resonance, $I_M = V_M/R$

and $V_L = I_M X_L = \frac{\omega L}{R}V_M$

or $V_L = \omega V_M$

At resonance the voltage drop across inductor or capacitor is Q times applied voltage.

Filter circuits: A filter circuit consists of a series combination of a resistor and capacitor (see Fig. 5.82). Filter circuit is used to eliminate high frequency or low frequency noise from a periodic signal. The input signal to the RC circuit is $V(t) = V_0 \sin\omega t$. The maximum input is given by

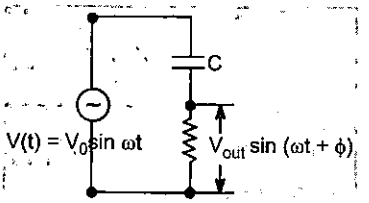


Fig. 5.82

$$V_{in} = IZ = I\sqrt{R^2 + X_C^2}$$
$$= I\sqrt{R^2 + (1/\omega C)^2}$$

Maximum output potential drop across the resistor is

$$V_{R_M} = \frac{RV_M}{\left(R^2 + \frac{1}{\omega C}\right)^2} = \frac{\omega RCV_M}{[1 + (\omega RC)^2]^{1/2}}$$

Now we analyse the expression for low and high frequency limits.

Case (i) When angular frequency approaches zero, the potential difference across resistor approaches zero; this is a DC limit.

Case (ii) For very high frequencies the denominator approaches

$$[1 + (\omega RC)^2]^{1/2} \simeq [(\omega RC)^2]^{1/2} \simeq \omega RC$$

Hence the peak potential difference across the resistor is

$$V_M \simeq \frac{\omega RCV_0}{\omega RC} = V_0$$

For high frequencies the impedance of the capacitor is negligible, the capacitor behaves as short circuited and most of the potential difference appears across the resistor in accordance with Kirchhoff's law.

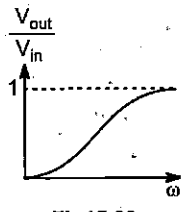


Fig. 5.83

Because the potential difference across the resistor is appreciable only at high frequency, it is called a **high pass filter.**

For the high pass filter, the voltage gain is

$$\text{Voltage gain} = \frac{V_{out}}{V_{in}} = \frac{\omega RC}{[1+(\omega RC)^2]^{1/2}}$$

The gain $\frac{V_{out}}{V_{in}}$ for a high pass filter.

Now we reverse the role of the resistor and capacitor so that the output is equal to the potential drop across the capacitor as shown in Fig. 5.84.

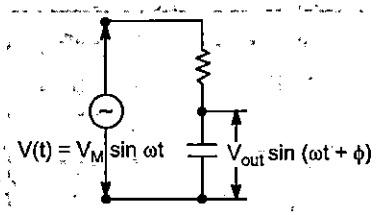


Fig. 5.84

$$V_{out} = I_M X_C = \frac{I_M}{\omega C}$$

$$V_{C_M} = \frac{V_M}{\frac{1}{X_C}(R^2+X_C^2)^{1/2}} = \frac{V_M}{[1+(\omega RC)^2]^{1/2}}$$

The expression shows how peak value of potential difference on capacitor varies with frequency, we analyse two limiting cases.

Case (i): If the angular frequency is zero,

$$V_{C_M} = V_M \qquad \text{[amplitude of source]}$$

For $\omega = 0$, we have DC situation, no current is present, a capacitor behaves as an open circuit for DC. Potential difference across resistor is zero and the capacitor is effectively connected in parallel with the DC battery (AC voltage with zero frequency).

Case (ii) For large values of the angular frequency of the source, the peak value of the potential difference across the capacitor approaches zero.

With the increase in angular frequency, the impedance of the capacitor decreases. At large ω, the capacitor has impedance tending to zero, it behaves as a short circuit.

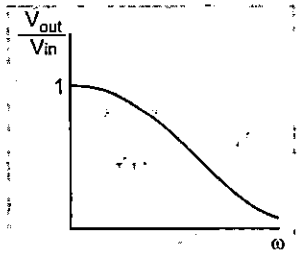


Fig. 5.85

We consider the voltage source to be an input and the potential difference across the capacitor as an output, the peak potential difference across the capacitor is appreciable only for low angular frequencies, this output is called a **low pass filter.**

$$\text{The voltage gain} = \frac{V_{out}}{V_{in}} = \frac{1}{[1+(\omega RC)^2]^{1/2}}.$$

Choke Coil

A choke coil is basically a series AC *LR* circuit, used for reducing current without power loss. In a DC circuit a rheostat is used to reduce current. A choke coil consists of an inductor with a soft iron core, applied in series with the main circuit. The current in an *LR* circuit is

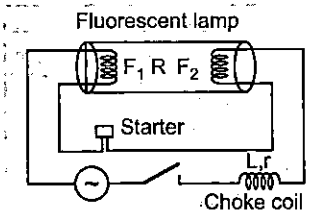


Fig. 5.86

$$I_M = \frac{V_M}{Z}$$

where $Z = \sqrt{(R+r)^2 + (\omega L)^2}$

Due to large inductance, the impedance of a circuit is increased, thereby reducing current. However, average power dissipated in an inductor is approximately zero, there is no appreciable energy loss.

$$P_{av.} = V_{rms} I_{rms} \cos\phi \simeq 0$$

and $$\cos\phi = \frac{r}{Z} = \frac{r}{\sqrt{r^2+\omega^2L^2}} \simeq \frac{r}{\omega L}$$

For large L, $\frac{r}{\omega L} \to 0$

Example 87

An LCR series circuit with 100Ω resistance is connected to an AC source of 200 V and angular frequency 300 rad/s. When only the capacitance is removed, the current lags behind the voltage by 60°. When only the inductance is removed, the current leads the voltage by 60°. Calculate the current and the power dissipated in the LCR circuit.

Solution: When capacitance is removed, the circuit becomes *LR* with

$$\tan\phi = \frac{X_L}{R}$$

i.e., $$X_L = R\tan\phi = 100\sqrt{3}\,\Omega$$

and when inductance is removed, the circuit becomes *CR* with

$$\tan\phi = \frac{X_C}{R}$$

i.e., $$X_C = R\tan\phi = 100\sqrt{3}\,\Omega$$

as here $X_L = X_C$, so the circuit is in series resonance and hence as $X = X_L - X_C = 0$, *i.e.*, $Z = \sqrt{R^2+X^2} = R$. So,

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{V_{rms}}{R}$$

$$= \frac{200}{100} = 2\text{A}$$

and $$P_{av.} = V_{rms} I_{rms} \cos\phi$$
$$= 200 \times 2 \times 1 = 400\text{W}$$

Example 88

An inductor 20×10^{-3} henry, a capacitor 100μF and a resistor 50 Ω are connected in series across a source of emf V = 10 sin 314 t. Find the energy dissipated in the circuit in 20 minutes. If resistance is removed from the circuit and the value of inductance is doubled, then find the variation of current with time in the new circuit.

Solution: Here, time of 1 cycle $T = 1/50$ s. So, we have to calculate the average energy as time $\gg T$.

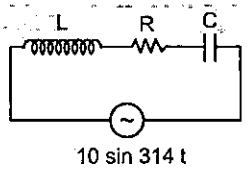


Fig. 5E.88

Energy consumed in time t

$$= V_{rms} I_{rms} \cos\phi t$$
$$= \frac{I_M}{\sqrt{2}} \cdot \frac{V_M}{\sqrt{2}} \cdot \frac{R}{Z} t$$
$$V = \frac{V_M^2 R}{2Z^2} t \qquad \left(\because I_M = \frac{V_M}{Z}\right)$$

Now, $$Z = \sqrt{R^2 + \left(\omega L \sim \frac{1}{\omega C}\right)^2}$$

$$= \sqrt{(50)^2 + \left(314 \times 20 \times 10^{-3} - \frac{1}{314 \times 100 \times 10^{-6}}\right)^2}$$

$$= \sqrt{3153.6} \approx 56\,\text{ohm}$$

$$\therefore \text{Energy consumed} = \frac{10^2 \times 50 \times 20 \times 60}{2 \times 3153.6}$$
$$= 951\text{ J}$$

When resistance is removed,

$$\cos\phi = \frac{R}{Z'} = 0 \quad \text{or} \quad \phi = \frac{\pi}{2}$$

$$Z' = \frac{1}{\omega C} - \omega L'$$
$$= \frac{1}{314 \times 10^{-4}} - 314 \times 40 \times 10^{-3}$$
$$= 19.3$$

$$I = \frac{V_M}{Z'} \sin(\omega t + \phi)$$
$$= \frac{10}{19.3} \sin\left(314t + \frac{\pi}{2}\right)$$
$$= 0.52 \cos 314\, t$$

Example 89

A box P and a coil Q are connected in series with an AC source of variable frequency. The emf of the source is constant at 10 V. Box P contains a capacitance of 1μF in series with a resistance of 32 Ω. Coil Q has a self-inductance 4.9 mH and a resistance of 68 Ω in series. The frequency is adjusted so that the maximum current flows in P and Q. Find the impedance of P and Q at this frequency. Also find the voltage across P and Q respectively.

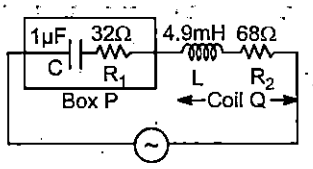


Fig. 5E.89

Solution: As this circuit is a series *LCR* circuit, current will be maximum at resonance, *i.e.*,

$$\omega = \frac{1}{\sqrt{LC}}$$
$$= \frac{1}{\sqrt{(4.9 \times 10^{-3})(10^{-6})}} = \frac{10^5}{7}\ \text{rad/s}$$

with $$I_M = \frac{V_M}{R} = \frac{10}{(32+68)} = \frac{1}{10}\ \text{A}$$

So the impedance,

$$Z_P = [R_1^2 + (1/\omega C)^2]^{1/2}$$
$$= [(32)^2 + (7/10^5 \times 10^{-6})^2]^{1/2}$$
$$= \sqrt{5924} = 77\Omega$$

and $$Z_Q = [R_2^2 + (\omega L)^2]^{1/2}$$
$$= [(68)^2 + (4.9 \times 10^{-3} \times 10^5/7)^2]^{1/2}$$
$$= \sqrt{9524} = 97.6\Omega$$

and hence, $$V_P = IZ_P = \frac{1}{10} \times (77) = 7.7\text{V}$$

and $$V_Q = IZ_Q = \frac{1}{10} \times (97.6) = 9.76\text{V}$$

Example 90

An LCR circuit has L = 10 mH, R = 3 Ω and C = 1 μF connected in series to a source of 15 cos ωt V. Calculate the current amplitude and the average power dissipated per cycle at a frequency that is 10% lower than the resonance frequency.

Solution: As here,

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$= \frac{1}{\sqrt{10^{-2}\times 10^{-6}}} = 10^4 \text{ rad/s}$$

so, $$\omega = \omega_0 - \frac{10}{100}\omega_0$$

$$= \frac{9}{10}\omega_0 = 9\times 10^3 \text{ rad/s}$$

and hence,

$$X_L = \omega L = 9\times 10^3 \times 10^{-2} = 90\Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{9\times 10^3 \times 10^{-6}} = 111.11\Omega$$

So, $$X = X_L - X_C = 90 - 111.11 = -21.11\Omega$$

and hence,

$$Z = \sqrt{R^2 + X^2} = \sqrt{3^2 + (-21.11)^2},$$

i.e., $$Z = \sqrt{9 + 445.63} = 21.32\Omega$$

and as here

$$V = 15\cos\omega t,$$

i.e., $$V_M = 15\text{V}$$

$$I_M = \frac{V_M}{Z} = \frac{15}{21.32} = 0.704\text{A}$$

The average power dissipated,

$$P_{av.} = V_{rms} I_{rms}\cos\phi$$

$$= (I_{rms}\times Z)\times I_{rms}\times\frac{R}{Z}$$

i.e., $$P_{av.} = I_{rms}^2 R = \frac{1}{2}I_0^2 R \qquad \left[\text{as } I_{rms} = \frac{I_0}{\sqrt{2}}\right]$$

so, $$P_{av.} = \frac{1}{2}\times(0.704)^2 \times 3 = 0.74\text{W}$$

Now as $$f = \frac{\omega}{2\pi} = \frac{9\times 10^3}{2\pi} \text{ cycle/s}$$

so, $$\frac{P_{av.}}{\text{cycle}} = \frac{0.74}{(9\times 10^3/2\pi)}$$

$$= \frac{2\pi\times 0.74}{9\times 10^3}\frac{\text{J}}{\text{cycle}}$$

i.e., $$\frac{P_{av.}}{\text{cycle}} = 5.16\times 10^{-4} \text{ J/cycle}$$

Example 91

A series LCR circuit containing a resistance of 120 Ω has angular resonance frequency 4×10^5 rad s^{-1}. At resonance the voltages across resistance and inductance are 60 V and 40 V respectively. Find the values of L and C. At what frequency the current in the circuit lags the voltage by 45° ?

Solution: At resonance as $X = 0$

$$I = \frac{V}{R} = \frac{60}{120} = \frac{1}{2}\text{A}$$

and as $$V_L = IX_L = I\omega L, \quad L = \frac{V_L}{I\omega}$$

so, $$L = \frac{40}{(1/2)\times 4\times 10^5} = 0.2\text{ mH}$$

and as $$\omega_0 = \frac{1}{\sqrt{LC}}, \qquad C = \frac{1}{L\omega_0^2}$$

i.e., $$C = \frac{1}{0.2\times 10^{-3}\times(4\times 10^5)^2} = \frac{1}{32}\mu\text{F}$$

Now in case of series *LCR* circuit,

$$\tan\phi = \frac{X_L - X_C}{R}$$

So current will lag the applied voltage by 45° if,

$$\tan 45° = \frac{\omega L - \frac{1}{\omega C}}{R}$$

i.e., $$1\times 120 = \omega\times 2\times 10^{-4} - \frac{1}{\omega(1/32)\times 10^{-6}}$$

i.e., $$\omega^2 - 6\times 10^5\omega - 16\times 10^{10} = 0$$

i.e., $$\omega = \frac{6\times 10^5 \pm \sqrt{(6\times 10^5)^2 + 64\times 10^{10}}}{2}$$

i.e., $$\omega = \frac{6\times 10^5 + 10\times 10^5}{2} = 8\times 10^5 \text{ rad/s}$$

Example 92

A current of 4 A flows in a coil when connected to a 12 V DC source. If the same coil is connected to a 12 V, 50 rad/s AC source a current of 2.4 A flows in the circuit. Determine the inductance of the coil. Also find the power developed in the circuit if a 2500 μF capacitor is connected in series with the coil.

Solution: In case of a coil as $Z = \sqrt{R^2 + \omega^2 L^2}$

i.e., $$I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + \omega^2 L^2}}$$

So when DC is applied as $\omega = 0$,

$$I = \frac{V}{R}$$

i.e., $$R = \frac{12}{4} = 3\Omega$$

and when AC is applied,

$$I = \frac{V}{Z}$$

i.e., $$Z = \left(\frac{V}{I}\right) = \left(\frac{12}{2.4}\right) = 5\Omega$$

or $$R^2 + X_L^2 = 5^2 \qquad (\text{as } Z = \sqrt{R^2 + X_L^2})$$

so, $$X_L^2 = 5^2 - R^2 = 5^2 - 3^2 = 4^2$$

i.e., $$X_L = 4\Omega$$

but as $$X_L = \omega L, \quad L = \frac{X_L}{\omega} = \frac{4}{50} = 0.08\Omega$$

Now when the capacitor is connected to the above circuit in series,

as $X_C = \dfrac{1}{\omega C} = \dfrac{1}{50 \times 2500 \times 10^{-6}}$

$= \dfrac{10^3}{125} = 8\Omega$

so, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$= \sqrt{3^2 + (4-8)^2} = 5\ \Omega$

and hence, $I = \dfrac{V}{Z} = \dfrac{12}{5} = 2.4\ \text{A}$

So, $P_{\text{av.}} = V_{\text{rms}} I_{\text{rms}} \cos\phi$

$= (I_{\text{rms}} \times Z) \times I_{\text{rms}} \times \left(\dfrac{R}{Z}\right)$

i.e., $P_{\text{av.}} = I_{\text{rms}}^2 R = (2.4)^2 \times 3 = 17.28\,\text{W}$

Symbolic Notation of Phasors

A vector quantity can be represented in terms of its components. A vector $\vec{\mathbf{A}}$ can be expressed symbolically in terms of complex number as

$$\vec{\mathbf{A}} = a + jb$$

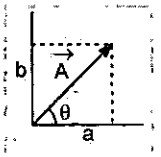


Fig. 5.87

where j is known as an operator. In mathematics x is referred to as real component and y as imaginary component but in AC circuit analysis these are known as in *phase* (active) quadrature (*reactive*) components respectively.

In Fig. 5.88 shown, the vectors $\vec{\mathbf{A}}_1, \vec{\mathbf{A}}_2, \vec{\mathbf{A}}_3$ and $\vec{\mathbf{A}}_4$ can be expressed as

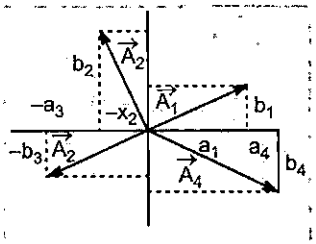


Fig. 5.88

$$\vec{\mathbf{A}}_1 = a_1 + jb_1$$

$$\vec{\mathbf{A}}_2 = -a_2 + jb_2$$

$$\vec{\mathbf{A}}_3 = -a_3 - jb_3$$

$$\vec{\mathbf{A}}_4 = a_4 - jb_4$$

Significance of Operator *j*

Symbol j is used to indicate the anticlockwise rotation of a vector through 90°. It is assigned a value $\sqrt{(-1)}$.

$$jj = j^2 = \sqrt{(-1)}\sqrt{(-1)} = -1$$

When operator j is operated on vector $\vec{\mathbf{E}}$, we get the new vector $j\vec{\mathbf{E}}$ which is displaced by 90° in counter-clockwise direction from $\vec{\mathbf{E}}$ (Fig. 5.89). Further application of j will give $j^2\vec{\mathbf{E}} = -\vec{\mathbf{E}}$ as shown.

If the operator j is applied to the vector $j^2\vec{\mathbf{E}}$, the result is $j^3\vec{\mathbf{E}} = -j\vec{\mathbf{E}}$. The vector $j^3\vec{\mathbf{E}}$ is 270° counter-clockwise from the reference axis and is directly opposite to $j\vec{\mathbf{E}}$. If the vector $j^3\vec{\mathbf{E}}$ is, in turn, operated on by j, the result will be

$$j^4\vec{\mathbf{E}} = [\sqrt{(-1)}]^4\vec{\mathbf{E}} = \vec{\mathbf{E}}$$

Hence, it is seen that successive applications of the operator j to the vector $\vec{\mathbf{E}}$ produce successive 90° steps of rotation of the vector in the counter-clockwise direction without in any way affecting the magnitude of the vector.

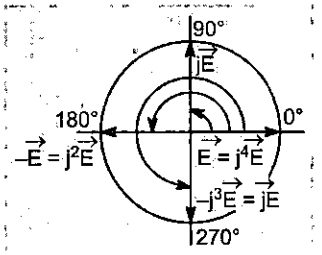


Fig. 5.89

It will also be seen from Fig. 5.89 that the application of $-j$ to $\vec{\mathbf{E}}$ yields $-j\vec{\mathbf{E}}$ which is a vector of identical magnitude but rotated 90° *clockwise* from $\vec{\mathbf{E}}$.

Summarising the above, we have

j = 90° ccw rotation $= \sqrt{(-1)}$

j^2 = 180° ccw rotation $= [\sqrt{(-1)}]^2 = -1;$

j^3 = 270° ccw rotation $= [\sqrt{(-1)}]^3 = -\sqrt{(-1)} = -j$

j^4 = 360° ccw rotation $= [\sqrt{(-1)}]^4 = +1;$

j^5 = 450° ccw rotation $= [\sqrt{(-1)}]^5 = \sqrt{(-1)} = j$

It should also be noted that $\dfrac{1}{j} = \dfrac{j}{j^2} = \dfrac{j}{-1} = -j.$

Conjugate Complex Numbers

Two complex numbers are said to be conjugate if they differ only in the algebraic sign of their quadrature components. Accordingly, the numbers $(a + jb)$ and $(a - jb)$ are conjugate. The sum of two conjugate numbers gives in-phase (or active) component and their difference gives quadrature (or reactive) component.

Complex Impedance

The complex number $e^{j\omega t}$ may be rewritten as

$$e^{j\omega t} = \cos\omega t + j\sin\omega t$$

The magnitude of $e^{j\omega t}$ is unity and its real part is $\cos\omega t$. We can therefore represent an alternating voltage with amplitude V_M and angular frequency ω by the real part of the complex number $V_M e^{j\omega t}$.

In Fig. 5.90 the complex number $V_M e^{j\omega t}$ is plotted as the point P on a diagram in which the real part is the horizontal coordinate and the imaginary part is the vertical coordinate. A diagram representing complex numbers in this way is called an *Argand diagram*.

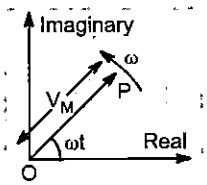


Fig. 5.90

The line OP from the origin to P has a length V_M and the angle between OP and the real axis is ωt, which is the phase of the voltage. OP rotates anticlockwise at a uniform angular speed ω about O. The projection of OP on to the real axis is $V_M \cos \omega t$, which is the actual alternating voltage. The imaginary part of $V_M e^{j\omega t}$ has no physical meaning, and it is only introduced to keep track of phase differences.

The sinusoidally varying complex number $V_M e^{j\omega t}$ represents phasor v.

$$V = V_M e^{j\omega t} = V_M \cos \omega t + jV_M \sin \omega t.$$

Because the complex number v contains information about the phase as well as the amplitude of the voltage, it is called a **phasor** and the Agrand diagram is referred to as a phasor diagram.

Currents may similarly be represented by phasors. If the phase difference between current and voltage is ϕ we represent the current by

$$I = I_M e^{j(\omega t+\phi)} = I_M[\cos(\omega t + \phi) + j\sin(\omega t + \phi)]$$

Relation Between the Phasors *I* and *V*

(a) Resistive circuits: Relation between the phasors *I* and *V* for resistive, capacitative and inductive components: The phase difference ϕ between current and voltage is zero, and the amplitude of the current is $I_M = V_M/R$. The phasor representing the current is

$$I = I_M e^{j\omega t} = \frac{V_M}{R} e^{j\omega t} = \frac{V}{R}.$$

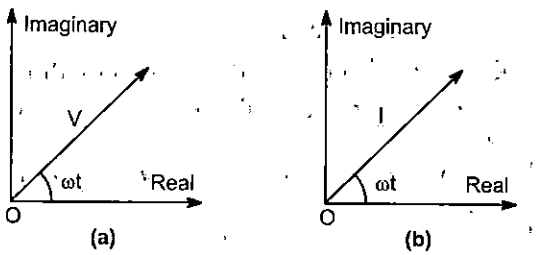


Fig. 5.91

The phasor equation $V = IR$ looks just the same as Ohm's law $V = IR$ for the actual current and voltage. The phasor diagrams for V and I are shown in Fig. 5.91 (a). Both phasors rotate about the origin at an angular speed ω and both have the same phase at all times.

(b) Capacitive circuits: For a capacitor the current amplitude is ωCV_0 and the current leads the voltage by the phase angle $\pi/2$. The phasor for the current is

$$I = I_M e^{j(\omega t+\pi/2)} = \omega CV_M e^{j(\omega t+\pi/2)}.$$

Splitting $e^{j\pi/2}$ into its real and imaginary parts, we have

$$e^{j\pi/2} = \cos(\pi/2) + j\sin(\pi/2) = j,$$

and current becomes

$$I = j\omega CV_M e^{j\omega t} = j\omega Cv$$

or

$$V = \frac{i}{j\omega C}$$

This equation again has the form of Ohm's law with the resistance R replaced by the complex number $1/j\omega C$. The phasors V and I now point in different directions. At $t = 0$ when V is real, I is purely imaginary because of the factor j. The two phasors are perpendicular to one another, and remain so at all times, as indicated in Fig. 5.92.

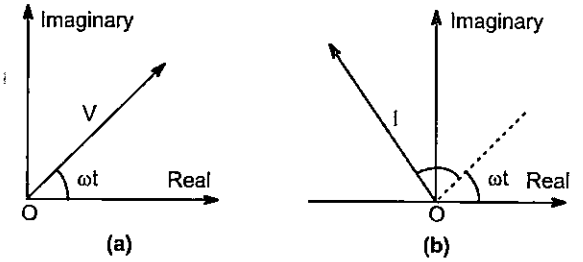


Fig. 5.92

(c) Inductive circuits: For a current passing through an idealized self-inductance, the phase difference between current and voltage is $(-\pi/2)$ and the amplitude of the current is $V_M/\omega L'$. Using

$$e^{-j\pi/2} = \cos(-\pi/2) + j\sin(-\pi/2) = -j,$$

the complex current is

$$I = I_M e^{j(\omega t-\pi/2)} = \frac{-j}{\omega L} V_M e^{j\omega t}$$

or, since $-j = -j^2/j = 1/j$, $V = I \times j\omega L$.

This has the same forms as Ohm's with the resistance R replaced by the complex number $j\omega L$. The phasors V and I are again perpendicular to one another as for the capacitor, but now the current lags behind the voltage (Fig. 5.93).

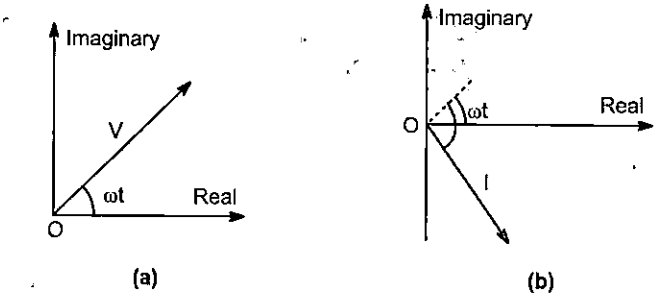


Fig. 5.93

For all three types of passive component the relation between voltage and current phasors is expressed in the same form as Ohm's law for resistors. This is why the use of complex numbers simplifies AC calculations. The ratio V/I is called the **complex impedance** and is denoted by Z. The complex impedance satisfies the equation

$$V = IZ$$

where $Z = R$ for resistance R,

$$Z = \frac{1}{j\omega C} \text{ for capacitance } C$$

and $Z = j\omega L$ for inductance L.

Kirchhoff's Rules for AC Circuits

When an AC network contains capacitance and inductance, the currents in different branches of the network may have different phases. The phase differences have to be taken into account when applying Kirchhoff's first rule at a junction in a circuit. Fig. 5.94 (a) shows two

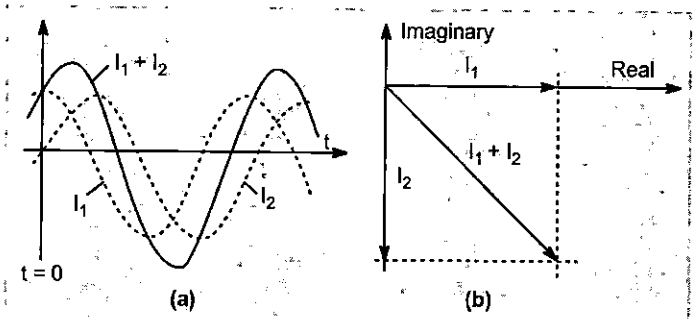


Fig. 5.94

alternating currents I_1 and I_2 that have the same amplitude but differ in phase by $\pi/2$. The sum of these currents has an amplitude $\sqrt{2}$ times that of I_1 and I_2, and is not in phase with either. Since the real parts of phasors are the same as the actual currents, we can write $I_3 = I_1 + I_2$. The argument may be extended to junctions of any number of connections, and we may say that the algebraic sum of the current phasors arriving at any junction is zero: Kirchhoff's first rule applies to the phasors for AC currents. The current phasors through all the components in any branch of a network must therefore be the same, since for any two components connected in series the current leaving one and entering the other must be the same.

Kirchhoff's second rule also applies to AC network, provided that induced emfs are taken into account. In the presence of changing magnetic fields the electric field is no longer conservative, but Faradays' law holds, and the line integral of the electric field is given by the equation

$$\oint \vec{\mathbf{E}}.\, d\mathbf{l} = -\frac{d\phi}{dt}.$$

The right-hand side of this equation is the rate of change of magnetic flux through the closed circuit around which the line integral is evaluated. For the networks we are representing by equivalent circuits, there are no fields except in the separate components. Induced emfs only occur because of the self-inductance and mutual inductance of coils. The emf due to a self-inductance L, for example, is represented as a complex voltage $j\omega LI$. When induced emfs are included in this way as complex voltages, Kirchhoff's second rule, that the sum of the voltages around any loop of the network is zero, it still true.

Illustration: A coil with self-inductance L and resistance R is connected to a generator delivering an AC voltage of amplitude V_M at an angular frequency ω, as represented by the equivalent circuit shown in Fig. 5.95. What is the phase difference between the voltage and the current in the coil ? In the figure one terminal of the generator is shown connected to earth (the symbol ⏚): this does not affect the calculations, but is included to remind

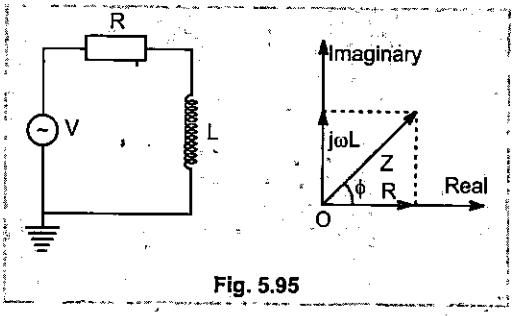


Fig. 5.95

that most real circuits do have earth connections. The current I through R and L is the same. Equating the voltage of the generator to the voltage drops across R and L,

$$V = I(j\omega L + R) = IZ \qquad ...(1)$$

where the complex impedance of the coil, including both its resistance and self-inductance, is

$$Z = j\omega L + R. \qquad ...(2)$$

Fig. 5.95 includes an Argand diagram showing how the real and imaginary parts of Z are added to give the complex impedance of the whole coil. From this figure we see that the magnitude of the impedance is $Z_0 = \sqrt{(\omega L)^2 + R^2}$ and that it makes an angle ϕ with the real axis given by $\tan\phi = \omega L/R$. The impedance Z may be written alternatively as $Z = Z_0 e^{j\phi}$, and

$$V = Z_0 e^{j\phi} I.$$

The angle $\phi = \tan^{-1}(\omega L/R)$ is thus the phase difference by which the current through the coil lags behind the voltage across it.

This result may be obtained mathematically using the complex impedance $Z = j\omega L + R$ as

$$Z = Z_0 e^{j\omega} = Z_0(\cos\phi + j\sin\phi).$$

The relation between the two forms is given as

$$Z_0 = \sqrt{(\omega L)^2 + R^2} \quad \text{and} \quad \tan\phi = \omega L/R.$$

- The impedance Z of the coil given by eqn. (2) is the sum of the impedances of the components L and R, which are in series in Fig. 5.95. Since the voltage and current phasors satisfy Kirchhoff's rules, complex impedances that are in series or in parallel may be combined in the same way as series and parallel resistors. For n impedances Z_j in series,

$$Z_s = \sum_{j=1}^{n} Z_j. \qquad ...(3)$$

Similarly for n impedances Z_j in parallel,

$$\frac{1}{Z_p} = \sum_{j=1}^{n} \frac{1}{Z_j}. \qquad ...(4)$$

The impedance of combination of parallel and series impedances can be worked out using eqns. (3) and (4) without having to use Kirchhoff's rules to evaluate all the currents in the network.

Example 93

For the circuit shown in Fig. 5E.93 find the expressions for the impedance of the circuit and phase of current.

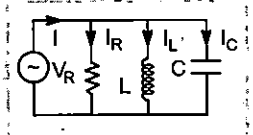


Fig. 5E.93

Solution: When L, C and R are connected in parallel to same source of AC, current in inductance lags while in capacitance leads the applied voltage by $(\pi/2)$. However, current and voltage in resistance are in the same phase, *i.e.*,

$$I_R = \frac{V}{R}\sin\omega t; \quad I_L = \frac{V}{X_L}\sin\left(\omega t - \frac{\pi}{2}\right)$$

and $$I_C = \frac{V}{X_C}\sin\left(\omega t + \frac{\pi}{2}\right)$$

and as all these are in parallel,

$$I = I_R + I_L + I_C$$

with $$X_L = \omega L \quad \text{and} \quad X_C = \frac{1}{\omega C}$$

So, $$I = \left[\frac{V_M}{R}\sin\omega t + V_M\left(\omega C - \frac{1}{\omega L}\right)\cos\omega t\right]$$

Now if we take

$$\frac{V_M}{R} = I_M \sin\phi$$

and $$V_M\left(\omega C - \frac{1}{\omega L}\right) = I_0\cos\phi$$

$$I = I_M \sin(\omega t - \phi)$$

with $$\tan\phi = \frac{\left(\omega C - \frac{1}{\omega L}\right)}{1/R} = R\left(\omega C - \frac{1}{\omega L}\right)$$

and, $$I_M = \frac{V_M}{Z} = V_M\left[\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2\right]^{1/2}$$

i.e., $$\frac{1}{Z} = \left[\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2\right]^{1/2}$$

or $$Z = \left[\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2\right]^{-1/2}$$

or $$Z = \left[\frac{1}{R^2} + \left(\frac{1}{X_C} - \frac{1}{X_L}\right)^2\right]^{-1/2}$$

$$\left[\text{as } X_C = \frac{1}{\omega C} \text{ and } X_L = \omega L\right]$$

Example 94

For a resistance R and capacitance C in series, the impedance is twice that of a parallel combination of the same elements. What is the frequency of applied emf ?

Solution: As shown in Fig. 5E.94, in case of series combination,

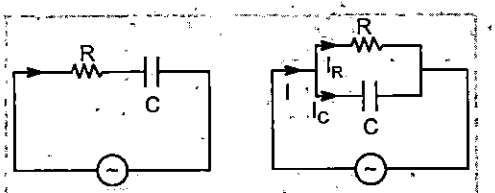


Fig. 5E.94

$$Z_s = \sqrt{R^2 + X_C^2} = [R^2 + (1/\omega C)^2]^{1/2}$$

In case of parallel combination, $I_R = \frac{V}{R}\sin\omega t$

and $$I_C = \frac{V}{X_C}\sin\left(\omega t + \frac{\pi}{2}\right)$$

So, $$I = I_R + I_C = \frac{V}{R}\sin\omega t + \frac{V}{X_C}\cos\omega t$$

i.e., $$I = I_M \sin(\omega t + \phi)$$

with $$I_M\cos\phi = \frac{V}{R} \quad \text{and} \quad I_M\sin\phi = \frac{V}{X_C}$$

So, $$I_M = \left[\left(\frac{V}{R}\right)^2 + \left(\frac{V}{X_C}\right)^2\right]^{1/2} = \frac{V}{Z_P}$$

i.e., $$\frac{1}{Z_p} = \left[\frac{1}{R^2} + \left(\frac{1}{X_C}\right)^2\right]^{1/2}$$

i.e., $$Z_p = \frac{R}{\sqrt{1 + \omega^2 C^2 R^2}}$$

and as according to given problem,

$$Z_s = 2Z_p; \quad \textit{i.e.}, \quad Z_s^2 = 4Z_p^2$$

i.e., $$\frac{(R^2\omega^2C^2 + 1)}{\omega^2C^2} = 4\frac{R^2}{(1 + R^2\omega^2C^2)}$$

i.e., $$(1 + R^2\omega^2C^2)^2 = 4R^2\omega^2C^2$$

or $$1 + R^2\omega^2C^2 = 2R\omega C$$

or $$(R\omega C - 1)^2 = 0$$

or $$\omega = \frac{1}{RC} \quad \textit{i.e.}, \quad f = \frac{1}{2\pi RC}$$

Example 95

A capacitor with capacitance $C = 1.0\,\mu F$ and a coil with active resistance $R = 0.10\Omega$ and inductance $L = 1.0\,mH$ are connected in parallel to a source of sinusoidal voltage $V = 31$ V. Find:

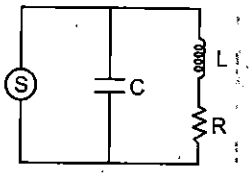


Fig. 5E.95

(a) the frequency ω at which the resonance sets in;
(b) the effective value of the fed current in resonance, as well as the corresponding currents flowing through the coil and through the capacitor.

Solution: The circuit is shown in Fig. 5E.95.

(a) The impedance of the circuit is

$$\frac{1}{Z} = \frac{1}{R + \sqrt{X_L}} + \frac{1}{-jX_C}$$

or
$$\frac{1}{Z} = \frac{1}{R + jX_L} + \frac{1}{-jX_C}$$

$$= \frac{1}{R + jX_L} + \frac{j}{-j^2 \frac{1}{\omega C}} \quad (\because \text{here } j^2 = -1)$$

$$\frac{1}{Z} = \frac{1}{R + j\omega L} + j\omega C$$

At resonance, impedance should be minimum. For this $\frac{d\left(\frac{1}{Z}\right)}{d\omega}$ should be maximum.

$$\therefore \quad \frac{d\left(\frac{1}{Z}\right)}{d\omega} = 0$$

or
$$\frac{d\left(\frac{1}{Z}\right)}{d\omega} = \frac{-jL}{(R + jL\omega)^2} + jC = 0$$

After solving, we get $R^2 + \omega^2 L^2 = \frac{L}{C}$.

$$\therefore \quad \omega = \omega_{res.} = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \qquad ...(1)$$

$$= 3 \times 10^4 \text{ rad/s}$$

(b)
$$I = \frac{VRC}{L}$$

$$I = 3 \times 10^{-3} \text{ A} = 3\,\text{mA}$$

Here
$$I_L = \frac{V}{\sqrt{L/C}} = V\sqrt{\frac{C}{L}} = 1.0\,\text{A}$$

Also,
$$I_C = \frac{V}{X_C} = V\omega C = VC\omega_{res.}$$

$$= VC\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$= V\sqrt{\frac{C}{L} - \left(\frac{RC}{L}\right)^2} = 1.0\,\text{A}$$

[from eqn. (1)]

Example 96

A capacitor with capacitance C and a coil with active resistance R and inductance L are connected in parallel to a source of sinusoidal voltage of frequency ω. (a) Find the phase difference between the current fed to the circuit and the source voltage. (b) A circuit consists of a capacitor with capacitance C and a coil with active resistance R and inductance L connected in parallel. Find the impedance of the circuit at frequency ω of alternating voltage.

Solution: (a) The circuit of the problem is shown in Fig. 5E.96. The impedance of the circuit is

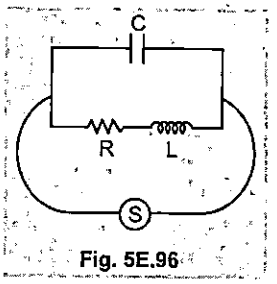


Fig. 5E.96

$$\frac{1}{2} = \frac{1}{-jX_C} + \frac{1}{R + jX_L}$$

$$= \frac{j}{\frac{-j^2}{\omega C}} + \frac{1}{R + j\omega L}$$

$$= \omega Cj + \frac{1}{R + j\omega L} \qquad (\because j^2 = -1)$$

or
$$\frac{1}{Z} = \frac{\omega RCj + \omega^2 LCj^2 + 1}{R + j\omega L}$$

$$= \frac{\omega RCj - \omega^2 LC + 1}{R + j\omega L}$$

$$\therefore \quad Z = \frac{R + j\omega L}{(1 - \omega^2 LC) + \omega RCj}$$

$$\therefore \quad Z = \frac{(R + j\omega L)\{(1 - \omega^2 LC) - \omega RCj\}}{\{(1 - \omega^2 LC) + \omega RCj\}\{(1 - \omega^2 LC) - \omega RCj\}}$$

$$Z = \frac{R - j\,[C\omega(R^2 + \omega^2L^2) - \omega L]}{(1 - \omega^2LC)^2 - (\omega RC)^2}$$

$$\therefore \tan\phi = \frac{\omega C(R^2 + \omega^2L^2) - \omega L}{R}$$

(b) From previous part of problem,

$$Z = \frac{R + j\omega L}{(1 - \omega^2LC) + \omega RCj}$$

$$\therefore \quad |Z| = \frac{|R + j\omega L|}{|1 - \omega^2LC + \omega RCj\,|}$$

$$\therefore \quad Z = |Z| = \sqrt{\frac{R^2 + \omega^2L^2}{\omega^2R^2C^2 + (1 - \omega^2LC)^2}}$$

Example 97

A black-box contains an inductor L, resistor R and a capacitor of capacitance C. It is observed that when black-box is connected through a DC source then current through the resistance is zero (at $t = 0$). Again when the box is connected through an AC source of rms value V_{rms} the current through the source is equal to the current through the resistor at resonance.

(i) Draw the circuit diagram, (ii) Calculate the equivalent impedance.

Solution: From clue (i) we have two possible combinations in both the cases $I_R = 0$ at $t = 0$.

Again from clue (ii) it is found that the current through the source is equal to the current through the resistor R. So we can easily short-listed combination (ii).

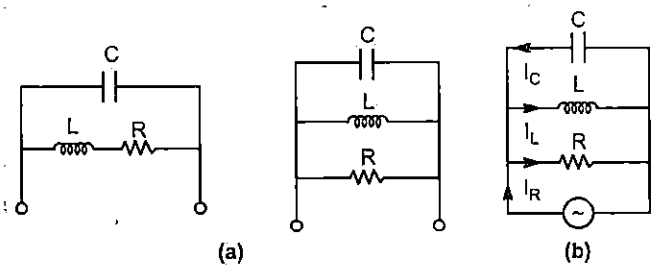


Fig. 5E.97

$I_L = I_C$ at resonance and both are in opposite phase. Hence combination (ii) is correct.

We have
$$\frac{1}{Z} = \frac{1}{R} + \frac{1}{j\omega L} - \frac{\omega C}{j}$$

$$\frac{1}{Z} = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)$$

$$\therefore \quad Z = \frac{1}{\sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2}}$$

Example 98

A capacitor with capacitance $C = 1\mu F$ and a coil with active resistance $R = 0.10\Omega$ and inductance $L = 1.0$ mH are connected in parallel to a source of sinusoidal voltage $V = 31$ V. Find:

(a) the frequency ω at which the resonance sets in,

(b) the effective value of the fed current in resonance, as well as the corresponding currents flowing through the coil and through the capacitor.

Solution:
$$\frac{1}{Z} = \frac{1}{R} + \frac{1}{j\omega L} - \frac{\omega C}{j}$$

$$\frac{1}{Z} = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)$$

$$\frac{1}{|Z|} = \sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2}$$

$$I = \frac{V}{|Z|}$$

$$= V\sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2}$$

For resonance frequency

$$\omega C = \frac{1}{\omega L}$$

$$\omega = 3 \times 10^4 \text{ rad/s}$$

$$I = \frac{V}{R} = \frac{31}{10} = 3.1 \text{ A}$$

$$I_L = \frac{V}{\omega L} = 1 \text{ A}$$

$$I_C = V\omega C = 1 \text{ A}$$

Example 99

In the circuit shown, switch S_1 was closed for a long time. At time $t = 0$ the switch is opened again. (a) Find the maximum potential difference across the plates of the capacitor after the switch is opened. Also find the angular frequency of oscillation of the charge on the capacitor.

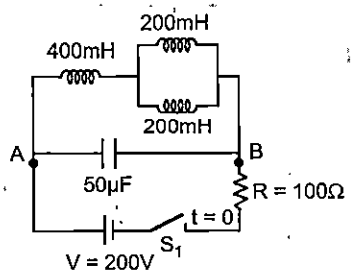


Fig. 5E.99

(b) Now the battery and the resistance (100Ω) are removed and an AC source with a peak emf of 200 V and angular frequency 100 rad/s is applied between A and B.
Find the peak value of the current drawn from the source, and also find the phase difference between the current and the source emf.

Solution: (a) The equivalent inductance $L = 500\,\text{mH}$.
Just before opening the switch, current through the inductors is

$$I_M = \frac{200}{100} = 2\text{A}$$

Potential drop across the resistor is $2 \times 100 = 200$ V
Hence, across capacitor potential difference is zero.

$\therefore$ Energy stored $u = \frac{1}{2}L(2)^2 = \frac{1}{2} \times 500 \times 10^{-3} \times 2^2 = 1\,\text{J}$

$$\therefore \quad \frac{1}{2}LI_M^2 = \frac{1}{2}CV_{\max}^2 = 1\text{ J}$$

$$\Rightarrow \quad V_{\max} = \sqrt{\frac{L}{C}}I_M$$

$$= 2 \times \sqrt{\frac{500 \times 10^{-3}}{50 \times 10^{-6}}} = 200\text{ V}$$

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{500 \times 10^{-3} \times 50 \times 10^{-6}}}$$

$$= 200\text{ rad/s}$$

(b) The impedance of the LC circuit at $\omega = 100$ rad/s is

$$Z = \left[\frac{-j}{\omega C} \times j\omega L \Big/ \left(j\omega L - \frac{j}{\omega C}\right)\right]$$

$$= \frac{j\omega L}{1 - \omega^2 LC} \quad (j^2 = -1)$$

With the given value of ω, $\omega^2 \times LC = \frac{1}{4}$

$$\therefore \quad Z = j \times 100 \times \frac{4}{3} \times 0.5 = j\frac{200}{3}\Omega$$

The peak current $= \frac{V_M}{|Z|} = 3$ A

and the current lags w.r.t. source emf by a phase of $(\pi/2)$.

Example 100

A capacitor with capacitance C whose interelectrode space is filled up with poorly conducting medium with active resistance R is connected to a source of alternating voltage $V = V_M \cos\omega t$. Find the time dependence of the steady-state current flowing in lead wires. The resistance of the wires is to be neglected.

Solution: If the space between electrodes is filled with poorly conducting material, then the system behaves as a parallel combination of resistance and capacitor, (shown in Fig. 5E.100).

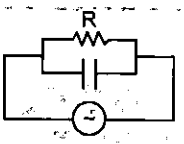


Fig. 5E.100

Here $X_C = \frac{1}{\omega C}$

$$\therefore \quad \frac{1}{Z} = \frac{1}{R} + \frac{1}{\omega C}(-j) \quad \text{(in complex representation)}$$

$$\text{or} \quad \frac{1}{Z} = \frac{1}{R} - \frac{1}{\omega C}j$$

$$\text{or} \quad \frac{1}{Z} = \frac{1}{R} + \omega Cj$$

$$\text{or} \quad \frac{1}{Z} = \frac{(1 + \omega CRj)}{R}$$

$$\therefore \quad Z = \frac{R}{(1 + \omega CRj)}$$

$$\therefore \quad Z = \frac{R}{\sqrt{1 + \omega^2 C^2 R^2}}$$

$$\therefore \quad I = \frac{V}{Z} = \frac{V_M \cos(\omega t + \phi)}{Z}$$

$$\therefore \quad I = I_M \cos(\omega t + \phi)$$

Here $I_M = \frac{V_M}{Z} = \frac{V_M}{R}\sqrt{1 + \omega^2 C^2 R^2}$

$$\because \quad \tan\phi = \frac{R}{X_C} = \frac{R}{\frac{1}{\omega C}}$$

$$\therefore \quad \tan\phi = \omega RC$$

Example 101

Prove that in a series LCR circuit, the frequencies f_1 and f_2 at which the current amplitude falls to $\frac{1}{\sqrt{2}}$ of the current at resonance are separated by an interval equal to $(R/2\pi L)$.

Solution: Current amplitude or peak value of the current in series LCR circuit is given by

$$I_M = \frac{V_M}{\sqrt{[R^2 + \{\omega L - (1/\omega C)\}^2]}} \quad \ldots(1)$$

while current at resonance is given by

$$I_R = \frac{V_M}{R}\left(\because \omega L = \frac{1}{\omega C}\right) \ldots(2)$$

Given that, at f_1 and f_2, the current amplitude falls to $\frac{1}{\sqrt{2}}$ of the current at resonance.

Hence $$\frac{V_M}{\sqrt{2}R} = \frac{V_M}{\sqrt{[R^2 + \{\omega L - (1/\omega C)\}^2]}} \quad ...(3)$$

This gives $$\omega_1 L - \left(\frac{1}{\omega_1 C}\right) = -R \quad ...(4)$$

and $$\omega_2 L - \left(\frac{1}{\omega_2 C}\right) = +R \quad ...(5)$$

Adding eqns. (4) and (5), we get

$$L(\omega_1 + \omega_2) = \left(\frac{\omega_1 + \omega_2}{\omega_1 \omega_2}\right)\frac{1}{C} \quad \text{or} \quad \omega_1 \omega_2 = \frac{1}{LC} \quad ...(6)$$

Subtracting eqns. (4) and (5), we get

$$(\omega_2 - \omega_1)L + \frac{(\omega_2 - \omega_1)}{\omega_1 \omega_2}\frac{1}{C} = 2R \quad ...(7)$$

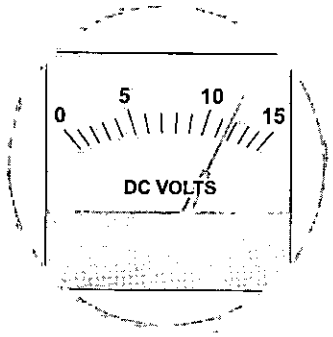
Putting the value of $\frac{1}{C}$ from eqn. (6),

$$(\omega_2 - \omega_1)L + \frac{(\omega_2 - \omega_1)}{\omega_1 \omega_2} \times \omega_1 \omega_2 L = 2R$$

or $$(\omega_2 - \omega_1) = \frac{R}{L}$$

or $$2\pi(f_2 - f_1) = \frac{R}{L}$$

$$\therefore \quad (f_2 - f_1) = \frac{R}{2\pi L}$$



PROBLEMS

Level 1 Only One Alternative is Correct

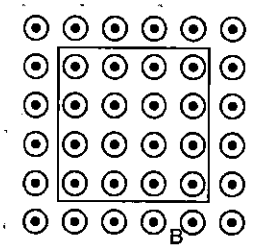
1. An electron is moving in a circular orbit of radius R with an angular acceleration α. At the centre of the orbit is kept a conducting loop of radius $r, (r << R)$. The e.m.f., induced in the smaller loop due to the motion of the electron is:

(a) zero, since charge on electron in constant

(b) $\frac{\mu_0 er^2}{4R}\alpha$ (c) $\frac{\mu_0 er^2}{4\pi R}\alpha$

(d) none of these

2. A uniform magnetic filed B is directed out of the page. A metallic wire has the shape of a square frame and is placed in the field as shown. While the shape of the wire is steadily transformed into a circle in the same plane, the current in the frame :



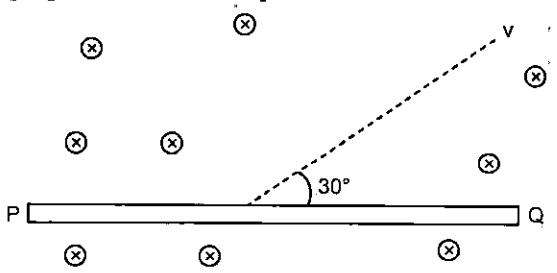
(a) is directed clockwise

(b) does not appear

(c) is directed counterclock wise

(d) is alternating

3. A conducting rod PQ of length $l = 2\,\text{m}$ is moving at a speed of $2\,\text{ms}^{-1}$ making an angle of 30° with its length. A uniform magnetic field $B = 2\,\text{T}$ exists in a direction perpendicular to the plane of motion. Then:



(a) $V_P - V_Q = 8\text{V}$ (b) $V_P - V_Q = 4\text{V}$

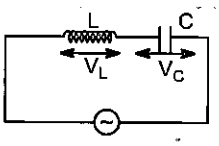
(c) $V_Q - V_P = 8\text{V}$ (d) $V_Q - V_P = 4\text{V}$

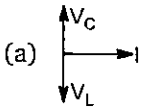
4. The dimension of the ratio of magnetic flux and the resistance is equal to that of:

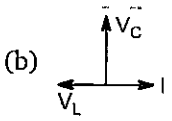
(a) induced emf (b) charge

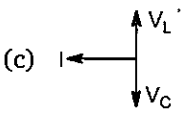
(c) inductance (d) current

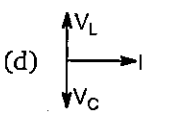
5. The current I, potential difference V_L across the inductor and potential difference V_C across the capacitor in circuit as shown in the figure are best represented vectorially as:



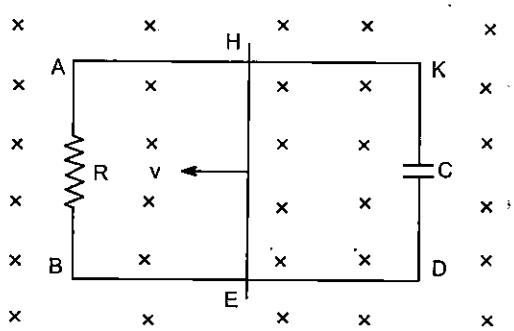
(a) 

(b) 

(c) 

(d) 

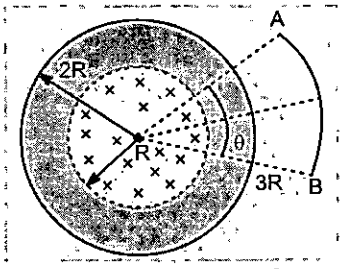
6. In the circuit shown in figure, a conducting wire HE is moved with a constant speed v towards left. The complete circuit is placed in a uniform magnetic field $\vec{\mathbf{B}}$ perpendicular to the plane of the circuit inwards. The current in $HKDE$ is :



(a) clockwise (b) anticlockwise

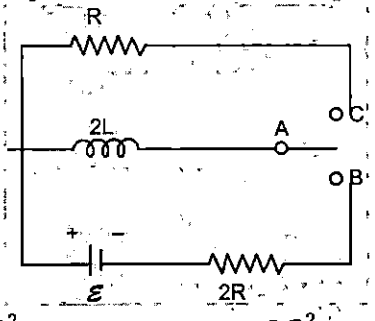
(c) alternating (d) zero

7. In the given figure two concentric cylindrical region in which time varying magnetic field is present as shown. From the centre to radius R magnetic field is perpendicular into the plane varying as $dB/dt = 2k_0$ and in a region from R to $2R$ magnetic field is perpendicular out of the plane varying as $dB/dt = 4k_0$. Find the induced emf across an are AB of radius $3R$.



(a) $6R^2k\theta$ (b) $5R^2k\theta$
(c) $7R^2k\theta$ (d) none of these

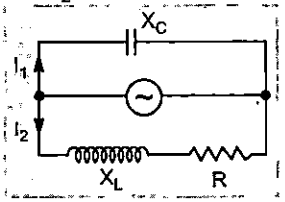
8. In the circuit shown, A is joined to B for a long, and then, A is joined to C. The total heat produced in R is:



(a) $\frac{L\mathcal{E}^2}{R^2}$ (b) $\frac{L\mathcal{E}^2}{2R^2}$
(c) $\frac{L\mathcal{E}^2}{4R^2}$ (d) $\frac{L\mathcal{E}^2}{8R^2}$

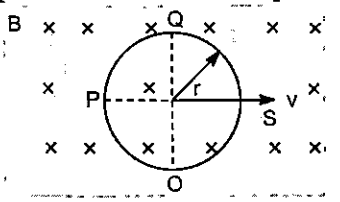
9. A capacitor $C = 2\mu F$ and an inductor with $L = 10$ H and coil resistance 5Ω are in series in a circuit. When an alternating current of r.m.s. value 2A flows in the circuit, the average power in watts in the circuit is:
(a) 100 (b) 50
(c) 20 (d) 10

10. In the shown AC circuit phase difference between currents I_1 and I_2 is:



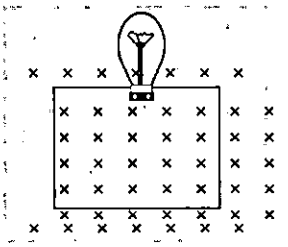
(a) $\frac{\pi}{2} - \tan^{-1}\frac{X_L}{R}$ (b) $\tan^{-1}\frac{X_L + X_C}{R}$
(c) $\frac{\pi}{2} + \tan^{-1}\frac{X_L}{R}$ (d) $\tan^{-1}\frac{X_L - X_C}{R} + \frac{\pi}{2}$

11. A conducting ring of radius r and resistance R rolls on a horizontal surface with constant velocity v. The magnetic field B is uniform and is normal to the plane of the loop. Choose the correct option :



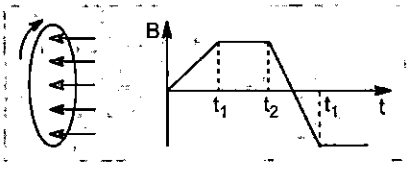
(a) The induced e.m.f. between O and Q is $2Brv$
(b) An induced current $I = \frac{2Bvr}{R}$ flows in the clockwise direction.
(c) An induced current $I = \frac{2Bvr}{R}$ flows in the anticlockwise direction
(d) No current flows.

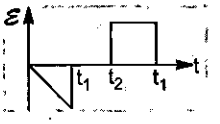
12. A square wire loop of 10.0 cm side lies at right angles to a uniform magnetic field of 20T. A 10 V light bulb is in a series with the loop as shown in the figure. The magnetic field is decreasing steadily to zero over a time interval Δt. The bulb will shine with full brightness if Δt is equal to time interval Δt. The bulb will shine with full brightness if Δt is equal to:

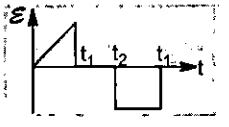


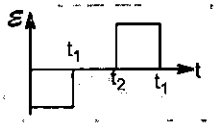
(a) 20 ms (b) 0.02 ms
(c) 2 ms (d) 0.2 ms

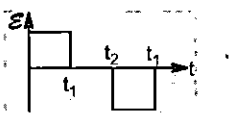
13. A wire loop is placed in a region of time varying magnetic field which is oriented orthogonally to the plane of the loop as shown in the figure. The graph shows the magnetic field variation as the function of time. Assume the positive emf is the one which drives a current int he clockwise direction and seen by the observer in the direction of B. Which of the following graphs best represents the induced emf as a function of time.



(a) 

(b) 

(c) 

(d) 

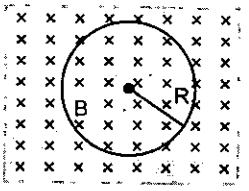
14. An n sided regular polygon is inscribed in a circular region of radius R using a wire of cross sectional radius r and resistivity ρ. If the magnetic field in the circular

region has a time rate of change given by $\frac{dB}{dt}$ then the current in the loop at any time is given by :

(a) $\frac{\pi R\frac{dB}{dt}r^2\cos(r/n)}{2\rho}$ (b) $\frac{\pi R\frac{dB}{dt}r^2\cos(r/n)}{\sqrt{2}\rho}$

(c) $\frac{\pi R\frac{dB}{dt}r^2\sin(r/n)}{2\rho}$ (d) $\frac{\pi R\frac{dB}{dt}r^2\cos(r/n)}{3\rho}$

15. A conducting loop of radius R is present in a uniform magnetic field B perpendicular the plane of the ring. If radius R varies as a function of time 't', as $R = R_0 + t$. The e.m.f.; induced in the loop is:



(A) $2\pi(R_0 + t)B$
(b) $\pi(R_0 + t)B$ clockwise
(c) $2\pi(R_0 + t)B$ anticlockwise
(d) zero

16. A vertical bar magnet is dropped from position on the axis of a fixed metallic coil as shown in Fig-I. In Fig- II magnet is fixed and horizontal coil is dropped. The acceleration of the magnet and coil are a_1 and a_2 respectively then:

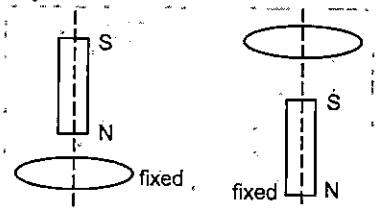


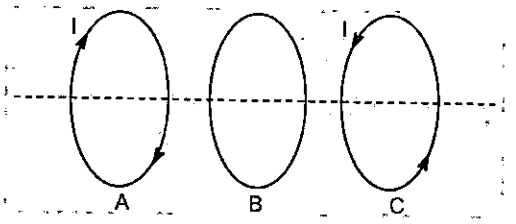
Fig.-I Fig.-II

(A) $a_1 > g, a_2 > g$ (b) $a_1 > g, a_2 < g$
(c) $a_1 < g, a_2 < g$ (d) $a_1 < g, a_2 > g$

17. A closed planar wire loop of area A and arbitrary shape is placed in a uniform magnetic field of magnitude B, with its plane perpendicular to magnetic field. The resistance of the wire loop is R. The loop is now turned upside down by 180° so that its plane again becomes perpendicular to the magnetic field. The total charge that must have flowed through the wire ring in the process is:

(a) $< AB/R$ (b) $= AB/R$
(c) $= 2AB/R$ (d) None of these

18. Three identical coils A, B and C carrying currents are placed coaxially with their planes parallel to one another. A and C carry currents as shown. B is kept fixed, while A and C both are moved towards B with the same speed. Initially, B is equally separated from A and C. The direction of the induced current in the coil B is :

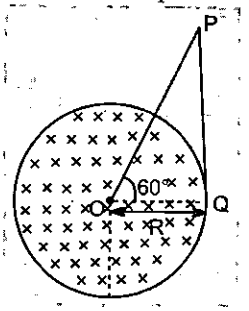


(a) same as that in coil A
(b) same as that in coil B
(c) zero
(d) none of these

19. The magnetic flux through a stationary loop with resistance R varies during interval of time T as $\phi =$ at $(T - t)$. The heat generated during this time neglecting the inductance of loop will be:

(a) $\frac{a^2T^3}{3R}$ (b) $\frac{a^2T^2}{3R}$
(c) $\frac{a^2T}{3R}$ (d) $\frac{a^2T^3}{R}$

20. In a cylindrical region having radius R, magnetic field varies with time as $B = a + bt$. OPQ is a triangular loop made of wire having resistance per unit length λ. Current induced in the loop is :



(a) $\frac{\pi bR(\sqrt{3}-1)}{12\sqrt{3}\lambda}$ (b) $\frac{\pi bR}{12\sqrt{3}(\sqrt{3}+1)\lambda}$
(c) $\frac{\pi bR(\sqrt{3}-1)}{6\sqrt{3}\lambda}$ (d) none of these

21. A simple LR circuit is connected to a battery at time $t = 0$. The energy stored in the inductor reaches half its maximum value at time :

(a) $\frac{R}{L}\ln\left[\frac{\sqrt{2}}{\sqrt{2}-1}\right]$ (b) $\frac{L}{R}\ln\left[\frac{\sqrt{2}-1}{\sqrt{2}}\right]$
(c) $\frac{L}{R}\ln\left[\frac{\sqrt{2}}{\sqrt{2}-1}\right]$ (d) $\frac{R}{L}\ln\left[\frac{\sqrt{2}-1}{\sqrt{2}}\right]$

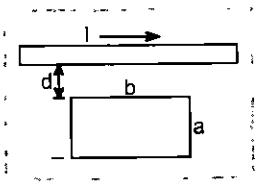
22. A long straight wire is parallel to one edge as in Fig. If the current in the long wire is varies in time as $I = I_0 e^{-t/\tau}$, what will be the induced emf in the loop?

(a) $\frac{\mu_0 bI}{\mu\tau}\ln\left(\frac{d+a}{d}\right)$

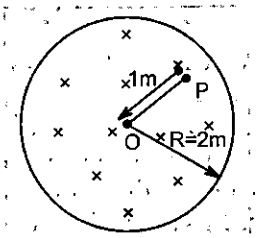
(b) $\frac{\mu_0 bI}{2\pi\tau}\ln\left(\frac{d+a}{d}\right)$

(c) $\frac{\mu_0 bI}{\pi\tau}\ln\left(\frac{d+a}{d}\right)$

(d) $\frac{\mu_0 bI}{\pi\tau}\ln\left(\frac{d}{d+a}\right)$



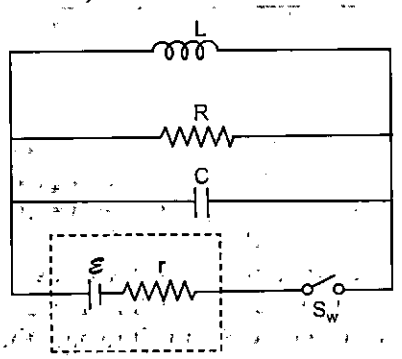
23. The magnetic field in a certain cylindrical region is changing with time according to the law $B = [16 - 4t^2]$ Tesla. The induced electric field at point P at time $t = 2$ sec.



(a) 8 Volt/m (b) 6 Volt/m

(c) 4 Volt/m (d) none of these

24. A pure inductor L, a capacitor C and a resistance R are connected across a battery of e.m.f. $\mathcal{E}$ and internal resistance r as shown in the figure. The switch S_w is closed at $t = 0$, select the correct alternative (s).



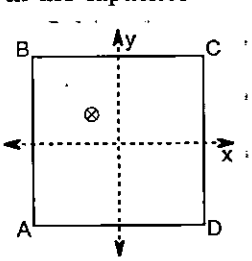
(a) current through resistance R is zero all the time

(b) current through resistance R is zero at $t = 0$ and $t \to \infty$

(c) maximum charge stored in the capacitor is CE

(d) maximum energy stored in the inductor is equal to the maximum energy stored in the capacitor

25. A square coil $ABCD$ is placed in x-y plane with its centre at origin. A along straight wire, passing through origin, carries a current in negative z-direction. Current in the wire increases with time. The induced current in the coil is:



(a) clockwise (b) anticlockwise

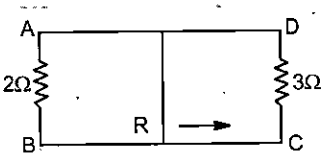
(c) zero (d) alternating

26. The dimensions of permeability of free space can be given by:

(a) $[MLT^{-2}A^{-2}]$ (b) $[MLA^{-2}]$

(c) $[ML^{-3}T^2A^2]$ (d) $[MLA^{-1}]$

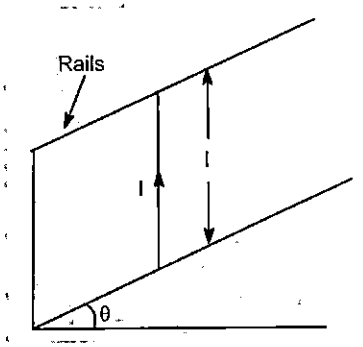
27. ---------- uniform magnetic field perpendicular to plane of loop. The magnetic induction is 0.1 tesla and resistance of connector (R) is 1 ohm. The sides AB and CD have resistance 2 ohm and 3 ohm respectively. Find the current in the connector during its motion with constant velocity one metre/sec.



(a) $\frac{1}{110}$A (b) $\frac{1}{220}$A

(b) $\frac{1}{55}$A (d) $\frac{1}{440}$A

28. A conducting wire of length l and mass m is placed on two inclined rails as shown in figure. A current I is flowing in the wire in the direction shown. When no magnetic field is present in the region, the wire is just on the verge of sliding. When a vertically upward magnetic field is switched on, the wire starts moving up the incline. The distance traveled by the wire as a function of time t will be :



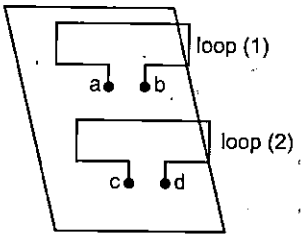
(a) $\frac{1}{2}\left[\frac{IBl}{m} - 2g\right]t^2$

(b) $\frac{1}{2}\left[\frac{IBl}{m}\times\frac{1}{\cos\theta} - 2g\sin\theta\right]t^2$

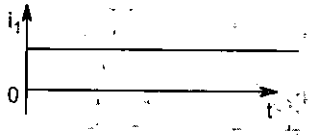
(c) $\frac{1}{2}\left[\frac{IBl}{m} - 2g\sin\theta\right]t^2$

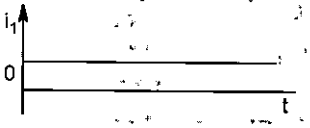
(d) $\frac{1}{2}\left[\frac{IBl}{m}\frac{\cos 2\theta}{\cos\theta} - 2g\sin\theta\right]t^2$

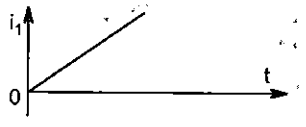
29. An electric current i_1 can flow either direction through loop (1) and induced current i_2 in loop (2). Positive i_1 is when current is from 'a' to 'b' in loop (1) and positive i_2 is when the current is from 'c' to 'd' in loop (2). In an experiment, the graph of i_2 against time 't' is as shown below;

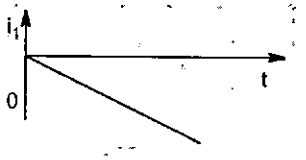


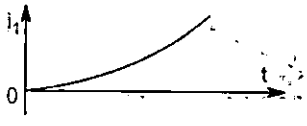
Which one(s) of the following graphs could have caused i_2 to behave as give above:

(a)


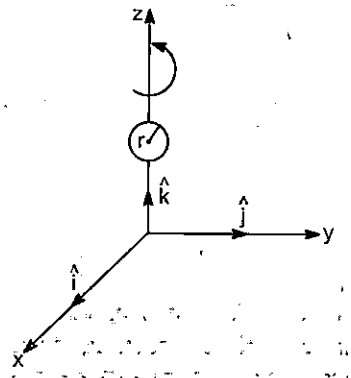
(b)


(c)


(d)


(e)


30. A circular loop wire of radius r rotates about the z-axis with angular velocity ω. The normal to the loop is always perpendicular to the z-axis. At time $t = 0$, the normal is parallel to the y-axis. An external magnetic field $\vec{\mathbf{B}} = B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}$ is applied. The e.m.f. $\mathcal{E}(t)$ induced in the loop is :



(a) $\pi r^2\omega B_y \sin\omega t$ (b) $\pi r^2\omega B_z \cos\omega t$

(c) $\pi r^2\omega B_z \sin\omega t$ (d) $\pi r^2\omega B_y \cos\omega t$

31. In the arrangement shown in give figure current from A to B is increasing in magnitude. Induced current in the loop will:

(a) have clockwise direction
(b) have anticlockwise direction
(c) be zero
(d) oscillate between clockwise and anticlockwise

32. A flexible wire loop in the shape of a circle has a radius that grows linearly with time. There is a magnetic field perpendicular to the plane of the loop that has a magnitude inversely proportional to the distance from the centre of the loop, $B(r) \propto \frac{1}{r}$. How does the e.m.f. $\mathcal{E}$ vary with time ?

(a) $\mathcal{E} \propto t^2$ (b) $\mathcal{E} \propto t$

(c) $\mathcal{E} \propto \sqrt{t}$ (d) $\mathcal{E}$ is constant

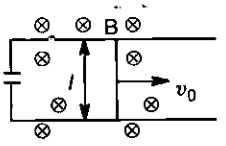
33. Two identical coaxial circular loops carry a current i each circulating in the same direction. If the loops approach each other:

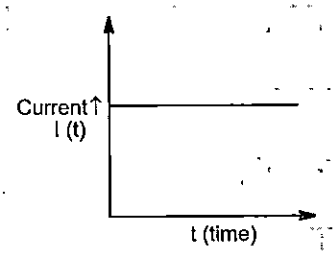
(a) the current in each will decrease
(b) the current in each will increase
(c) the current in each will remain the same
(d) the current in one will increase and in other will decrease

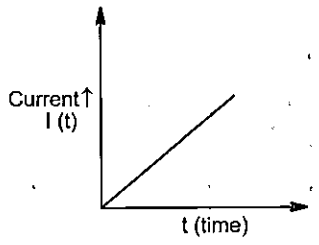
34. The resistance of a circular coil is 50 turns & 10 cm diameter is 5Ω What must be the potential difference across the ends of the coil so as to nullify the horizontal component of the earth's magnetic filed [$B_H = \pi \times 10^{-5}$ T] at the centre of the coil ? How should the coil be placed to achieve this result ?

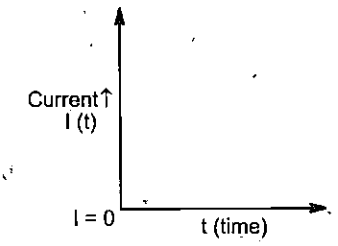
(a) 0.5V with plane of coil normal to the magnetic meridian
(b) 0.5V with plane of coil in the magnetic meridian
(c) 0.25V with plane of coil normal to the magnetic meridian
(d) 0.25V with plane of the coil in the magnetic meridian

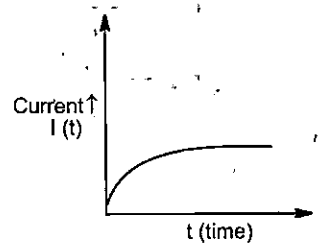
35. Two infinitely long conducting parallel rails are connected through a capacitor C as shown in the figure. A conductor of length l is moved with constant speed v_0. Which of the following graph truly depicts the variation of current through the conductor with time?



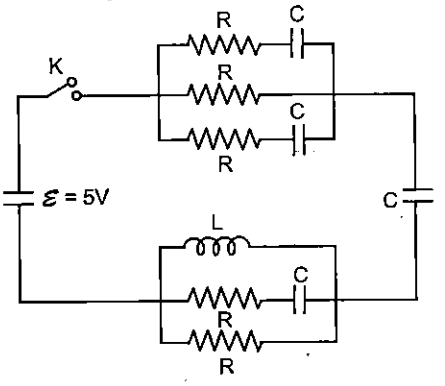
(a)


(b)


(c)


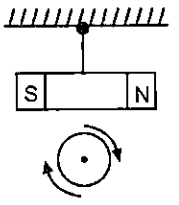
(d)


36. The current passing through the battery immediately after key (K) is closed [it is given that initially all the capacitors are uncharged (given that $R = 6\Omega$ and $C = 4\mu$ F)] is :



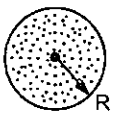
(a) 1A (b) 5 A
(c) 3 A (d) 2 A

37. A negative charge is given to a non-conducting loop and the loop is rotated in the plane of paper about its centre as shown in figure. The magnetic field produced by the ring affects a small magnet placed above the ring in the same plane:



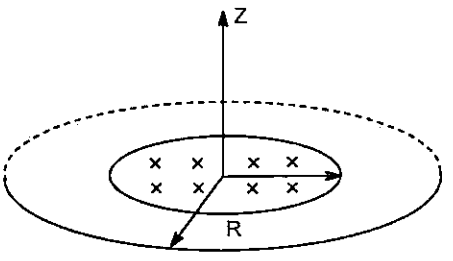
(a) the magnet does not rotate
(b) the magnet rotates clockwise as seen from below
(c) the magnet rotates anticlockwise as seen from below
(d) no effect on magnet is there

38. In the diagram shown a time varying non uniform magnetic field passes through a circular region of radius R. The magnetic field is directed outwards and it is a function of radial distance 'r' and time 't' according to the relation $B = B_0 rt$. What is the induced electric field strength at a radial distance $R/2$ from the center ?



(a) $B_0 R^2/12$ (b) $B_0 R^2/6$
(c) $2B_0 R^2/3$ (d) $B_0 R^2/16$

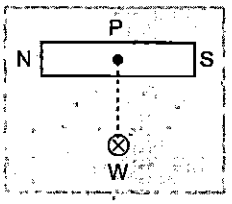
39. A line charge λ per unit length is pasted uniformly on to the rim of a wheel of mass m and radius R. The wheel has light non-conducting spokes and is free to rotate about a vertical axis as shown in figure. A uniform magnetic field extends over a radial region given by $B = -B_0 \vec{k}\,(r \le a; a < R) = 0$ (otherwise). What is the angular velocity of the wheel when this field is suddenly switched off ?



(a) $\frac{-2B\pi a^2 r}{mR}\hat{k}$ (b) $\frac{-B\pi a^2 r}{3mR}\hat{k}$

(c) $\frac{-B\pi a^2 \lambda}{2mR}\hat{k}$ (d) $\frac{-B\pi a^2 \lambda}{2mR}\hat{k}$

40. Figure shows a bar magnet and a long straight wire W, carrying current into the plane of paper. Point P is the point of intersection of axis of magnet and the line of shortest distance between magnet and the wire. If P is the mid-point of the magnet, then which of the following statements is correct?



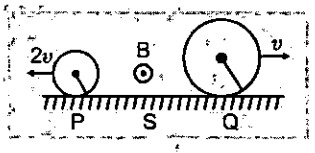
(a) magnet experiences a torque in the clockwise direction

(b) magnet experiences a torque in anticlockwise direction

(c) magnet experiences a force, normal to the line of shortest distance

(d) magnet experiences a force along the line of shortest distance

41. Two conducting rings P and Q of radii r and $2r$ rotate uniformly in opposite directions with centre of mass velocities $2v$ and v respectively on a conducting surface S. There is a uniform magnetic field of magnitude B perpendicular to the plane of the rings. The potential difference between the highest points of the two rings is:



(a) zero (b) $4Bvr$

(c) $8Bvr$ (d) $16Bvr$

42. A long metal bar of 30 cm length is aligned along a north south line and moves eastward at a speed of 10 ms^{-1}. A uniform magnetic field of 4.0 T points vertically downwards. If the south end of the bar has a potential of 0 V, the induced potential at the north end of the bar is:

(a) + 12 V

(b) – 12 V

(c) 0 V

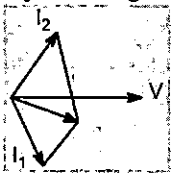
(d) cannot be determined since there is not closed circuit

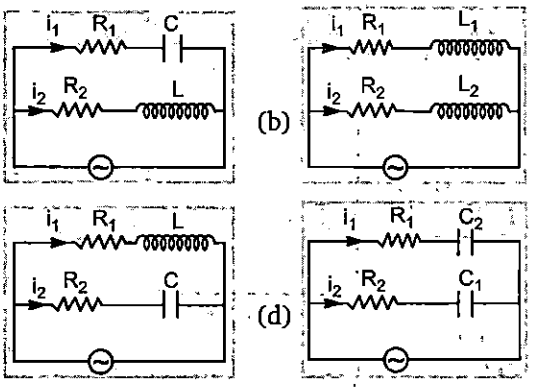
43. The magnetic field in a region is given by $\vec{B} = B_0\left(1+\frac{x}{a}\right)\hat{k}$. A square loop of edge length d is placed with its edge along the x- and y-axes. The loop is moved with a constant velocity $\vec{v} = v_0\hat{i}$. The e.m.f. induced in the loop is :

(a) $\frac{v_0 B_0 d^2}{a}$ (b) $\frac{v_0 B_0 d^3}{a^2}$

(c) $v_0 B_0 d$ (d) zero

44. Select the correct circuit-diagram for a phasor diagram as shows in the adjacent Fig.





45. A thin wire of length 2 m is perpendicular to the x-y plane. It is moved with velocity $\vec{v} = (2\hat{i} + 3\hat{j} + \hat{k})$ m/s through a region of magnetic induction $\vec{B} = (\hat{i} + 2\hat{j})$ Wb / m^2. Then potential difference induced between the ends of the wire:

(a) 2 volts (b) 4 volts

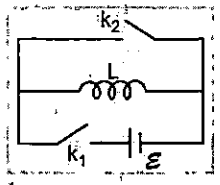
(c) 0 volts (d) none of these

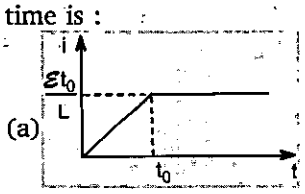
46. The magnetic flux density B is changing in magnitude at a constant rate dB/dt. A given mass m of copper, drawn into a wire of radius a and formed into a circular loop of radius r is placed perpendicular to the field B. The induced current in the loop is i. The resistivity of copper is r and density is d. The value of the induced current i is :

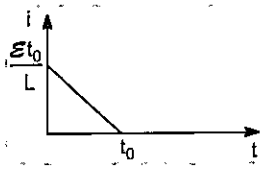
(a) $\frac{m}{2\pi\rho d}\frac{dB}{dt}$ (b) $\frac{m}{4\pi a^2 r}\frac{dB}{dt}$

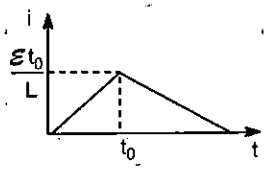
(c) $\frac{m}{4\pi a d}\frac{dB}{dt}$ (d) $\frac{m}{4\pi\rho d}\frac{dB}{dt}$

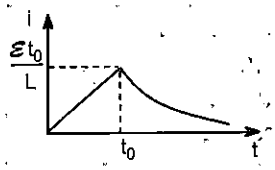
47. In the circuit shown, switch k_2 is open and switch k_1 is closed at $t = 0$. At time $t = t_0$, switch k_1 is opened and switch k_2 is simultaneously closed. The variation of inductor current with time is :



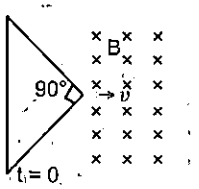


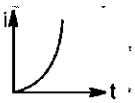
(b) 

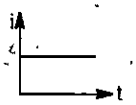
(c) 

(d) 

48. The figure shows an isosceles triangle wire frame with apex angle equal to $\pi/2$. The frame starts entering into the region of uniform magnetic field B with constant velocity v at $t = 0$. The longest side of the frame is perpendicular to the direction of velocity. If i is the instantaneous current through the frame then choose the alternative showing the correct variation of i with time:



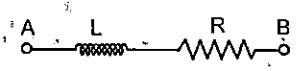
(a) 

(b) 

(c)

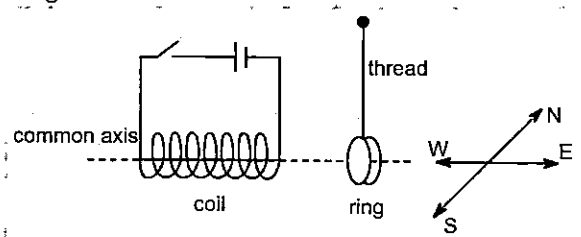
(d)

49. In an A.C. sub-circuit (See in figure), the resistance $R = 0.2\Omega$. At a certain instant $V_A - V_B = 0.5V$, $I = 0.5A$, $\frac{\Delta I}{\Delta t} = 8A/s$. The inductance of the coil is:



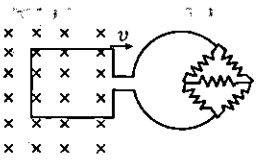
(a) 0.01H (b) 0.02H
(c) 0.5H (d) 0.5H

50. An aluminium ring hangs vertically from a thread with its axes pointing east-west. A coil is fixed near to the ring and coaxial with it :



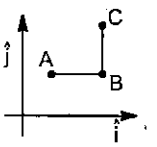
What is the initial motion of the aluminium ring when the current in the coil is switched on ?
(a) moves towards E (b) moves towards W
(c) moves towards N (d) moves towards S

51. A square metal loop of side 10 cm and resistance 1Ω is moved with a constant velocity partly inside a magnetic field of 2 Wbm^{-2}, directed into the paper, as shown in the figure. This loop is connected to a network of five resistors each of value 3Ω. If a steady current of 1 mA flows in the loop, then the speed of the loop is:



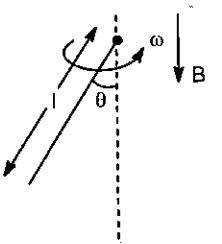
(a) 0.5 cms^{-1} (b) 1 cms^{-1}
(c) 2 cms^{-1} (d) 4 cms^{-1}

52. There is a uniform magnetic field B normal to the xy plane. A conductor ABC has length $AB = l_1$, parallel to the x-axis, and length $BC = l_2$, parallel to the y-axis. ABC moves in the xy-plane with velocity $v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}}$. The potential difference between A and C is proportional to:



(a) $v_x l_1 + v_y l_2$ (b) $v_x l_2 + v_y l_1$
(c) $v_x l_2 - v_y l_1$ (d) $v_x l_1 - v_y l_2$

53. A rod of length l rotates in the form of a conical pendulum with an angular velocity ω about its axis as shown in figure. The rod makes an angle θ with the axis. The magnitude of the motional e.m.f. developed across the two ends of the rod is :



(a) $\frac{1}{2}B\omega l^2$ (b) $\frac{1}{2}B\omega l^2 \tan^2\theta$
(c) $\frac{1}{2}B\omega l^2 \cos^2\theta$ (d) $\frac{1}{2}B\omega l^2 \sin^2\theta$

54. Two parallel long straight conductors lie on a smooth surface. Two other parallel conductors rest on them at right angles so as to form a square of side a initially. A uniform magnetic field B exists at right angles to the plane containing the conductors. They all start moving out with a constant velocity v. If r is the resistance per unit length of the wire the current in the circuit will be:

(a) $\frac{Bv}{r}$ (b) $\frac{Br}{v}$
(c) Bvr (d) Bv

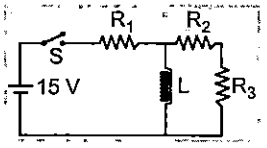
55. A metallic rod of length L and mass M is moving under the action of two unequal forces F_1 and F_2 (directed opposite to each other) acting at is length. Ignore gravity and any external magnetic field. If specific charge of electrons is (e/m), then the potential

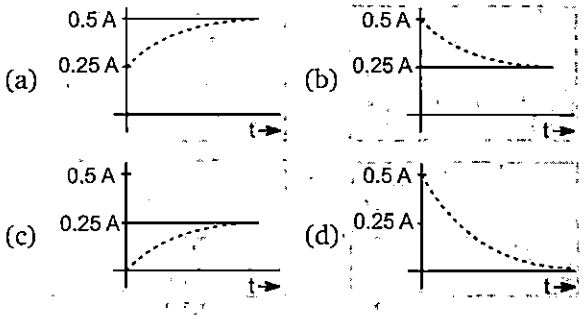
difference between the ends of the rod is steady state must be:

(a) $|F_1 - F_2| mL/eM$ (b) $(F_1 - F_2) mL/eM$

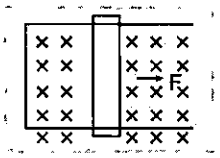
(c) $[mL/eM] \ln [F_1/F_2]$ (d) None of these

56. The figure shows a battery with emf 15V in a circuit with $R_1 = 30\Omega$, $R_2 = 10\Omega$, $R_3 = 20\Omega$ and $L = 3.0H$. The switch S is initially in the open position and is then closed at time $t = 0$. Then the graph which shows the correct variation of current through battery after switch S is closed





57. A rod closing the circuit shown in figure moves along a U shaped wire at a constant speed v under the action of the force F. The circuit is in a uniform magnetic field perpendicular to the plane. Find F if the rate of heat of heat generation in the circuits is Q :



(a) $F = Qv$ (b) $F = \frac{Q}{v}$

(c) $F = \frac{v}{Q}$ (d) $F = \sqrt{Qv}$

58. A square loop of side a and resistance R is moved in the region in the region of uniform magnetic field B(loop remaining completely inside field), with a velocity v through a distance x. The work done is:

(a) $\frac{Bl^2vx}{R}$ (b) $\frac{2B^2l^2vx}{R}$

(c) $\frac{4Bl^2vx}{R}$ (d) none of these

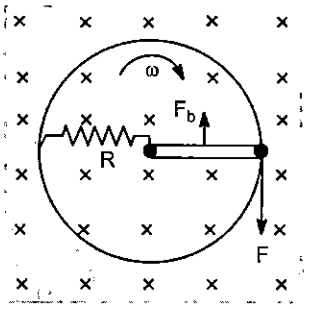
59. A metallic ring of mass m and radius r with a uniform metallic spoke of same mass m and length r is rotated about its axis with angular velocity ω in a perpendicular uniform magnetic field B as shown. The central end of the spokes is connected to the rim of the wheel through a resistor R as shown. The resistor does not rotate, its one end is always at the center of the ring and the other end is always in contact with the ring. A force F as shown is needed to maintain constant angular velocity of the wheel. F is equal to (the ring and the spoke has zero resistance) :

(a) $\frac{B^2\omega r^2}{8R}$

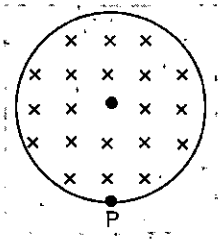
(b) $\frac{B^2\omega r^2}{2R}$

(c) $\frac{B^2\omega r^3}{2R}$

(d) $\frac{B^2\omega r^3}{4R}$



60. A uniform magnetic field of induction B is confined to cylindrical region of radius R. The magnetic field is increasing at a constant rate of $\frac{dB}{dt}$ (Ts^{-1}). An electron of charge q, placed at the point P on the periphery of the field, experiences an acceleration:



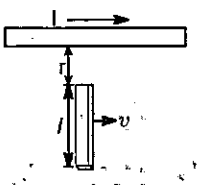
(a) $\frac{1}{2}\frac{eR}{m}\frac{dB}{dt}$ towards left

(b) $\frac{1}{2}\frac{eR}{m}\frac{dB}{dt}$ towards right

(c) $\frac{eR}{m}\frac{dB}{dt}$ towards left

(d) zero

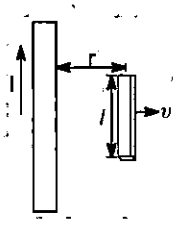
61. A conducting rod of length l moves with velocity v a direction parallel to a long wire carrying a steady current I. The axis of the rod is maintained perpendicular to the wire with near end a distance r away as shown in the figure. Find the emf induced in the rod:



(a) $\frac{\mu_0 Iv}{\pi}\ln\left(\frac{r+l}{r}\right)$ (b) $\frac{2\mu_0 Iv}{\pi}\ln\left(\frac{r+l}{r}\right)$

(c) $\frac{\mu_0 Iv}{\pi}\ln\left(\frac{l}{r+l}\right)$ (d) $\frac{\mu_0 Iv}{2\pi}\ln\left(\frac{r+l}{r}\right)$

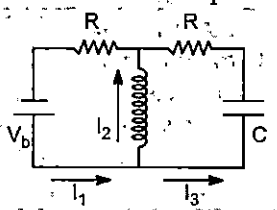
62. A conducting rod moves with constant velocity v perpendicular to the long, straight wire carrying a current I as shown compute that the emf generated between the ends of the rod:



(a) $\frac{\mu_0 vIl}{\pi r}$ (b) $\frac{\mu_0 vIl}{2\pi r}$

(c) $\frac{2\mu_0 vIl}{\pi r}$ (d) $\frac{\mu_0 vIl}{4\pi r}$

63. Given the circuit above, which of these choices is a possible system of equations representing the complete Kirchoff's law description of the circuit ?



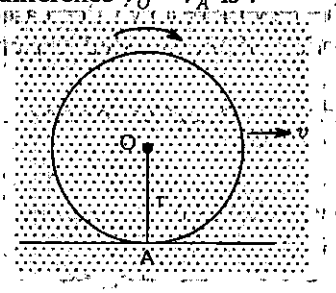
(a) $V_b = I_1 R$, $Q = CV_b$, $V_b = -L\frac{dI_2}{dt}$

(b) $V_b - L\frac{dI_2}{dt} - RI_1 = 0$, $-\frac{Q}{C} - R(I_1 - I_2) + L\frac{dI_2}{dt} = 0$, $I_3 = I_1 - I_2$

(c) $V_b - L\frac{dI_2}{dt} - RI_2 = 0$, $-QC - R(I_1 + I_2) + L\frac{dI_2}{dt} = 0$

(d) $V_b - L\frac{dI_2}{dt} + RI_1 = 0$, $-\frac{Q}{C} - RI_3 - L\frac{dI_2}{dt} = 0$, $I_1 = I_2 + I_3$

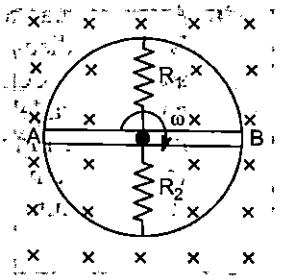
64. A conducting ring of radius r with a conducting spoke is in pure rolling on a horizontal surface in a region having a uniform magnetic field B as shown, n being the velocity of the centre of the ring. Then the potential difference $V_O - V_A$ is :



(a) $\frac{Bvr}{2}$ (b) $\frac{3Bvr}{2}$

(c) $\frac{-3Bvr}{2}$ (d) $\frac{Bvr}{2}$

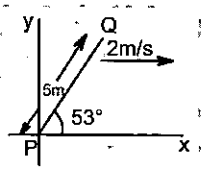
65. AB is a resistanceless conducting rod which forms a diameter of a conducting ring of radius r rotating in a uniform magnetic field B as shown. The resistors R_1 and R_2 do not rotate. Then current through the resistor R_1 is :



(a) $\frac{B\omega r^2}{2R_1}$ (b) $\frac{B\omega r^2}{2R_2}$

(c) $\frac{B\omega r^2}{2R_1R_2}(R_1+R_2)$ (d) $\frac{B\omega r^2}{2(R_1+R_2)}$

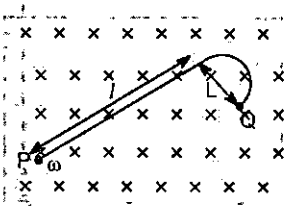
66. A conducting rod PQ of length 5 m oriented as shown in figure is moving with velocity $(2\text{m/s})\hat{\mathbf{i}}$ without any rotation in a uniform magnetic field $(3\hat{\mathbf{j}} + 4\hat{\mathbf{k}})$ Tesla. Emf induced in the rod is:



(a) 32 volts (b) 40 volt

(c) 50 volts (d) none of these

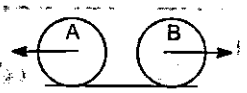
67. When a 'J' shaped conducting rod is rotating in its own plane with constant angular velocity ω about one of its and P, in a uniform magnetic field $\vec{\mathbf{B}}$ directed normally into the plane of paper) then magnitude of emf induced across it will be:



(a) $B\omega\sqrt{L^2 + l^2}$ (b) $\frac{1}{2}B\omega L^2$

(c) $\frac{1}{2}B\omega(L^2 + l^2)$ (d) $\frac{1}{2}B\omega l^2$

68. Two identical conducting rings A and B of radius R are in pure rolling over two horizontal conduting planes with same speed (of centre of mass) v but in opposite direction. A constant magnetic field B is present pointing into the plane of paper. Then, the potential difference between the highest points of the two rings is :



(a) zero (b) $2Bvr$

(c) $4Bvr$ (d) none of these

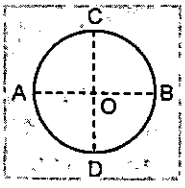
69. A semi-circular wire of radius R rotates with constant angular frequency ω in its own plane about one end in a uniform magnetic field of induction B perpendicular to its length. The e.m.f. induced across the ends is :

(a) $\frac{B\omega R^2}{2}$ (b) $2B\omega R^2$

(c) $2B\omega R$ (d) $B\omega R$

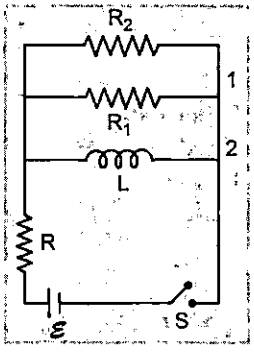
70. A rectangular coil of wire rotates about an axis which is perpendicular to a uniform magnetic field at a steady rate. Consider the instant when the plane of the coil is parallel to the magnetic field lines. At that instant the induced electromotive force is :
(a) minimum
(b) maximum
(c) zero
(d) constant, the same at all times.

71. A vertical conducting ring of radius R falls vertically with a speed v in a horizontal uniform magnetic field B. which is perpendicular to the plane of the ring ?



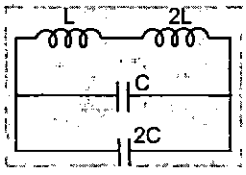
(a) A and B are the same potential
(b) C and D are at the same potential
(c) current flows in clockwise direction
(d) current flows in anticlockwise direction

72. Switch S is closed for a long time at $t = 0$. If it is opened, then :



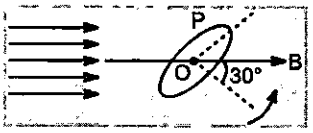
(a) total heat produced in resistor R after opening the switch is $\frac{1}{2}\frac{LV}{R^2}$
(b) total heat produced in resistor R_1 after opening the switch is $\frac{1}{2}\frac{LV^2}{R^2}\left(\frac{R_1}{R_1+R_2}\right)$
(c) heat produced in resistor R_1 after opening the switch is $\frac{1}{2}\frac{R_2LV^2}{(R_1+R_2)R^2}$
(d) no heat will be produced in R_1

73. The frequency of oscillation of current in the inductance is :



(a) $\frac{1}{3\sqrt{LC}}$ (b) $\frac{1}{6\pi\sqrt{LC}}$
(c) $\frac{1}{\sqrt{LC}}$ (d) $\frac{1}{2\pi\sqrt{LC}}$

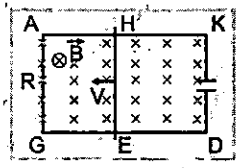
74. A plane circular coil P of area A is placed in a uniform magnetic induction field B such that the normal to the coil plane makes an angle of 30° with the direction of the field. The coil is now rotated in the field by "wobbling" such that the normal always makes angle 30° with the field during rotation, the angular velocity of rotation being ω. Then the e.m.f. induced in the coil will be :



(a) ωAB (b) $\omega BA\cos\theta$
(c) $\omega BA\sin\theta$ (d) zero

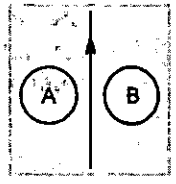
75. The magnetic field in a region is given by $\vec{B} = B_0\left(1+\frac{x}{a}\right)\hat{k}$. A square loop of edge length d is placed with its edge along x and y axis. The loop is moved with constant velocity $\vec{v} = v_0\hat{i}$. The emf induced in the loop is:
(a) $\frac{v_0B_0d^2}{a}$ (b) $\frac{v_0B_0d^2}{2a}$
(c) $\frac{v_0B_0a^2}{d}$ (d)None of these

76. In the circuit shown in figure, a conducting wire HE is moved with a constant speed V towards left. The complete circuit is placed in a uniform magnetic field $\vec{B}$ perpendicular to the plane of the circuit directed in inward direction. The current in $HKDE$ is:



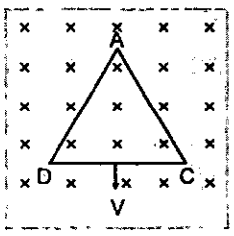
(a) clockwise (b) anticlockwise
(c) alternating (d) zero

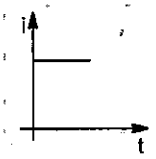
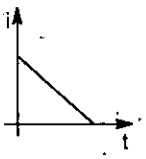
77. A and B are two metallic rings placed at opposite sides of an infinitely long straight conducting wire as shown. If current in the wire is slowly decreased, the direction of the induced current will be :

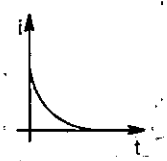
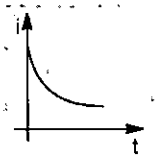


(a) clockwise in A and anticlockwise in B
(b) anticlockwise in A and clockwise in B
(c) clockwise in both A and B
(d) anticlockwise in both A and B

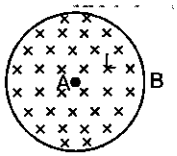
78. An equilateral triangular loop ADC of some finite magnetic field $\vec{B}$ as shown in the figure. At time $t = 0$, side DC of loop is at edge of the magnetic field. Magnetic field is perpendicular to the paper inwards (or perpendicular to the plane of the coil). The induced current *versus* time graph will be as:



(a)  (b) 

(c)  (d) 

79. A copper rod AB of length L, pivoted at one end A, rotates at constant angular velocity ω, at right angles to a uniform magnetic field of induction B. The e.m.f., developed between the mid-point C of the rod and end B is:



(a) $\frac{B\omega l^2}{4}$ (b) $\frac{B\omega l^2}{2}$

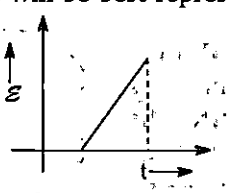
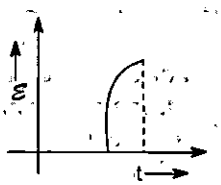
(C) $\frac{3B\omega l^2}{4}$ (d) $\frac{3B\omega l^2}{8}$

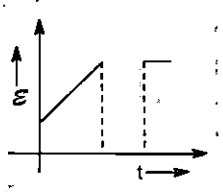
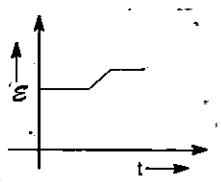
80. A rod of length l with uniformly distributed charge Q is rotated about one end with a constant frequency f. Its magnetic moment is :

(a) $\pi f Q l^2$ (b) $\frac{\pi f Q l^2}{3}$

(c) $\frac{2\pi f Q l^2}{3}$ (d) $2\pi f Q l^2$

81. Radius of a circular ring is changing with time and the coil is placed in a uniform magnetic field perpendicular to its plane. The variation of r with time t is shown in figure. Then the induced e.m.f. $\mathcal{E}$ with time will be best represented by :

(a)  (b) 

(c)  (d) 

82. A ring of resistance 10Ω, radius 10 cm and 100 turns is rotated at a rate 100 revolutions per second about a fixed axis which is perpendicular to a uniform magnetic field of induction 10 mT. The amplitude of the current in the loop will be nearly (Take : $\pi^2 = 10$):

(a) 200 A (b) 2 A

(c) 0.002 A (d) none of these

83. For L-R circuit, the time constant is equal to:

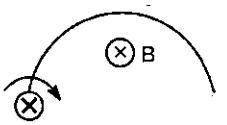
(a) twice the ratio of the energy stored in the magnetic field to the rate of dissipationof energy in the resistance

(b) ratio of the energy stored in the magnetic field to the rate of dissipation of energy in the resistance

(c) half the ratio of the energy stored in the magnetic field to the rate of dissipation of energy in the resistance

(d) square of the ratio of the energy stored in the magnetic field to the rate of dissipation of energy in the resistance

84. A semicircular wire of radius R is rotated with constant angular velocity about an axis passing through one end and perpendicular to the plane of the wire. There is a uniform magnetic field of strength B. The induced e.m.f. between the ends is :



(a) $B\omega R^2/2$ (b) $2B\omega R^2$

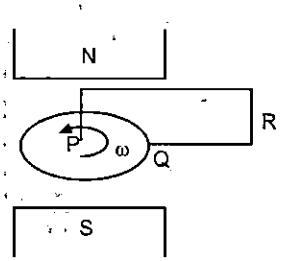
(c) is variable (d) none of these

85. A rectangular coil of single turn, having area A, rotates in a uniform magnetic field B an angular velocity ω about an axis perpendicular to the field. If initially the plane of coil is perpendicular to the field, then the average induced e.m.f., when it has rotated through 90° is:

(a) $\frac{\omega BA}{\pi}$ (b) $\frac{\omega BA}{2\pi}$

(c) $\frac{\omega BA}{4\pi}$ (d) $\frac{2\omega BA}{\pi}$

86. A metal disc rotates freely, between the poles of a magnet in the direction indicated. Brushes P and Q makes contact with the edge of the disc and the metal axle. What current, if any, flows through R?



(a) A current from P to Q

(b) a current from Q to P

(c) no current, because the emf in the disc is opposed by the back emf

(d) no current,because the emf induced in one side of the disc is opposed by the emf induced in the other side.

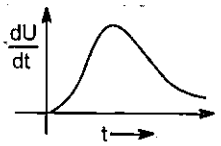
(e) no current, because no radial emf is induced in the disc

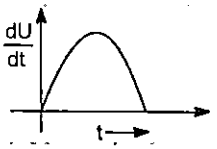
87. Resonance occurs in a series L-C-R circuit when the frequency of the applied e.m.f. is 1000 Hz. Then :

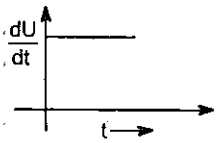
(a) when $f = 900$ Hz, the circuit behaves as a capacitive circuit

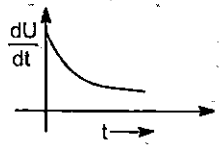
(b) the impedance of the circuit is maximum at $f = 1000\,\text{Hz}$
(c) at resonance the voltages across L and current in C differ in phase by 180°
(d) if the value of C is doubled resonance occurs at $f = 2000\,\text{Hz}$.

88. Rate of increment of energy in an inductor with time in series LR circuit getting charge with a battery of e.m.f. of $\mathcal{E}$ is best represented by (inductor has initially zero current :

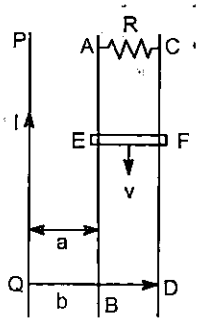
(a) 

(b) 

(c) 

(d) 

89. PQ is an infinite current-carrying conductor. AB and CD are smooth conducting rods on which a conductor EF moves with constant velocity v as shown. The force needed to maintain constant speed of EF is :



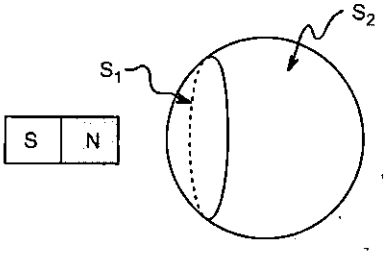
(a) $\frac{1}{vR}\left[\frac{\mu_0 Iv}{2\pi}\ln\left(\frac{b}{a}\right)\right]^2$

(b) $\left[\frac{\mu_0 Iv}{2\pi}\ln\left(\frac{b}{a}\right)\right]^2\frac{1}{vR}$

(c) $\left[\frac{\mu_0 Iv}{2\pi}\ln\left(\frac{b}{a}\right)\right]^2\frac{v}{R}$

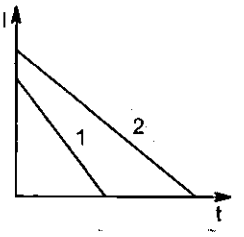
(d) $\frac{V}{R}\left[\frac{\mu_0 Iv}{2\pi}\ln\left(\frac{b}{a}\right)\right]^2$

90. A spherical surface is split into two parts, S_1 and S_2, by a circular loop. The sphere is placed near a bar magnet, as shown below.



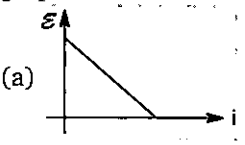
Through which of the two parts is the magnitude of the magnetic flux larger ?
(a) S_1
(b) S_2
(c) The magnitude of the flux is the same for both
(d) Cannot tell without more information about magnetic field

91. Two identical inductance currents that vary with time according to linear laws (as shown in figure). In which of two inductance is the self induction emf greater?



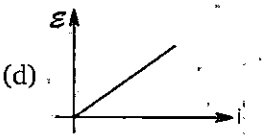
(a) 1
(b) 2
(c) same
(d) data are insufficient to decide

92. In an L-R circuit connected to a battery of constant e.m.f., $\mathcal{E}$ switch S is closed at time $t = 0$. If $\mathcal{E}$ denotes the magnitude of induced e.m.f., across inductor and i the current at any time t. Then which of the following graphs shows the variation of $\mathcal{E}$ with i?

(a) 

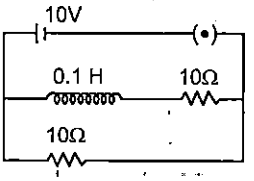
(b)

(c)

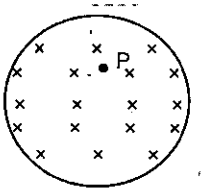
(d) 

93. In the adjoining circuit, initially the switch S is open. The switch 'S' is closed at $t = 0$. The difference between the maximum and minimum current that can flow in the circuit is:

(a) 2 Amp
(b) 3 Amp
(c) 1 Amp
(d) nothing can be concluded

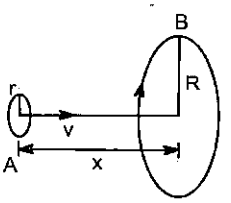


94. Fig. shows a uniform magnetic field B confined to a cylindrical volume and is increasing at a constant rate. The instantaneous acceleration experienced by an electron placed at P is:



(a) zero
(b) towards right
(c) towards left
(d) upwards

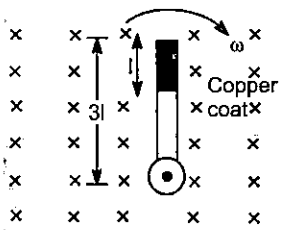
95. Loop A of radius $r << R$ moves towards loop B with a constant velocity v in such a way that their planes are always parallel. What is the distance between the two loops (x) when the induced e.m.f. in loop A is maximum ?



(a) R (b) $\frac{R}{\sqrt{2}}$

(c) $\frac{R}{2}$ (d) $R\left(1-\frac{1}{\sqrt{2}}\right)$

96. A wooden stick of length $3l$ is rotated about an end with constant angular velocity ω in a uniform magnetic field B perpendicular to the plane of motion. If the upper 1/3 of its length is coated with copper, the potential difference across the whole length of the stick is :



(a) $\frac{9B\omega l^2}{2}$ (b) $\frac{4B\omega l^2}{2}$

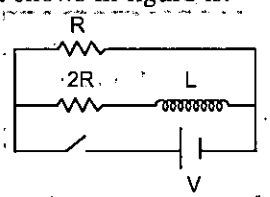
(c) $\frac{5B\omega l^2}{2}$ (d) $\frac{B\omega l^2}{2}$

97. A current of 2A is increasing at a rate of 4 A/s through a coil of inductance $2H$. The energy stored in the inductor per unit time is:

(a) 2 J/s (b) 1 J/s

(c) 16 J/s (d) 4 J/s

98. The ratio of time constant in charging and discharging in the circuit shows in figure is:



(a) 1 : 1 (b) 2 : 2

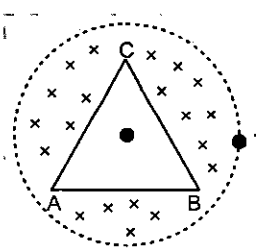
(c) 2 : 3 (d) 1 : 3

99. A gold rod of length l is accelerated in the horizontal direction with an acceleration a_0. The rod is held between two perfectly insulating clamps. Calculate the electric field set up in the rod. Take the mass of electron as m.

(a) $E=\frac{ma_0}{e}$ (b) $E=ma_0l$

(c) zero (d) none of these

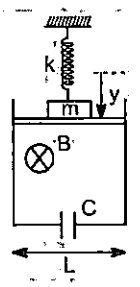
100. A triangular wire frame (each side = 2m) is placed in a region of time variant magnetic field having $dB/dt = \sqrt{3}$ T/s . The magnetic field is perpendicular to the plane of the triangle. The base of the triangle AB has a resistance 1Ω while the other two sides have resistance 2Ω each. The magnitude of potential difference between the points A and B will be:



(a) 0.4 V (b) 0.6 V

(c) 1.2 V (d) None of these

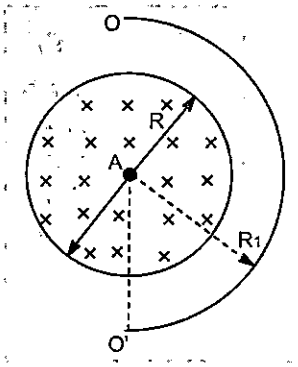
101. A block is attached to the ceiling by a spring that has a force constant $k = 200$N/m. A conducting rod is rigidly attached to the block. The combined mass of the block and the rod is $m = 0.3$kg. The rod can slide without friction along two vertical parallel rails, which are a distance $L = 1$m apart. A capacitor of known capacitance $C = 500\mu$m is attached to the rails by the wires. The entire system is placed in a uniform magnetic field $B = 20$T directed as shown. Find the angular frequency (in red/sec) of the vertical oscillations of the block. Neglect the self-inductance and electrical resistance of the rod and all wires.



(a) $20\sqrt{5}$ (b) 20

(c) $\frac{20\sqrt{5}}{3}$ (d) none of these

102. There is a uniform magnetic field B in circular region of radius R as shown in figure, whose magnitude changes at the rate of dB/dt. The e.m.f. induced across the ends of a circular concentric conducting arc of radius R_1 having an angle θ as shown($\angle OAO' = \theta$) is :



(a) $\frac{\theta}{2\pi}R_1^2\frac{dB}{dt}$ (b) $\frac{\theta}{2}R^2\frac{dB}{dt}$

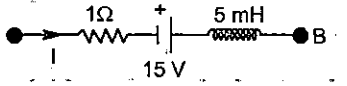
(c) $\frac{\theta}{2\pi}R^2\frac{dB}{dt}$ (d) none of these

103. A square conducting loop of side L is situated in gravity-free space. A small conducting circular loop of radius r ($r << L$) is placed at the centre of the square loop, with its plane perpendicular to the plane of the square loop. The mutual inductance of the two coils is:

(a) $\frac{2\sqrt{2}\mu_0 I}{L}r^2$ (b) $\frac{\sqrt{2}\mu_0 I_0}{L}r^2$

(c) zero (d) none of these

104. The network shown in the figure is part of a complete circuit. If at a certain instant, the current I is 5A and it is decreasing at a rate of 10^3 As^{-1} then $V_B - V_A$ equals:



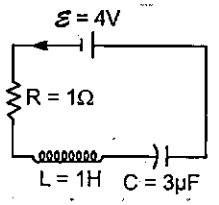
(a) 20 V (b) 15 V

(c) 10 V (d) 5 V

105. In Problem 104, if I is reversed in direction, then $V_B - V_A$ equals:
(a) 5 V (b) 10 V
(c) 15 V (d) 20 V

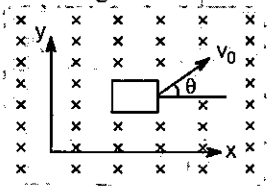
106. A long solenoid of N turns has a self inductance L and area of cross section A. When a current I flows through the solenoid, the magnetic field inside it has magnitude B. The current i is equal to:
(a) BAN/L (b) $BANL$
(c) BN/AL (d) D/ANL

107. The current in the given circuit is increasing with a rate $a = 4$ amp/s. The charge on the capacitor at an instant when the current in the circuit is 2 amp will be:



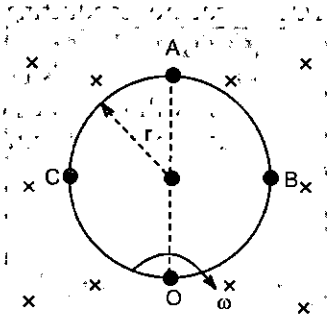
(a) 4μC (b) 5μC
(c) 6μC (d) none of these

108. In the space shown a non-uniform magnetic field $\vec{B} = B_0(1+x)\hat{k}$ tesla is present. A closed loop of small resistance, placed in the xy-plane is given velocity v_0. The force due to magnetic field on the loop is :



(a) zero
(b) along $+x$ direction
(c) along $-x$ direction
(d) along $+y$ direction

109. In figure, there is a conducting ring having resistance R placed in the plane of paper in a uniform magnetic field B_0. If the ring is rotating about in the plane of paper about an axis passing through point O and perpendicular to the plane of paper with constant angular speed ω in clockwise direction, then:

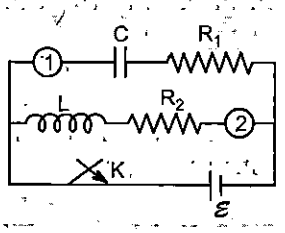


(a) point A will be at higher potential than O
(b) the potential of point B and C will be same
(c) the current in the ring will be zero
(d) the current in the ring will be $\frac{2B_0\omega r^2}{R}$

110. L, C and R represent physical quantities inductance, capacitance and resistance. The combination which has the dimensions of frequency is:
(a) $\frac{1}{RC}$ and $\frac{R}{L}$ (b) $\frac{1}{\sqrt{RC}}$ and $\sqrt{\frac{R}{L}}$
(c) $\sqrt{LC}$ (d) $\frac{C}{L}$

111. A long straight wire of circular cross-section is made of a non-magnetic material. The wire is of radius a. The wire carries a current I which is uniformly distributed over its cross-section. The energy stored per unit length in the magnetic field contained within the wire is:
(a) $U = \frac{\mu_0 I^2}{8\pi}$ (b) $U = \frac{\mu_0 I^2}{16\pi}$
(c) $U = \frac{\mu_0 I^2}{4\pi}$ (d) $U = \frac{\mu_0 I^2}{2\pi}$

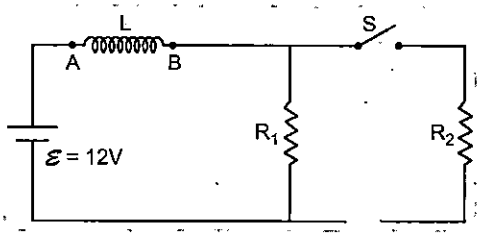
112. In the circuit of figure, (1) and (2) are ammeters. Just after key K is pressed to complete the circuit, the reading is :



(a) maximum in both 1 and 2
(b) zero in both 1 and 2
(c) zero in 1, minimum in 2
(d) maximum in 1, zero in 2

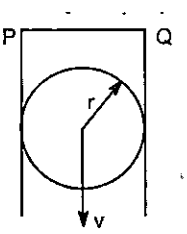
113. A coil of inductance 5H is joined to a cell of emf 6V through a resistance 10Ω at time $t = 0$. The emf across the coil at time $t = \ln \sqrt{2}$ s is:
(a) 3 V (b) 1.5 V
(c) 0.75 V (d) 4.5 V

114. The circuit shown has been operating for a long time. The instant after the switch in the circuit labeled S is opened, what is the voltage across the inductor V_L and which labeled point (A or B) of the inductor is at a higher potential ? Take $R_1 = 4.0\Omega$, $R_2 = 8.0\Omega$ and $L = 2.5H$.



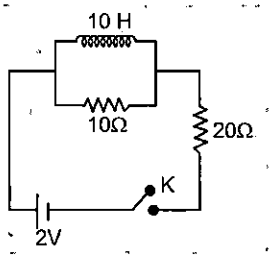
(a) $V_L = 12$V; Point A is at the higher potential
(b) $V_L = 12$V; Point B is at the higher potential
(c) $V_L = 6$V; Point A is at the higher potential
(d) $V_L = 6$V; Point B is at the higher potential

115. A vertical ring of radius r and resistance R slips vertically between two frictionless and resistanceless vertical rails (see figure). The rails are joined at the top. There is a uniform magnetic field B perpendicular to the plane of ring and the rails. When the speed of the ring is v, the current in the section PQ is :



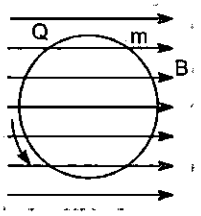
(a) zero (b) $2Brv/R$
(c) $4Brv/R$ (d) $8Brv/R$

116. Two resistors of 10Ω and 20Ω and an ideal inductor of 10H are connected to a 2 V battery as shown. The key K is inserted at time $t = 0$. The initial ($t = 0$) and final ($t \to \infty$) currents through battery are:



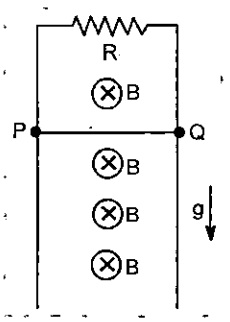
(a) $\frac{1}{15}$A, $\frac{1}{10}$A (b) $\frac{1}{10}$A, $\frac{1}{15}$A
(c) $\frac{2}{15}$A, $\frac{1}{10}$A (d) $\frac{1}{15}$A, $\frac{2}{25}$A

117. A thin uniform ring of mass m and electric charge Q uniformly distributed rotates around an axis perpendicular to its plane and going through its center. The angular momentum of the ring is 7.5×10^{-4} kg- m^2/s. The ring is in a homogeneous magnetic field of a field strength of $0.1T$ and the lines of the magnetic induction are parallel with the plane of the ring. Torque exerted on the ring is ? [The specific charge (charge-mass ratio)of the ring is $Q/m = 10^{-5}$ C/kg].



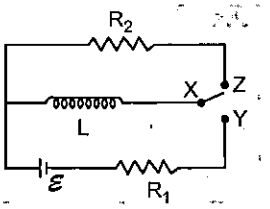
(a) 3.75×10^{-10}(N - m) upward along the plane
(b) 3.75×10^{-10}(N - m) downward along the plane
(c) 7.5×10^{-10}(N - m) upward along the plane
(d) none of these

118. A rod PQ of mass m and length l can slide without friction on two vertical conducting semi-infinite rails. It is given a velocity v_0 downwards, so that it continues to move downwards with the same speed v_0 on its own at any later instant of time. Assuming g to be constant everywhere, the value of v_0 is :



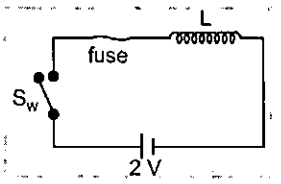
(a) $\frac{mgR}{2B^2L^2}$ (b) $\frac{mgR}{B^2L^2}$
(c) 0 (d) None of these

119. In the circuit shown, X is joined to Y for a long time, and then X is joined to Z. The total heat produced in R_2 is:



(a) $\frac{L\mathcal{E}^2}{2R_1^2}$ (b) $\frac{L\mathcal{E}^2}{2R_2^2}$
(c) $\frac{L\mathcal{E}^2}{2R_1R_2}$ (d) $\frac{L\mathcal{E}^2R_2}{2R_1^2}$

120. In the circuit shown, the cell is ideal. The coil has an inductance of 4H and zero resistance. F is a fuse of zero resistance and will blow when the current through it reaches 5A. The switch is closed at $t = 0$. The fuse will blow:

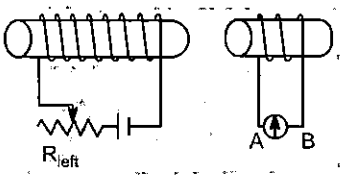


(a) just after $t = 0$ (b) after 2 s
(c) after 5 s (d) after 10 s

121. A small coil of radius r is placed at the centre of a large coil of radius R, where $R >> r$. The coils are coplanar. The coefficient of mutual inductance between the coils is:

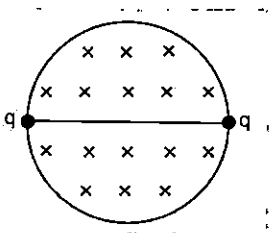
(a) $\frac{\mu_0\pi r}{2R}$ (b) $\frac{\mu_0\pi r^2}{2R}$
(c) $\frac{\mu_0\pi r^2}{2R^2}$ (d) $\frac{\mu_0\pi r}{2R^2}$

122. For the solenoids shown in the diagram (which are assumed to be close to each other), the resistance of the left-hand circuit is slowly increased. In which direction does the current flow through galvanometer in the right-hand circuit ?



(a) B to A
(b) A to B
(c) there is no current
(d) cannot be determined

123. A cylindrical region of uniform magnetic field exists perpendicular to the plane of paper which is incresing at a constant rate $\frac{dB}{dt} = \alpha$. The diameter of the cylindrical region is l. A non-conducting rigid rod of length l having two charged particles is kept fixed on the diameter of

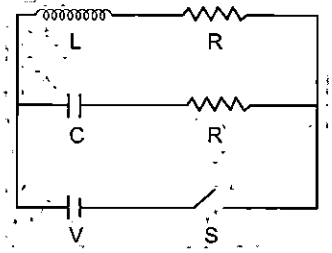


cylindrical region w.r.t. inertial frame. If two charged particles having charges θ each is kept fixed at the ends of the non-conducting rod. The net force on any one of the charge q is :

(a) $\frac{ql\alpha}{4}$ (b) $\frac{ql\alpha}{2}$

(c) zero (d) $ql\alpha$

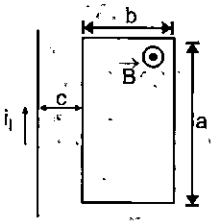
124. In the circuit shown in the figure, $R = \sqrt{\frac{L}{C}}$. Switch S is closed at time $t = 0$. The current through C and L would be equal after a time t equal to:



(a) CR (b) $CR \ln(2)$

(c) $\frac{L}{R\ln(2)}$ (d) LR

125. The mutual inductance between the rectangular-loop and the long straight wire as shown in figure is M.



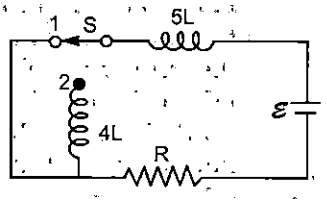
(a) M = Zero

(b) $M = \frac{\mu_0 a}{2\pi}\ln\left(1+\frac{c}{b}\right)$

(c) $M = \frac{\mu_0 b}{2\pi}\ln\left(\frac{a+c}{b}\right)$

(d) $M = \frac{\mu_0 a}{2\pi}\ln\left(1+\frac{b}{c}\right)$

126. In the circuit shown in figure, the switch S is shifted to position 2 from position 1 at $t = 0$, having been in position 1 for a long time. The current in the circuit just after shifting of switch will be it (battery and both the inductors are ideal)



(a) $\frac{4}{5}\frac{\mathcal{E}}{R}$ (b) $\frac{5}{4}\frac{\mathcal{E}}{R}$

(c) $\frac{5}{9}\frac{\mathcal{E}}{R}$ (d) $\frac{\mathcal{E}}{R}$

127. A solenoid of inductance L and resistance r is connected in parallel to a resistance R. A battery of e.m.f. $\mathcal{E}$ and of negligible internal resistance r is connected across this parallel combination. At $t = 0$ switch S is opened:

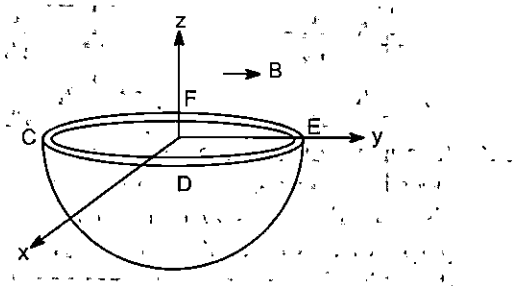
(a) Current in the inductor just after opening of the switch is $= \frac{\mathcal{E}(r+R)}{rR}$

(b) Total energy dissipated in the solenoid and the resistor long time after opening of the switch is $\frac{1}{2}L\frac{\mathcal{E}^2(R+r)^2}{r^2R^2}$

(c) The amount of heat generated in the solenoid due to the opening of the switch is $\frac{\mathcal{E}^2 L}{2r(r+R)}$

(d) The amount of heat generated in the solenoid due to the opening of the switch is $\frac{\mathcal{E}^2 L}{2R(r+R)}$

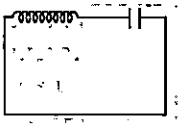
128. A rigid circular massless frame carrying current I_0 is fixed to a uniform non-conducting hemispherical shell of mass m and the system lies in a gravity free space. The wire from lies in x-y plane and a uniform magnetic field is switched on along y-axis. The angular acceleration of the shell at this instant is :



(a) $\frac{12\pi BI}{5m}$ (b) $\frac{12\pi BI}{m}$

(c) $\frac{12\pi BI}{3m}$ (d) none of above

129. In an L-C circuit, the capacitor has maximum charge q_0. The value of $\left|\frac{di}{dt}\right|_{max}$ is:



(a) $\frac{q_0}{LC}$ (b) $\frac{q_0}{\sqrt{LC}}$

(c) $\frac{q_0}{LC} - 1$ (d) none of these

130. In a L-R decay circuit, the initial current at $t = 0$ is I. The total charge that has flows through the resistor till the energy in the inductor has reduced to one-fourth its initial value, is:

(a) LI/R (b) $LI/2R$

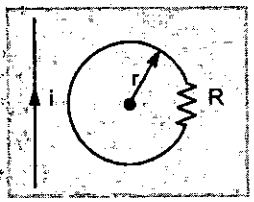
(c) $LI/\sqrt{2}R$ (d) None of these

131. A long straight wire is placed along the axis of a circular ring of radius R. The mutual inductance of this system is:

(a) $\frac{\mu_0 R}{2}$ (b) $\frac{\mu_0 \pi R}{2}$

(c) $\frac{\mu_0}{2}$ (d) zero

132. In figure, the mutual inductance of a coil and a very long straight wire is M, the coil has resistance R and the self inductance L. There is an infinite wire which lies in the same plane as that of the coil.

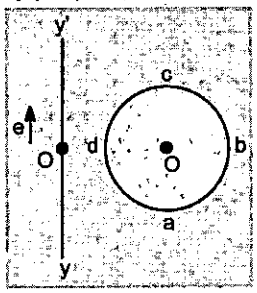


The current in the wire varies according to the law $i = at$, where a is a constant and t is the time, the time dependence of current in the coil is :

(a) $\frac{M}{aR}$ (b) $MaRe^{-Rt/L}$

(c) $\frac{M}{R}e^{-tR/L}$ (d) $\frac{Ma}{R}(1-e^{-tR/L})$

133. An electron moves on a straight line path YY' as shown in figure. A coil is kept on the right such that YY' is in the plane of the coil. At the instant when the electron gets closest to the coil (neglect self-induction of the coil):



(a) the current in the coil flows clockwise
(b) the current in the coil flows anticlockwise
(c) the current in the coil is zero
(d) the current in the coil does not change the direction as the electron crosses point O

134. An inductor coil stores U energy when i current is passed through it and dissipates energy at the rate of P. The time constant of the circuit, when this coil is connected across a battery of zero internal resistance, is:

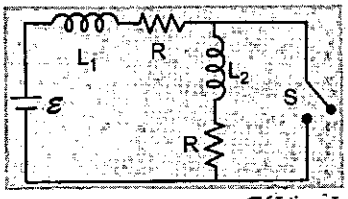
(a) $\frac{4U}{P}$ (b) $\frac{U}{P}$

(c) $\frac{2U}{P}$ (d) $\frac{2P}{U}$

135. A capacitor of capacitance 2μF is charged to a potential difference of 12 V. It is then connected across an inductor of inductance 0.6 mH. The current in the circuit when the potential difference across the capacitor is 6 V is:

(a) 3.6 A (b) 2.4 A
(c) 1.2 A (d) 0.6 A

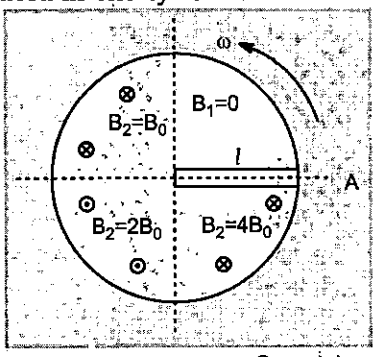
136. The switch S shown in figure is closed for $t < 0$ and is opened at $t = 0$. When currents through L_1 and L_2 are first equal, their common value is :



(a) $\frac{\mathcal{E}}{R}$ (b) $\frac{\mathcal{E}(L_2+L_1)}{RL_1}$

(c) $\frac{\mathcal{E}L_1}{R(L_1+L_2)}$ (d) $\frac{\mathcal{E}}{R}\frac{(L_1+L_2)}{L_2}$

137. A thin conducting rod OA of length l is rotated about end O with constant angular frequency ω. Then magnetic field in the quadrants are $B_1 = 0$; $B_2 = B_0$; $B_3 = 2B_0$; $B_4 = 4B_0$. B_2 and B_4 are into the plane of the paper whereas B_3 is out of the plane. The average emf induced in one cycle is :



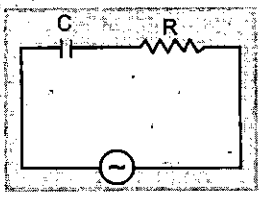
(a) $\frac{13}{16}\omega Bl^2$ (b) $\frac{3}{8}\omega Bl^2$

(c) $\frac{1}{6}\omega Bl^2$ (d) $\frac{3}{4}\omega Bl^2$

138. Left $F = 50$ Hz, and $C = 100$ μF in an AC circuit containing a capacitor only. If the peak value of the current in the circuit is 1.57 A at $t = 0$. The expression for the instantaneous vlotage across the capacitor will be:

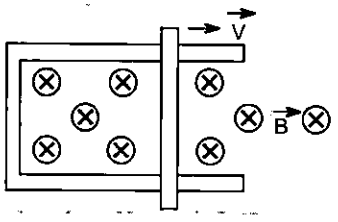
(a) $\mathcal{E} = 50\sin(100\pi t - \pi/2)$
(b) $\mathcal{E} = 100\sin(50\pi t)$
(c) $\mathcal{E} = 50\sin 100\pi t$
(d) $\mathcal{E} = 50\sin(100\pi t + \pi/2)$

139. In the circuit shown if the emf of source at an instant is 5V, the potential difference across capacitor at the same instant is 4V. The potential difference across R at that instant may be:



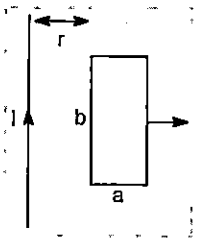
(a) 3V (b) 9V
(c) $\frac{3}{\sqrt{2}}$V (d) none of these

140. A rod lies across frictionless rails in a uniform magnetic field $\vec{B}$ as shown in figure. The rod moves to the right with speed v. In order for the induced e.m.f. in the circuit to be zero, the magnitude of the magnetic field should :



(a) not change
(b) increase linearly with time
(c) decrease linearly with time
(d) decrease non-linearly with time

141. A rectangular loop of wire with dimensions shown in figure is coplanar with a long wire carrying current I. The distance between the wire and the left side of the loop is r. The loop is pulled to the right as indicated. What are the directions of the induced current in the loop and the magnetic forces on the left and the right sides of the loop when the loop is pulled ?

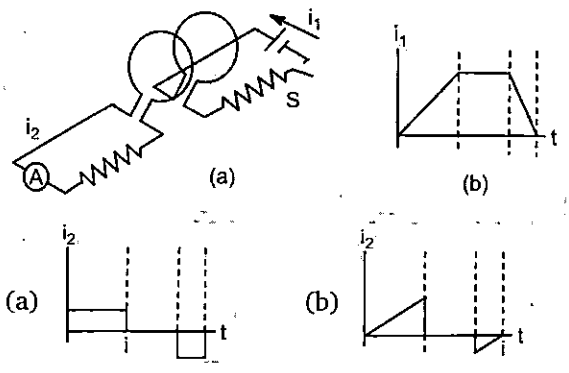


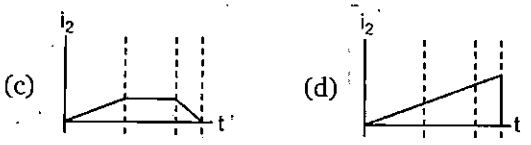
Induced current	Force on left side	Force on right side
(a) Counterclockwise	To the left	To the left
(b) Counterclockwise	To the right	To the left
(c) Clockwise	To the right	To the left
(d) Clockwise	To the left	To the right

142. The power in ac circuit is given by $P = \mathcal{E}_{rms} I_{rms} \cos\phi$ The value of $\cos\phi$ in series LCR circuit at resonance is:

(a) zero (b) 1
(c) $\frac{1}{2}$ (d) $\frac{1}{\sqrt{2}}$

143. The current through the coil in figure (a) varies as shown in figure (b). Which graph in best shows the ammeter A reading as a function of time ?

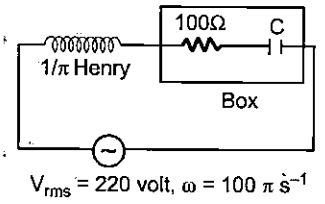




144. When 100 V DC is applied across a solenoid a current of 1 A flows in it. When 100 V AC is applied across the same coil, the current drops to 0.5 A. If the frequency of the AC source is 50 Hz, the impedance and inductance of the solenoid are:

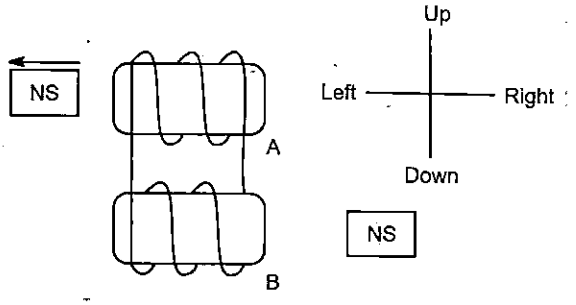
(a) 100 Ω, 0.93 H (b) 200 Ω, 1.0 H
(c) 10 Ω, 0.86 H (d) 200 Ω, 0.55 H

145. In the circuit, as shown in the figure, if the value of R.M.S current is 2.2 ampere, the power factor of the box is:



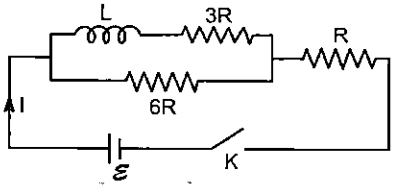
(a) $\frac{1}{\sqrt{2}}$ (b) 1
(c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{2}$

146. A bar magnet was pulled away from a hollow coil A as shown in figure. As the south pole came out of the coil, the bar magnet next to hollow coil B experienced a magnetic force :



(a) to the right (b) to the left
(c) upwards (d) equal to zero

147. In the given circuit diagram, the key K is switched on at $t = 0$. The ratio of the current I through the cell at $t = 0$ to that at $t = \infty$ will be :



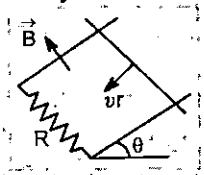
(a) 3 : 1 (b) 1 : 3
(c) 1 : 2 (d) 2 : 1

148. An alternating current $I = I_0 \sin \omega t$ is flowing in a circuit. The ratio of rms current in the interval of 0 to T the average current in the circuit for the time interval from $T/8$ to $3T/8$ (where T is time period) is :

(a) $\frac{\pi}{2\sqrt{2}}$ (b) $\frac{\pi}{2}$

(c) $\frac{\pi}{4}$ (d) $\frac{\sqrt{2}}{4}\pi$

149. A copper rod of mass m slides under gravity on two smooth parallel rails l distance aparted set at an angle θ to the horizontal (see figure). At the bottom, the rails are joined by a resistance R. There is a uniform magnetic field perpendicular to the plane of the rails. The terminal velocity of the rod is :



(a) $\frac{mgR\cos\theta}{B^2l^2}$ (b) $\frac{mgR\sin\theta}{B^2l^2}$

(c) $\frac{mgR\tan\theta}{B^2l^2}$ (d) $\frac{mgR\cot\theta}{B^2l^2}$

150. In A.C. circuit when A.C ammeter is connected it reads i current if a student uses D.C. ammeter in place of A.C. ammeter the reading in the D.C. ammeter will be:

(a) $\frac{i}{\sqrt{2}}$ (b) $\sqrt{2}i$

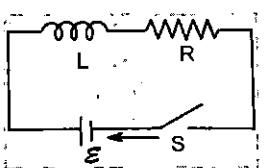
(c) $0.637i$ (d) zero

151. An inductive circuit contains resistance of 10Ω and an inductance of 2.0 H. If an AC voltage of 120 V and frequency 60 Hz is applied to this circuit, the current would be nearly:

(a) 0.8 A (b) 0.48 A

(c) 0.16 A (d) 0.32 A

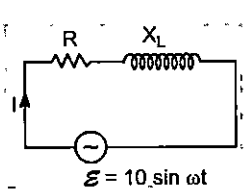
152. In the circuit shown in figure, switch S is closed at time $t = 0$. The charge that passes through the battery in one time constant is :



(a) $\frac{eR^2\mathcal{E}}{L}$ (b) $\mathcal{E}\left(\frac{L}{R}\right)$

(c) $\frac{\mathcal{E}L}{eR^2}$ (d) $\frac{eL}{\mathcal{E}R}$

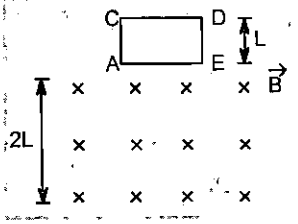
153. An A.C. circuit having supply voltage $\mathcal{E}$ consists of a resistor of resistance 3Ω and an inductor of reactance 4Ω as shown in the figure. The voltage across the inductor at $t = \mu / \omega$ is:



(a) 2 volts (b) 10 volts

(c) zero (d) 4.8 volts

154. A square coil $ACDE$ with its plane vertical is released from rest in a horizontal uniform magnetic field $\vec{B}$ of length $2L$ (see figure). The acceleration of the coil is :



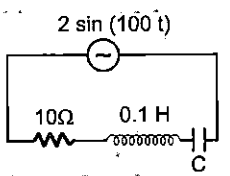
(a) less than g for all the time till the loop crosses the magnetic field completely

(b) less than g when it enters the field and greater than g when it comes out of the field

(c) g all the time

(d) less than g when it enters and comes out of the field but equal to g when it is within the field

155. The power factor of the circuit is $1/\sqrt{2}$. The capacitance of the circuit is equal to:



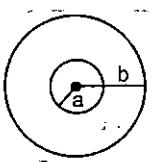
(a) 400 μF

(b) 300 μF

(c) 500 μF

(d) 200 μF

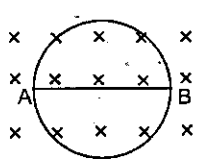
156. Two concentric and coplanar coils have radii a and b ($>> a$) as shown in figure. Resistance of the inner coil is R. Current in the outer coil is increased from 0 to i, then the total charge circulating the inner coil is :



(a) $\frac{\mu_0 ia^2}{2Rb}$ (b) $\frac{\mu_0 iab}{2R}$

(c) $\frac{\mu_0 iab}{2a}\frac{\pi b^2}{R}$ (d) $\frac{\mu_0 ib}{2\pi R}$

157. The radius of the circular conducting loop shown in figure is R. Magnetic field is decreasing at a constant rate α. Resistance per unit length of the loop is ρ.

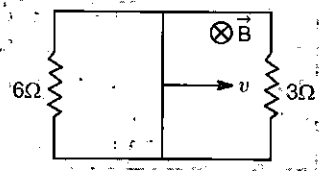


Then, the current in wire AB is (AB is one of the diameters):

(a) $\frac{R\alpha}{2\rho}$ from A to B (b) $\frac{R\alpha}{2\rho}$ from B to A

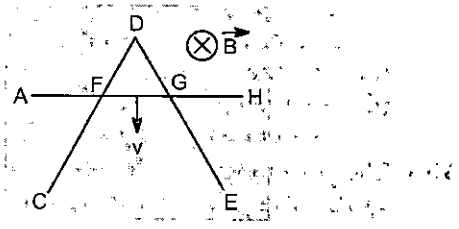
(c) $\frac{2R\alpha}{2\rho}$ from A to B (d) zero

158. A rectangular loop with a sliding connector of length $l = 1.0$ m is situated in a uniform magnetic field $B = 2$T perpendicular to the plane of loop. Resistance of connector is $r = 2\Omega$. Two resistances of 6 and 3Ω are connected as shown in figure. The external force required to keep the connector moving with a constant velocity $v = 2$m/s is :



(a) 6N (b) 4N
(c) 2N (d) 1N

159. A long conducting wire AH is moved over a conducting triangular wire CDE with a constant velocity v in a uniform magnetic field $\vec{B}$ directed into the plane of the paper. Resistance per unit length of each wire is r. Then :

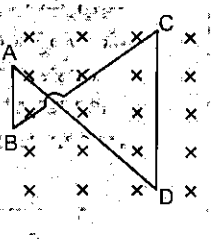


(a) a constant clockwise induced current will flow in the closed loop
(b) an increasing anticlockwise induced current will flow in the closed loop
(c) a decreasing anticlockwise induced current will flow in the closed loop
(d) a constant anticlockwise induced current will flow in the closed loop

160. A current i_0 is flowing through an $L-R$ circuit of time constant t_0. The source of the current is switched off at time $t = 0$. Let r be the value of $(-di/dt)$ at time $t = 0$. Assuming this rate to be constant, the current will reduce to zero in a time interval of :

(a) t_0 (b) et_0
(c) $\frac{t_0}{e}$ (d) $\left(1-\frac{1}{e}\right)t_0$

161. A conducting wire frame is placed in a magnetic field which is directed into the plane of the paper (see figure). The magnetic field is increasing at a constant rate. The directions of induced currents in wires AB and CD are :

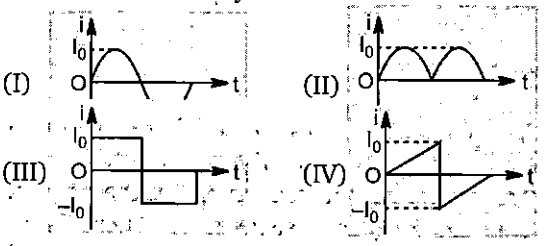


(a) B to A and D to C (b) A to B and C to D
(c) A to B and D to C (d) B to A and C to D

162. In case of induced electric field due to a time-dependent magnetic field in a cylindrical region, which of the statement is correct ?

(a) It obeys the Gauss' law for electric flux.
(b) The work done by induced electric field while moving a charged particle in a closed loop is zero.
(c) The induced electric field and time varying magnetic field are parallel each other at any instant of time
(d) Electrostatic potential can be defined for the induced electric field.

163. If I_1, I_2, I_3 and I_4 are the respective r.m.s. Values of the time varying current as shown in the four cases I, II, III and IV. Then identify the correct relations:

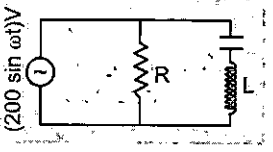


(a) $I_1 = I_2 = I_3 = I_4$
(b) $I_3 > I_1 = I_2 > I_4$
(c) $I_3 > I_4 > I_2 = I_1$
(d) $I_3 > I_2 > I_1 > I_4$

164. A metal rod of resistance 20Ω is fixed along a diameter of a conducting ring of radius 0.1m and lies on x-y plane. There is a magnetic field $\vec{B} = (50T)\hat{k}$. The ring rotates with an angular velocity $\omega = 20$ rad/s about its axis. An external resistance of 10Ω is connected across the centre of the ring and rim. The current through external resistance is :

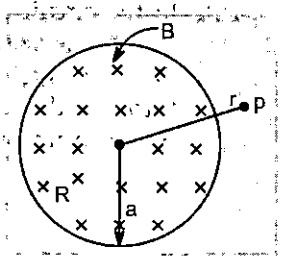
(a) $\frac{1}{4}$ (b) $\frac{1}{2}$
(c) $\frac{1}{3}$ (d) 0

165. In the circuit diagram shown, $X_C = 100\Omega$, $X_L = 200\Omega$ and $R = 100\Omega$. The effective current through the source is:



(a) 2 A (b) $\sqrt{2}$ A
(c) 0.5 A (d) $2\sqrt{2}$ A

166. A uniform but time-varying magnetic field $B(t)$ exists in a circular region of radius a and is directed into the plane of the paper as shown in figure. The magnitude of the induced electric field at point P at a distance from the centre of the circular region :



(a) is zero (b) decreases as $\frac{1}{r}$
(c) increases as r (d) decreases as $\frac{1}{r^2}$

167. The effective value of current $i = 2 \sin 100\pi t + 2\sin(100\pi T + 30°)$ is:

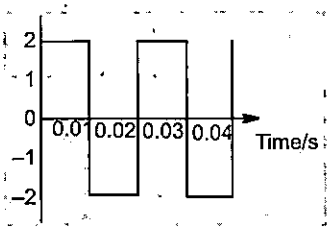
(a) $\sqrt{2}$ A (b) $2\sqrt{2+\sqrt{3}}$
(c) 4 (d) none of these

168. A metal disc of radius a rotates with a constant angular velocity ω about its axis. The potential difference between the centre and the rim of the disc is (m = mass of electron, e = charge on electron) :

(a) $\frac{m\omega^2a^2}{e}$ (b) $\frac{1}{2}\frac{m\omega^2a^2}{e}$

(c) $\frac{e\omega^2a^2}{2m}$ (d) $\frac{e\omega^2a^2}{m}$

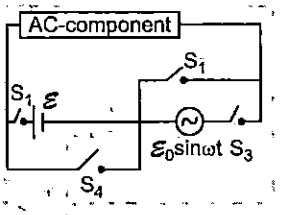
169. The direct current which would give the same heating effect in an equal constant resistance as the current shown in figure, *i.e.*, the r.m.s., current, is:



(a) zero (b) $\sqrt{2}$ A

(c) 2 A (d) $2\sqrt{2}$ A

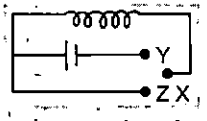
170. In the given circuit if switches S_3 and S_4 are open, keeping S_1 and S_2 closed, the value of I is I_0. If switches S_1 and S_2 are open, keeping S_3 and S_4 closed, the value of I is $I_0 \sin \omega t$. Now, if switches rms current through the AC component, during one complete :



(a) $I_0\left(1+\frac{1}{\sqrt{2}}\right)$ (b) $I_0\left(1-\frac{1}{\sqrt{2}}\right)$

(c) $\frac{I_0}{\sqrt{2}}$ (d) $\sqrt{\frac{3}{2}}I_0$

171. In the circuit shown in figure, the coil has inductance and resistance. When X is joined to Y, the time constant is t during the growth of current :



When the steady state is reached, heat is produced in the coil at a rate P. X is now joined to Z.

(a) The total produced in the coil is $P\tau$

(b) The total heat produced in the coil is $\frac{1}{2}P\tau$

(c) The total heat produced in the coil is $2P\tau$

(d) The data given are not sufficient to reach a conclusion

172. Power factor of an L-R series circuit is 0.6 and that of a C-R series circuit is 0.5. If the element (L, C and R) of the two circuits are joined in series the power factor of this circuit is found to be 1. The ratio of the resistance in the L-R circuit to the resistance in the C-R circuit is:

(a) 6/5 (b) 5/6

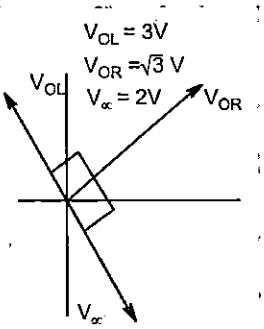
(c) $\frac{4}{3\sqrt{3}}$ (d) $\frac{3\sqrt{3}}{4}$

173. The length of a thin wire required to manufacture a solenoid of inductance L and length l, if the cross-sectional diameter is considered less than its length is :

(a) $\sqrt{\frac{4\pi lL}{\mu_0}}$ (b) $\sqrt{\frac{\pi lL}{\mu_0}}$

(c) $\sqrt{\frac{2\pi lL}{\mu_0}}$ (d) $\sqrt{\frac{4\pi lL}{3\mu_0}}$

174. The given figure represents the phasor diagram of a series L-C-R circuit connected to an A.C. source. At the instant 't' when the source voltage is given by $V = V_0 \cos \omega t'$, the current in the circuit will be:



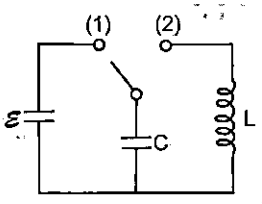
(a) $I = I_0 \cos(\omega t' + \pi/6)$

(b) $I = I_0 \cos(\omega t' - \pi/6)$

(c) $I = I_0 \cos(\omega t' + \pi/3)$

(d) $I = I_0 \cos(\omega t' - \pi/3)$

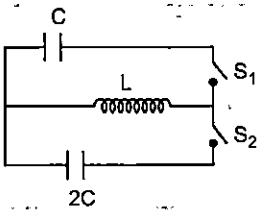
175. In the electrical network at $t < 0$ (see figure), key was placed on (1) till the capacitor got fully charged. Key is placed on (2) at $t = 0$. Time when the energy in both the capacitor and the inductor will be same for the first time is :



(a) $\frac{\pi}{4}\sqrt{LC}$ (b) $\frac{3\pi}{4}\sqrt{LC}$

(c) $\frac{\pi}{3}\sqrt{LC}$ (d) $\frac{2\pi}{3}\sqrt{LC}$

176. A capacitor of capacitance C is charged to potential V_0 and is connected in circuit as shown in the figure. Switch S_1 is closed at $t = 0$. After time $t = \frac{\pi\sqrt{LC}}{6}$, switch S_1 is opened while S_2 is closed. If initially both the switches were open and capacitor of capacitance $2C$ was uncharged, the maximum charge stored on $2C$ would be :



(a) $CV_0/\sqrt{2}$ (b) $CV_0/2$

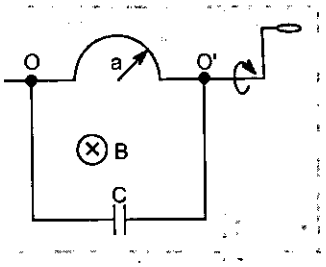
(c) $CV_0/4$ (d) none of these

177. The total heat produced in the resistor R in an RL circuit when the current in the inductor decreases from I_0 to 0 is :

(a) LI_0^2 (b) $\frac{1}{2}LI_0^2$

(c) $\frac{3}{2}LI_0^2$ (d) $\frac{1}{3}LI_0^2$

178. A copper rod is bent into a semi-circle of radius a and at ends straight parts are bent along diameter of the semi-circle and are passed through fixed, smooth and conducting ring O and O' as shown in figure. A capacitor having capacitance C is connected to the rings. The system is located in a uniform magnetic field of induction B such that axis of rotation OO' is perpendicular to the field direction. At initial moment of time ($t = 0$), plane of semi-circle was normal to the field direction and the semi-circle is set in rotation with constant angular velocity ω. Neglecting resistance and inductance of the circuit. The current flowing through the circuit in function of time is :



(a) $\frac{1}{4}\pi\omega^2 a^2 CB\cos\omega t$ (b) $\frac{1}{2}\pi\omega^2 a^2 CB\cos\omega t$

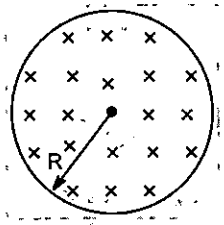
(c) $\frac{1}{4}\pi\omega^2 a^2 CB\sin\omega t$ (d) $\frac{1}{2}\pi\omega^2 a^2 CB\sin\omega t$

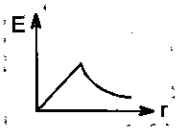
179. The phase difference between current and voltage in an AC circuit is $\pi/4$ radian. If the frequency of AC is 50 Hz, then the phase difference is equivalent to the time difference:

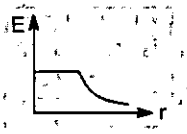
(a) 0.78 s (b) 15.7 ms

(c) 0.25 s (d) 2.5 ms

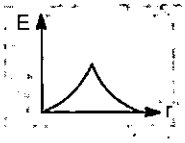
180. A cylindrical space of radius R is filled with a uniform magnetic induction B parallel to the axis of the cylinder. If B changes at a constant rate, the graph showing the variation of induced electric field with distance r from the axis of cylinder is :



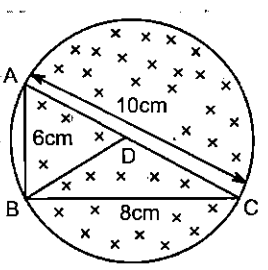
(a) 

(b) 

(c)

(d) 

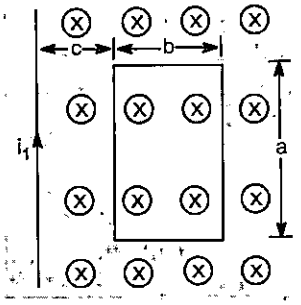
181. ABC is a triangular frame made out of a metallic wire. BD is a median of the triangular frame joining the vertex B to the side AC, (Also made of the same metallic wire). It is known that $AB = 6$cm, $BC = 8$cm, and $AC = 10$cm. The cross sectional area of the wire is $1\,mm^2$ and its resistivity is $24\,n\Omega m$. The triangle ABC lies in a cylindrical region of magnetic field such that the intersection of the surface of the cylinder with the plane containing the frame form the circumcircle of triangle ABC. The magnetic field in the region varies at the rate of 0.263 T/s. What is the magnitude of the induced current in mA in the median BD of the frame?



(a) 5 mA (b) 10 mA

(c) 15 mA (d) 20 mA

182. Figure shows a rectangular coil near a long wire. The mutual inductance of the combination is :



(a) $\frac{\mu_0 a}{2\pi}\ln\left(1-\frac{b}{c}\right)$ (b) $\frac{\mu_0 a}{2\pi}\ln\left(1+\frac{b}{c}\right)$

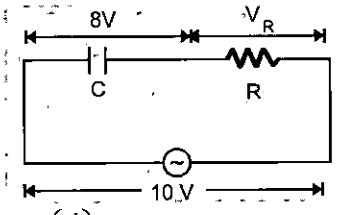
(c) $\frac{\mu_0 a}{\pi}\ln\left(1+\frac{b}{c}\right)$ (d) $\frac{\mu_0 a}{\sqrt{2}\pi}\ln\left(1+\frac{b}{c}\right)$

183. Charge Q is uniformly distributed on a thin insulating ring of mass m which is initially at rest. To what angular velocity will the ring be accelerated when a magnetic field B, perpendicular to the plane of the ring, is switched on ?

(a) $-\frac{QB}{2m}$ (b) $-\frac{3QB}{2m}$

(c) $-\frac{QB}{m}$ (d) $-\frac{QB}{4m}$

184. In a series C-R circuit shown in figure, the applied voltage is 10 V and the voltage across capacitor is found to be 8 V. Then the voltage across R, and the phase difference between current and the applied voltage will respectively be:



(a) 6V, $\tan^{-1}\left(\frac{4}{3}\right)$ (b) 3V, $\tan^{-1}\left(\frac{3}{4}\right)$

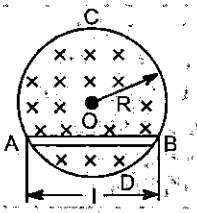
(c) 6V, $\tan^{-1}\left(\frac{5}{3}\right)$ (d) none of these

185. Find the inductance of a unit length of two parallel wires each of radius a whose centres are a distance d apart carry equal currents in opposite directions. Neglect the flux within the wire. The wires carry current in opposite directions:

(a) $\frac{\mu_0 l}{2\pi}\ln\left(\frac{d-a}{a}\right)$ (b) $\frac{\mu_0 l}{\pi}\ln\left(\frac{d-a}{a}\right)$

(c) $\frac{3\mu_0 l}{\pi}\ln\left(\frac{d-a}{a}\right)$ (d) $\frac{\mu_0 l}{3\pi}\ln\left(\frac{d-a}{a}\right)$

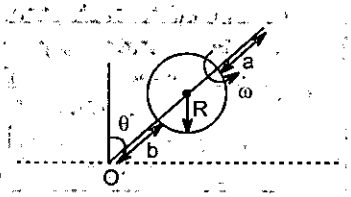
186. A uniform magnetic field of induction B fills a cylindrical volume of radius R. A rod AB of length $2l$ is placed as shown in figure. If B is changing at the rate dB/dt, the e.m.f. that is produced by the changing magnetic field and that acts between the ends of the rod is :



(a) $\frac{dB}{dt}l\sqrt{R^2-l^2}$ (b) $\frac{dB}{dt}l\sqrt{R^2+l^2}$

(c) $\frac{1}{2}\frac{dB}{dt}l\sqrt{R^2-l^2}$ (d) $\frac{1}{2}\frac{dB}{dt}\sqrt{R^2+l^2}$

187. A top of mass M is spinning with an angular velocity ω on a smooth horizontal surface about an axis OA, which makes an angle θ with the vertical as shown in the Fig. Modeling the top as a sphere of radius R, with two light nails of lengths a and b: then choose the correct statement :



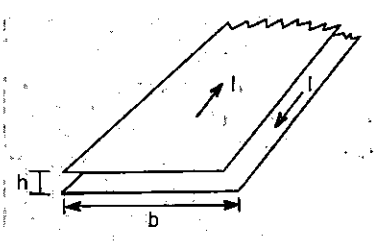
(a) Magnitude of its instantaneous rate of change of angular momentum about the contact point O with the ground is $Mg(R+b)\sin\theta$.

(b) The angular acceleration of the top must be zero.

(c) The angular acceleration of the top will have a constant direction as well as magnitude.

(d) The angular acceleration vector of the top is always directed vertically upwards.

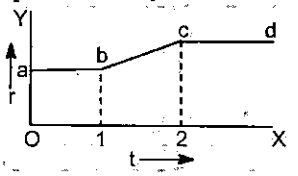
188. Calculate the inductance of a unit length of a double tape line as shown in figure. If the tapes are separated by a distance h which is considerably less than their width b:

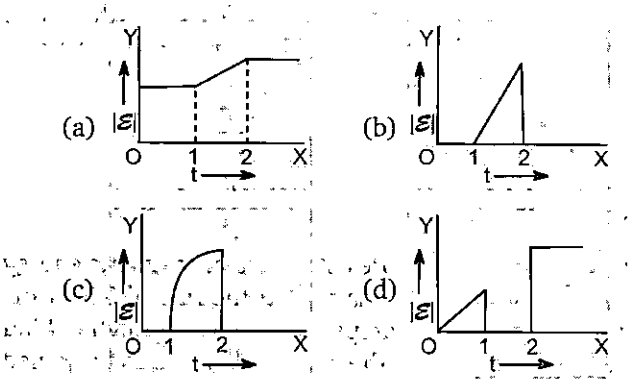


(a) $\frac{\mu_0 h}{b}$ (b) $\frac{\mu_0 h}{2b}$

(c) $\frac{2\mu_0 h}{b}$ (d) $\frac{\sqrt{2}\mu_0 h}{b}$

189. A flexible wire bent in the form of a circle is place in a uniform magnetic field perpendicularly to the plane of the coil. The radius of the coil changes as shown in figure. The graph of magnitude of induced e.m.f. in the coil is represented by





190. In series L-R circuit $X_L = 3R$. Now a capacitor with $X_C = R$ is added in series. Ratio of new to old power factor is:

(a) 1 (b) 2

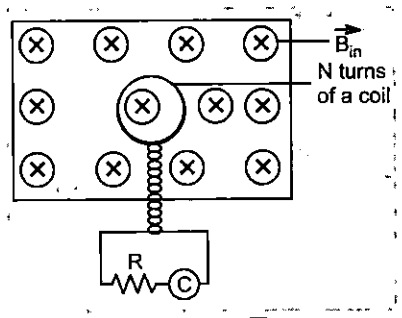
(c) $\frac{1}{\sqrt{2}}$ (d) $\sqrt{2}$

191. In a long straight solenoid with cross-sectional radius a and number of turns per unit length n, the current varies with the rate I A/s. The magnitude of the eddy current field strength as a function of distance r from the solenoid axis is :

(a) $\frac{1}{2}\frac{nIa^2}{\mu_0 r}$ (b) $\frac{1}{2}\frac{Ia^2}{\mu_0 r}$

(c) $\frac{nIa^2}{\mu_0 r}$ (d) $\frac{1}{2}\frac{\mu_0 nIa^2}{r}$

192. A flip coil consists of N turns of circular coils which lie in a uniform magnetic field. Plane of the coils is perpendicular to the magnetic field as shown in figure. The coil is connected to a current integrator which measures the total charge passing through it. The coil is turned through 180° about the diameter. The charge passing through the coil is:



(a) $\frac{NBA}{R}$ (b) $\frac{\sqrt{3}NBA}{2R}$

(c) $\frac{NBA}{\sqrt{2}R}$ (d) $\frac{2NBA}{R}$

193. The approximate formula expressing the formula inductance of two thin coaxial loops of the same radius a when their centres are separated by a distance l with $l >> a$ is :

(a) $\frac{1}{2}\frac{\mu_0 \pi a^4}{l^3}$ (b) $\frac{1}{2}\frac{\mu_0 a^4}{l^2}$

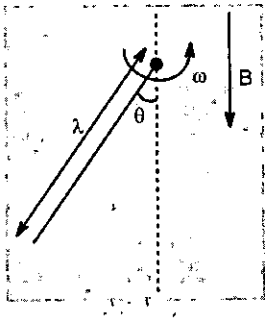
(c) $\frac{\mu_0}{4\pi}\frac{\pi a^4}{l^2}$ (d) $\frac{\mu_0}{\pi}\frac{a^4}{l^3}$

194. An induction coil stores 32 joules of magnetic energy and dissipates energy as heat at the rate of 320 watts. When a current of 4 amperes is passed through it. Find the time constant of the circuit when the coil is joined across a battery:

(a) 0.2 s (b) 0.1 s

(c) 0.3 s (d) 0.4 s

195. A rod of length λ rotates in the form of a conical pendulum with an angular velocity ω about its axis as shown in the Fig. The rod makes an angle θ with the axis. The magnitude of the motional emf developed across two ends of the rod is :



(a) $\frac{1}{2}B\omega l^2$ (b) $\frac{1}{2}B\omega l^2 \tan^2\theta$

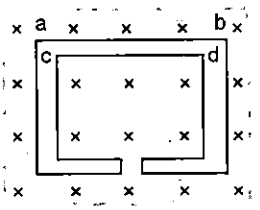
(c) $\frac{1}{2}B\omega l^2 \cos^2\theta$ (d) $\frac{1}{2}B\omega l^2 \sin^2\theta$

196. A toroid is along a coil of wire, wound over a circular core. The coefficient of self-inductance of the toroid is given by (radius = r), when the magnetic field is within it is uniform and $R >> r$:

(a) $L = \frac{\mu_0 N r^2}{2R}$ (b) $L = \frac{\mu_0 N r}{2R}$

(c) $L = \frac{\mu_0 N r^2}{R}$ (d) $L = \frac{\mu_0 N^2 r^2}{2R}$

197. A wire is bent to form the double loop shown in figure. There is a uniform magnetic field directed into the plane of the loop. If the magnitude of this field is decreasing, the current will flow from:



(a) a to b and c to d (b) b to a and d to c

(c) a to b and d to c (d) b to a and c to d

198. A series LCR circuit with $L = 0.12$ H, $C = 480$ nF, $R = 23\Omega$ is connected to a 230 V variable frequency supply. The maximum value I_{max} of the current amplitude and the corresponding source frequency ν are respectively.

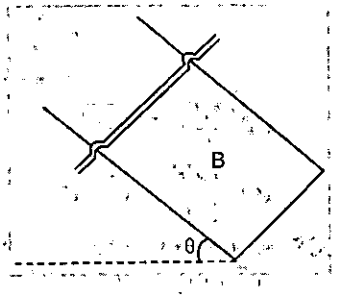
(a) $I_{max} = 10$ Å, $\nu = 600$ Hz

(b) $I_{max} = 10A$, $\nu = 663$ Hz

(c) $I_{max} = 14.1A$, $\nu = 600$ Hz

(d) $I_{max} = 14.1A$, $\nu = 663$ Hz

199. A wire of length l, mass m and resistance R slides without any friction down the parallel conducting rails of negligible resistance (See figure) The rails are connected to each other at the bottom by a resistanceless rail parallel to the wire so that the wire and rails form a closed rectangular conducting loop. The plane of the rails makes an angle θ with the horizontal and a uniform vertical magnetic field of induction B exists throughout the region. The steady state velocity of the wire is:



(a) $\frac{mg}{R}\frac{\sin\theta}{B^2 l^2 \cos^2\theta}$ (b) $\frac{mg}{R}\frac{\sin^2\theta}{B^2 l^2 \cos^2\theta}$

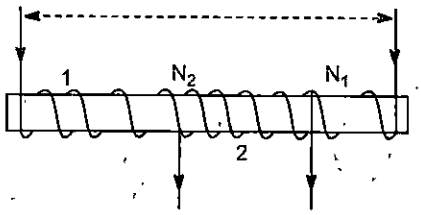
(c) $\frac{mgR\sin\theta}{B^2 l^2 \cos^2\theta}$ (d) $mgR\frac{\sin^2\theta}{B^2 l^2 \cos\theta}$

200. The inductance L of a solenoid of length l, whose windings are made of material of density D and resistivity ρ, is (the winding resistance is R) :

(a) $\frac{\mu_0}{4\pi l}\frac{Rm}{\rho D}$ (b) $\frac{\mu_0}{4\pi R}\frac{lm}{\rho D}$

(c) $\frac{\mu_0}{4\pi l}\frac{R^2 m}{\rho D}$ (d) $\frac{\mu_0}{2\pi R}\frac{lm}{\rho D}$

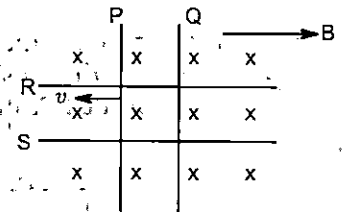
201. A long solenoid of length L, cross section A having N_1 turns has wound about its centre a small coil of N_2 turns as shown in figure. Then, the mutual inductance of two circuits is :



(a) $\frac{\mu_0 A(N_1/N_2)}{L}$ (b) $\frac{\mu_0 A(N_1 N_2)}{L}$

(c) $\mu_0 A N_1 N_2 L$ (d) $\frac{\mu_0 A N_1^2 N_2}{L}$

202. Two identical conductors P and Q are placed on two frictionless rails R and S in a uniform magnetic field directed into the plane. If P is moved in the direction shown in figure with a constant speed then rod Q:



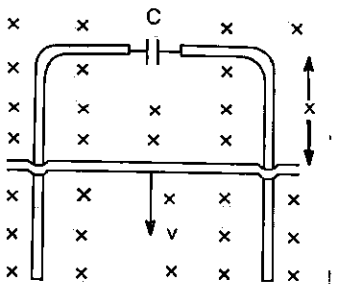
(a) will be attracted towards P
(b) will be repelled away from P
(c) will remains stationary
(d) may be repelled or attracted towards P

203. A thin nonconducting ring of mass m carrying a charge q can freely rotate about its axis. Initially, the ring is at rest and no magnetic field is present. Then a uniform field of magnetic induction was switched on, which was perpendicular to the plane of the ring and increased with time as a given function $B(t)$. The angular velocity $\omega(t)$ of the ring as a function of the field $B(t)$ will be given by :

(a) $\omega(t) = \frac{qB(t)}{m}$ (b) $\omega(t) = \frac{qB(t)}{2m}$

(c) $\omega(t) = \frac{qB(t)}{2\pi m}$ (d) $\omega = 0$

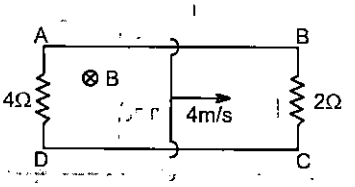
204. A conductor of length l and mass m can slide without any friction along the two vertical conductors connected at the top through a capacitor (see figure). A uniform magnetic field B is set up $\perp$ to the plane of paper. The voltage across the capacitor in terms of distance x through which it falls is :



(a) $Bl2gx$ (b) $Bl\sqrt{gx}$

(c) $Bl\sqrt{2gx}$ (d) $Bx\sqrt{2gl}$

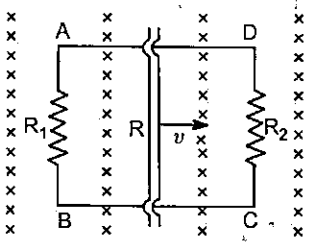
205. A sliding wire of length 0.25 m and having a resistance of 0.5Ω moves along conducting guiding rails AB and CD with a uniform speed of 4 m/s. A magnetic field of 0.5 T exists normal to the plane of $ABCD$ directed into the page. The guides are short-circuited with resistances of 4Ω and 2Ω as shown. The current through the sliding wire is :



(a) 0.27 A (b) 0.37 A

(c) 1.0 A (d) 0.72 A

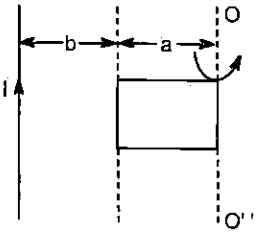
206. A rectangular loop with a sliding conductor of length l is located in a uniform magnetic field perpendicular to the plane of the loop (See figure). The magnetic induction is B. The conductor has a resistances R. The sides AB and CD have resistance R_1 and R_2, respectively. The current through the conductor during its motion to the right with a constant velocity v is:



(a) $\frac{Blv(R_1+R_2)}{R_1(R_1+R_2)}$ (b) $\frac{Bl^2v}{R_1+R_1R_2}$

(c) $\frac{Blv(R_1+R_2)}{R_1R_2+R(R_1+R_2)}$ (d) $\frac{Bl^2v}{R_1R_2+R(R_1+R_2)}$

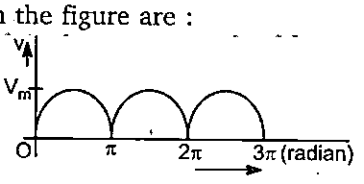
207. A square loop of side a and a straight infinite conductor are placed in the same plane with the two sides of a square parallel to the conductor (See figure). The inductance and the resistance are equal to L and R. The frame is turned through 180° about axis OO'. The charge that flows in the square loop is:



(a) $\frac{\mu_0}{\pi R}\ln\frac{a+b}{b}$ (b) $\frac{\mu_0}{\pi R}\ln\left(\frac{2a+b}{b}\right)$

(c) $\frac{\mu_0 Ia}{2\pi R}\ln\frac{2a+b}{b}$ (d) $\ln\left(\frac{a+b}{b}\right)\frac{\mu_0 I}{2\pi R}$

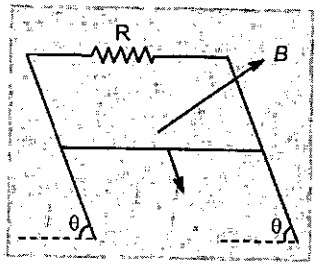
208. The average and effective values for the waveshape shown in the figure are :



(a) $\frac{2}{\pi}V_m$ and $\frac{V_m}{2}$ (b) $\frac{V_m}{\pi}$ and $\frac{V_m}{\sqrt{2}}$

(c) $\frac{2}{\pi}V_m$ and $\frac{V_m}{\sqrt{2}}$ (d) $\frac{V_m}{\pi\sqrt{2}}$ and $\frac{V_m}{\sqrt{2}}$

209. A conducting wire of mass m slides down two smooth conducting bars, set at an angle θ to the horizontal as shown in figure. The separation between the bars is l. the system is located in the magnetic field B, perpendicular to the plane of the sliding wire and bars. The velocity of the wire is :



(a) $\frac{mgR\sin\theta}{B^2l^2}$ (b) $\frac{mgR\sin\theta}{Bl^3}$

(c) $\frac{mgR\theta}{B^2l^5}$ (d) $\frac{mgR\sin\theta}{Bl^4}$

210. A line charge (λ per unit length) is in the form of a circular wheel of radius 'a' and is free to rotate in a horizontal plane. There is a magnetic field inside $B = B_0k$ extending upto a radius $b(b < a)$. If the magnetic field is switched off, the angular velocity ω of the wheel is given by (I is the moment of inertia)

(a) $\pi a^2b\lambda B_0/I$ clockwise as seen from above

(b) $\pi ab^2\lambda B_0/I$ clockwise as seen from above

(c) $\pi ab^2\lambda B_0/I$ anti-clockwise as seen form above

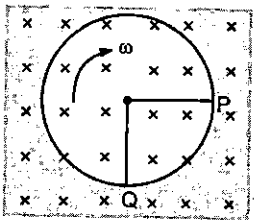
(d) $\pi a^2b\lambda B_0/I$ anti-clockwise as seen form above

211. A series circuit has a resistance of 60Ω and an impedance of 135Ω. When the total potential difference is 120V, the power consumed in the circuit will be :

(a) 47.4W (b) 95W

(c) zero (d) 52.3W

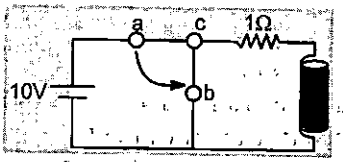
212. A conducting ring of radius r is rolling without slipping with a constant angular velocity ω (See figure). If the magnetic field strength is B and is directed into the page then, the e.m.f. induced across PQ is :



(a) $B\omega r^2$ (b) $\frac{B\omega r^2}{2}$

(c) $4B\omega r^2$ (d) $\frac{\pi^2r^2B\omega}{8}$

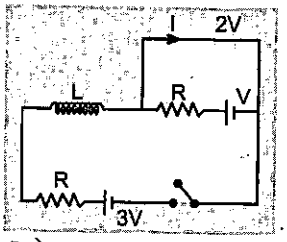
213. In the given circuit, the switch is closed to the position be from the earlier position of ac $t = 0$. The current in the inductor after 2s of closing the switch between b and c is :



(a) $\frac{1}{10e^2}$ (b) $10e^{-2}$

(c) $10e$ (d) $10e^2$

214. In the L-R circuit shown, what is the variation of the current I as a function of time? The switch is closed at time $t = 0$ sec.



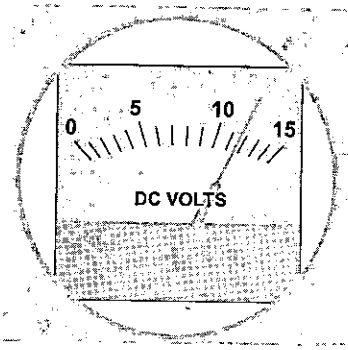
(a) $\frac{V}{R}\left(1-e^{-\frac{Rt}{L}}\right)$ (b) $\frac{V}{R}e^{-\frac{Rt}{L}}$

(c) $-\frac{V}{R}e^{-\frac{Rt}{L}}$ (d) None of these

ANSWERS

Level-1: Only One Alternative is Correct

1.	(b)	2.	(a)	3.	(b)	4.	(b)	5.	(d)	6.	(d)	7.	(b)	8.	(c)
9.	(c)	10.	(c)	11.	(a)	12.	(a)	13.	(c)	14.	(a)	15.	(c)	16.	(c)
17.	(a)	18.	(c)	19.	(a)	20.	(a)	21.	(a)	22.	(b)	23.	(a)	24	(b)
25.	(c)	26.	(a)	27.	(b)	28.	(d)	29.	(d)	30.	(a)	31.	(a)	32.	(d)
33.	(a)	34.	(c)	35.	(c)	36.	(a)	37.	(b)	38.	(a)	39.	(d)	40.	(d)
41.	(c)	42.	(a)	43.	(a)	44.	(c)	45.	(a)	46.	(d)	47.	(a)	48.	(d)
49.	(d)	50.	(a)	51.	(c)	52.	(c)	53.	(d)	54.	(a)	55.	(a)	56.	(a)
57.	(b)	58.	(d)	59.	(d)	60.	(a)	61.	(d)	62.	(b)	63.	(b)	64.	(c)
65.	(a)	66.	(a)	67.	(c)	68.	(c)	69.	(b)	70	(b)	71.	(b)	72.	(c)
73.	(b)	74.	(d)	75.	(a)	76.	(d)	77.	(b)	78.	(b)	79.	(d)	80.	(b)
81.	(a)	82.	(b)	83.	(a)	84.	(b)	85.	(d)	86.	(a)	87.	(a)	88.	(a)
89.	(a)	90.	(c)	91.	(a)	92.	(a)	93.	(c)	94.	(b)	95.	(c)	96.	(c)
97.	(c)	98.	(b)	99.	(c)	100.	(a)	101.	(b)	102.	(b)	103.	(c)	104.	(b)
105.	(c)	106.	(a)	107.	(c)	108.	(c)	109.	(c)	110.	(a)	111.	(b)	112.	(d)
113.	(a)	114.	(d)	115.	(d)	116.	(a)	117.	(a)	118.	(b)	119.	(a)	120.	(d)
121.	(b)	122.	(a)	123.	(c)	124.	(b)	125.	(d)	126.	(c)	127.	(c)	128.	(a)
129.	(a)	130.	(b)	131.	(d)	132.	(d)	133.	(c)	134.	(c)	135.	(d)	136.	(c)
137.	(b)	138.	(c)	139.	(b)	140.	(d)	141.	(d)	142.	(b)	143.	(a)	144.	(d)
145.	(a)	146.	(a)	147.	(a)	148.	(c)	149.	(b)	150.	(d)	151.	(c)	152.	(c)
153.	(d)	154.	(d)	155.	(c)	156.	(a)	157.	(d)	158.	(c)	159.	(d)	160.	(a)
161.	(a)	162.	(a)	163.	(b)	164.	(c)	165.	(a)	166.	(b)	167.	(d)	168.	(b)
169.	(c)	170.	(d)	171.	(b)	172.	(d)	173.	(a)	174.	(b)	175.	(a)	176.	(a)
177.	(b)	178.	(b)	179.	(d)	180.	(a)	181.	(b)	182.	(b)	183.	(a)	184.	(a)
185.	(b)	186.	(a)	187.	(a)	188.	(a)	189.	(b)	190.	(d)	191.	(d)	192.	(d)
193.	(a)	194.	(a)	195.	(d)	196.	(d)	197.	(c)	198.	(b)	199.	(c)	200.	(a)
201.	(b)	202.	(a)	203.	(b)	204.	(c)	205.	(a)	206.	(c)	207.	(c)	208.	(c)
209.	(a)	210.	(b)	211.	(a)	212.	(d)	213.	(b)	214.	(c)				



SOLUTION

Level-1: Only One Alternative is Correct

2. (a) $4L = 2\pi r$

$$\frac{2L}{\pi} = r$$

$$A = L^2 \qquad A = \pi\left(\frac{2L}{\pi}\right)^2$$

$$A = \pi\frac{4L^2}{\sqrt{2}} \qquad \phi = B\frac{dA}{dt}$$

Clockwise lenz law

Alternative:

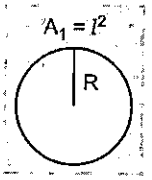
$$A_1 = l^2$$

$$2\pi R = 4l$$

$$R = \frac{2l}{\pi}$$

$$A_2 > A_1$$

$$A_2 = \pi R^2 = \pi \times \frac{4l^2}{\pi^2} = l^2\left(\frac{4}{\pi}\right) \quad \therefore$$



$\phi_2 > \phi_1$

So current flows clockwise (by lenz law)

3. (b) e.m.f. induced across the rod PQ is

$$\mathcal{E} = \vec{\mathbf{B}} \cdot (\vec{\mathbf{l}} \times \vec{\mathbf{v}})$$

$$= Blv \sin\theta$$

$$= 2 \times 2 \times 2 \times \sin 30$$

$$\mathcal{E} = 4\text{V}$$

Free electrons of the rod shift towards right due to force $q(\vec{\mathbf{v}} \times \vec{\mathbf{B}})$

Thus end P is at higher potential

or $V_P - V_Q = 4\text{V}$

6. (a) Potential difference across capacitor

$$\therefore \quad V = Bvl = \text{constant}$$

Therefore, change stored in the capacitor is also constant. Thus, current through the capacitor is zero.

7. (b) $\oint \vec{\mathbf{E}}.d\vec{\mathbf{l}} = 10\pi R^2 k$

$$E6\pi R = 10\pi R^2 k$$

$$E = \frac{10}{6}Rx = \frac{5}{3}Rk$$

Induced emf across $AB = \frac{5}{3}Rk3R\theta = 5R^2k\theta$

8. (c) I_0 = Peak value $= \dfrac{\mathcal{E}}{2R}$

Total heat produced across R is $H = \frac{1}{2}LI_0^2$

$$H = \frac{1}{2}(2L)\frac{\mathcal{E}^2}{4R^2} \Rightarrow H = \frac{L\mathcal{E}^2}{4R^2}$$

11. (a) $\mathcal{E}(t) = -\dfrac{d\phi_m}{dt}; \mathcal{E}(t) = -\dfrac{d}{dt}[Ba]$

$$\mathcal{E}(t) = -a\frac{dB}{dt}$$

$$\mathcal{E}(t) = -a\beta < 0$$

$[\because \beta > 0]$

The e.m.f. is constant and negative, so that induced electric field points around the ring F_2 towards F_1. So face F_1 will develop an excess positive charge.

18. (c) Because A and C are at equal distance from B, and their flux across B is in opposite direction, so at any time flux in B will be zero. Hence no. e.m.f. is induced.

21. (c) $U_{\max} = \frac{1}{2}LI_0^2 i, U = \frac{U_{\max}}{2}$

$$\Rightarrow \quad \frac{1}{2}LI^2 = \frac{1}{2}\left[\frac{1}{2}LI_0^2\right]$$

$$\Rightarrow \quad I_0^2[I - e^{-t/\tau}]^2 = \frac{I_0^2}{2}$$

$$\Rightarrow \quad I - e^{-t/\tau} = I - \frac{1}{\sqrt{2}} = \frac{\sqrt{2}-1}{\sqrt{2}}$$

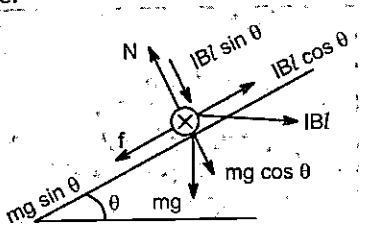
$$\Rightarrow \quad -\frac{t}{\tau} = \ln\left(\frac{\sqrt{2}-1}{\sqrt{2}}\right)$$

$$\Rightarrow \quad t = \tau \ln\left(\frac{\sqrt{2}}{\sqrt{2}-1}\right)$$

$$\Rightarrow \quad t = \frac{L}{R}\ln\left(\frac{\sqrt{2}}{\sqrt{2}-1}\right)$$

24. (b) At $t = 0$, charge on C is zero, so p.d. across C is zero, so also across R the p.d. is zero. Hence there is no current in R. At $t = \infty$, current through L is maximum and constant, so p.d. across L is zero, therefore p.d. across R is zero. Hence no current in R.

28. (d) The front view of the arrangement is as shown in figure.



From initial condition, $mg\sin\theta = \mu mg\cos\theta$

$\Rightarrow \quad \mu = \tan\theta$

$ma = IBl\cos\theta - mg\sin\theta - \mu N$

$N = mg\cos\theta + IBl\sin\theta$

$\Rightarrow \quad a = \frac{IBl}{m}\cos\theta - 2g\sin\theta - \frac{IBl}{m}\frac{\sin^2\theta}{\cos\theta}$

$= \frac{IBl}{m}\frac{\cos 2\theta}{\cos\theta} - 2g\sin\theta$

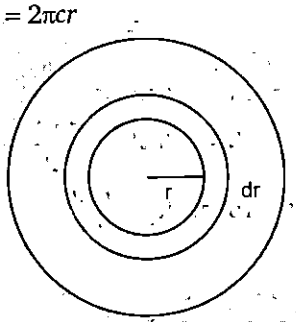
Now, $s = \frac{1}{2}at^2 = \frac{1}{2}\left[\frac{IBl}{m}\frac{\cos 2\theta}{\cos\theta} - 2g\sin\theta\right]t^2$

30. (a) Since area vector rotates in xy plane, therefore flux is due to by only.

At any instant $\phi(t) = \pi r^2 B_y\cos\omega t$

$$|E_{ind}| = \pi r^2 B_y\omega\sin\omega t$$

32. (d) $\phi = 2\pi cr$



$$\mathcal{E} = \frac{d\phi}{dt} = 2\pi c\frac{dr}{dt} \qquad [r = r_0 + kt]$$

So $\mathcal{E}$ is constant

36. (a) $R_{eq} = \frac{5R}{6} \Rightarrow I = \frac{6\mathcal{E}}{5R} = 1\text{A}$

38. (a) Flux for loop at radius $R/2$ at time t

$$\phi = \int_0^{R/2} B2\pi r\,dr = B_0 t\,2\pi\left[\frac{r^3}{3}\right]^{R/2}$$

$$= B_0 2\pi\frac{1}{3}\left(\frac{R^3}{8}\right)(t)$$

$\vec{E}\cdot d\vec{l}$ for loop

$$E(2\pi R/2) = -\frac{d\phi}{dt} = -B_0 2\pi\frac{R^3}{24}\cdot 1$$

$$\Rightarrow \quad E = -\frac{B_0R^2}{12} \qquad (-\text{ means clockwise})$$

39. (b) $I\vec{\omega} = 0 = -\int\frac{1}{2\pi R}\frac{d\phi}{dt}\times QRdt\,\hat{k}z = -\lambda R$

$$\int d\phi = -B\lambda\pi a^2 R\hat{k} \Rightarrow \vec{\omega} = \frac{-B\pi a^2\lambda}{2mR}\hat{k}$$

43. (a) $|\mathcal{E}| = \frac{d\phi}{dt} \Rightarrow |\mathcal{E}| = A\frac{dR}{dt}$

$$|\mathcal{E}| = (d^2)\frac{d}{dt}\left[B_0\left(1+\frac{x}{a}\right)\right]$$

$$|\mathcal{E}| = (d^2)B_0\left(0+\frac{1}{a}\frac{dx}{dt}\right)$$

$$\Rightarrow \quad |\mathcal{E}| = \frac{B_0d^2}{a}\left(\frac{dx}{dt}\right)$$

$$|\mathcal{E}| = \frac{B_0d^2v_0}{a}$$

44. (c) Basic concept of phasor.

46. (d) $\mathcal{E} = \frac{d\phi}{dt} \Rightarrow i = \frac{\mathcal{E}}{R} = \frac{1}{R}\frac{d}{dt}(BA) = \frac{A}{R}\frac{dB}{dt}$

where πr^2 = area of the loop of radius r and R = resistance of the loop of length $(2\pi r)$ and area of cross section πa^2.

$$R = \frac{\rho l}{\pi a^2} = \frac{\rho(2\pi r)}{\pi a^2}$$

Further mass of wire is $m = (\pi a^2)(2\pi r)(d)$

$$i = \frac{(\pi a^2)(\pi r^2)}{\rho(2\pi r)}\frac{dB}{dt}$$

$$i = \frac{(\pi a^2)(2\pi r)}{4\pi\rho}\frac{dB}{dt} \Rightarrow i = \frac{m}{4\pi\rho d}\frac{dB}{dt}$$

47. (a) $L = \frac{dI}{dt} = \mathcal{E}$ or $I = \frac{\mathcal{E}t}{L}\ \forall 0 \le t \le t_0$

49. (d) $V_A - V_B = V_L + V_R = \frac{LdI}{dt} + IR$

$= L\times 8 + 0.5\times 0.2 = 0.5\text{V}$

$8L = 0.5 - 0.1 = 0.4;\ L = 0.5\text{H}$

50. (a) When the switch is closed, the current flows through the coil as shown and thus sets up a magnetic flux f through the coil linking the ring. The motion of the ring will be such that, by Lenz's law, it opposes the sudden increase in flux linkage from the west. Hence, the ring moves towards E (east).

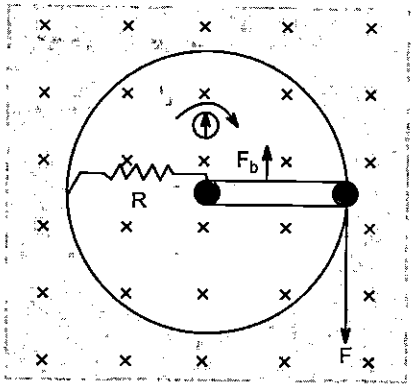
53. (d) $\mathcal{E} = \int(\vec{v}\times\vec{B})d\vec{l} \Rightarrow \mathcal{E} = \frac{1}{2}Bl^2\omega\sin^2\theta$

56. (a) Current through battery exponentially changes from

$$\frac{\mathcal{E}}{R_1+R_2+R_3} \text{ to } \frac{\mathcal{E}}{R_1}$$

59. (d) $F_b = BIL$

Induced current :



$$I = \frac{(B\omega r^2/2)}{r}$$

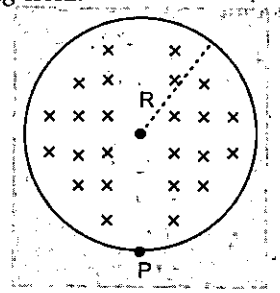
$$\therefore \quad F_b = B\left(\frac{B\omega r^2}{2R}\right)r = \frac{B^2\omega r^3}{2R}$$

To maintain constant angular velocity,

$$F(r) = F_b(r/2)$$

$$\Rightarrow \quad F = \frac{F_b}{2} = \frac{B^2\omega r^3}{4R}$$

60. (a) If we consider the cylindrical surface to be a ring of radius R, there will be an induced e.m.f. due changing field.



$$\int \vec{E}.d\vec{l} = \frac{d\phi}{dt} = -A\frac{dB}{dt}$$

$$\Rightarrow \quad E(2\pi R) = -A\frac{dB}{dt} = -\pi R^2\frac{dB}{dt}$$

$$\Rightarrow \quad E = \frac{R}{2}\frac{dB}{dt}$$

$\therefore$ Force on the electron

$$F = -Ee = -\frac{eR}{2}\frac{dB}{dt}$$

$$\Rightarrow \text{Acceleration} = \frac{1}{2}\frac{eR}{m}\frac{dB}{dt}$$

As the field is increasing and is being directed inside the paper, there will be anticlockwise induced current (in order to oppose the cause) in the ring (assumed). Hence there will be force towards left on the electron.

63. (b) By Kirchoff law at point A

$$I_1 = I_2 + I_3$$

$$I_3 = I_1 - I_2$$

$$\text{Loop } (I) \; -\frac{Q}{C} - R(I_1 - I_2) + L\frac{dI_2}{dt} = 0$$

$$\text{Loop } (II) \; V_b - \frac{LdI_2}{dt} - RI_1 = 0$$

Alternative:

$$I_1 = I_2 + I_3 \Rightarrow I_3 = I_1 - I_2 \quad \ldots(1)$$

$$\text{and} \quad -V_b + I_1R + L\frac{dI_2}{dt} = 0 \quad \ldots(2)$$

$$-L\frac{dI_2}{dt} + I_3R + \frac{Q}{C} = 0 \quad \ldots(3)$$

64. (c) Considering pure rolling of OA about A, the induced e.m.f. across OA will be

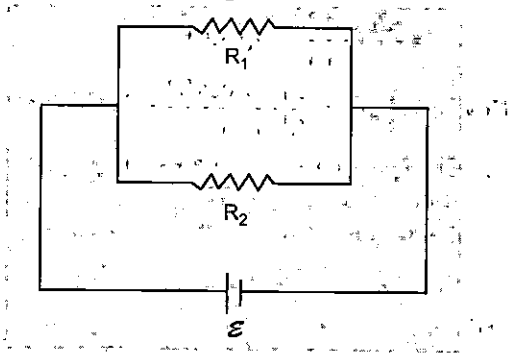
$$|\vec{\mathcal{E}}| = \frac{B\omega(r)^2}{2}$$

From Lenz's law, O will be negative end, while A will be the positive end.

Hence,

$$V_o - V_A = -\frac{3B\omega r^2}{2}$$

65. (a) The equivalent diagram is

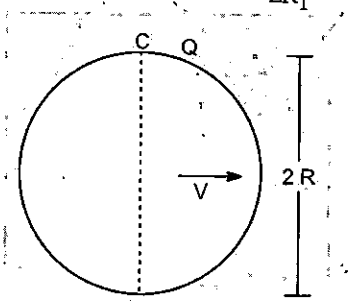


The induced e.m.f. across the centre and any point on the circumference is

$$|\vec{\mathcal{E}}| = \frac{1}{2}B\omega l^2 = \frac{B\omega r^2}{2}$$

$\therefore$ Current through $R_1 = \frac{B\omega r^2}{2R_1}$

68. (c)



Considering a projected length $2R$ on the ring in vertical plane.

This length will move at a speed v perpendicular to the field.

This results in an induced e.m.f. :

$\mathcal{E} = Bv(2R)$ in the ring.

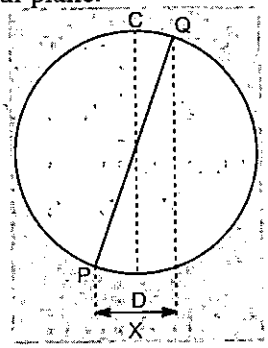
In ring A, $\mathcal{E}A = B(-v)(2R)$

In ring B, $\mathcal{E}B = B(v)(2R)$

Therefore, potential difference between A and B
$= B(v)(2R) - B(-v)(2R) = 4BvR$

Note : *There will be no p.d. across the diameter due to rotation.*

71. (b) When the ring falls vertically, there will be an induced e.m.f. across A and B ($\mathcal{E} = Bv(2r)$).
Note that there will be a potential difference across any two points on the ring, and the line joining these has a projected length in the horizontal plane. For example, between points P and Q there is a projected length x in the horizontal plane.



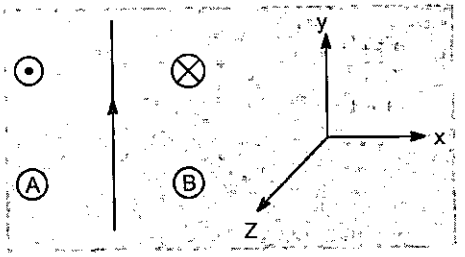
$\therefore$ p.d. across P and Q is
$V = Bvx$
But for points C and D, $x = 0$
Therefore, p. d. $= 0$

72. (c) Just before opening the switch, the current in the inductor is $\mathcal{E}/R$. Energy stored in it $= \frac{1}{2}L\left(\frac{\mathcal{E}}{R}\right)^2$
This energy will dissipate in the resistors R_1 and R_2 in the ratio $\frac{1}{R_1}$ and $\frac{1}{R_2}$.

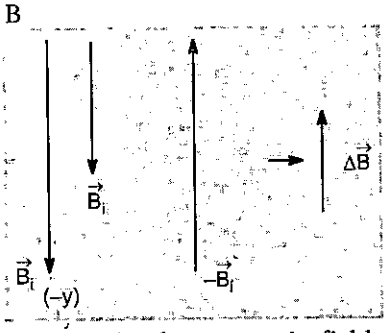
73. (b) Equivalent inductance
$L_{eq} = L + 2L = 3L$
$C_{eq} = C + 2C = 3C$
Frequency of oscillation
$$f = \frac{1}{2\pi\sqrt{L_{eq}C_{eq}}} = \frac{1}{6\pi\sqrt{LC}}$$

74. (d) It appears as if some flux changes must occur as the lines of magnetic induction field B are cut across by the rotating coil. If we resolve the given field parallel and perpendicular to the plane of the coil, it is easy to see that the flux due to perpendicular component remains constant and that due to parallel component does not enter the coil during its rotation. Hence, no induced e.m.f. and current will be produced.

77. (b) The fields at A and B are out of the paper and inside the paper, respectively.

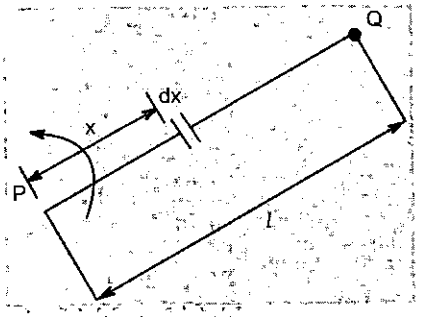


As the current in the straight wire decreases, the field also decreases.
For B



The change in the magnetic field which causes induced current ($\Delta\vec{\mathbf{B}}$) is along (+) z direction.
Hence, induced e.m.f. and hence current should be such as to oppose this change $\Delta\vec{\mathbf{B}}$.
Hence, induced e.m.f. should be along $-z$ direction which results in a clockwise current in B. Similarly, there will be anticlockwise current in A.

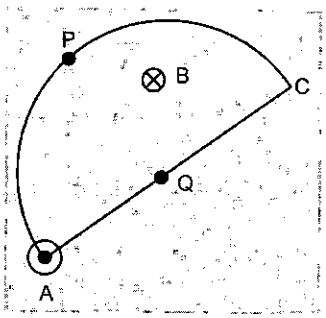
80. (b)



Charge on the differential element dx, $dq = \frac{Q}{l}.dx$
Equivalent current, $di = fdq$
$\therefore$ magnetic moment of this element,
$dm = (di)NA(N = 1)$
$= (\pi x^2)f\frac{Q}{l}dx$
$\Rightarrow$ $m = \int_0^\mu d\mu = \frac{\pi fQ}{l}\int_0^l x^2 dx,$
$\mu = \frac{1}{3}\pi fQl^2.$

81. (a) $\mathcal{E} = \frac{BdA}{dt} = \frac{Bd}{dt}(\pi r^2) = B2\pi r\frac{dr}{dt}$

84. (b) We connect a conducting wire from A to C and complete the semicircular loop. The e.m.f. in the semicircular loop is zero because its magnetic flux does not change.



$\therefore$ e.m.f. of section APC + e.m.f. of section $CQA = 0$

$\therefore$ e.m.f. of section APC + e.m.f. of section $AQC = 2BR^2\omega$

88. (a) Rate of increment of energy in the inductor

$$= \frac{dU}{dt} = \frac{d}{dt}\left(\frac{1}{2}Li^2\right) = Li\frac{di}{dt}$$

Current in the inductor at time is :

$$I = i_0\left(1 - e^{-\frac{t}{\tau}}\right) \text{ and } \frac{dI}{dt} = \frac{i_0}{\tau}e^{-\frac{t}{\tau}}$$

$$\frac{dU}{dt} = \frac{Li_0^2}{\tau}e^{-\frac{t}{\tau}}\left(1 - e^{-\frac{t}{\tau}}\right)$$

$$\frac{dU}{dt} = 0 \text{ at } t = 0 \text{ and } t = \infty$$

89. (a) Induced e.m.f. $\int_a^b Bvdx = \int_a^b \frac{\mu_0 I}{2\pi x}Bvdx$

$\Rightarrow$ Induced e.m.f. $= \frac{\mu_0 Iv}{2\pi}\ln\left(\frac{b}{a}\right)$

$\Rightarrow$ Power dissipated $= \frac{\mathcal{E}^2}{R}$

Also, power $= Fv \Rightarrow F = \frac{\mathcal{E}^2}{vR}$

$\Rightarrow F = \frac{1}{vR}\left[\frac{\mu_0 Iv}{2\pi}\ln\left(\frac{b}{a}\right)\right]^2$

90. (c) The magnetic flux through any closed surface is zero and so the magnetic flux through S_1 and S_2 is zero. This means that the flux through S_1 is equal to the negative of that through S_2, and so the magnetic flux through both must be equal in magnitude.

95. (c) $\phi_A = \frac{\mu_0 i\pi R^2}{2\pi(R^2 + x^2)^{3/2}}\pi r^2$

$\Rightarrow \mathcal{E}_A = -\frac{d\phi}{dt}$

$= \frac{\mu_0 i\pi}{2}R^2r^2(-3/2)(R^2 + x^2)^{-5/2}2x$

$\mathcal{E}_A$ is maximum when $\frac{d\mathcal{E}_A}{dx} = 0$

$\Rightarrow \frac{d}{dx}\frac{x}{(R^2 + x^2)^{5/2}} = 0$

or $(R^2 + x^2)^{5/2} - \frac{5x}{2}(R^2 + x^2)^{3/2}2x = 0$

or $R^2 + x^2 - 5x^2 = 0$

or $x = \frac{R}{2}$

96. (c) When the rod rotates, there will be an induced current in the rod. The given situation can be treated as if a rod A of length $3l$ is rotating in clockwise direction, while another rod B of length $2l$ is rotating in the anticlockwise direction with the same angular speed ω

As $\mathcal{E} = \frac{1}{2}B\omega l^2$

For A : $\mathcal{E}_A = \frac{1}{2}B\omega(3l)^2$

and $\mathcal{E}_B = \frac{1}{2}B(-\omega)(2l)^2$

Resultant induced e.m.f. will be :

$$\mathcal{E} = \mathcal{E}_A + \mathcal{E}_B = \frac{1}{2}B\omega l^2(9 - 4)$$

$$\mathcal{E} = \frac{5}{2}B\omega l^2$$

99. (c) Magnetic field is out of the plane and as the electron moves, the field decreases. From Lenz's rule, the current in the coil is anticlockwise.

101. (b) Let the mass have descended a distance y and be at speed v. The changing magnetic flux through the circuit loop leads to a Faraday emf BLv. This emf is related to the charge on the capacitor as

$$q = CBLv$$

Differentiating with respect to time gives the current in the loop

$$I = CBL\frac{dv}{dt}$$

The magnetic force on the current carrying bar (upward if the bar is descending) is given by

$$F_{mag} = BIL = CB^2L^2\frac{dv}{dt}$$

The net downward force on the bar and Newton's second law of motion gives

$$mg - ky - CB^2L^2\frac{dv}{dt} = m\frac{d^2y}{dt^2}$$

Transforming to the new variable $u = y - \frac{mg}{k}$ allows the equation of motion to be written in the form

$$\frac{d^2u}{dt^2} + \frac{k}{m + CB^2L^2}u = 0$$

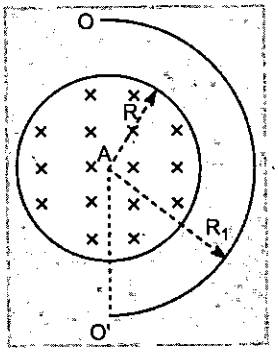
This is the familiar simple harmonic motion equation with angular frequency

$$\omega = \sqrt{\frac{k}{m + CB^2L^2}} = 20$$

102. (b) Required e.m.f. $= \pi R^2 \times \frac{dB}{dt} \times \frac{\theta}{2\pi}$

$$\therefore \quad \mathcal{E} = \frac{R^2}{2R_1}\left(\frac{dB}{dt}\right)$$

$$\text{e.m.f.} = \frac{\theta}{2\pi}\frac{R^2}{2R_1}\left(\frac{dB}{dt}\right)2\pi R_1 = \frac{\theta}{2}R^2\left(\frac{dB}{dt}\right)$$



103. (c) The magnetic field produced by the square loop is parallel to the plane of the circular loop. Hence, the mutual inductance is zero.

108. (c) Use Lenz's law. Induced e.m.f. of the current opposes the change in flux through it.

109. (c) The change in magnetic flux is zero, hence the current in the ring will be zero.

112. (d) At $t = 0$, for the purpose of current calculation in circuit, inductor can be assumed as open and capacitor as short circuited.

115. (d) When a ring moves in a magnetic field perpendicular to its plane, replace the ring by a diameter perpendicular to the direction of motion. The e.m.f. is induced across this diameter. In the question current flowing in the ring will be through the two semicircular portions, in parallel.

Induced e.m.f., $\mathcal{E} = B(2r)\vec{v}\,l$

Resistance of each half of ring $= R/2$

As these are in parallel, the equivalent resistance $= R/4$

Current in the circuit $= \frac{B(2r)v}{(R/4)} = \frac{8Brv}{R}$

117. (a) $\omega = \frac{\text{angular momentum}}{mR^2\,(\text{M.I.})} = \frac{J}{mR^2}$

Ring is equivalent to a current loop of current $I = \frac{Q}{2\pi}\omega$

magnetic moment of loop

$$\vec{\mu} = nI\vec{A} = (1)\left(\frac{Q}{2\pi}\right)\pi R^2\hat{n} = \left(\frac{Q}{2}\omega R^2\right)\hat{n}$$

($\hat{n}$ is normal to plane outward)

Torque $\vec{\tau} = \vec{\mu} \times \vec{B}$

$$= \frac{Q}{2}\omega R^2 B \text{ (up the plane)}$$

$$= \frac{Q}{2}R^2\left(\frac{J}{mR^2}\right)B = \frac{1}{2}\left(\frac{Q}{m}\right)(J)(B)$$

$$= \frac{1}{2} \times 10^{-5} \times 7.5 \times 10^{-4} \times 0.1$$

$$= 3.75 \times 10^{-10}\,\text{N upward along the plane}$$

118. (b) $\frac{B^2 v_0 L^2}{R} = mg \Rightarrow v_0 = \frac{mgR}{B^2L^2}$

119. (a) The current in L for steady state $= \frac{\mathcal{E}}{R_1}$

$\therefore$ Energy stored in $L\,(E_1) = \frac{1}{2}LI_0^2 = \frac{1}{2}L\frac{\mathcal{E}^2}{R_1^2}$

122. (a) Increasing the resistance causes a decrease in the current on the left. This reduces the field strength for the solenoid $B = \mu_0 nI$. By the right-hand rule, the field through the solenoid is directed to the RIGHT. Hence, in the right-hand circuit, since there will be fewer field lines directed to the right, there is an induced electric field that will produce a current in the wires to "replace" the lost field lines. This means that current will be directed with the same orientation as the current in the left circuit.: the galvanometer will deflect to the LEFT.

123. (c) $\left|\int \vec{E}\cdot d\vec{l}\right| = \left|\frac{d\phi}{dt}\right|;\ E2\pi\frac{l}{2} = \pi\frac{l^2}{4}\frac{dB}{dt}$

$$E = \frac{l}{4}\alpha$$

$F = \frac{ql\alpha}{4}$ is the electric force on the charge but net force of the particle is zero.

126. (c) Let i_1 be the current in the circuit before shifting

$$i_1 = \frac{\mathcal{E}}{R} \qquad \text{(i)}$$

Since, the flux associated with the inductors will be same just before and just after shifting

$$\therefore \quad i_1 5L = i_2 9L \quad \Rightarrow \quad i_2 = \frac{5}{9}\frac{\mathcal{E}}{R}$$

127. (c) Applying KVL in the outer loop we get $I_0 r - \mathcal{E} = 0$

$$\Rightarrow \quad I = \frac{\mathcal{E}}{r}$$

$\therefore$ Initial energy in solenoid $= U_0 = \frac{1}{2}LI_0^2 = \frac{\mathcal{E}^2 L}{2r^2}$.

This energy will be dissipated in the form of heat in r and R after opening of the switch. Since the same current flows through these resistances and the thermal power in them is $i^2 r$ and $i^2 R$[i varies with time] respectively, therefore heat generated in each resistor is directly proportional to its resistance.

$\therefore$ Heat generated in solenoid

$$= \frac{r}{r+R}U_0 = \left[\frac{r}{r+R}\right]\frac{\mathcal{E}^2 L}{2r^2} = \frac{\mathcal{E}^2 L}{2r(r+R)}$$

128. (a) $\tau = \pi R^2 IB$

$$I_C = \frac{2}{3}MR^2 - \frac{MR^2}{4} = \frac{5MR^2}{12}$$

$$\alpha = \frac{\tau}{I_C} = \frac{\pi R^2 IB}{(5MR^2/12)} = \frac{12\pi IB}{5M}$$

132. (d) $E = \frac{MdI}{dt} = Ma$

$I = \frac{Ma}{R}(1 - e^{-tR/L})$

133. (c) Current in the YY' direction is from Y' to Y but the current is constant and hence the magnetic flux through the coil is constant. Therefore the current in the coil is zero.

136. (c) Constancy of flux implies that $\frac{\mathcal{E}}{R}L_1 = I(L_1 + L_2)$

i.e., $I = \frac{\mathcal{E}L_1}{R(L_1 + L_2)}$

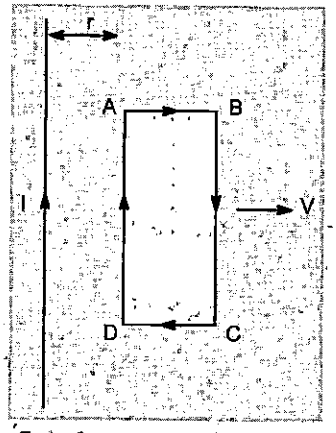
137. (b)

$$\mathcal{E}_{avg} = \frac{\frac{0.\pi}{2\omega} + \frac{\omega Bl^2}{2}\cdot\frac{\pi}{2\omega} - \frac{\omega 2Bl^2}{2}\cdot\frac{\pi}{2\omega} + \frac{\omega 4Bl^2}{2}\cdot\frac{\pi}{2\omega}}{\frac{2\pi}{\omega}} = \frac{3}{8}\omega Bl^2$$

140. (d) $Blvt = \text{constant}$

$B = \frac{C}{lvt}$

141. (d)

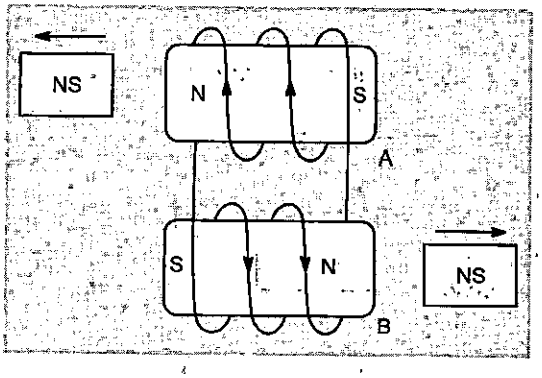


As the flux decreases, to maintain flux current in the loop is clockwise. Force on DA due to the long wire is towards left while on BC is towards right.

143. (a) $\phi \propto \frac{di_1}{dt}$

Hence, $i_2 = M\frac{di_1}{dt}$ (direction already indicated)

146. (a)



147. (a) At $t = 0$, the branch containing L will offer infinite resistance while the branch containing the capacitor will be effectively a short circuit. Hence, $(R)_{t=0} = \frac{\mathcal{E}}{R}$. Similarly at $t = \infty$, L will offer zero resistance whereas C will be an open circuit. Hence effective resistance

$$= R + \frac{6\times 3R}{6+3} = 3R \quad (I)_{t=\infty} = \frac{\mathcal{E}}{3R}$$

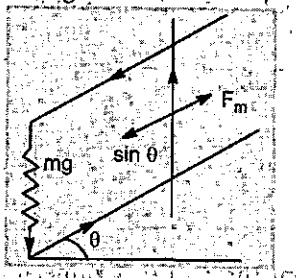
$\therefore$ The required ratio $= \frac{\mathcal{E}}{R}\times\frac{3R}{\mathcal{E}} = 3:1$

148. (c) $i_{rms} = \frac{I_0}{\sqrt{2}}$

$$i_{avg} = \frac{\int Idt}{\int dt} = \frac{\int_{T/8}^{3T/8} I_0 \sin\omega t}{\frac{3T}{8} - \frac{T}{8}}$$

$$i_{avg} = \frac{2\sqrt{2}I_0}{\pi}$$

149. (b) Terminal velocity is attained when magnetic force is equal to $mg\sin\theta$



$\therefore \quad F_m = mg\sin\theta$

or $IlB = mg\sin\theta$

or $\left(\frac{\mathcal{E}}{R}\right)lB = mg\sin\theta$

or $\frac{(Bv_T l)}{R}lB = mg\sin\theta$

$v_T = \frac{mgR\sin\theta}{B^2l^2}$

152. (c) The current at time t is given by

$I = i_0(1 - e^{-t/\tau})$

Here $i_0 = \mathcal{E}/R$ and $\tau = \frac{L}{R}$

$\therefore \quad q = \int_0^{\tau} idt = \int_0^t i_0(1 - e^{-t/\tau})\,dt$

$$= \frac{i_0 t}{e} = \frac{\left(\frac{\mathcal{E}}{R}\right)\left(\frac{L}{R}\right)}{e} = \frac{\mathcal{E}L}{eR^2}$$

154. (d) When the coil is within the field, there is no change in the magnetic flux passing through it. Thus no current will be induced and the acceleration will be g. But according to Lenz's law, the induced current will oppose its motion when it

enters or leaves the field. Therefore, acceleration will be less than g.

156. (a) $dq = \frac{d\phi}{R}$

$\phi_1 = 0$

$\phi_f = \left(\frac{\mu_0}{2\pi}\frac{I}{b}\right)(\pi a^2) = \frac{\mu_0 I a^2}{2b}$

$\therefore \quad d\phi = \frac{\mu_0 I a^2}{2Rb}$

So, $dq = \frac{I_0 I a^2}{2Rb}$

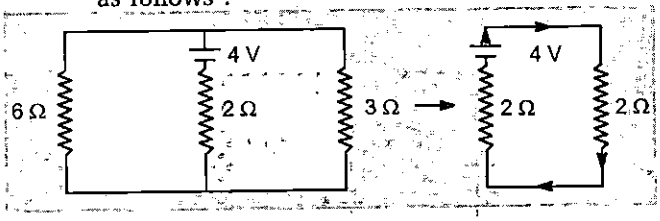
157. (d) According to Lenz's Law, e.m.f. of same magnitude in clockwise direction is induced in the two loops into which the figure is divided. So, current is induced in the clockwise direction in the outer boundary but no current is there in wire AB.

158. (c) Motional e.m.f.

$\mathcal{E} = Bvl$

$\mathcal{E} = (2)(2)(1) = 4\text{V}$

This acts as a cell of e.m.f. $\mathcal{E} = 4\text{V}$ and internal resistance $r = 2\Omega$. The simple circuit can be drawn as follows :



$\therefore$ Current through the connector

$I = \frac{4}{2+2} = 1\text{A}$

Magnetic force on connector

$F_m = IlB$

$= (1)(1)(2)$

$= 2\text{N}$ (towards left)

Therefore, to keep the connector moving with a constant velocity, a force of 2N will have to be applied towards right.

159. (d) Magnetic flux in $\otimes$ direction through the coil is increasing. Therefore, induced current will produce magnetic field in $\odot$ direction. Thus the current in the loop is anticlockwise. Magnitude of induced current at any instant of time is

$I = \frac{\mathcal{E}}{R}$

$= \frac{Bv(FG)}{\rho(FG + GD + DF)}$

When the wire AH moves downwards FG, GD and DF all increase in the same ratio. Therefore, I is constant.

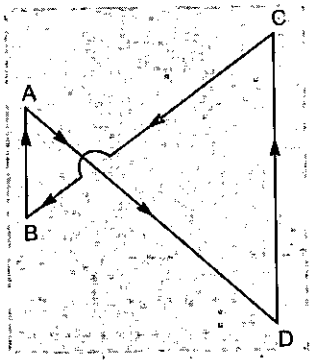
160. (a) $I = I_0 e^{-t/t_0}$

$\left(-\frac{dI}{dt}\right) = \frac{I_0}{t_0} e^{-t/t_0}$

$\left(-\frac{dI}{dt}\right) = \frac{i_0}{t_0} = r$ (at $t = 0$)

$\therefore$ The desired time is $\frac{i_0}{r}$ or t_0.

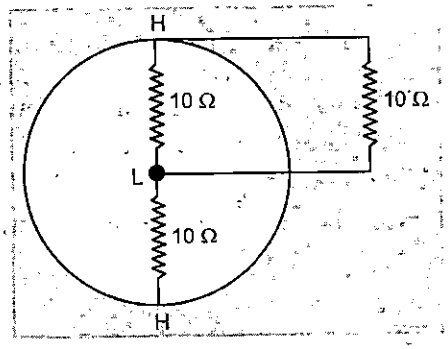
161. (a) Magnetic field in $\otimes$ direction in increasing. Therefore, induced current will produce magnetic field in $\odot$ direction. Thus current in both the loops should be anticlockwise. But as the area of the loop on the right side is more, induced e.m.f. in this will be more compared to the left side loop. $\left(\mathcal{E} = -\frac{d\phi}{dt} = -S\frac{dB}{dt}\right)$. Therefore, net current in the complete loop will be in a direction shown in figure.



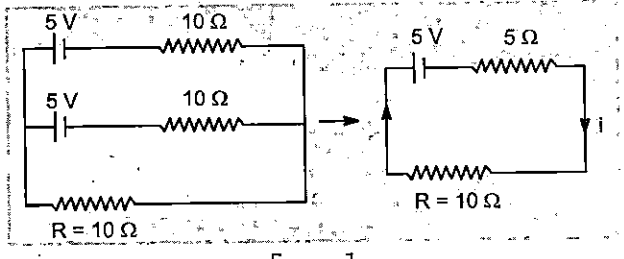
164. (c) Potential difference between centre of the ring and the rim is

$V = \frac{1}{2}B\omega R^2$

$= \frac{1}{2}(50)(2)(0.1)^2 = 5\text{V}$



Now the circuit can be drawn as follows :



$\therefore \quad I = \frac{5}{10+5} = \frac{1}{3}\text{A}$

166. (b) For $r \geq a$

$$\oint \vec{E} \cdot d\vec{l} = \left|\frac{d\phi}{dt}\right| = S\left|\frac{dB}{dt}\right|$$

or $$E(2\pi r) = \pi a^2 \left|\frac{dB}{dt}\right|$$

$$\therefore \quad E = \frac{a^2}{2r}\left|\frac{dB}{dt}\right|$$

$\therefore$ Induced electric field $\propto 1/r$

For $r \leq a$

$$E(2\pi r) = \pi r^2 \left|\frac{dB}{dt}\right|;$$

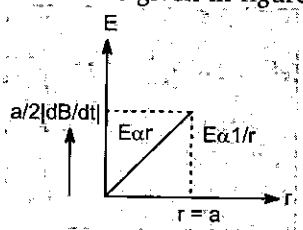
or $$E = \frac{r}{2}\left|\frac{dB}{dt}\right|$$

or $$E \propto r$$

At $r = a$,

$$E = \frac{a}{2}\left|\frac{dB}{dt}\right|$$

Therefore, variation of E with r (distance from the centre) will be as given in figure.



168. (b) Let E be the electric field at a distance r from the centre of the disc. Then

$$eE = m\omega^2 r$$

or $$E = \frac{m\omega^2 r}{e}$$

$$\therefore \quad \text{P.D.} = \int_{r=0}^{r=a} E\,dr$$

$$= \int_0^a \frac{m\omega^2 r}{e} = \frac{m\omega^2 a^2}{2e}$$

170. (d) From the given data, one can conclude that the AC-component has to be resistance. Effective current is rms current $= i_{eff}$

$$i_{eff} = \sqrt{\frac{\int_0^T (I_0 + I_0 \sin \omega t)^2}{T}}$$

171. (b) Let L and R be the inductance and the resistance of the coil, respectively. Let $\mathcal{E}$ = e.m.f. of the cell.

Steady state current, $I_0 = \mathcal{E}/R$

$$P = I_0^2 R = \frac{\mathcal{E}^2}{R}$$

Energy stored in the coil

$$= \frac{1}{2}LI_0^2 = \frac{1}{2}L\left(\frac{\mathcal{E}^2}{R^2}\right);$$

$$= \frac{1}{2}\left(\frac{L}{R}\right)\left(\frac{\mathcal{E}^2}{R}\right) = \frac{1}{2}\tau P$$

173. (a) Magnetic field of a solenoid on its axis is given as

$$B = \mu_0 nI = \mu_0\left(\frac{N}{l}\right)I$$

$$N\phi = NBA = N\left[\mu_0\left(\frac{N}{l}\right)I\right]\pi r^2$$

$$\frac{N\phi}{L} = \frac{\mu_0 N^2 \pi r^2}{l}$$

$$L = \frac{\mu_0 N^2 r^2 \pi}{l}$$

Total length of the wire, $S = 2\pi rN$

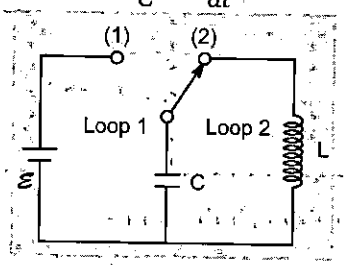
$$L = \frac{\mu_0 N^2 r^2 \pi}{l}\frac{4\pi}{4\pi} = \frac{\mu_0 (2\pi rN)^2}{4\pi l}$$

$$L = \frac{\mu_0 S^2}{4\pi l} = \frac{\mu_0}{4\pi}\frac{S^2}{l}$$

$$S = \sqrt{\frac{4\pi lL}{\mu_0}}$$

175. (a) When switch 2 is closed, applying KCL in loop 2, we get

$$\frac{q}{C} + L\frac{dI}{dt} = 0$$



$$L\frac{d^2q}{dt^2} + \frac{q}{C} = 0$$

$$\frac{d^2q}{dt^2} = \frac{-q}{CL} = 0$$

$$\Rightarrow \quad \frac{d^2q}{dt^2} = \omega^2 q = 0$$

Hence, $$q = q_0 \cos \omega t, \omega = \sqrt{\frac{1}{LC}}$$

and $$I = -q_0 \omega \sin \omega t$$

U_C = Energy stored in the capacitor

$$= \frac{q^2}{2C} = \frac{q_0^2 \cos^2 \omega t}{2C}$$

U_L = Energy stored in the inductor

$$= \frac{1}{2}LI^2 = \frac{1}{2}Lq_0^2\omega^2 \sin^2 \omega t$$

when $$U_C = U_L$$

$$\frac{q_0^2 \cos^2 \omega t}{2C} = \frac{q_0^0 \omega^2 L \sin^2 \omega t}{2}$$

$$\cot^2 \omega t = \omega^2 LC = 1$$

For minimum value of t, $\omega t = \frac{\pi}{4}$

Hence, $t = \frac{\pi}{4}\sqrt{LC}$

176. (a) $Q = Q_0 \cos\omega t = CV_0 \cos\frac{1}{\sqrt{LC}}t$

$$I = \frac{CV_0}{\sqrt{LC}}\sin\frac{1}{\sqrt{LC}}t$$

Maximum energy stored in

$$2C = \frac{1}{2}LI^2 = \frac{1}{2}\times L\frac{C^2V_0^2}{LC}\times\frac{1}{2}$$

$$\frac{Q_{max}^2}{2C} = \frac{CV_0^2}{4} \Rightarrow Q_{max} = \frac{CV_0}{\sqrt{2}}$$

177. (b) The power dissipated in the resistor,

$$P = \frac{dW}{dt} = I^2R$$

Since the current through resistor varies with time we must integrate.

The total energy produced as heat in the resistor

$$W = \int_0^\infty I^2Rdt$$

The current in an RL circuit is

$$I = I_0e^{-(R/L)t}$$

$$W = \int_0^\infty I_0^2e^{-2R/L}Rdt$$

We can integrate by substituting

$$W = I^2R\frac{L}{2R}\int_0^\infty e^{-x}dx = \frac{1}{2}LI_0^2[-e^{-x}]_0^\infty = \frac{1}{2}LI_0^2$$

Note that the total heat produced equals the energy $(1/2)LI_0^2$ originally stored in the conductor.

178. (b) When the copper rod is rotated, flux linked with the circuit varies with time.

Therefore, an e.m.f. is induced in the circuit.

At time t, plane of semi-circle makes angle ωt with the plane of rectangular part of the circuit. Hence, component of the magnetic induction normal to plane of semi-circle is equal to $B\cos\omega t$.

Flux linked with semi-circular part is

$$\phi_1 = \frac{1}{2}\pi a^2\cos\omega t$$

Let area of rectangular part of the circuit be A.

$\therefore$ Flux linked with this part is

$$\phi_2 = BA$$

$\therefore$ Total flux linked with the circuit is

$$\phi = \frac{1}{2}\pi a^2B\cos(\omega t) + BA$$

$\therefore$ Induced e.m.f. in the circuit,

$$\varepsilon = -\frac{d\phi}{dt} = \frac{1}{2}\pi\omega a^2B\sin(\omega t)$$

Since resistance of the circuit is negligible, therefore potential difference across the capacitor is equal to induced e.m.f. in the circuit.

$\therefore$ Charge on the capacitor at time t is $q = Ce$

$$= \frac{1}{2}\pi\omega a^2CB\sin(\omega t)$$

But current $I = \frac{dq}{dt} = \frac{1}{2}\pi\omega^2a^2CB\cos(\omega t)$

Due to flow of current, semi-circle experiences a momentum. Therefore, power is required to keep the semi-circle rotating with constant angular velocity. In fact, power required to rotate the semi-circle is equal to electrical power generated in the circuit.

$\therefore$ Power required ,

$$P = \varepsilon I = \frac{1}{8}\pi^2\omega^3a^4CB^2\sin(2\omega t)$$

180. (a) For $r \le R$

$|\vec{\varepsilon}|$ = Magnitude of induced e.m.f. $= \frac{d\phi_B}{dt}$

$\therefore$ $|\vec{\varepsilon}| = \oint\vec{E}d\vec{l} = \frac{d\phi_B}{dt}$ $(\because \phi_B = \vec{B}.\vec{A}.)$

$$\therefore E(2\pi r) = A\frac{dB}{dt} = \pi r^2\frac{dB}{dt}$$

$\therefore$ $E \propto r \Rightarrow$ shows straight line

For $r \ge R$

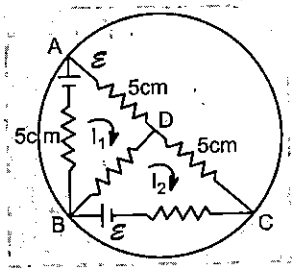
$$E(2\pi r') = \pi R^2\frac{dB}{dt}$$

$$\therefore E \propto \frac{1}{r}$$

$\Rightarrow$ shows rectangular hyperbola

181. (b) Induced EMF in the loop ABD or $BDC = \frac{d\phi}{dt} = A\frac{dB}{dt}$

$$\varepsilon = \frac{1}{4}\times 8\times 6\times 10^{-4}\times 0.263 = 12\times 0.263\times 10^{-4}$$



Say the resistance per cm of wire is R.

Now applying KVL to loops ADB and BDC.

$$5I_1 + 5(I_1 - I_2) + 6I_1 = \frac{\varepsilon}{R} \text{ or } 16I_1 - 5I_2 = \frac{\varepsilon}{R}$$

Similarly $13I_2 - 5(I_1 - I_2) = \frac{\varepsilon}{R}$ or $18I_2 - 5I_2 = \frac{\varepsilon}{R}$

Solving this we get $I_1 = \frac{23}{263}\frac{\varepsilon}{R}$, $I_2 = \frac{2I}{263}\frac{\varepsilon}{R}$

$$\Rightarrow i_2 - i_1 = \frac{2}{263}\frac{\varepsilon}{R}$$

$$= \frac{2}{263} \times 12 \times 0.236 \times 10^{-4} \times \frac{1 \times 1 \times 10^{-6}}{24 \times 10^{-9} \times 1 \times 10^{-2}}$$

$$= 0.01\,\text{A} = 10\,\text{mA}$$

182. (b) Let current i_1 in the straight wire be upward. Then the magnetic field due to the straight wire has magnitude $B_1 = \mu_0 i_1/2\pi r$ at a distance r. In accordance with right hand rule B_1 points inward to the plane of page. We consider a differential strip of thickness dr, area $dA_2 = adr$. Magnetic flux through area dA, $d\phi_B = B_1(adr)$.

Total flux through the loop,

$$\phi_{B_{12}} = \int B_1 dA_2 = \int_c^{c+b} \frac{\mu_0 i_1}{2\pi r} a\,dr$$

$$= \frac{\mu_0 i_1 a}{2\pi} \int_c^{c+b} \frac{dr}{r}$$

$$= \frac{\mu_0 i_1 a}{2\pi} \ln\left(\frac{c+b}{c}\right)$$

Therefore mutual inductance,

$$M = M_{12} = \frac{\phi_{12}}{i_1} = \frac{\mu_0 a}{2\pi} \ln\left(1 + \frac{b}{c}\right)$$

183. (a) In accordance with Faraday's law of electromagnetic induction, the changing magnetic field induces an electric field in the ring. Let us imagine the ring to be divided into differential elements of length ds and denote the tangential component of the induced electric field by E_t. The charge on element ds of the ring is $dQ = Q\frac{ds}{2\pi r}$ where r is the radius of the ring. The force exerted on it is $dF_t = dQE_t$, and the resultant torque is $d\tau = rdF_t$.

Thus the total torque experienced by the ring is

$$t = \int d\tau = \int rQ \frac{ds}{2\pi r} E_t = \frac{Q}{2\pi} \int E_1 ds$$

The induced electromotive force along the ring is directly proportional to the rate of change in the magnetic flux, we have

$$\int E_1 ds = -\frac{d\phi}{dt} = -\pi r^2 \frac{dB}{dt}$$

As a result of the torque, the ring, which has a moment of inertia $I = mr^2$, starts to spin with angular acceleration a. During a time interval dt its angular velocity changes by

$$d\omega = \alpha dt = \frac{\tau}{I} dt$$

$$= \frac{Q}{2\pi}\left(-\pi r^2 \frac{dB}{dt}\right)\frac{1}{mr^2} dt = -\frac{Q}{2m} dB$$

Since the magnetic field strength increases from zero to B, the final angular velocity of the ring will be

$$\omega = -\frac{QB}{2m}$$

$\Rightarrow$ The negative sign shows that the direction of the angular velocity vector is opposite to the magnetic induction if Q is positive.

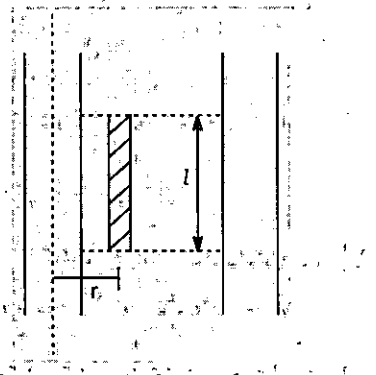
$\Rightarrow$ The final angular velocity does not depend on the radius of the ring, the time over which the magnetic flux changes, or even on how the magnetic flux increases with time.

$\Rightarrow$ In our calculation we ignored the magnetic field produced by the rotating ring.

$\Rightarrow$ Except in the case of a *cylindrical symmetric* uniform field, it is not possible to find the actual value of the induced electric field within the ring because the geometrical structure of the magnetic field is unknown and we do not know the position of the ring in the magnetic field. We can determine the total induced electromotive force, but not the electric field itself.

185. (b) Flux through the strip

$$\phi = \int_a^{d-a} \frac{\mu_0 I}{2\pi r}(ldr) = \frac{\mu_0 Il}{2\pi} \ln\left(\frac{d-a}{a}\right)$$



The other wire produces the same result, so the total flux through the dotted rectangle is

$$\phi_{\text{total}} = \frac{\mu_0 Il}{\pi} \ln\left(\frac{d-a}{a}\right)$$

The inductance of ϕ_{total},

$$L = \frac{\phi_{\text{total}}}{I} = \frac{\mu_0 l}{\pi} \ln\left(\frac{d-a}{a}\right)$$

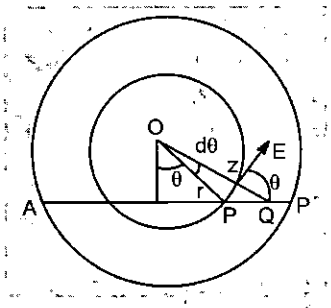
186. (a) Consider a point on the circumference of a circle of radius r ($r < R$). Let E be the electric field along the tangents to the circle. Then $\mathcal{E}$ (electromotive force)

$$= \oint \vec{\mathcal{E}} . d\vec{1} = \mathcal{E}.2\pi r$$

$$\mathcal{E} = -\frac{d\phi_B}{dt} = -\frac{d}{dt}(\pi r^2 B) = -\pi r^2 \frac{dB}{dt}$$

$$\mathcal{E} \times 2\pi r = -\pi r^2 \frac{dB}{dt} \Rightarrow \mathcal{E} = -\frac{1}{2} r \frac{dB}{dt}$$

The minus sign suggests that the induced electric field acts to oppose the change in the magnetic field.



Now consider a point P in the rod and a small distance $dl = PQ$ along AB. The electric field at P is $E = 1/2\, r\, dB/dt$ along the tangent to the circle as shown in figure.

Elementary work done

$$= (E\cos\theta)\, dl = Edl\cos\theta$$

Draw perpendicular PN from P on OQ

Then $PN = rd\theta = dl\cos\theta$

Elementary work done

$$= Erd\theta = \left(\frac{1}{2} r\frac{dB}{dt}\right) rd\theta = \left(\frac{1}{2} r^2 d\theta\right)\frac{dB}{dt}$$

But $\frac{1}{2} r^2 d\theta$ = area of the tiangle OPQ

Total work done in taking unit charge from A to B

$= \frac{dB}{dt} \times$ summation of the areas of elementary triangle

$$= \frac{dB}{dt} \times \text{area}\, OAB$$

= P. D. between the ends of $A.B.$

$$V = \frac{dB}{dt} \times \frac{1}{2} \times 2 \times l\sqrt{R^2 - l^2} = \frac{dB}{dt} l\sqrt{R^2 - l^2}$$

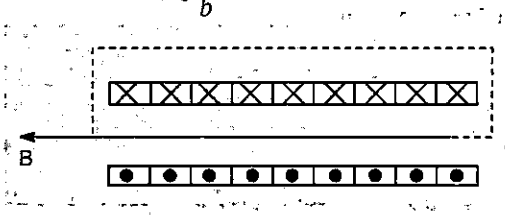
187. (a) $\tau = \frac{dL}{dt} = Mg\sin\theta\,(R + b)$

As the top will keep on precessing the torque will also keep changing the direction.

188. (a) $\oint \vec{\mathbf{B}}\, d\vec{\mathbf{l}} = \mu_0 I$

$$Bb = \mu_0 I$$

$$B = \mu_0 \frac{I}{b}$$



Magnetic flux passing through this double tape

$$\phi = BA = B(lh) \Rightarrow \phi = \frac{\mu_0 I}{b} lh$$

$$\frac{\phi}{I} = \frac{\mu_0 lh}{b} \Rightarrow L = \frac{\mu_0 lh}{b} \Rightarrow \frac{L}{l} = \frac{\mu_0 h}{b}$$

189. (b) In the $r - t$ graph, it is clear that from a to b there is no change in radius and hence no change in area and magnetic flux. Same is the situation from c to d.

Now, $$|\mathcal{E}| = \frac{d}{dt}(\phi)$$

$$|\mathcal{E}| = B\frac{d}{dt}(\pi r^2)$$

$$|\mathcal{E}| = 2\pi Br\frac{dr}{dt}$$

Since $r \propto t$, $\therefore \frac{dr}{dt} = \text{constant}$

$\therefore$ $|\mathcal{E}| \propto r$

191. (d) Magnitude of the circular magnetic field produced is

$$\vec{\mathbf{E}}\, d\vec{\mathbf{l}} = -\frac{d\phi}{dt}$$

$$E2\pi r = \pi r^2 \frac{dB}{dt}$$

$$E = \frac{1}{2} r\frac{dB}{dt}$$

Now $B = \mu_0 nI$

$$E = \frac{1}{2} r\frac{d}{dt}(\mu_0 nI)$$

$$E = \frac{1}{2} r\mu_0 nI' \left(I' = \frac{dI}{dt}\right)$$

where $r < a$

But when $r > a$

$$\int \vec{\mathbf{E}}\, d\vec{\mathbf{l}} = -\frac{d\phi}{dt}$$

$$B2\pi r = \pi a^2 \frac{dB}{dt}$$

$$E = \frac{1}{2}\frac{a^2}{r}\frac{d}{dt}(\mu_0 nI)$$

or $$E = \frac{1}{2}\frac{\mu_0 na^2}{r} I$$

192. (d) When the flip coil is rotated, the magnetic flux through it changes, resulting in an induced e.m.f. $\mathcal{E}$ and induced current $I = \mathcal{E}/R$, where R is the total resistance of the circuit.

As $$I = \frac{dQ}{dt}$$

$$Q = \int dQ = \int Idt$$

From Faraday's Law, induced e.m.f.

$$|\mathcal{E}| = \frac{d\phi_B}{dt}$$

$$Q = \int Idt = \int \frac{E}{r} dt$$

$$= \frac{1}{R}\int \frac{d\phi_B}{dt} dt$$

$$= \frac{1}{R}\int d\phi_B = \frac{\Delta\phi_B}{R}$$

This is a generalized result and can used to determine charge through a circuit in which an e.m.f. and a current is induced.

Initial flux through the coil, $\phi_{Bi} = +NBA$

Final flux through the coil, $\phi_{Bf} = -NBA$

When the coil is turned through 180° its flux reverses; the angle between magnetic field and area vector is reversed.

$$\Delta\phi_B = \phi_{Bf} - \phi_{Bi} = NBA - (-NBA) = 2NBA$$

$$Q = \frac{2NBA}{R}$$

193. (a) Let I = current in one loop. This magnetic flux at the centre of the other coaxial loop at a distance l from the centre of the first loop is

$$B = \frac{\mu_0}{4\pi}\frac{2I\pi a^2}{(a^2+l^2)^{3/2}}$$

where $p_m = I\pi a^2$

= magnetic moment of loop

The flux through the other loop is

$$\phi_{12} = B\pi a^2$$

$$= \frac{\mu_0}{4\pi}\frac{2\pi a^2 I}{(a^2+l^2)^{3/2}}\pi a^2$$

or $$\frac{\phi_{12}}{I} = \frac{\mu_0}{4\pi}\frac{2\pi^2 a^4}{2(a^2+l^2)^{3/2}}$$

$$= \frac{\mu_0 \pi a^4}{2(a^2+l^2)^{3/2}}$$

$$\therefore \quad \phi_{12} = M_{12} I$$

$$\therefore \quad M_{12} = \frac{\phi_{12}}{I} = \frac{1}{2}\frac{\mu_0 \pi a^4}{l^3} \qquad \therefore\ a \ll 1$$

195. (d) $h_{avg} = \int \frac{dh}{\mu} = \int_0^1 \frac{dy}{\mu_0(1+y)} = \frac{\ln(h+1)}{\mu_0}$

196. (d) $L = \frac{\phi}{I}$, $\phi = NAB$

$$B = \mu_0 nI$$

Where $n = \frac{R}{2\pi R}$

$$\therefore \quad \phi = N\pi r^2\left(\mu_0 \frac{N}{2\pi R} I\right)$$

$$\phi = \frac{\mu_0 N^2 r^2 I}{2R}$$

$$L = \frac{\phi}{I} = \frac{\mu_0 N^2 r^2 I}{2R \times I} = \frac{\mu_0 N^2 r^2}{2R}$$

197. (c) By Lenz's law clockwise current is induced in both loops. Greater the area, large will be the induced e.m.f. Therefore, outer loop has greater area.

198. (b) The current amplitude is maximum at resonant frequency ν for which

$$\omega = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{0.12 \times 480 \times 10^{-9}}}$$

Corresponding frequency is

$$\nu = \frac{\omega}{2\pi}\frac{1}{2\pi}\sqrt{\frac{10^8}{5.76}} = 663\,\text{Hz}$$

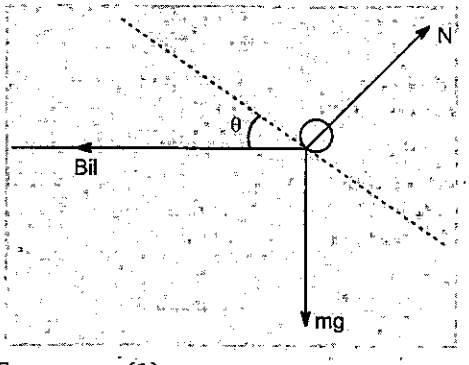
Maximum value of current amplitude is

$$I_{max} = \frac{V}{R} = \frac{230}{23} = 10\,\text{A}$$

199. (c) $Bil\cos\theta = mg\sin\theta$

Here induced e.m.f. across slider is $Bil\cos\theta$ (1)

$\therefore$ induced current $I = \frac{Blv\cos\theta}{R}$



From eqn. (1)

$$Bl\cos\theta\frac{Blv\cos\theta}{R} = mg\sin\theta$$

$$\therefore \quad v = \frac{mgR\sin\theta}{B^2 l^2\cos^2\theta}$$

200. (a) For a solenoid, $L = \mu_0 N^2 \frac{A}{l}$. If x is length of wire and a is area of cross-section, then

$$R = \frac{\rho x}{a} \quad \text{and} \quad m = a.x.D$$

$$Rm = \frac{\rho x}{a} axD, \quad \therefore \quad x = \sqrt{\frac{Rm}{\rho D}}$$

Also, $x = 2\pi rN$, $N = \frac{x}{2\pi r}$ $\left(\therefore L = \frac{\mu_0 N^2 A}{l}\right)$

$$\therefore \quad L = \mu_0\left(\frac{x}{2\pi r}\right)^2\frac{\pi r^2}{l} = \frac{\mu_0}{4\pi l}\frac{Rm}{\rho D}$$

201. (b) We know that

$$\phi_2 = B.A = \mu_0\frac{N_1}{l}i_1 A$$

By definition of mutual Inductance $M = \frac{N_2\phi_2}{i_1}$

By putting the value of ϕ_2, we get $M = \frac{\mu_0 N_1 N_2 A}{l}$

203. (b) The changing magnetic flux through the ring (due to changing field B) produces and electrical field along the ring (tangential to the ring at every point), the magnitude of the electrical field being $E = \frac{-R}{2}\frac{dB}{dt}$, the minus sign indicating that the electrical field opposes the change of the magnetic field. This electric field produces a tangential force

F on the charge q carried by the ring, $|F| = |E|.q = \frac{Rq}{2}\frac{dB}{dt}$.

Angular acceleration $= \frac{d\omega}{dt} = \frac{|F|R}{I} = \frac{|F|R}{mR^2}$, where I is the moment of the ring (Since the ring is nonconducting, the force on the charge tending to produce a current act on the ring itself)

Hence, $\frac{d\omega}{dt} = \frac{1}{mR}\frac{Rq}{2}\frac{dB}{dt} = \frac{q}{2m}\frac{dB}{dt}$

Integrating, $|\omega| = \frac{q}{2m}\left|\vec{\mathbf{B}}(t)\right|$

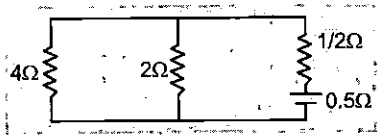
204. (c) $\mathcal{E}$, the voltage across capacitor = induced e.m.f. across the slider = Blv

As capacitor does not permit flow of current, there is magnetic force on the rod to prevent its fall.

$v^2 = 2gx, v = \sqrt{2gx}$

$\therefore \quad \mathcal{E} = Bl\sqrt{2gx}$

205. (a)



$I = \frac{0.5}{11/6} = \frac{3}{11}$ A

206. (c) Induced e.m.f. = Blv, R is internal resistance of seat of e.m.f., *i.e.* of rod

Total resistance of circuit $= R + \frac{R_1R_2}{R_1 + R_2}$

$\therefore \quad I = \frac{Blv}{R + \frac{R_1R_2}{(R_1 + R_2)}}$

$= \frac{Blv(R_1 + R_2)}{R_1R_2 + R(R_1 + R_2)}$

207. (c) $I = \frac{\mathcal{E} - L\frac{dI}{dt}}{R}, \quad \therefore RI = \mathcal{E} - \frac{dI}{dt}L$

(taking integration on both sides w.r.t. 't')

$\therefore \quad \int RIdt = \int \mathcal{E}dt - \int L\frac{dI}{dt}dt$

$\therefore \quad Rq = \int -\frac{d\phi}{dt}dt - L[I]_i^f = \phi_i - L[I_f - I_i]$

$= \phi_i - \phi_f$

$\therefore \quad q = \frac{\phi_i - \phi_f}{R}$ ... (i)

But $\phi_i = \frac{\mu_0 Ia}{2\pi}\int_b^{a+b}\frac{dx}{x} = \frac{\mu_0 Ia}{2\pi}\ln\frac{a+b}{b}$

$\phi_f = \frac{\mu_0 Ia}{2\pi}\ln\frac{2a+b}{a+b}$

$\therefore \quad \frac{\phi_i - \phi_f}{R} = \frac{\mu_0 Ia}{2\pi R}\ln\frac{2a+b}{b}$

Then, $q = \frac{\mu_0 Ia}{2\pi R}\ln\frac{2a+b}{b}$

208. (c) $V_{av} = \frac{V_m}{\pi}\int_0^{\pi}\sin\theta d\theta = \frac{2V_m}{\pi}$

$V_{eff}^2 = \frac{V_m^2}{\pi}\int_0^{\pi}\sin^2\theta d\theta = \frac{V^2m}{\pi}$

209. (a) Component of weight along the inclined plane $= mg\sin\theta$

Again, $F = Bil = B\frac{Blv}{R}l = \frac{B^2l^2v}{R}$

Now, $\frac{B^2l^2v}{R} = mg\sin\theta$

or $v = \frac{mgR\sin\theta}{B^2l^2}$

210. (b) $E(2\pi a) = \frac{B_0}{t}\pi b^2$; $\tau = \lambda(2\pi a)aE$; $\omega = \frac{\tau}{I}t$

211. (a) Impedance is given by $Z^2 = R^2 + (X_L - X_C)^2$

Hence, $Z = \sqrt{(60)^2 + (X_L - X_C)^2} = 135$

Then, $|X_L - X_C| = \sqrt{(135)^2 - (60)^2} = 120.9\Omega$

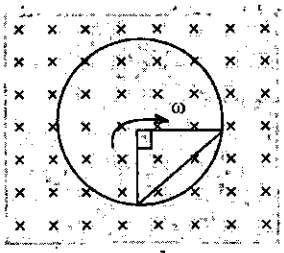
Also, $|\tan\theta| = \frac{|X_L - X_C|}{R} = \frac{120.9}{60}$ and $|\theta| = 63.6°$

The power is

$P = \frac{V^2}{Z}\cos\theta = \frac{(120)^2}{135}\cos(63.6)° = 47.4$ W

212. (d) $|\vec{\mathcal{E}}|$ = Magnitude of induced e.m.f.

$= \frac{Bl^2}{2}\omega$ ...(1)

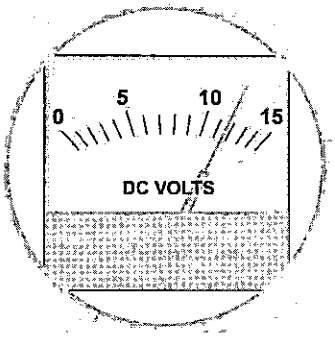


$\theta = \frac{\pi}{2} = \frac{l}{r}$

$\therefore \quad |\vec{\mathcal{E}}| = l = \frac{\pi}{2}r$ ...(2)

Using (2) in (1), we get

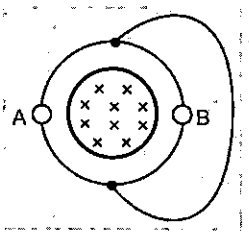
$\therefore \quad |\vec{\mathcal{E}}| = B\frac{\left(\frac{\pi}{2}r\right)^2\omega}{2} = \frac{\pi^2r^2B\omega}{8}$



PROBLEMS

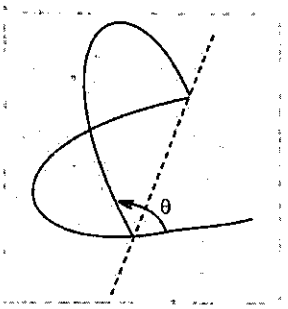
Level ② More than One Alternative is/are Correct

1. A very long solenoid perpendicular to the page generates a magnetic field into the page whose magnitude increases with time. This induces an emf in a conducting wire loop around the solenoid which lights two identical bulbs connected in series along the wire. Now two points diametrically opposed on the wire loop are shorted with another wire lying to the right of bulb B in the plane of the page. After the shorting wire is inserted.



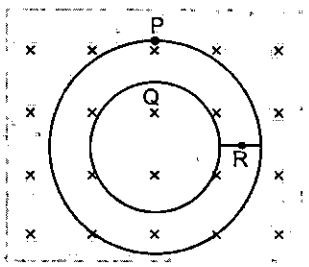
(a) bulb A goes out
(b) bulb B gets brighter
(c) bulb B goes out
(d) bulb A gets brighter

2. A uniform circular loop of radius a and resistance R is placed perpendicular to a uniform magnetic field B. One half of the loop is rotated about the diameter with angular velocity ω as shown. Then, the current in the loop is :



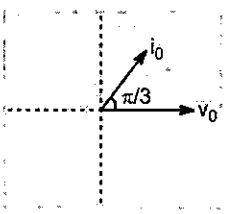
(a) zero, when θ is zero
(b) $\frac{\pi a^2 B\omega}{2R}$, when θ is zero
(c) zero, when $\theta = \pi/2$
(d) $\frac{\pi a^2 B\omega}{2R}$, when $\theta = \pi/2$

3. Figure shown plane figure made of a conductor located in a magnetic field along the inward normal to the plane of the figure. The magnetic field starts diminishing. Then the induced current:



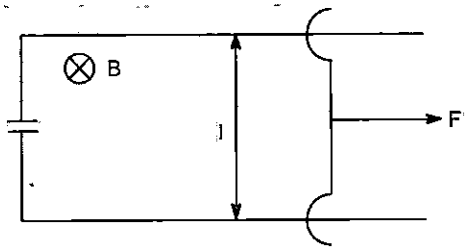
(a) at point P is clockwise
(b) at point Q is anticlockwise
(c) at point Q is clockwise
(d) at point R is zero

4. For an LCR series circuit phasors of current i and applied voltage $v = v_0 \sin\omega t$ are shown in diagram at $t = 0$. Which of the following is/are correct



(a) At $t = \frac{\pi}{2\omega}$, instantaneous power supplied by source is negative
(b) From $t > 0$ to $t < \frac{2\pi}{3\omega}$ average power supplied by source is positive
(c) At $t = \frac{5\pi}{6\omega}$, instantaneous power supplied by source is negative
(d) If ω is increased slightly, angle between the two phasors decreases.

5. A conducting wire of length l and mass m can slide without friction on two parallel rails and is connected to capacitance C. The whole system lies in a magnetic field B and a constant force F is applied to the rod. Then :



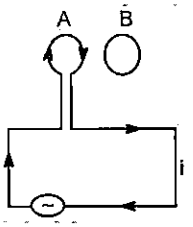
(a) the rod moves with constant velocity

(b) the rod moves with an acceleration of $\frac{F}{m+B^2l^2C}$

(c) there is constant charge on the capacitor.

(d) charge on the capacitor increases with time

6. Two circular coils A and B are facing each other as shown in figure. The current i through A can be altered:



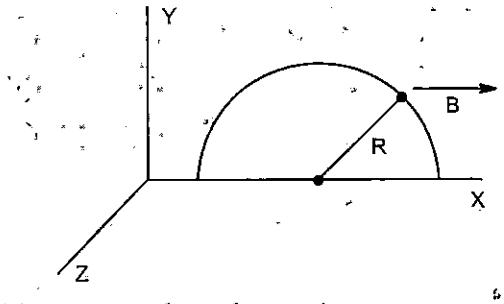
(a) there will repulsion between A and B if i is increased

(b) there will be attraction between A and B if i is increased

(c) there will be neither attraction nor repulsion when i is changed

(d) attraction or repulsion between A and B depends on the direction of current. It does not depend whether the current is increased or decreased.

7. A semicircle conducing ring of radius R is placed in the xy plane, as shown in the figure. A uniform magnetic field is set up along the x-axis. No emf, will be induced in the ring. If



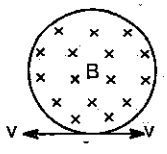
(a) it moves along the x-axis

(b) it moves along the y-axis

(c) it moves along the z-axis

(d) it remain stationary

8. A circular conducting loop of radius r_0 and having resistance per unit length λ as shown in the fig. is placed in a magnetic field B which is constant in space and time. The ends of the loop are crossed and pulled in opposite directions with a velocity v such that the loop always remains circular and the radius of the loop goes on decreasing, then :



(a) Radius of the loop changes with r as $r = r_0 - vt/\pi$

(b) EMF induced in the loop as a function of time is $\mathcal{E} = 2Bv[r_0 - vt/\pi]$

(c) Current induced in the loop is $I = \frac{Bv}{2\pi\lambda}$

(d) Current induced in the loop is $I = \frac{Bv}{\pi\lambda}$

9. The e.m.f., induced in a coil of wire, which is rotating in a magnetic field, does not depend on:

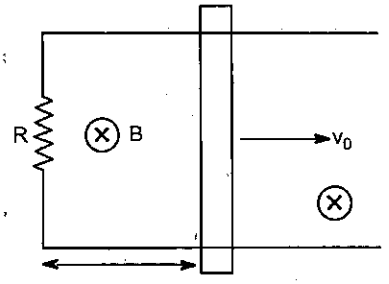
(a) the angular speed of rotation

(b) the area of the coil

(c) the number of turns on the coil

(d) the resistance of the coil

10. A conducting rod of length l is moved at constant velocity v_0 on two parallel, conducting, smooth, fixed rails, which are placed in a uniform constant magnetic field B perpendicular to the plane of the rails as shown in figure. A resistance R is connected between the two ends of the rail. Then, which of the following is/are correct ?



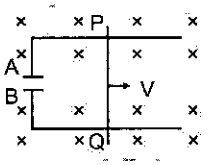
(a) The thermal power dissipated in the resistor is equal to the rate of work done by an external person pulling the rod

(b) If applied external force is doubled, then a part of the external power increases the velocity of the rod

(c) Lenz's law is not satisfied if the rod is accelerated by an external force

(d) If resistance R is doubled, then power required to maintain the constant velocity V_0 becomes half

11. A conducting rod PQ of length L = 1.0 m is moving with a uniform speed v = 20 m/s in a uniform magnetic field B = 4.0 T directed into the paper A capacitor of capacity $C = 10\mu F$ is connected as shown in figure. Then:



(a) $q_A = +800\mu C$ and $q_B = -800\mu C$

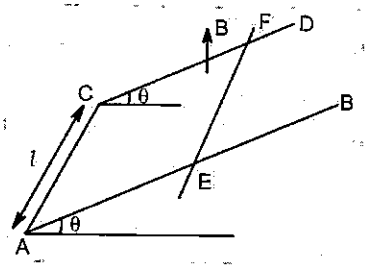
(b) $q_A = -800\mu C$ and $q_B = +800\mu C$

(c) $q_A = 0 = q_B$

(d) charged stored in the capacitor increases exponentially will time

12. AB and CD are smooth parallel rails, separated by a distance l, and inclined to the horizontal at an angle θ. A uniform magnetic field of magnitude B, directed

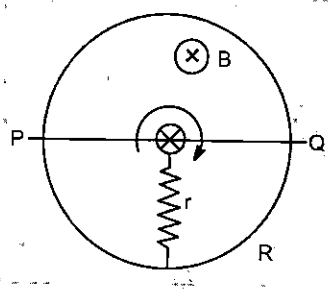
vertically upwards, exists in the region. *EF* is a conductor of mass *m*, carrying a current *i*. For *EF* to be in equilibrium,



(a) *i* must flow from *E* to *F*
(b) $Bil = mg \tan\theta$
(c) $Bil = mg \sin\theta$
(d) $Bil = mg$

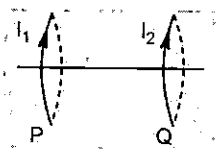
13. In the previous question (12), if *B* is normal to the plane of the rails:
(a) $Bil = mg \tan\theta$
(b) $Bil = mg \sin\theta$
(c) $Bil = mg \cos\theta$
(d) equilibrium cannot be reached

14. In figure, *R* is a fixed conducting ring of negligible resistance and radius *a*. *PQ* is a uniform rod of resistance *r*. It is hinged at the centre of the ring and rotated about this point in clockwise direction with a uniform angular velocity ω There is a uniform magnetic field of strength *B* pointing inward and *r'* is a stationary resistance. Then :



(a) current through *r* is zero
(b) current through *r* is $\frac{2B\omega a^2}{5r}$
(c) direction of current in external resistance *r* is from centre to circumference.
(d) Direction of current in external resistance *r* is from circumference to center

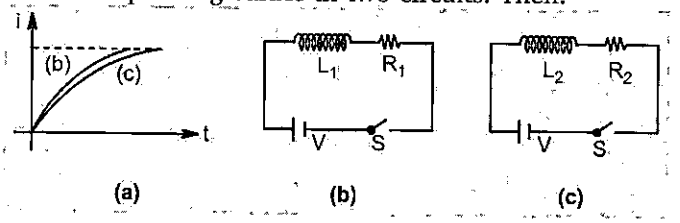
15. Two circular coils *P* and *Q* are fixed coaxially & carry currents I_1 and I_2 respectively:



(a) if $I_2 = O$ and *P* moves towards *Q*, a current in the same direction as I_1 is induced in *Q*
(b) if $I_1 = O$ and *Q* moves towards *P*, a current in the opposite direction to that of I_2 is induced in *P*.
(c) when $I_1 \neq 0$ and $I_2 \neq 0$ are in the same direction then the two coils tend to move apart.
(d) When $I_1 \neq 0$ and $I_2 \neq 0$ are in opposite directions then the coils tends to move apart.

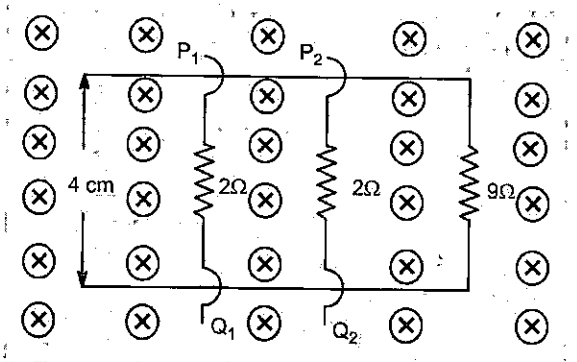
16. A bar magnet is moved along the axis of copper ring placed far away from the magnet. Looking from the side of the magnet, an anticlockwise current is found to be induced in the ring. Which of the following may be true?
(a) The south pole faces the ring and the magnet moves towards it.
(b) The north pole faces the ring and the magnet moves towards it.
(c) The south pole faces the ring and the magnet moves away from it.
(d) The north pole faces the ring and the magnet moves away from it.

17. Current growth in two *L-R* circuits (b) and (c) as shown in Fig. (a). Let L_1, L_2, R_1 and R_2 be the corresponding values in two circuits. Then:



(a) $R_1 > R_2$ (b) $R_1 = R_2$
(c) $L_1 > L_2$ (d) $L_1 < L_2$

18. In the figure shown, the wires P_1Q_1 and P_2Q_2 are made to slide on the rails with same speed of 5 cms^{-1}. In this region a magnetic field of 1 T exists. The electric current in the 9Ω resistance is :

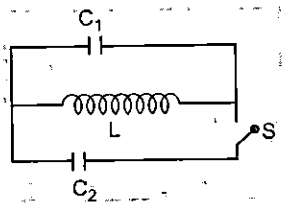


(a) zero if both wires slide towards left
(b) zero if both wires slide in opposite directions
(c) 0.2 mA if both wires move towards left
(d) 0.2 mA if both wires move in opposite directions

19. For *L-R* circuit, the time constant is equal to:
(a) twice the ratio of the energy stored in the magnetic field to the rate of the dissipation of energy in the resistance.
(b) the ratio of the energy stored in the magnetic field to the rate of dissipation of energy in the resistance.
(c) half of the ratio of the energy stored in the magnetic field to the rate of dissipation of energy in the resistance.

(d) square of the ratio of the energy stored in the magnetic field to the rate of dissipation energy in the resistance.

20. At a moment ($t = 0$) when charge on capacitor C_1 is zero, the switch is closed. If I_0 be the current through inductor at that instant, for $t > 0$,



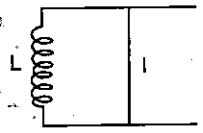
(a) maximum current through inductor equals $I_0/2$.

(b) maximum current through inductor equals $\frac{C_1 I_0}{C_1 + C_2}$

(c) maximum charge on $C_1 = \frac{C_1 I_0 \sqrt{LC_1}}{C_1 + C_2}$

(d) maximum charge on $C_1 = I_0 C_1 \sqrt{\frac{L}{C_1 + C_2}}$

21. Two parallel resistanceless rails are connected by an inductor of inductance L at one end as shown in figure. A magnetic field B exists in the space which is perpendicular to the plane of the rails. Now a conductor of length l and mass m is placed transverse on the rail and given an impulse J towards the rightward direction. Then select the correct option (s):



(a) Velocity of the conductor is half of the initial velocity after a displacement of the conductor $d = \sqrt{\frac{3J^2L}{4B^2l^2m}}$

(b) Velocity of the conductor is half of the initial velocity after a displacement of the conductor $d = \sqrt{\frac{3J^2L}{B^2l^2m}}$

(c) Current flowing through the inductor at the instant when velocity of the conductor is half of the initial velocity is $i = \sqrt{\frac{3J^2}{4Lm}}$

(d) Current flowing through the inductor at the instant when velocity of the conductor is half of the initial velocity is $i = \sqrt{\frac{3J^2}{mL}}$

22. Coefficient of mutual inductance for the given coils does not depend on :

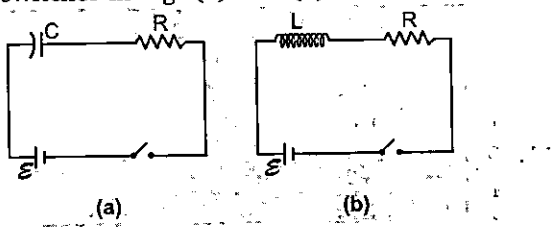
(a) their relative orientation
(b) their geometry
(c) Rate at which current changes in the coil
(d) the medium in which the coil lies

23. No potential difference develops across the length of a rod moving in a uniform magnetic field when

(a) $\vec{V} \| \vec{B} \| \vec{l}$ (b) $\vec{V} \| \vec{B}$

(c) $\vec{V} \| \vec{l}$ (d) $\vec{B} \| \vec{l}$

24. The switches in Fig. (a) and (b) are closed at $t = 0$



(a) The charge on C just after $t = 0$ is $\mathcal{E}C$.
(b) The charge on C long after $t = 0$ is $\mathcal{E}C$
(c) The current in L just after $t = 0$ is $\mathcal{E}/R$
(d) The current in L long after $t = 0$ is $\mathcal{E}/R$

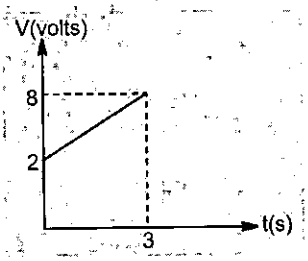
25. An L-R circuit with a battery is connected at $t = 0$. Which of the following quantities is not zero just after the circuit:

(a) current in the circuit
(b) magnetic field energy in the inductor
(c) power delivered by the battery
(d) emf induced in the inductor

26. The symbols L, C, R represent inductance, capacitance and resistance respectively. Dimension of frequency are given by the combination:

(a) $1/RC$ (b) R/L
(c) $\frac{1}{\sqrt{LC}}$ (d) C/L

27. A circuit element is placed in a closed box. At time $t = 0$, constant current generator supplying a current of 1 amp, is connected across the box. Potential difference across the box varies according to graph shown in figure. The element in the box is:



(a) resistance of 2Ω
(b) battery of emf 6 V
(c) inductance of 2H
(d) capacitance of 0.5 F

28. An A.C. Source of voltage V and of frequency 50 Hz is connected to an inductor of 2H and negligible resistance. A current of r.m.s., value I flows in the coil. When the frequency of the voltage is changed to 400 Hz keeping the magnitude of V the same, the current is now:

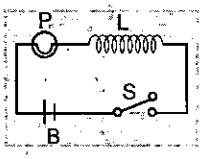
(a) $8I$ in phase with
(b) $4I$ and leading by 90° from V
(c) $I/4$ and lagging by 90° from V
(d) $I/8$ and lagging by 90° from V

29. In a series *R-L-C* circuit, the frequency of the source is half of the resonance frequency. The name of the circuit will be:
(a) capacitive (b) inductive
(c) purely resistive (d) data insufficient

30. The reactance of a circuit is zero. It is possible that the circuit contains:
(a) an inductor and a capacitor
(b) an inductor but no capacitor
(c) a capacitor but no inductor
(d) neither an inductor nor a capacitor

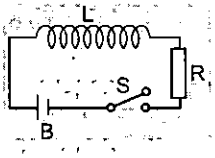
31. Two coil *A* and *B* have coefficient of mutual inductance $M = 2H$. The magnetic flux passing through coil *A* changes by 4 Weber in 10 seconds due to the change in current in *B*. Then
(a) change in current in *B* in this time interval is 0.5 A
(b) the change in current in *B* in this time interval is 2A
(c) the change the current in *B* in this time interval is 8A
(d) a change in current of 1A in coil *A* will produce a change in flux passing through B by 4 Weber.

32. In figure, a lamp *P* is in series with an iron-core inductor *L*. When the switch *S* is closed, the brightness of the lamp rises relatively slowly to its full brightness than it would do without the inductor. This is due to:



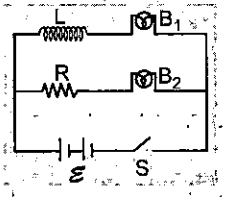
(a) the low resistance of *P*
(b) the induced-emf in *L*
(c) the low resistance of *L*
(d) the high voltage of the battery *B*

33. In figure, the switch *S* is closed so that a current flows in the iron-core inductor which has inductance *L* and the resistance *R*. When the switch is open, a spark is obtained in it at the contact. The spark is due to:



(a) a slow flux change in *L*
(b) a sudden increase in the emf of the battery *B*
(c) a rapid flux change in *L*
(d) a rapid flux change in *R*

34. An inductor *L*, a resistance *R* and two identical bulbs B_1 and B_2 are connected to a battery through a switch *S* as shown in the figure. The resistance of coil having inductance *L* is also *R*. Which of the following statement gives the correct description of the happenings when the switch *S* is closed?



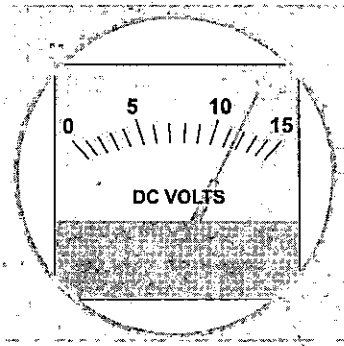
(a) The bulb B_2 lights up earlier than B_1 and finally both the bulbs shine equally bright.
(b) B_1 light up earlier and finally both the bulbs acquire equal brightness.
(c) B_2 lights up earlier and finally B_1 shines brighter than B_2.
(d) B_1 and B_2 light up together with equal brightness all the time.

35. A circuit has three elements, a resistance of 11Ω, a coil of inductive reactance 120Ω and a capacitive reactance of 120Ω in series and connected to an A.C. Source of 110 V, 60 Hz. Which of the three elements have minimum potential difference?
(a) Resistance
(b) Capacitance
(c) Inductor
(d) All will have equal potential difference

ANSWERS

Level-2: More than One Alternative is/are Correct

1.	(c, d)	**2.**	(a, d)	**3.**	(a, c, d)	**4.**	(b, c, d)	**5.**	(b, d)	**6.**	(a)
7.	(a, b, c, d)	**8.**	(a, b, d)	**9.**	(d)	**10.**	(a, b, d)	**11.**	(a)	**12.**	(a, b)
13.	(b)	**14.**	(b, d)	**15.**	(b, d)	**16.**	(b, c)	**17.**	(b, d)	**18.**	(b, c)
19.	(a)	**20.**	(d)	**21.**	(a, c)	**22.**	(c)	**23.**	(a, b, c, d)	**24.**	(b, d)
25.	(d)	**26.**	(a, b, c)	**27.**	(d)	**28.**	(d)	**29.**	(a)	**J.**	(a, d)
31.	(b)	**32.**	(b)	**33.**	(c)	**34.**	(a)	**35.**	(a)		



SOLUTION

Level-2: More than One Alternative is/are Correct

1. (c, d)
Bulb B has been shorted.

2. (a, d)
Flux changes due to the rotation of the semi-circle.

5. (b, d)

$$i = \frac{dq}{dt} = \frac{d}{dt}(CvBl) = CBl\frac{dv}{dt} = CBla$$

$$\therefore \quad F - CB^2la = ma$$

$$\Rightarrow \quad a = \frac{F}{M + B^2l^2C}$$

$\Rightarrow$ e.m.f. increases

$\Rightarrow$ charge increases

8. (a, b, d)
Perimeter is decreasing at a rate of $2v$

$$\therefore \quad \frac{d}{dt}(2\pi r) = 2v$$

$$\Rightarrow \quad \frac{dr}{dt} = \frac{v}{\pi}$$

$$\therefore \quad r = (r_0 - \frac{v}{\pi}t)$$

$$\phi = B \cdot \pi r^2$$

$$\Rightarrow \quad \mathcal{E} = \left|\frac{-d\phi}{dt}\right| = B \cdot 2\pi \cdot r\frac{dr}{dt}$$

$$\therefore \quad \mathcal{E} = 2B\pi(r_0 - \frac{v}{\pi}t)\frac{v}{\pi}$$

$$= 2Bv(r_0 - \frac{v}{\pi}t)$$

$$I = \frac{\phi}{R} = \frac{2Bvr}{\lambda \cdot 2\pi r} = \frac{Bv}{\pi\lambda}$$

10. (a, b, d)
(a) Rate of work done by external agent is

$\frac{d\mathcal{E}}{dt} = \frac{BILdx}{dt} = BILv$ and thermal power dissipated in the resistor $= \mathcal{E}I = (BvL)I$ clearly both are equal.

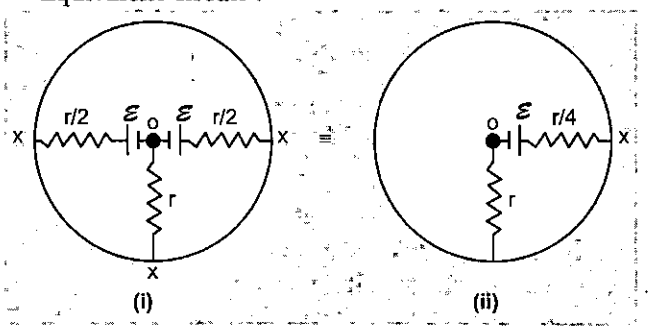
(b) If applied external force is doubled, the rod will experience a net force and hence acceleration. As a result velocity increases.

Since, $I = \frac{\mathcal{E}}{R}$

(d) On doubling R, current and hence required power become half.

Since, $P = BIL_v$

14. (b, d)
Equivalent circuit :



Induced e.m.f. $\mathcal{E} = \frac{B\omega r^2}{2} = \left(\frac{B\omega a^2}{2}\right)$ ($\because$ radius $= a$)

By nodal equation :

$$4\left(\frac{x - \mathcal{E}}{r}\right) + \left(\frac{x - 0}{r}\right) = 0$$

$$5x = 4\mathcal{E}$$

$$\Rightarrow \quad x = \frac{4\mathcal{E}}{5} = \frac{2B\omega a^2}{5r}$$

and $$I = \frac{x}{\mathcal{E}} = \frac{2B\omega a^2}{5r}$$

Also direction of current in (i) will be toward negative terminal, *i.e.*, from rim to origin Alternating, by equivalent of cells figure.

$$I = \frac{\mathcal{E}}{r + \frac{r}{4}} = \frac{4\mathcal{E}}{5r}$$

18. (b, c)
Each wire can be replaced by a battery whose e.m.f. is equal to

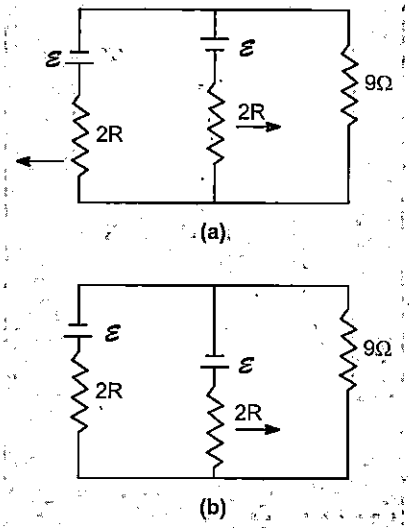
$$Blv = 1 \times 4 \times 10^{-2} \times 5 \times 10^{-2}$$
$$= 20 \times 10^{-4}\,\text{V}$$

The polarity of the battery can be given by Fleming's right hand rule. When both wire move in opposite direction, the circuit diagram looks like as shown in figure.

The effective e.m.f. of the two batteries shown in the diagram is zero.

So, (b) is correct and (d) is wrong.

When both wires move towards left, the circuit diagram looks like as shown in figure.



Effective e.m.f. of two batteries shown in $\mathcal{E} = (20 \times 10^{-4}\,\text{V})$ and internal resistance is 1Ω

Hence, current in the circuit is

$$i = \frac{20 \times 10^{-4}}{10} = 0.2\,\text{mA}$$

Hence, (c) is correct and (a) is wrong.

21. (a, c)

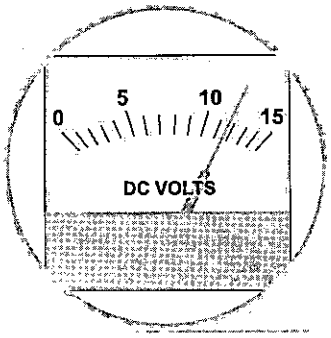
$$L\frac{di}{dt} = Bvl$$

$$i = \frac{Bl}{L}x$$

and $$\frac{B^2l^2x}{L} = -mv\frac{dv}{dx}$$

$$d = \sqrt{\frac{3v_0^0 mL}{4B^2l^2}}$$

where $$v_0 = \frac{J}{M}$$

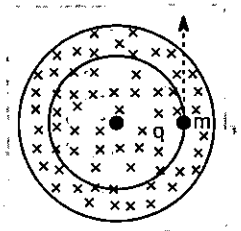


PROBLEMS

Level 3 Comprehension Based Problems

Passage 1

The centre cross-section of a long cylindrical region containing uniform but time varying magnetic field B is shown in the Fig. A particle of unknown constant mass and variable positive charge moves in a plane in such a way so that the radius of the circle remains constant.



Now answer the following questions.

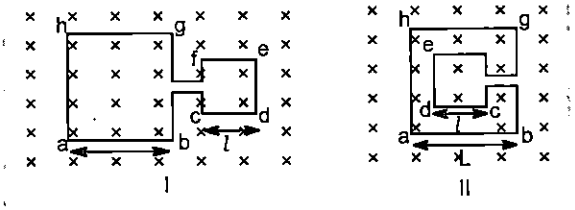
1. If the magnetic field is increased by 2%, the speed will:
(a) increase by 2% (b) increase by 1%
(c) decrease by 2% (d) decrease by 1%

2. In the above question, the charge will :
(a) increase by 2% (b) increase by 1%
(c) decrease by 2% (d) decrease by 1%

3. Choose the correct option :
(a) Both charge and mass can be constant
(b) If mass is constant, charge must change
(c) If mass is constant, charge must increase
(d) None of these are correct

Passage 2

The adjoining figure shows two different arrangements in which two square wire frame are placed in a uniform, constantly decreasing magnetic field B.



1. The value of magnetic flux in each case is given by:
(a) Case I : $\Phi = \pi(\Lambda^2 + l^2)B$; Case II: $\Phi = \pi(L^2 - l^2)B$
(b) Case I : $\Phi = \pi(L^2 + l^2)B$; Case II: $\Phi = \pi(L^2 + l^2)B$
(c) Case I : $\Phi = (L^2 + l^2)B$; Case II : $\Phi = (L^2 + l^2)B$
(d) Case I : $\Phi = (L + l^2)B$; Case II : $\Phi = \pi(L - l)^2 B$

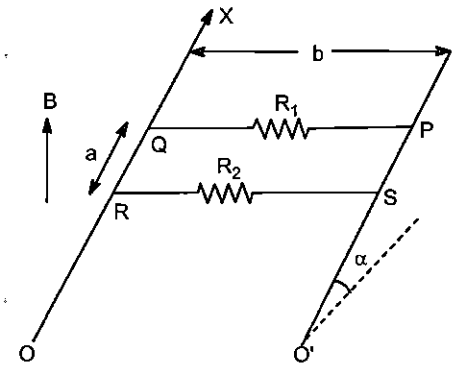
2. The direction of induced current in the case I is:
(a) from a to b and from c to d
(b) from a to b and from f to e
(c) from be to a and from d to c
(d) from b to a and from e to f

3. The direction of induced current in the case II is:
(a) from a to b and from c to d
(b) from b to a and from f to e
(c) from b to a and from c to d
(d) from a to b and from d to c

4. If I_1 and I_2 are the magnitudes of induced current in the case I and II, respectively, then:
(a) $I_1 = I_2$
(b) $I_1 > I_2$
(c) $I_1 < I_2$
(d) nothing can be said

Passage 3

Two resistors R_1 and R_2 can slide without friction along two parallel metal guides directed at an angle α to the horizontal and separated by a distance b. The guides are not connected at the bottom as shown. The entire system is placed in an upward magnetic field which decreases with time as $\frac{dB}{dt} = -a$, and also it decreases along the guides as $\frac{dB}{dt} = -\beta, x = 0$ and x- axis is along the guide as shown in figure. The resistors are made to slide with a constant velocity v downwards.



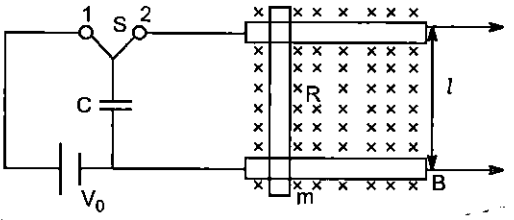
1. The e.m.f. induced due to decrease of B with time only and the direction of current in loop, respectively:
(a) $ab\alpha$ (anticlockwise)
(b) $\frac{a^2}{b}\alpha$ (clockwise)
(c) $ab\alpha$ (clockwise)
(d) $\frac{a^2}{b}\alpha$ (anticlockwise)

2. The e.m.f. induced due to decrease in B due to change in position only :
(a) βabv (clockwise)
(b) βabv (anticlockwise)
(c) $\beta^2 av$ (clockwise)
(d) $\beta^2 abv$ (anticlockwise)

3. The net current in loop is :
(a) $\frac{ab\alpha + ab\beta v}{R_1 + R_2}$ (b) $\frac{ab\alpha - ab\beta v}{R_1 + R_2}$
(c) $\frac{a^2 / b\alpha + \beta^2 av}{R_1 + R_2}$ (d) $\frac{a^2/b\alpha + \beta^2 av}{\frac{R_1 R_2}{R_1 + R_2}}$

Passage 4

One end of a horizontal fixed track of gauge l and negligible resistance is connected to a capacitor of capacitance C charged to voltage V_0. The inductance of the assembly is negligible. The system is palced in a homogenous, vertical magnetic field of induction B as shown in the Fig.



A frictionless conducting rod of mass m and negligible resistance is palced perpendicularly on to the track. The polarity of the capacitor is such that the rod is repelled from the capacitor. Now the switches is turned from 1 to 2.

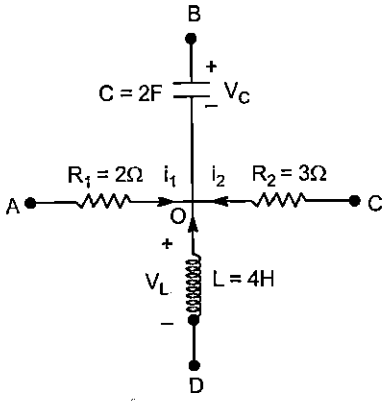
1. What is the maximum velocity of the rod ?
(a) $\frac{BlCV_0}{m + B^2 l^2 C}$ (b) $\frac{2BlCV_0}{m + B^2 l^2 C}$
(c) $\frac{BlCV_0}{2(m + B^2 l^2 C)}$ (d) none of these

2. Find the minimum charge on the capacitor :
(a) $\frac{B^2 l^2 CV_0}{2m + B^2 l^2 C}$ (b) $\frac{B^2 l^2 C^2 V_0}{m + B^2 l^2 C}$
(c) $\frac{2B^2 l^2 C^2 V_0}{m + B^2 l^2 C}$ (d) none of these

3. Find the maximum efficiency of this electromagnetic gun :
(a) $\frac{1}{\left[\frac{\sqrt{m}}{Bl\sqrt{C}} + \frac{Bl\sqrt{C}}{\sqrt{m}}\right]^2}$
(b) $\frac{1}{\left[\frac{\sqrt{m}}{Bl\sqrt{C}} + \frac{2Bl\sqrt{C}}{\sqrt{m}}\right]^2}$
(c) $\frac{1}{\left[\frac{\sqrt{m}}{2Bl\sqrt{C}} + \frac{Bl\sqrt{C}}{\sqrt{m}}\right]}$
(d) none of these

Passage 5

In figure, $i_1 = 10e^{-2t}A$, $i_2 = 4A$ and $V_C = 3e^{-2t}V$.

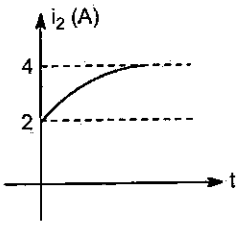


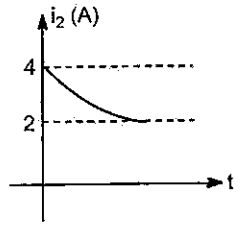
1. The current i_L is :

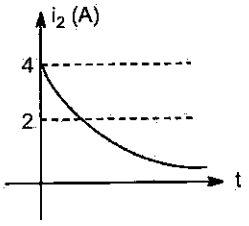
(a) $[2-2(1-e^{-2t})]$ A (b) $[2+2(1-e^{-2t})]$ A

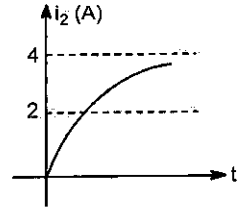
(c) $[3-2(1-e^{-2t})]$ A (d) $[2+3(1-e^{-2t})]$ A

2. The variation of current in the inductor with time can be represented as :

(a)


(b)


(c)


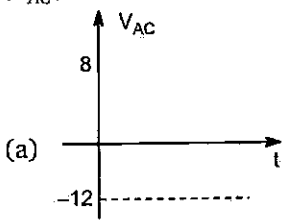
(d)


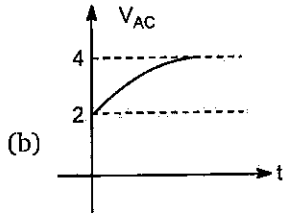
3. The potential difference across inductor V_L is :

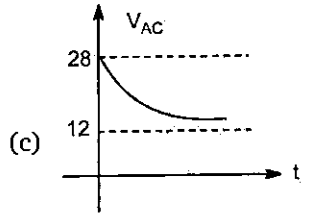
(a) $8e^{-2t}$V (b) $9e^{-2t}$V

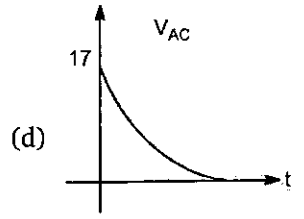
(c) $16e^{-2t}$V (d) $18e^{-2t}$V

4. The variation of potential difference across A and C (V_{AC}) with time can be represented as :

(a)


(b)


(c)


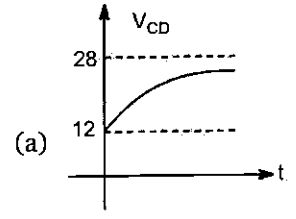
(d)


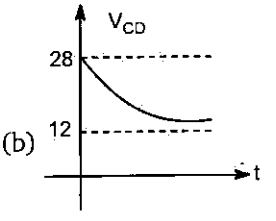
5. The potential difference across $AB(V_{AB})$ is :

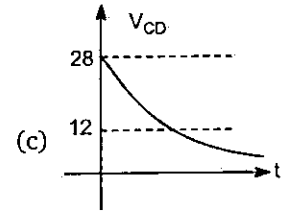
(a) $8e^{-2t}$V (b) $\frac{1}{2}e^{-3t}$V

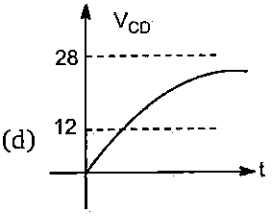
(c) $17e^{-2t}$V (d) $16e^{-2t}$V

6. The variation of potential difference across C and D (V_{CD}) with time can be expressed as :

(a)


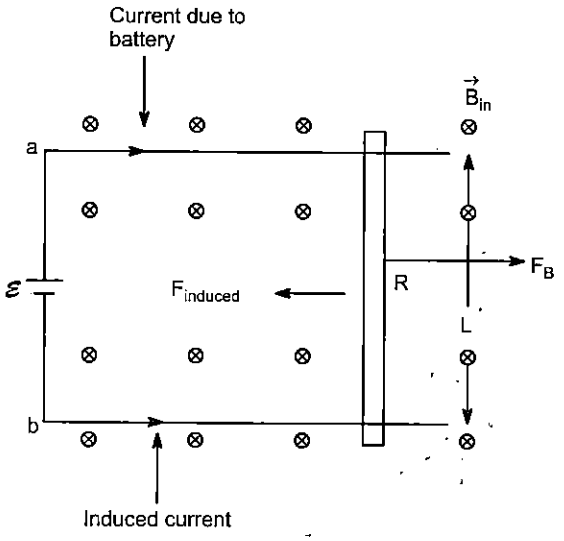
(b)


(c)


(d)


Passage 6

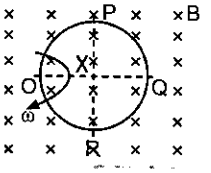
In figure shown, the rod has a resistance R, the horizontal rails have negligible friction. A battery of e.m.f. $\mathcal{E}$ and negligible internal resistance is connected between points a and b. The rod is initially at rest.



1. The force on the rod as a function of the speed v (where $\tau = mR/Bl^2$) is :
   (a) $\frac{\mathcal{E}}{Bl}(1-e^{-t/\tau})$ (b) $\frac{\mathcal{E}}{Bl}(1+e^{-t/\tau})$
   (c) $\frac{3}{2}\frac{\mathcal{E}}{Bl}(1-e^{-t/\tau})$ (d) $\frac{\mathcal{E}}{2Bl}(1-e^{-t/\tau})$
2. After some time the rod will approach a terminal speed. Find an expression for it.
   (a) $\frac{3}{2}\frac{\mathcal{E}}{Bl}$ (b) $\frac{\mathcal{E}}{2Bl}$
   (c) $\frac{\mathcal{E}}{Bl}$ (d) $\frac{2\mathcal{E}}{Bl}$
3. The current when the rod attains its terminal speed is
   (a) $\frac{2\mathcal{E}}{R}$ (b) $\frac{\mathcal{E}}{R}$
   (c) $\frac{3}{2}\frac{\mathcal{E}}{R}$ (d) $\frac{\mathcal{E}}{2R}$

Passage 7

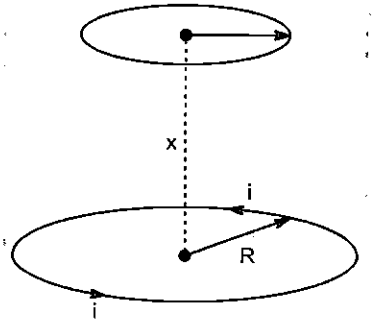
A conducting ring of radius a is rotated about a point O on its periphery as shown in the figure. in a plane perpendicular to uniform magnetic field B which exists everywhere. The rotational velocity is ω



1. Select the correct statement(s) related to the potential of the points P, Q and R:
   (a) $V_P - V_O > 0$ and $V_R - V_O < 0$
   (b) $V_P = V_R > V_O$
   (c) $V_O > V_P = V_Q$
   (d) $V_Q - V_P = V_P - V_O$
2. Select the correct statement(s) related to the magnitude of potential differences:
   (a) $V_P - V_O = \frac{1}{2}B\omega a^2$
   (b) $V_P - V_Q = \frac{1}{2}B\omega a^2$
   (c) $V_Q - V_O = 2B\omega a^2$
   (d) $V_P - V_R = 2B\omega a^2$
3. Select the correct statement(s) related to the induced current in the ring:
   (a) Current flows from $Q \longrightarrow P \longrightarrow O \longrightarrow R \longrightarrow Q$
   (b) Current flows from $Q \longrightarrow R \longrightarrow O \longrightarrow P \longrightarrow Q$
   (c) Current flows from $Q \longrightarrow P \longrightarrow O$ and from $Q \longrightarrow R \longrightarrow O$
   (d) No current flows

Passage 8

Figure shows two parallel and coaxial loop. The smaller loop (radius r) is above the larger loop (radius R), by distance $x >> R$. The magnetic field due to current i in the larger loop is nearly constant throughout the smaller loop. Suppose that x is increasing at a constant rate of $dx/dt = v$.



1. Determine the magnetic flux through the smaller loop as a function of x :

(a) $\frac{\mu_0 iR^2\pi r^2}{(R^2+x^2)^{3/2}}$ (b) $\frac{\mu_0 iR^2\pi r^2}{2(R^2+x^2)^{3/2}}$

(c) $\frac{2\mu_0 iR^2\pi r^2}{(R^2+x^2)^{3/2}}$ (d) $\frac{\sqrt{2}\mu_0 iR^2\pi r^2}{(R^2+x^2)^{3/2}}$

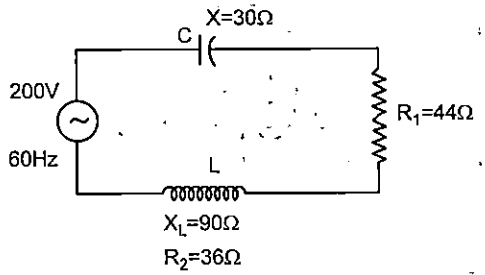
2. The induced e.m.f. and the direction of the induced current in the smaller loop is :

(a) $\frac{\mu_0\pi iR^2r^2}{x^4}v$ (b) $\frac{\mu_0\pi iR^2r^2}{2x^4}v$

(c) $\frac{3}{2}\frac{\mu_0\pi iR^2r^2}{x^4}v$ (d) $\frac{\mu_0\pi iR^2r^2}{3x^4}v$

Passage 9

A series circuit connected across a 200V, 60Hz line consists of a capacitive reactance 30Ω a noninductive resistor of 44Ω and a coil of inductive reactance 90Ω and resistence 36Ω as shown in the diagram.



1. The potential difference across the coil is :

(a) 100V (b) 194V

(c) 97V (d) Zero

2. The power used in the circuit is :

(a) 320W (b) 144W

(c) 160W (d) 96W

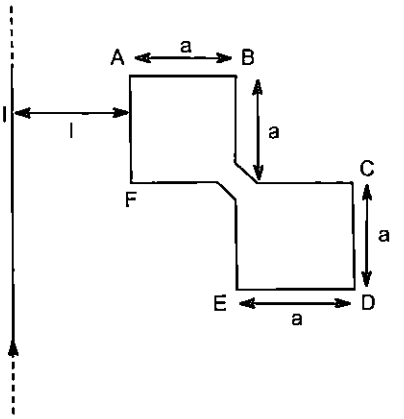
3. The power dissipated in the inductance coil is :

(a) zero (b) 320W

(c) 144W (d) 160W

Passage 10

In figure, there is a conducting loop *ABCDEF* of resistance λ per unit length placed near a long straight current-carrying wire. The dimensions are shown in the figure. The long wire lies in the plane of the loop. The current in the long wire varies as $I = I_0(t)$:



1. The mutual inductance of the pair is :

(a) $\frac{\mu_0 a}{2\pi}\ln\left(\frac{2a+l}{l}\right)$ (b) $\frac{\mu_0 a}{2\pi}\ln\left(\frac{2a-l}{l}\right)$

(c) $\frac{2\mu_0 a}{\pi}\ln\left(\frac{a+l}{l}\right)$ (d) $\frac{\mu_0 a}{\pi}\ln\left(\frac{a+l}{l}\right)$

2. The e.m.f. induced in the closed loop is :

(a) $\frac{\mu_0 I_0 a}{2\pi}\ln\left(\frac{2a+l}{l}\right)$ (b) $\frac{\mu_0 I_0 a}{2\pi}\ln\left(\frac{2a-l}{l}\right)$

(c) $\frac{2\mu_0 I_0 a}{\pi}\ln\left(\frac{a+l}{l}\right)$ (d) $\frac{\mu_0 I_0 a}{\pi}\ln\left(\frac{a+l}{l}\right)$

3. The heat produced in the loop in time t is :

(a) $\frac{\left[\frac{\mu_0}{2\pi}\ln\left(\frac{a+l}{l}\right)l_0\right]^2 at}{4\lambda}$

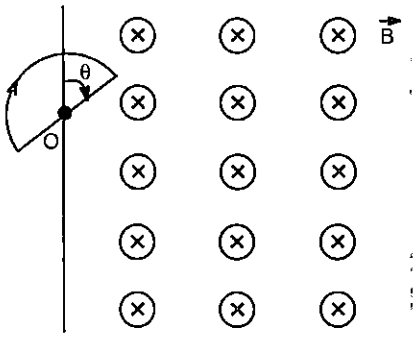
(b) $\frac{\left[\frac{\mu_0}{2\pi}\ln\left(\frac{2a+l}{l}\right)l_0\right]^2 at}{8\lambda}$

(c) $\frac{\left[\frac{3\mu_0}{\pi}\ln\left(\frac{a+l}{l}\right)l_0\right]^2 at}{3\lambda}$

(d) $\frac{\left[\frac{\mu_0}{2\pi}\ln\left(\frac{3a+l}{l}\right)l_0\right]^2 at}{6\lambda}$

Passage 11

A wire loop enclosing a semicircle of radius R is located on the boundary of a uniform magnetic field B. At the moment $t = 0$, the loop is set into rotation with constant angular acceleration α about an axis O conducting with the line of vector on the boundary. The clockwise e.m.f. direction is taken to be positive.

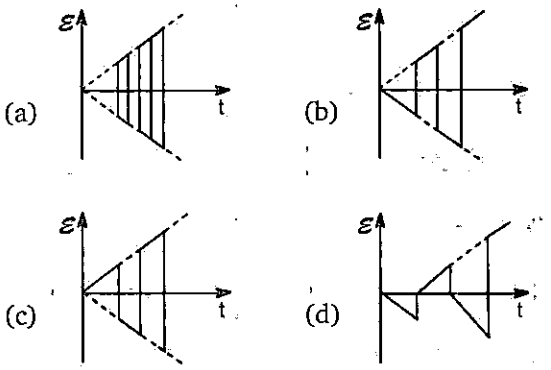


1. The variation of e.m.f. as a function of time is :

(a) $\frac{1}{2}BR^2\alpha t$ (b) $\frac{3}{2}BR^2\alpha tA$

(c) $\sqrt{3}BR^2\alpha t$ (d) $\frac{BR^2\alpha t}{\sqrt{2}}$

2. The variation of e.m.f. as a function of time is :



Passage 12

A metal disc of radius $a = 25\,cm$ rotates with a constant angular velocity $\omega = 130\ rad/s$ about its axis.

1. If there is no external magnetic field, the potential difference between the centre and the rim of the disc will be :

(a) zero (b) 3.0×10^{-9} V

(c) 1×10^{-9} V (d) 1.5×10^{-9} V

2. If an external uniform magnetic field of induciton $B = 5.0\,mT$ be directed perpendicular to the rotating disc, the potential difference between the centre and the rim disc will be :

(a) zero (b) 10 mV

(c) 20 mV (d) 30 mV

3. If the disc were not a metal, but a non-conducting material and rotates with a constant angular velocity ω and a constant magnetic induction field B acts perpendicular to it, then which of the following statements will be true ?

(a) an e.m.f. will be set up between the centre and rim of the disc

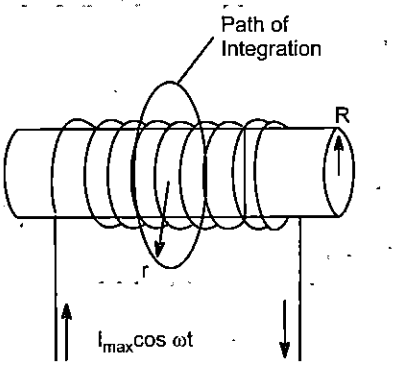
(b) the disc will tend to slow down and set-up equal and opposite charges on its faces

(c) the disc will be polarised radially with opposite charge developed near the centre and the rim respectively

(d) The disc will continue to rotate with constant angular velocity ω and no charges will be developed in it .

Passage 13

A long solenoid of radius R has n turns of wire per unit length and carries a time varying current that varies sinusoidally as $I = I_{max}\cos\omega t$, where I_{max} is the maximum current and ω is the angular frequency of the alternating current source (shown in figure)



1. The magnitude of the induced electric field inside the solenoid, a distance $r < R$ from its long central axis.

(a) $\frac{3\mu_0 nI_{max}\omega}{2}r\sin\omega t$ (b) $\frac{\mu_0 nI_{max}\omega}{2}r\cos\omega t$

(c) $\mu_0 nI_{max}\omega r\sin\omega t$ (d) $\frac{\mu_0 nI_{max}\omega}{2}r\sin\omega t$

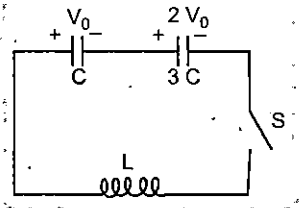
2. The magnitude of electric field outside the solenoid at a distance $r > R$ from its long central axis is :

(a) $\frac{\mu_0 nI_{max}\omega R^2}{2r}\sin\omega t$ (b) $\frac{2\mu_0 nI_{max}\omega R^2}{r}\sin\omega t$

(c) $\frac{\mu_0 nI_{max}\omega R^2}{3r}\sin\omega t$ (d) $\frac{3\mu_0 nI_{max}\omega R^2}{2r}\sin\omega t$

Passage 14

Two capacitors of capacitance C and $3C$ are charged to potential difference V_0 and $2V_0$, respectively, and connected to an inductor of inductance L as shown in figure. Initially the current in the inductor is zero. Now, the switch S is closed.



1. The maximum current in the inductor is :

(a) $\frac{3V_0}{2}\sqrt{\frac{3C}{L}}$ (b) $V_0\sqrt{\frac{3C}{L}}$

(c) $2V_0\sqrt{\frac{3C}{L}}$ (d) $V_0\sqrt{\frac{C}{L}}$

2. Potential difference across capacitor of capacitance C when the current in the circuit is maximum is

(a) $\frac{V_0}{4}$ (b) $\frac{3V_0}{4}$

(c) $\frac{5V_0}{4}$ (d) none of these

3. Potential difference across capacitor of capacitance $3C$ when the current in the circuit is maximum is :

(a) $\frac{V_0}{4}$ (b) $\frac{V_0}{4}$

(c) $\frac{5V_0}{4}$ (d) none of these

Passage 15

The magnetic field within a long, straight solenoid with a circular cross section and radius R is increasing at a rate of dB/dt.

1. The rate of change of flux through a circle with radius r_1 inside the solenoid, normal to the axis of the solenoid, and with center on the solenoid axis is :

(a) $\sqrt{2}\pi r_1^2\frac{dB}{dt}$ (b) $\frac{1}{2}\pi r_1^2\frac{dB}{dt}$

(c) $\pi r_1^2\frac{dB}{dt}$ (d) $\frac{3}{2}\pi r_1\frac{dB}{dt}$

2. The magnitude of the induced electric field inside the solenoid at a distance r_1 from its axis is :

(a) $\frac{r_1}{2}\frac{dB}{dt}$ (b) $r_1\frac{dB}{dt}$

(c) $\frac{3r_1}{2}\frac{dB}{dt}$ (d) $\frac{r_1}{\sqrt{2}}\frac{dB}{dt}$

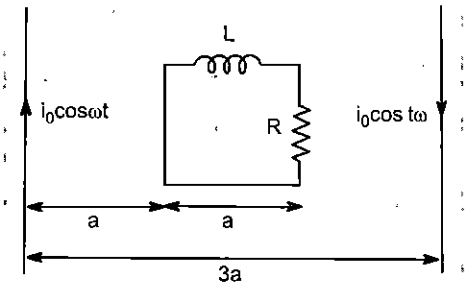
3. The magnitude of the induced electric field outside the solenoid at a distance r_2 from the axis is :

(a) $\frac{R^2}{r_2}\frac{dB}{dt}$ (b) $\frac{R^2}{2r_2}\frac{dB}{dt}$

(c) $\frac{3R^2}{2r_2}\frac{dB}{dt}$ (d) $\frac{R^2}{\sqrt{2}r_2}\frac{dB}{dt}$

Passage 16

In figure, a square loop consisting of an inductor of inductance L and resistor of resistance R is placed between two long parallel wires. The two long straight wires have time varying current of magnitude $I = I_0\cos\omega t$ but the directions of current in them are opposite.



1. Total magnetic flux in this loop is :

(a) $\frac{\mu_0 Ia}{\pi}\ln 2$

(b) $\frac{2\mu_0 Ia}{\pi}\ln 2$

(c) $\frac{4\mu_0 Ia}{\pi}\ln 2$

(d) $\frac{\mu_0 Ia}{2\pi}\ln 2$

2. Magnitude of e.m.f. in this circuit only due to flux change associated with two long straight current carrying wires will be :

(a) $\frac{\mu_0 a\ln 2I_0\omega}{\pi}\sin\omega t$

(b) $\frac{2\mu_0 a\ln 2I_0\omega}{\pi}\sin\omega t$

(c) $\frac{\mu_0 a\ln 2I_0\omega}{2\pi}\cos\omega t$

(d) $\frac{\mu_0\ln 2I_0\omega}{\pi}\cos\omega t$

3. The instantaneous current in the circuit will be :

(a) $\frac{2\mu_0 a\ln 2I_0\omega}{\pi\sqrt{R^2+\omega^2L^2}}\sin$

(b) $\frac{2\mu_0 a\ln 2I_0\omega}{\pi\sqrt{R^2+\omega^2L^2}}\sin(\omega t-\phi)$

(c) $\frac{\mu_0 a\ln 2I_0\omega}{\pi\sqrt{R^2+\omega^2L^2}}\sin\omega t$

(d) $\frac{\mu_0 a\ln 2I_0\omega}{\pi\sqrt{R^2+\omega^2L^2}}\sin(\omega t-\phi)$

$\left(\text{where } \tan\phi=\frac{\omega L}{R}\right)$

Passage 17

A thin non-conducting ring of mass m, radius a, carrying a charge q can rotate freely about its own axis which is vertical. At the initial moment the ring was at rest and no magnetic field was present. At instant $t = 0$, a uniform magnetic field is switched on which is vertically downwards and increases with time according to the law $B = B_0 t$. Neglecting magnetism induced due to rotational motion of the ring. Now answer the following questions.

1. The angular acceleration of the ring and its direction of rotation as seen from above :

(a) $\frac{Eq}{2ma}$, anticlockwise (b) $\frac{Eq}{ma}$, anticlockwise

(c) $\frac{2Eq}{ma}$ clockwise (d) $\frac{Eq}{ma}$ clockwise

2. The power developed by the forces acting on the ring, as a function of time :

(a) $\frac{E^2q^2}{2m}t$ (b) $\frac{E^2q^2}{m}t$

(c) $\frac{2E^2q^2}{m}t$ (d) $\frac{\sqrt{2}E^2q^2}{m}t$

MATCHING TYPE PROBLEMS

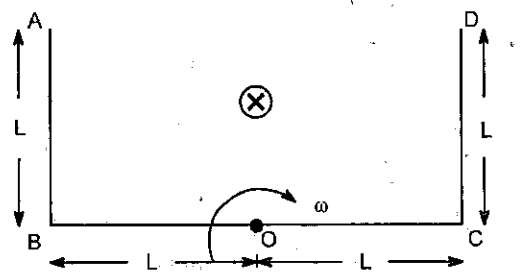
1. Match the dimensional formulae with the corresponding units.

	Column-I		Column-II
(a)	$ML^2T^{-2}A^{-1}$	(p)	farad
(b)	$M^{-1}L^{-2}T^4A^2$	(q)	weber
(c)	$ML^2T^{-2}A^{-2}$	(r)	ohm
(d)	$ML^2T^{-3}A^{-2}$	(s)	henry

2. Column I gives situations involving a charged particle which may be realized under the condition given in column II. Match the situations in column I with the conditions in column II

	Column-I		Column-II
(a)	Increase in speed of a charged particle.	(p)	Electric field uniform in space and constant in time.
(b)	Exert a force on an electron initially at rest.	(q)	Magnetic field uniform in space and constant in time.
(c)	Move a charged particle in a circle with uniform speed	(r)	Magnetic field uniform in space but varying with time.
(d)	Accelerate a moving charged particle	(s)	Magnetic field non-uniform in space but constant with time

3. A frame $ABCD$ is rotating with the angular velocity ω about an axis which passing through the point O perpendicular to the plane of paper as shown in the figure. A uniform magnetic field $\vec{B}$ is applied into the plane of the paper in the region as shown. Match the following.

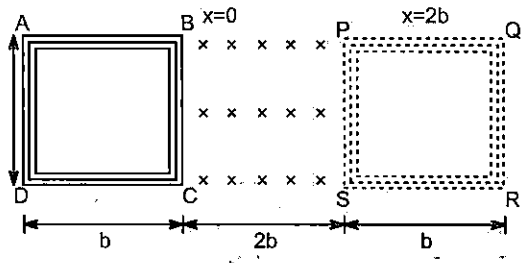


	Column-I		Column-II
(a)	Potential difference between A and O.	(p)	zero
(b)	Potential difference between O and D	(q)	$\frac{B\omega L^2}{2}$
(c)	Potential difference between C and D	(r)	$B\omega L^2$
(d)	Potential difference between A and D	(s)	constant

4. Consider an LC oscillating circuit and oscillations of a spring block system. Let Q and V be the initial charge and potential on the capacitor, I be the current in the circuit at time t, $\frac{dI}{dt}$ be the rate of change of current, U_C and U_L be the maximum energy across the capacitor and the inductor. Further if mass of block is m, spring constant of spring is k, displacement of block or extension in the spring is x, acceleration of block at any instant is a, F be the restoring force, K be the kinetic energy of block and U be the potential energy of spring, then match the pairs in Column-I to those in Column-II.

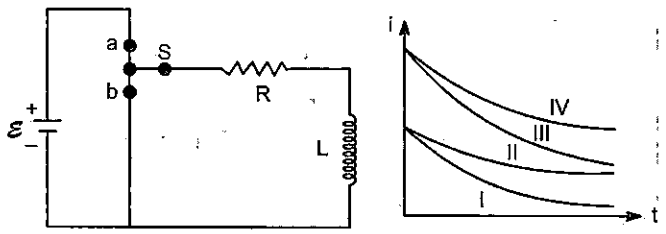
	Column-I		Column-II
(a)	L, C	(p)	x, F
(b)	$I, \frac{dI}{dt}$	(q)	$m, \frac{1}{k}$
(c)	Q, V	(r)	K, U
(d)	U_C, U_L	(s)	V, a
		(t)	U, K

5. A rectangular loop of wire with dimensions l and b has N turns and a total resistance R. The loop moves with constant velocity from AB to PQ in a region of uniform magnetic field as shown. Let x be the distance of AB from PQ.



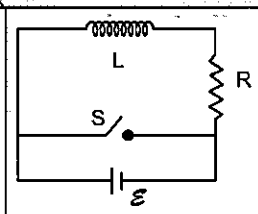
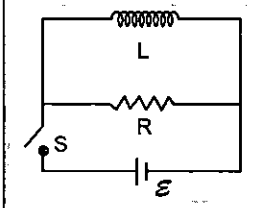
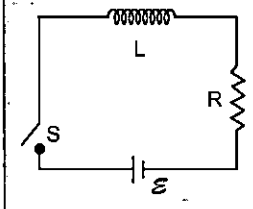
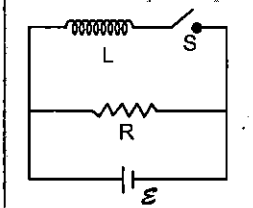
	Column-I Flux linked with the loop		Column -II E.m.f. induced in the
(a)	$0 < x < b$, $BNlx$	(p)	$BvlN$
(b)	$b < x < 2b$, $BNlb$	(q)	0
(c)	$2b < x < 3b$, $BNl(3b-x)$	(r)	$-BvlN$
(d)	$x > 3b$	(s)	BvN

6. The switch S in the circuit is connected with point a for a very long time, then it is shifted to position b. The resulting current through the inductor is shown by curves in the graph for four sets of values for the resistance R and inductance L (given in column I). Which set corresponds with which curve ?

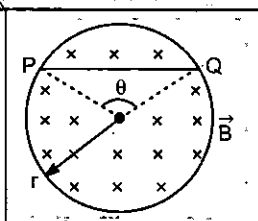
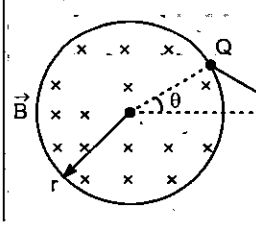


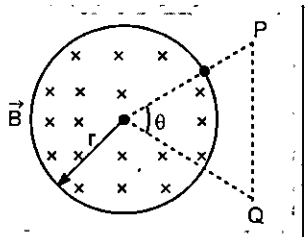
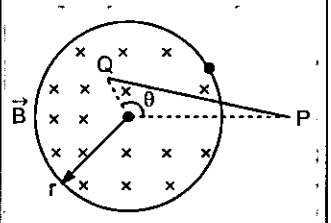
	Column-I		Column-II
(a)	R_0 and L_0	(p)	I
(b)	$2R_0$ and L_0	(q)	II
(c)	R_0 and $2L_0$	(r)	III
(d)	$2R_0$ and $2L_0$	(s)	IV

7. In Column I some circuits are given. In all the circuits except in (i), switch S remains closed for long time and then it is opened at $t = 0$ while for (i), the situation is reversed. column II tells something about the circuit quantities. Match the entries of column I with the entries of column II.

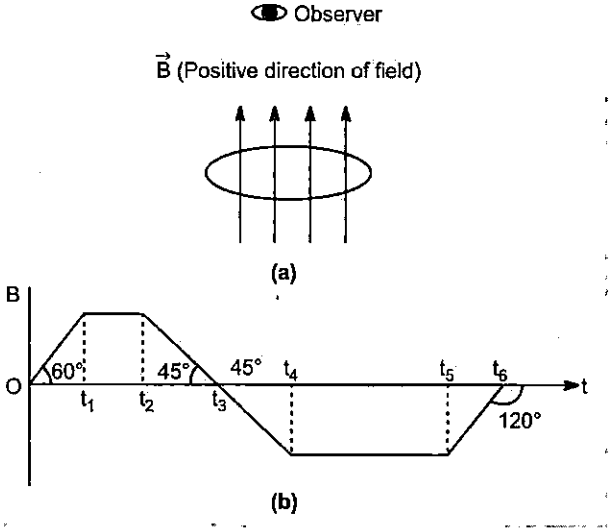
	Column-I		Column-II
(a)		(p)	Induced e.m.f. can be greater than $\mathcal{E}$.
(b)		(q)	Induced e.m.f. would be less than $\mathcal{E}$.
(c)		(r)	Finally, energy stored in inductor is zero.
(d)		(s)	Finally, energy stored in inductor is non-zero

8. Column I shows the cylindrical region of radius r where a downward magnetic field $\vec{B}$ exists, where $\vec{B}$ is increasing at the rate of $\frac{dB}{dt}$. A rod PQ is placed in different citation as shown. Match the column I with the correct statement in column II regarding the induced e.m.f. in rod.

	Column-I		Column-II
(a)		(p)	Induced e.m.f. in rod PQ is $\frac{1}{2}r^2\theta\frac{dB}{dt}$
(b)		(q)	Induced e.m.f. in rod PQ is less than $\frac{1}{2}r^2\theta\frac{dB}{dt}$

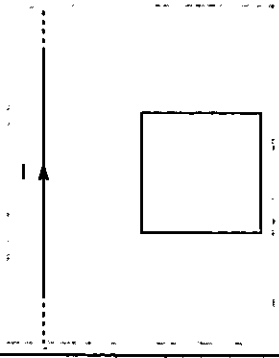
(c)		(r)	End P is positive with respect to point Q
(d)		(s)	End Q is positive with respect to point P.

9. A conducting loop is held in a magnetic field such that the field is oriented perpendicular to the area of the loop as shown in figure (a). At any instant magnetic flux density over the entire area has the same value but it varies with time as shown in figure (b).



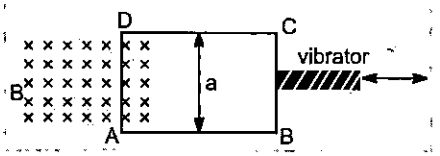
	Column-I		Column-II
(a)	Induced current in the coil is in the clockwise sense	(p)	For $t_2 < t < t_3$
(b)	Induced current in the coil is in the anticlockwise sense	(q)	For $t_3 < t < t_4$
(c)	Induced current is zero	(r)	For $t_5 < t < t_6$
(d)	Induced current is maximum	(s)	For $t_4 < t < t_5$

10. A square loop of perimeter $4l$, is placed near a long straight wire carrying a current I as shown. Match phenomenon in Column-I with the consequences in Column-II.



	Column-I		Column-II
(a)	I is increased	(p)	Induced current in loop is clockwise
(b)	I is decreased	(q)	Induced current in loop is anticlockwise
(c)	If loop is moved away from the wire	(r)	wire will attract the loop
(d)	If loop is moved towards the wire	(s)	wire will repel the loop

11. A uniform magnetic induction field B acts in the region perpendicular to the plane of paper and into it. $ABCD$ is a square coil of side a, of resistance R and of N turns, attached to a vibrator such that, as the vibrator vibrates, the coil is drawn in and out of the magnetic field with a periodic time T. Consider each of the cases given in Column-I and the matching solutions jumbled up in Column-II. Let x be the coordinate of end of coil at any instant and $\frac{dx}{dt} = v$



	Column-I		Column-II
(a)	The e.m.f. induced in the coil at instant t is	(p)	$(N^2B^2a^2v^2)/R$
(b)	The induced current in the coil at instant t is	(q)	$(N^2B^2a^2v)/R$
(c)	The external force acting on a current element is	(r)	$NBa(dx/dt)$
(d)	The power output of the force on the element is	(s)	$\left(\frac{NBav}{R}\right)$

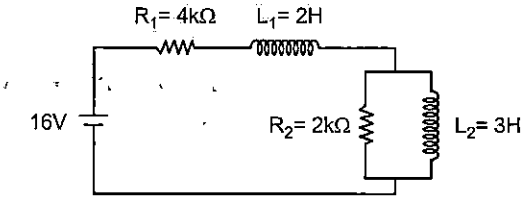
12. Column-I gives certain situations in which a straight metallic wire of resistance R is used and Column-II gives some resulting effects. Match the statements in Column-I with the statements in Column-II.

	Column-I		Column-II
(a)	A charged capacitor is connected to the ends of the wire.	(p)	A constant current flows through the wire
(b)	The wire is moved perpendicular to its length with a constant velocity in a uniform magnetic field perpendicular to the plane of motion.	(q)	Thermal energy is generated in the wire
(c)	The wire is placed in a constant electric field that has a direction along the length of the wire.	(r)	A constant potential difference develops between the ends of the wire.
(d)	A battery of constant emf is connected to the ends of the wire.	(s)	Charges of constant magnitude appear at the ends of the wire.

13. An LCR series circuit has a current which lags behind the applied voltage by ϕ. The voltage across the inductance has a maximum value equal to twice the maximum of the voltage across the capacitor. $E_L = 30\sin(100t)$. If $R = 20\Omega$, then match the items given in Column-I with that in Column-II.

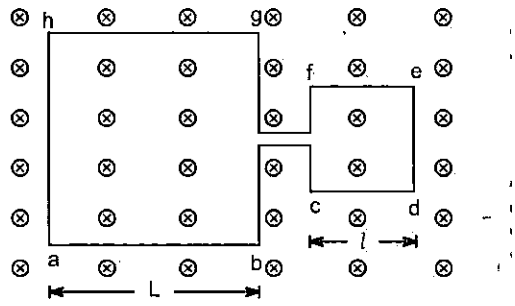
	Column-I		Column-II
(a)	Reactance of capacitor if $\phi = 45°$.	(p)	20Ω
(b)	Reactance of inductor if $\phi = 45°$.	(q)	$20\sqrt{3}\Omega$
(c)	Impedance of circuit if $\phi = 45°$.	(r)	40Ω
(d)	Reactance of circuit if $\phi = 60°$.	(s)	$20\sqrt{2}\Omega$

14. In the circuit shown, $L_1 = 2\text{H}$, $L_2 = 3\text{H}$, $R_1 = 4\text{k}\Omega$ and $R_2 = 2\text{k}\Omega$. In steady state. The items of Column-I to the values in Column-II.

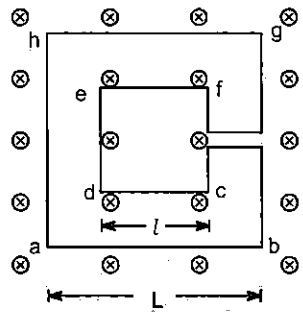


	Column-I		Column-II
(a)	The current through inductor L_1 in mA.	(p)	16
(b)	The voltage across inductor L_2 in volt.	(q)	24
(c)	The energy stored in inductor L_1 in μ J.	(r)	4
(d)	The energy stored in inductor L_2 in μ J.	(s)	0

15. The figure shows two different arrangements in which two square wire frames are placed in a uniform constantly decreasing magnetic field B.



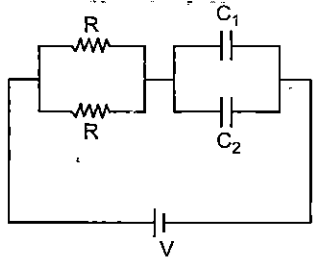
FRAME I

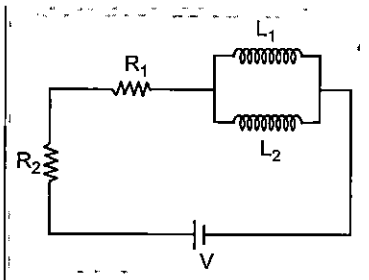
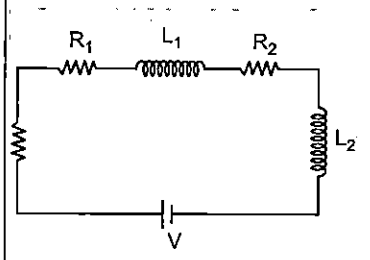
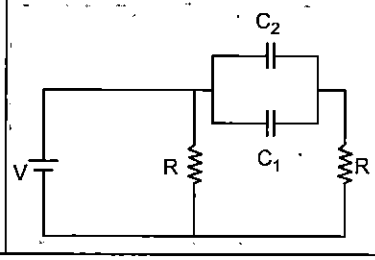


FRAME II

	Column-I		Column-II
(a)	Magnetic flux in wire frame I	(p)	$(L^2 - l^2)B$
(b)	Magnetic flux in wire frame II	(q)	$(L^2 + l^2)B$
(c)	Direction of induced current in wire frame I	(r)	from b to a and from f to e
(d)	Direction of induced current in wire frame II	(s)	from b to a and from c to d
		(t)	from b to a and from d to c

16. Match the circuit with their respective time constants.

	Column-I		Column-II
(a)		(p)	$\dfrac{(L_1 + L_2)}{(R_1 + R_2)}$

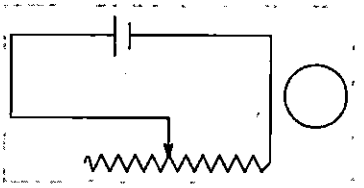
(b)		(q)	$\frac{R}{2}(C_1+C_2)$
(c)		(r)	$R(C_1+C_2)$
(d)		(s)	$\frac{\left(\frac{L_1L_2}{L_1+L_2}\right)}{R_1+R_2}$

ASSERTION AND REASON

(a) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
(b) Statement-1 is true, statement-2 is NOT the correct explanation for statement-1.
(c) Statement-1 is true, statement-2 is false.
(d) Statement-1 is false, statement-2 is true.

1. **Statement-1:** When a circuit having large inductance is switched off sparking occurs at the switch.
Statement-2: Emf induced in an inductor is given by $|\mathcal{E}| = L\left|\frac{di}{dt}\right|$.

2. **Statement-1:** When resistance of rheostat is increased, clockwise current is induced in the ring.
Statement-2: Magnetic flux through the ring is out of the page and decreasing.



3. **Statement-1:** The mutual inductance of two concentric conducting rings of different radii is maximum if the rings are also coplanar.
Statement-2: For two coaxial conducting rings of different radii, the magnitude of magnetic flux in one ring due to current in other ring is maximum when both rings are coplanar.

4. **Statement-1:** When a conducting closed loop is placed with its plane perpendicular to an external magnetic field which changes with time, an induced current flows in the loop.
Statement-2: The changing magnetic field creates an electrical field which causes the induced e.m.f. in the loop.

5. **Statement-1:** When a square loop made of thin conducting wire is moved across an external uniform magnetic field parallel to one side with a uniform speed, no current is produced in it even through the loop cuts the magnetic flux due to the field.
Statement-2: The total magnetic flux through the loop remains constant during the motion.

6. **Statement-1:** The work done in taking a unit charge around a closed loop in an induced electrical field is not zero.
Statement-2: Lenz's law is not consistent with the law of conservation of energy.

ANSWERS

Level-3: Comprehension Based Problems

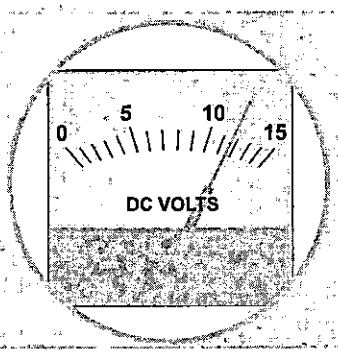
Passage-1:	**1.** (b)	**2.** (d)	**3.** (b)		**Passage-2:**	**1.** (c)	**2.** (c)	**3.** (b)	**4.** (b)
Passage-3:	**1.** (a)	**2.** (b)	**3.** (b)		**Passage-4:**	**1.** (a)	**2.** (b)	**3.** (a)	
Passage-5:	**1.** (b)	**2.** (a)	**3.** (c)		**Passage-6:**	**1.** (a)	**2.** (c)	**3.** (b)	
	4. (a)	**5.** (c)	**6.** (b)						
Passage-7:	**1.** (b,d)	**2.** (c)	**3.** (d)		**Passage-8:**	**1.** (b)	**2.** (c)		
Passage-9:	**1.** (b)	**2.** (a)	**3.** (c)		**Passage-10:**	**1.** (a)	**2.** (a)	**3.** (b)	
Passage-11:	**1.** (a)	**2.** (b)			**Passage-12:**	**1.** (b)	**2.** (c)	**3.** (d)	
Passage-13:	**1.** (d)	**2.** (b)			**Passage-14:**	**1.** (a)	**2.** (c)	**3.** (c)	
Passage-15:	**1.** (c)	**2.** (a)	**3.** (b)		**Passage-16:**	**1.** (a)	**2.** (b)	**3.** (d)	
Passage-17:	**1.** (b)	**2.** (b)							

Matching Type Problems

1. (a)-q; (b)-p; (c)-s; (d)-r
2. (a)-p, r; (b)-p, r; (c)-q, s; (d)-p, q, r, s
3. (a)-p, s; (b)-q, s; (c)-r; (d)-p, s
4. (a)-q; (b)-s; (c)-p; (d)-t
5. (a)-r; (b)-q; (c)-p; (d)-q
6. (a)-r; (b)-p; (c)-s; (d)-q
7. (a)-r; (b)-r; (c)-p, r; (d)-p, s
8. (a)-q, r; (b)-p, s; (c)-r, p; (d)-q, s
9. (a)-r; (b)-p, q; (c)-s; (d)-r
10. (a)-q, s; (b)-p, r; (c)-p, r; (d)-q, s
11. (a)-r; (b)-s; (c)-q; (d)-p
12. (a)-q; (b)-r, s; (c)-s; (d)-p, q, r
13. (a)-p; (b)-r; (c)-s; (d)-q
14. (a)-r; (b)-s; (c)-p; (d)-q
15. (a)-q; (b)-p; (c)-t; (d)-r
16. (a)-q; (b)-s; (c)-p; (d)-r

Assertion and Reason

1.	(a)	**2.**	(c)	**3.**	(a)	**4.**	(a)	**5.**	(a)	**6.**	(c)



SOLUTION

Level-3: Comprehension Based Problems

Passage-1

1. (b) 2. (d) 3. (b)

$$E \times 2\pi R = \frac{d}{dt}(B \times \pi R^2) \Rightarrow E = \frac{R}{2}\frac{dB}{dt}$$

Now, $$qE = m\frac{dv}{dt} \Rightarrow \frac{q}{2}R.\frac{dB}{dt} = m\frac{dv}{dt}$$

$$\Rightarrow \quad \frac{1}{2}dB = \left(\frac{m}{qR}\right)dv$$

$$\Rightarrow \quad \frac{dv}{v} = \frac{1}{2}\frac{dB}{B} \quad ...(i) \qquad [\text{Using } R = \frac{mv}{qB}]$$

$$= \frac{1}{2} \times 2\% = 1\%$$

Also, $$R = \frac{mv}{qB} \Rightarrow \frac{v}{v} = \frac{dq}{q} + \frac{dB}{B}$$

$$\Rightarrow \quad \frac{dq}{q} = -\frac{1}{2}\frac{dB}{B} = -1\%$$

Passage-3

1. (a) $$\phi = abBe = -\frac{d\phi}{dt},$$

$$= ab\left(-\frac{dB}{dt}\right) = ab\alpha \text{ anticlockwise as seen from above.}$$

2. (b) $\mathcal{E} = \beta abv$ in clockwise sense.

3. (b) Applying principle of superposition.

Passage-4

1. (a) $$BlV_{max} = \frac{Q_{max}}{C}$$

$$m\frac{dV}{dt} = Ma = Bil = -Bl\frac{dQ}{dt}$$

$$mv_{max} = BLl(Q_0 - Q_{max})$$

$$mv_{max} = \frac{BlCV_0}{m + B^2l^2C}$$

2. (b) $$C_{max} = \frac{B^2l^2C^2V_0}{m + B^2l^2C}$$

3. (a) $$= \frac{\frac{1}{2}mv_{max}^2}{\frac{1}{2}CV_0^2}$$

Passage-5

1. (b) Instantaneous current in the capacitor,

$$q = CV_C = (2)(3e^{-2t}) = 6e^{-2t}\text{A}$$

Current, $$i_C = \frac{dq}{dt}$$

$$= -12e^{-2t}\text{A}$$

Current flows from B to O.

From KVL, we have

$$i_L = i_1 + i_2 + i_C$$

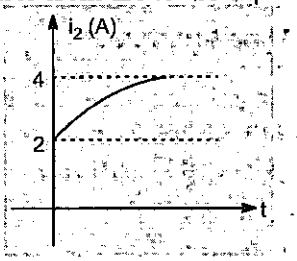
$$= 10e^{-2t} + 4 - 12e^{-2t}$$

$$= (4 - 2e^{-2t})\text{A}$$

$$= [2 + 2(1 - e^{-2t})]\text{A}$$

2. (a) i_L *vs.* time graph is as shown in figure.

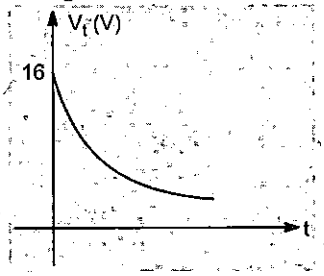
i_L increases from 2A to 4A exponentially.



3. (c) $$V_L = L\frac{di_L}{dt}$$

$$(4)\frac{d}{dt}(4 - 2e^{-2t}) = 16e^{-2t}\text{V}$$

V_L decreases exponentially from 16 A to 0 as shown in figure



4. (a) From *KVL*, we have

$$V_A - i_1R_1 + i_2R_2 = V_C$$

$$V_A - V_C = i_1R_1 - i_2R_2$$

Substituting the values, we have

$$V_{AC} = (10e^{-2t})(2) - (4)(3)$$

$$V_{AC} = (20e^{-2t} - 12)\text{V}$$

At $t = 0, V_{AC} = 8\text{V}$

At $t = \infty, V_{AC} = -12\text{V}$

Therefore, V_{AC} decreases exponentially from 8 V to −12V.

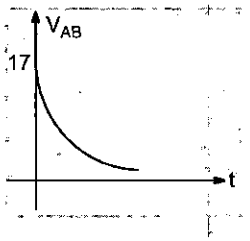
5. (c) $V_A - i_1R_1 + V_C = V_B$

$V_{AB} = V_A - V_B = i_1R_1 - V_C$

Substituting the value we have,

$V_{AB} = (10e^{-2t})(2) - 3e^{-2t}$

$V_{AB} = 17e^{-2t}\text{V}$



Thus, V_{AB} decreases exponentially from 17 V to 0.

6. (b) $V_C - i_2R_2 - i_L = V_D$

$V_{CD} = V_C - V_D = i_2R_2 + i_L$

Substituting the values we have,

$V_{CD} = (4)(3) + 16e^{-2t}$

$V_{CD} = (12 + 16e^{-2t})\text{V}$

At $t = 0, \quad V_{CD} = 28\text{V}$

and at $t = \infty, \quad V_{CD} = 12\text{V}$

i.e., V_{CD} decreases exponentially from 28V to 12V

Passage-6

1. (a) The current of the battery at any instant, $I = \mathcal{E}/R$.

The magnetic force due to this current

$$F_B = IBL = \frac{\mathcal{E}Bl}{R}$$

This magnetic force will accelerate the rod from its position of rest. The motional e.m.f. developed in the rod is Blv;

The induced current,

$$I_{\text{induced}} = \frac{Blv}{R}$$

The magnetic force due to the induced current.

$$F_{\text{induced}} = I_{\text{induced}} = \frac{B^2l^2v}{R}$$

From Fleming's left hand rule, force F_B is to the right and F_{induced} is to the left. Net force on the rod $= F_B - F_{\text{induced}}$ From Newton's second law.

$$F_B = F_{\text{induced}} = m\frac{dv}{dt}$$

$$\frac{\mathcal{E}Bl}{R} - \frac{B^2l^2v}{R} = m\frac{dv}{dt}$$

On separating variables and integrating speed from v_0 to v and time from 0 to t, we have

$$\frac{dv}{\mathcal{E} - Bvl} = \frac{Bl}{mR}dt$$

$$\int_0^v \frac{dv}{(\mathcal{E} - Bvl)} = \frac{Bl}{mR}\int_0^t dt$$

$$\ln\left(\frac{\mathcal{E} - Bvl}{\mathcal{E}}\right) = -\frac{B^2l^2}{mR}t$$

$$\frac{E - Bvl}{\mathcal{E}} = e - \frac{B^2l^2}{mR}t$$

$$v = \frac{\mathcal{E}}{Bl}(1 - e^{-t/\tau}) \quad \text{where } t = \frac{mR}{(Bl)^2}$$

2. (c) The rod will attain a terminal velocity at $t \to \infty$, *i.e.*,

when $e^{-t/\tau} = 0$, the velocity is independent of time.

$$v_T = \frac{\mathcal{E}}{Bl}$$

3. (b) The induced current $I_{\text{induced}} = Blv/R$. When the rod has attained terminal speed,

$$I_{\text{induced}} = \frac{Bl}{R} \times \left(\frac{E}{Bl}\right) = \mathcal{E}/R$$

The current of battery and the induced current are of same magnitude, hence net current through the circuit is zero.

Passage-8

1. (b) Magnetic field on the axis of a circular coil is given by

$$B = \frac{\mu_0 iR^2}{2(R^2 + x^2)^{3/2}}$$

Since $x >> R$, therefore, magnetic field at the centre of the smaller loop is

$$B \approx \frac{\mu_0 iR^2}{2x^3}$$

Flux linked with coil is

$$\phi = B(\pi r^2) = \frac{\mu_0 \pi i R^2 r^2}{2x^3}$$

2. (c) From Faraday's law we have

$$\mathcal{E} = -\frac{df}{dt} = \frac{3}{2}\frac{\mu_0 \pi i R^2 r^2}{x^4}v$$

As the external magnetic field at the position of smaller coil is decreasing, the induced magnetic field is parallel to external field.

Hence the direction of current is anticlockwise when seen from the top.

Passage-9

1. (b) Here, $Z = \sqrt{(R_1 + R_2)^2 + (X_L - X_C)^2}$

$= \sqrt{(44 + 36)^2 + (90 - 30)^2} = 100\Omega$

The impedence of the coil is, $Z_C = \sqrt{R_2^2 + X_L^2}$

$= \sqrt{(36)^2 + (90)^2} = 97\Omega$

Hence, potential differences across the coil is

$\text{P.D.} = 2(97) = 194\text{V}$

2. (a) Power used in the circuit $= VI\cos\phi$

Now, $\cos\phi = \frac{R}{Z} = \frac{80}{100} = 0.80$

Also, current $= I = \frac{V}{Z} = \frac{200}{100} = 2\text{A}$

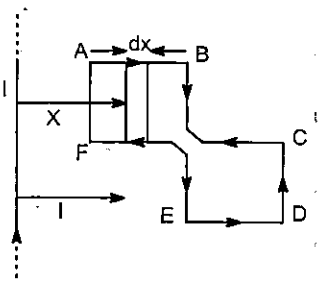
Hence, Power $= (200)(2)(0.8) = 320\text{W}$

3. (c) The power dissipated in the induction coil

$= I^2R_2 = (2)^2(36) = 144\text{W}$

Passage-10

1. (a) Consider a strip at a distance x from the wire of thickness dx. Magnetic flux associated with this strip



$$\Delta f = B(x)a\,dx = \frac{\mu_0 / a}{2\pi x}dx$$

$$\phi = \frac{\mu_0 / a}{2\pi}\left[\int_{l}^{a+l}\frac{dx}{x} + \int_{a+l}^{2a+l}\frac{dx}{x}\right]$$

$$= \frac{\mu_0 / a}{2\pi}\ln\left(\frac{2a+l}{l}\right)$$

2. (a) $M = \dfrac{Q}{I_{(\text{Primary})}} \Rightarrow M = \dfrac{\mu_0 a}{2\pi}\ln\left(\dfrac{2a+l}{l}\right)$

$$\mathcal{E} = -M\frac{dI}{dt}$$

$$\mathcal{E} = -MI_0 = -\frac{\mu_0 I_0 a}{2\pi}\ln\left(\frac{2a+l}{l}\right)$$

3. (b) Heat produced $= \dfrac{\mathcal{E}^2}{R}t = \dfrac{\left[\dfrac{m_0}{2\pi}\ln\left(\dfrac{2a+l}{l}\right)l_0\right]^2 at}{\Omega}$

Passage-11

1. (a) EMF is induced in the loop because area inside the magnetic field is continually changing.

From $q = 0$ to π, 2π to 3π, 4π to 5π, the loop begins to enter the magnetic field. Thus the magnetic field passing through the loop is increasing. Hence, current in the loop is anticlockwise, and for $\theta = \pi$ to 2π; 3π to 4π, 5π to 6π, etc. magnetic field passing through the loop is decreasing. Hence current in the loop is clockwise.

Angular acceleration of the loop is constant, therefore angle turned in time t is $\theta = 1/2\,\alpha t^2$ and time taken,

$$t = \sqrt{2\pi/\theta}$$

t_1 = time taken to rotate angle

$$\pi = \sqrt{\frac{2\pi}{\alpha}}$$

From $t = 0$ to $t = t_1$, e.m.f. is negative,

t_2 = time taken to rotate angle $2\pi = \sqrt{\dfrac{4\pi}{\alpha}}$

From $t = t_1$ to $t = t_2$, e.m.f. is positive

t_n = time taken to rotate angle $n\pi = \sqrt{\dfrac{2n\pi}{\alpha}}$

Area inside the field is $A = \dfrac{1}{2}R^2\theta$

$$= \frac{1}{4}R^2\alpha t^2$$

So flux passing through the loop,

$$\phi = BA = \frac{1}{4}BR^2\alpha t^2$$

$$\mathcal{E} = \left|\frac{d\phi}{dt}\right| = \frac{1}{2}BR^2\alpha t; \quad e \propto t$$

2. (b) *i.e.*, $\mathcal{E}$-t graph is a straight line passing through the origin. $\mathcal{E} - t$ equation with sign can be written as

$$\mathcal{E} = (-1)^n\left(\frac{1}{2}BR^2\alpha t\right)$$

Hence $n = 1,2,3....$ is the number of half revolutions that the loop performs at the given moment.

The $\mathcal{E} - t$ graph is as shown in figure (b).

Passage-12

1. (b) $eE = m\omega^2 r \Rightarrow E = \dfrac{m\omega^2 r}{e}$

$$\Delta V = \int_0^r E\,dr = \frac{m\omega^2 r^2}{2e} = 3\times 10^{-9}\ \text{V}$$

2. (c) In this case the given external field $B = 5.0\times 10^{-3}$ T

Hence induced e.m.f. $\mathcal{E} = \left(\dfrac{1}{2}\omega r^2\right)B$

$$= \frac{(130)(25\times 10^{-2})^2 \times 5\times 10^{-3}}{2}\ \text{V} = 20\ \text{mV}$$

3. (d) A uncharged non-conducting ring or disc does not develop any changes on account of an external magnetic field.

Passage-13

1. (d) $\oint \vec{\mathbf{E}}.d\vec{\mathbf{s}} = \dfrac{d}{dt}(B\pi r^2) = -\pi R^2\dfrac{dB}{dt}$

$$\oint \vec{\mathbf{E}}.d\vec{\mathbf{s}} = E(2\pi r) = -\pi R^2\frac{dB}{dt} \qquad ...(i)$$

The magnetic field inside the long solenoid is given by $B = m_0 nl$. When we substitute $I = I_{\max}\cos\omega t$ into this equation and then substitute the result into equation (i), we find that

$$E(2\pi r) = -\pi R^2\mu_0 nI_{\max}\frac{d}{dt}(\cos\omega t)$$

$$= \pi R^2\mu_0 nI_{\max}\sin\omega t$$

$$E = \frac{\mu_0 nI_{\max}\omega R^2}{2r}\sin\omega t \quad (\text{for } r > R)$$

Hence, the electric field varies sinusoidally with time and its amplitude falls off as 1/2 outside the solenoid.

Proceeding as above, we find that

$$E(2\pi r) = -\pi r^2\frac{dB}{dt} = \pi r^2\mu_0 nI_{\max}\sin\omega t$$

$$E = \frac{\mu_0 nI_{\max}w}{2r}r\sin\omega t \qquad (\text{for } r < R)$$

This shows that the amplitude of the electric field induced inside the solenoid by the changing magnetic flux through the solenoid increases linearly with r and varies sinusoidally with time.

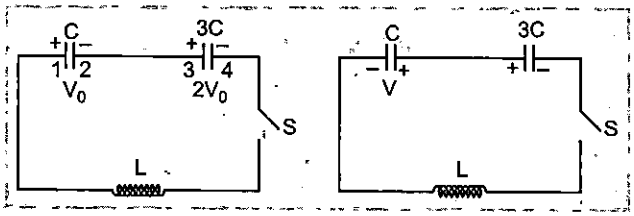
Passage-14

1. (a) Loss in energy of capacitor = Energy store in Inductor

$$\Rightarrow \frac{1}{2}VC_0^2+\frac{1}{2}3C(2V_0)^2-\frac{1}{2}\times 4CV^2=\frac{1}{2}LI$$

$$\Rightarrow I=\frac{3}{2}V_0\sqrt{\frac{3V_0}{L}}$$

2. (c) 3. (c)



When current is maximum $\frac{dI}{dt}=0$

$\therefore$ e.m.f. across $L=0$ so potential difference across the capacitor will be same.

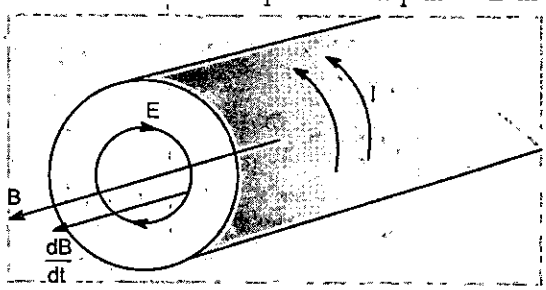
From the law of conservation of charge on plate 2 and 3.

$$3CV+CV=6CV_0-CV_0 \qquad \Rightarrow V=\frac{5}{4}V_0$$

Passage-15

1. (c) $\frac{d\phi_B}{dt}=A\frac{dB}{dt}=\pi r_1^2\frac{dB}{dt}$

2. (a) $E=\frac{1}{2\pi r_1}\frac{d\phi_B}{dt}=\frac{\pi r_1^2}{2\pi r_1}\frac{dB}{dt}=\frac{r_1}{2}\frac{dB}{dt}$

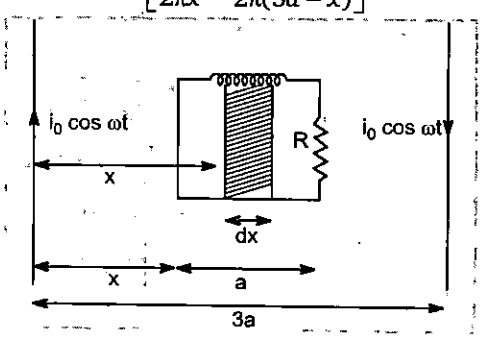


3. (b) $E=\frac{1}{2\pi r_2}\frac{d\phi_B}{dt}=\frac{\pi R^2}{2\pi r_2}\frac{dB}{dt}=\frac{R^2}{2r_2}\frac{dB}{dt}$

Passage-16

1. (a) $d\phi=BdA$

$$d\phi=\left[\frac{\mu_0 I}{2\pi x}+\frac{\mu_0 I}{2\pi(3a-x)}\right]adx$$



$$\phi=\frac{\mu_0 I}{2\pi}\left[\int_a^{2a}\frac{dx}{x}+\int_a^{2a}\frac{dx}{(3a-x)}\right]a;$$

$$\phi=\frac{\mu_0 Ia}{\pi}\ln 2$$

2. (b) Magnitude of e.m.f. in this circuit :

$$\mathcal{E}=\left|\frac{d\phi}{dt}\right|=\frac{\mu_0 a(\ln 2)}{\pi}\left|\frac{dI}{dt}\right|$$

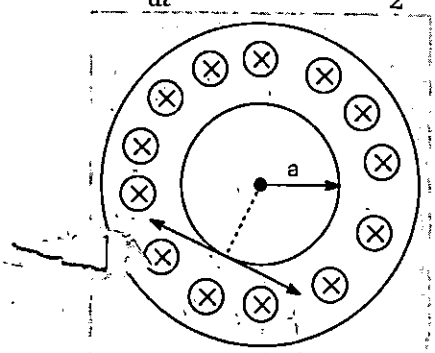
$$\mathcal{E}=\frac{\mu_0 \ln 2}{\pi}I_0\omega\sin\omega t$$

3. (d) A.C. current, $I=\frac{\mu_0 a\ln 2 I_0\omega}{p\sqrt{R^2+\omega^2L^2}}\sin(\omega t-\phi)$

Passage-17

1. (b) $E=\frac{r}{2}\frac{dB}{dt}$

$\frac{dB}{dt}=B_0 \qquad \Rightarrow E=\frac{a}{2}B_0$



$$d\tau=dF.a=(dqE).a$$

$$\tau=\int d\tau=Ea\int dq=Eaq$$

$$\tau=I\alpha$$

$$\alpha=\frac{\tau}{I}=\frac{Eaq}{(ma^2)}=\frac{Eq}{ma}$$

$$\alpha=\frac{Eq}{ma} \text{ anticlockwise}$$

2. (b) Power $P=\tau.w=(Eaq)(\alpha t)=Eaq\frac{Eq}{ma}t$

$$P=\frac{E^2q^2}{m}t$$

Assertion and Reason

6. Lenz's law is consistent with the law of conservation of energy. An induced electrical field is non-conservative.